Review & Exercise

Basic Concepts

- Linear Time-Invariant System
  - Principle of Superposition
  - Property of homogeneity
  - constant parameters

Mathematic Models of Control Systems

- Ordinary Differential Equation

$$y^{(n)}(t)+a_n y^{(n-1)}(t)+\cdots\cdots+a_1 y(t)$$

$$=b_{m+1}u^{(m)}(t)+b_m u^{(m-1)}(t)+\cdots\cdots+b_1 u(t)$$

$u(t)$ input $\qquad$ $y(t)$ output

Mathematatic Models of Control Systems

- Transfer Function

Zero initial state:

$$u(0) = \dot{u}(0) = \cdots\cdots = u^{(m-1)}(0) = 0$$

$$y(0) = \dot{y}(0) = \cdots\cdots = y^{(n-1)}(0) = 0$$

Laplace Transformation:

$$L(y(t)) = Y(s)$$

$$L(u(t)) = U(s)$$

$$s \equiv \frac{d}{dt} \qquad \frac{1}{s} \equiv \int_{0^-}^{t} dt$$

Mathematic Models of Control Systems

Laplace Transformation of the system:

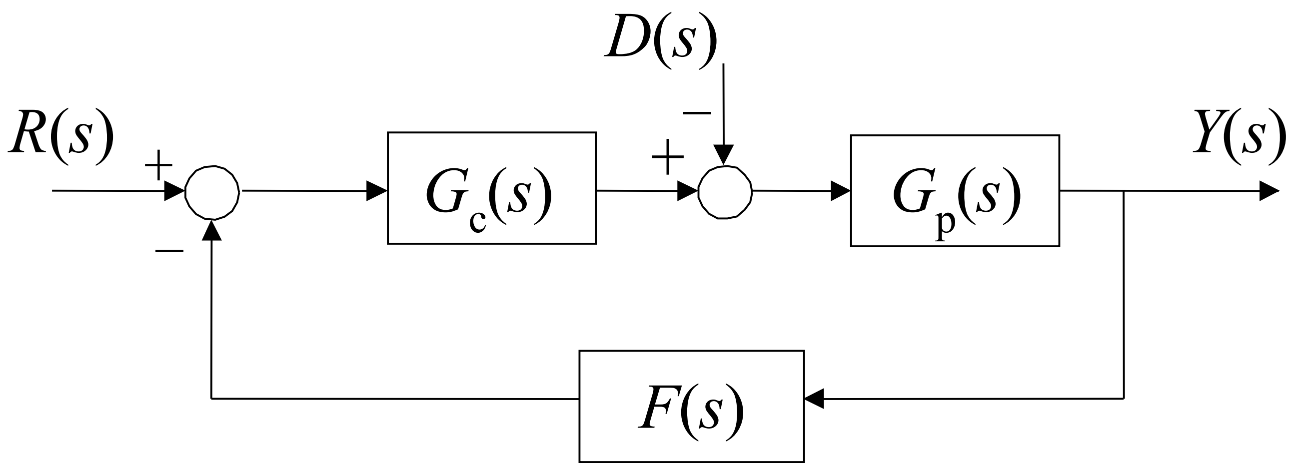
$$[s^n + a_n s^{n-1} + \cdots\cdots + a_1 s]Y(s)$$

$$= [b_{m+1} s^m + b_m s^{m-1} + \cdots\cdots + b_1 s]U(s)$$

Transfer function of the system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{m+1} s^m + b_m s^{m-1} + \cdots + b_1}{s^n + a_n s^{n-1} + \cdots + a_1} \qquad (n \geq m)$$

Typical Configuration of a feedback control system



Open-loop TF: $$G_0(s) = G_c(s)G_p(s)F(s)$$

Closed-loop TF from input to output:

$$G(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)F(s)}$$

For a unit feedback system

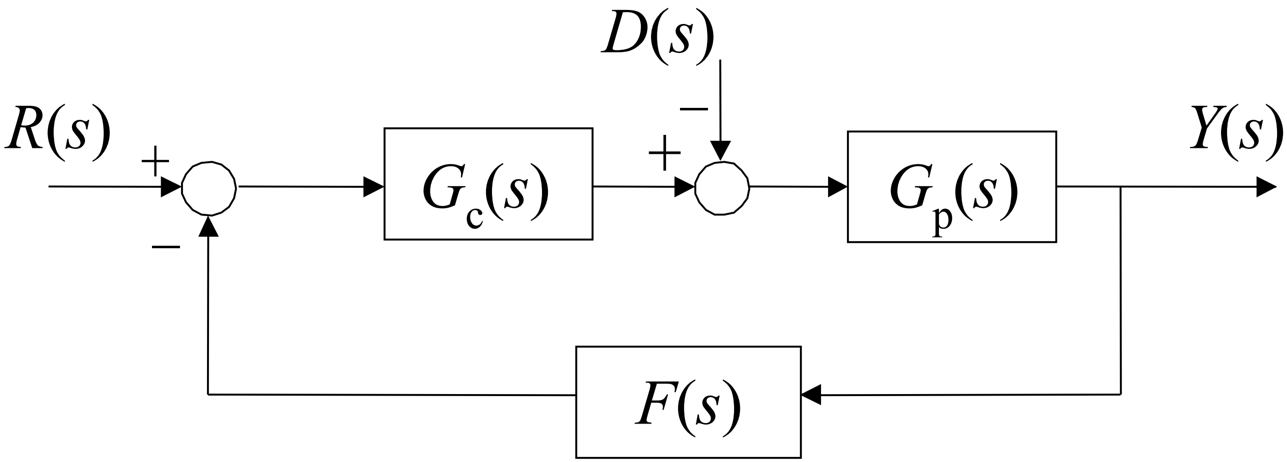
Open-loop TF:

$$G_0(s) = G_c(s)G_p(s)$$

Closed-loop TF from input to output:

$$G(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)}$$

$$G(s) = \frac{G_0(s)}{1+G_0(s)}$$

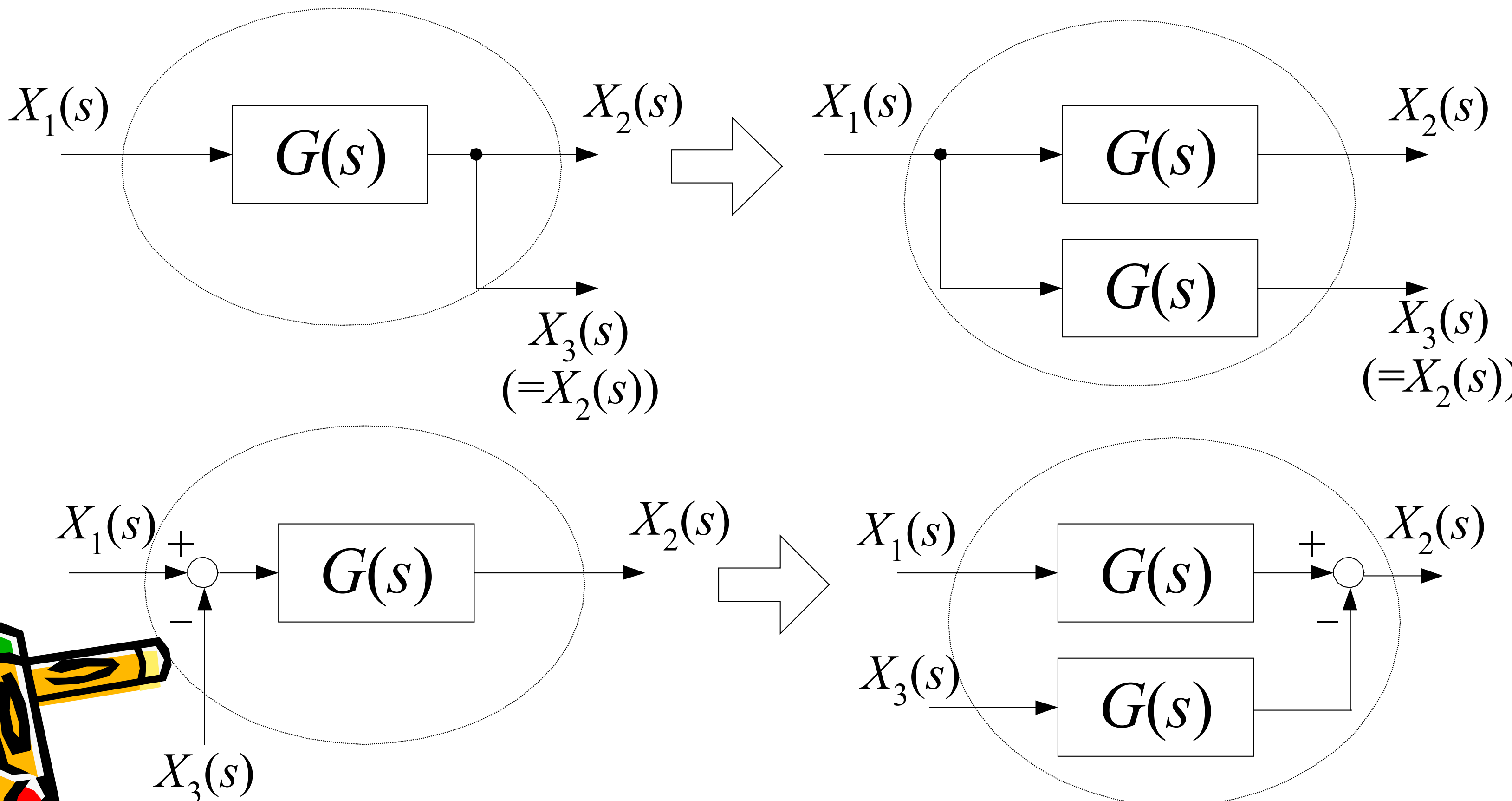


Closed-loop TF from disturbance to output:

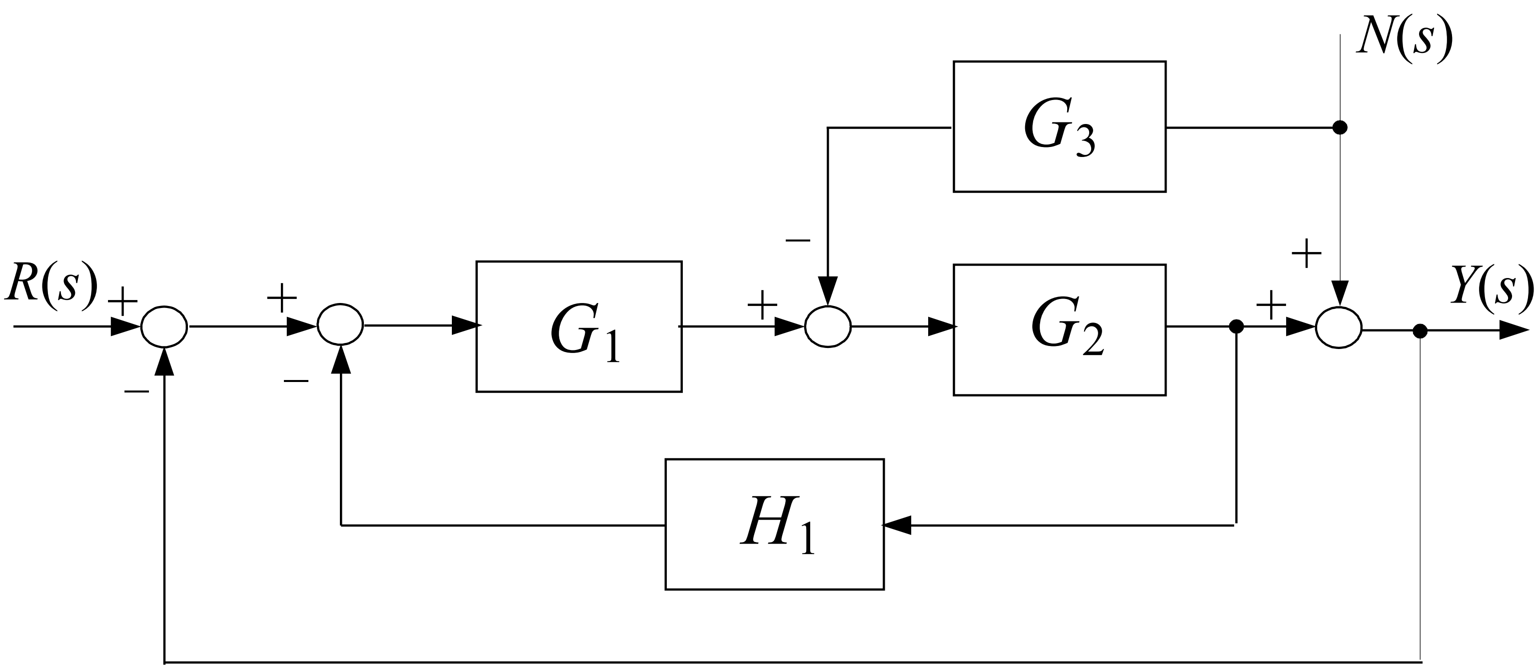
$$G_{\mathrm{D}}(s) = \frac{Y(s)}{D(s)} = \frac{-G_{\mathrm{p}}(s)}{1 + G_{\mathrm{c}}(s) G_{\mathrm{p}}(s) F(s)}$$

