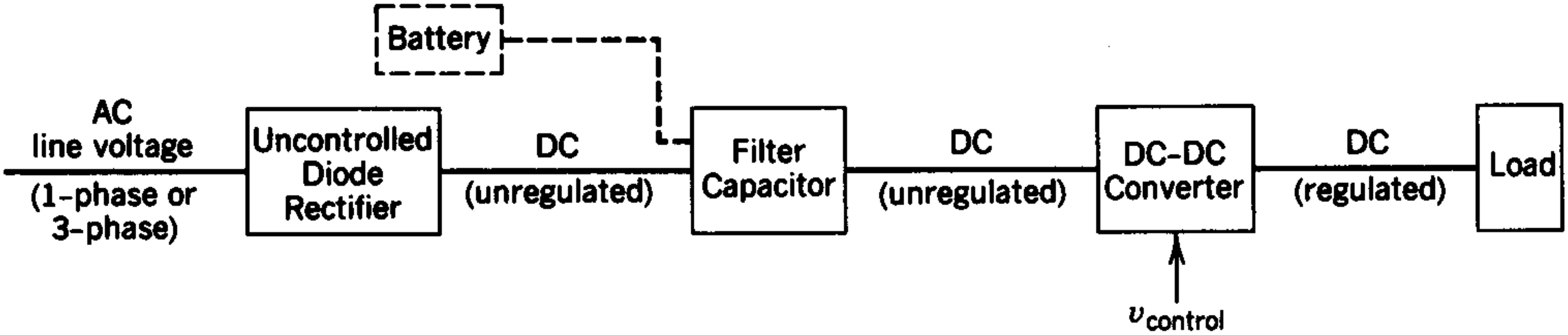
Chapter 4

DC-DC Converters
(DC Choppers)

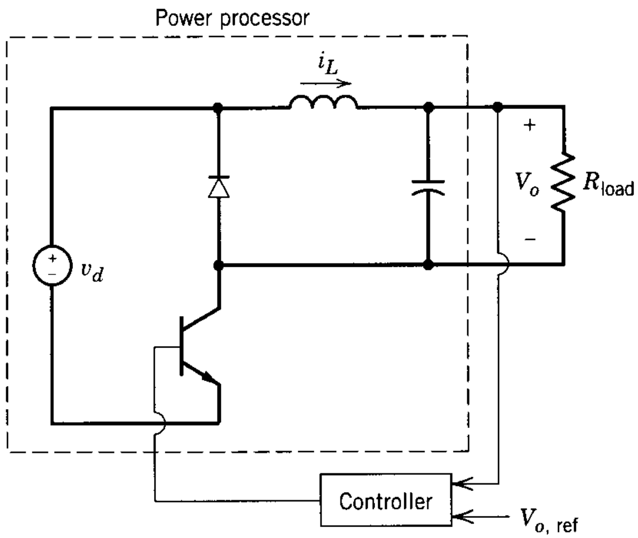
DC choppers = DC-to-DC converters

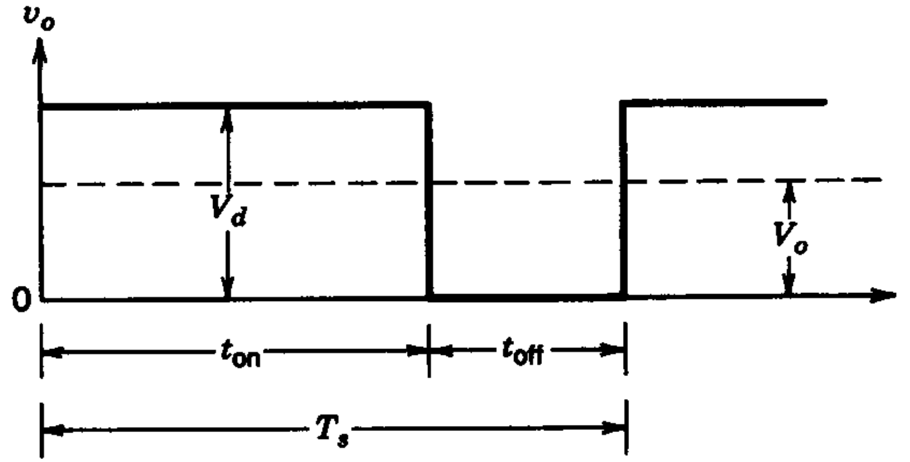
A chopper can be considered as a DC equivalent to an AC transformer with a continuously variable turns ratio. Like a transformer, it can be used to step-down or step-up a DC voltage source.

A DC-DC Converter System



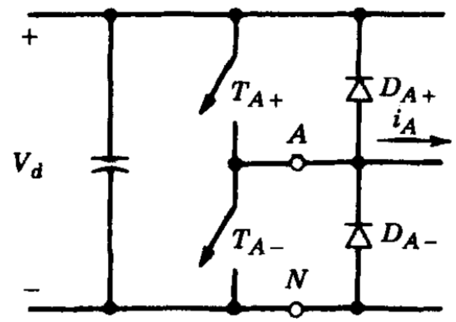
Basic Principle





$$V_o = \frac{1}{T_s}\int_0^{t_{on}} v_0 dt = \frac{t_{on}}{T_s} V_d = f_s t_{on} V_d = D V_d$$

D – duty ratio

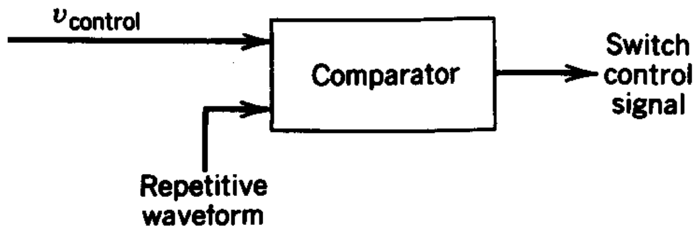


Basic Unit of PWM circuit

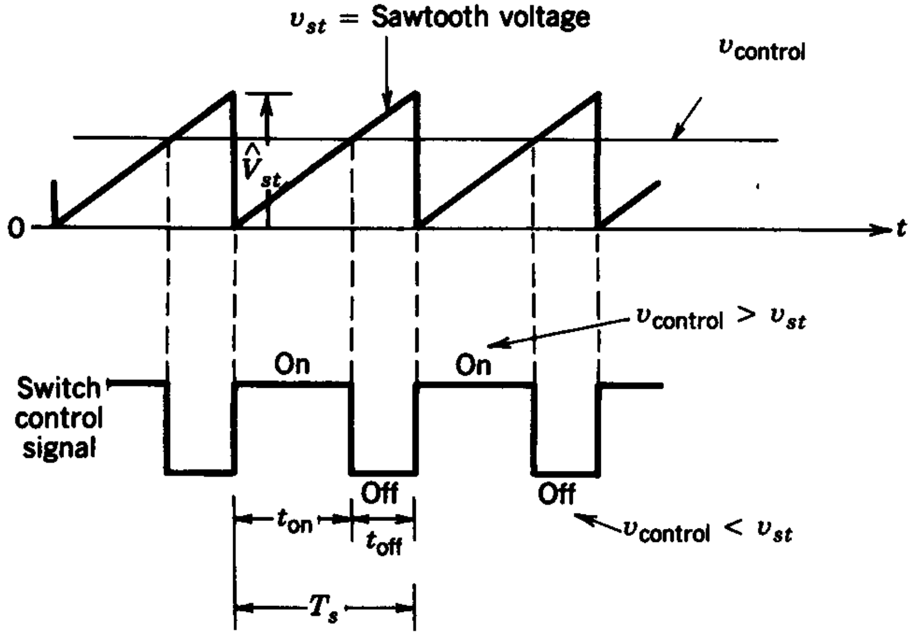
2 methods to control the output voltage:

1) **frequency fixed, on-duration adjusted -- PWM**
2) *Both frequency and on-duration adjusted.*

Modulation of DC-DC converters by PWM



(a)



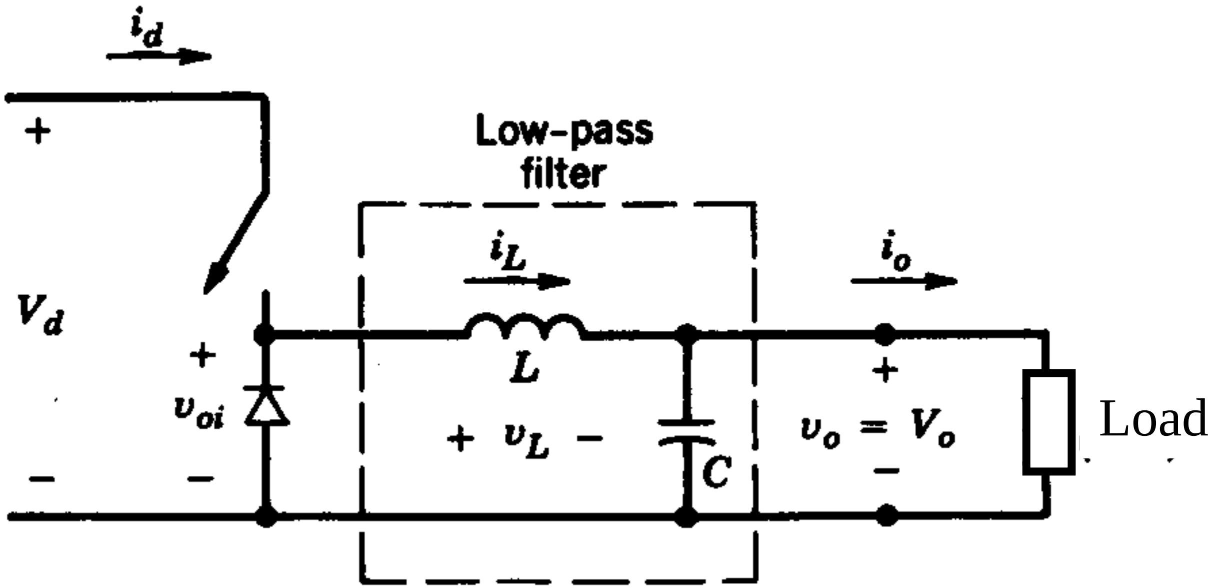
(switching frequency $f_s = \frac{1}{T_s}$)

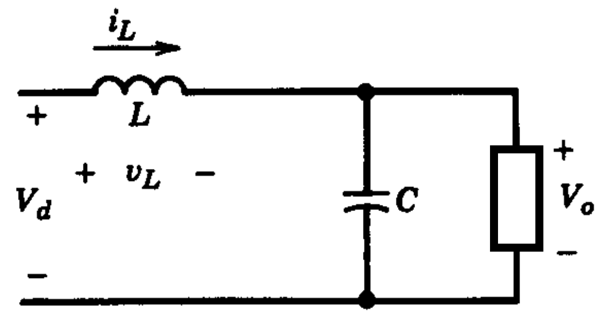
(b)

Basic topologies of DC-DC Converters:

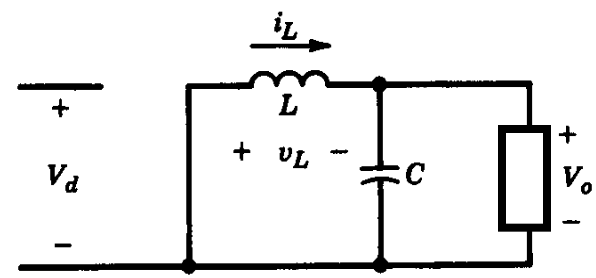
1. Buck regulators (step-down)
2. Boost regulators (step-up)
3. Buck-Boost regulators
4. Cuk regulators
5. Full-bridge regulators

a key term: PWM (pulse-width modulation)

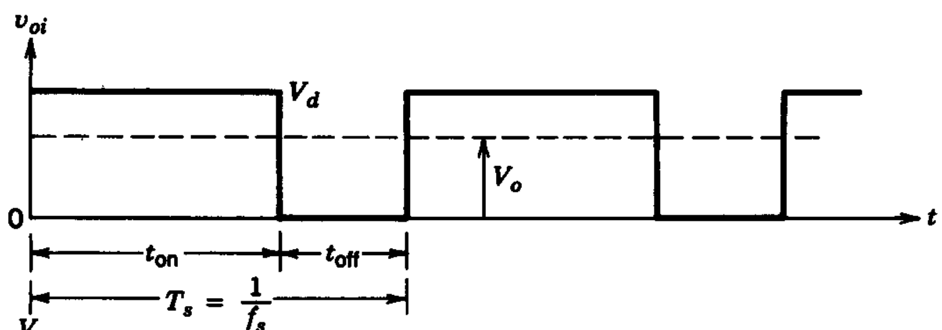


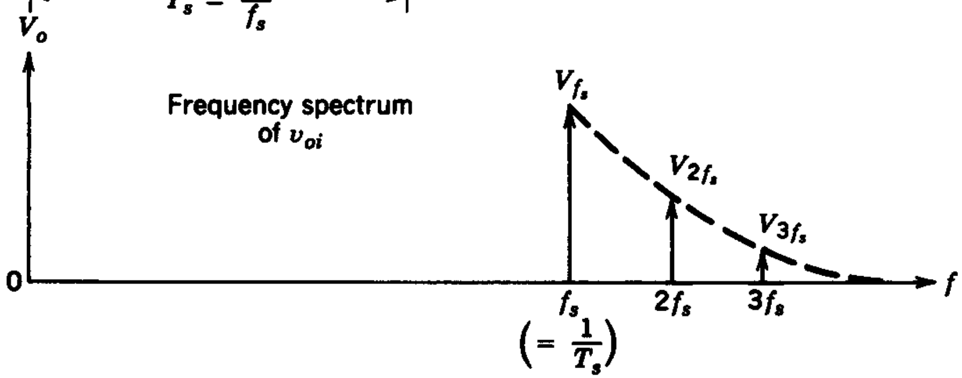


Mode 1 — on



Mode 2 — off





5-1. Buck Regulators

Focus on steady state

Continuous-conduction

inductor current positively continuous

C1: Capacitor voltage can be seen as constant unless the purpose is for voltage ripple calculation (capacitor large enough)

