指南车——方向的开环自动调节系统

我国古代指南车的发明，传说始于皇帝（公元前 2698 - 2599 年）或周公时代（公元前 1100 年左右）。据刘仙洲先生考证，最早可推至西汉（公元前 200 年），最保守的说法也在东汉时代（公元 78 - 139 年），因此是远早于飞球调速器的。

根据宋史上的记载，指南车是用一辆双轮独辕车组成，由马匹来拉动，车内采用一种能自动离合的齿轮系统，车厢外壳上层放置了一个木刻的仙人，无论车轮向哪个方向转弯，仙人的手臂都会指向正南方。李约瑟等国外学者根据中国古代的记载推测，指南车利用的是差动齿轮。图 1 是一种差动齿轮。它的作用是测出指南车左和右两轮的转速之差。这个差由第三轴（图 1 右方轴）的转动表示出来，表明车辆转了弯，也就是车厢站立的木仙人所指的方向有了误差。其余的齿轮组成控制部分，即依据此误差来转动木仙人的轴，使其恢复到原来的位置。即木仙人所指的方向不变。

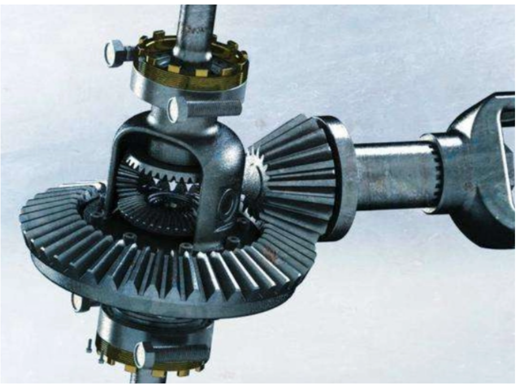


图 1: 一种差动齿轮

在指南车中，差动齿轮测出的是方向误差。因此指南车是一个利用误差控制的、负反馈的闭环控制系统，方框图见图 2。

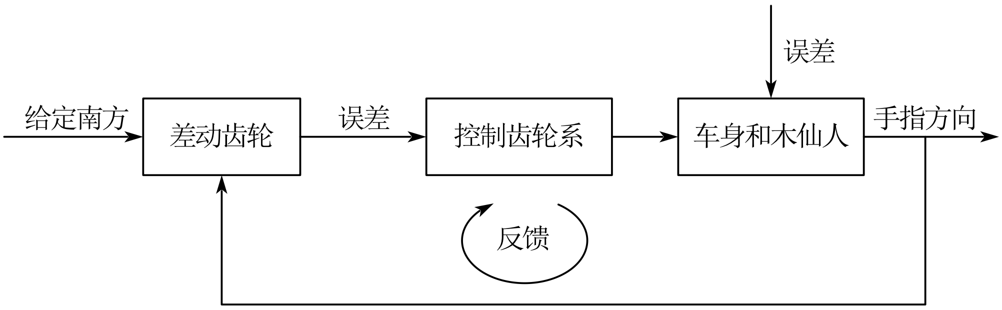


图 2: 指南车的方框图

Week 2 Homework

P34,1 **Solution:**

(1) Use the Laplace transform, we get:

$$s^3Y(s) + 3s^2Y(s) + 4sY(s) + Y(s) = 2sU(s) + U(s)$$

Then, we can get the transfer function of the system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s+1}{s^3+3s^2+4s+1}$$

(2) Use the Laplace transform, we get:

$$s^4Y(s) + 6s^2Y(s) + 10sY(s) + 3Y(s) = 7U(s)$$

Then, we can get the transfer function of the system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{7}{s^4+6s^2+10s+3}$$

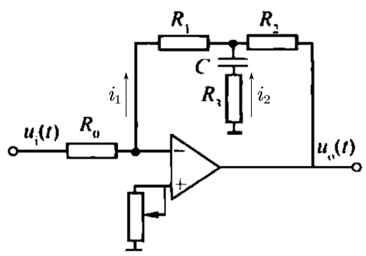
(3) Use the Laplace transform, we get:

$$s^3Y(s) + 2s^2Y(s) + 8sY(s) + Y(s) + \frac{5}{s}Y(s) = 3sU(s) + U(s)$$

Then, we can get the transfer function of the system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3s+1}{s^3+2s^2+8s+\frac{5}{s}+1} = \frac{3s^2+s}{s^4+2s^3+8s^2+s+5}$$

P34,2(b) **Solution:** Suppose the branch current is i_1 and i_2 , as shown in the figure:



We can get the equations:

$$\begin{cases} U_o(s) = -R_1I_1(s) - R_2(I_1(s) + I_2(s)) \\ I_1(s) = \frac{U_i(s)}{R_0} \\ I_2(s) = \frac{\frac{R_1}{R_0}U_i(s)}{\frac{1}{sC}+R_3} = \frac{R_1}{R_0}\frac{U_i(s)}{\frac{1}{sC}+R_3} \end{cases}$$

Then we have:

$$U_o(s) = -\frac{R_1+R_2}{R_0}U_i(s) - \frac{R_1R_2}{R_0}\frac{1}{\frac{1}{sC}+R_3}U_i(s)$$

