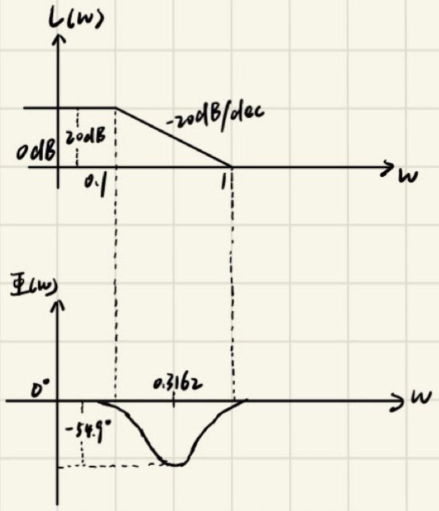
自控作业5.

1: (1): $G(j\omega)=10\,\dfrac{j\omega+1}{10j\omega+1}$



⇒低频段: $L(\omega)=20\lg 10=20dB$ slope=0

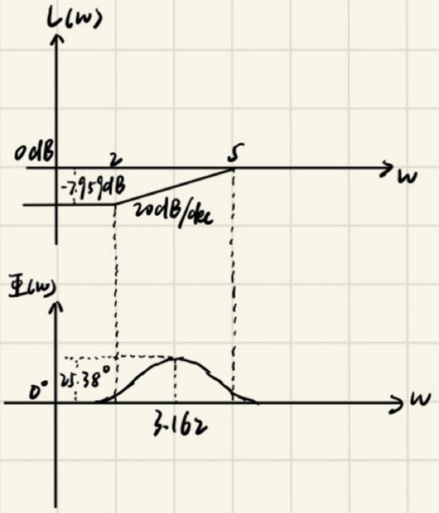
转折点: $\omega_1=0.1$ 后. slope=-20dB/dec

$\omega_2=1$ 后. slope=0dB/dec

$\alpha=0.1$. $\Phi_m=\sin^{-1}\dfrac{\alpha-1}{\alpha+1}=-54.9^\circ$

⇒滞后校正装置.

(2). $G(j\omega)=\dfrac{2}{5}\cdot\dfrac{0.5j\omega+1}{0.2j\omega+1}$



⇒低频段: $L(\omega)=20\lg\frac{2}{5}=-7.959dB$ slope=0

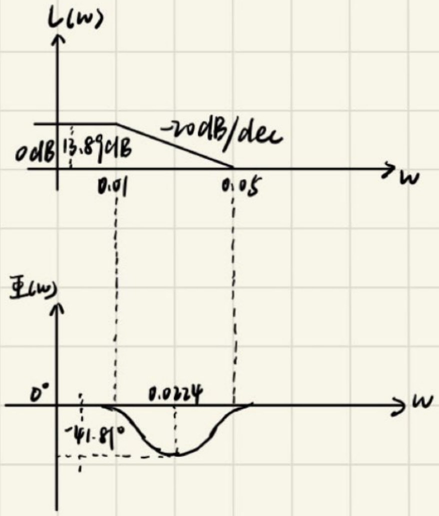
转折点: $\omega_1=2$ 后. slope=+20dB/dec

$\omega_2=5$ 后. slope=0dB/dec

$\alpha=2.5$. $\Phi_m=\sin^{-1}\dfrac{\alpha-1}{\alpha+1}=25.38^\circ$

⇒超前校正装置.

