Chapter 2

Mathematical Models of Systems

Second Lesson

Review

- Basic concepts about control systems
- Key mechanisms for control
- History of the development of control theory
- Basic information about this course

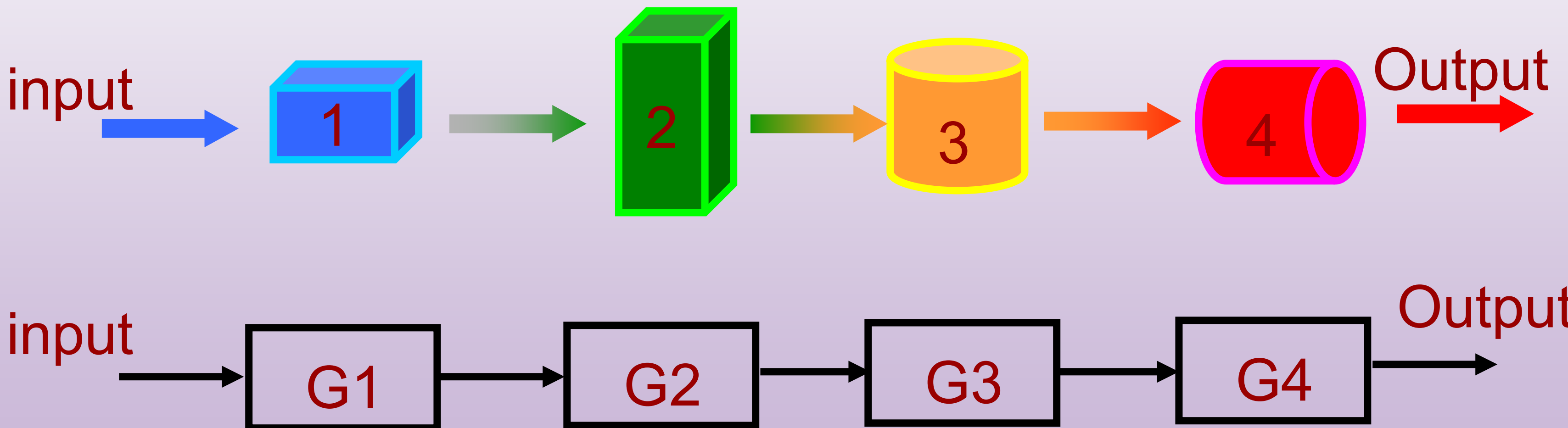
- Basics about LTI systems
- Linearization of a nonlinear system
- Linear system modeling
- Laplace transformation
- Transfer function of a linear system

Outlines

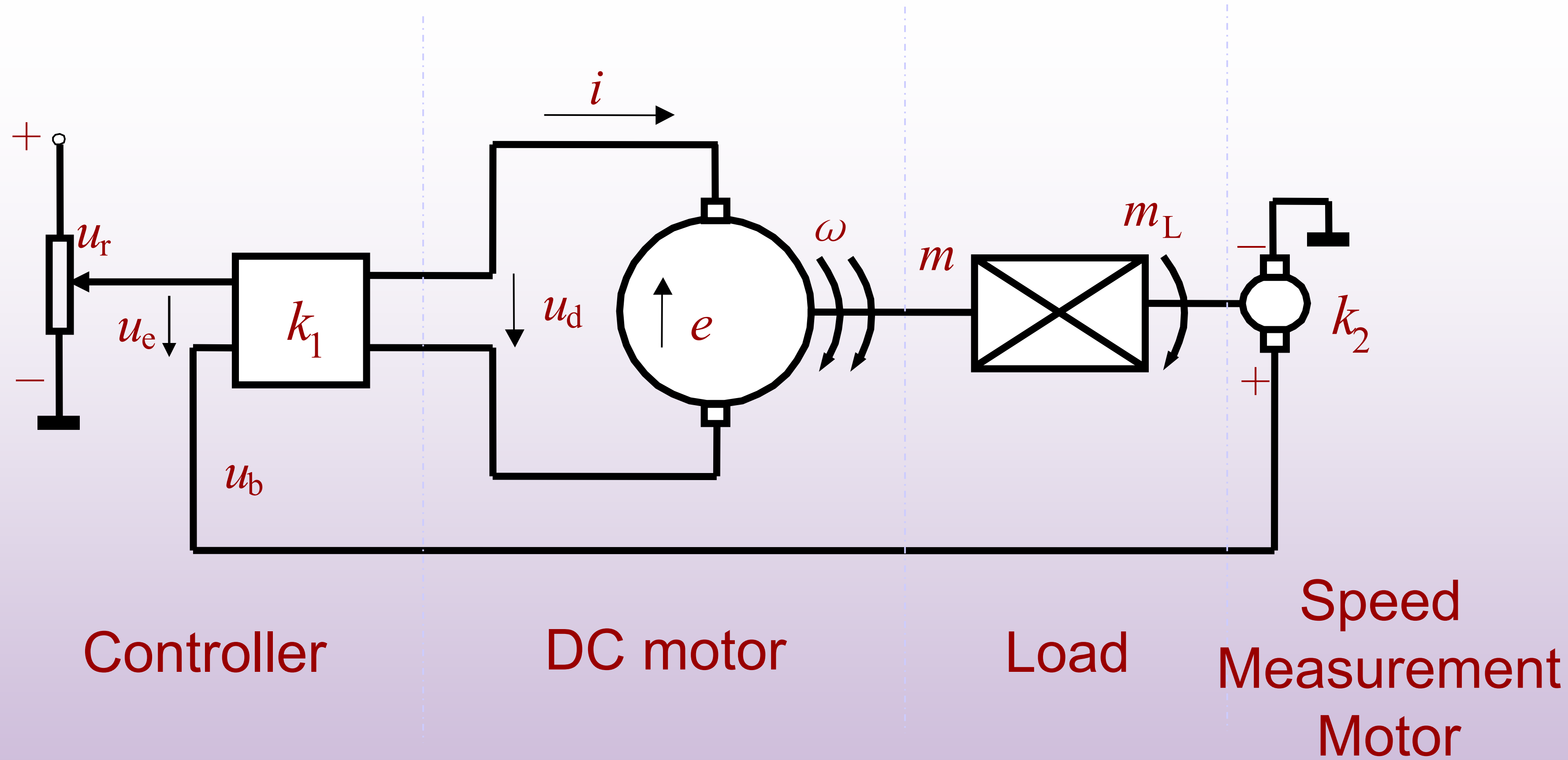
- **Block diagram of a linear system**
- **Block diagram transformation**
- **Signal-flow graph**
- **Gain formula (Mason Formula)**
- **State space model**
- **State equation versus transfer function – from SE to TF**

