Review & Exercise

For Modern Control and Digital Control Theory

Chapter 6 State Feedback Control Systems

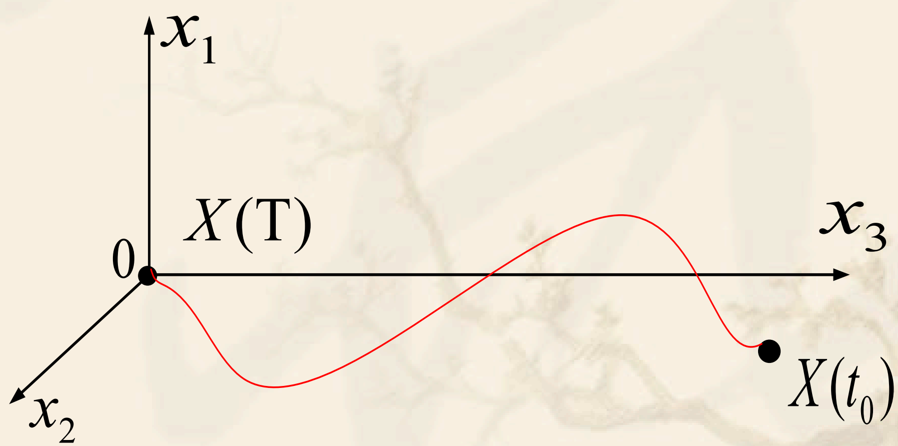
Controllability

a. Definition

b. Criterion

c. CCF

$$\left.\begin{array}{c} LTI \\ \\ u(t) \\ \\ T \\ \\ x(t_0) \neq 0 \end{array}\right\} \Rightarrow x(T) = 0$$



$$S = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}_{n\times np}$$

$$rank\left[S\right] = n$$

$$\dot{x} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u$$

$$\lambda_i \neq \lambda_j$$

$$b_i \neq 0$$

CCF:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

Example1： please determine a、b、c to make the following system controllable.

$$\dot{X} = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} X + \begin{bmatrix} a \\ b \\ c \end{bmatrix} u$$

Example1：please determine a、b、c to make the following system controllable.

$$\dot{X}=\begin{bmatrix}\lambda & 1 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}X+\begin{bmatrix}a\\ b\\ c\end{bmatrix}u$$

Solution: calculate the controllability matrix

$$A = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \qquad A^2 = \begin{bmatrix} \lambda^2 & 2\lambda & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix}$$

$$S = \begin{bmatrix} a & a\lambda + b & a\lambda^2 + 2b\lambda \\ b & b\lambda & b\lambda^2 \\ c & c\lambda & c\lambda^2 \end{bmatrix}$$

$$S = \begin{bmatrix} a & a\lambda + b & a\lambda^2 + 2b\lambda \\ b & b\lambda & b\lambda^2 \\ c & c\lambda & c\lambda^2 \end{bmatrix}$$

Because S has two identical rows, the system is uncontrollable no mater what values a, b and c take.

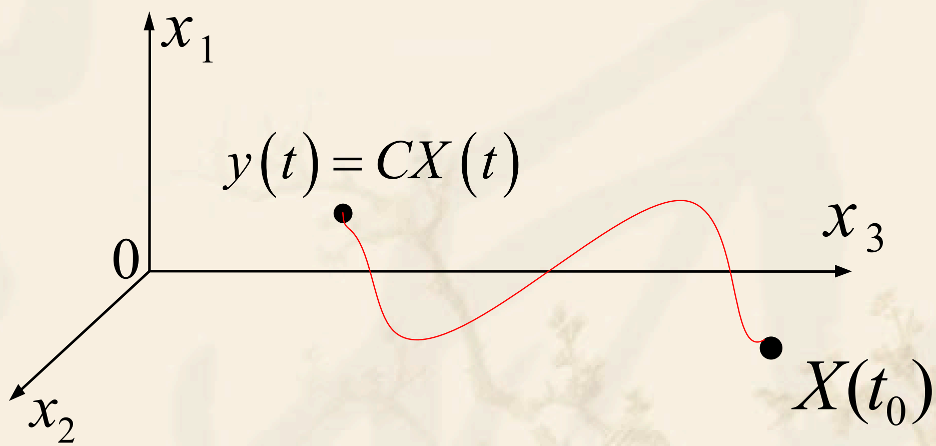
Observability

a. Definition

b. Criterion

c. OCF

$$\left.\begin{aligned} &LTI \\ &u(t) \\ &t_0 \to T \\ &y(T) \neq 0 \end{aligned}\right\} \Longrightarrow x(t_0)$$



$$V = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}_{nq\times n}$$

$$rank\left[V\right] = n$$

$$\dot{X} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} X + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u$$

$$\lambda_i \neq \lambda_j$$

$$y = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} X$$

$$c_i \neq 0$$

$$\dot{X} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_1 \\ 1 & 0 & \cdots & 0 & -a_2 \\ 0 & 1 & \cdots & 0 & -a_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_n \end{bmatrix} X + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} X$$

Will linear transformation change:

1. Controllability
2. Observability
3. Characteristic roots
4. Transfer function (matrix)

投票 最多可选1项

设置

Will linear transformation change the observability of a control system?

A Yes

B No

提交

Will linear transformation change the controllability of a control system?

A Yes

B No

提交

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设置

Will linear transformation change the characteristic roots of a control system?

A Yes

B No

提交

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设置

Will linear transformation change the transfer function of a control system?

A Yes

B No

提交

Example2：please determine a、b to make the following system both controllable and observable.

$$\dot{X} = \begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} X$$

作答

Example2：please determine a、b to make the following system both controllable and observable.

$$\dot{X} = \begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} X$$

Solution：

Controllability matrix：

$$S=\begin{bmatrix} b & Ab \end{bmatrix}=\begin{bmatrix} 1 & a-1 \\ -1 & -b \end{bmatrix}$$

,

$$\det\left[S\right]\neq 0$$

$$b\neq a-1$$

Observability matrix：

$$V=\begin{bmatrix} c^{\mathrm{T}} \\ c^{\mathrm{T}}A \end{bmatrix}=\begin{bmatrix} 1 & -1 \\ a & 1-b \end{bmatrix}$$

$$\det\left[V\right]\neq 0$$

$$1-b\neq -a$$

Controllability and observability vs. zero-pole cancellation

Controllable & observable

$\Longleftrightarrow$ No zero-pole cancellation

Transfer function

$\Longleftrightarrow$ controllable & observable modes

Pole placement

- Condition
- Undetermined coefficients

Questions

- Can pole-placement change the controllability of a LTI system？

- Can pole-placement change the observability of a LTI system？

Example 3: A given system with a transfer function of $G(s) = 1/s^2$

(1) Please design its state feedback so that the compensated system has a damping ratio of $\zeta = 0.7$ and an undamped natural frequency of $\omega_n = 5\ s^{-1}$

(2) Sketch the state diagram of the compensated system, and find its transfer function.

作答

Example 3: A given system with a transfer function of $G(s)=1/s^2$

(1) Please design its state feedback so that the compensated system has a damping ratio of $\zeta=0.7$ and an undamped natural frequency of $\omega_n=5\ s^{-1}$

(2) Sketch the state diagram of the compensated system, and find its transfer function.

Considering: The original system is a second-order system, state-feedback would not change the order of the system.

State-feedback will change the characteristic roots of the system, consequently change the dynamics of the system.

Solution: (1) set up state space model for the system

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2}$$

CCF of the system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

After introduce state-feedback

$$A - BK = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$$

The new characteristic equation is

$$\left|sI-(A-BK)\right| = s^2 + k_2 s + k_1 = 0$$

The desired characteristic equation is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

So

$$K = \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} \omega_n^2 & 2\zeta\omega_n \end{bmatrix} = \begin{bmatrix} 25 & 7 \end{bmatrix}$$

(2) The transfer function is

$$G_{new}(s) = C(sI - (A - BK))^{-1} B$$

What is C ?

