10.(1) 判断能控性

$$S=\begin{bmatrix}0 & 1\\ 1 & 2\end{bmatrix} \quad \mathrm{rank}[S]=2 \text{ 故能控}$$

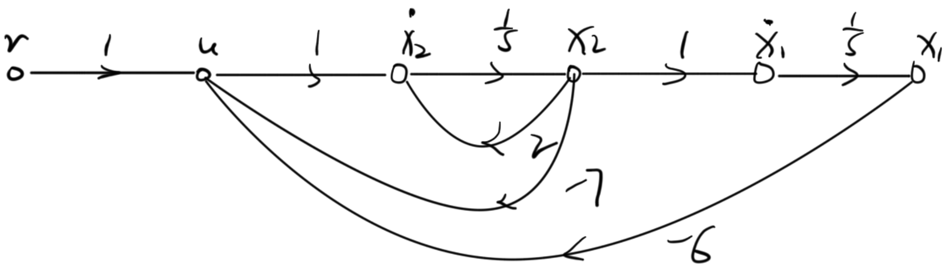
设反馈 $k^T=[k_1 \ \ k_2]$

则有 $\dot{x}=(A-bk^T)x+br=\begin{bmatrix}0 & 1\\ -k_1 & 2-k_2\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}r$

$$\det(sI-A+bk^T)=\begin{vmatrix}s & -1\\ k_1 & s+k_2-2\end{vmatrix}=s^2+(k_2-2)s+k_1$$

期望特征多项式为 $(s+2)(s+3)=s^2+5s+6$ 得 $k_2=7$ $k_1=6$ 即 $k^T=[6 \ \ 7]$

状态反馈后的状态图为



11.(2) 先判断能观性, $V=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\mathrm{rank}[V]=2$ 能观, 观测器极点可任意配置

令 $g=\begin{bmatrix}g_1\\ g_2\end{bmatrix}$ 则 $A-gc^T=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}-\begin{bmatrix}g_1\\ g_2\end{bmatrix}[1 \ \ 0]=\begin{bmatrix}-g_1 & 1\\ -2-g_2 & -3\end{bmatrix}$

$$\det(sI-A+gc^T)=\begin{vmatrix}s+g_1 & -1\\ 2+g_2 & s+3\end{vmatrix}=(s+g_1)(s+3)+2+g_2=s^2+(g_1+3)s+3g_1+g_2+2$$

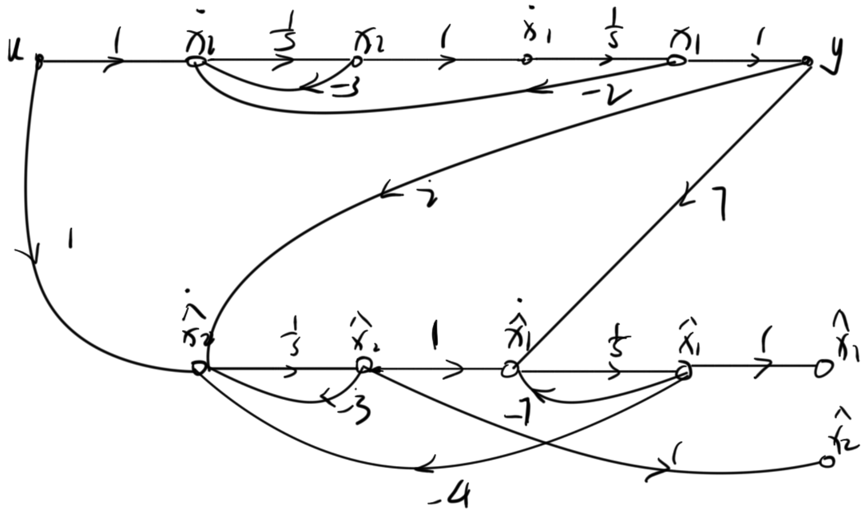
期望观测器特征多项式为 $(s+5)^2=s^2+10s+25$

则 $\begin{cases}g_1+3=10\\ 3g_1+g_2+2=25\end{cases} \Rightarrow \begin{cases}g_1=7\\ g_2=2\end{cases}$ 则 $g=\begin{bmatrix}7\\ 2\end{bmatrix}$