C2: $I_C = 0$ ($I_L = I_0$), $V_L = 0$ in a cycle

Assuming i_L rises linearly by ΔI_L in time t_{on}

Assuming i_L drops linearly by ΔI_L in time t_{off}

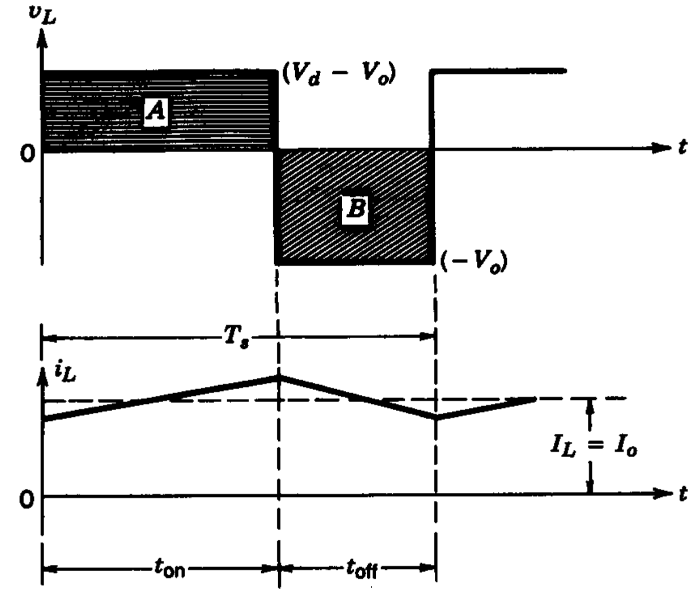
$$V_d - V_o = L\frac{\Delta I_L}{t_{on}} \qquad -V_o = -L\frac{\Delta I_L}{t_{off}}$$

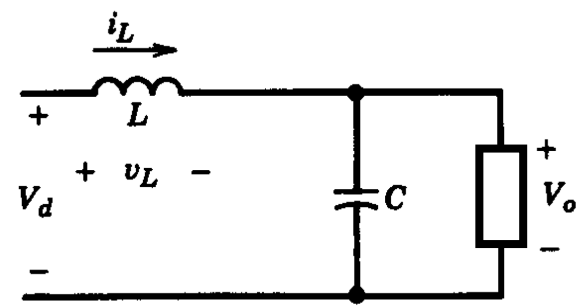
$$V_o = DV_d$$

equal to that before LC

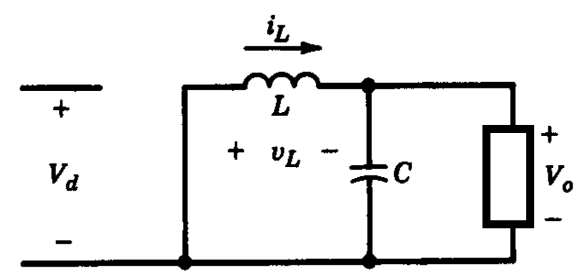
$$V_d * I_d = V_o * I_o \qquad I_d = DI_o$$

$$\Delta I_L = \frac{V_d D(1-D)}{f_s L}$$



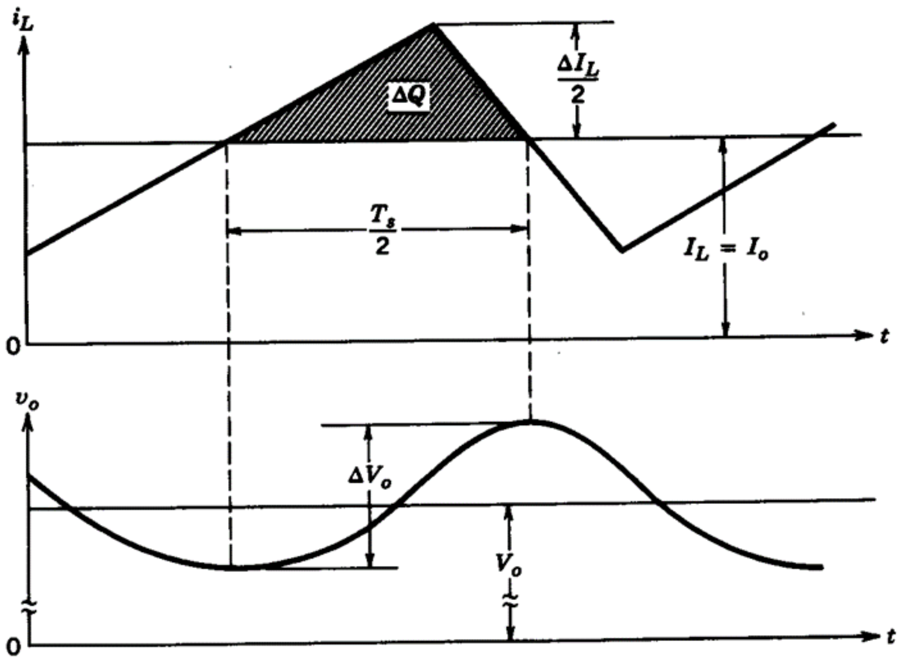


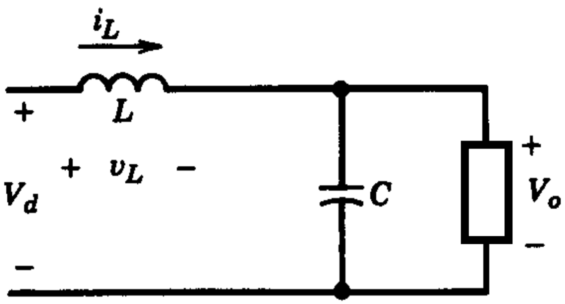
Mode 1 — on



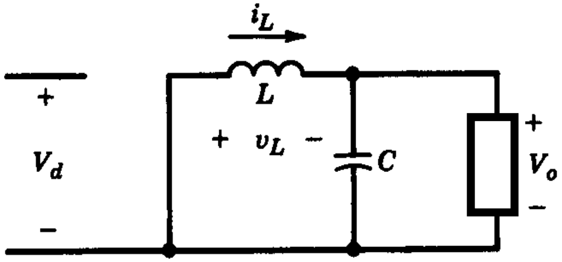
Mode 2 — off

Output Voltage Ripple





Mode 1 — on

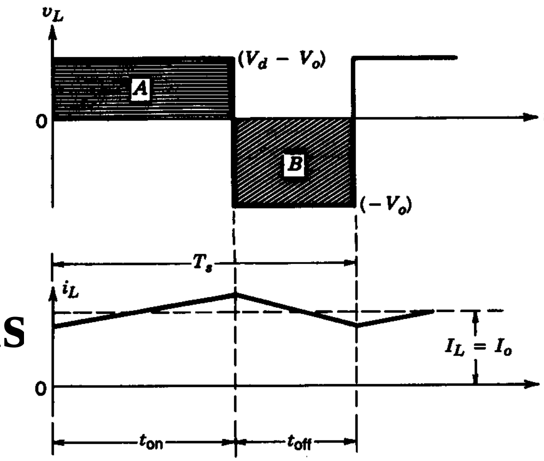


Mode 2 — off

$$\Delta V_o = \frac{V_d D(1-D)}{8LCf_s^2}$$

Discontinuous-conduction

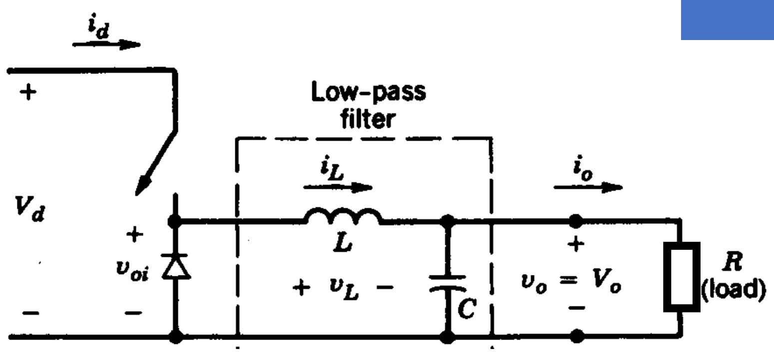
inductor current is discontinuous

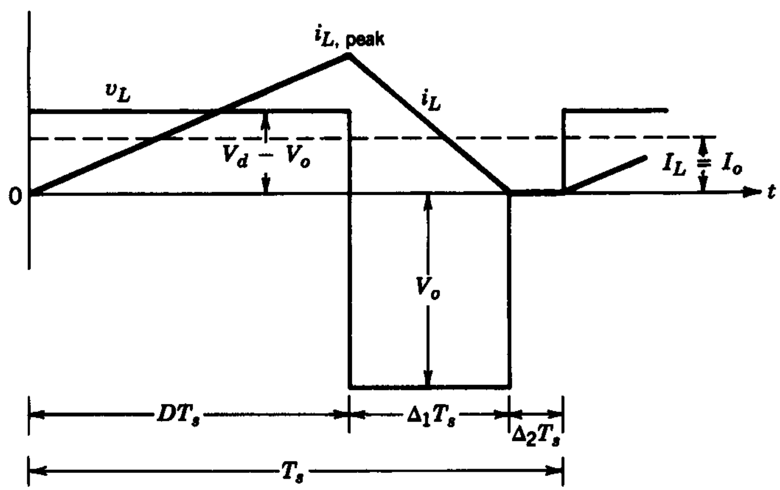


Boundary between continuous and discontinuous

$$I_{LB} = I_{oB} = 0.5\Delta I_L$$

$$\Delta I_L = \frac{V_d D(1-D)}{f_s L}$$

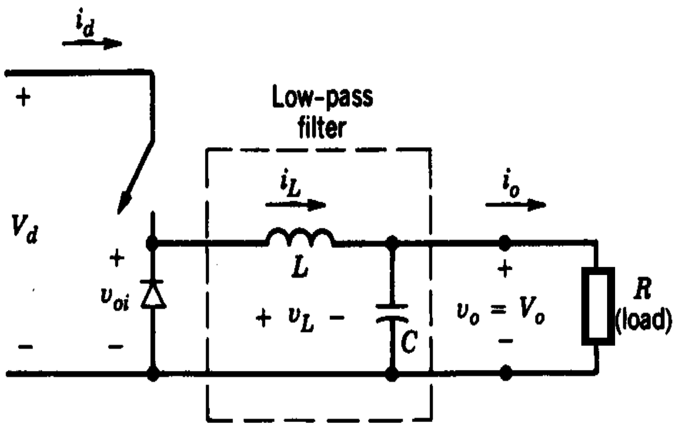




In steady state, the average inductor voltage must be zero.

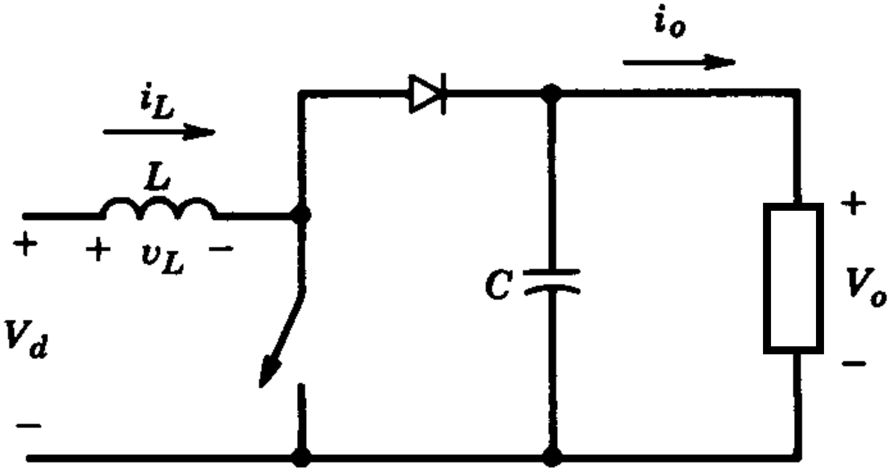
$$\frac{V_o}{V_d} = \frac{D}{D + \Delta_1}$$

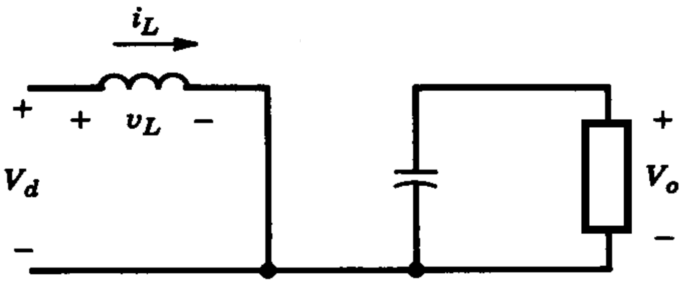
$$\Delta_1 = \frac{2 f_s L}{V_d D} I_o$$



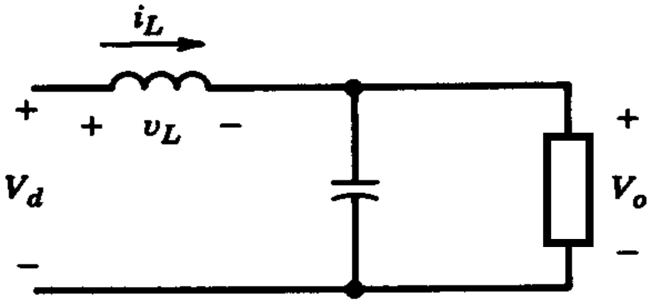
Characteristics of Buck circuits:

1. step-down ;
2. simple, requiring only one *device*;
3. high efficient (>90%);
4. the input current discontinuous and a smoothing input filter normally required;
5. providing only positive output voltage and unidirectional output current.