Block Diagrams

- Complex control systems can be represented by block diagrams
- Block diagrams consist of unidirectional, operational blocks that represent the transfer function of the components or subsystems



Transfer function of dc motor system



Input: u_r m_L

Output: ω

Transfer function of dc motor system

Controller:

$$u_\mathrm{d} = k_1(u_\mathrm{r} - u_\mathrm{b}) = k_1 u_\mathrm{e} \qquad G_1(s) = \frac{U_\mathrm{d}(s)}{U_\mathrm{e}(s)} = K_1$$

DC motor :

$$u_\mathrm{d} = e + Ri + L\frac{di}{dt}$$

$$e = k_\mathrm{e}\omega$$

$$m = k_\mathrm{m} i$$

Load :

$$m - m_\mathrm{L} = J\frac{d\omega}{dt}$$

Speed measurement motor :

$$u_\mathrm{b} = k_2\omega \qquad G_3(s) = \frac{U_\mathrm{b}(s)}{\Omega(s)} = K_2$$

Dynamic equation of motor-load

$$\frac{L}{R}\cdot\frac{JR}{K_\mathrm{e}K_\mathrm{m}}\cdot\frac{d^2\omega}{dt^2}+\frac{JR}{K_\mathrm{e}K_\mathrm{m}}\cdot\frac{d\omega}{dt}+\omega=\frac{u_\mathrm{d}}{K_\mathrm{e}}-\frac{L}{R}\cdot\frac{R}{K_\mathrm{e}K_\mathrm{m}}\cdot\frac{dm_\mathrm{L}}{dt}-\frac{R}{K_\mathrm{e}K_\mathrm{m}}\cdot m_\mathrm{L}$$

Set: $T_\mathrm{a}=\frac{L}{R}$ $\quad T_\mathrm{m}=\frac{JR}{K_\mathrm{e}K_\mathrm{m}}$

$$T_\mathrm{a}T_\mathrm{m}\frac{d^2\omega}{dt^2}+T_\mathrm{m}\frac{d\omega}{dt}+\omega=\frac{u_\mathrm{d}}{K_\mathrm{e}}-\frac{R}{K_\mathrm{e}K_\mathrm{m}}(T_\mathrm{a}\frac{d}{dt}+1)m_\mathrm{L}$$

Transfer function of the system:

When no load: $$G_{21}(s)=\frac{\Omega(s)}{U_\mathrm{d}(s)}=\frac{1/K_\mathrm{e}}{T_\mathrm{a}T_\mathrm{m}s^2+T_\mathrm{m}s+1}$$