(3). $G(j\omega)=5\cdot\dfrac{20j\omega+1}{100j\omega+1}$



⇒低频段: $L(\omega)=20\lg 5=13.98dB$ slope=0

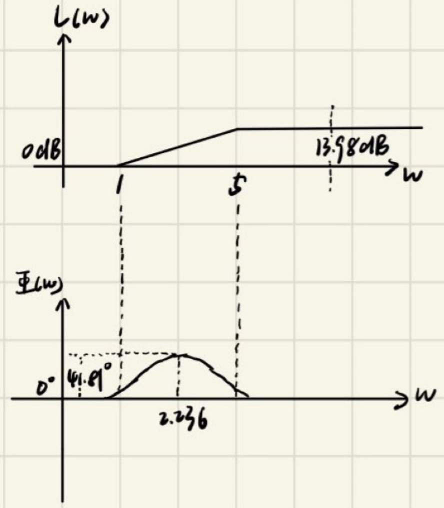
转折点: $\omega_1=0.01$ 后. slope=-20dB/dec

$\omega_2=0.05$ 后. slope=0dB/dec

$\alpha=0.2$. $\Phi_m=\sin^{-1}\dfrac{\alpha-1}{\alpha+1}=-41.81^\circ$

⇒滞后校正装置.

(4): $G(jw)=\dfrac{jw+1}{0.2jw+1}$



⇒低频段: $L(w)=20\lg 1=0dB$ slope=0

转折点: $w_1=1$ 处. slope=+20dB/dec

$w_2=5$ 处. slope=0dB/dec

$\alpha=5$ $\Phi_m=\sin^{-1}\dfrac{\alpha-1}{\alpha+1}=41.81°$

⇒超前校正装置

2: 原传递函数: $G_0(s)=\dfrac{10}{s^2+10s}$ ⇒ $G(s)=\dfrac{10}{s^2+10s+10}$, $\zeta=\dfrac{\sqrt{10}}{2}$, $w_n=\sqrt{10}$

此时为过阻尼. $\sigma\%=0$. $t_s=\dfrac{4}{\zeta w_n}=0.8s$. $D(s)=s^2+10s+10$, $s_1=-1.127$. $s_2=-8.873$.

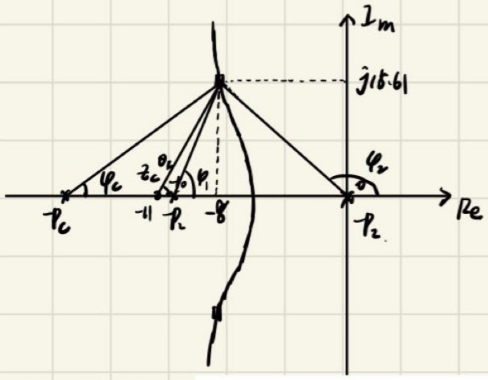
原开环传递函数$G_0(s)$极点为: $-p_1=-10$. $-p_2=0$.

根据动态性能: $\begin{cases}\sigma\%=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}\le 20\% \\ t_s=\dfrac{4}{\zeta w_n}\le 0.5s\end{cases}$ ⇒ $\begin{cases}\zeta=0.456 \\ w_n=17.54\end{cases}$ ⇒期望极点 $s_{1,2}=-\zeta w_n\pm w_n\sqrt{1-\zeta^2}=-8\pm j15.61$

由积分环节$v=1$. 稳态精度满足要求. 而动态性能不满足. 故引入超前校正网络$G_c(s)=k_c\cdot\dfrac{s+z_c}{s+p_c}$.

取: $-z_c=-11$.

有: $-\varphi_1-\varphi_2-\varphi_c+\theta_c=-180°$ ⇒ $-\arctan\dfrac{15.61}{2}-(90°+\arctan\dfrac{8}{15.61})-\arctan\dfrac{15.61}{p_c-8}+\arctan\dfrac{15.61}{3}=-180°$