Mode 1 — on



Mode 2 — off

5-2. Boost Regulators

Continuous-conduction

C1 and C2 are still feasible

Assuming i_L rises linearly by ΔI_L in time t_{on}

Assuming i_L drops linearly by ΔI_L in time t_{off}

$$V_d = L\frac{\Delta I_L}{t_{on}}$$

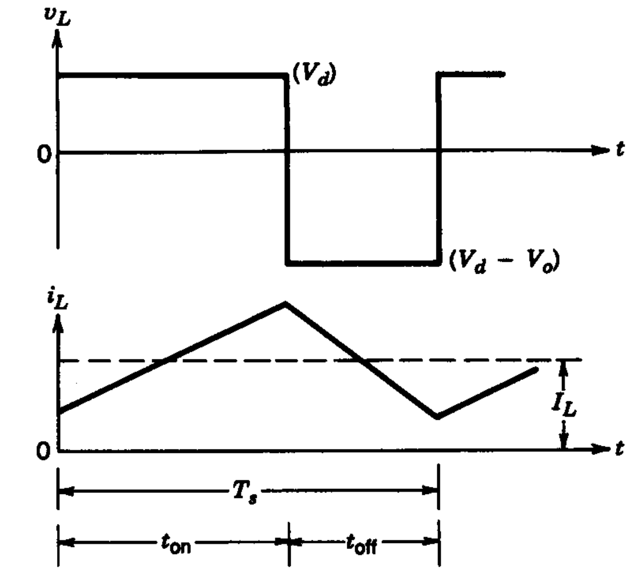
$$V_d - V_o = -L\frac{\Delta I_L}{t_{off}}$$

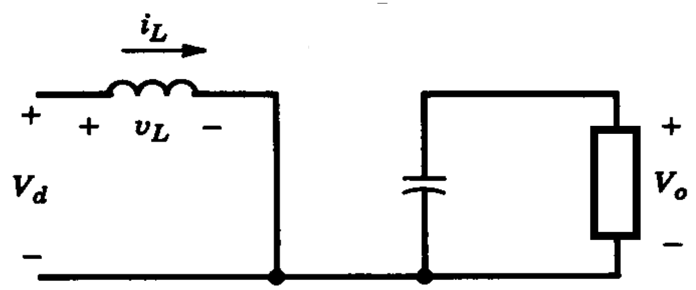
$$V_o = \frac{V_d}{1-D}$$

$$V_d * I_d = V_o * I_o$$

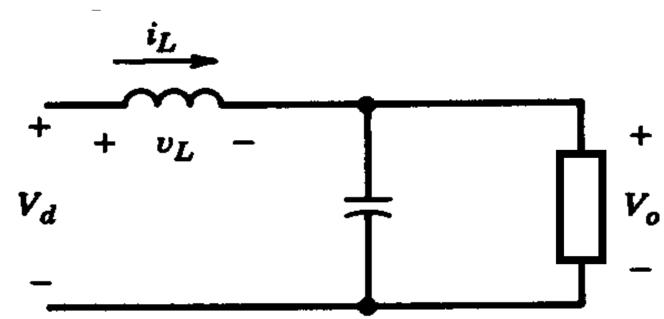
$$I_d = \frac{I_o}{1-D}$$

$$\Delta I_L = \frac{V_d D}{f_s L}$$



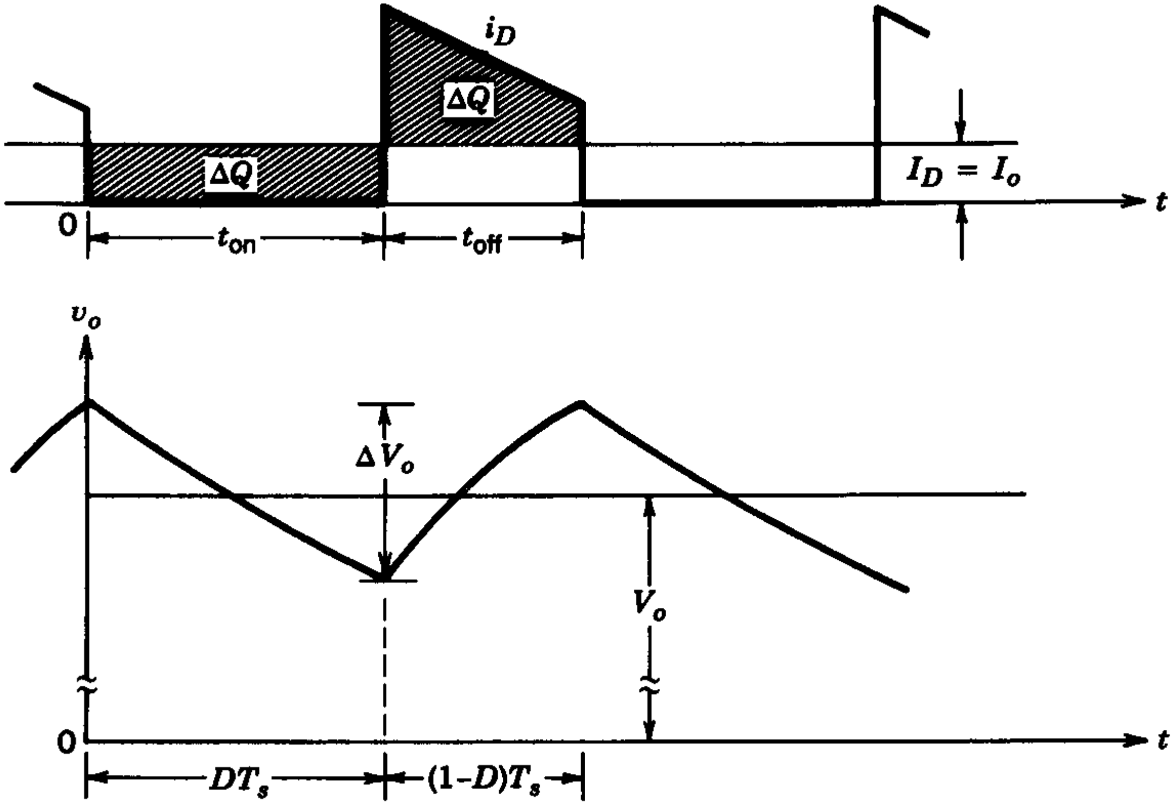


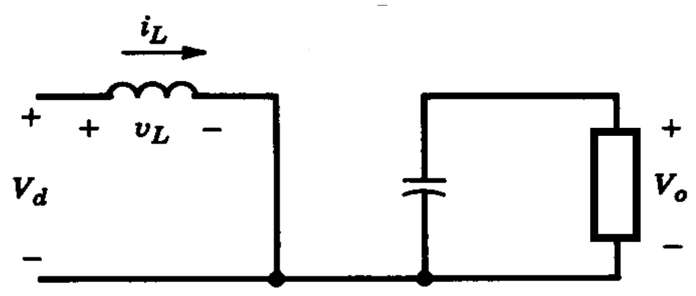
Mode 1 — on



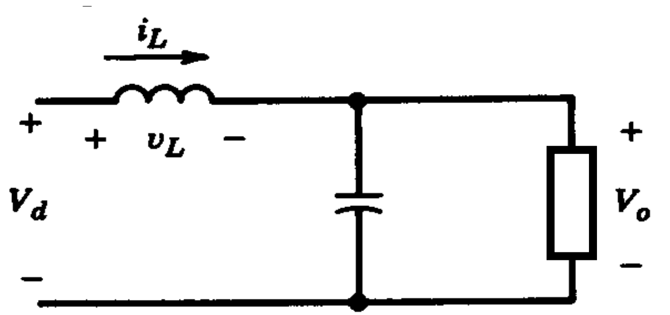
Mode 2 — off

Output Voltage Ripple





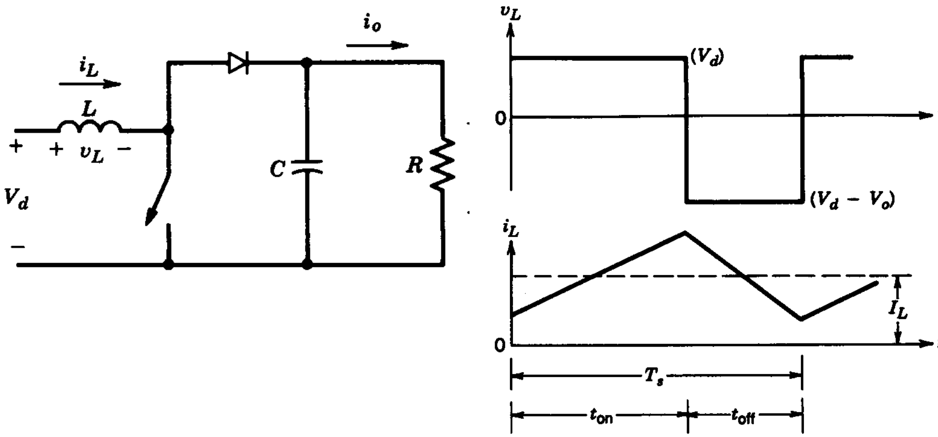
Mode 1 — on



Mode 2 — off

$$\Delta V_o = \frac{I_o D}{f_s C}$$

Discontinuous-conduction

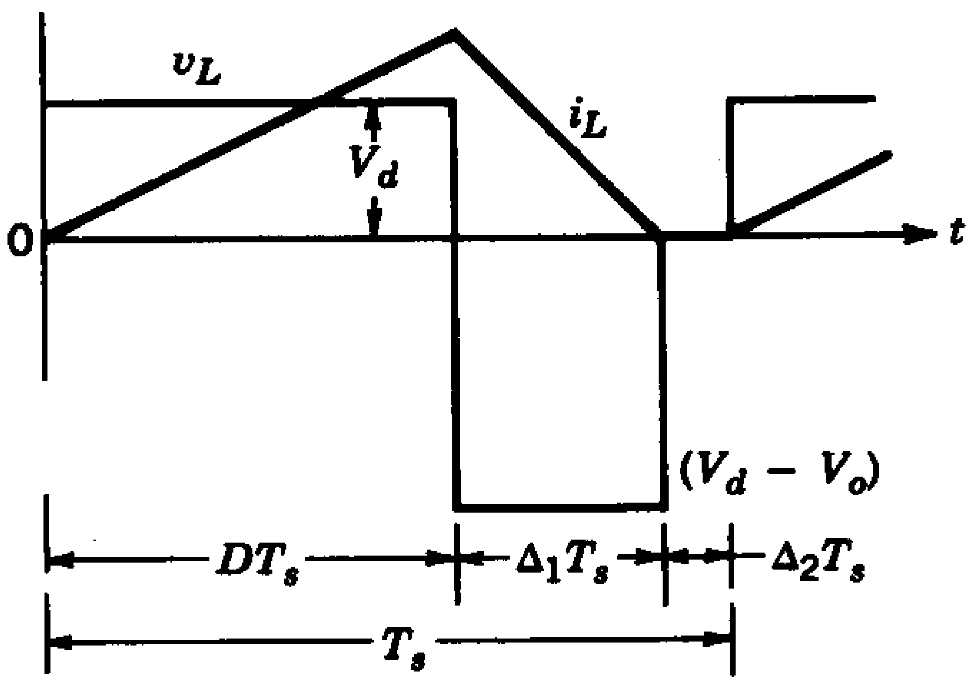


Boundary between continuous and discontinuous

$$I_{LB} = 0.5\Delta I_L$$

$$I_{oB} = I_{LB}(1-D)$$

$$\Delta I_L = \frac{V_d D}{f_s L}$$

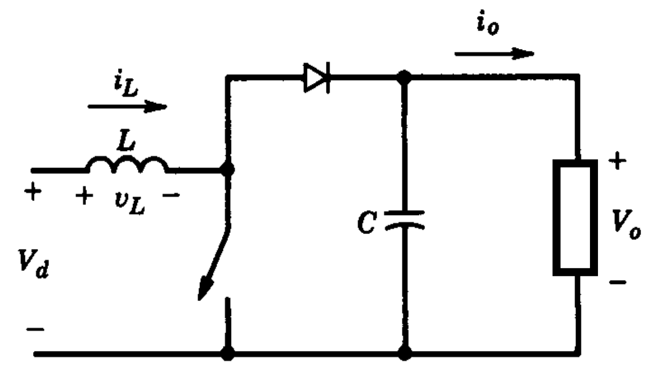


In steady state, the average inductor voltage must be zero.

$$\frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

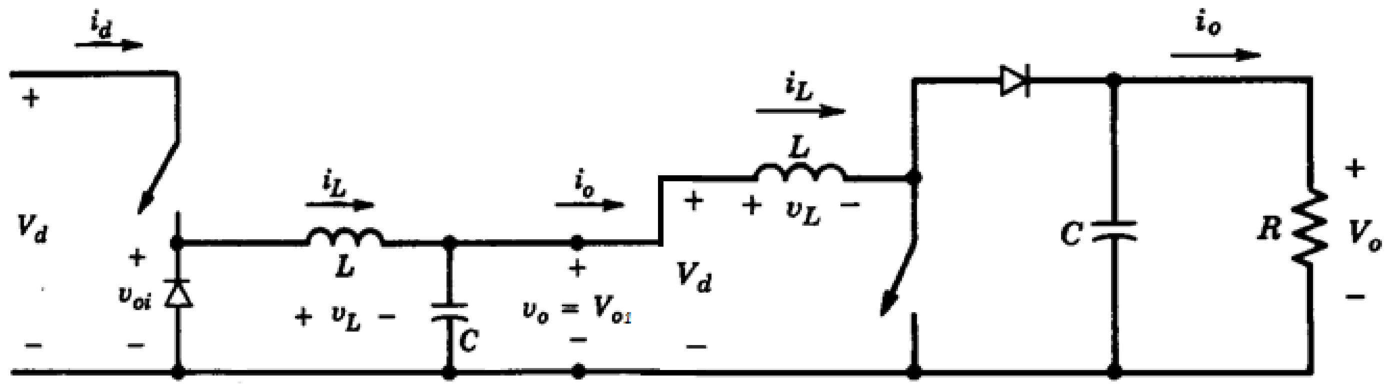
$$\Delta_1 = \frac{2 f_s L}{V_d D} I_o$$