Block diagram transformation

Keep the input-output relationship unchanged

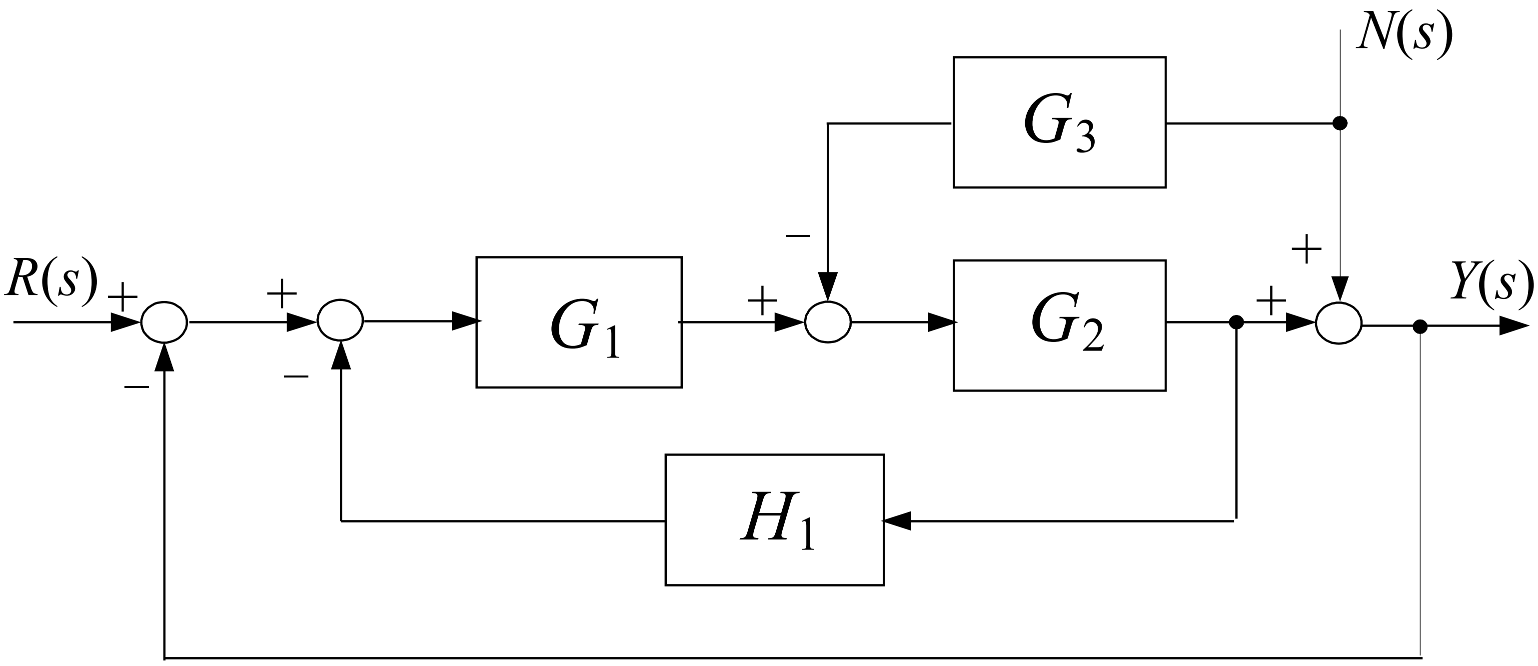


Q1: Please find the transfer functions of the given system

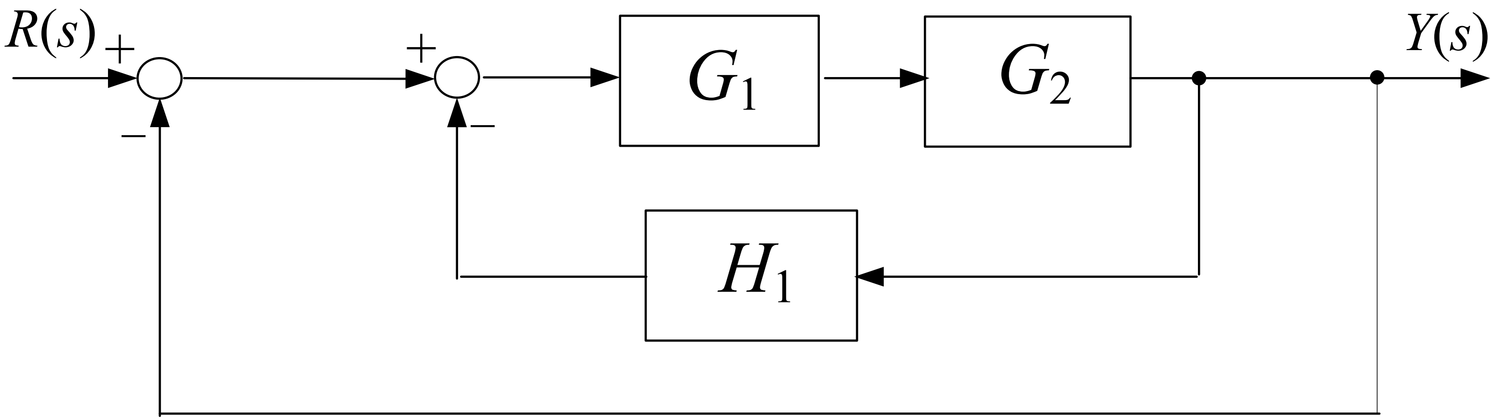


作答

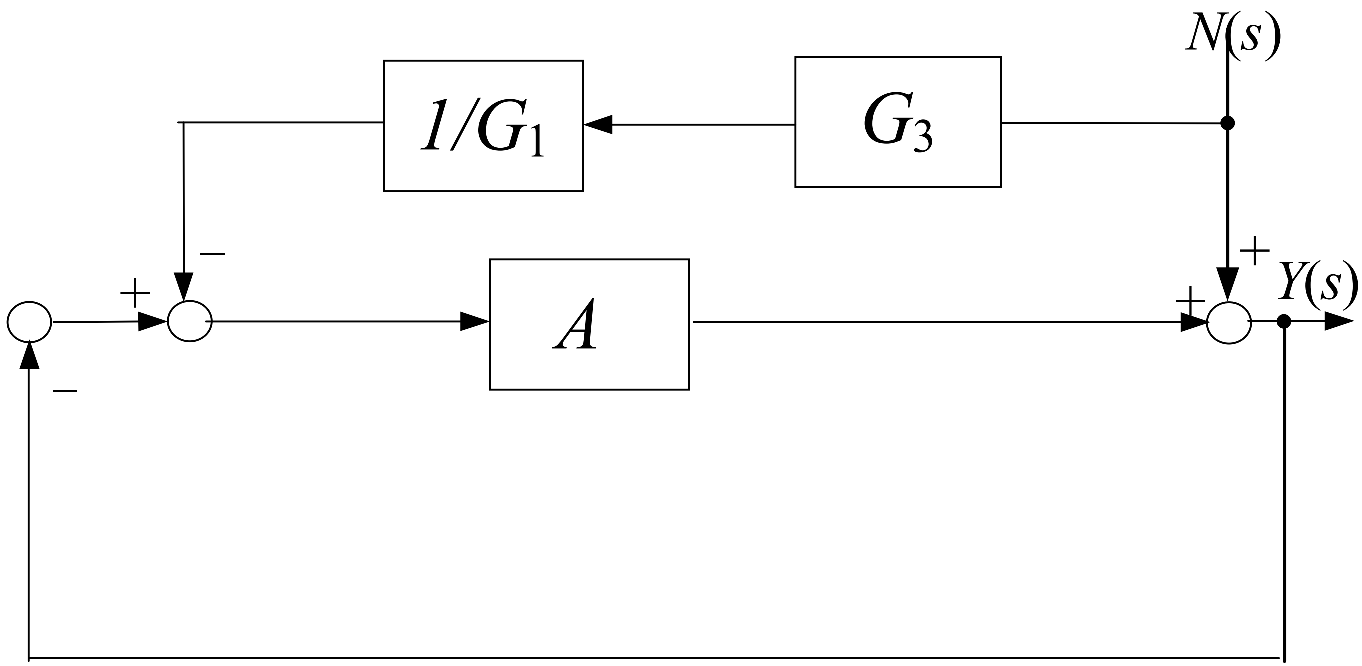
Q1: Please find the transfer functions of the given system



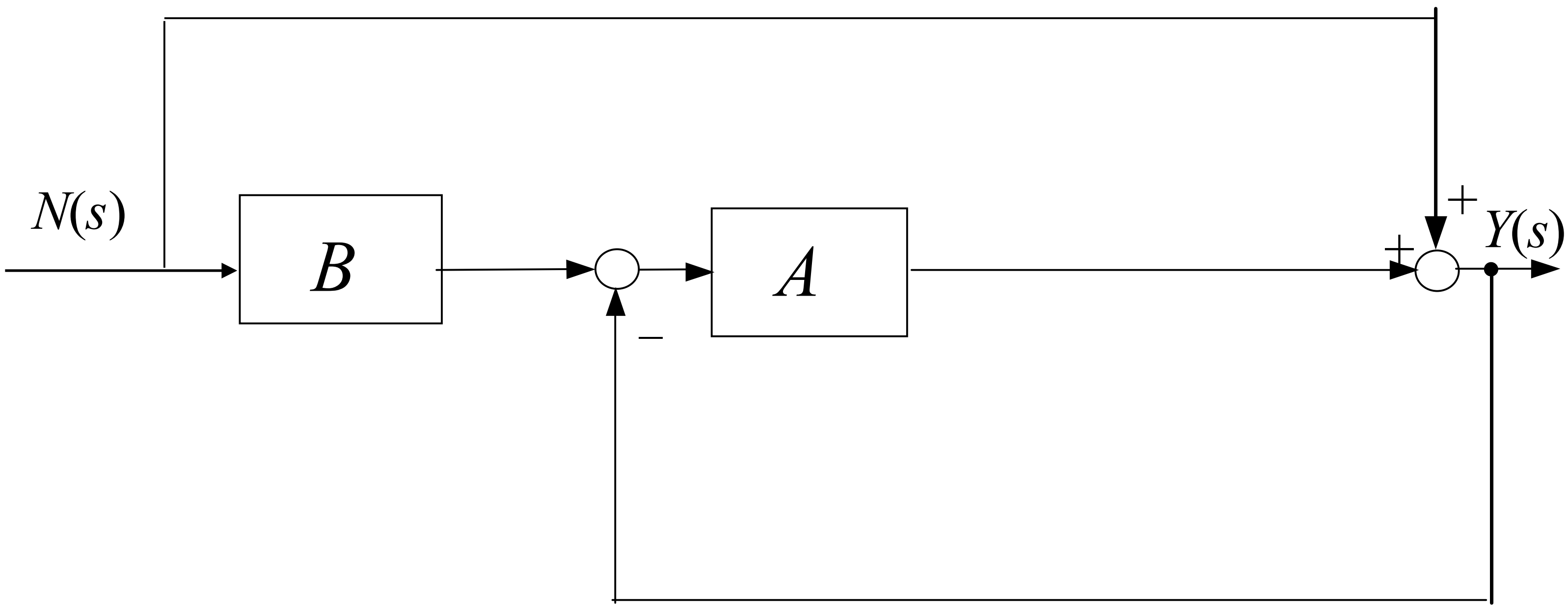
multi-input single-output



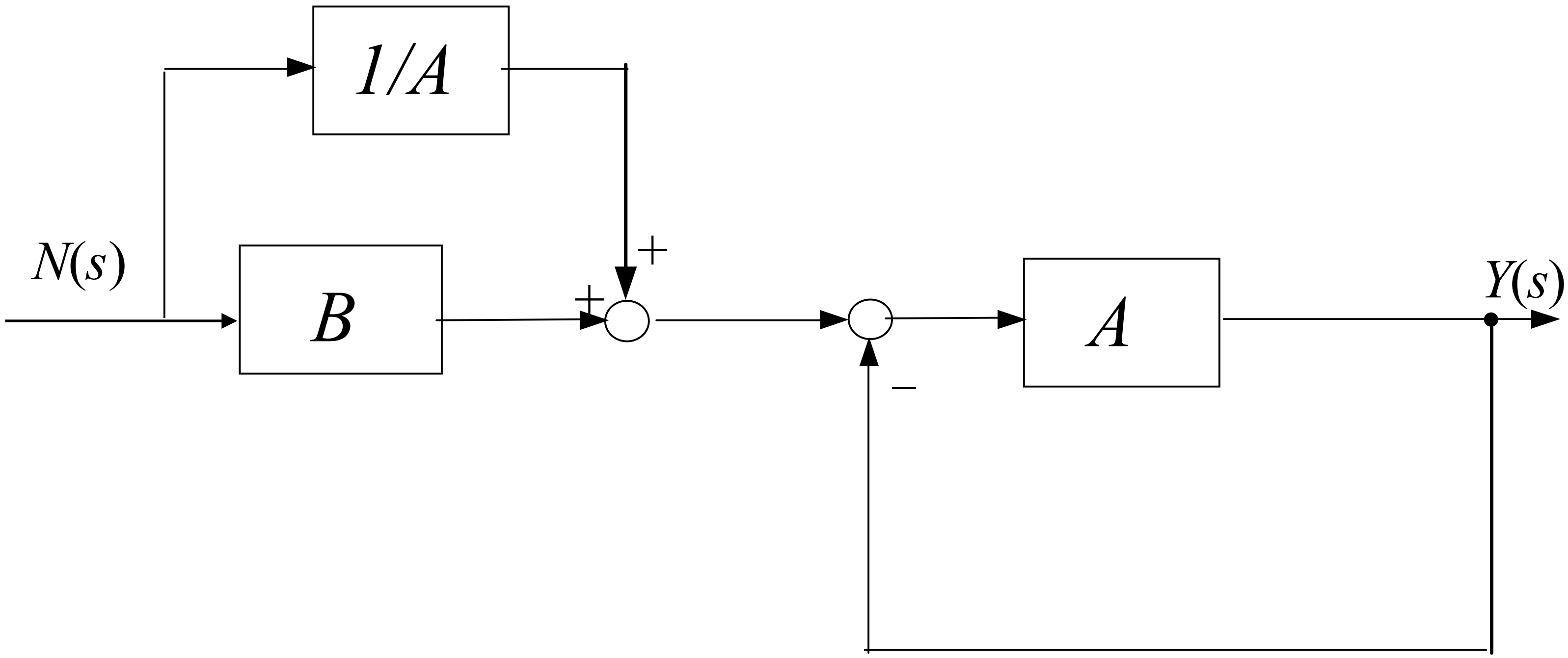
$$\frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_1 + G_1 G_2}$$



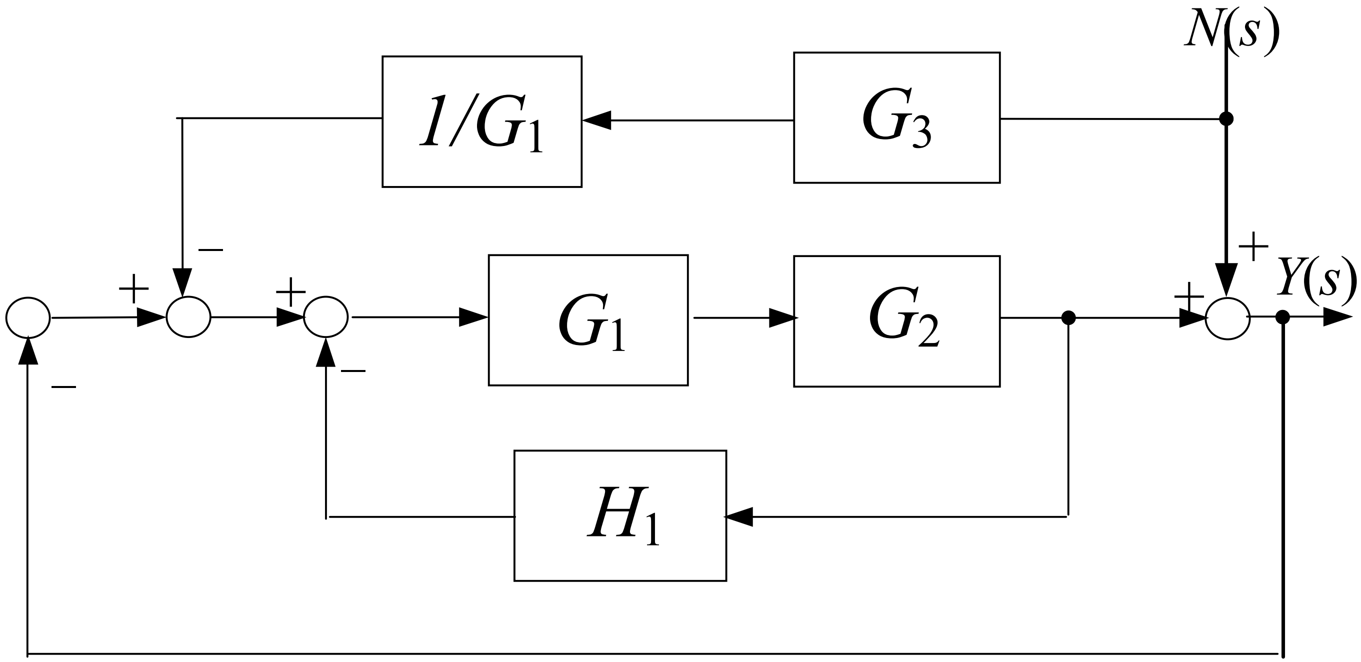
$$A = \frac{G_1 G_2}{1 + G_1 G_2 H_1}$$



$$A = \frac{G_1 G_2}{1 + G_1 G_2 H_1} \qquad B = -\frac{G_3}{G_1}$$

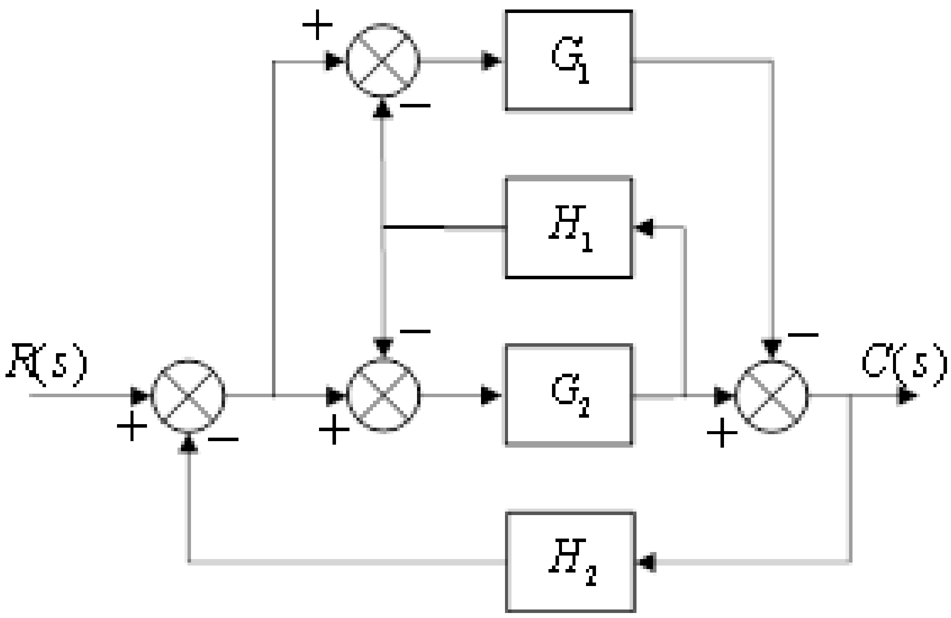


$$G = \left(B + {}^{1}\!/\!_{A}\right)\left(\frac{A}{1+A}\right) = \frac{1+AB}{1+A}$$

