Homework 2

P36,10(3) **Solution:** The transfer function of this system is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s^3 + 8s^2 + 17s + 1}{s^3 + 6s^2 + 11s + 1} = 1 + \frac{2s^2 + 6s}{s^3 + 6s^2 + 11s + 1}$$

Then, we can get the state equation and output equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 6 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$$

P36,12(3) **Solution:** The transfer function of this system is

$$\frac{Y(s)}{U(s)} = \frac{s^3 + 8s^2 + 12s + 9}{s^3 + 7s^2 + 14s + 8} = \frac{s^2 - 2s + 1}{s^3 + 7s^2 + 14s + 8} + 1$$

Controllable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$$

Observable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -14 \\ 0 & 1 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u$$

P37,13(b) Remove the $1/s$ branch from the state diagram. Based on the modified state diagram, with $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$ and y as output variables, and x_1 , x_2 , x_3 and u as input variables, write the state equations and output equations.

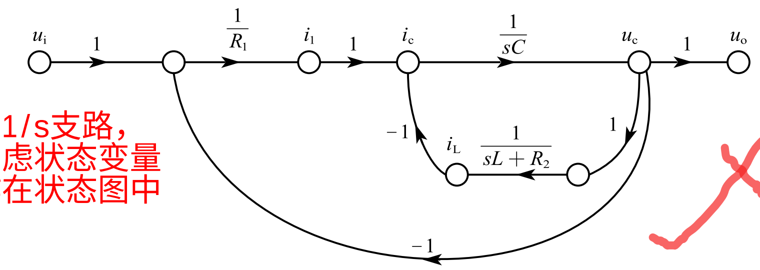
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-(a_2+a_3)}{1+a_0a_3} & -a_1 & \frac{1-a_0a_2}{1+a_0a_3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} \frac{1}{1+a_0a_3} & 0 & \frac{a_0}{1+a_0a_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cancel{+u}$$

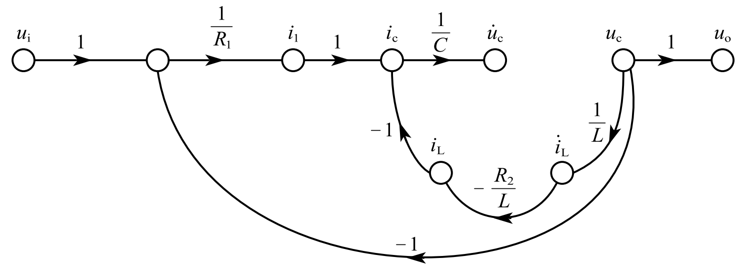
输出方程不含u项(被1/s隔断，y与u无直接关联)

P37,15 **Solution:** Draw a signal flow diagram for the system, as shown in the figure:

作状态图时需含有1/s支路，
并且题目中要求考虑状态变量
初始值，也需要作在状态图中



Rewrite the $\frac{1}{Ls+R_2}$ branch and $\frac{1}{sC}$ branch in the form including the integral branch $1/s$, then remove the $1/s$ branch from the state diagram. Based on the modified state diagram, with $\dot{u}_\mathrm{c}$, $\dot{i}_\mathrm{L}$, u_o as output variables, and u_c , i_L , u_i as input variables, write the state equations and transfer equations.



$$\begin{bmatrix}\dot{u}_\mathrm{c}\\ \dot{i}_\mathrm{L}\end{bmatrix} = \begin{bmatrix}-\frac{1}{R_1C} & -\frac{1}{C}\\ \frac{1}{L} & -\frac{R_2}{L}\end{bmatrix}\begin{bmatrix}u_\mathrm{c}\\ i_\mathrm{L}\end{bmatrix} + \begin{bmatrix}\frac{1}{R_1C}\\ 0\end{bmatrix}u_\mathrm{i}$$

$$u_\mathrm{o} = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}u_\mathrm{c}\\ i_\mathrm{L}\end{bmatrix}$$

[1 0]

P37,17(3) **Solution:**

$$(sI-A)^{-1} = \begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 0 & 0 & s\end{bmatrix}^{-1} = \frac{1}{s^3}\begin{bmatrix}s^2 & s & 1\\ 0 & s^2 & s\\ 0 & 0 & s^2\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{s} & \frac{1}{s^2} & \frac{1}{s^3}\\ 0 & \frac{1}{s} & \frac{1}{s^2}\\ 0 & 0 & \frac{1}{s}\end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix}1 & t & \frac{t^2}{2}\\ 0 & 1 & t\\ 0 & 0 & 1\end{bmatrix}$$