Characteristics of Boost circuits:



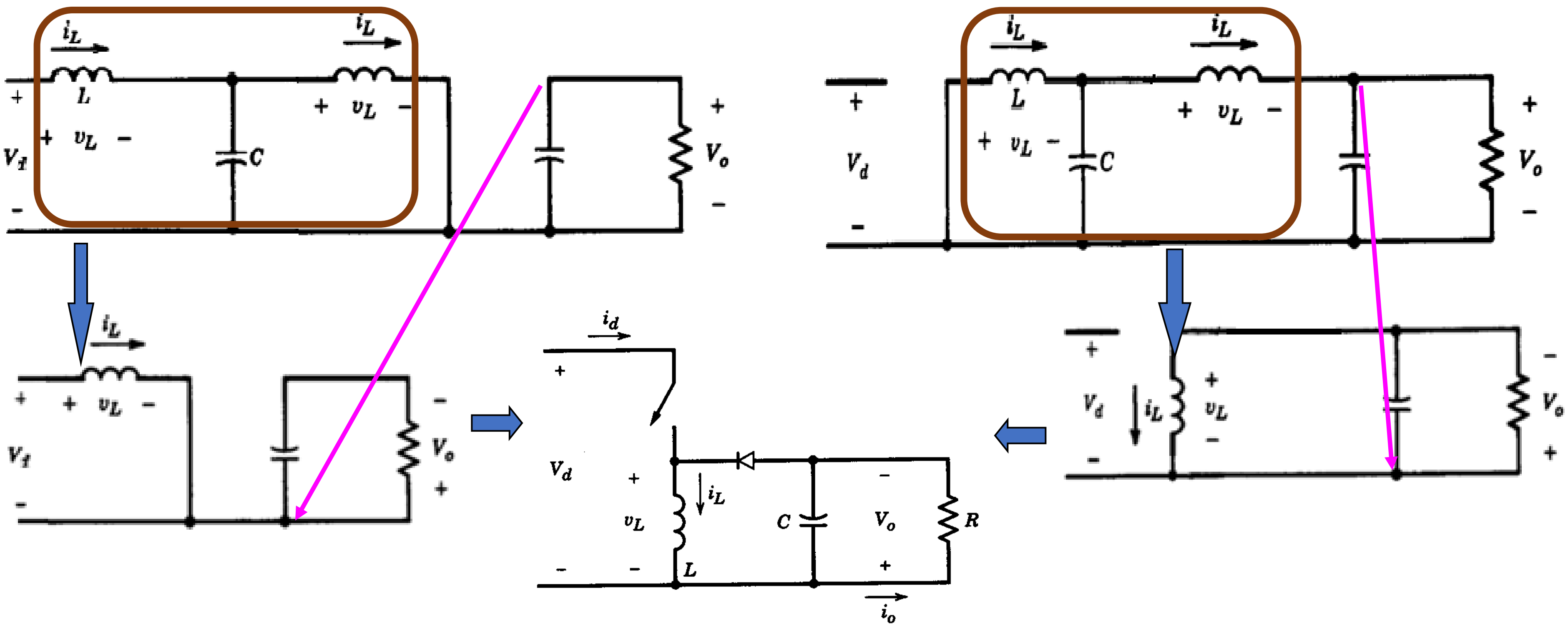
1. step-up;
2. simple, requiring only one device;
3. high efficient;
4. input current continuous;
5. output voltage very sensitive to changes in duty cycle k, making it difficult to stabilize the regulator;
6. average output current less than the average inductor current by a factor of (1-D) and a much higher rms current flowing through the filter capacitor, resulting in the use of a larger filter capacitor than those of a Buck regulator.

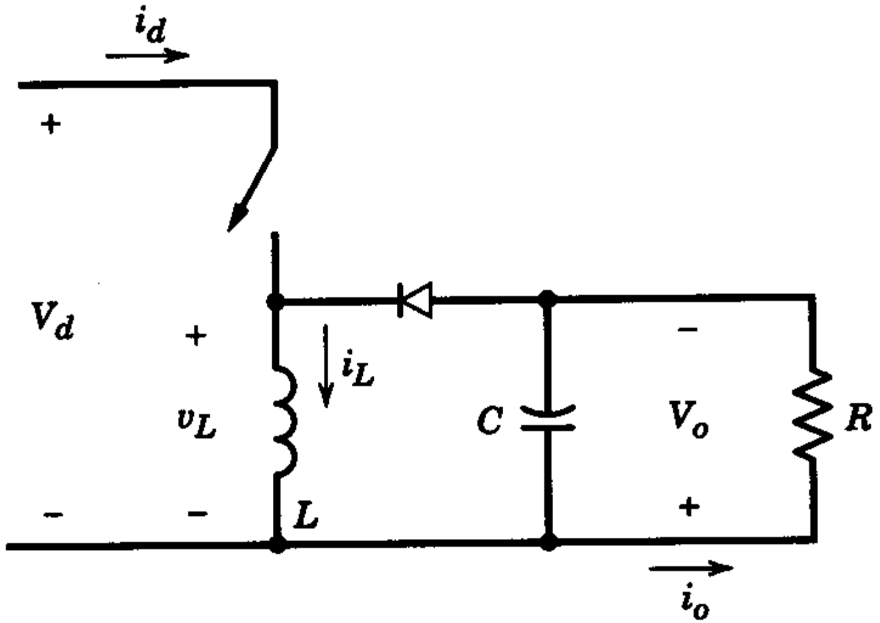
Supplement 4-1 Buck-Boost: Cascade of Buck and Boost