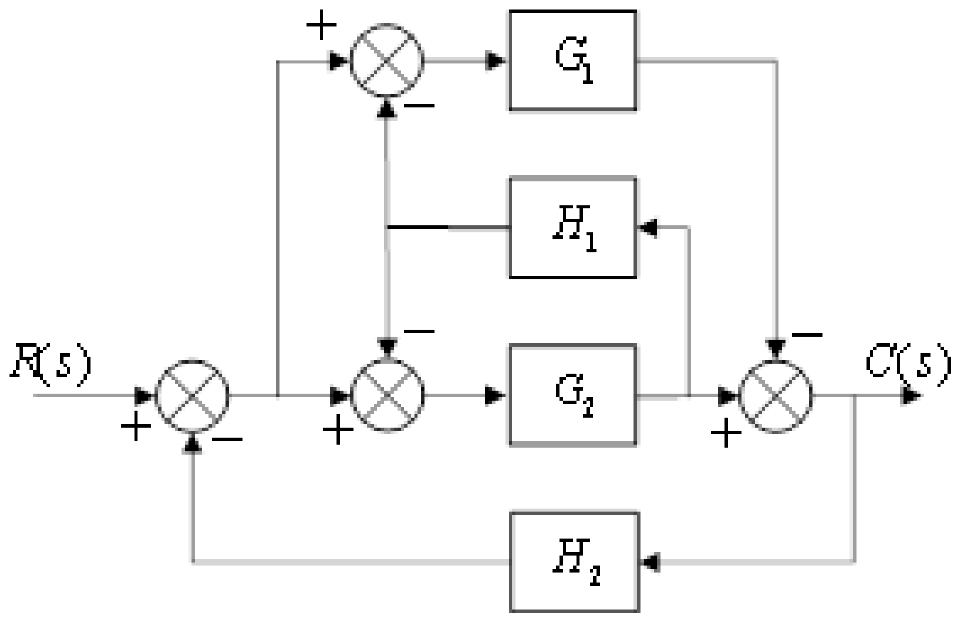


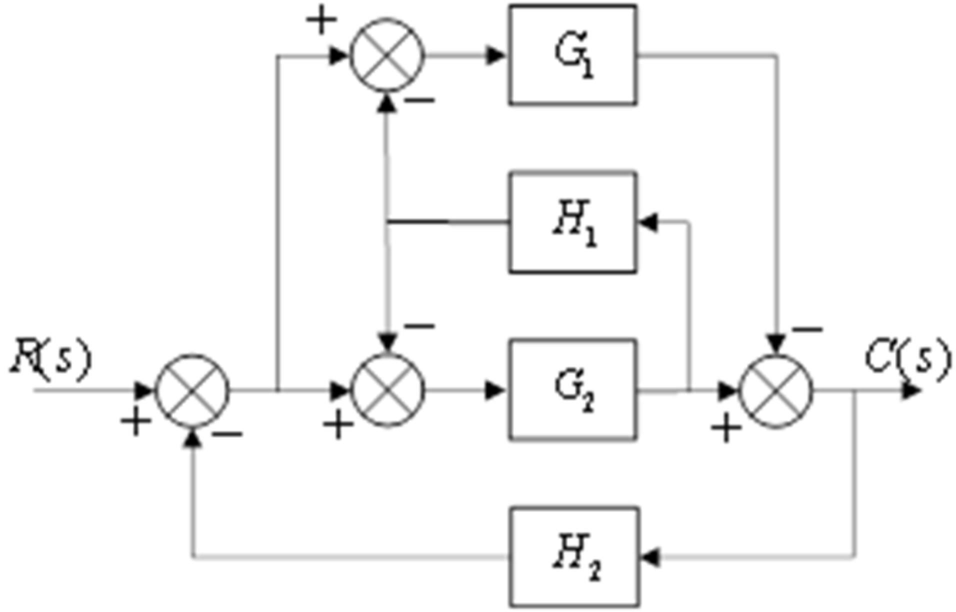
$$\frac{Y(s)}{N(s)} = \frac{1 + G_1 G_2 H_1 - G_2 G_3}{1 + G_1 G_2 H_1 + G_1 G_2}$$

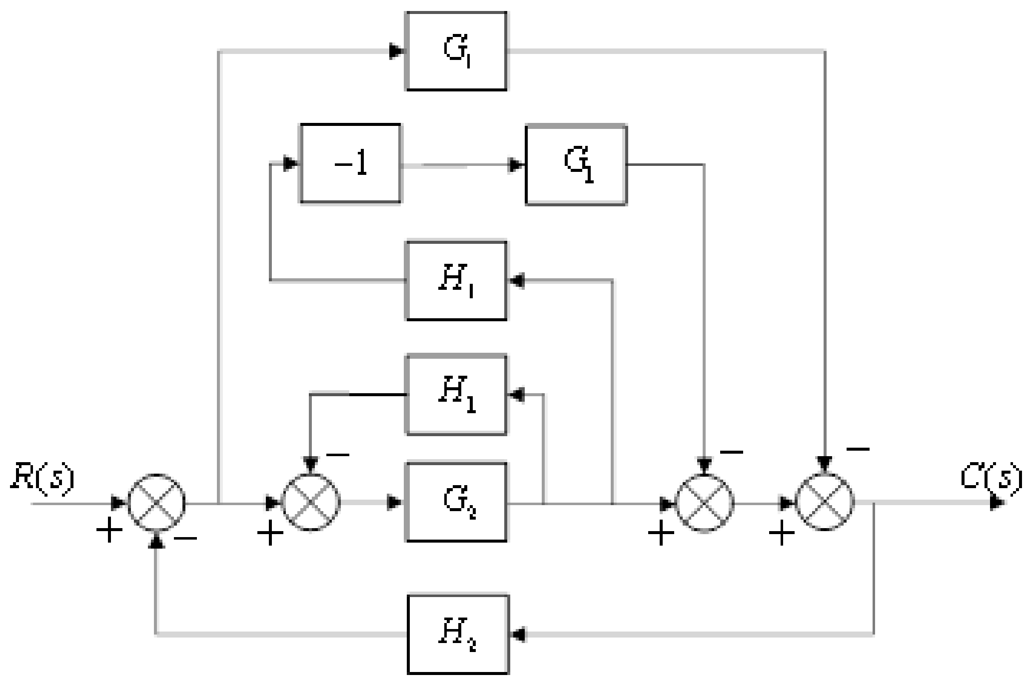
Q2: Please find the transfer functions of the given system

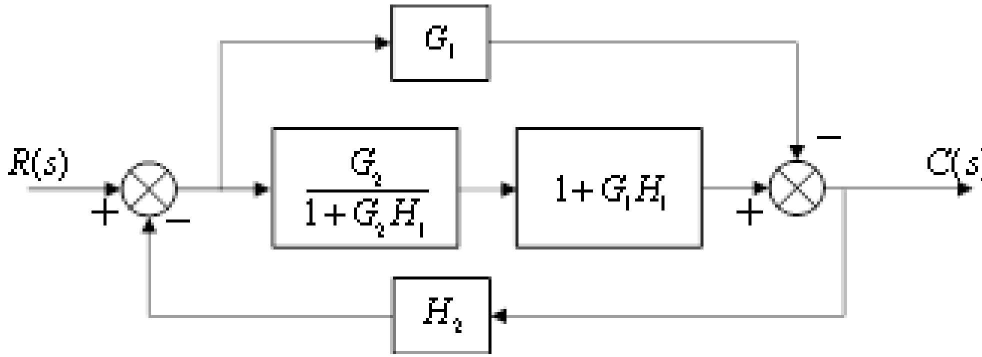


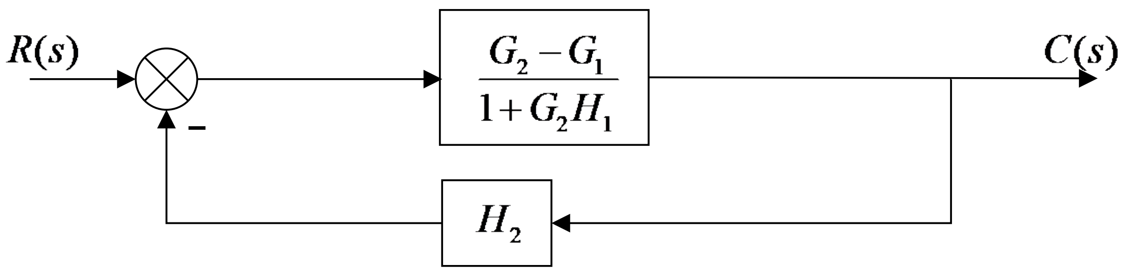
Q2: Please find the transfer functions of the given system

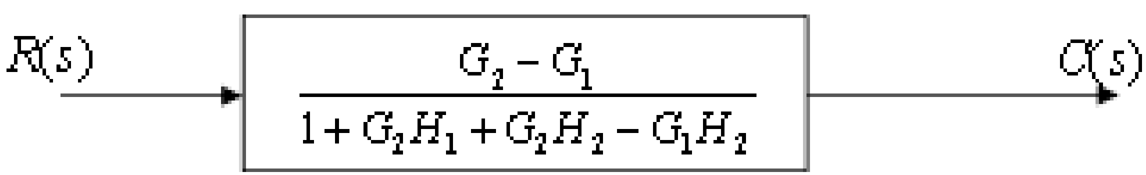



R(s)
G1
H1
G2
H2
C(s)

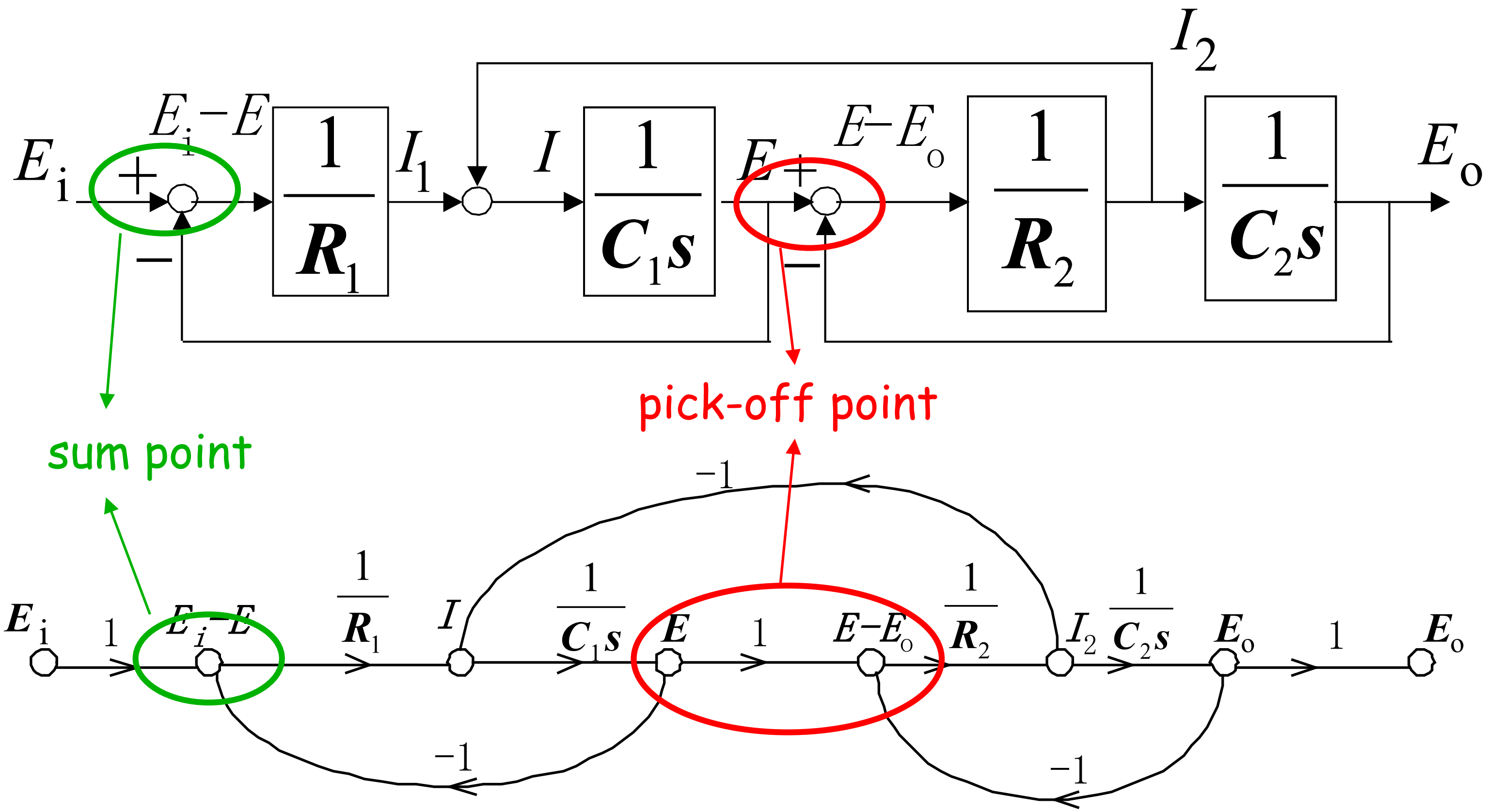

R(s)
G1
−1
G1
H1
H1
G2
H2
C(s)


G_1
$R(s)$
$\frac{G_2}{1+G_2H_1}$
$1+G_1H_1$
$C(s)$
H_2
+
−
+
−


$R(s)$
$\frac{G_2-G_1}{1+G_2H_1}$
$C(s)$
H_2
-


$R(s)$
$\frac{G_2-G_1}{1+G_2H_1+G_2H_2-G_1H_2}$
$C(s)$

Signal Flow Graph



Mason Formula

$$G = \frac{\sum_k G_k \Delta_k}{\Delta}$$

G Gain between input and output

$$\Delta = 1 - \sum L_i + \sum L_a L_b - \sum L_\alpha L_\beta L_\gamma + \cdots$$

L_i Gain of an individual loop

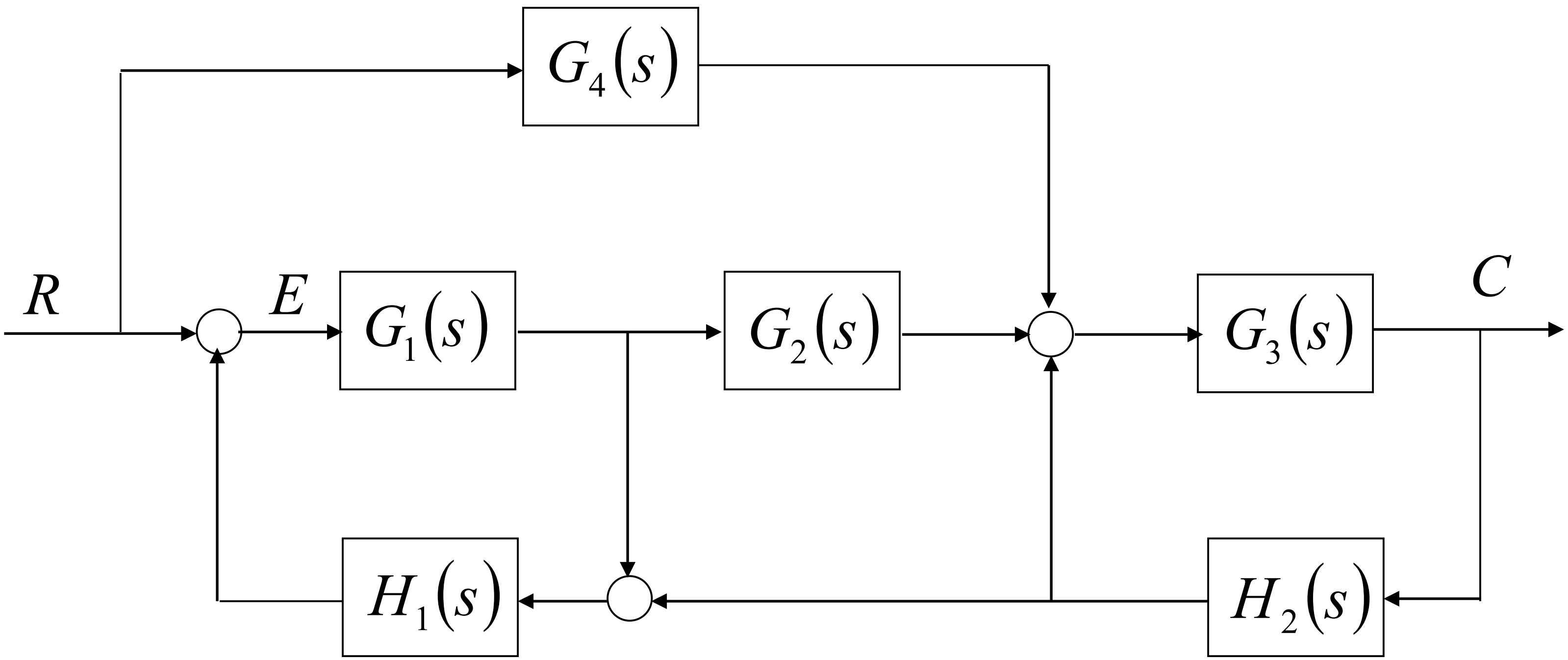
$L_a L_b$ Gain product of two nontouching loops

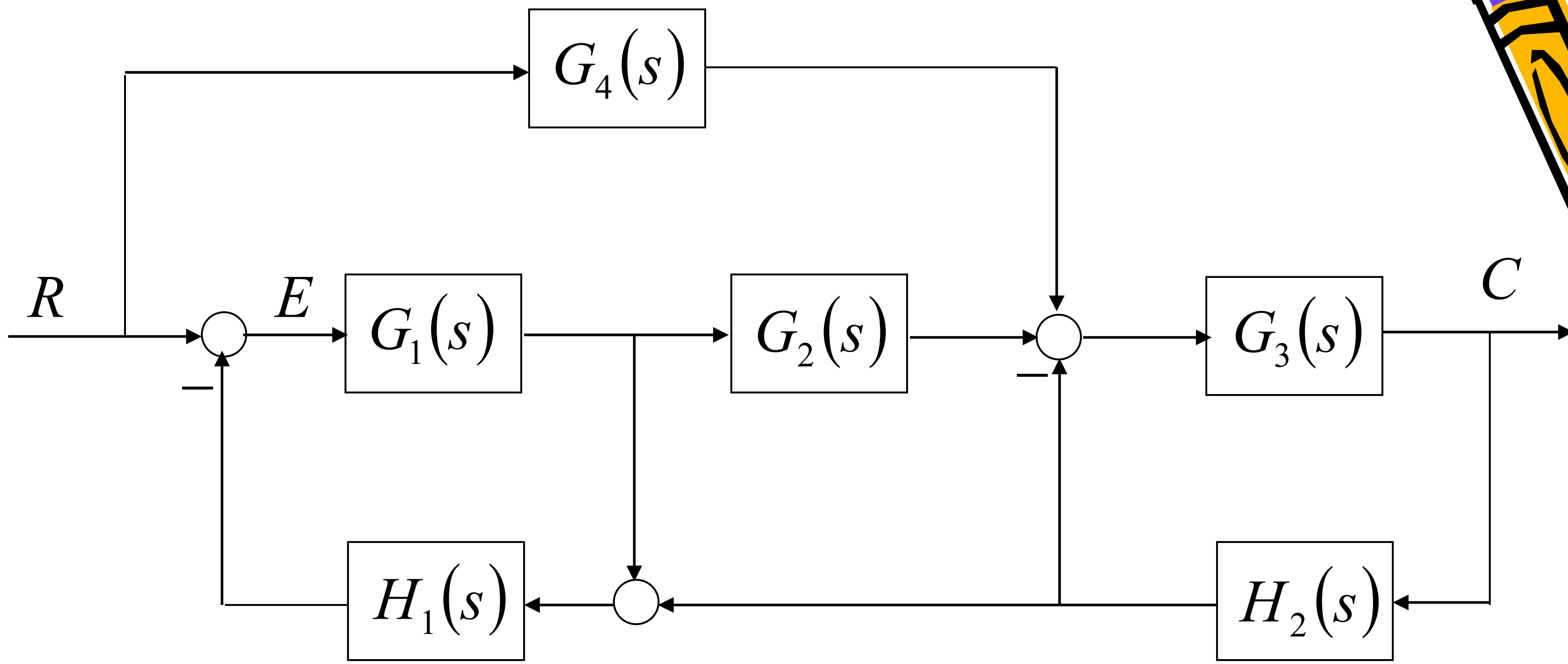
$L_\alpha L_\beta L_\gamma$ Gain product of any three nontouching loops