P70,1(2) **Solution:**

$$|sI-A| = \begin{vmatrix}s & -1 & 0\\ 0 & s & -1\\ -250 & 0 & s+5\end{vmatrix} = s^3+5s^2-250$$

$$G(s) = c^{\mathrm{T}}(sI - A)^{-1}b$$
$$= \frac{1}{s^3+5s^2-250}\begin{bmatrix}-5 & 1 & 0\end{bmatrix}\begin{bmatrix}s(s+5) & s+5 & 1 \\ 250 & s(s+5) & s \\ 250s & 250 & s^2\end{bmatrix}\begin{bmatrix}0\\0\\50\end{bmatrix}$$
$$= \frac{50(s-5)}{(s^2+10s+50)(s-5)} = \frac{50}{s^2+10s+50}$$

说明稳定性时简要说明一下有什么极点，零极点相消情况（参考书本P41例题3.2）

The system transfer function has common factors that cancel out in the numerator and denominator. However, the factors that cancel correspond to characteristic roots in the right-half plane. Therefore, the system is BIBO stable and not asymptotically stable.

P71,4(3) **Solution:** The Routh table is

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & 0 \approx \delta^+ & 6 & 0 \\ s^2 & 4-\frac{12}{\delta+} \approx -\frac{12}{\delta+} & 10 & 0 \\ s^1 & 6 & 0 & 0 \\ s^0 & 10 & 0 & 0 \end{array}$$

The first column of the Routh table changes sign twice, indicating that there are two positive roots in the right-half plane. Therefore, the system is unstable.

P71,5(3) **Solution:** The Routh table is

$$\begin{array}{c|ccc} s^5 & 1 & 8 & 7 \\ s^4 & 4 & 8 & 4 \\ s^3 & 6 & 6 & 0 \\ s^2 & 4 & 4 & 0 \\ s^1 & 0 & 0 & 0 \\ s^0 & 4 & 0 & 0 \end{array}$$

In the case where the entire row s^1 in the Routh table is zero, an auxiliary polynomial can be obtained from the preceding row s^2. Subsequently, the derivative of the auxiliary polynomial is calculated, and the resulting data is used to construct a new row, replacing the row that was entirely zero. The Routh table is then continued with these modifications. The transformed Routh table is as follows, with no change in the sign of coefficients in the first column, indicating that the system does not have characteristic roots in the right-half plane of the s-plane.

$$\begin{array}{c|ccc} s^5 & 1 & 8 & 7 \\ s^4 & 4 & 8 & 4 \\ s^3 & 6 & 6 & 0 \\ s^2 & 4 & 4 & 0 \\ s^1 & 8 & 0 & 0 \\ s^0 & 4 & 0 & 0 \end{array}$$

The roots of the auxiliary equation are also roots of the original characteristic equation, $4s^2+4 = 4(s+j)(s-j) = 0$ The auxiliary equation has a complex conjugate pair of roots $\pm j$, indicating that the system has a pair of roots on the imaginary axis. Therefore, the system is not asymptotically stable and does not possess engineering stability.

P71,5(2) Characteristic polynomial is

$$s(s-1)(s+5) + K(s+1) = s^3 + 4s^2 + (K-5)s + K$$

The Routh table is

分类讨论，考虑一下s^1为全零行的情况。

$$\begin{array}{c|cc} s^3 & 1 & K-5 \\ s^2 & 4 & K \\ s^1 & \frac{3K-20}{4} & 0 \\ s^0 & K & 0 \end{array}$$

The condition for system stability can be obtained from the Routh table is $K > 0$ and $3K - 20 > 0$, i.e. $K > \frac{20}{3}$. The system is stable when $K > \frac{20}{3}$.

P71,8(1) **Solution:** Characteristic polynomial is

$$(0.1s + 1)(0.2s + 1) + 20 = 0$$

According to the Routh criterion, the system is stable. The system is a Type O system with $K = 20$. Static position error coefficient is

$$K_{\mathrm{p}} = \lim_{s\to 0} G_0(s) = 20$$

Static velocity error coefficient is

$$K_{\mathrm{v}} = \lim_{s\to 0} sG_0(s) = 0$$

When the input signal is $r(t) = 1(t)$,

$$e_{\mathrm{ss}} = \frac{1}{1+K_{\mathrm{p}}} = \frac{1}{21}$$

When the input signal is $r(t) = t$,

$$e_{\mathrm{ss}} = \frac{1}{K_{\mathrm{v}}} \to \infty$$

P71,8(2) Characteristic polynomial is

$$s(s + 2)(s + 10) + 200 = 0$$

i.e. $s^3 + 12s^2 + 20s + 200 = 0$. According to the Routh criterion, the system is stable. The system is a Type I system with $K = \lim_{s\to 0} sG_0(s) = \frac{200}{2\times 10} = 10$. Static position error coefficient is