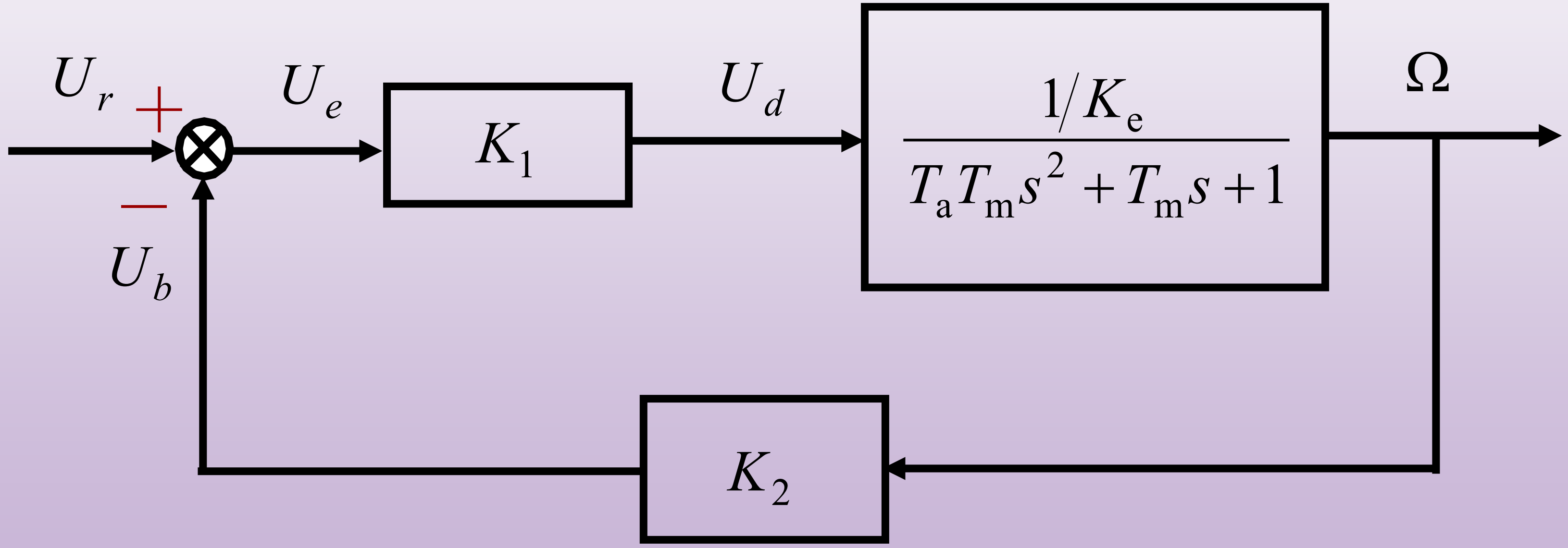
Transfer function of the system:

When $u_d = 0$: $$G_{22}(s)=\frac{\Omega(s)}{M_L(s)}=-\frac{1/K_\mathrm{e}\cdot R/K_m\cdot(T_a s+1)}{T_\mathrm{a}T_\mathrm{m}s^2+T_\mathrm{m}s+1}$$

How to construct block diagram of a system?

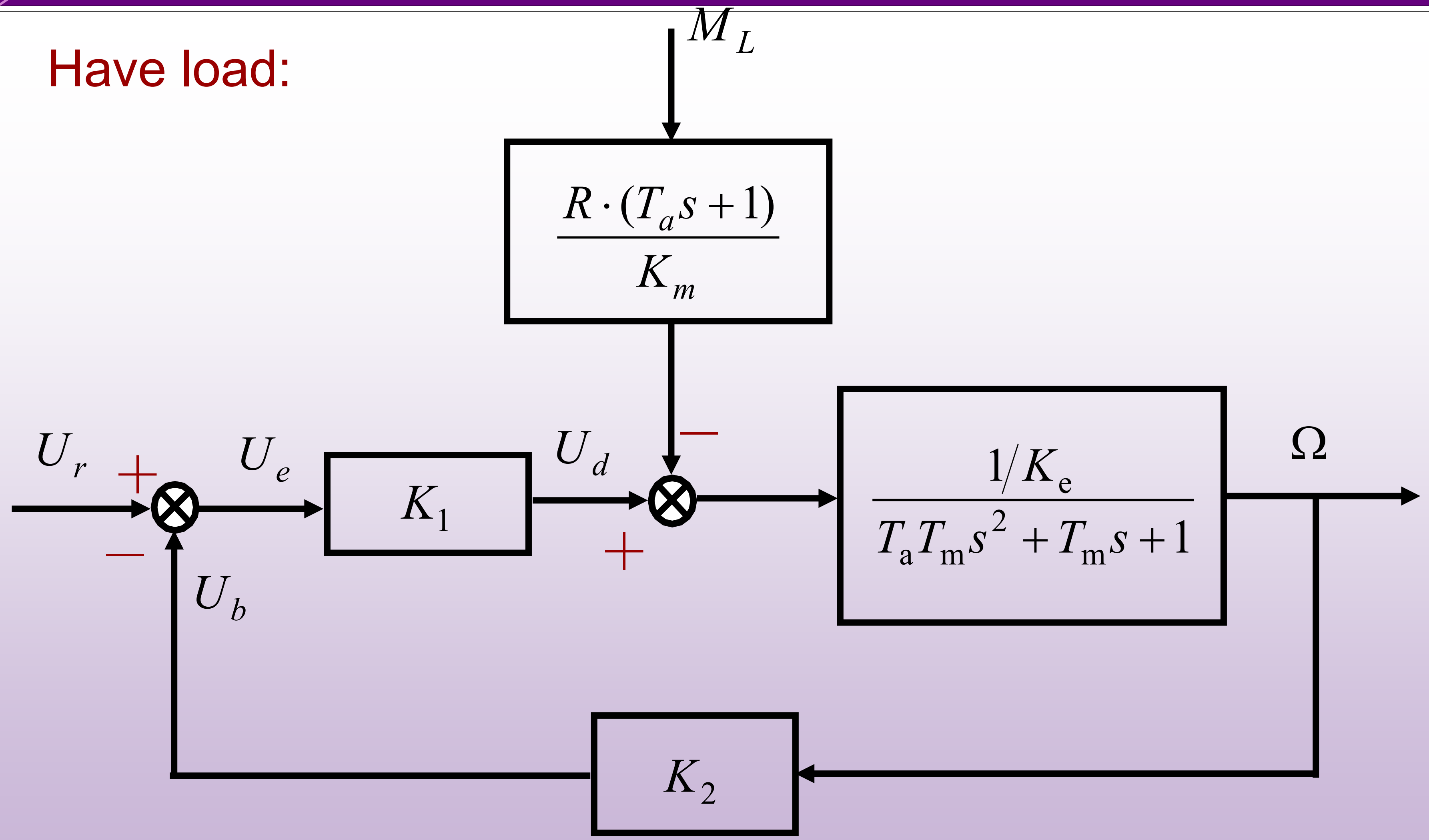
– Construct blocks of subsystems
– Connect blocks according to the signal flow