G_k Gain of the *k*th forward path between input and output

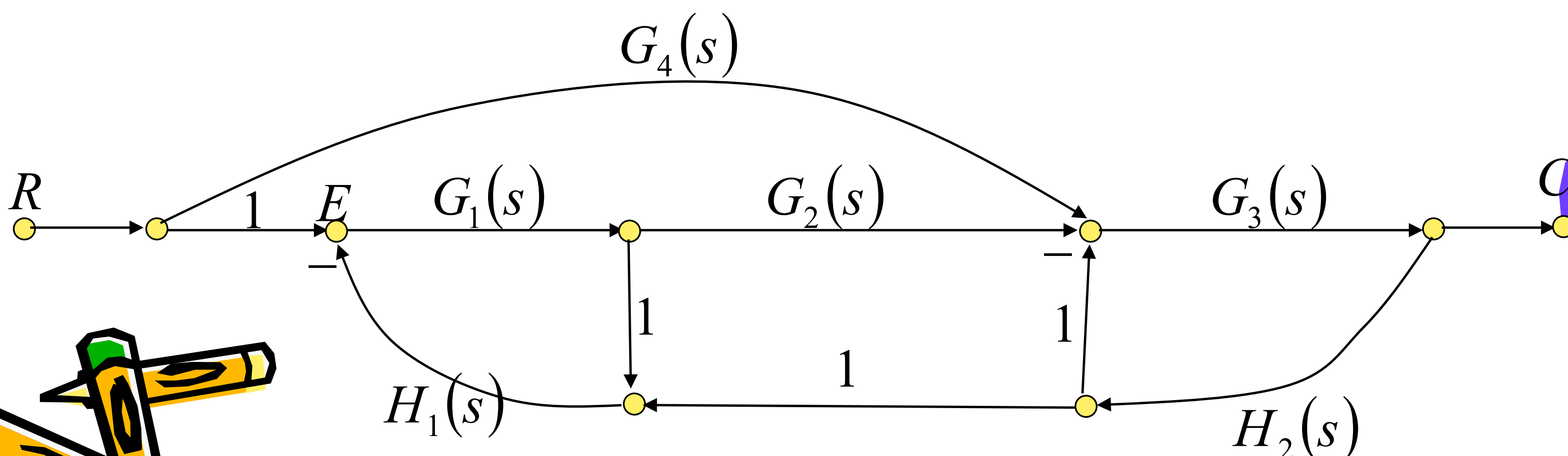
Δ_k The left part of Δ that is nontouching of the *k*th forward path

Q3: please construct the SFG of the following system, then use mason formula to find the TF.

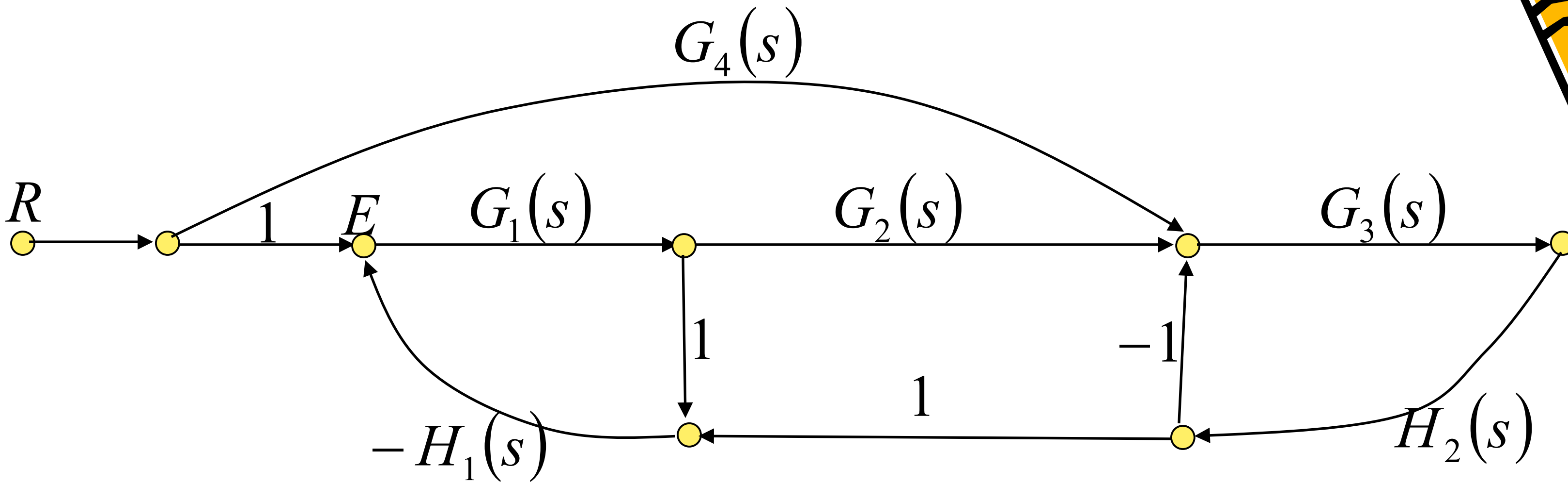




$G_4(s)$
R
E
$G_1(s)$
$G_2(s)$
C
$G_3(s)$
–
–
$H_1(s)$
$H_2(s)$



$G_4(s)$
R
1
E
$G_1(s)$
$G_2(s)$
$G_3(s)$
C
–
–
1
1
1
$H_1(s)$
$H_2(s)$



Loops: $L_1 = -G_1(s)H_1(s)$

$$L_2 = -G_3(s)H_2(s)$$

$$L_3 = -G_1(s)G_2(s)G_3(s)H_1(s)H_2(s)$$

Nontouching loops: L_1, L_2 $\quad L_1L_2 = G_1(s)G_3(s)H_1(s)H_2(s)$

$$\begin{aligned}\Delta &= 1 - \sum L_a + \sum L_b L_c \\ &= 1 + G_1(s)H_1(s) + G_3(s)H_2(s) + G_1(s)G_2(s)G_3(s)H_1(s)H_2(s) \\ &\quad + G_1(s)G_3(s)H_1(s)H_2(s)\end{aligned}$$

Forward path: $P_1 = G_1(s)G_2(s)G_3(s)$ $\quad \Delta_1 = 1$

$P_2 = G_3(s)G_4(s)$ $\quad \Delta_2 = 1 + G_1(s)H_1(s)$

Transfer function:

$$G(s) = \frac{P_1 + P_2\Delta_2}{\Delta}$$

State Space Equation

SISO:

$$\dot{X} = AX + Bu$$
$$y = CX$$

u, y are scalars

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^{\mathrm{T}}$$

State Space Equation

MIMO:

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

U: p×1

Y: q×1

1. All the state space models of systems have the same form
2. The number of the state variables are determined by the order of the system
3. The choosing of state variables is not unique
4. Linear transformation can change the state variable set

From SE to TF

SISO:

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B$$

MIMO:

$$Y(s) = C(sI - A)^{-1} BU(s) = G(s)U(s)$$

From TF to SE

Controllability Canonical Form

Observability Canonical Form

Diagonal Canonical Form

Controllability Canonical Form

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{m+1}s^m + b_m s^{m-1} + \cdots + b_1}{s^n + a_n s^{n-1} + \cdots + a_1}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}_{n\times n} X + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n\times 1} u$$

$$y = \begin{bmatrix} b_1 & b_2 & \cdots & b_{m+1}, & 0 & \cdots & 0 \end{bmatrix}_{1\times n} X$$

$$\frac{Y(s)}{V(s)} \cdot \frac{V(s)}{U(s)} = \frac{b_{m+1}s^m + b_m s^{m-1} + \cdots + b_1}{s^n + a_n s^{n-1} + \cdots + a_1}$$

$$\frac{Y(s)}{V(s)} = b_{m+1}s^m + b_m s^{m-1} + \cdots + b_1$$

$$\frac{V(s)}{U(s)} = \frac{1}{s^n + a_n s^{n-1} + \cdots + a_1}$$

$$Y(s) = (b_{m+1}s^m + b_m s^{m-1} + \cdots + b_1) \cdot V(s)$$

$$U(s) = (s^n + a_n s^{n-1} + \cdots + a_1) \cdot V(s)$$

$$x_1 = v$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\vdots$$

$$\dot{x}_n = -a_1 x_1 - a_2 x_2 - \cdots - a_n x_n + u$$

$$y = b_1 x_1 + b_2 x_2 + \cdots + b_{m+1} x_{m+1}$$

Q4: Please find the step response of the following system

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [2 \quad 0] X$$

Initial condition $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Solution:

$$sX(s) - X\left(0\right) = AX(s) + BU(s)$$

$$X(s) = (sI - A)^{-1} X(0) + (sI - A)^{-1} BU(s)$$

$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}^{-1} = \frac{1}{(s+1)(s+3)} \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\frac{3}{2}}{s+1} - \dfrac{\frac{1}{2}}{s+3} & \dfrac{\frac{1}{2}}{s+1} - \dfrac{\frac{1}{2}}{s+3} \\ -\dfrac{\frac{3}{2}}{s+1} + \dfrac{\frac{3}{2}}{s+3} & -\dfrac{\frac{1}{2}}{s+1} + \dfrac{\frac{3}{2}}{s+3} \end{bmatrix}$$

$$(sI-A)^{-1}BU(s)=\begin{bmatrix} \dfrac{\frac{3}{2}}{s+1}-\dfrac{\frac{1}{2}}{s+3} & \dfrac{\frac{1}{2}}{s+1}-\dfrac{\frac{1}{2}}{s+3} \\ -\dfrac{\frac{3}{2}}{s+1}+\dfrac{\frac{3}{2}}{s+3} & -\dfrac{\frac{1}{2}}{s+1}+\dfrac{\frac{3}{2}}{s+3} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{s}$$

$$=\begin{bmatrix} \dfrac{1}{s(s+1)(s+3)} \\ \dfrac{1}{(s+1)(s+3)} \end{bmatrix}=\begin{bmatrix} \dfrac{\frac{1}{3}}{s}-\dfrac{\frac{1}{2}}{s+1}+\dfrac{\frac{1}{6}}{s+3} \\ \dfrac{\frac{1}{2}}{s+1}-\dfrac{\frac{1}{2}}{s+3} \end{bmatrix}$$

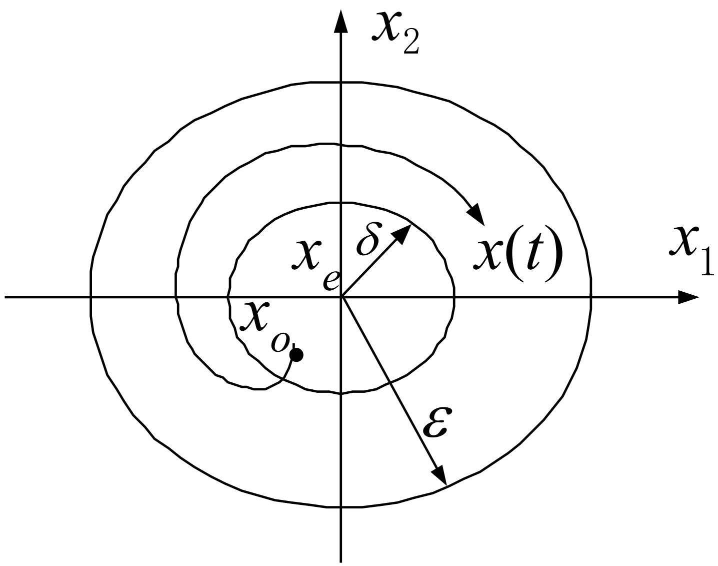
$$X=\begin{bmatrix}\frac{3}{2}e^{-t}-\frac{1}{2}e^{-3t} & \frac{1}{2}e^{-t}-\frac{1}{2}e^{-3t}\\ -\frac{3}{2}e^{-t}+\frac{3}{2}e^{-3t} & -\frac{1}{2}e^{-t}+\frac{3}{2}e^{-3t}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}\frac{1}{3}-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}\\ \frac{1}{2}e^{-t}-\frac{1}{2}e^{-3t}\end{bmatrix}$$

$$X=\begin{bmatrix}\frac{1}{3}+e^{-t}-\frac{1}{3}e^{-3t}\\ -e^{-t}+e^{-3t}\end{bmatrix}$$

$$y=\begin{bmatrix}2 & 0\end{bmatrix}X=\begin{bmatrix}2 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{3}+e^{-t}-\frac{1}{3}e^{-3t}\\ -e^{-t}+e^{-3t}\end{bmatrix}=\frac{2}{3}+2e^{-t}-\frac{2}{3}e^{-3t}$$

Stability of a Control System

Liapunov stability



BIBO stability

Routh table

$$a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0$$

s^n	a_0	a_2	a_4	$\cdots$
s^{n-1}	a_1	a_3	a_5	$\cdots$
s^{n-2}	$b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$	$b_2 = \dfrac{a_1 a_4 - a_0 a_5}{a_1}$	$b_3 = \cdots$	$\cdots$
s^{n-3}	$c_1 = \dfrac{b_1 a_3 - a_1 b_2}{b_1}$	$c_2 = \dfrac{b_1 a_5 - a_1 b_3}{b_1}$	$c_3 = \cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
s^0	$\cdots$	0	0	0

Q5：Use Routh table to check the stability of the following system

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

作答

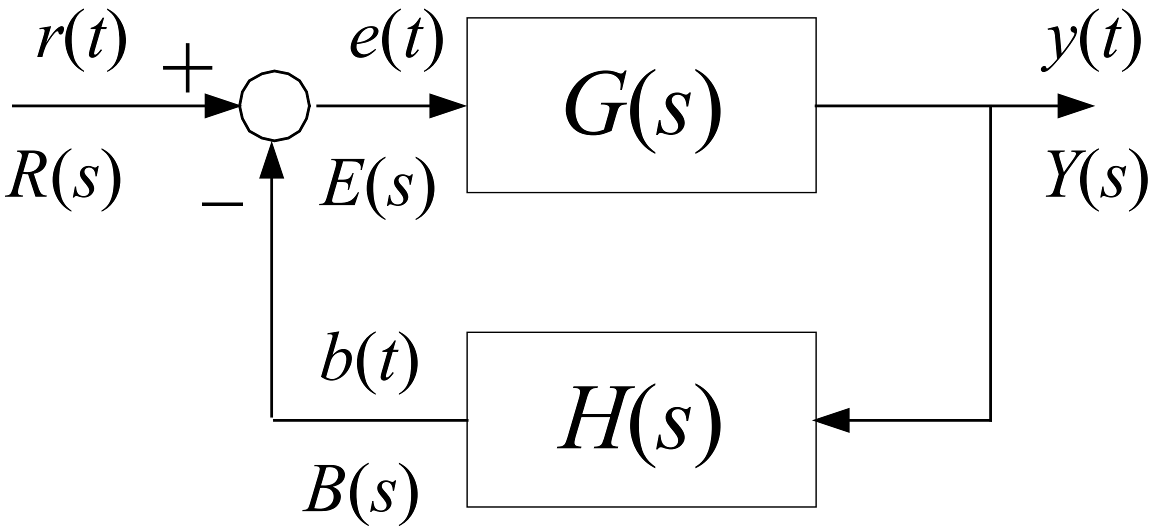
Q5: Use Routh table to check the stability of the following system

$$s^5+4s^4+8s^3+8s^2+7s+4=0$$

A: Routh table

s^5	1	8	7	
s^4	4	8	4	
s^3	$\frac{4\times 8-1\times 8}{4}=6$	$\frac{4\times 7-1\times 4}{4}=6$	0	
s^2	$\frac{6\times 8-4\times 6}{6}=4$	4	0	$\rightarrow$ Auxiliary polynomial, $4s^2+4$
s^1	0	0	0	$\Downarrow$ Derivative
s^1	8	0	0	$\leftarrow$ construct new row, $8s$
s^0	4	0	0	

