清華大学 数学作业纸

班级：电24 姓名：孟如锦 编号：3X 科目：自控 第 1 页

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10.(3)

$$TF:\ G(s)=\frac{Y(s)}{U(s)}=\frac{s^3+8s^2+17s+1}{s^3+6s^2+11s+1}=1+\frac{2s^2+6s}{s^3+6s^2+11s+1}$$

CCF:

the state equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-1&-11&-6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

the output equation:

$$y=\begin{bmatrix}0&6&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+u$$

12.(3)

$$\frac{Y(s)}{U(s)}=\frac{s^3+8s^2+12s+9}{s^3+7s^2+14s+8}=1+\frac{s^2+1-2s}{s^3+7s^2+14s+8}$$

① CCF:

the state equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-8&-14&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

the output equation:

$$y=\begin{bmatrix}1&-2&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+u$$

② OCF:

the state equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&0&-8\\1&0&-14\\0&1&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1\\-2\\1\end{bmatrix}u$$

the output equation:

$$y=\begin{bmatrix}0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+u$$

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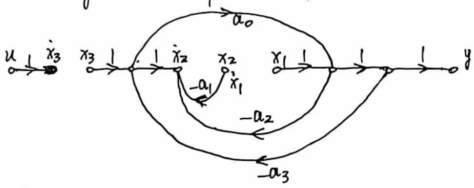
13. get rid of "$\frac{1}{s}$" paths:



Loops: $-a_0a_3$

Mason Formula: the state equation is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\ \frac{-a_2-a_3}{1+a_0a_3}&-a_1&\frac{1-a_0a_2}{1+a_0a_3}\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

the output equation:

$$y=\begin{bmatrix}\frac{1}{1+a_0a_3}&0&\frac{a_0}{1+a_0a_3}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

15.

$$\begin{cases}u_i=i_1R_1+u_c\\ i_1=i_L+C\frac{du_c}{dt}\\ u_o=L\frac{di_L}{dt}+i_LR_2=u_c\end{cases}$$

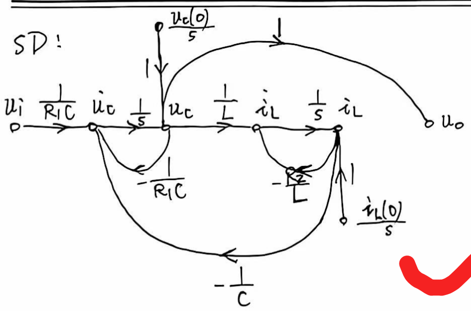
$\Rightarrow$ the state equation:

$$\begin{bmatrix}\dot{u}_c\\ \dot{i}_L\end{bmatrix}=\begin{bmatrix}-\frac{1}{R_1C}&-\frac{1}{C}\\ \frac{1}{L}&-\frac{R_2}{L}\end{bmatrix}\begin{bmatrix}u_c\\ i_L\end{bmatrix}+\begin{bmatrix}\frac{1}{R_1C}\\0\end{bmatrix}u_i$$

the output equation:

$$u_o=\begin{bmatrix}1&0\end{bmatrix}\begin{bmatrix}u_c\\ i_L\end{bmatrix}$$

SD：



17.(3)

$$(sI-A)^{-1}=\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 0 & s \end{bmatrix}^{-1}=\frac{1}{s^3}\begin{bmatrix} s^2 & s & 1 \\ 0 & s^2 & s \\ 0 & 0 & s^2 \end{bmatrix}$$

$$\cancel{\Phi(t)=} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} & \frac{1}{s^3} \\ 0 & \frac{1}{s} & \frac{1}{s^2} \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

$$\Phi(t)=\mathcal{L}^{-1}\left[(sI-A)^{-1}\right]=\begin{bmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

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1.(2) a

CCF： ~~50~~

$$\frac{Y(s)}{50U(s)}=\frac{s-5}{s^3+5s^2-250}$$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{50(s-5)}{s^3+5s^2-250}=\frac{50}{s^2+10s+50}$$

the poles of $G(s)$ all lie in the left-half s-plane. so the system is BIBO stable.

But $G(s)$ has a cancelled pole, which is 5, so the system is not ~~ay~~ asymptotic stable.