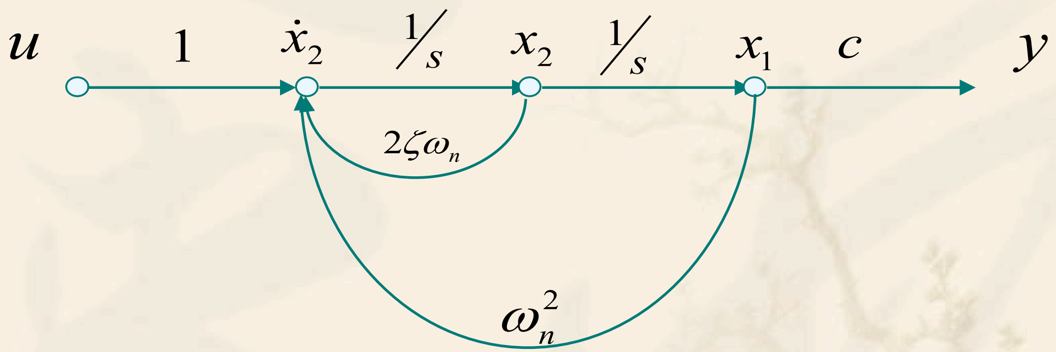
Assume $y = \begin{bmatrix} c & 0 \end{bmatrix} X$ $\qquad C = \begin{bmatrix} c & 0 \end{bmatrix}$

$$\begin{aligned} G_{new}(s) &= C(sI - (A - BK))^{-1} B \\ &= \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{c}{s^2 + 2\zeta\omega_n s + \omega_n^2} \end{aligned}$$

$$u = r - BK$$

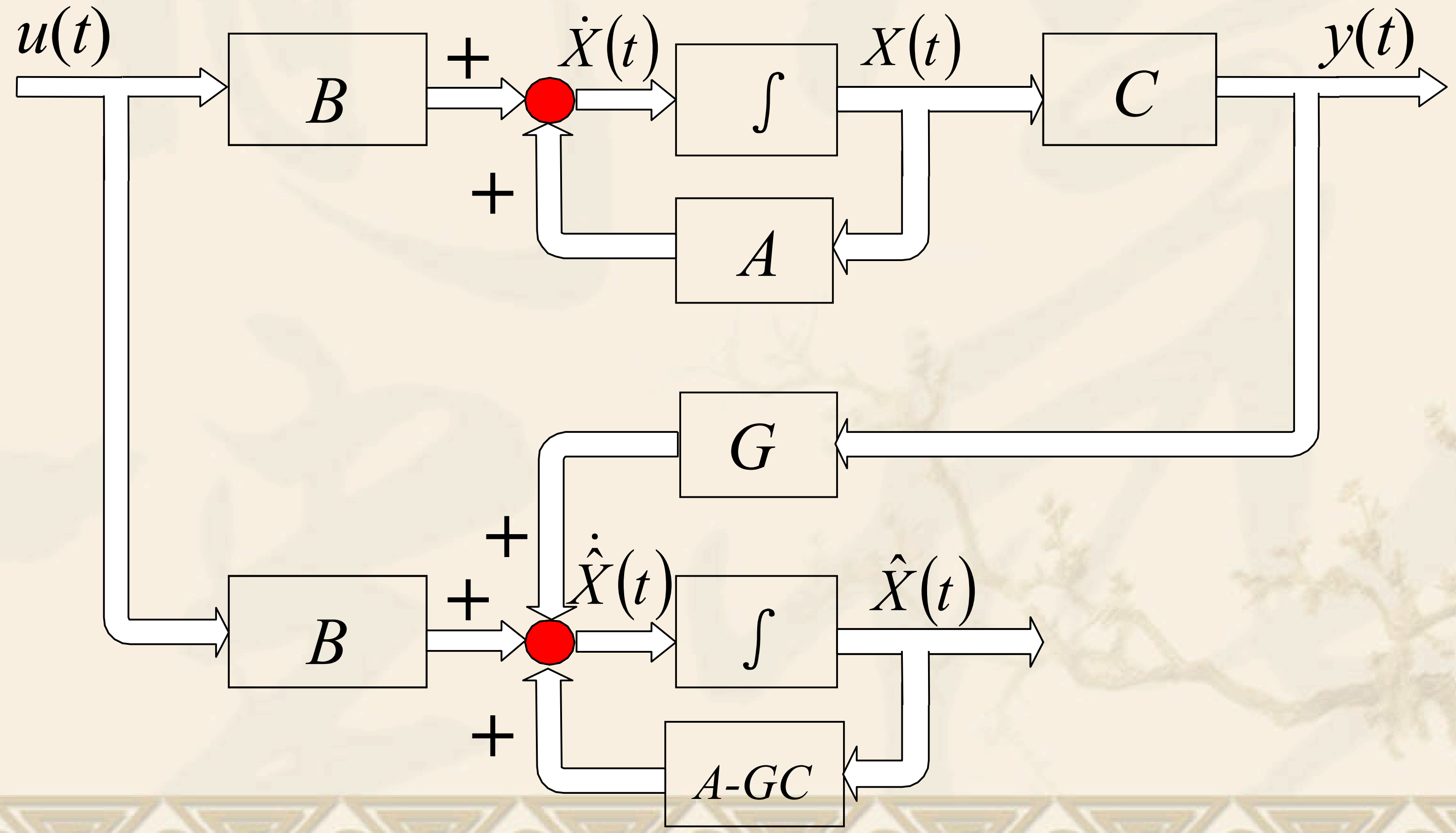
The new state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega_n^2 & 2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$



State observer (estimator)

$$\dot{\hat{X}} = (A - GC)\hat{X} + Bu + Gy$$



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Example 4 (1) please design a state observer with poles at -3 and -4 for the given system; (2) please give the state equation of the observer; (3) if the state feedback $u = K^T \hat{X} + V$, where $K^T = [-2, -3]$, $\hat{X}$ is the estimated state, please give out the transfer function of the closed-loop system with the controlled process, state estimator and state feedback.

$$\dot{X} = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} X$$

作答

Example 4 (1) please design a state observer with poles at -3 and -4 for the given system; (2) please give the state equation of the observer; (3) if the state feedback $u = r - K^T \hat{X}$, where $K^T = [-2, -3]$, $\hat{X}$ is the estimated state, please give out the transfer function of the closed-loop system with the controlled process, state estimator and state feedback.

$$\dot{X} = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} X$$

Solution： ① check the observability of the system

$$V = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$$

$$rank\left[V\right] = 2$$

the system is observable

the desired characteristic equation of the observer:

$$(s+3)\left(s+4\right) = s^2 + 7s + 12$$

suppose the gain matrix

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

the state equation of the observer

$$\dot{\hat{X}} = \left(A - GC\right)\hat{X} + bu + Gy$$

characteristic equation of the observer

$$f\left(s\right) = \det\left[sI - \left(A - GC\right)\right] = \det\begin{bmatrix} s - 1 + 2g_1 & -1 + g_1 \\ 2g_2 & s + 2 + g_2 \end{bmatrix}$$

$$= s^2 + \left(2g_1 + g_2 + 1\right)s + 4g_1 + g_2 - 2$$

compare the two characteristic equations

$$f^*(s) = s^2 + 7s + 12$$

$$f(s) = s^2 + (2g_1 + g_2 + 1)s + 4g_1 + g_2 - 2$$

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

② the state equation of the observer

$$\dot{\hat{X}} = (A - GC)\hat{X} + bu + Gy$$

$$\dot{\hat{X}} = \begin{bmatrix} -7 & -3 \\ 4 & 0 \end{bmatrix}\hat{X} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u + \begin{bmatrix} 4 \\ -2 \end{bmatrix}y$$

$$\hat{y} = \begin{bmatrix} 2 & 1 \end{bmatrix}\hat{X}$$

③ the transfer function of the closed-loop system

the transfer function of the closed-loop system with the observer should be identical to that of the system using the real state feedback signals.