则观测器方程为

$$\dot{\hat{x}}=\begin{bmatrix}-7 & 1\\ -4 & -3\end{bmatrix}\hat{x}+\begin{bmatrix}0\\ 1\end{bmatrix}u+\begin{bmatrix}7\\ 2\end{bmatrix}y$$

状态图如下,



12、 $S=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $V=\begin{bmatrix}1&1\\1&-6\end{bmatrix}$ $\text{rank}[S]=2$, $\text{rank}[V]=2$，能控且能观

极点配置，设 $k^T=[k_1\ \ k_2]$ 则有 $A-bk^T=\begin{bmatrix}0&0\\1&-6\end{bmatrix}-\begin{bmatrix}1\\0\end{bmatrix}[k_1\ \ k_2]=\begin{bmatrix}-k_1&-k_2\\1&-6\end{bmatrix}$

$\det(sI-A+bk^T)=\begin{vmatrix}s+k_1&k_2\\-1&s+6\end{vmatrix}=s^2+(6+k_1)s+6k_1+k_2$

期望特征多项式为 $(s+4-j6)(s+4+j6)=s^2+8s+52$

有 $\begin{cases}6+k_1=8\\6k_1+k_2=52\end{cases}\Rightarrow\begin{cases}k_1=2\\k_2=40\end{cases}$ 则 $k^T=[2\ \ 40]$

设计观测器 令 $g=\begin{bmatrix}g_1\\g_2\end{bmatrix}$ 则 $A-gc^T=\begin{bmatrix}0&0\\1&-6\end{bmatrix}-\begin{bmatrix}g_1\\g_2\end{bmatrix}[1\ \ 1]=\begin{bmatrix}-g_1&-g_1\\1-g_2&-6-g_2\end{bmatrix}$

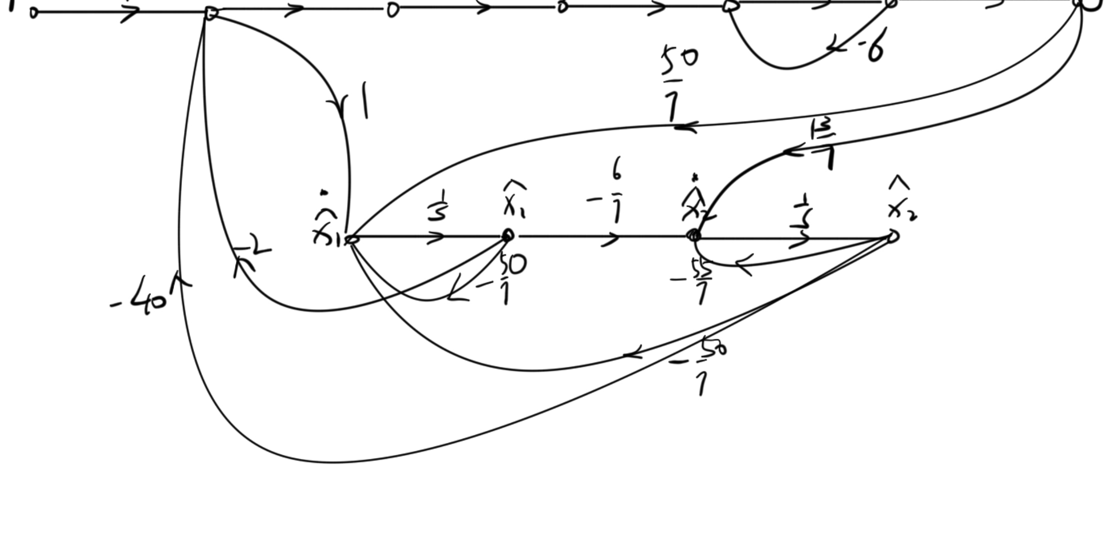
$\det(s-A+gc^T)=\begin{vmatrix}s+g_1&g_1\\g_2-1&s+g_2+6\end{vmatrix}=s^2+(g_1+g_2+6)s+g_1g_2+6g_1-g_1g_2+g_1$