No load



Transfer function of dc motor system

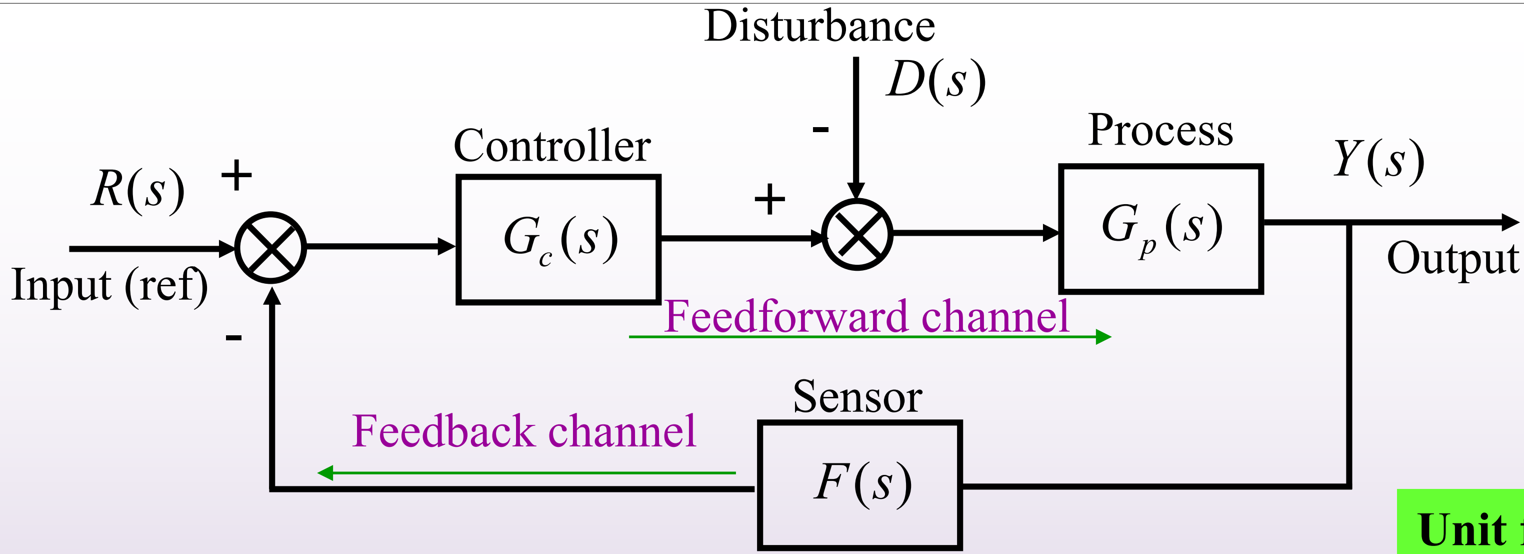
Have load:



Transfer function of common blocks

- Proportional block $\quad \boldsymbol{k}$
- Inertial block（1st order） $\quad \dfrac{1}{Ts+1}$
- Oscillation block（2nd order） $\quad \dfrac{1}{T_1T_2s^2+T_2s+1}$
- Integration block $\quad \dfrac{1}{s}$
- Lead-lag block $\quad \dfrac{T_2s+1}{T_1s+1}$
- Time delay block $\quad e^{-\tau s}$

Block Diagram Representation of Feedback Systems



Unit feedback

$$G(s) = \frac{G_0(s)}{1 + G_0(s)}$$

Open-loop TF: $$G_0(s) = G_c(s)G_p(s)F(s)$$

Closed-loop TF: $$G(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)F(s)}$$

TF from D to Y: $$G_D(s) = \frac{Y(s)}{D(s)} = \frac{-G_p(s)}{1 + G_c(s)G_p(s)F(s)}$$