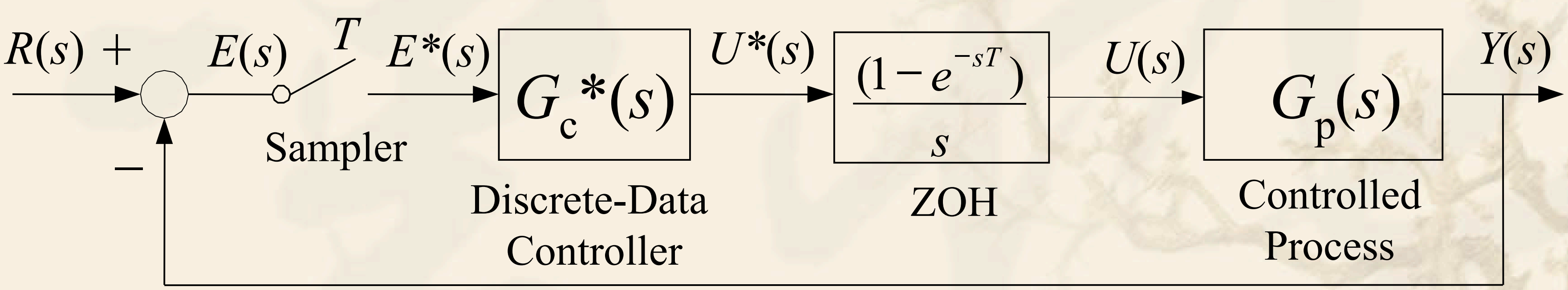
$$A-BK^T=\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}-\begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}-2 & -3\end{bmatrix}=\begin{bmatrix}3 & 4\\ 0 & -2\end{bmatrix}$$

$$G(s)=C\left[sI-(A-BK^T)\right]^{-1}B$$

$$=\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}s-3 & -4\\ 0 & s+2\end{bmatrix}^{-1}\begin{bmatrix}1\\ 0\end{bmatrix}=\frac{2}{s+1}$$

Chapter 7 Digital Control Systems

- characteristics：discrete-data processing block
- sampler
- ZOH



Z transformation

- Definition
- Theorems
- Methods for Z transformation
- Methods for inverse Z transformation

Definition

$$X(z) = Z[x(kT)] = \sum_{k=0}^{\infty} x(kT) z^{-k}$$

$$X(z) = x(0) + x(T)z^{-1} + x(2T)z^{-2} + \cdots$$

weighted summation of z’s power series

Example 5： please find the Z transformation of

$$E(s)=\frac{1-e^{-s}}{s^2(s+1)}$$

作答

Example 5： please find the Z transformation of

$$E(s)=\frac{1-e^{-s}}{s^2(s+1)}$$

Solution： perform the partial fraction expansion

$$E(s)=(1-e^{-s})(\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s+1})$$

check the look-up table：

$$E(z)=(1-z^{-1})\left[\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right]$$

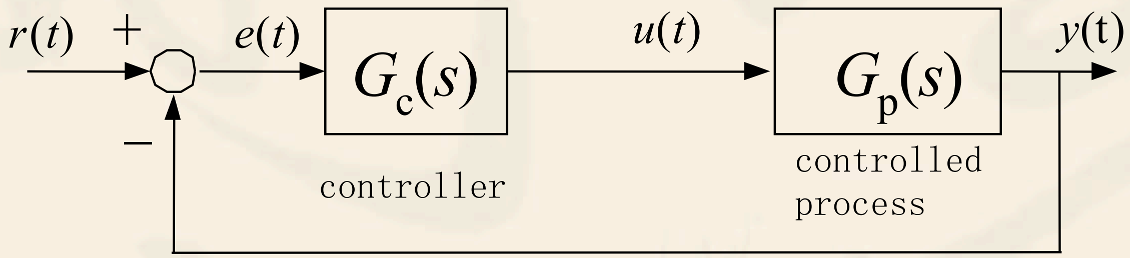
$$=\frac{1-(T+1)e^{-T}+(T-1+e^{-T})z}{z^2-(1+e^{-T})z+e^{-T}}$$

Methods for inverse Z transformation

1. power series method
2. residue method
3. look-up table

Discretize a continuous system

- Why discretization？



Analysis ⇨ Modeling

Synthesis ⇨ Implementation

Discretization in modeling

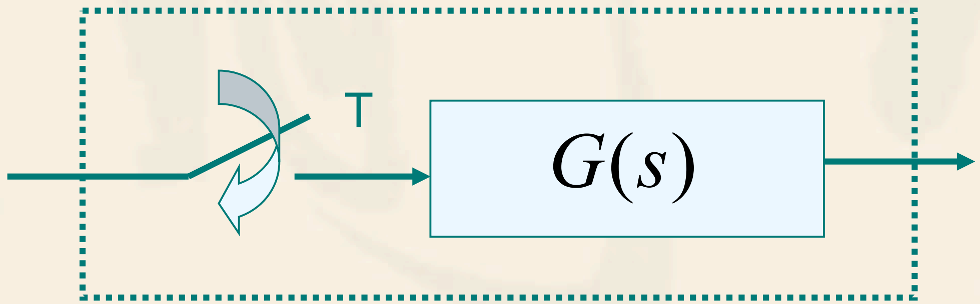
Transfer function $\Longrightarrow$ z-transfer function

state equation $\Longrightarrow$ discrete state equation

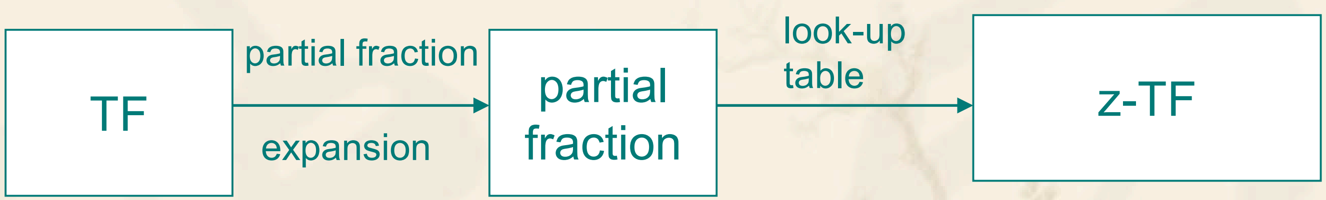
From TF to zTF

- Add ideal sampler
- Add ideal sampler + ZOH

Ideal sampler

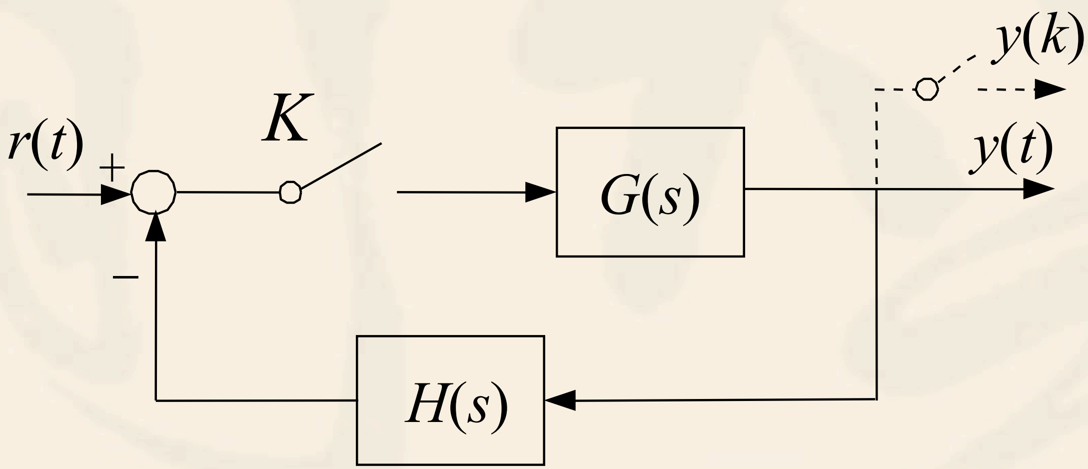


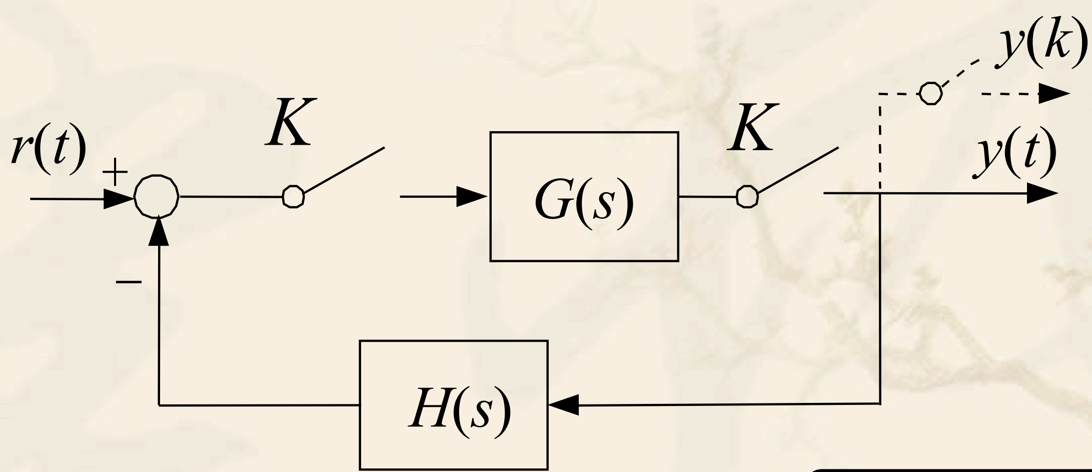
$$Z[G(s)] = G(z)$$



主观题 10分

Please give out the z-transfer functions of the following block diagrams.