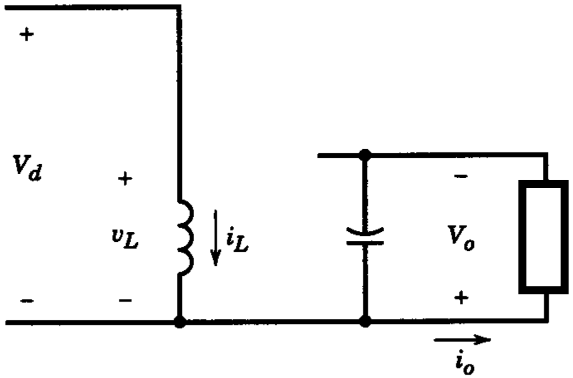


Mode 1 — on

Mode 2 — off

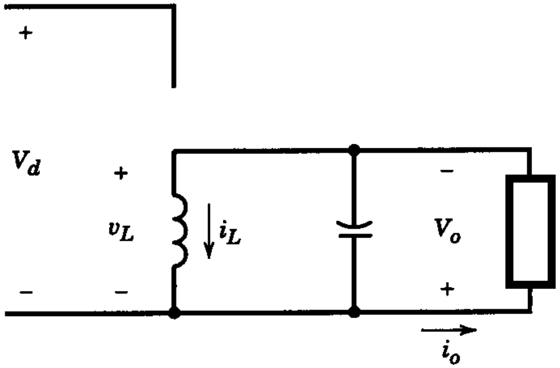






Mode 1 — on

5-3. Buck-Boost Regulators



Mode 2 — off

Continuous-conduction

Assuming i_L rises linearly by ΔI_L in time t_{on}

Assuming i_L drops linearly by ΔI_L in time t_{off}

$$V_d = L\frac{\Delta I_L}{t_{on}}$$

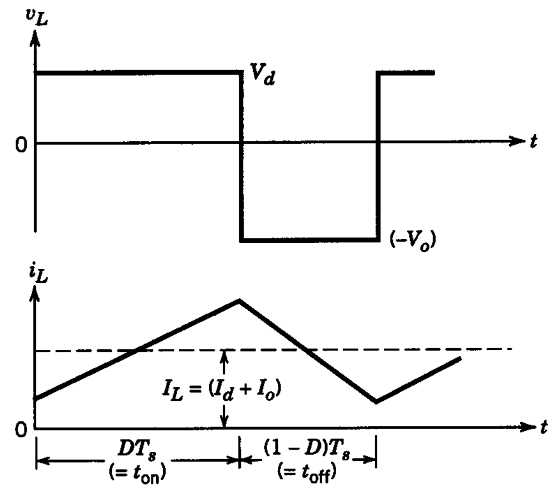
$$V_o = L\frac{\Delta I_L}{t_{off}}$$

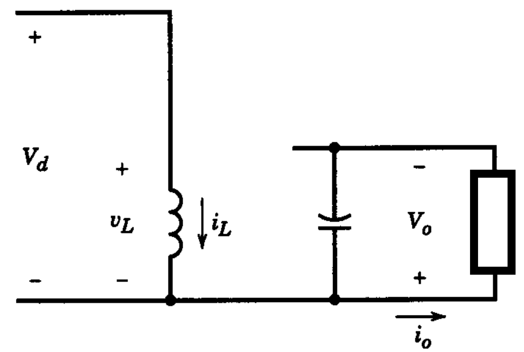
$$V_o = \frac{V_d D}{1-D}$$

$$V_d * I_d = V_o * I_o$$

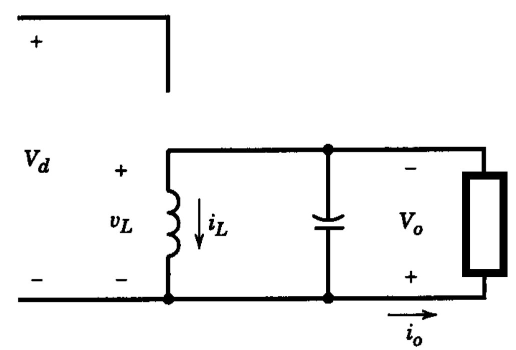
$$I_d = \frac{I_o D}{1-D}$$

$$\Delta I_L = \frac{V_d D}{f_s L}$$



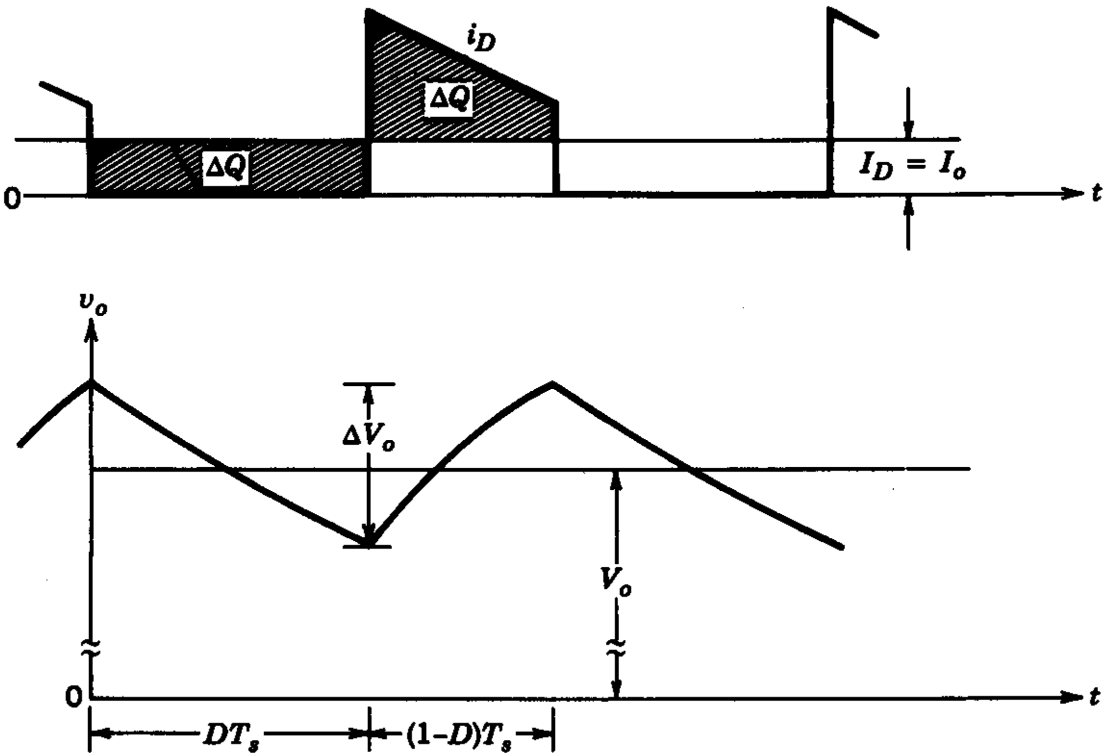


Mode 1 — on

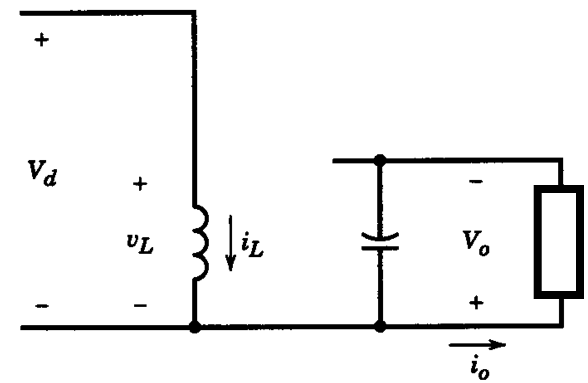


Mode 2 — off

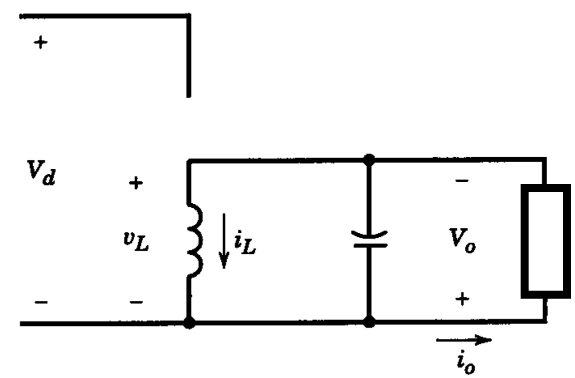
Output Voltage Ripple



$$\Delta V_o = \frac{I_o D}{f_s C}$$

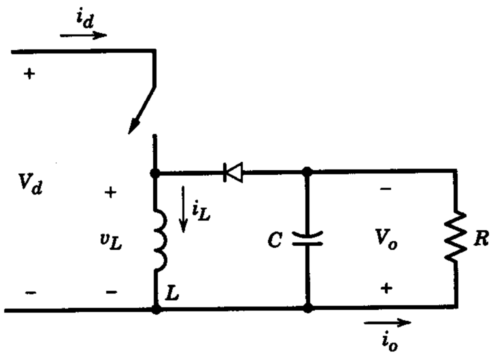


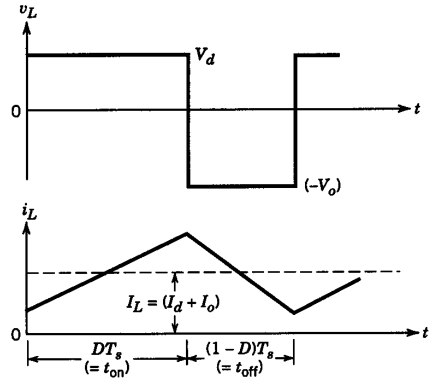
Mode 1 — on



Mode 2 — off

Discontinuous-conduction



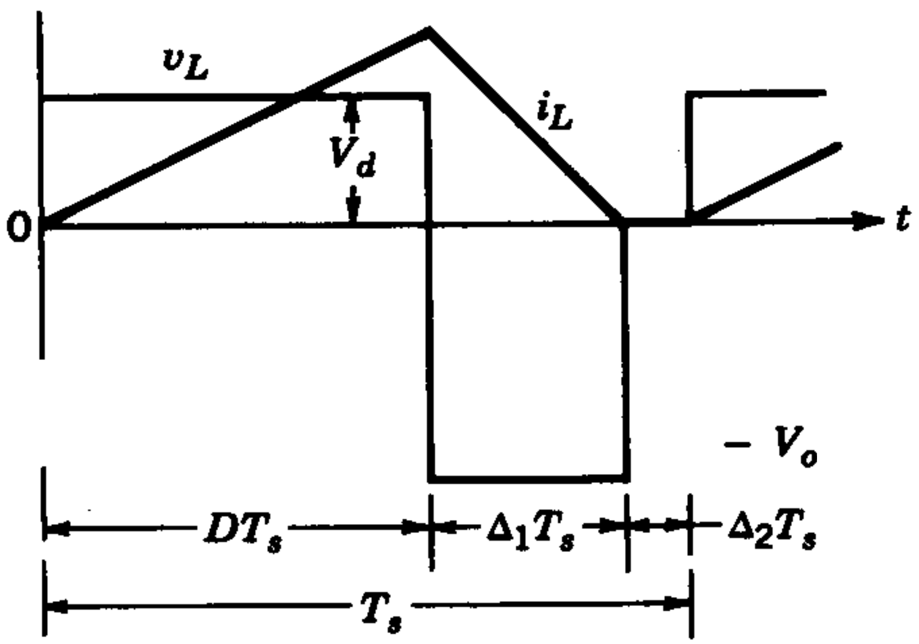


Boundary between continuous and discontinuous

$$I_{LB} = 0.5\Delta I_L$$

$$I_{oB} = I_{LB}(1\text{-}D)$$

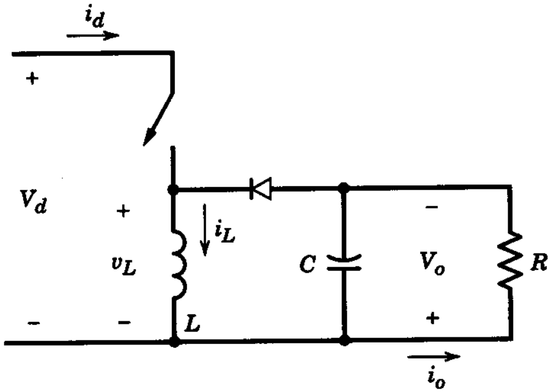
$$\Delta I_L = \frac{V_d D}{f_s L}$$



In steady state, the average inductor voltage must be zero.

$$\frac{V_o}{V_d} = \frac{D}{\Delta_1}$$

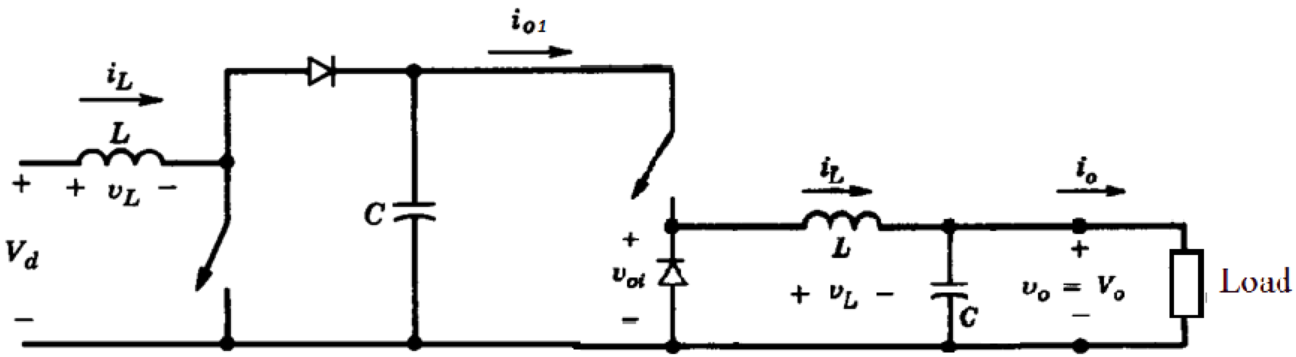
$$\Delta_1 = \frac{2 f_s L}{V_d D} I_o$$



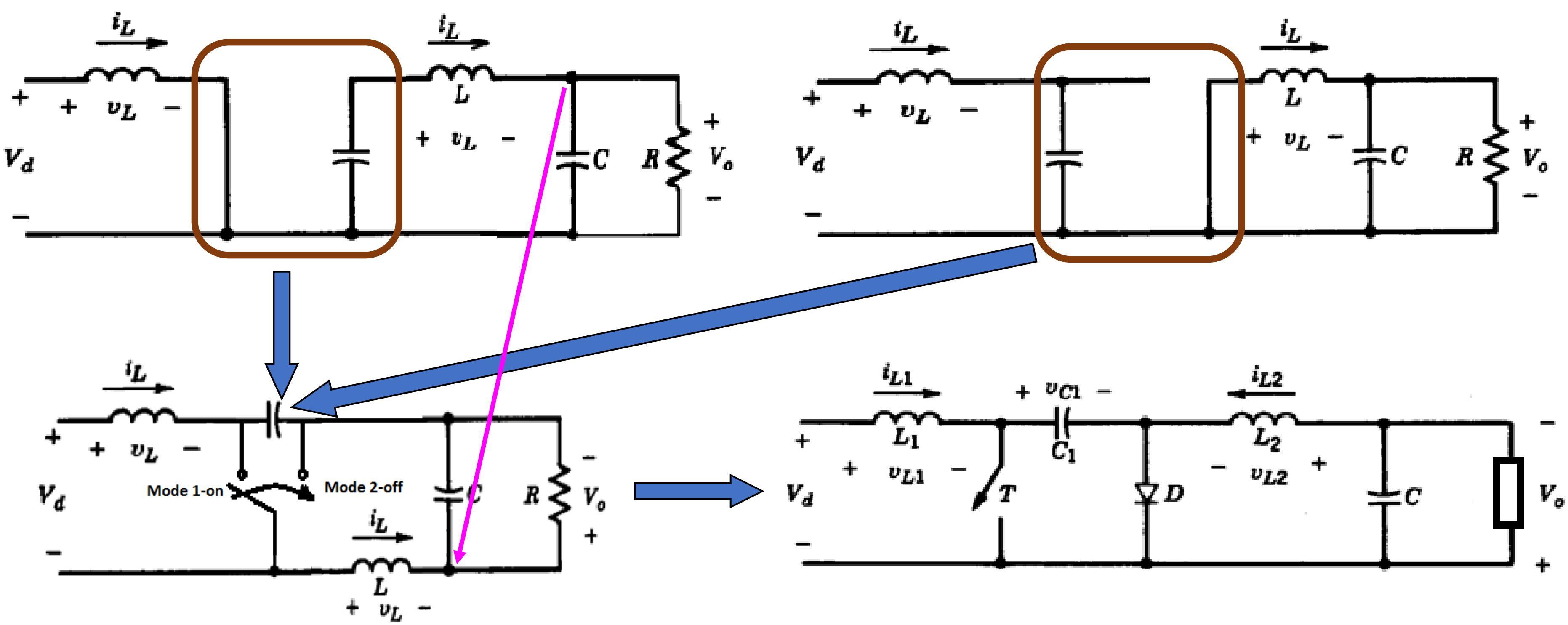
Characteristics of Buck-Boost circuits:

1. step-down or –up;
2. simple, requiring only one device;
3. high efficient;
4. polarity of output voltage reversed;
5. the input current discontinuous and a high peak current flowing through the device.

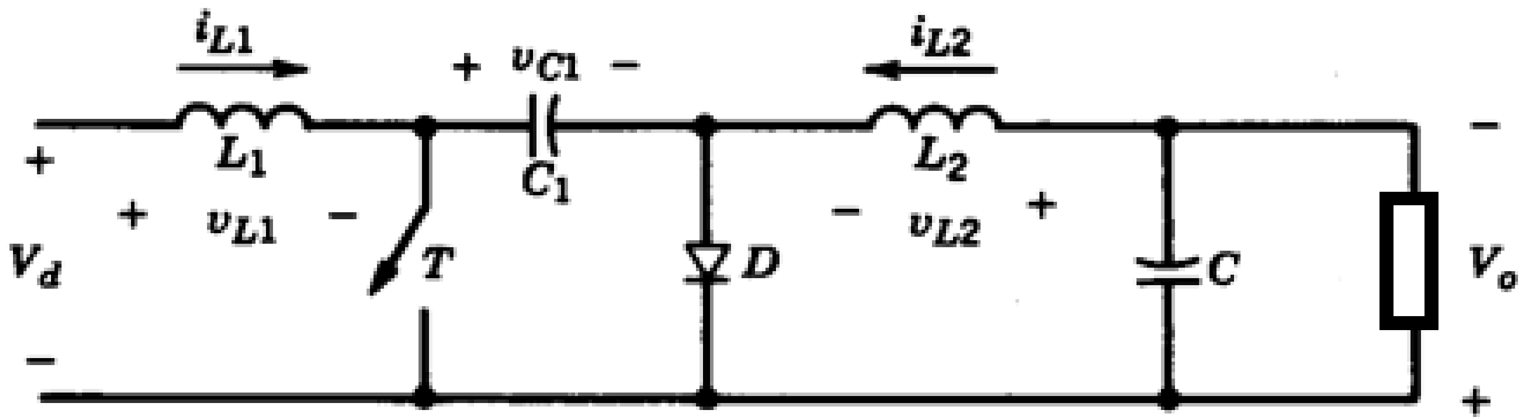
Supplement 4-2 Cuk: Cascade of Boost and Buck

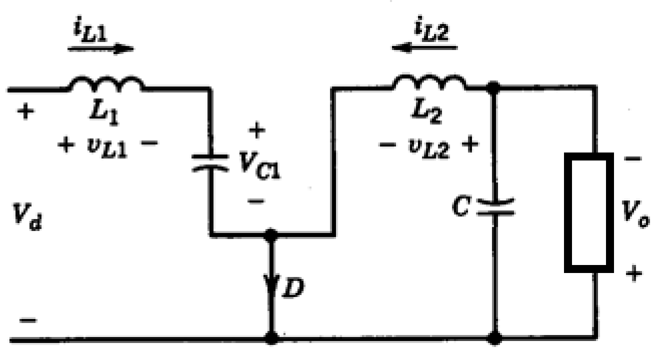


Mode 1 — on Mode 2 — off

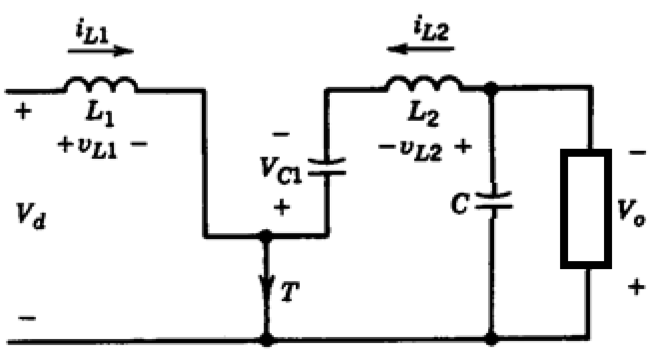


5-4. Cuk Regulators





Mode 1 — off



Mode 2 — on

Continuous-conduction

$$V_d - V_{c1} = -L_1 \frac{\Delta I_{L1}}{t_{off}}$$

$$V_d = L_1 \frac{\Delta I_{L1}}{t_{on}}$$

$$V_o = L_2 \frac{\Delta I_{L2}}{t_{off}}$$

$$V_{c1} - V_o = L_2 \frac{\Delta I_{L2}}{t_{on}}$$

$$\mathrm{V_{c1}} = \frac{\mathrm{V_d}}{1-\mathrm{D}}$$

$$\mathrm{V_{c1}} = \frac{\mathrm{V_o}}{\mathrm{D}}$$

$$V_o = \frac{DV_d}{1-D}$$

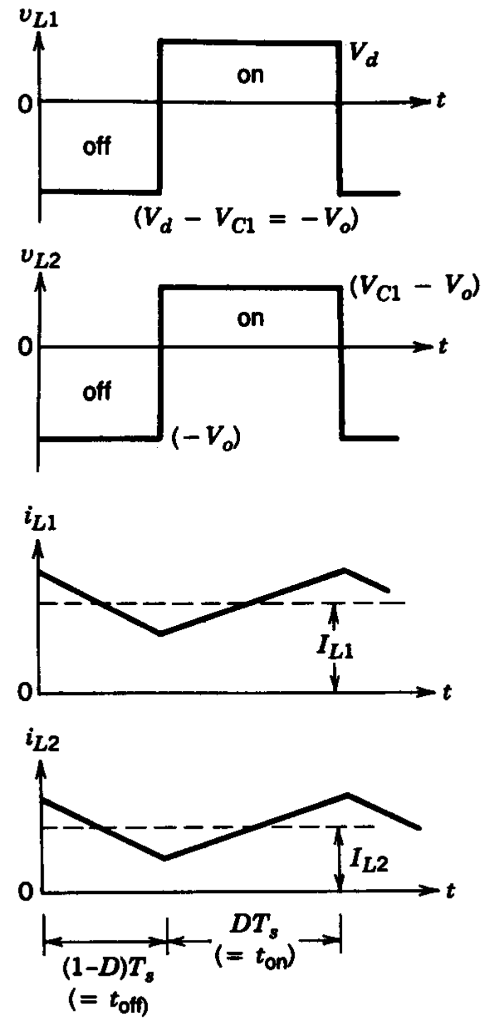
$$I_d = \frac{DI_o}{1-D}$$

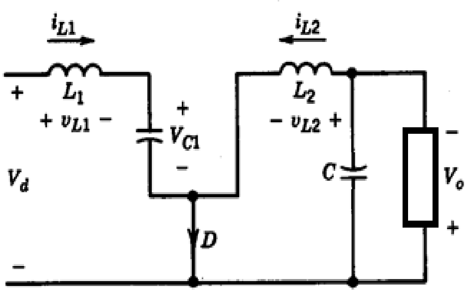
$$\Delta I_{L1} = \frac{DV_d}{f_s L_1}$$