期望的观测器特征多项式为 $(s+5)(s+10)=s^2+15s+50$

则有 $\begin{cases}g_1+g_2+6=15\\7g_1=50\end{cases}$ 则 $\begin{cases}g_1=\frac{50}{7}\\g_2=\frac{13}{7}\end{cases}$ $g=\begin{bmatrix}\frac{50}{7}\\\frac{13}{7}\end{bmatrix}$

观测器方程 $\dot{\hat{x}}=\begin{bmatrix}-\frac{50}{7}&-\frac{50}{7}\\-\frac{6}{7}&-\frac{55}{7}\end{bmatrix}\hat{x}+\begin{bmatrix}1\\0\end{bmatrix}u+\begin{bmatrix}\frac{50}{7}\\\frac{13}{7}\end{bmatrix}y$

状态图如下



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1.(1) 若结果收敛则 $Z[1-e^{-at}] = Z[1(t)] - Z[e^{-at}] = \frac{z}{z-1} - \frac{z}{z-e^{-aT}} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$

(4) 查表7.1 有 $Z[te^{-at}] = \frac{Tz(e^{-aT})}{(z-e^{-aT})^2}$

2(3) $m=n$. $G(z) = 1 + \frac{1.5z-0.5}{z^2-1.5z+0.5} = 1 + X_0(z)$

$$\frac{X_0(z)}{z} = \frac{1.5z-0.5}{z^3-1.5z^2+0.5z} = -\frac{1}{z} + \frac{2}{z-1} - \frac{1}{z-0.5}$$

则 $G(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$

则 $x(k) = Z^{-1}[G(z)] = 2 - 0.5^k \quad (k \geq 0)$

(4) $G(z) = \frac{-z}{z-1} + \frac{z}{z-2}$

则 $x(k) = Z^{-1}[G(z)] = -1 + 2^k \quad (k \geq 0)$

3.(1) $G(z) = Z\left[\frac{a}{s(s+a)}\right] = Z\left[\frac{1}{s} - \frac{1}{s+a}\right] = \frac{z}{z-1} - \frac{z}{z-e^{-aT}} = \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$

(2) $G(s) = \frac{\frac{1}{a}}{s^2} - \frac{\frac{1}{a^2}}{s} + \frac{\frac{1}{a^2}}{s+a}$

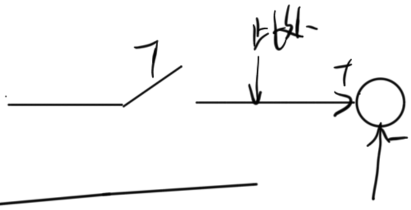
则 $G(z) = Z[G(s)] = \frac{1}{a}\frac{Tz}{(z-1)^2} - \frac{1}{a^2}\frac{z}{z-1} + \frac{1}{a^2}\frac{z}{z-e^{-aT}} = \frac{(aT-1)z^2 + z + (z^2 - aTz - z)e^{-aT}}{a^2(z-1)^2(z-e^{-aT})}$

4.(a) $G_1(s)$ 移动到 $R(s)$ 与 $H(s)$ 处

则有 $\frac{Y(z)}{RG_1(z)} = \frac{G_2(z)}{1+G_2(z)G_1H(z)}$ 但写不出 $G(z)$ 有 $Y(z) = \frac{RG_1(z)G_2(z)}{1+G_2(z)G_1H(z)}$

(b) $G(z) = \frac{G_1G_2(z)}{1+G_1G_2(z)H(z)}$

(c) $G(z) = \frac{G_1(z)G_2(z)}{1+G_1(z)G_2(z)H(z)}$

(d) 将 $G_2(s)$ 移到 $H(s)$ 与 

处

则 $\frac{Y(z)}{G_2(z)R(z)} = \frac{1}{1+(G_2H(z)+G_1(z)G_2(z))}$

则 $G(z) = \frac{Y(z)}{R(z)} = \frac{G_2(z)}{1+G_2H(z)+G_1(z)G_2(z)}$