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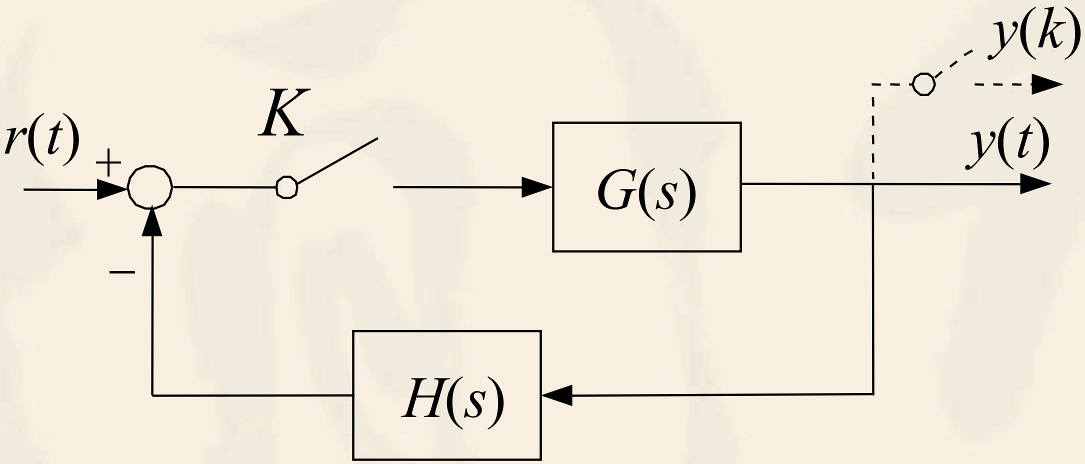
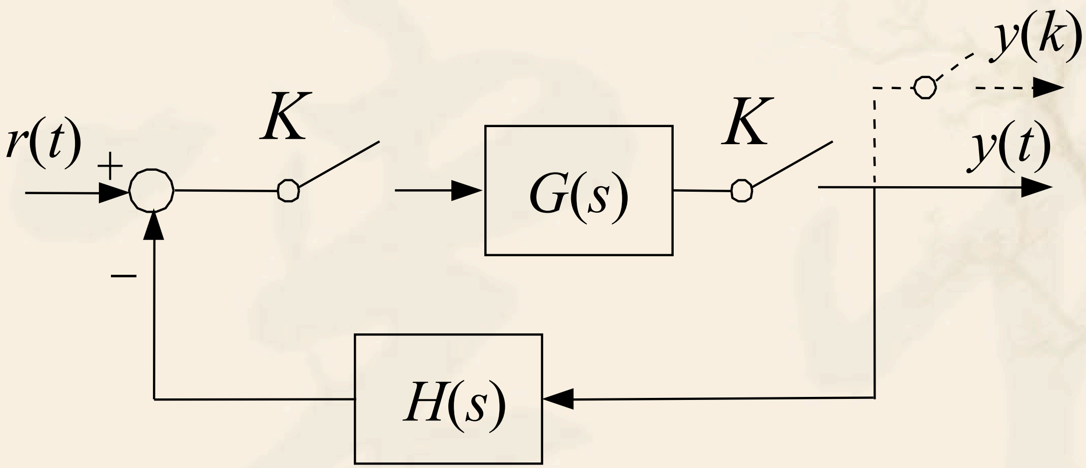
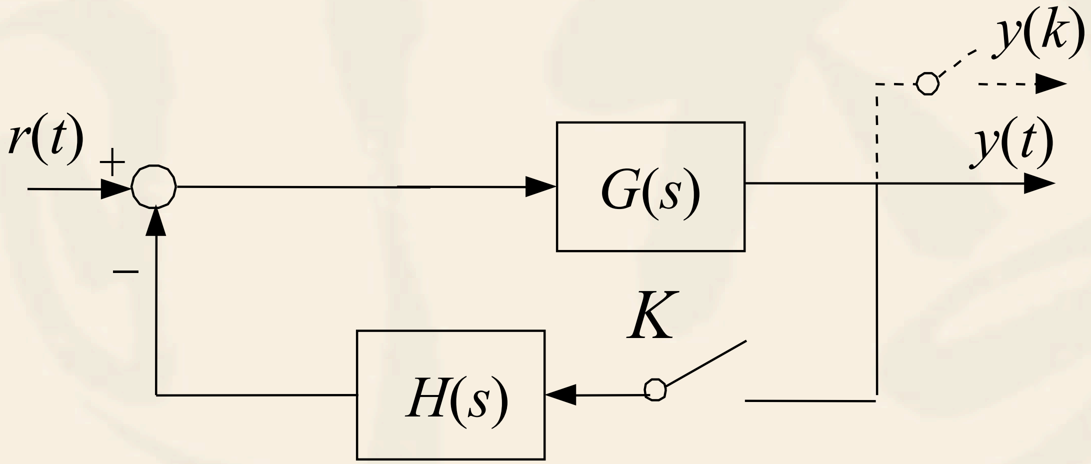


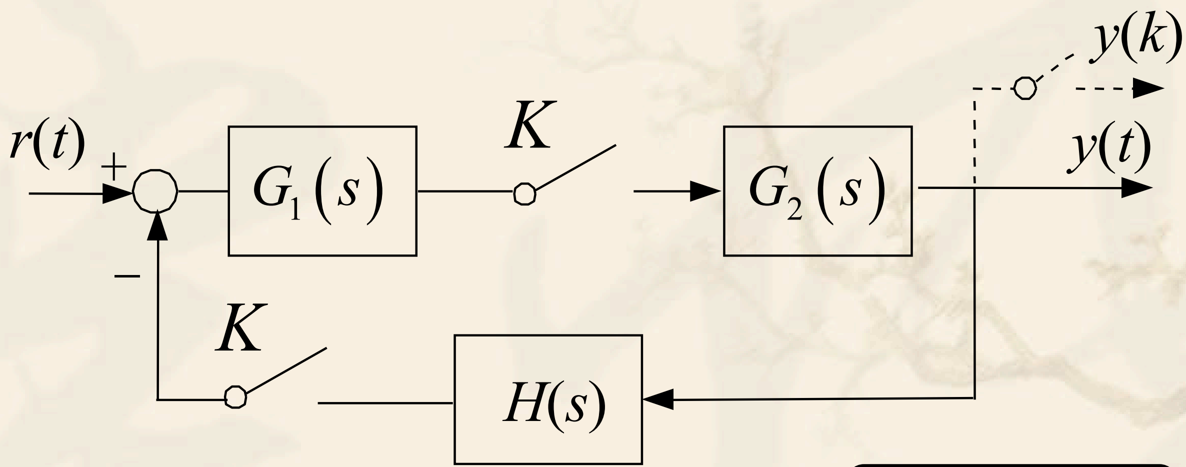
$$Y(z)=\frac{G(z)R(z)}{1+GH(z)}$$



$$Y(z)=\frac{G(z)R(z)}{1+G(z)H(z)}$$

Please give out the z-transfer functions of the following block diagrams.