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- **Block diagram of a linear system**
- **Block diagram transformation**
- **Signal-flow graph**
- **Gain formula (Mason Formula)**
- **State space model**
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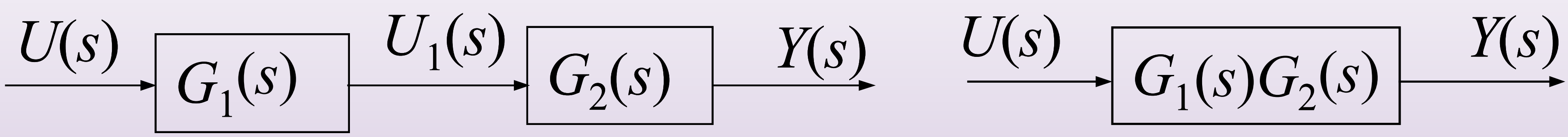
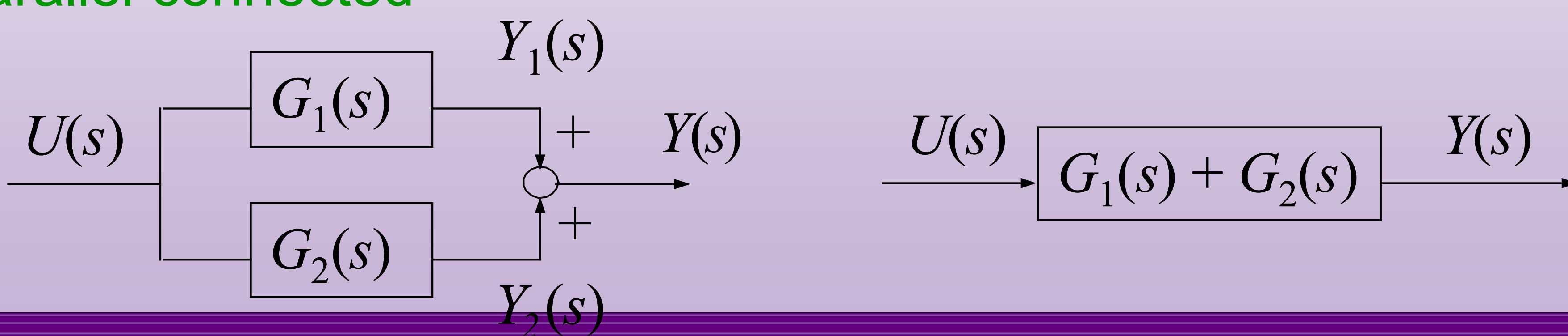
Block Diagram Transformation

- The principle for block diagram transformation is to keep the input-output relationship of the representations constant before and after reduction.

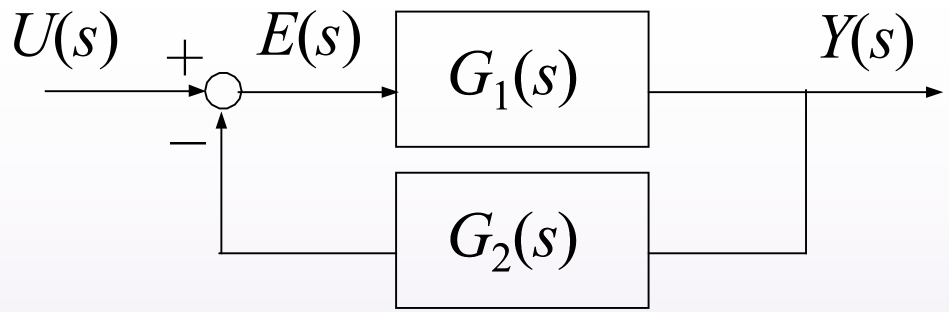
series connected



parallel connected



Negative feedback:

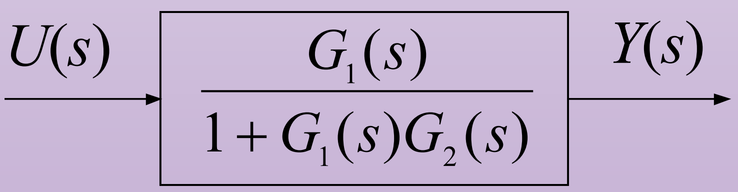


$$E(s) = U(s) - G_2(s)Y(s) \qquad Y(s) = G_1(s)E(s)$$

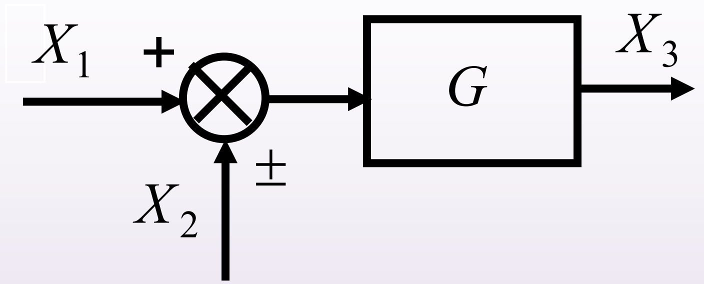
$$Y(s) = G_1(s)[U(s) - G_2(s)Y(s)] = G_1(s)U(s) - G_1(s)G_2(s)Y(s)$$