Steady-state Error



Desirable value real value

$$e(t) = y_r(t) - y(t)$$

$$e(t) = r(t) - b(t)$$

steady-state error

$$e_{ss} = \lim_{t \to \infty} e(t)$$

Final-value theorem

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

Condition:

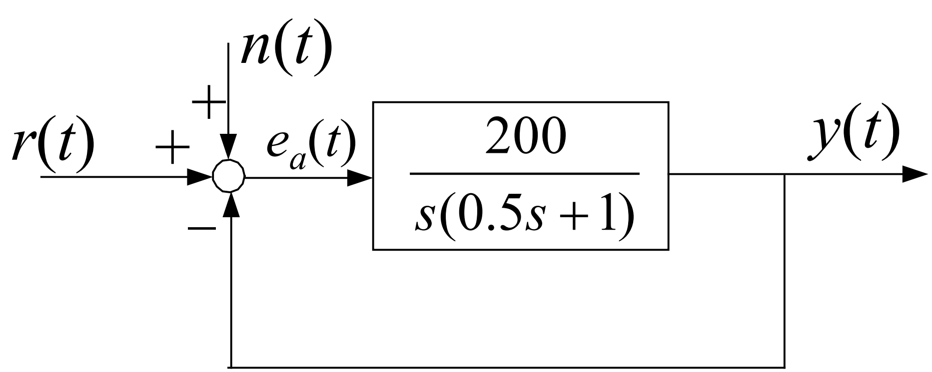
poles of the $sE(s)$ stay at the left half s-plane

the limit of e(t) does exist

主观题 10分

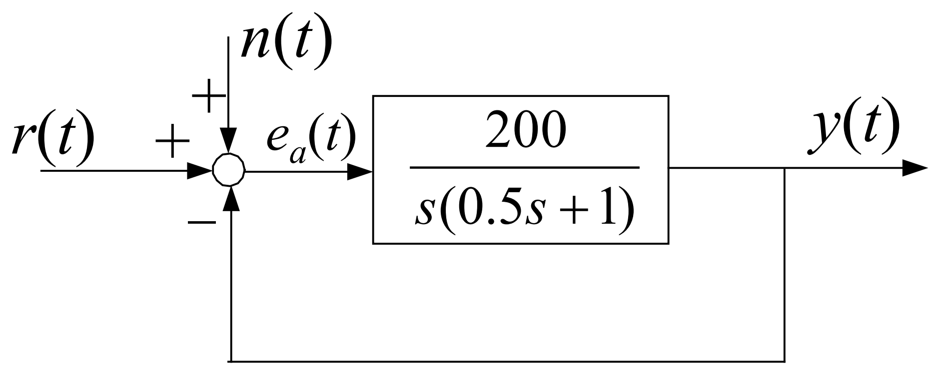
设置

Q6: please find the steady-state error of the given system when the input $r(t)=1(t)$, and the disturbance $n(t)=0.1\times 1(t)$, the system steady-state error is defined as $e(t)=r(t)-y(t)$.



作答

Q6: please find the steady-state error of the given system when the input $r(t)=1(t)$, and the disturbance $n(t)=0.1\times 1(t)$, the system steady-state error is defined as $e(t)=r(t)-y(t)$.



Notice: the definition of the steady-state error is given. Don't take it for granted that $e_a(t)$ is the steady-state error of the system.

(1) Check the stability of the system。 The closed-loop TF is,

$$G(s)=\frac{\dfrac{200}{s(0.5s+1)}}{1+\dfrac{200}{s(0.5s+1)}}=\frac{200}{0.5s^2+s+200} \qquad \text{stable}$$

(2)Calculate the steady-state error according to the final-value theorem。

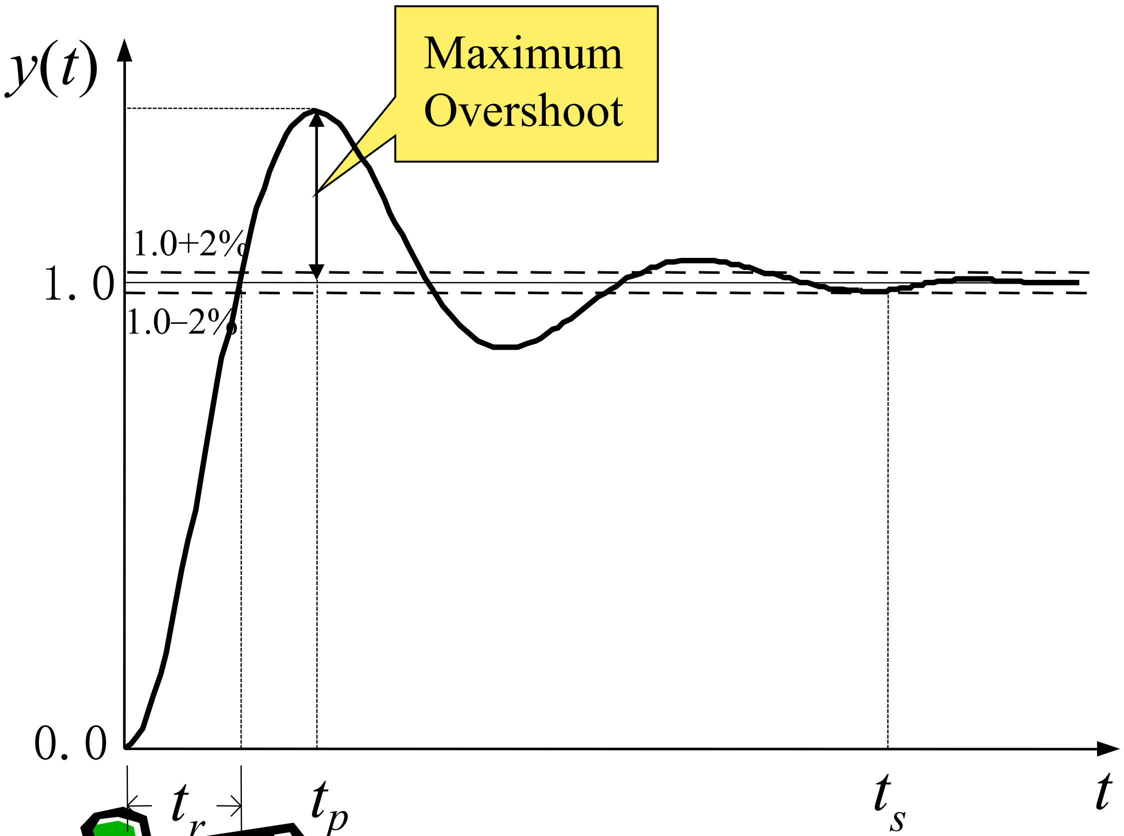
$$E(s)=R(s)\text{-}Y(s)$$

$$E(s)=R(s)-G(s)[R(s)+N(s)]=\frac{1}{s}-\frac{200}{0.5s^2+s+200}\left(\frac{1}{s}+\frac{0.1}{s}\right)$$

$$=\frac{1}{s}\cdot\frac{0.5s^2+s-20}{0.5s^2+s+200}$$

$$e_{ss}=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\cdot\frac{1}{s}\cdot\frac{0.5s^2+s-20}{0.5s^2+s+200}=-0.1$$

Dynamic Performance Indices

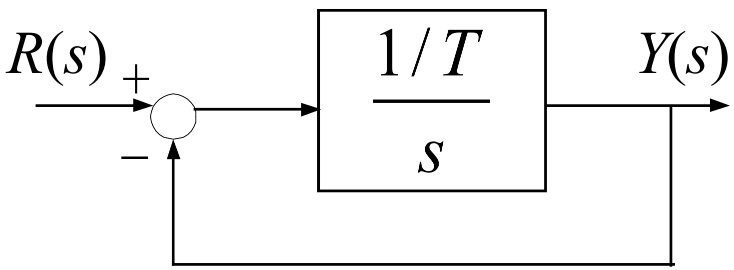


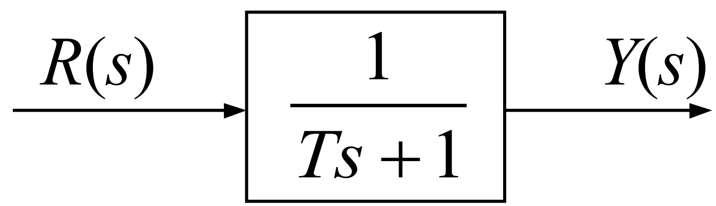
$\sigma\%$	Maximum Overshoot
t_r	Rise Time
t_p	Peak Time
t_s	Settling Time

First-order system

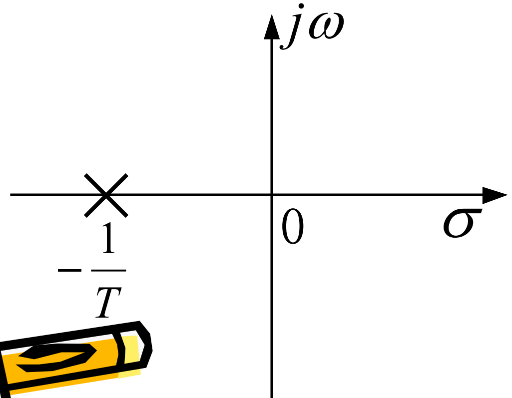
$$G(s)=\frac{Y(s)}{R(s)}=\frac{\frac{1}{T}}{s+\frac{1}{T}}=\frac{1}{Ts+1}$$

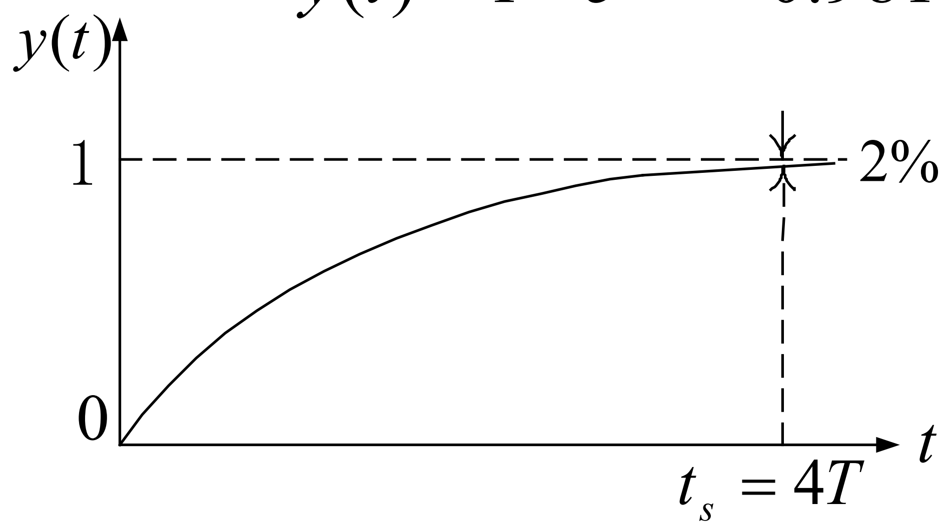
$$y(t)=1-e^{-\frac{1}{T}t}$$





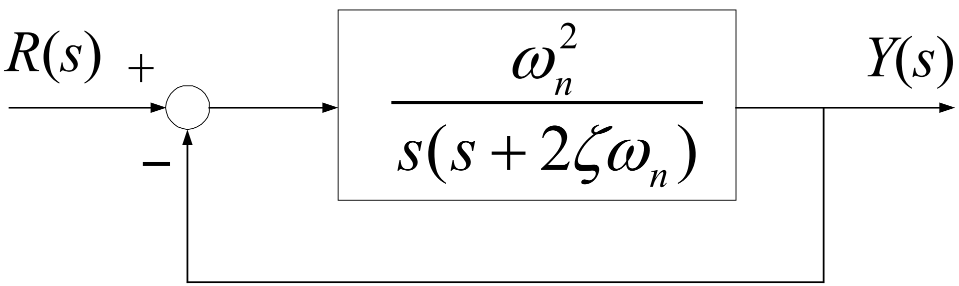
$$y(t)=1-e^{-4t}\approx 0.9817$$

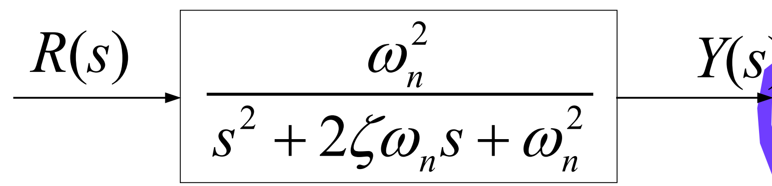




Second-order system

$$G(s)=\frac{Y(s)}{R(s)}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{1}{T^2s^2+2\zeta Ts+1}$$





$T=\dfrac{1}{\omega_n}$ Time constant

ω_n Natural undamped frequency

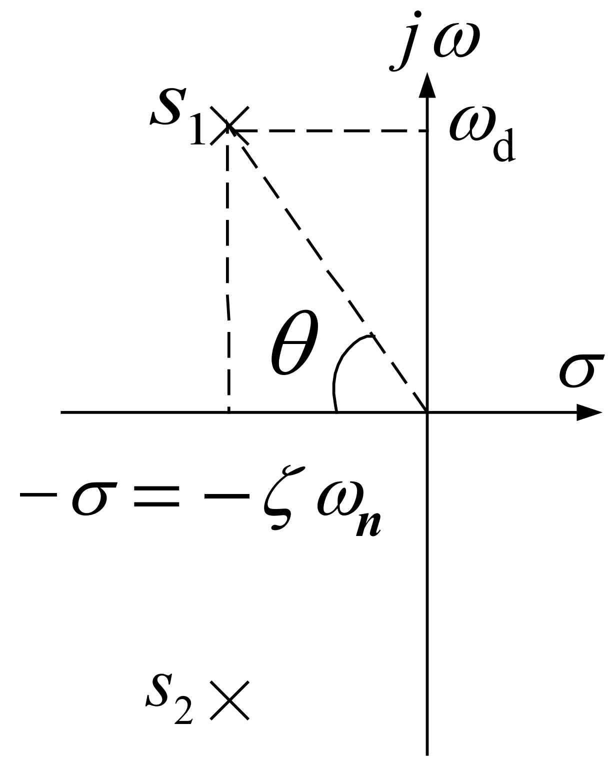
ζ Damping ratio

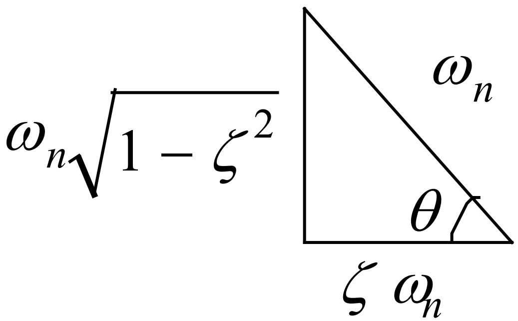
$$0 < \zeta < 1$$

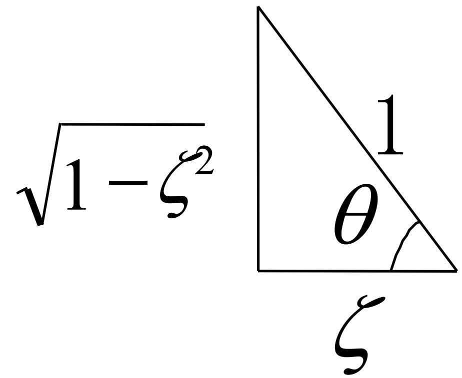
$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\sigma \pm j\omega_d$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$



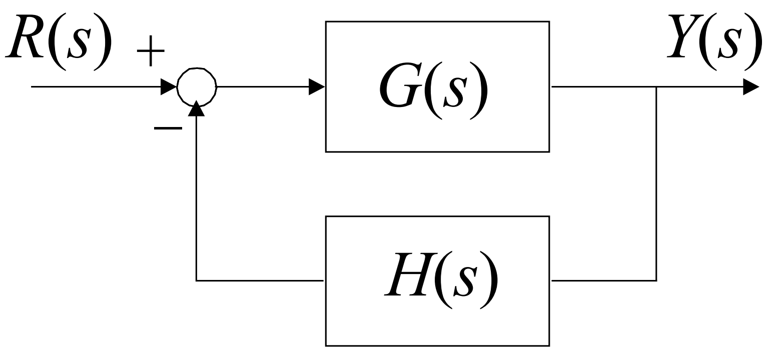




overshoot $$\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$$

settling time $$t_s = \frac{4}{\zeta\omega_n}$$

Root Loci



Closed-loop TF of the system:

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+G_0(s)}$$

The characteristic equation: $1+G_0(s)=0$ or $G_0(s)=-1$

Condition on magnitude if a point is a root of the characteristic equation:

$$|G_0(s)| = 1$$

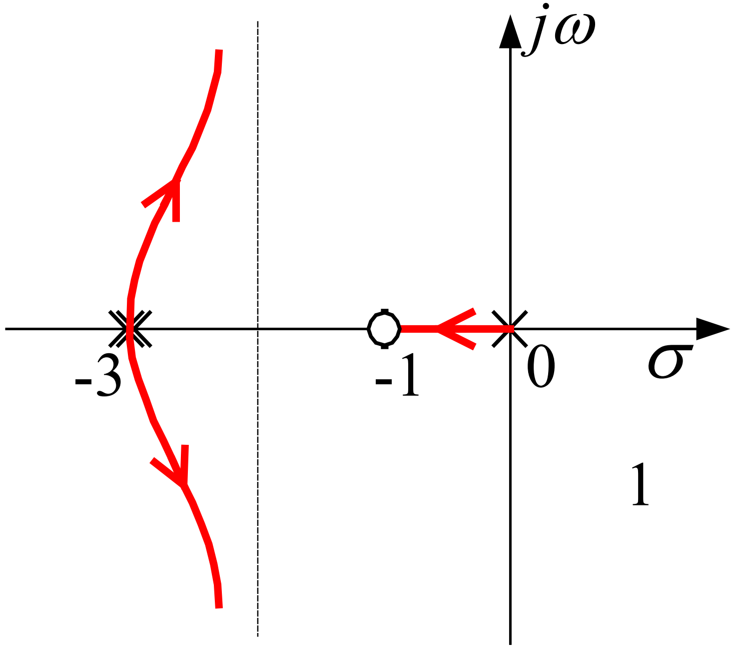
Condition on angle: $\angle G_0(s) = (2k+1)\pi$