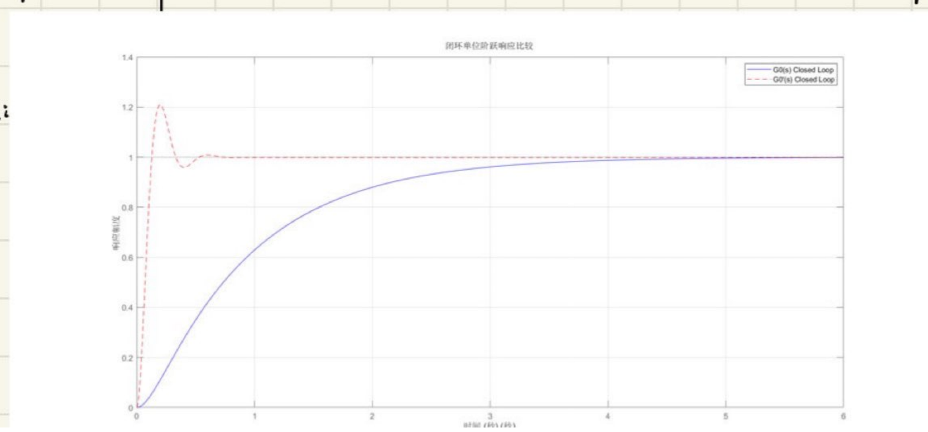
∴ $-82.7°-117.1°-\arctan\dfrac{15.61}{p_c-8}+79.12°=-180°$

得: $p_c=-17.27$

又: $\left|k_c\cdot\dfrac{s+11}{s+17.27}\cdot\dfrac{10}{s(s+10)}\right|_{s=-8\pm j15.61}=1$

得 $k_c=31.53$ ⇒ $G_c(s)=31.53\cdot\dfrac{s+11}{s+17.27}$.

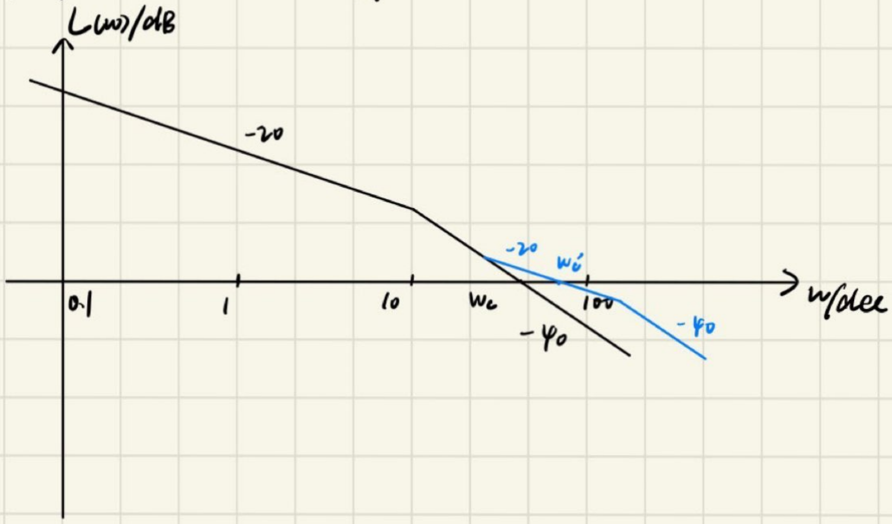
Matlab模拟:



7: 低频段: $L(\omega)=20\lg K=20\cdot\lg 200=46.02$、积分环节 $r=1$、$slope=-20dB/dec$

转折频率: $\omega_1=10$ 后. $slope=-40dB/dec$

原开环传递函数Bode图:



$\Rightarrow$: $L(10)=40\cdot\lg\frac{\omega_c}{10}=L(1)-20\cdot\lg\frac{10}{1}$ $\Rightarrow \omega_c=44.72\ rad/s<50rad/s$

$\gamma=180^\circ+\Phi(\omega_c)=180^\circ-90^\circ-\arctan(\frac{0.1\times44.72}{1})=12.6^\circ<45^\circ$

动态性能不符要求，设超前传递函数 $G_c(s)=k_c\cdot\frac{1}{\alpha}\cdot\frac{\alpha Ts+1}{Ts+1}$，取 $k_c\cdot\frac{1}{\alpha}=1$