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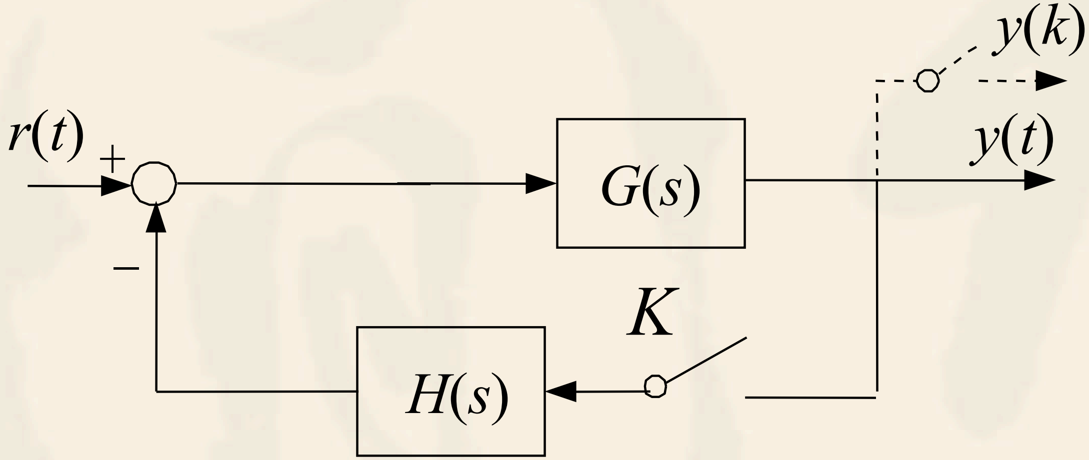
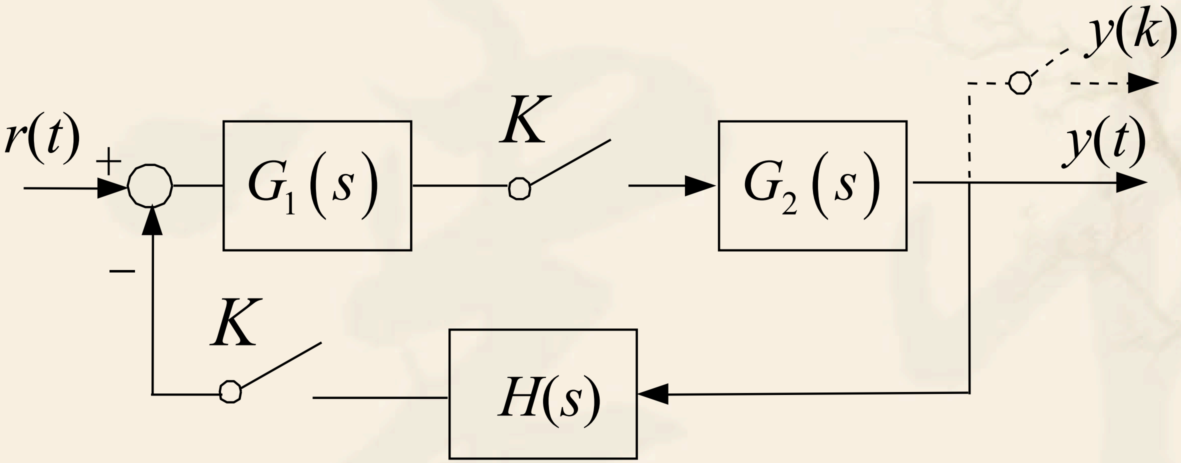
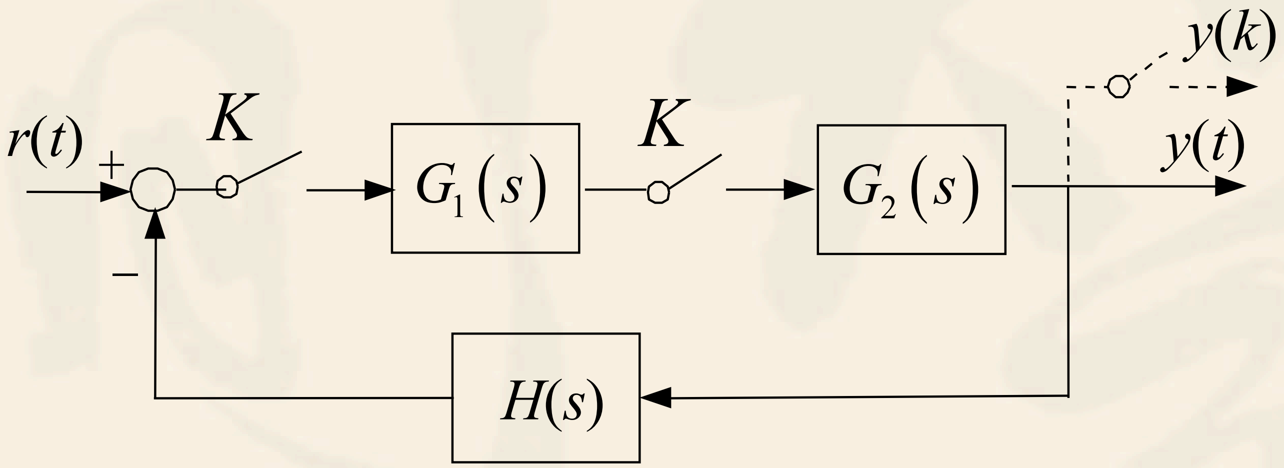


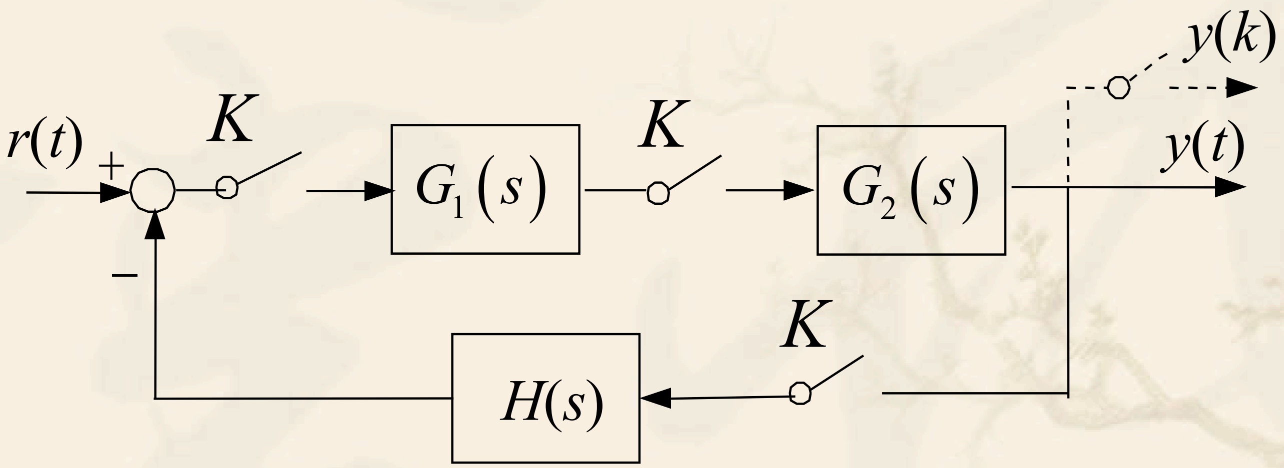
$$Y(z)=\frac{GR(z)}{1+GH(z)}$$



$$Y(z)=\frac{G_2(z)G_1R(z)}{1+G_1(z)G_2H(z)}$$

Please give out the z-transfer functions of the following block diagrams.