Odd multiples of π radians

Principles of constructing root loci

1. Starting and ending point
2. Number of branches
3. Asymptotic lines
4. Symmetry
5. Root loci on the real axis
6. Break-away point
7. Angle of departure and arrival
8. Intersection with the imaginary axis

Q7: please construct the root loci of the unit feedback system with the given open loop transfer function.



$$G_0(s) = \frac{k(s+1)}{s(s+3)^2}$$

Open-loop zeros and poles:

z_1=-1, p_1=0, $p_{2,3}$=—3

Intersection of asymptotes and real axis

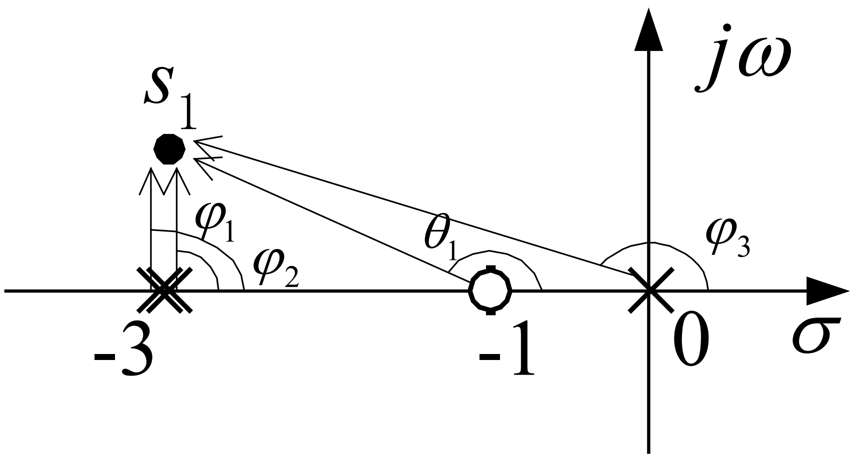
$$F = \frac{-3-3+1}{3-1} = \frac{-5}{2} = -2.5$$

Angle between asymptotes

$$\alpha = \pm 90^0$$

departure angle at -3:

Take a point s_1 right above -3 and very close to -3:

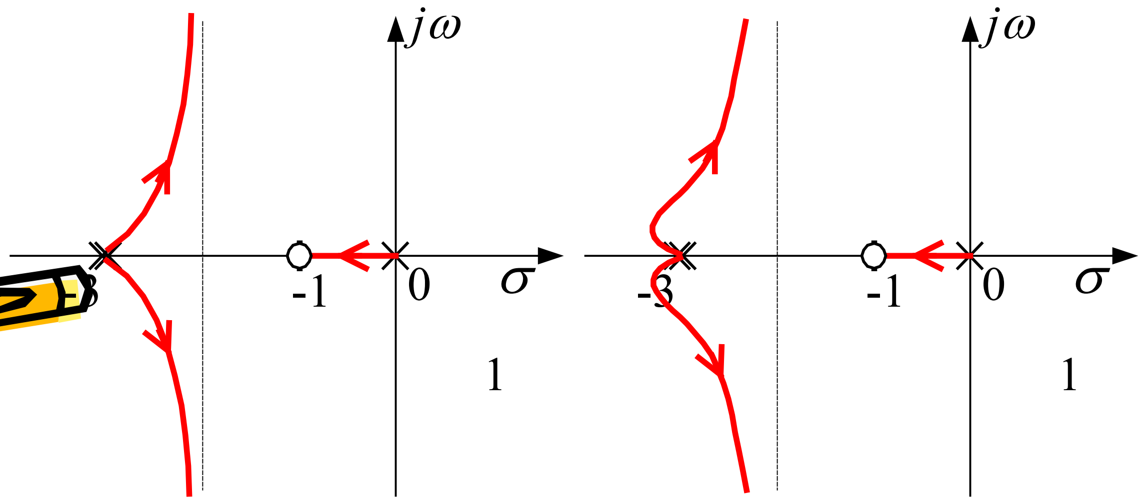


Check the angle condition

$$-\varphi_1 - \varphi_2 - \varphi_3 + \theta_1$$
$$= -90^0 - 90^0 - 180^0 + 180^0$$
$$= -180^0$$

s_1 is on the root loci

So, the following two departure angles are not correct



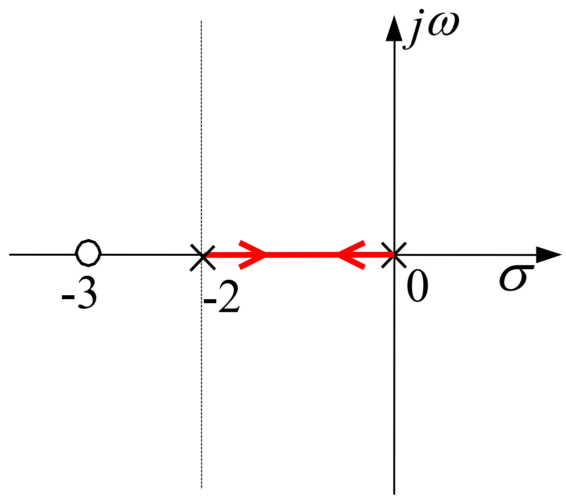
Generally speaking, root loci are vertical to the real axis

Q8: A system has the given open-loop transfer function. Please:
(1) sketch the root loci of the system;
(2) find the open-loop gain and damping ratio where the system has the biggest oscillation;
(3) when the open-loop gain is 2, please find the unit step response.

$$G_0(s) = \frac{K(s+3)}{s(s+2)}$$

A: Open-loop zeros and poles:

z_1=-3， p_1=0， p_2=-2

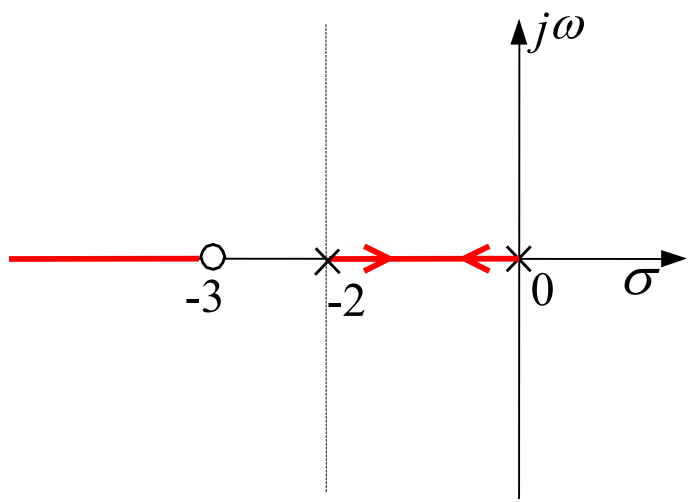


Intersection of asymptotes and real axis

$$F = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} \frac{0-2-(-3)}{2-1} = 1$$

Angle between asymptotes

$$\varphi_\alpha = \frac{(2k+1)\pi}{n-m} = \pi \quad (k=0)$$



Breakaway point:

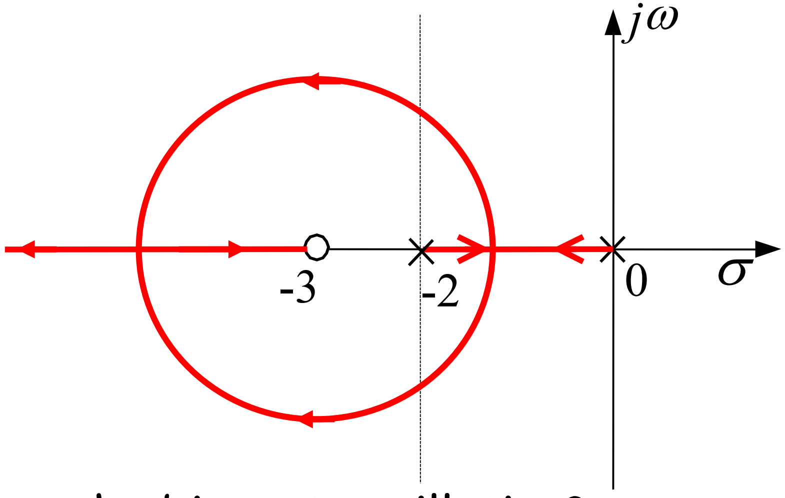
$$\frac{dK}{ds} = 0$$

$$s^2 + 2s + Ks + 3K = 0$$

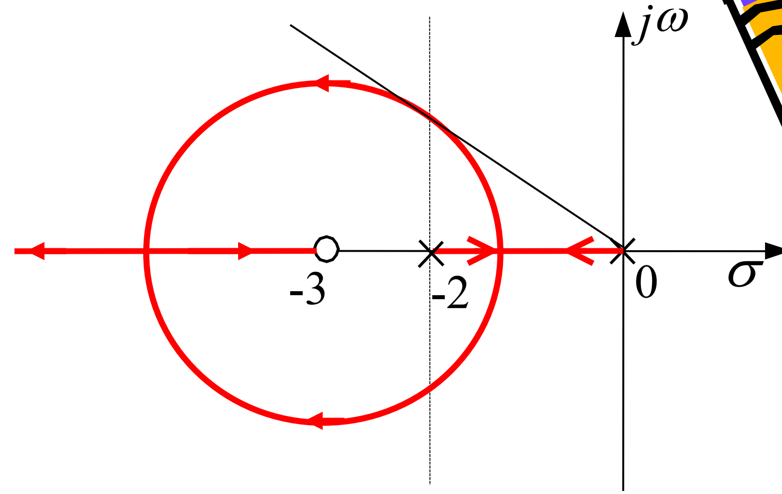
$$K = -\frac{s^2 + 2s}{s + 3}$$

$$\frac{dK}{ds} = s^2 + 6s + 6 = 0$$

$$s_1 = -1.27, \qquad s_2 = -4.73$$



When does the system have the biggest oscillation?



$$\sin\beta = \frac{1.73}{3} = 0.577$$

$$\zeta = \cos\beta = 0.817$$

$$\omega_n = \sqrt{3^2 - 1.73^2} = 2.45$$

$$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -2 + j1.41$$

$$K = \frac{|s_1 - p_1||s_1 - p_2|}{|s_1 - z|} = \frac{2.45 \times 1.41}{1.73} = 2$$

Answer of question 3 is omitted

Frequency-domain analysis

Frequency response

Encircle, Enclosure

Principle of argument

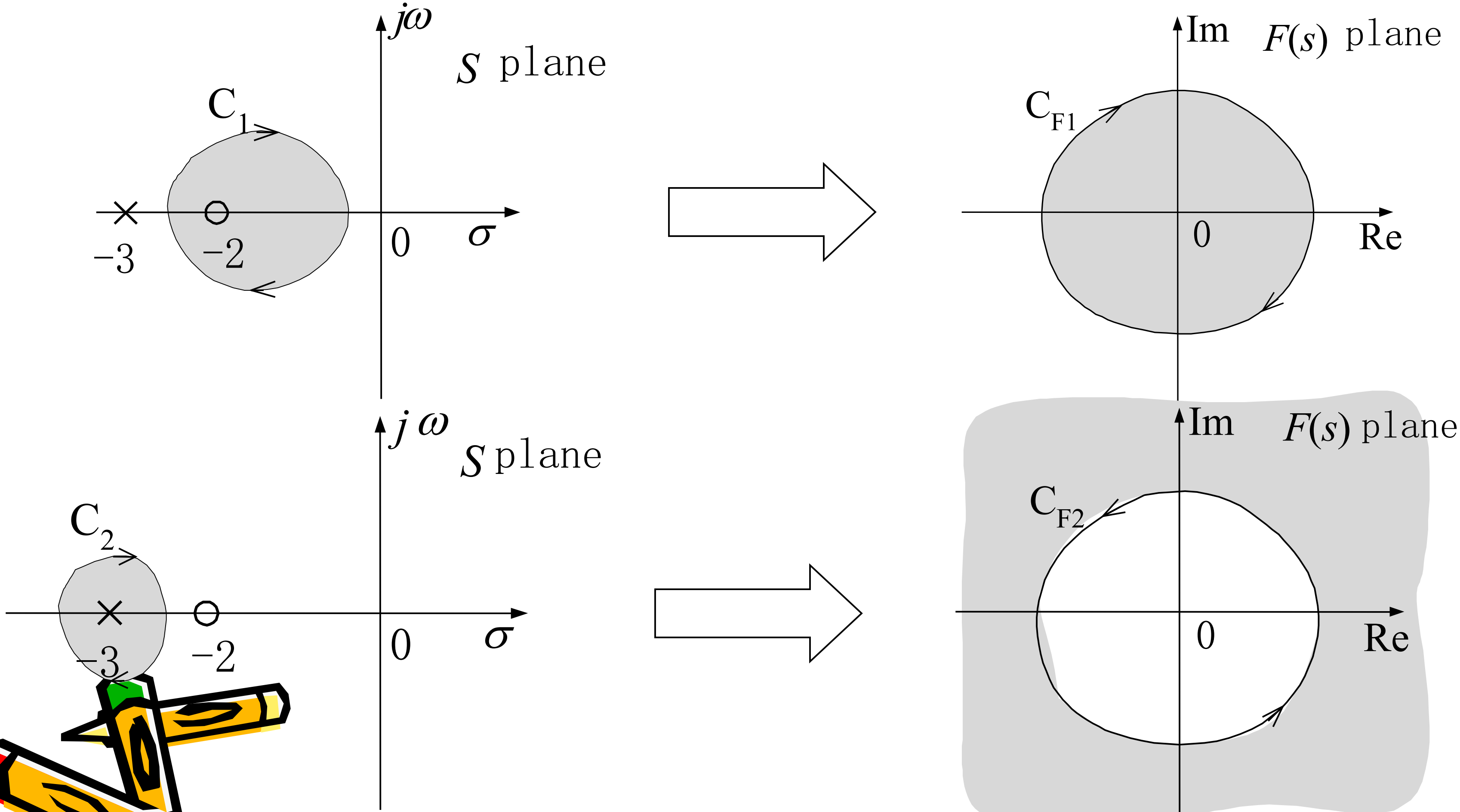
Nyquist path

Nyquist criterion

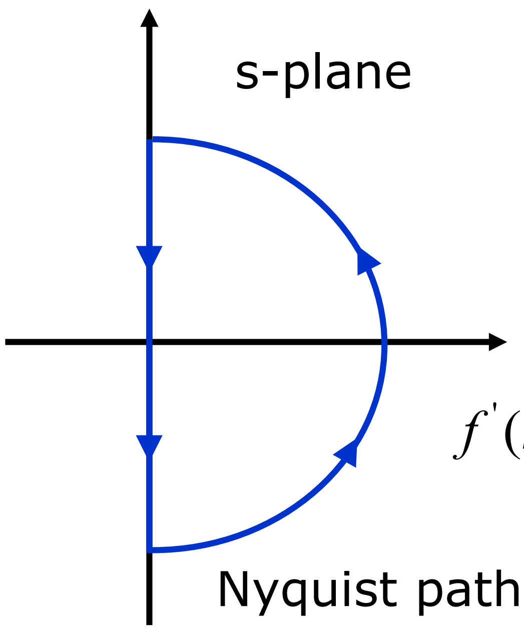
Principle of argument

Complex function mapping

$$F(s) = \frac{s+2}{s+3}$$

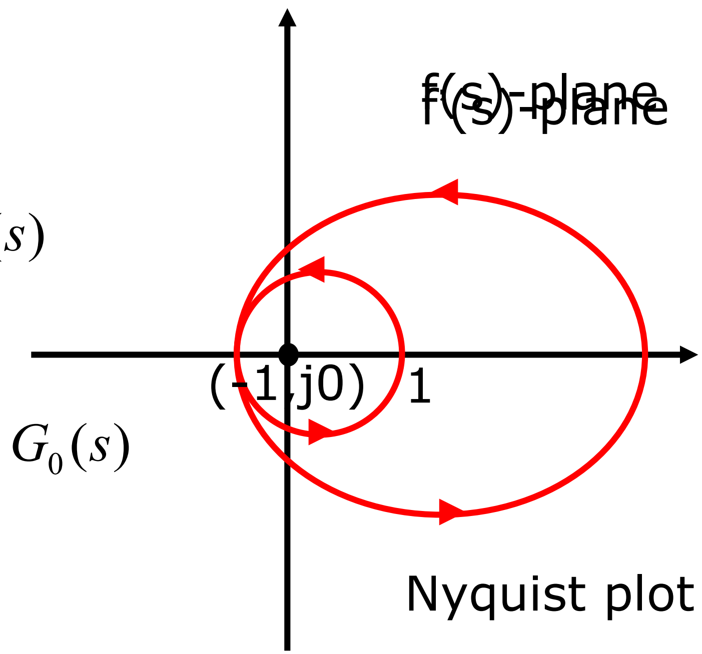


Set $f(s) = 1 + G_0(s) = \dfrac{D_c(s)}{D_0(s)}$ when s varies along the Nyquist path, a corresponding locus is got in the f(s)-plane



$$f(s) = 1 + G_0(s)$$

$$f^{'}(s) = f(s) - 1 = G_0(s)$$



(-1,j0) is the critical point

N=Z-P

N = number of encirclements of the (-1,j0) point made by the $G_0(s)$ plot

Z = number of zeros of $1+G_0(s)$ that are inside the Nyquist path

P = number of poles of $1+G_0(s)$ that are inside the Nyquist path

For closed-loop stability, Z must equal zero

Q9: please construct the nyquist plot of the system with the following open-loop TF, then determine its stability

$$G_0(s) = \frac{k}{s^2(Ts+1)}$$

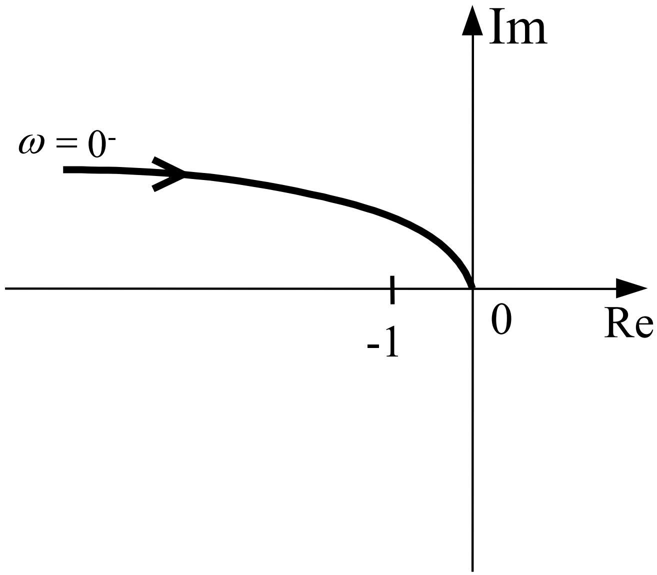
When $\omega=0$ $G(0)=\infty\angle-180^0$