$$\Delta I_{L2} = \frac{DV_d}{f_s L_2}$$

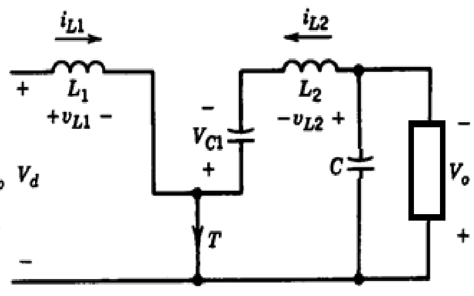
$$\Delta V_{c1} = \frac{I_d(1-D)}{f_s C_1}$$

$$\Delta V_{c2} = \frac{DV_d}{8 C_2 L_2 f_s^2}$$

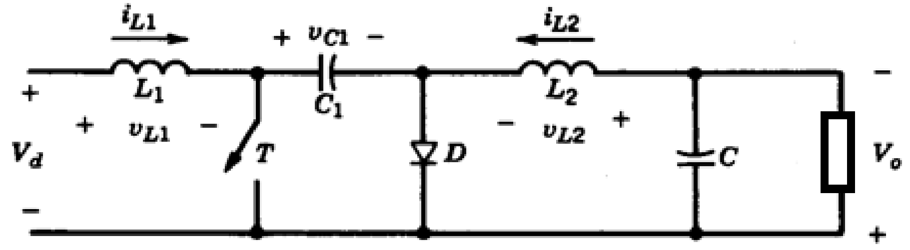




Model 1 off



Model 2 on



Characteristics of Cuk circuits:

1. step-down or –up;
2. requiring only one device;
3. high efficient;
4. input current continuous;
5. ripple current of C_1 high;
6. additional capacitor and inductor.

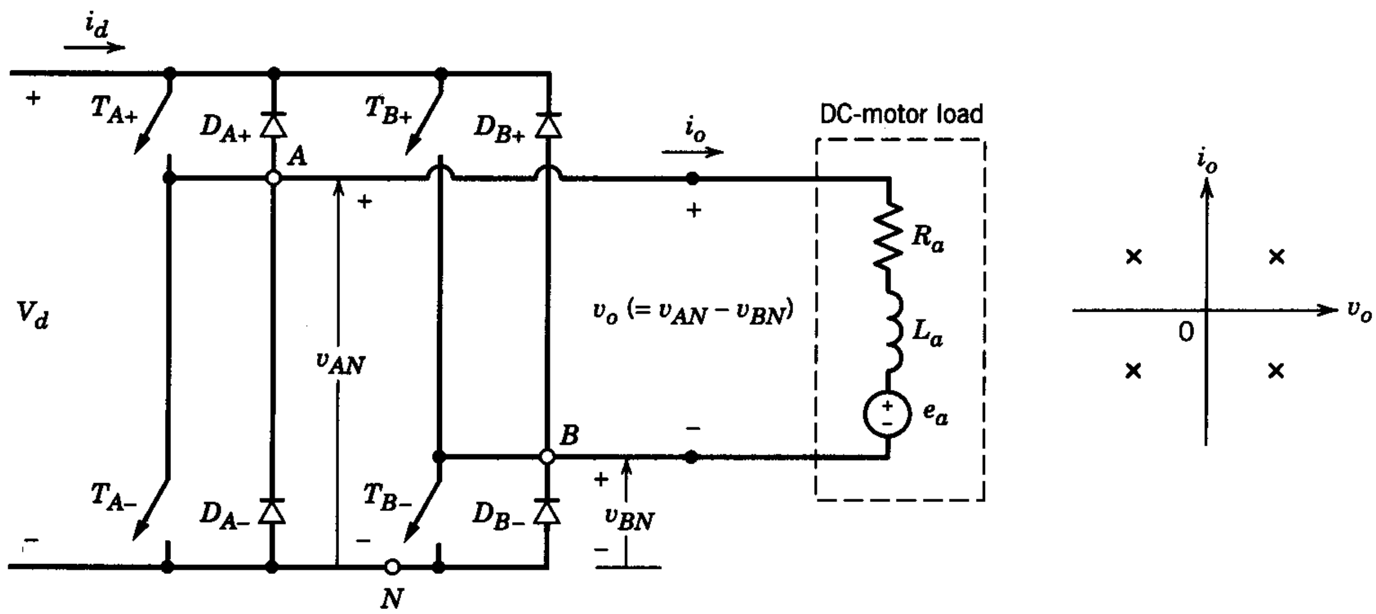
Summary:

- Buck (step-down), Boost (step-up), Buck-Boost (step-up & -down), Cuk (Boost-Buck, step-up & -down)
- Topologies of these DC choppers and circuits in different modes
- Key waveforms and calculations based on two rules

> ***DC components of input/output voltage/current***
> ***DC components of capacitor voltage and inductor current***
> ***Ripples of capacitor voltage and inductor current***

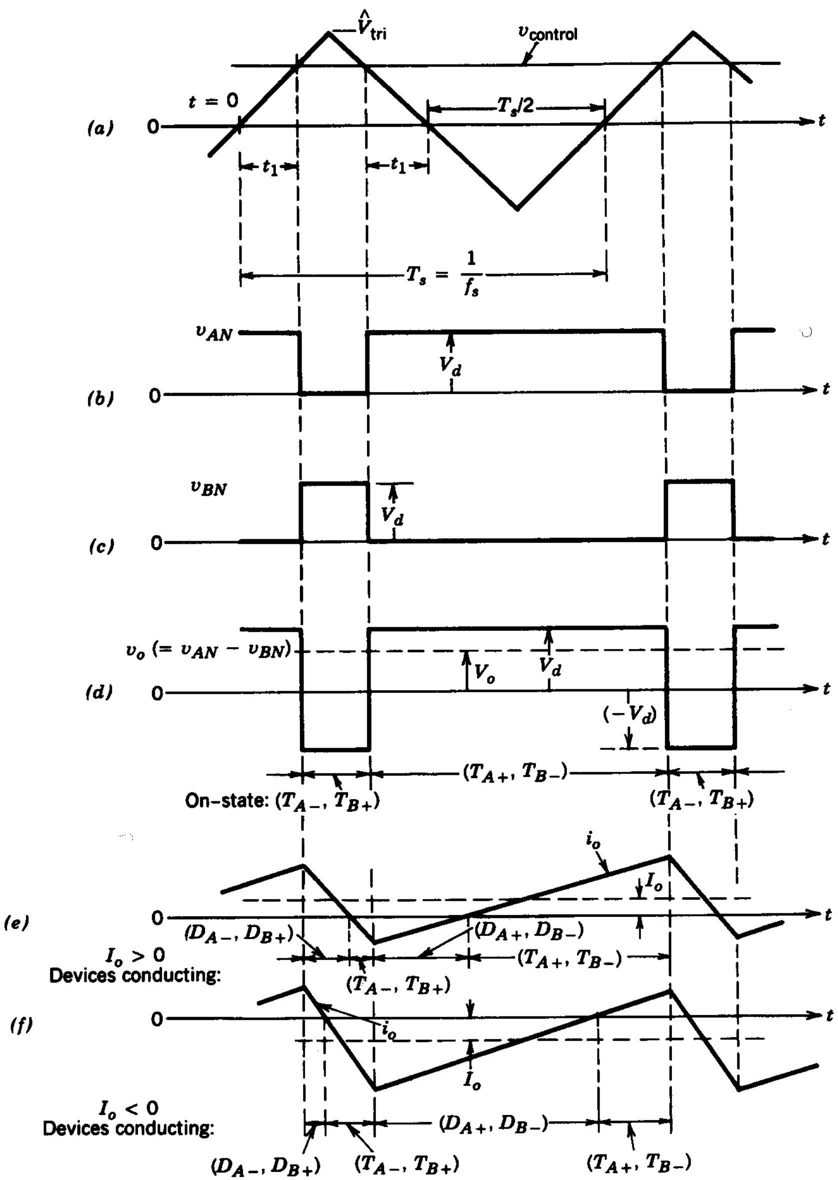
- Design of DC choppers
- Difference between continuous- and discontinuous-conduction

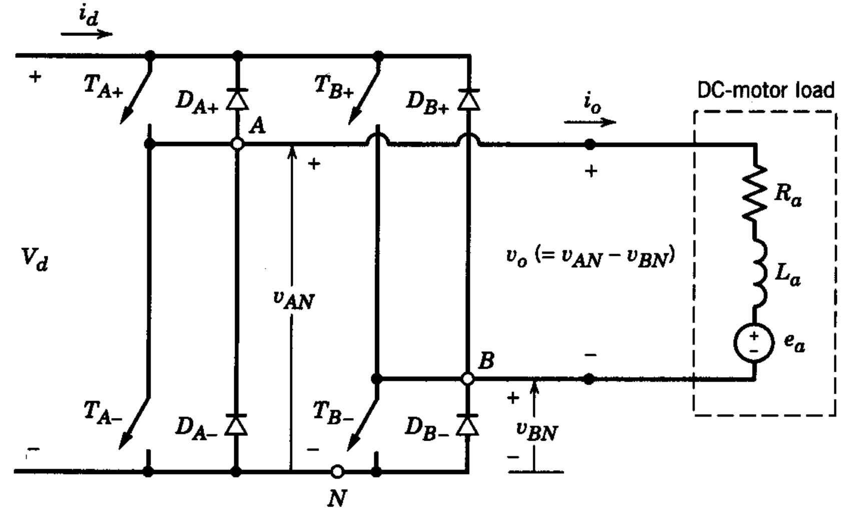
5-5. Full-bridge Regulators



Four quadrant operation is possible.

PWM
with Bipolar Voltage Switching





$v_{control} > v_{tri}$

T_{A+}, T_{B-} on: $v_{AN} = V_d$, $v_{BN} = 0$;

(T_{A-}, T_{B+} off) $v_o = V_d$

$i_o > 0$ T_{A+}, T_{B-} conduct; $i_d = i_o$

$i_o < 0$ D_{A+}, D_{B-} conduct; $i_d = i_o$

$v_{control} < v_{tri}$

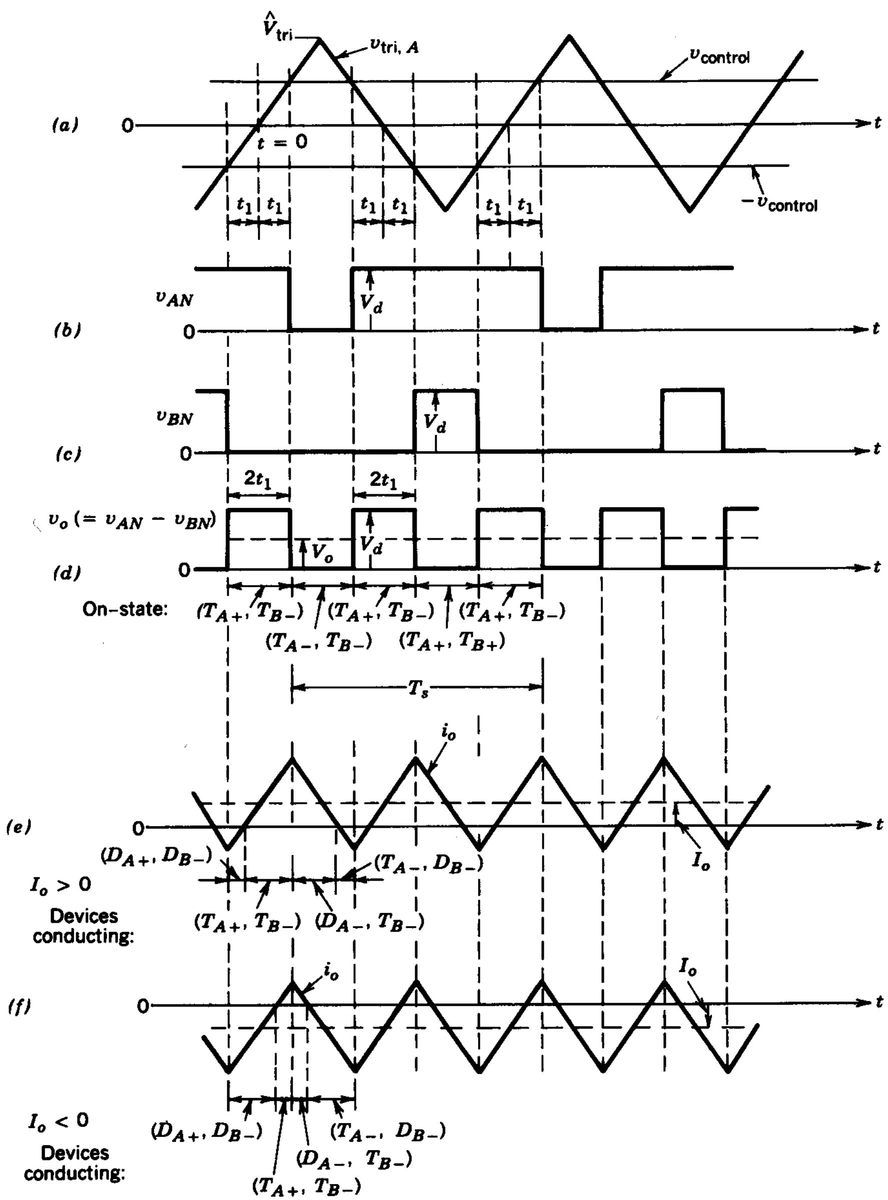
T_{A-}, T_{B+} on: $v_{AN} = 0$, $v_{BN} = V_d$;