考虑 $\Phi_m=45^\circ-12.6^\circ+10^\circ=42.4^\circ$ $\Rightarrow \Phi_m=\frac{\alpha-1}{\alpha+1}=\sin42.4^\circ=0.6743$. 得 $\alpha=5.141$

在 ω_c' 处提升幅值 $=\frac{1}{2}\times20\lg\alpha=40\times\lg\frac{\omega_c'}{\omega_c}$ $\Rightarrow \omega_c'=67.34\ rad/s>50rad/s$

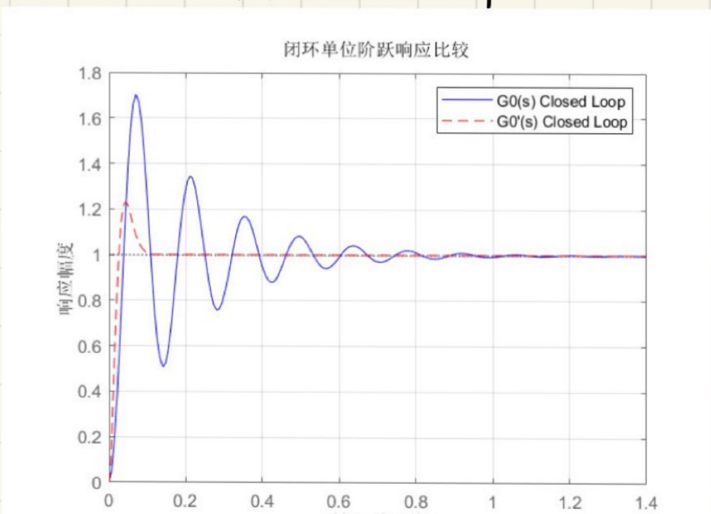
$T=\frac{1}{\omega_c'\sqrt{\alpha}}=\frac{1}{67.34\cdot\sqrt{5.141}}=0.0066$. $\alpha T=0.0337$

得开环传递函数: $G_c(s)=\frac{\alpha Ts+1}{Ts+1}=\frac{0.0337s+1}{0.0066s+1}=5.141\cdot\frac{s+29.67}{s+151.5}$, $G_0'(s)=5.141\cdot\frac{s+29.67}{s+151.5}\cdot\frac{2000}{s(s+10)}$

修正后: $\gamma=180^\circ-90^\circ-\arctan(\frac{0.1\times67.34}{1})+\arctan(\frac{67.34}{29.67})-\arctan(\frac{67.34}{151.5})=50.7^\circ>45^\circ$.

修正后系统满足要求. 在原Bode图中作出修正后系统部分:

Matlab模拟:



8: $e_{ss}=\lim_{s\to0}\frac{s\cdot\frac{1}{s^2}}{1+G_0(s)}=\lim_{s\to0}\frac{1}{s+sG_0(s)}=\frac{1}{k}\le0.06\Rightarrow k\geq16.67$

积分环节 $r=1$. 低频段 slope $=-20dB/dec$

转折频率 $w_1=1$ 后. slope $=-40dB/dec$

转折频率 $w_2=100$ 后. slope $=-60dB/dec$

考虑: $\gamma=180°+\Phi(w_c)=180°-90°-\arctan(w_c)-\arctan(0.01w_c)$.