When $\omega=\infty$ $G(j\infty)=0\angle-270^0$

Tendency: when ω vary from 0 to ∞

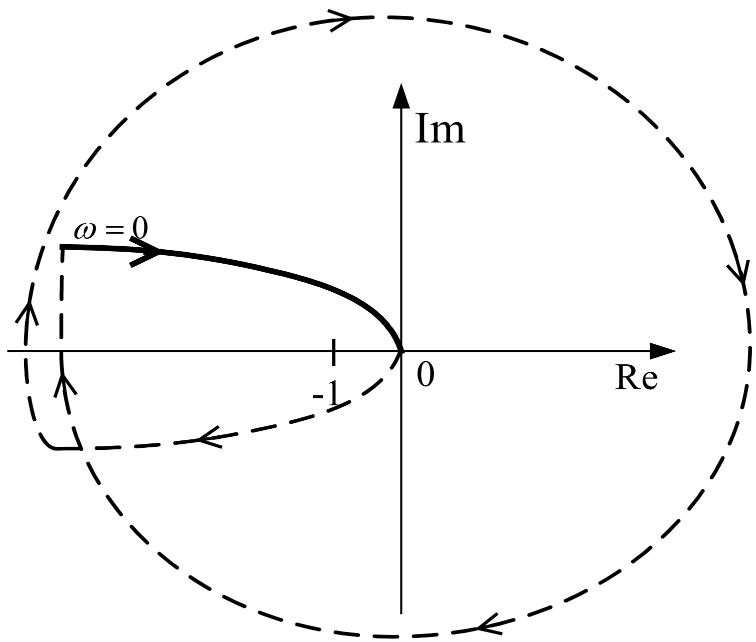
the magnitude of $G(j\omega)$ vary from ∞ to 0 monotonically

the angle of $G(j\omega)$ vary from -180^0 to -90^0 monotonically



the open loop system has two integration blocks,

So, at the infinity point, the points on the Nyquist plot, which are corresponding to 0^- and 0^+, should get closed after $180^0 \times 2 = 360^0$



N=2, n=0, m=2

Unstable

- Bode plot is another graphical expression of the frequency response of a system

- The Bode plot of the function $G(j\omega)$ is composed of two plots, one with the amplitude of the $G(j\omega)$ in decibels (dB) versus $\lg\omega$ or ω, and the other with the phase of $G(j\omega)$ in degrees as a function of $\lg\omega$ or ω.

Q10: please construct the Bode plot of the following system

$$G(s)=\frac{8(s+0.1)}{s(s^2+s+1)(s^2+4s+25)}=\frac{0.032(10s+1)}{s(s^2+s+1)(\frac{1}{25}s^2+\frac{4}{25}s+1)}$$

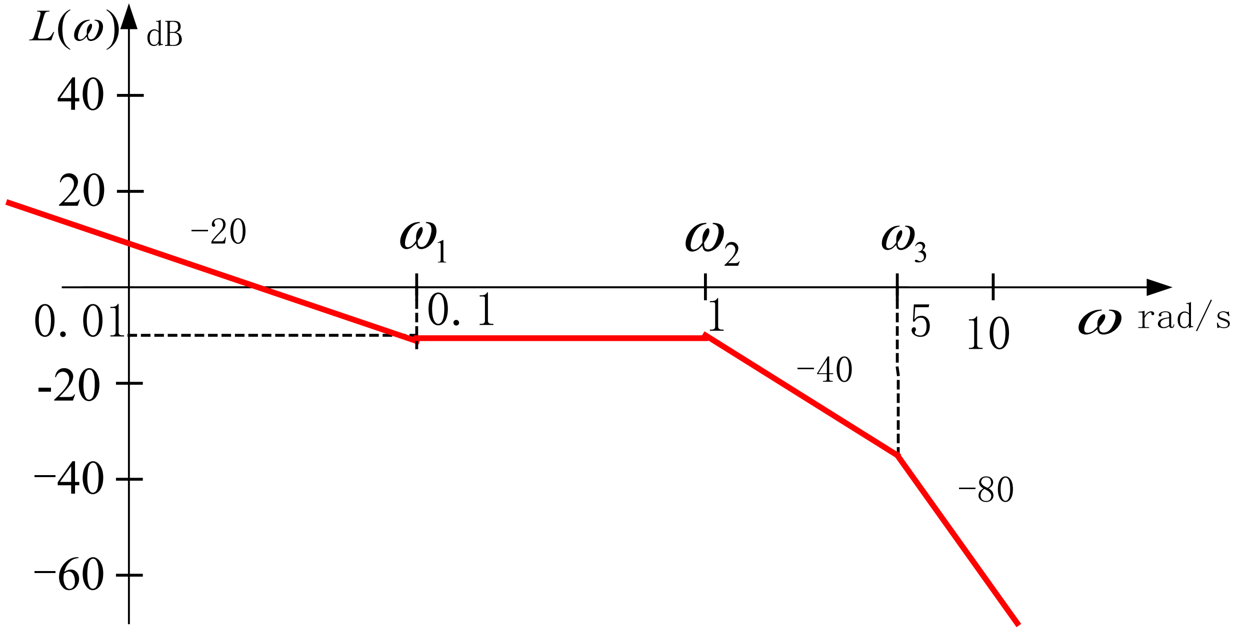
(1) Corner frequency: $\omega_1=0.1 \qquad +20\text{db/dec}$

$\omega_2=1 \qquad -40\text{db/dec}$

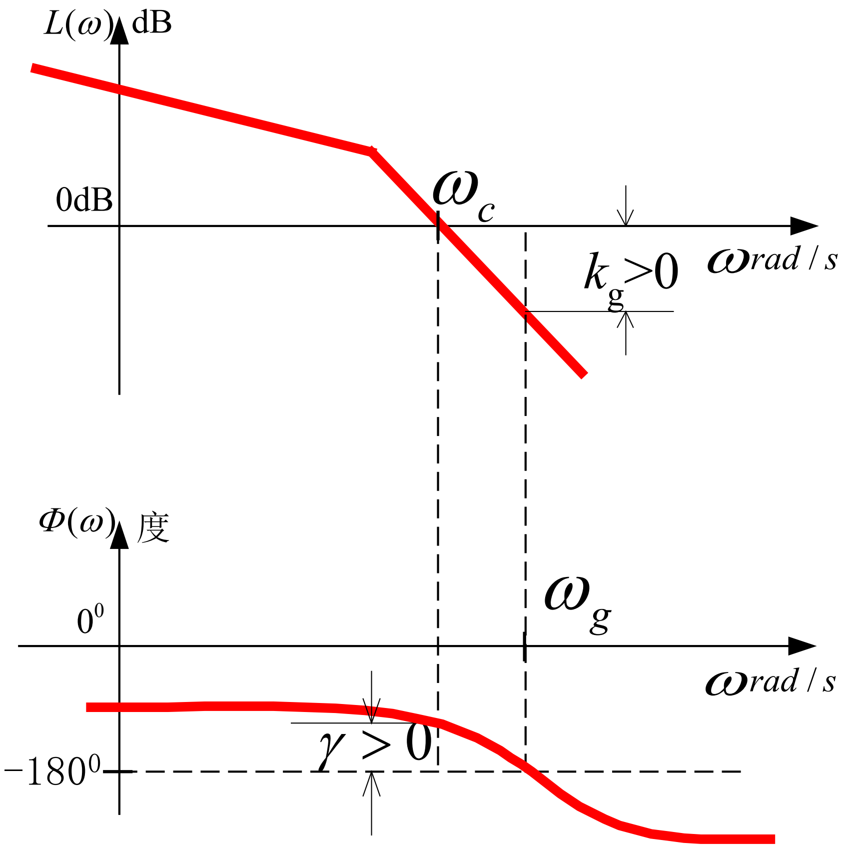
$\omega_3=5 \qquad -40\text{db/dec}$

(2) Low-frequency band $\quad \omega=0.1$

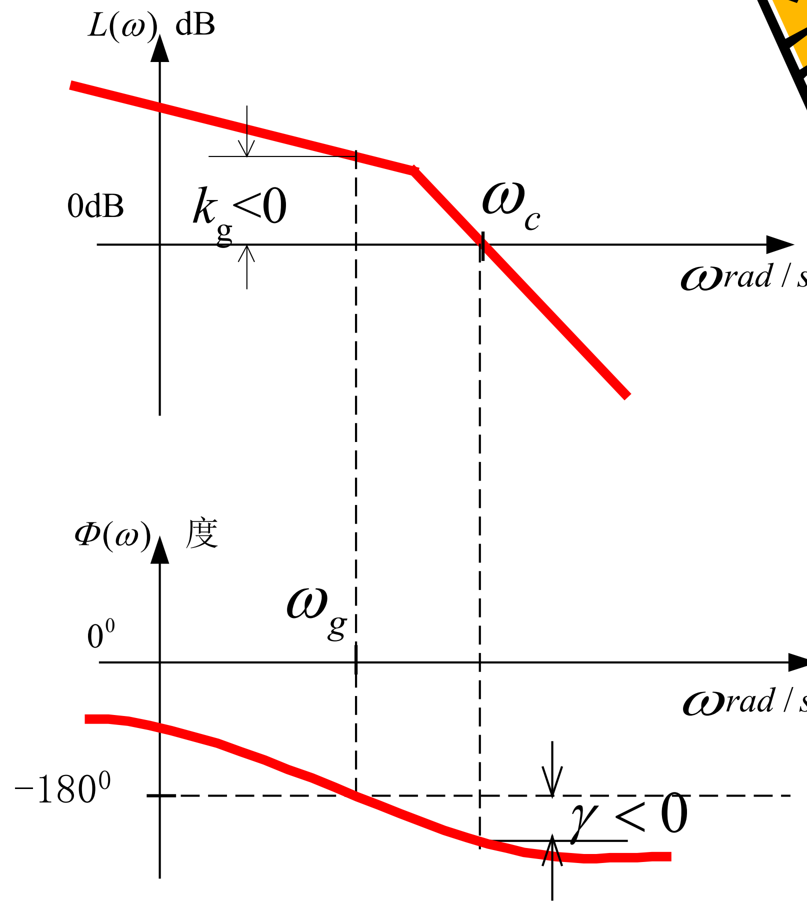
$$L(\omega)=20\lg(\frac{k}{\omega})=20\lg(\frac{0.032}{0.1})=-10\text{dB}$$


L(ω) dB
40
20
-20
-40
-60
−20
−40
−80
ω1
ω2
ω3
0. 01
0. 1
1
5
10
ω rad/s

Gain Margin and Phase Margin

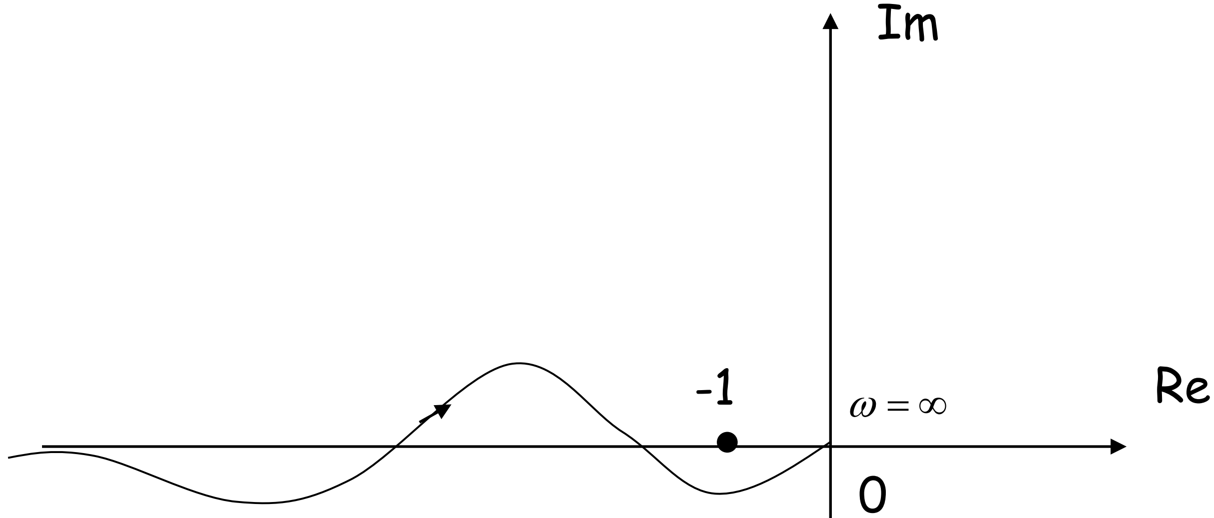


Stable



unstable

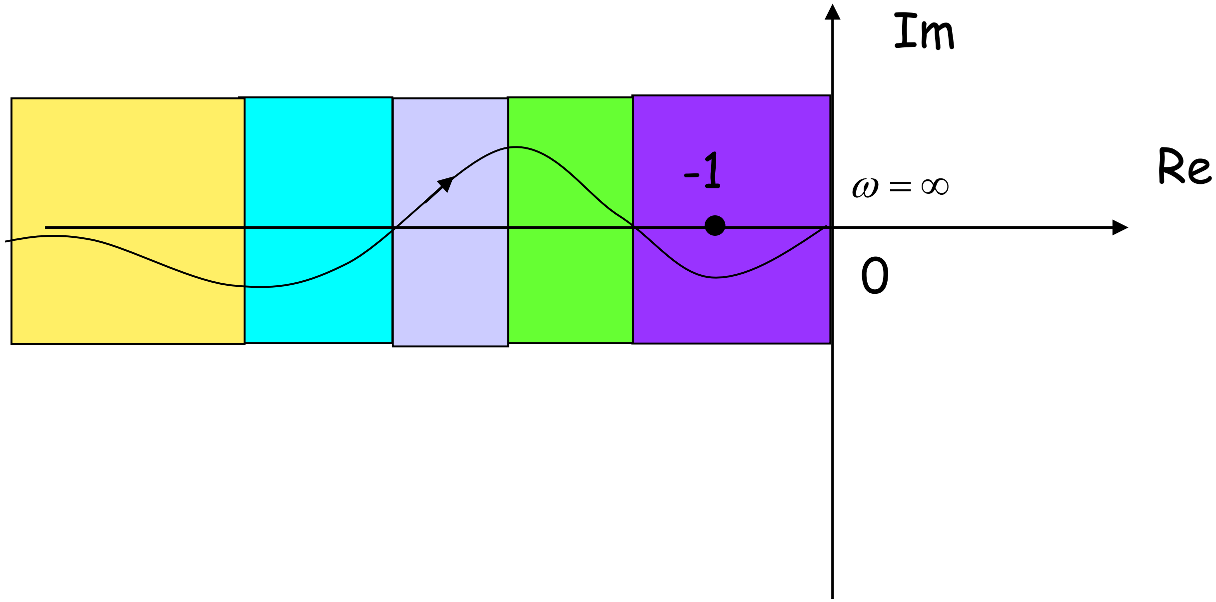
Q11: The frequency response of the open-loop transfer function of a minimum phase system is given as follows, please
(1) sketch the open-loop transfer function of the system;
(2) use Nyquist criterion to estimate the stability of the system;
(3) mark the gain-crossover, phase-crossover point, the phase margin



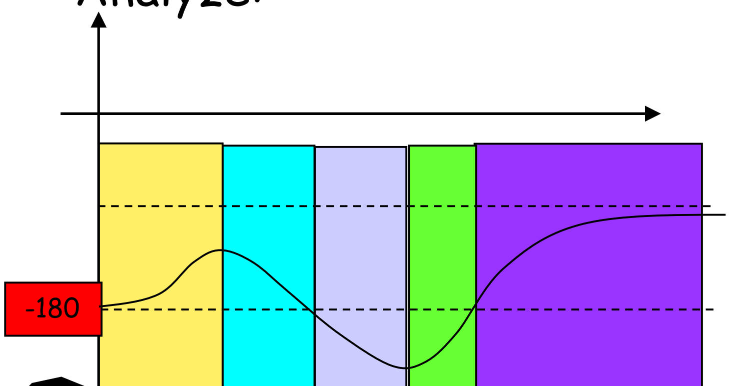
Start where?