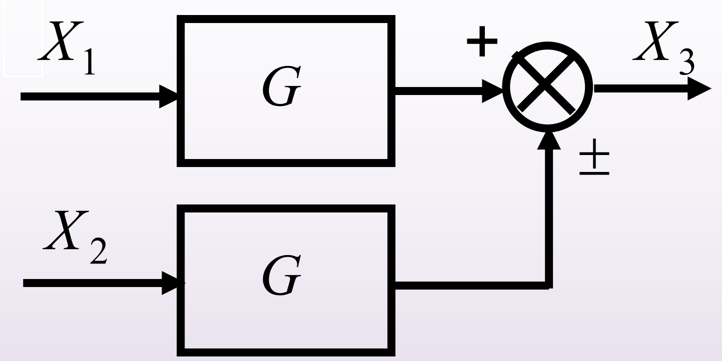
$$[1 + G_1(s)G_2(s)]Y(s) = G_1(s)U(s)$$

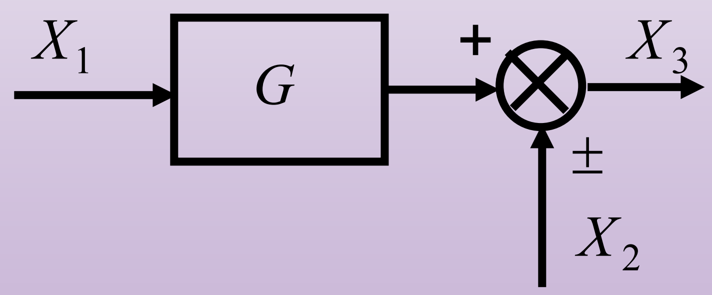
$$\frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 + G_1(s)G_2(s)} \Rightarrow$$