取 $w_c=0.8\Rightarrow\gamma=180°-90°-38.66°-0.46°=50.88°>45°$

此时: $-20\lg\frac{1}{w_c}=20\lg k\Rightarrow k=0.8<16.67$

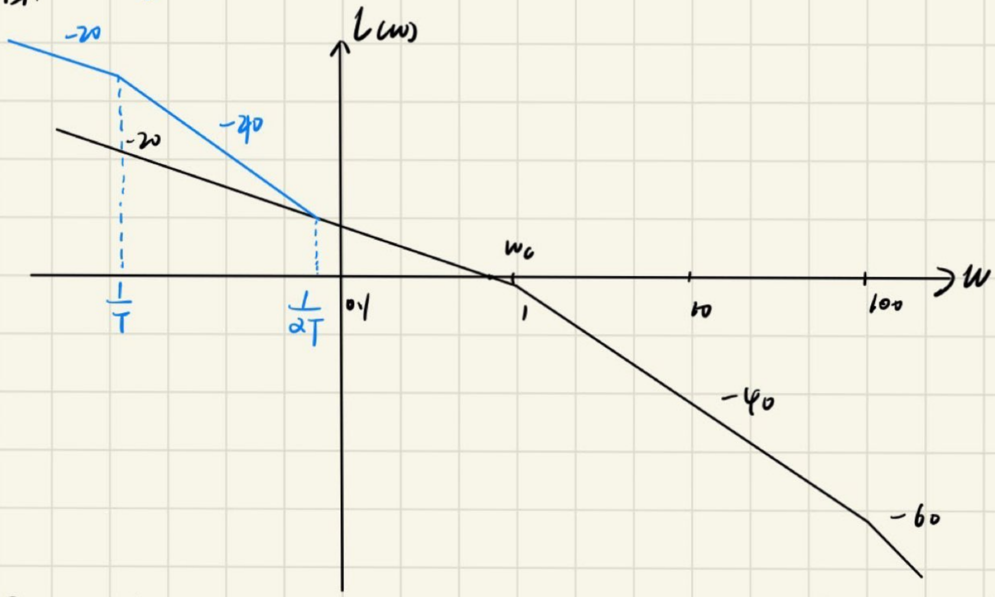
稳态性能不符. 引入串联滞后校正 $G_c(s)=k_c\cdot\frac{\alpha Ts+1}{Ts+1}=k_c\alpha\cdot\frac{s+\frac{1}{\alpha T}}{s+\frac{1}{T}}$

取: $k_c=\frac{1}{\alpha}=\frac{16.67}{0.8}=20.83\quad(\alpha=0.048)$

$\frac{1}{\alpha T}=\frac{w_c}{10}=0.08\Rightarrow T=260.4$

得 $G_c(s)=20.83\cdot\frac{12.5s+1}{260.4s+1}\Rightarrow G_0'(s)=16.67\cdot\frac{12.5s+1}{s\cdot(s+1)\cdot(0.01s+1)(260.4s+1)}$.

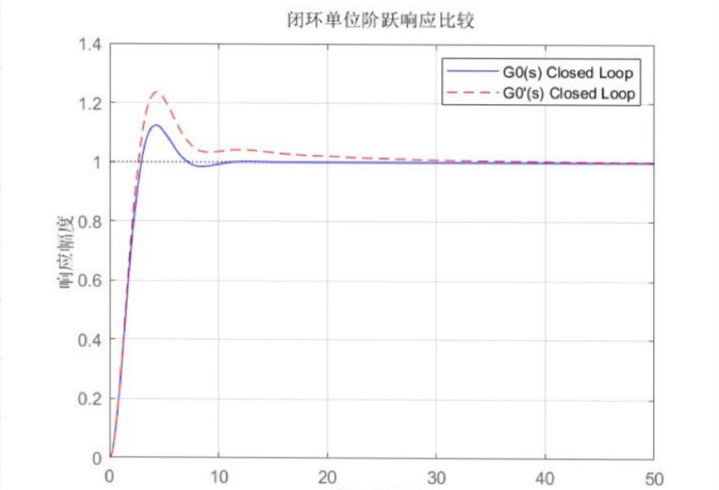
幅值Bode图:



此时: $\gamma=180°-90°-\arctan(0.8)-\arctan(0.008)+\arctan(12.5\times0.8)-\arctan(260.4\times0.8)=45.44°>45°$

设计满足要求.

