10.

(1) 从状态方程中可以看出只有x_1可控，x_2极点为-2，不可控.

状态反馈 $A-bk^T=\begin{bmatrix}0 & 1\\ -k_1 & 2-k_2\end{bmatrix}$

$\det(sI-(A-bk^T))=s^2+(k_2-2)s+k_1$

目标极点方程 $(s+2)(s+3)=s^2+5s+6$.

比较，可得 $k_1=6$ $k_2=7$ $k^T=[6\ \ 7]$

11.

(2) $S=[b\ \ Ab]=\begin{bmatrix}0 & 1\\ 1 & -3\end{bmatrix}$ $V=\begin{bmatrix}c^T\\ c^TA\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ 可控、可观，系统/观测器独立设

由 $\dot{\tilde{x}}=(A-Gc)\tilde{x}$ 令 $G=\begin{bmatrix}g_1\\ g_2\end{bmatrix}$

有 $(A-Gc)=\begin{bmatrix}-g_1 & 1\\ -g_2-2 & -3\end{bmatrix}$

$\det(sI-(A-Gc))=s^2+(3+g_1)s+3g_1+g_2+2$.

目标极点方程 $(s+5)(s+5)=s^2+10s+25$

比较，可得 $g_1=7$ $g_2=2$. $g=\begin{bmatrix}7\\ 2\end{bmatrix}$

12. $S=[b\ \ Ab]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $V=\begin{bmatrix}c^T\\ c^TA\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & -6\end{bmatrix}$ 可控、可观，观测器和系统独立配置、

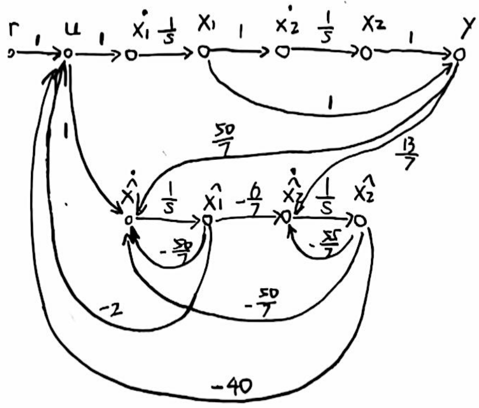
原系统 $\dot{x}=(A-bk^T)x+bk^T\tilde{x}+br$ 设 $k^T=[k_1\ \ k_2]$

系统极点 $A-bk^T=\begin{bmatrix}-k_1 & -k_2\\ 1 & -6\end{bmatrix}$

$\det(sI-(A-bk^T))=s^2+(k_1+6)s+6k_1+k_2$.

目标极点方程 $(s+4+j6)(s+4-j6)=s^2+8s+52$

对比 $k_1=2$ $k_2=40$ $k^T=[2\ \ 40]$



观测器 $\dot{\tilde{x}}=(A-Gc)\tilde{x}$ $G=\begin{bmatrix}g_1\\ g_2\end{bmatrix}$

极点 $A-Gc=\begin{bmatrix}-g_1 & -g_1\\ 1-g_2 & -6-g_2\end{bmatrix}$

$\det(sI-(A-Gc))=s^2+(g_1+g_2+6)s+7g_1$

目标极点方程 $(s+5)(s+10)=s^2+15s+50$.

对比 $g_1=\frac{50}{7}$ $g_2=\frac{13}{7}$ $G=\begin{bmatrix}\frac{50}{7}\\ \frac{13}{7}\end{bmatrix}$

根据 $\dot{x}=Ax+bu=\begin{bmatrix}0 & 0\\ 1 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u$ $u=r-k^T\hat{x}=r-[2\ \ 40]\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}$

$\dot{\tilde{x}}=(A-Gc)\tilde{x}+bu+Gy=\begin{bmatrix}-\frac{50}{7} & -\frac{50}{7}\\ -\frac{6}{7} & -\frac{55}{7}\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u+\begin{bmatrix}\frac{50}{7}\\ \frac{13}{7}\end{bmatrix}y$

1. (1) $x(k)=1-e^{-akT_s}$

$$X(z)=Z\left[1-e^{-akT_s}\right]=\frac{z}{z-1}-\frac{z}{z-e^{-aT_s}}=\frac{z(1-e^{-aT_s})}{(z-1)(z-e^{-aT_s})}.$$

(4) $x(k)=kT_s e^{-akT_s}$

首先,有 $$Z[kT_s]=\frac{zT_s}{(z-1)^2}$$

则 $$Z\left[kT_s e^{-aT_s\cdot k}\right]=\frac{ze^{aT_s}T_s}{(ze^{aT_s}-1)^2}=\frac{zT_s(e^{-aT_s})}{(z-e^{-aT_s})^2}$$

2. (3) $$G(z)=1+\frac{1.5z-0.5}{z^2-1.5z+0.5}=1+z\frac{1.5z-0.5}{z(z-0.5)(z-1)}=1+\left(\frac{-1}{z}+\frac{-1}{z-0.5}+\frac{2}{z-1}\right)z$$

$$=-\frac{z}{z-0.5}+\frac{2z}{z-1}$$

$$Z^{-1}[G(z)]=-0.5^k+2$$

(4) $$G(z)=z\frac{1}{(z-1)(z-2)}=z\left(\frac{1}{z-2}-\frac{1}{z-1}\right)=\frac{z}{z-2}-\frac{z}{z-1}$$

$$Z^{-1}[G(z)]=2^k-1$$

3. (1) $$G(s)=\frac{1}{s}-\frac{1}{s+a}\iff G(z)=\frac{z}{z-1}-\frac{z}{z-e^{-aT}}=\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$$

(2) $$G(s)=\frac{1}{s^2(s+a)}=\frac{1}{s}\left(\frac{1}{a}\cdot\frac{1}{s}-\frac{1}{a}\frac{1}{s+a}\right)=\frac{1}{as^2}-\frac{1}{a^2}\left(\frac{1}{s}-\frac{1}{s+a}\right)=\frac{1}{as^2}-\frac{1}{a^2s}+\frac{1}{a^2(s+a)}$$

$$\iff G(z)=\frac{Tz}{a(z-1)^2}-\frac{1}{a^2}\frac{z}{z-1}+\frac{1}{a^2}\frac{z}{z-e^{-aT}}$$

4. (a) $u(t)=r(t)-h(t)*y(t)$

$y(t)=\left[u(t)*g_1(t)\right]*g_2(t)$

离散化. $U(z)=R(z)-H(z)Y(z).$

$Y(z)=UG_1(z)G_2(z)$

$$\Rightarrow\quad Y(z)=\frac{RG_1(z)G_2(z)}{1+HG_1(z)G_2(z)}$$

$$\frac{Y(z)}{R(z)}=\frac{RG_1(z)G_2(z)}{R(z)[1+HG_1(z)G_2(z)]}$$

(b) $U(z)=R(z)-H(z)Y(z)$

$Y(z)=G_1G_2(z)\cdot U(z)$

$$\Rightarrow\quad\frac{Y(z)}{R(z)}=\frac{G_1G_2(z).}{1+G_1G_2(z)\cdot H(z)}$$

(c) 全部离散 $$\frac{Y(z)}{R(z)}=\frac{G_1(z)G_2(z)}{1+G_1(z)G_2(z)H(z)}.$$

(d) $U_1(z)=R(z)-G_1(z)Y(z)$

$U_2(z)=U_1(z)-H(z)Y(z)$

$Y(z)=G_2U_2(z)$

$$\Rightarrow\quad\frac{Y(z)}{R(z)}=\frac{G_2(z)}{1+G_2(z)G_1(z)+HG_2(z)}$$