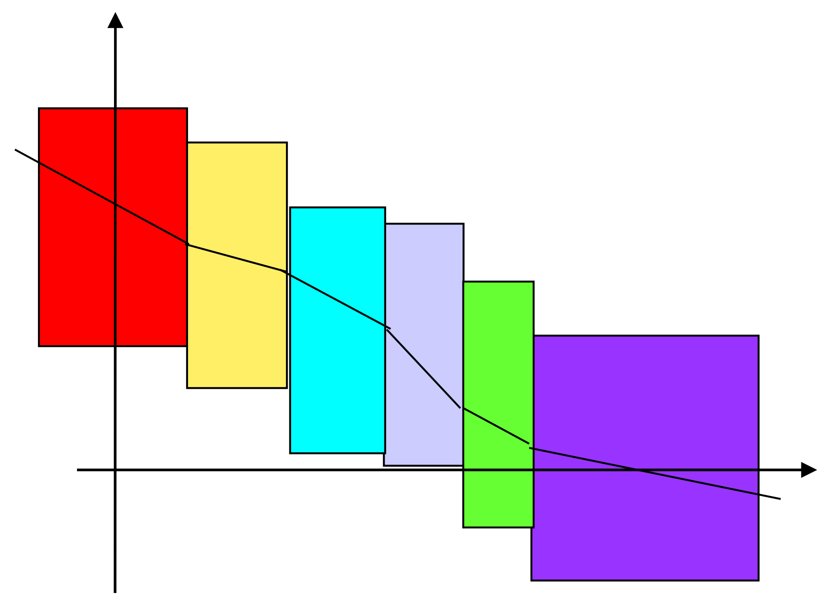
Bode plot?

phase or magnitude plot?

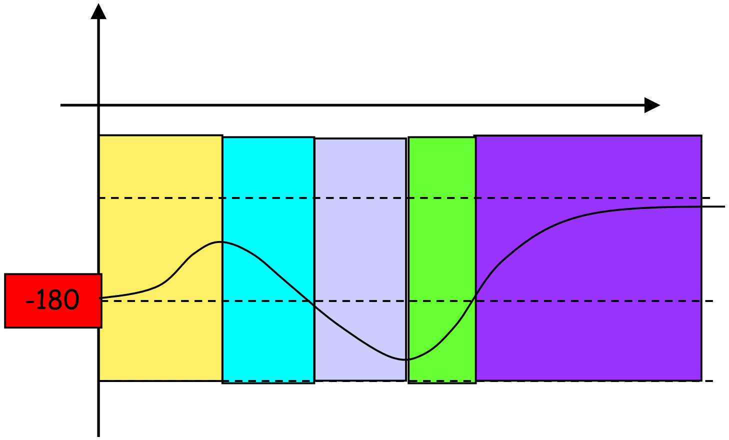


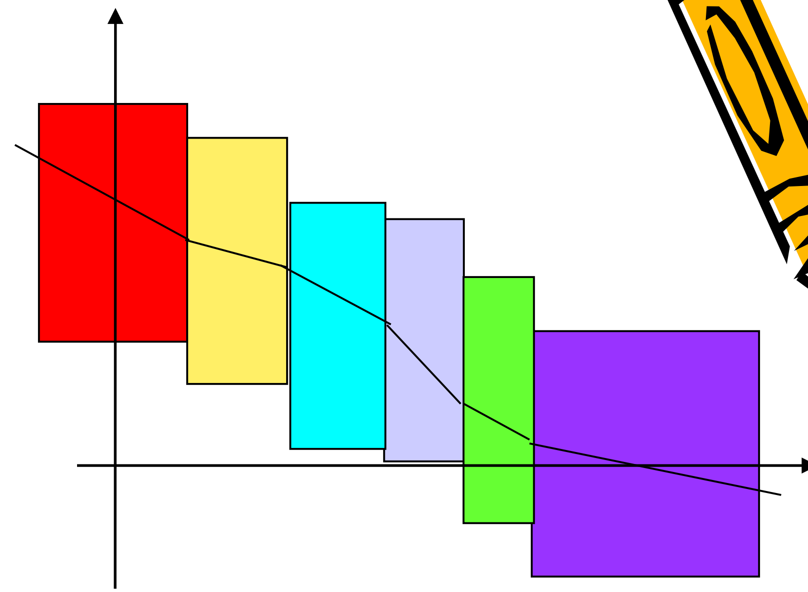
Analyze:





Analyze:



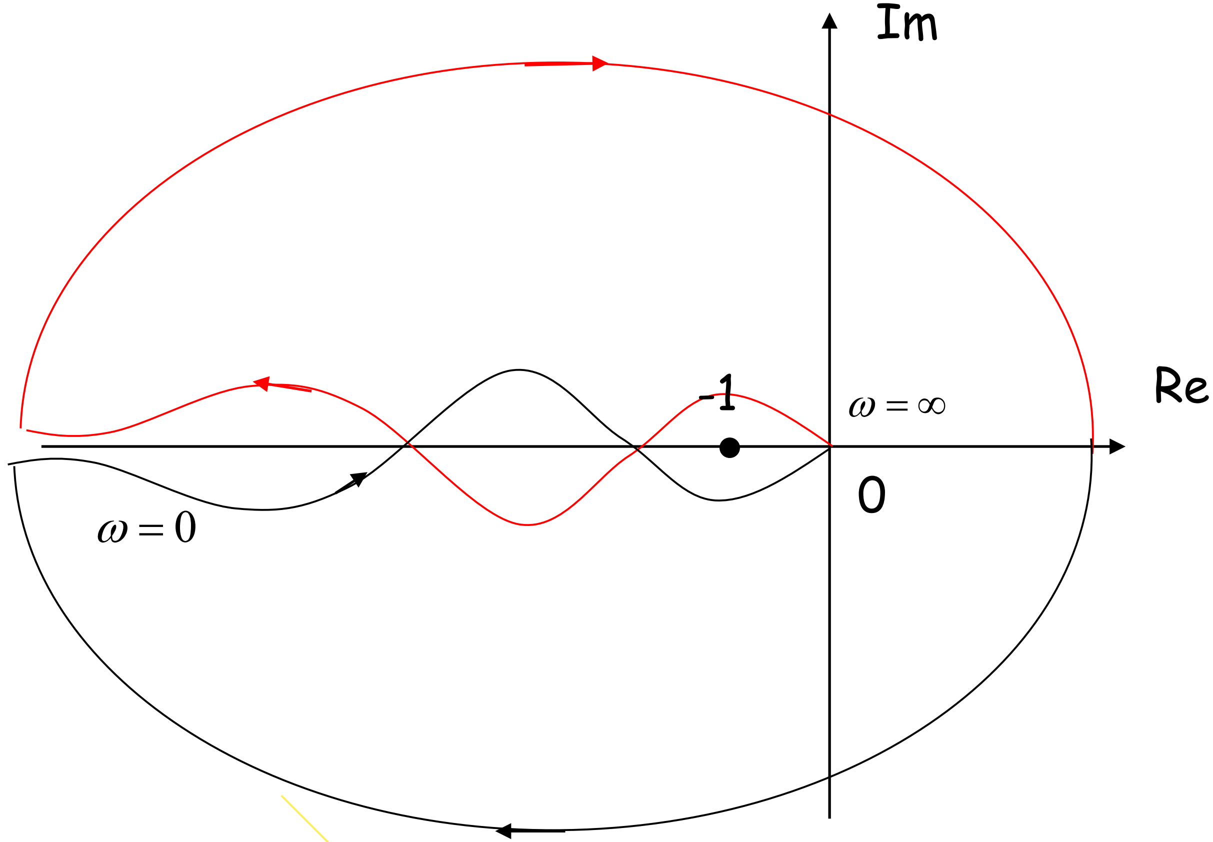


Conclusion:

(1) type II system
(2) three PD blocks
(3) two inertial blocks

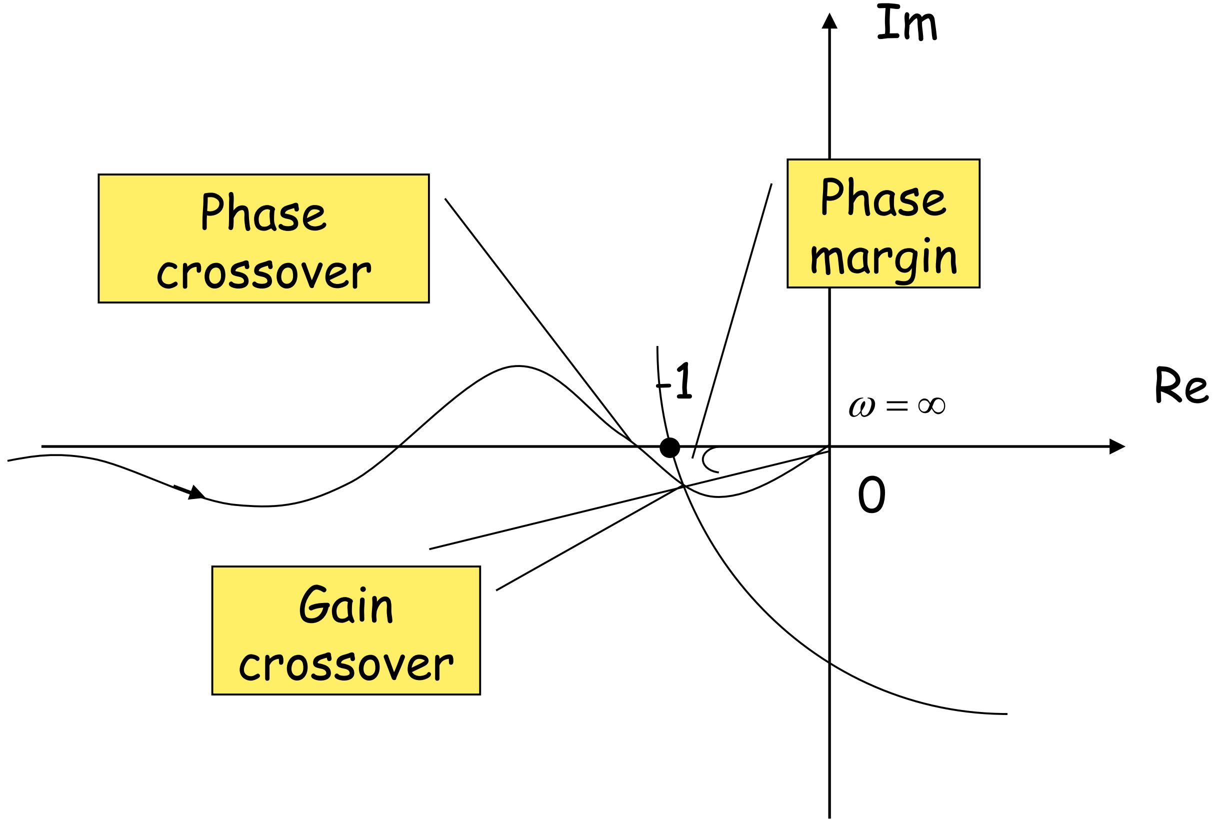
$$G(s) = \frac{K(T_1 s+1)(T_4 s+1)(T_5 s+1)}{s^2 (T_2 s+1)(T_3 s+1)}$$

$$T_1 > T_2 > T_3 > T_4 > T_5$$



N=Z-P

N=0, P=0, so Z=0, system is stable


Im
Phase crossover
Phase margin
-1
Re
$\omega = \infty$
0
Gain crossover

Q12: The open loop transfer function of a unit feedback system is as follows. Please design a phase lead controller to make the ramp error constant of the system to be $K_v = 100$, and the phase margin $\gamma \geq 45$

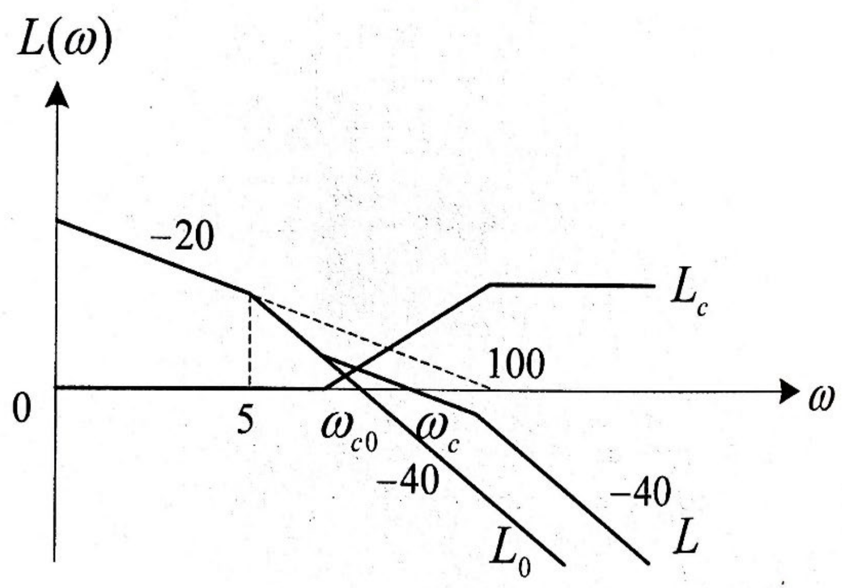
$$G_0(s) = \frac{100K}{s\left(\frac{s}{5}+1\right)}$$

A: determine the open loop gain:

$$K_v = \lim_{s\to 0} sG_0(s) = \lim_{s\to 0} s\cdot\frac{100K}{s\left(\frac{s}{5}+1\right)} = 100K = 100$$

$$K = 1$$

Sketch the magnitute plot:



Calculate the cross over frequency:

$$G_0(s) = \frac{100}{s\left(\frac{s}{5}+1\right)}$$

$$\left|G_0(j\omega)\right| = \begin{cases} \dfrac{100}{\omega} & \omega < 5 \\ \dfrac{100}{\omega \cdot \dfrac{\omega}{5}} & \omega \geqslant 5 \end{cases}$$

$$\omega_{c0} = \sqrt{500} = 22.4 rad / s$$

The phase margin is:

$$\gamma_0 = 180^\circ + \angle G_0\left(j\omega_{c0}\right) = 180^\circ - 90^\circ - \tan^{-1} 0.2\omega_{c0} = 12.6^\circ < 45^\circ$$

The phase lead controller is:

$$G(s) = \frac{1+aTs}{1+Ts}$$

The phase need to be added is:

$$\varphi_m = \gamma - \gamma_0 + \left(5 \sim 10^\circ\right) = 45^\circ - 12.6 + 7.6^\circ = 40^\circ$$

Calculate a: $$a=\frac{1+\sin\varphi_m}{1-\sin\varphi_m}=4.6$$

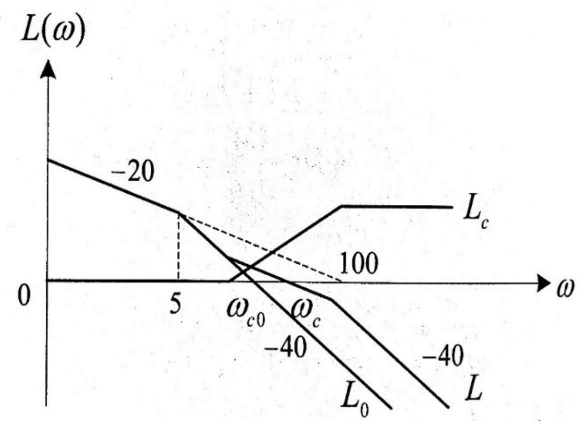
The maximum leading phase should be added to the new cross-over frequency:

$$-10\lg a=-40\lg\frac{\omega_c}{\omega_{c0}}$$

$$-10\lg 4.6=-40\lg\frac{\omega_c}{22.4}$$

$$\omega_c=32.8\text{rad/s}$$

$$T=\frac{1}{\omega_c\sqrt{a}}=\frac{1}{32.8\times\sqrt{4.6}}=0.0141$$



The phase lead controller is:

$$G_c(s) = \frac{1+0.0634s}{1+0.0141s}$$

Double check the phase margin of the compensated system:

$$\gamma = 180^\circ - 90^\circ + \tan^{-1} 0.0634\omega_c - \tan^{-1} 0.2\omega_c - \tan^{-1} 0.0141\omega_c = 48.2^\circ > 45^\circ$$

All the requirements have been satisfied!