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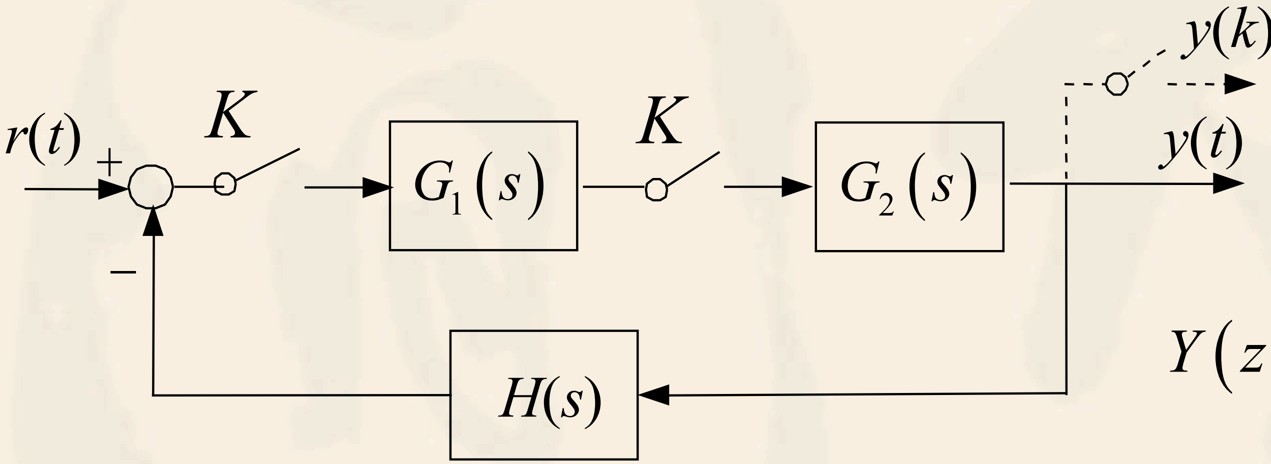
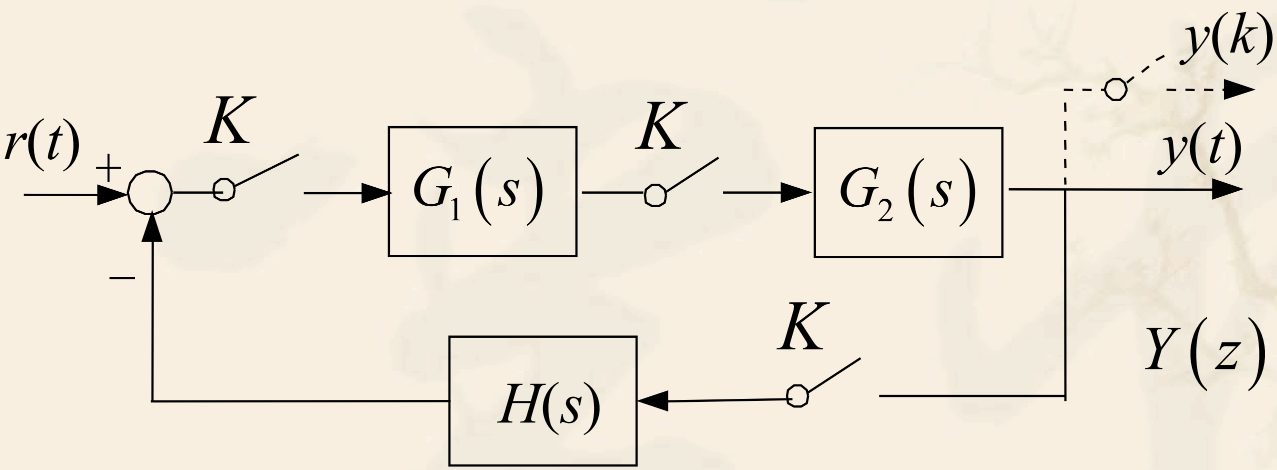
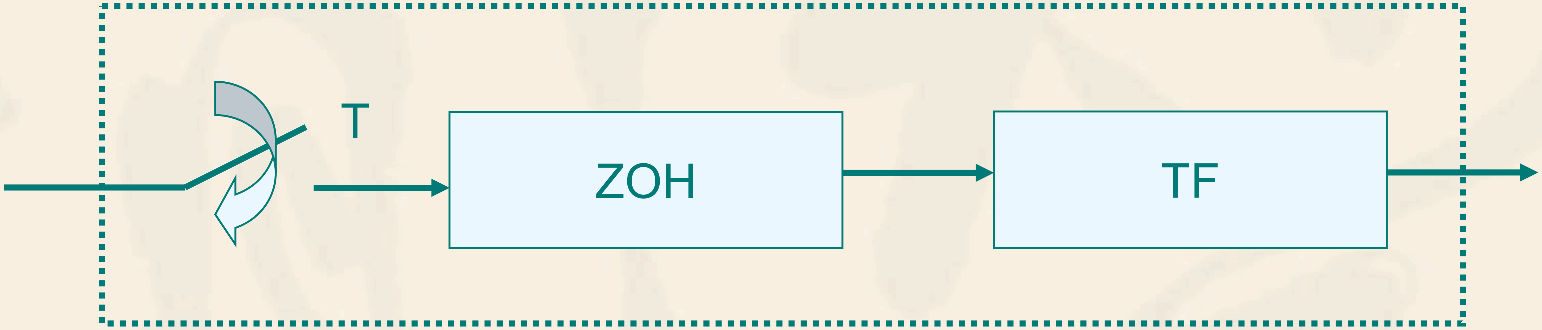


$$Y(z)=\frac{G_1(z)G_2(z)R(z)}{1+G_1(z)G_2H(z)}$$



$$Y(z)=\frac{G_1(z)G_2(z)R(z)}{1+G_1(z)G_2(z)H(z)}$$

- sample + ZOH



TF of ZOH： $$\frac{1-e^{-Ts}}{s}$$

z transfer function： $$Z\left[\frac{1-e^{-Ts}}{s}\cdot G(s)\right]=G(z)$$

From SE to DSE

SE

$$\dot{X}(t) = AX(t) + BU(t)$$
$$Y(t) = CX(t) + DU(t)$$

DSE

$$X(k+1) = GX(k) + HU(k)$$
$$Y(k) = CX(k) + DU(k)$$

$$G = e^{AT} \qquad H = \int_0^T e^{At}\,dt\,B$$

Discretization in implementation

- Zero-pole matching
- Bilinear transformation

Approximation method

- zero-pole matching

$$s = -a \Longrightarrow z = e^{-aT}$$

$$s = \infty \Longrightarrow z = -1$$

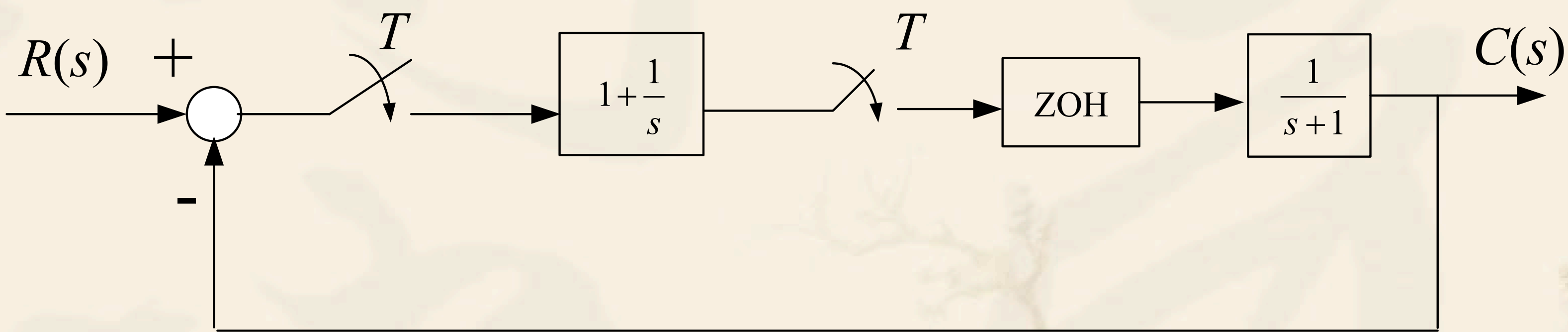
$G_c(0) = D_c(1)$ $\Longrightarrow$ Determine the gain

- bilinear transformation

$$z \approx \frac{1 + \frac{T}{2}s}{1 - \frac{T}{2}s}$$

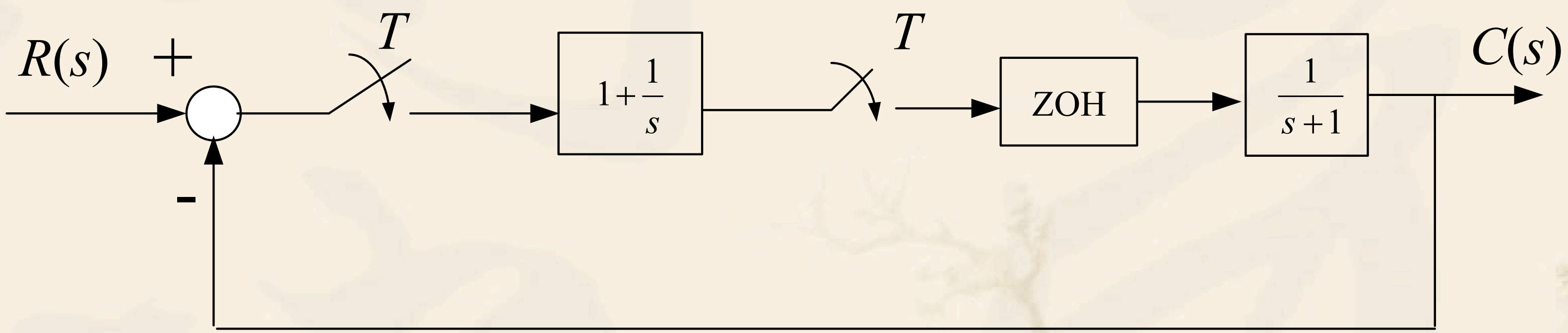
$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

Example 6：please find the z-transfer function of the given system，where T=1s.



作答

Example 6：please find the z-transfer function of the given system，where T=1s.