Matlab模拟:



P149-150:

1.(2): $B=\begin{bmatrix}0\\1\end{bmatrix}$. $AB=\begin{bmatrix}0\\2\end{bmatrix}$. $A^2B=\begin{bmatrix}0\\4\end{bmatrix}$

$S=[B \quad AB]=\begin{bmatrix}0&0\\1&2\end{bmatrix}$.

rank[S] = 1 < 3. 故系统不能控.

(4): $B=\begin{bmatrix}1&2\\0&0\\3&0\end{bmatrix}$ $AB=\begin{bmatrix}-2&-4\\0&0\\-9&0\end{bmatrix}$ $A^2B=\begin{bmatrix}4&8\\0&0\\27&0\end{bmatrix}$

$S=\begin{bmatrix}1&2&-2&-4&4&8\\0&0&0&0&0&0\\3&0&-9&0&27&0\end{bmatrix}$

rank[S] = 2 < 3. 故系统不能控.

2(1): $B=\begin{bmatrix}b\\-1\end{bmatrix}$ $AB=\begin{bmatrix}ab-1\\-b\end{bmatrix}$ $S=\begin{bmatrix}b&ab-1\\-1&-b\end{bmatrix}$

系统能控 $\Leftrightarrow$ rank[S] = 2 $\Leftrightarrow$ $-b^2+ab-1\neq0$ ∴故: $ab-b^2\neq1$

3.(3): $C^T=[0 \quad 1 \quad 1]$ $C^TA=[0 \quad 2 \quad -3]$. $C^T\cdot A^2=[0 \quad 4 \quad 9]$

$V=\begin{bmatrix}0&1&1\\0&2&-3\\0&4&9\end{bmatrix}$ rank[V] = 2 < 3. 故系统不能观.

(4): $C^T=\begin{bmatrix}1&0&0\\0&-1&0\end{bmatrix}$ $C^TA=\begin{bmatrix}1&0&-1\\1&2&0\end{bmatrix}$ $C^TA^2=\begin{bmatrix}-2&0&-2\\-1&-4&-1\end{bmatrix}$

$V=\begin{bmatrix}1&0&0\\0&-1&0\\1&0&-1\\1&2&0\\2&0&-2\\-1&-4&-1\end{bmatrix}$ rank[V] = 3. 故系统能观.

6(a): 并联:

$$x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad u=u_1=u_2, \quad y=y_1+y_2$$

故:

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ -3 & -4 & 0 \\ 0 & 0 & -1 \end{bmatrix} x+\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u, \quad y=\begin{bmatrix} 2 & 1 & 1 \end{bmatrix} x$$

$$S=\begin{bmatrix} 0 & 1 & -4 \\ 1 & -4 & 13 \\ 1 & -1 & 1 \end{bmatrix}$$ rank[S]=2<3. 系统不能控.

$$V=\begin{bmatrix} 2 & 1 & 1 \\ -3 & -2 & -1 \\ 6 & 5 & 1 \end{bmatrix}$$ rank[V]=2<3. 系统不能观

$$G_1(s)=c_1^T\cdot(sI_2-A_1)^{-1}\cdot b_1=\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}=\frac{1}{(s+1)(s+3)}\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}=\frac{2+s}{(s+1)(s+3)}$$

$$G_2(s)=c_2^T\cdot(s-A_2)^{-1}b_2=1\cdot\frac{1}{s+1}\cdot 1=\frac{1}{s+1}$$

$$\Rightarrow G(s)=G_1(s)+G_2(s)=\frac{2s+5}{(s+1)(s+3)}$$

(b). 串联: $x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $u=u_1$, $y=y_2$, $u_2=y_1$

$$\Rightarrow \dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ -3 & -4 & 0 \\ 2 & 1 & -1 \end{bmatrix} x+\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u, \quad y=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

$$S=\begin{bmatrix} 0 & 1 & -4 \\ 1 & -4 & 13 \\ 0 & 1 & -3 \end{bmatrix}, \quad V=\begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -1 \\ -5 & -3 & 1 \end{bmatrix}$$ rank[S]=rank[V]=3. 故系统能观能控.

$$G(s)=G_1(s)\cdot G_2(s)=\frac{s+2}{(s+1)^2(s+3)}.$$

9.(1):

$$S=[b\ \ Ab\ \ A^2b]=\begin{bmatrix}0&-1&-4\\0&0&0\\1&3&8\end{bmatrix}$$ rank[S]=2<3. 系统不能控.