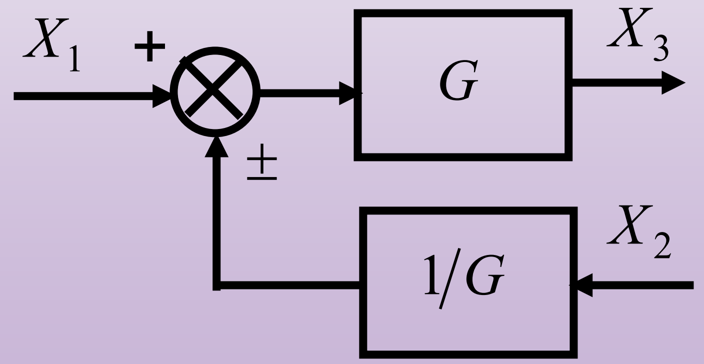


Moving a summing point

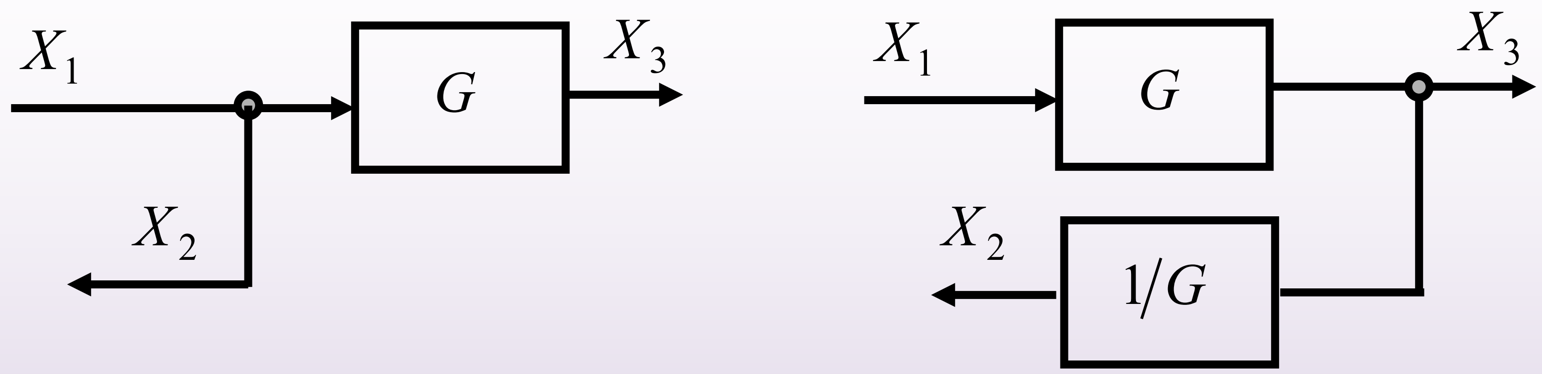


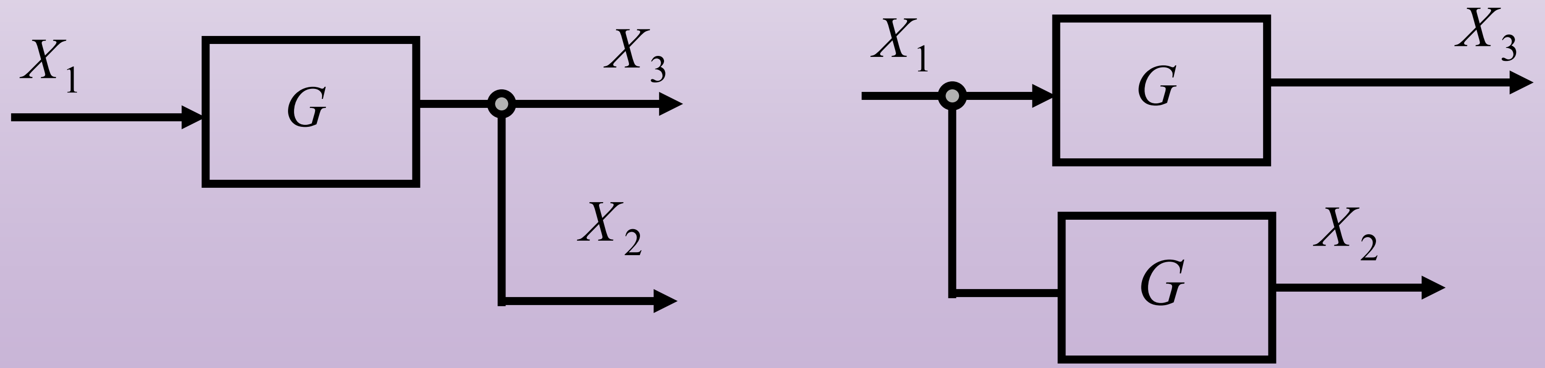