Solution：the open-loop z-transfer function is

$$G_0(z) = Z\left[1+\frac{1}{s}\right] \bullet Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{1}{s+1}\right]$$

$$= (1+\frac{z}{z-1}) \bullet (1-z^{-1}) \bullet Z\left[\frac{1}{s(s+1)}\right]$$

$$= \frac{2z-1}{z}\cdot Z\left[\frac{1}{s}-\frac{1}{s+1}\right]$$

$$= \frac{2z-1}{z-1}\cdot\frac{1-e^{-T}}{z-e^{-T}}$$

z-transfer function of the closed-loop system：

$$\frac{C(z)}{R(z)} = \frac{G_0(z)}{1+G_0(z)}$$

$$= \frac{1.264z - 0.632}{z^2 - 0.104z - 0.264}$$

Stability test of digital control systems

1. poles locate within unit circle
2. Routh criterion can be used after bilinear or w-transformation

Example 7：check the stability of the given digital control system.

$$G(z)=\frac{1}{z^2-0.5}$$

Example 7：check the stability of the given digital control system.

$$G(z) = \frac{1}{z^2 - 0.5}$$

Solution: the characteristic equation is

$$f(z) = z^2 - 0.5$$

~~miss z term, system is unstable.~~

$$f(z) = z^2 - 0.5 = 0$$

perform w-transformation

$$z = \frac{w+1}{w-1}$$

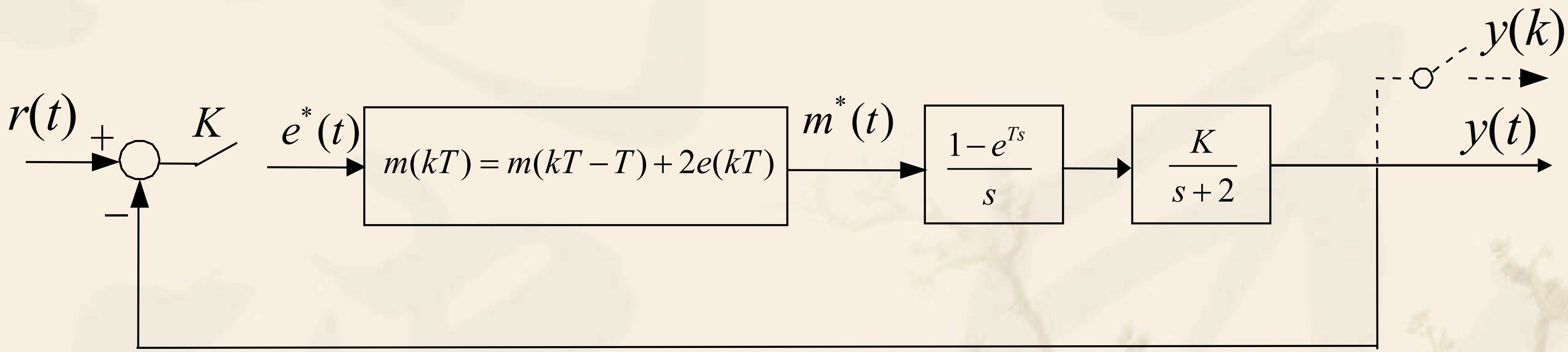
$$(\frac{w+1}{w-1})^2 - 0.5 = 0$$

s^2	1	1
s^1	6	
s^0	$5/6$	

$$w^2 + 6w + 1 = 0$$

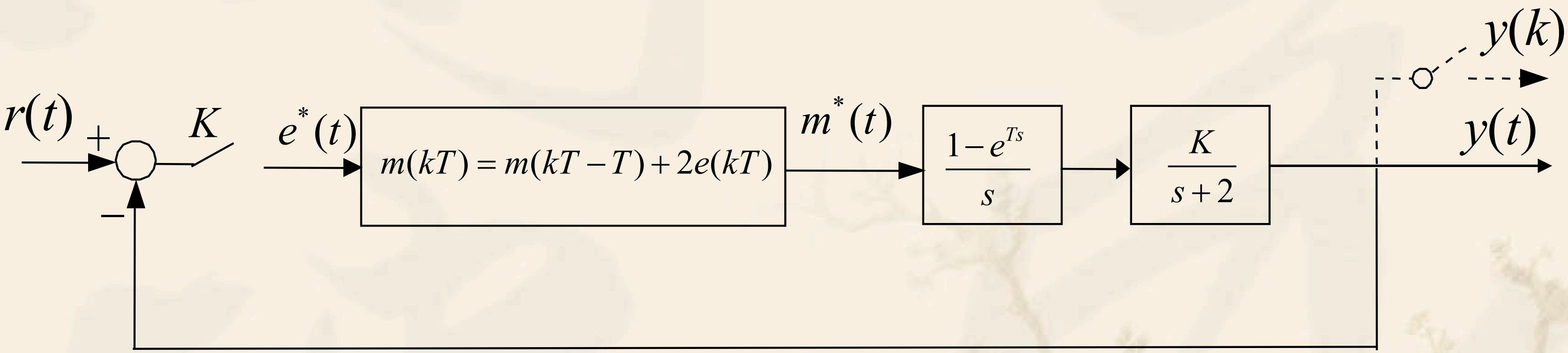
system is stable!

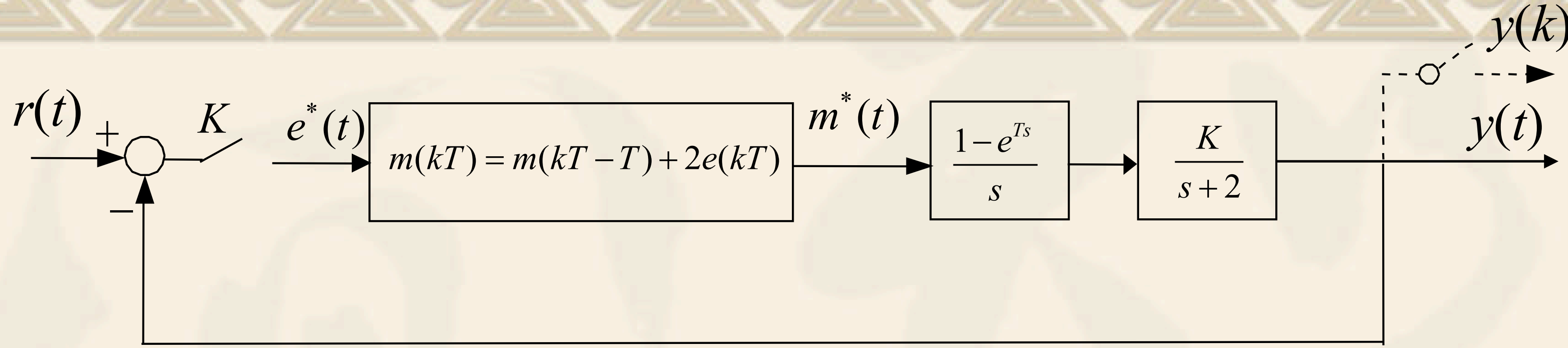
Example 8: a discrete-data system has the following structure, where T=0.5s, please find the z-transfer function of the system and analyze its steady-state error when the input is a unit-step function.



作答

Example 8: a discrete-data system has the following structure, where T=0.5s, please find the z-transfer function of the system and analyze its steady-state error when the input is a unit-step function.



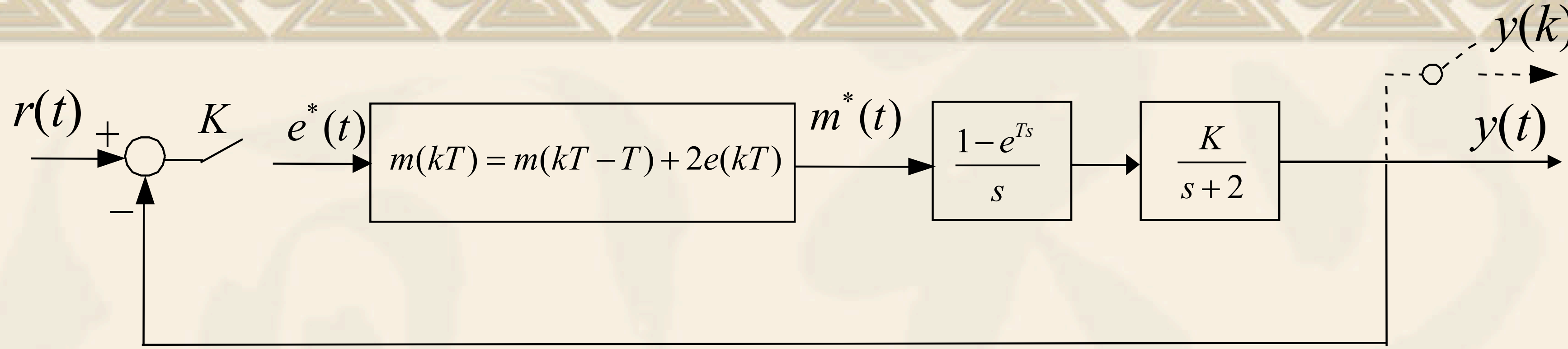


Solution: the z-transfer function of digital controller is

$$m(kT) = m(kT-T) + 2e(kT)$$

$$M(z) = z^{-1}M(z) + 2E(z)$$

$$\frac{M(z)}{E(z)} = \frac{2z}{z-1}$$



The open-loop z-transfer function of the system is

$$G(z) = D(z) \bullet Z\left[\frac{1-e^{Ts}}{s} \cdot \frac{K}{s+2}\right]$$

$$= \frac{2z}{z-1} \cdot K(1-z^{-1}) \cdot Z\left[\frac{0.5}{s} - \frac{0.5}{s+2}\right]$$

$$= \frac{0.632Kz}{(z-1)(z-0.367)}$$

Type 1 system, so it is errorless to the unit-step input.