$$K_{\mathrm{p}} = \lim_{s\to 0} G_0(s) = \lim_{s\to 0} \frac{K}{s} \to \infty$$

Static velocity error coefficient is

$$K_{\mathrm{v}} = \lim_{s\to 0} sG_0(s) = K = 10$$

When the input signal is $r(t) = 1(t)$,

$$e_{\mathrm{ss}} = \frac{1}{1+K_{\mathrm{p}}} = 0$$

When the input signal is $r(t) = t$,

$$e_{\mathrm{ss}} = \frac{1}{K_{\mathrm{v}}} = \frac{1}{10}$$

P71,8(3) Characteristic polynomial is

$$s^2(2s + 1)(s + 2) + 5(3s + 1) = 0$$

i.e. $2s^4 + 5s^3 + 2s^2 + 15s + 5 = 0$. According to the Routh table

$$\begin{array}{c|ccc} s^4 & 2 & 2 & 5 \\ s^3 & 5 & 15 & 0 \\ s^2 & -4 & 5 & 0 \\ s^1 & \frac{85}{4} & 0 & 0 \\ s^0 & 5 & 0 & 0 \end{array}$$

The sign changes twice in the first column, indicating two roots in the right-half plane, rendering the system unstable. Consequently, the system lacks stable states and has no steady-state error.

P72,9(4) The system is a second-order system with the characteristic polynomial s^2+4 , yielding $\omega_\text{n}=2$ and $\zeta=0$. The system is in an undamped state. In this case, $\delta\%=100\%$. The unit step response of the system exhibits sinusoidal oscillations and does not converge to a constant steady-state value. Therefore, it is not possible to define a settling time.

P72,10 Overshoot $\delta\%=\frac{5-4}{4}\times 100\%=25\%$, time to peak $t_\text{p}=0.4\text{s}$, from

$$\delta\%=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}=25\%$$

9-12题中：超调量和阻尼系数的符号一般用σ和ξ，而非δ和ζ；建议统一保留三位小数。

We can get $\zeta=0.4$, from

$$t_\text{p}=\frac{\pi}{\omega_\text{n}\sqrt{1-\zeta^2}}=0.4\text{s}$$

We can get $\omega_\text{n}=8.585$. The open-loop transfer function of system is

$$G_0(s)=\frac{\omega_\text{n}^2}{s(s+2\zeta\omega_\text{n})}=\frac{73.7}{s(s+6.856)}$$

P72,11 Overshoot $\delta\%=\frac{6-5}{5}\times 100\%=20\%$, time to peak $t_\text{p}=0.6\text{s}$, $K_1=\lim_{t\to\infty}y(t)=5$, from

$$\delta\%=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}=\cancel{25\%}\ 20\%$$

We can get $\zeta=0.46$, from

$$t_\text{p}=\frac{\pi}{\omega_\text{n}\sqrt{1-\zeta^2}}=\cancel{0.4\text{s}}\ 0.6\text{s}$$

We can get $\omega_\text{n}=5.90$. The open-loop transfer function of system is

$$G_0(s)=\frac{\omega_\text{n}^2}{s(s+2\zeta\omega_\text{n})}=\frac{\frac{\omega}{2\zeta}}{s(\frac{s}{2\zeta\omega_\text{n}}+1)}$$

Thus $K_2=\frac{\omega_\text{n}}{2\zeta}=6.41\quad T=\frac{1}{2\zeta\omega_\text{n}}=0.18$.

P72,12 Assuming the system is a second-order system, comparing the standard second-order system unit step response expression

$$y(t)=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_\text{n}t}\sin(\omega_\text{d}t+\theta)$$

We can get the corresponding parameters

$$\frac{1}{\sqrt{1-\zeta^2}}=1.25,\ \ \zeta\omega_\text{n}=1.2$$

$$\omega_\text{d}=\omega_\text{n}\sqrt{1-\zeta^2}=1.6,\ \ \cos\theta=\cos 53.1^\circ=\zeta$$

Thus, we can calculate the damping ratio ζ and natural frequency of oscillation ω_n of the system.

$$\zeta=0.6,\ \ \omega_\text{n}=2$$

Overshoot

$$\delta\%=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}=0.095\times 100\%=9.5\%$$

Settling time

$$t_\text{s}=\frac{4}{\zeta\omega_\text{n}}=\frac{4}{0.6\times 2}=3.33\text{s}$$

The open-loop transfer function of system is

$$G_0(s)=\frac{\omega_\text{n}^2}{s(s+2\zeta\omega_\text{n})}=\frac{4}{s(s+2.4)}$$

The closed-loop transfer function of system is

$$G(s)=\frac{G_0(s)}{1+G_0(s)}=\frac{4}{s^2+2.4s+4}$$