变换矩阵 $P=\begin{bmatrix}0&-1&0\\0&0&1\\1&3&0\end{bmatrix}$. $P^{-1}=\begin{bmatrix}3&0&1\\-1&0&0\\0&1&0\end{bmatrix}$

$$\dot{x}'=P^{-1}\cdot A\cdot P x'+P^{-1}\cdot b\cdot u$$

$$=\begin{bmatrix}3&0&1\\-1&0&0\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}1&2&-1\\0&1&0\\1&-4&3\end{bmatrix}\cdot\begin{bmatrix}0&-1&0\\0&0&1\\1&3&0\end{bmatrix}x'+\begin{bmatrix}3&0&1\\-1&0&0\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\1\end{bmatrix}\cdot u$$

$$\Rightarrow\begin{bmatrix}\dot{x}_1'\\\dot{x}_2'\\\dot{x}_3'\end{bmatrix}=\begin{bmatrix}0&-4&2\\1&4&-2\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}\cdot u$$

$$y=c^T\cdot P\cdot x'=[1\ \ -1\ \ 1]\cdot\begin{bmatrix}0&-1&0\\0&0&1\\1&3&0\end{bmatrix}\cdot x'=[1\ \ 2\ \ -1]\begin{bmatrix}x_1'\\x_2'\\x_3'\end{bmatrix}$$

式中 x_1'. x_2' 为能控部分. x_3' 为不能控部分.

$$V=\begin{bmatrix}c^T\\c^T A\\c^T A^2\end{bmatrix}=\begin{bmatrix}1&-1&1\\2&-3&2\\4&-7&4\end{bmatrix}$$ rank[V]=2<3. 系统不能观

逆矩阵 $Q^{-1}=\begin{bmatrix}1&-1&1\\2&-3&2\\1&0&0\end{bmatrix}$. $Q=\begin{bmatrix}0&0&1\\2&-1&0\\3&-1&-1\end{bmatrix}$

$$\dot{x}'=Q^{-1}\cdot A\cdot Q\cdot x'+Q^{-1}\cdot b\cdot u$$

$$=\begin{bmatrix}1&-1&1\\2&-3&2\\1&0&0\end{bmatrix}\cdot\begin{bmatrix}1&2&-1\\0&1&0\\1&-4&3\end{bmatrix}\cdot\begin{bmatrix}0&0&1\\2&-1&0\\3&-1&-1\end{bmatrix}\cdot x'+\begin{bmatrix}1&-1&1\\2&-3&2\\1&0&0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\cdot u$$

$$\Rightarrow\begin{bmatrix}\dot{x}_1'\\\dot{x}_2'\\\dot{x}_3'\end{bmatrix}=\begin{bmatrix}0&1&0\\-2&3&0\\1&-1&2\end{bmatrix}\cdot\begin{bmatrix}x_1'\\x_2'\\x_3'\end{bmatrix}+\begin{bmatrix}1\\2\\0\end{bmatrix}\cdot u$$

$$y=c^T\cdot Q\cdot x'=[1\ \ -1\ \ 1]\cdot\begin{bmatrix}0&0&1\\2&-1&0\\3&-1&-1\end{bmatrix}\cdot x'=[1\ \ 0\ \ 0]\cdot\begin{bmatrix}x_1'\\x_2'\\x_3'\end{bmatrix}.$$

式中 x_1'. x_2' 为能观部分. x_3' 为不能观部分.