Then, we can get the transfer function of the system:

$$
\begin{aligned}
G(s) = \frac{U_{\mathrm{o}}(s)}{U_{\mathrm{i}}(s)} &= -\frac{R_1+R_2}{R_0} - \frac{R_1R_2}{R_0}\frac{1}{\frac{1}{sC}+R_3} \\
&= -\frac{(R_1R_2+R_1R_3+R_2R_3)Cs+R_1+R_2}{R_0R_3Cs+R_0}
\end{aligned}
$$

P34,3(c) **Solution:** Suppose the displacement at the top of μ_2 is $x(t)$, then we can get the equations:

$$
\begin{cases}
k_1(x_{\mathrm{i}}(t)-x(t)) + \mu_1(\dot{x}_{\mathrm{i}}(t)-\dot{x}(t)) = \mu_2(\dot{x}(t)-\dot{x}_{\mathrm{o}}(t)) \\
k_2x_{\mathrm{o}}(t) = \mu_2(\dot{x}(t)-\dot{x}_{\mathrm{o}}(t))
\end{cases}
$$

The initial conditions are $x_{\mathrm{i}}(0) = x_{\mathrm{o}}(0) = 0$ and $\dot{x}_{\mathrm{i}}(0) = \dot{x}_{\mathrm{o}}(0) = 0$.

Use the Laplace transform to simplify the equations:

$$
\begin{cases}
(k_1+\mu_1s)X_{\mathrm{i}}(s) - (k_1+\mu_1s)X(s) = \mu_2sX(s) - \mu_2sX_{\mathrm{o}}(s) \\
k_2X_{\mathrm{o}}(s) = \mu_2sX(s) - \mu_2sX_{\mathrm{o}}(s)
\end{cases}
$$

Cancel the $X(s)$, we can get:

$$((k_1+\mu_1s+\mu_2s)(\mu_2s+k_2) - u_2^2s^2)X_{\mathrm{o}}(t) = (k_1+\mu_1s)\mu_2sX_{\mathrm{i}}(t)$$

Then, we can get the transfer function of the system:

$$G(s) = \frac{X_{\mathrm{o}}(s)}{X_{\mathrm{i}}(s)} = \frac{\mu_1\mu_2s^2+k_1\mu_2s}{\mu_1\mu_2s^2+(k_1\mu_2+k_2\mu_1+k_2\mu_2)s+k_1k_2}$$

P35,5(b) **Solution:** Individual loops: $-g_1h_3$, $-g_2h_2$, $-g_3h_1$, $-g_4g_5h_1h_2h_3$, $-h_4$, two non-touching loops: $-g_1h_3$ and $-g_3h_1$, $-h_4$ and $-g_3h_1$, $-h_4$ and $-g_2h_2$, $-h_4$ and $-g_1h_3$, three non-touching loops: $-h_4$, $-g_1h_3$, $-g_3h_1$, forward paths: $g_1g_2g_3$, $\Delta_1 = 1+h_4$ and $-g_4g_5$, $\Delta_2 = 1+g_2h_2$, according to Mason's formula:

$$
\begin{aligned}
G &= \frac{y}{r} \\
&= \frac{g_1g_2g_3(1+h_4)+g_4g_5(1+g_2h_2)}{1+g_1h_3+g_2h_2+g_3h_1+g_4g_5h_1h_2h_3+h_4+g_1h_3g_3h_1+g_1h_3h_4+g_2h_2h_4+g_3h_1h_4+g_1h_3g_3h_1h_4}
\end{aligned}
$$

P35,6(c) **Solution:** Individual loops: G_1 , $-G_2$, $-G_1G_2$, $-G_1G_2$, $-G_1G_2$, forward paths: $-G_1$, G_2 , G_1G_2 , G_1G_2 , $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$, according to Mason's formula:

$$
\begin{aligned}
G &= \frac{Y}{R} \\
&= \frac{G_2-G_1+G_1G_2+G_1G_2}{1+G_2-G_1+G_1G_2+G_1G_2+G_1G_2} \\
&= \frac{G_2-G_1+2G_1G_2}{1+G_2-G_1+3G_1G_2}
\end{aligned}
$$

P36,9 **Solution:** Individual loops: $-G_1$, $-G_2$, G_3G_4 , two non-touching loops: $-G_1$ and $-G_2$.

The paths from $R_1(s)$ to $Y_1(s)$: G_1 , $-G_3G_4$.

The paths from $R_2(s)$ to $Y_2(s)$: G_2 , $-G_3G_4$.

The paths from $R_1(s)$ to $Y_2(s)$: G_3 .

The paths from $R_2(s)$ to $Y_1(s)$: G_4 .

According to Mason's formula:

$$\frac{Y_1(s)}{R_1(s)} = \frac{G_1(1+G_2) - G_3G_4}{1+G_1+G_2-G_3G_4+G_1G_2}$$
$$\frac{Y_2(s)}{R_1(s)} = \frac{G_3}{1+G_1+G_2-G_3G_4+G_1G_2}$$
$$\frac{Y_1(s)}{R_2(s)} = \frac{G_4}{1+G_1+G_2-G_3G_4+G_1G_2}$$
$$\frac{Y_2(s)}{R_2(s)} = \frac{G_2(1+G_1) - G_3G_4}{1+G_1+G_2-G_3G_4+G_1G_2}$$