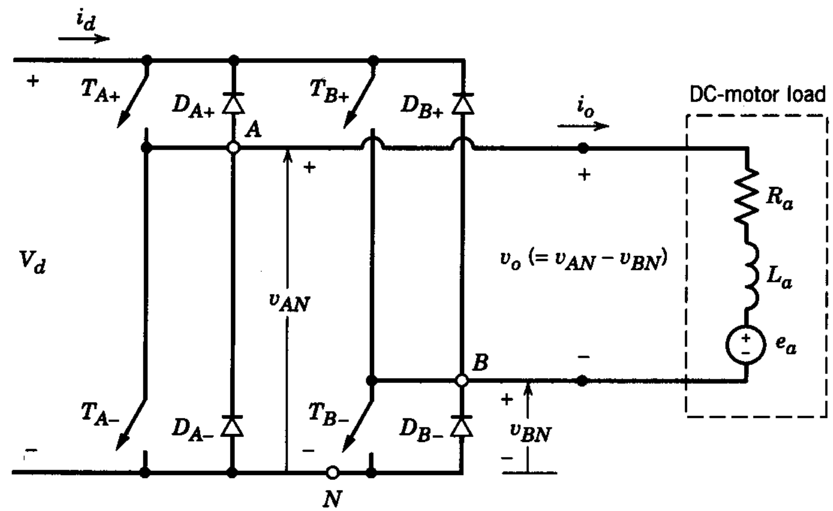
(T_{A+}, T_{B-} off) $v_o = -V_d$

$i_o < 0$ T_{A-}, T_{B+} conduct; $i_d = -i_o$

$i_o > 0$ D_{A-}, D_{B+} conduct; $i_d = -i_o$

PWM
with Unipolar Voltage Switching





Leg A:

$v_{control} > v_{tri}$ T_{A+} on (T_{A-} off) $v_{AN} = V_d$;

$v_{control} < v_{tri}$ T_{A-} on (T_{A+} off) $v_{AN} = 0$;

Leg B:

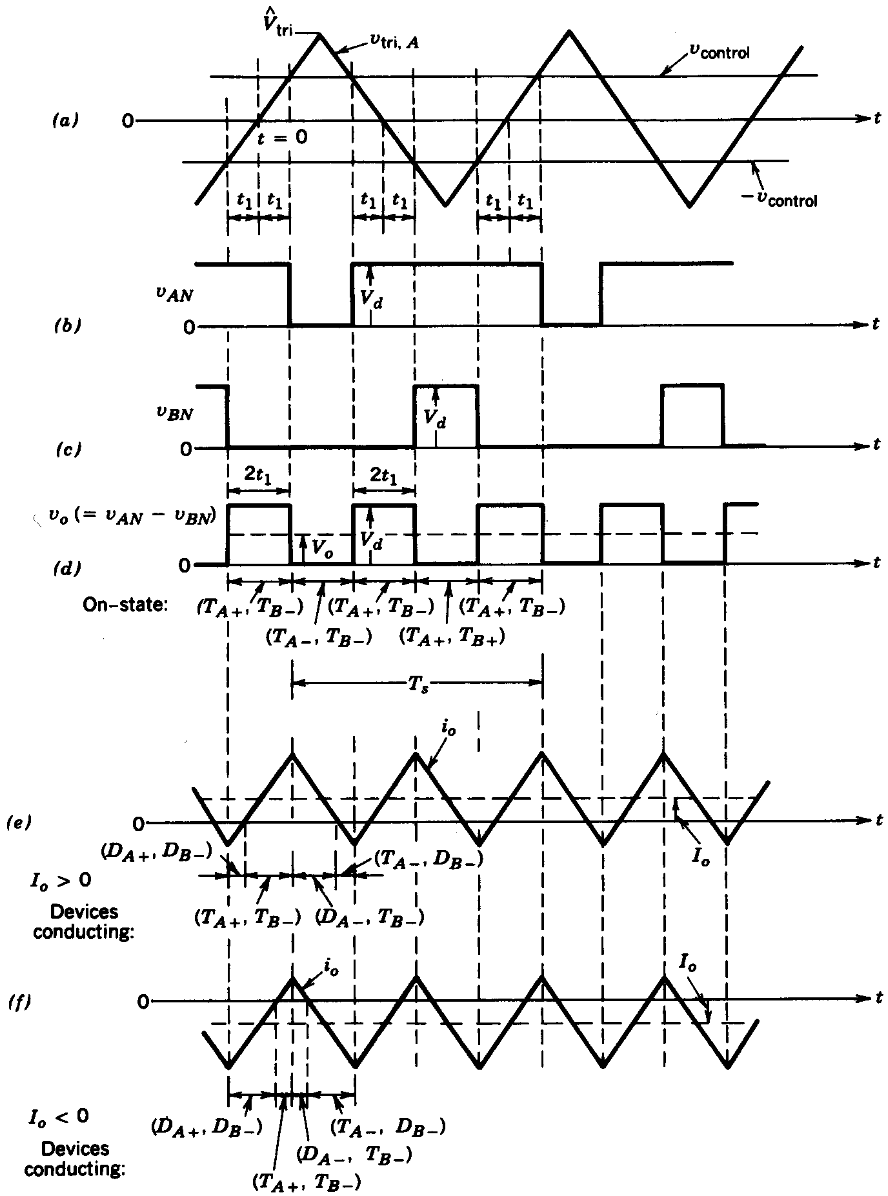
$-v_{control} > v_{tri}$ T_{B+} on (T_{B-} off) $v_{BN} = V_d$;

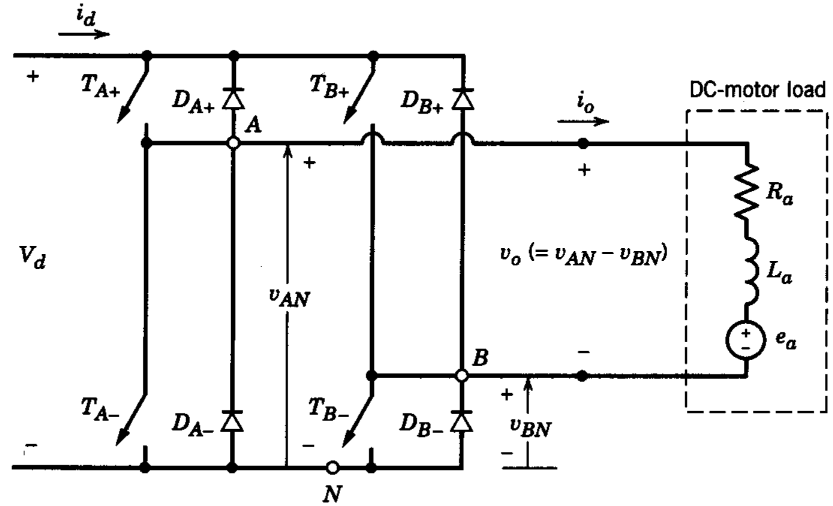
$-v_{control} < v_{tri}$ T_{B-} on (T_{B+} off) $v_{BN} = 0$;

$$v_o = v_{AN} - v_{BN}$$

“effectively” doubling f_s

PWM
with Unipolar Voltage Switching





Leg A:

T_{A+} on: $\boldsymbol{i_o} > 0$ T_{A+} conducts; $\boldsymbol{i_{d1}} = \boldsymbol{i_o}$

$\boldsymbol{i_o} < 0$ D_{A+} conducts; $\boldsymbol{i_{d1}} = \boldsymbol{i_o}$

T_{A-} on: $\boldsymbol{i_o} < 0$ T_{A-} conducts; $\boldsymbol{i_{d1}} = -\boldsymbol{i_o}$

$\boldsymbol{i_o} > 0$ D_{A-} conducts; $\boldsymbol{i_{d1}} = -\boldsymbol{i_o}$

Leg B:

T_{B+} on: $\boldsymbol{i_o} < 0$ T_{B+} conducts; $\boldsymbol{i_{d2}} = -\boldsymbol{i_o}$

$\boldsymbol{i_o} > 0$ D_{B+} conducts; $\boldsymbol{i_{d2}} = -\boldsymbol{i_o}$

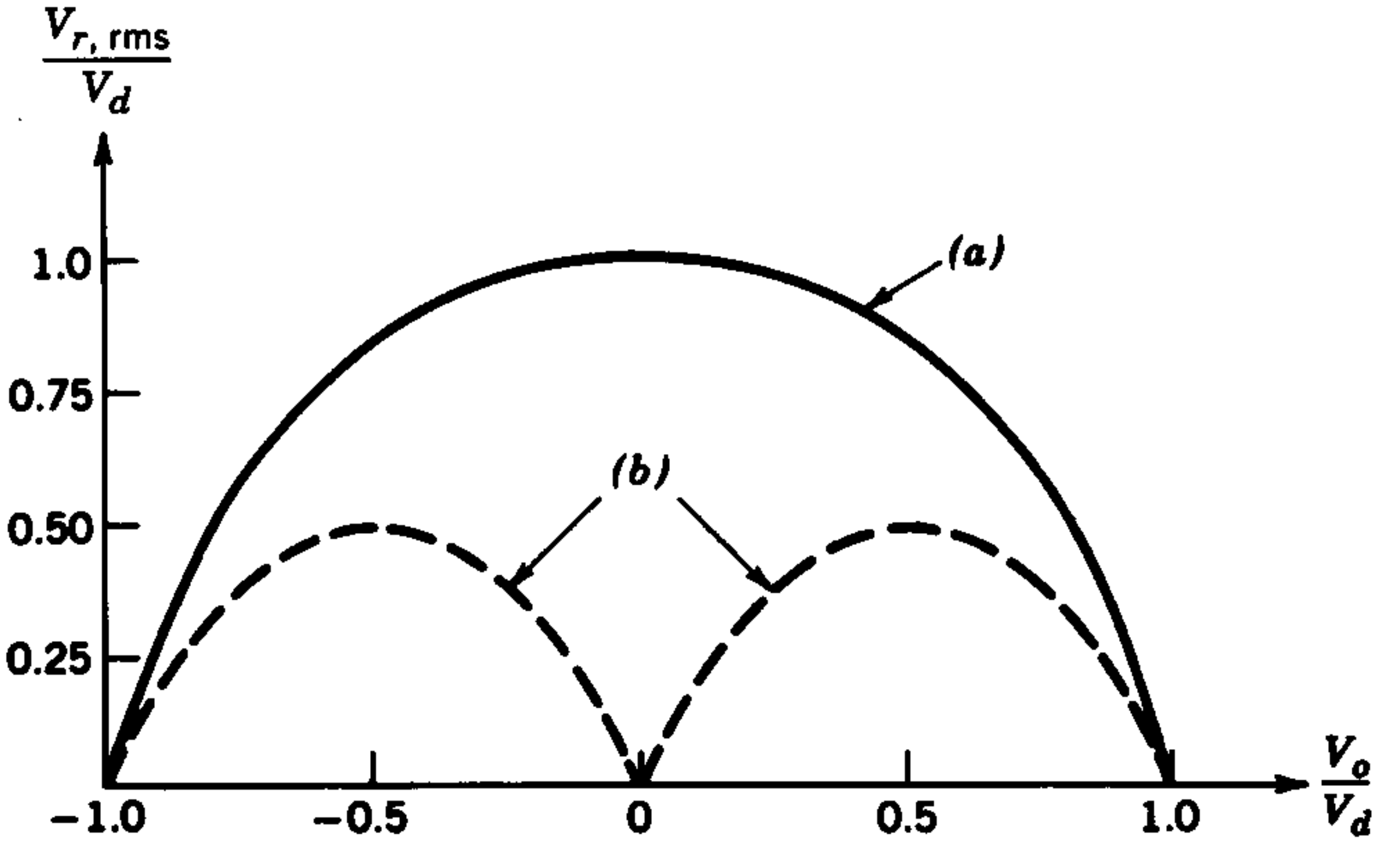
T_{B-} on: $\boldsymbol{i_o} > 0$ T_{B-} conducts; $\boldsymbol{i_{d2}} = \boldsymbol{i_o}$

$\boldsymbol{i_o} < 0$ D_{B-} conducts; $\boldsymbol{i_{d2}} = \boldsymbol{i_o}$

$$\boldsymbol{i_d} = \boldsymbol{i_{d1}} + \boldsymbol{i_{d2}}$$

Output Ripple of Full-bridge Regulators

using PWM with (a) bipolar and (b) unipolar voltage switching



Supplement 4-3 Simulation of the Cuk by Matlab

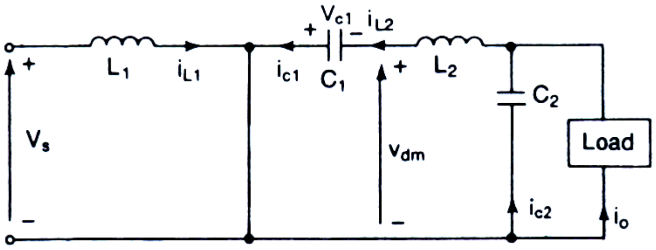
The working procedure of the Cuk regulators can be represented by two modes:

- Mode 1

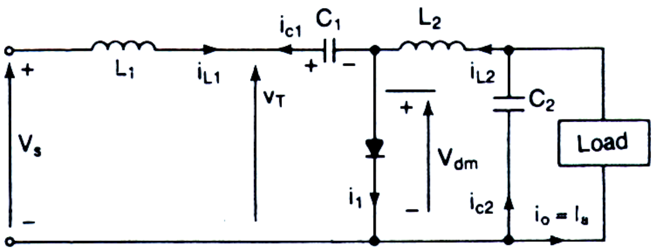
$$\begin{cases} \dfrac{di_{L_1}}{dt} = \dfrac{V_s}{L_1} \\ \dfrac{di_{L_2}}{dt} = \dfrac{1}{L_2}(v_{c1} - v_{c2}) \\ \dfrac{dv_{c1}}{dt} = -\dfrac{i_{L_2}}{C_1} \\ \dfrac{dv_{c2}}{dt} = \dfrac{1}{C_2}(\dfrac{v_{c2}}{R} - i_{L_2}) \end{cases}$$

- Mode 2

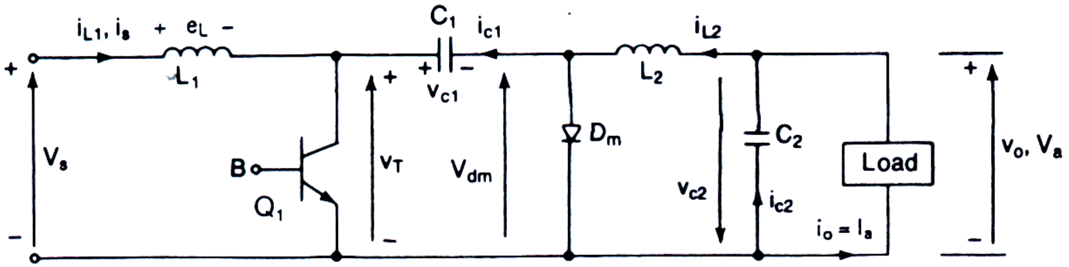
$$\begin{cases} \dfrac{di_{L_1}}{dt} = \dfrac{1}{L_1}(V_s - v_{c1}) \\ \dfrac{di_{L_2}}{dt} = \dfrac{v_{c2}}{L_2} \\ \dfrac{dv_{c1}}{dt} = \dfrac{i_{L_1}}{C_1} \\ \dfrac{dv_{c2}}{dt} = \dfrac{1}{C_2}(\dfrac{v_{c2}}{R} - i_{L_2}) \end{cases}$$



Mode 1



Mode 2



The load is assumed as a pure resistor R.

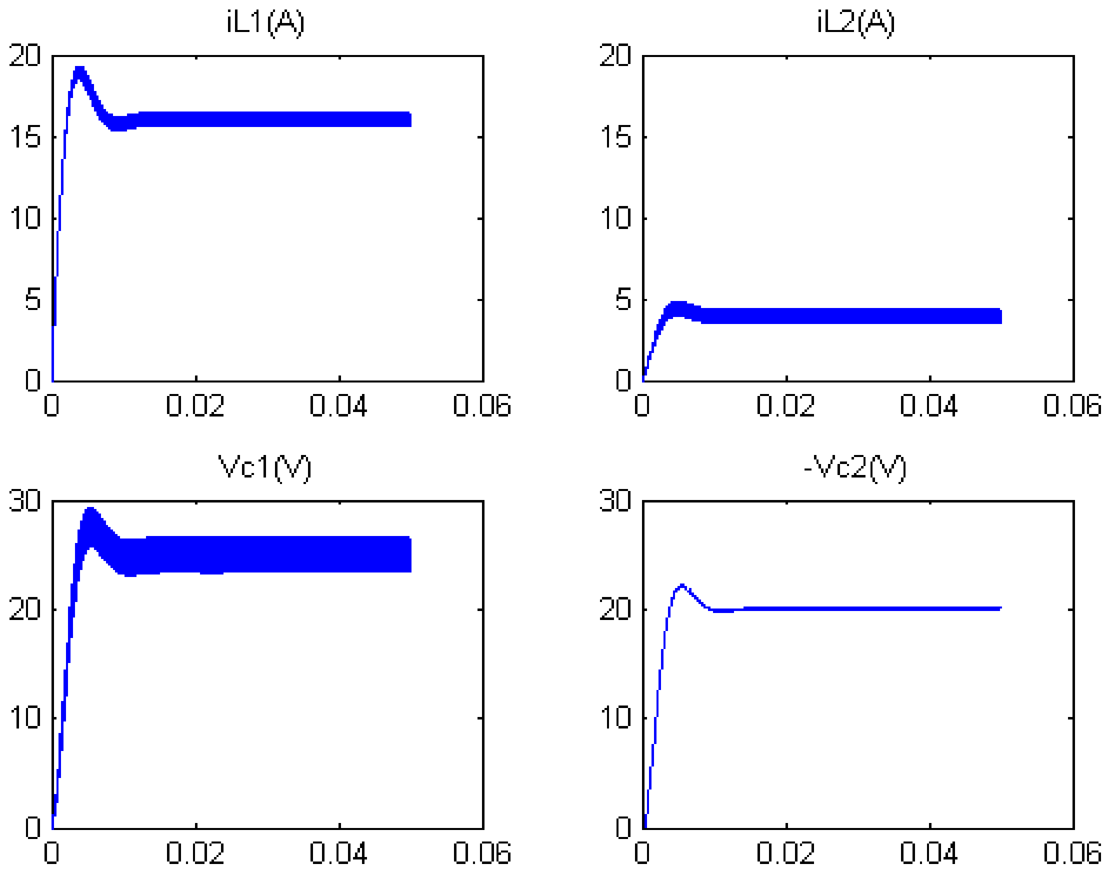
Simulation Parameters:

V_s=5V, C_1=C_2=100uF， L_1=L_2=0.5mH;

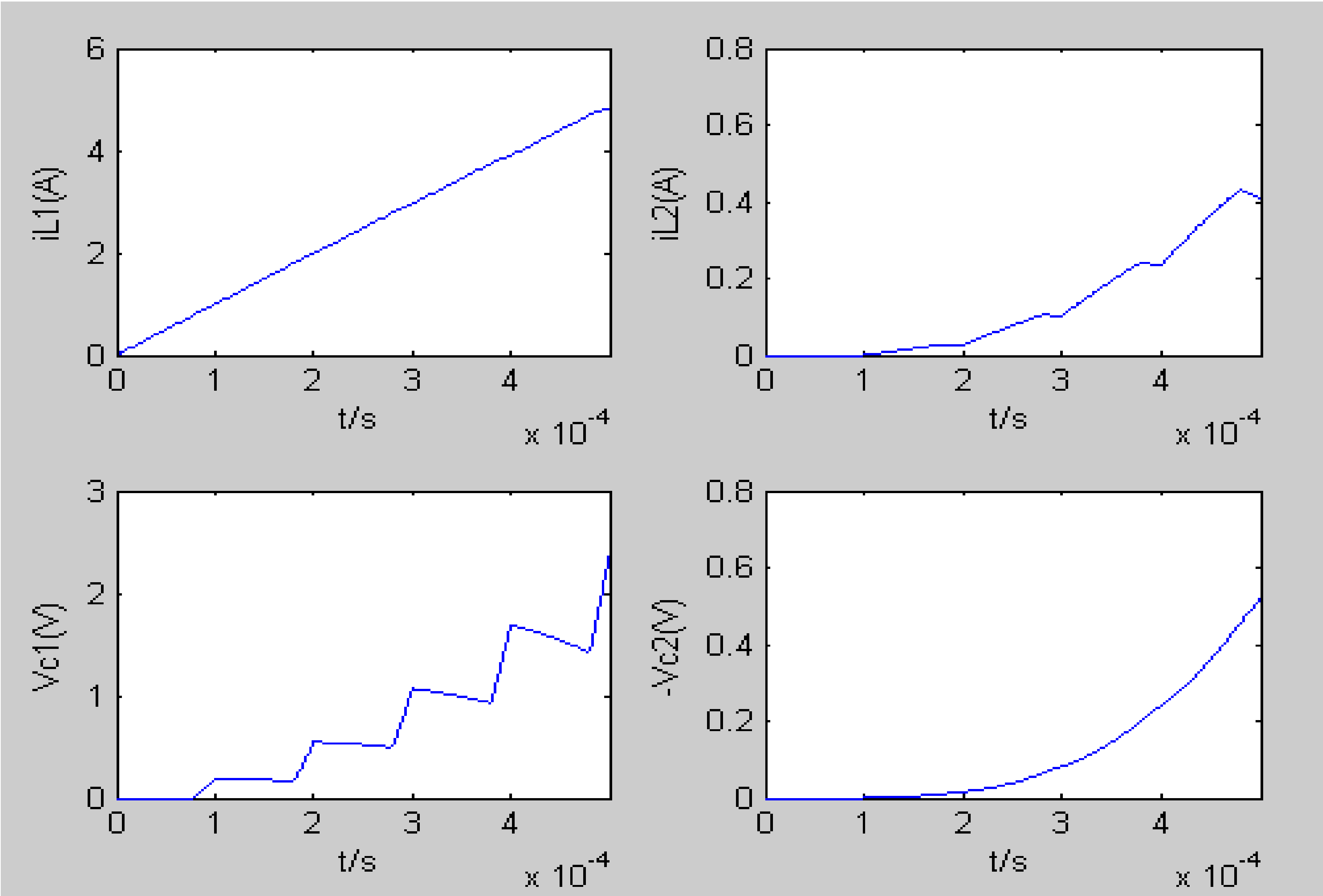
R_{load}=5Ω;

f_s=10kHz， D=0.8。

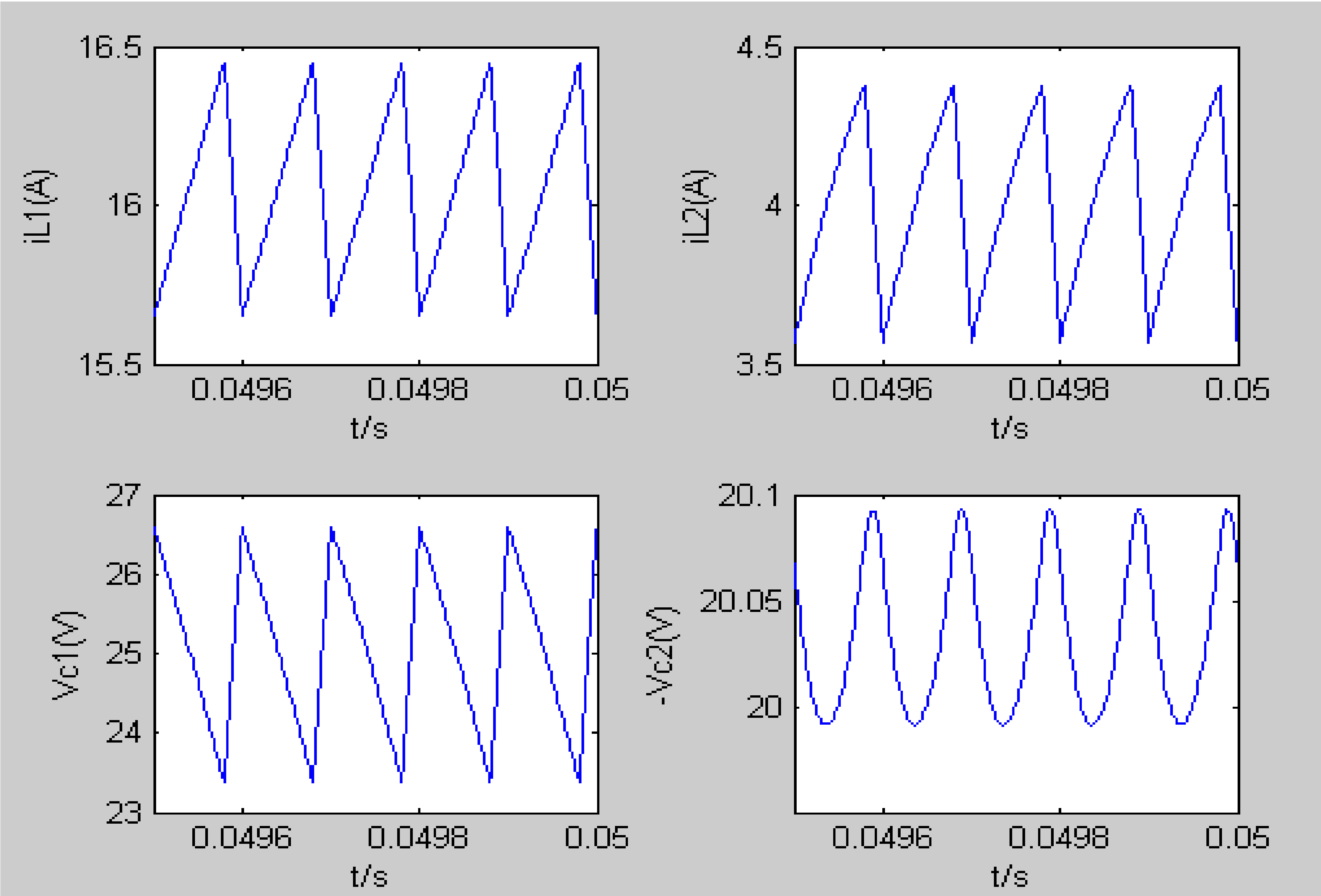
The functions used to solve the state equations in Matlab: ode23, ode45



The first five cycles



Five cycles in steady state



The End