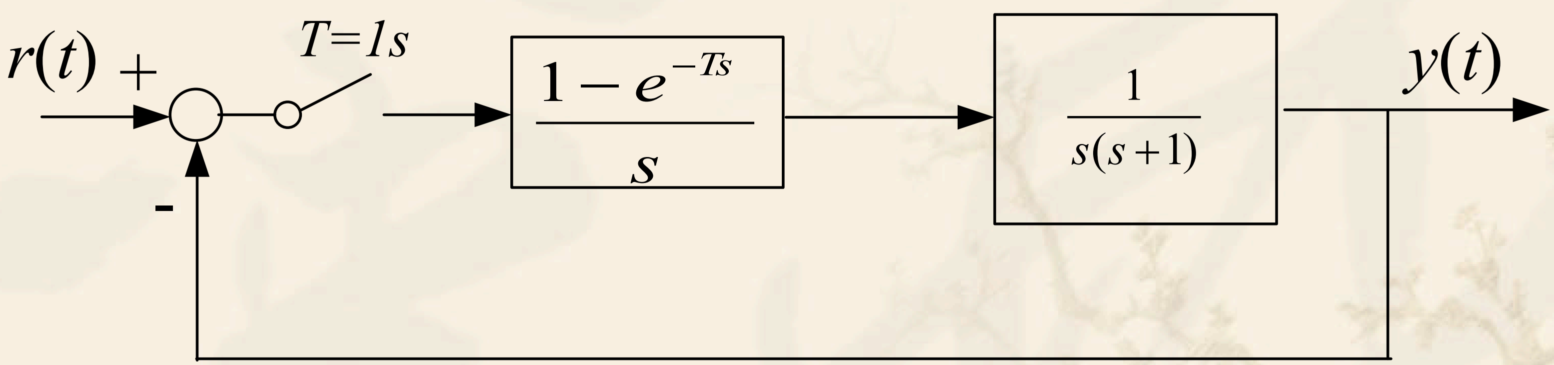
设置

Example 9: The given discrete-data system is shown as follows. The sampling period is T=1. Please

(1) Estimate the stability of the system;

(2) Find the z-transfer function of the system, and give out the difference equation;

(3) If y(0)=0, y(1)=0.386, please find y(2) and y(3) when the input is unit step.



作答

Example 9: The given discrete-data system is shown as follows. The sampling period is T=1. Please

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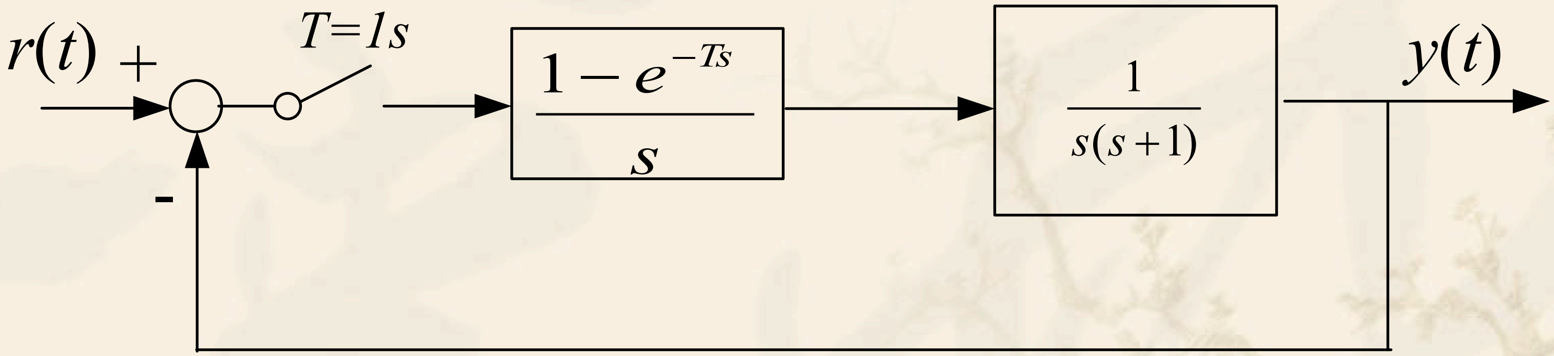

A: The open loop z-transfer function is:

$$G_0(z) = Z\left[\frac{1-e^{-sT}}{s}\cdot\frac{1}{s(s+1)}\right]$$

$$= (1-z^{-1})\cdot Z\left[\frac{1}{s^2(s+1)}\right] = (1-z^{-1})\cdot Z\left[\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s+1}\right]$$

$$= (1-z^{-1})\cdot\left[\frac{z}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-1}}\right]$$

$$= \frac{0.368z+0.264}{z^2-1.368z+0.368}$$

The characteristic equation is:

$$1+G_0(z)=0$$

$$z^2-z+0.632=0$$

The characteristic roots are:

$$z_{1,2}=0.5\pm j0.618$$

$$\left|z_{1,2}\right|=\left|0.5\pm j0.618\right|=0.795<1$$

So, the system is stable.

(2) Find the z-transfer function of the system, and give out the difference equation;

The closed-loop z-transfer function is:

$$G(z)=\frac{G_0(z)}{1+G_0(z)}=\frac{0.368z+0.264}{z^2-z+0.632}$$

$$G(z)=\frac{Y(z)}{R(z)}=\frac{0.368z+0.264}{z^2-z+0.632}$$

$$(z^2-z+0.632)Y(z)=(0.368z+0.264)R(z)$$

Real Translation (Time advance)

$$Z\left[x(k+m)\right]=z^m X(z)-z^m x(0)-z^{m-1}x(1)-\cdots-zx(m-1)$$

Do not have enough information!

Real Translation (Time delay) $Z\left[x(k-m)\right] = z^{-m}X\left(z\right)$

$$G(z) = \frac{Y(z)}{R(z)} = \frac{0.368z + 0.264}{z^2 - z + 0.632}$$

$$G(z) = \frac{Y(z)}{R(z)} = \frac{0.368z^{-1} + 0.264z^{-2}}{1 - z^{-1} + 0.632z^{-2}}$$

$$(1 - z^{-1} + 0.632z^{-2})Y(z) = (0.368z^{-1} + 0.264z^{-2})R(z)$$

$$\Downarrow$$

$$y(k) - y(k-1) + 0.632y(k-2) = 0.368r(k-1) + 0.264r(k-2)$$

$$y(k) = 0.368r(k-1) + 0.264r(k-2) + y(k-1) - 0.632y(k-2)$$

(3) If y(0)=0, y(1)=0.386, please find y(2) and y(3) when the input is unit step.

$$y(k) = 0.368r(k-1) + 0.264r(k-2) + y(k-1) - 0.632y(k-2)$$

$$y(2) = 0.368r(1) + 0.264r(0) + y(1) - 0.632y(0) = 1$$

$$y(3) = 0.368r(2) + 0.264r(1) + y(2) - 0.632y(1) = 1.4$$

Sample Questions for Final Exam

Identify the following statements true or false. If a statement is false, please give the reason or the right answer.

A PI controller always decreases the steady-state error of a LTI system. (   )

Sample Questions for Final Exam

Generally speaking, to decrease the steady-state error of a LTI system, the gain of ______ (low or high)-band frequency should be ______ (decreased or increased). To speed up the response a LTI system, the gain of ______ -band frequency should be ______ .

Sample Questions for Final Exam

（6） **The open-loop transfer function of a minimum phase unit negative feedback system has the following frequency response:**

$$\varphi(\omega) = -90^\circ - \arctan\frac{\omega}{2} - \arctan\omega$$

Please:

a) Find the open-loop transfer function with phase margin equal to 30° **.**

b) When a series compensation block ($G_c(s) = \frac{K_c(Ts+1)}{s+1}$ **) is added to the system, the phase margin is raised to** 60° **with the gain crossover frequency** ω_c **unchanged. Please determine** K_c, T **. (14%)**

Thank you all

for

being with me in this semester!