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4. (3) Routh's Tabulation:

s^5	1	2	11
s^4	2	4	10
s^3	$0 \approx \delta^+$	6	0
s^2	$4-\frac{12}{\delta^+} \approx -\frac{12}{\delta^+}$	10	0
s^1	6	0	0
s^0	10	0	0

signs of elements in the first column changes twice, so the system is not ~~syst~~ stable and has 2 roots in the right s-plane.

(4) Routh's Tabulation:

s^5	1	8	7
s^4	4	8	4
s^3	6	6	0
s^2	4	4	0
s^1	$\cancel{0}$ 8	0	0
s^0	4	0	0

the auxiliary equation: $4s^2+4=0$

roots symmetric with respect to the origin of the s-plane: $\pm j$

5. (2) the characteristic ~~fuc~~ function:

$s(s-1)(s+5)+K(s+1) = s^3+4s^2+(K-5)s+K$

Routh's Tabulation:

s^3	1	K-5
s^2	4	K
s^1	$\frac{3K-20}{4}$	0
s^0	K	0

if the system is stable,

$\begin{cases} 3K-20>0 \\ K>0 \end{cases} \Rightarrow K > \frac{20}{3}$

8.

(1) the characteristic function:

$(0.1s+1)(0.2s+1)+20 = 0.02s^2+0.3s+21=0$

$\Rightarrow s_{1,2} = \frac{-15 \pm j5\sqrt{159}}{2}$

so the system is stable. ~~N~~ type 0.

$k_p = \lim_{s\to 0} G_o(s) = 20$, $k_v = \lim_{s\to 0} sG_o(s) = 0$

$r(t) = 1(t)$, $e_{ss} = \frac{1}{1+k_p} = \frac{1}{21}$.

$r(t) = t$, $e_{ss} = \frac{1}{k_v} = \infty$

(2) the characteristic function:

$s(s+2)(s+10)+200 = s^3+12s^2+20s+200=0$.

Routh's Tabulation:

s^3	1	20
s^2	12	200
s^1	$\cancel{4}\ \frac{10}{3}$	0
s^0	200	0

so the system is stable.

open loop gain $k=200$, $r=1$, type 1.

$k_p = \lim_{s\to 0} G_o(s) \to \infty$

$k_v = \lim_{s\to 0} sG_o(s) = 10$

$r(t) = 1(t)$, $e_{ss} = 0$.

$r(t) = t$, $e_{ss} = \frac{1}{k_v} = \frac{1}{10}$

(3) the characteristic function:

$s^2(2s+1)(s+2)+5(3s+1) = 2s^4+5s^3+2s^2+15s+5=0$

Routh's Tabulation:

s^4	2	2	5
s^3	5	15	0
s^2	-4	5	0
s^1	$\frac{85}{4}$	0	0
s^0	5	0	0

the system is unstable, so it doesn't have the steady-stable errors.

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9. (4) $s^2+4=0$, $s_{1,2} = \pm j2$

$\omega_n = 2$, $\zeta = 0$

$\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 100\%$

$t_s = \frac{4}{\zeta\omega_n} \to \infty$

10. $\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = \frac{5-4}{4} \times 100\% = 25\%$

$\Rightarrow \zeta \approx 0.4037$

$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.4 \Rightarrow \cancel{t_p}\ \omega_n \approx 8.585$

$G_o(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \approx \frac{73.70}{s(s+6.932)}$

11. $\begin{cases} \sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 20\%, \\ t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.6 \end{cases} \Rightarrow \begin{cases} \zeta \approx 0.4559 \\ \omega_n \approx 5.883 \end{cases}$

$G_o(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} = \frac{k_2}{s(Ts+1)}$. so

$T = \frac{1}{2\zeta\omega_n} \approx 0.1864$

$k_2 = T\omega_n^2 \approx 6.452$

the ~~stat~~ steady-state value is 5, so $k_1 = 5$

12.

(2) $\frac{1}{\sqrt{1-\zeta^2}} = 1.25 \Rightarrow \zeta = 0.6$

$\zeta\omega_n = 1.2 \Rightarrow \omega_n = 2$

(3) $\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% \approx 9.478\%$

$t_s = \frac{4}{\zeta\omega_n} = \frac{10}{3}s$

(1) $G_o(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} = \frac{4}{s(s+2.4)}$

$G(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{4}{s^2+2.4s+4}$