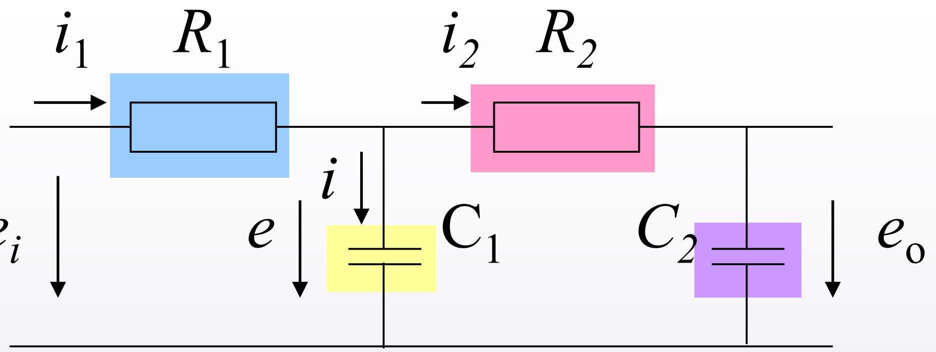


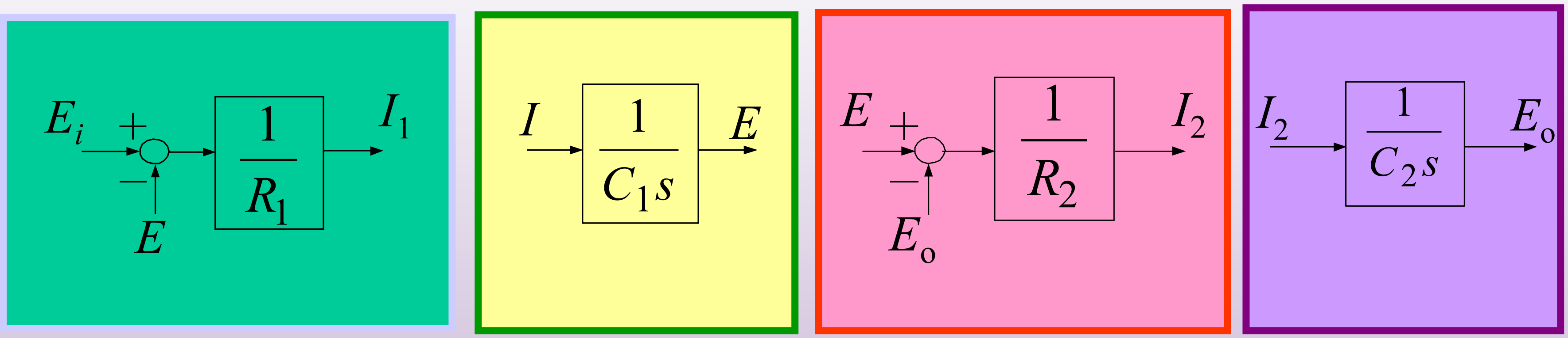
Moving a pickoff point

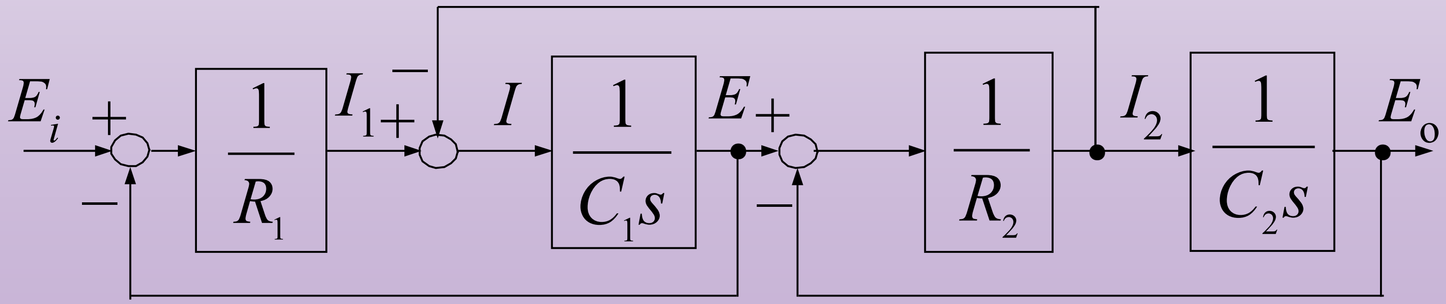


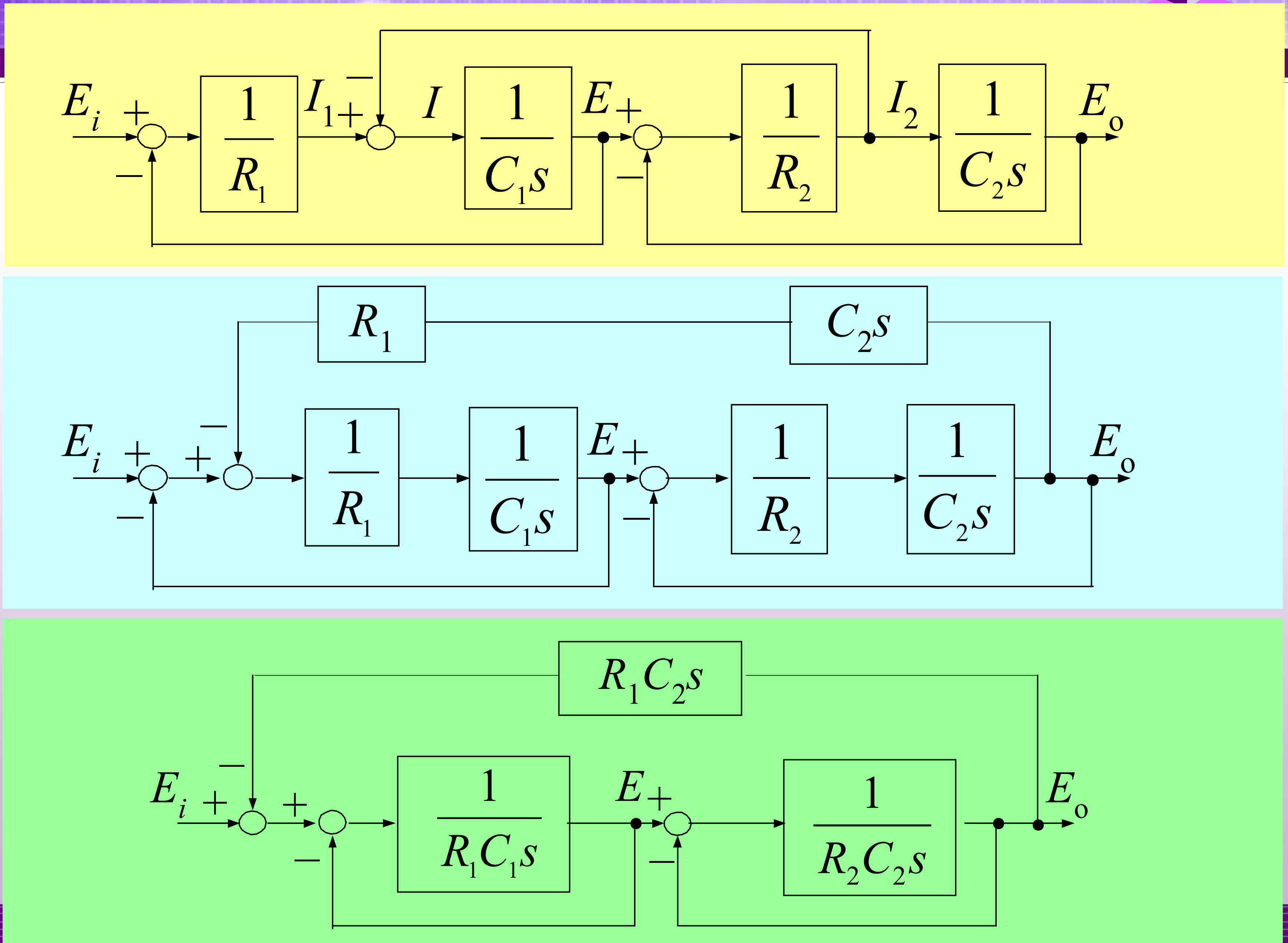


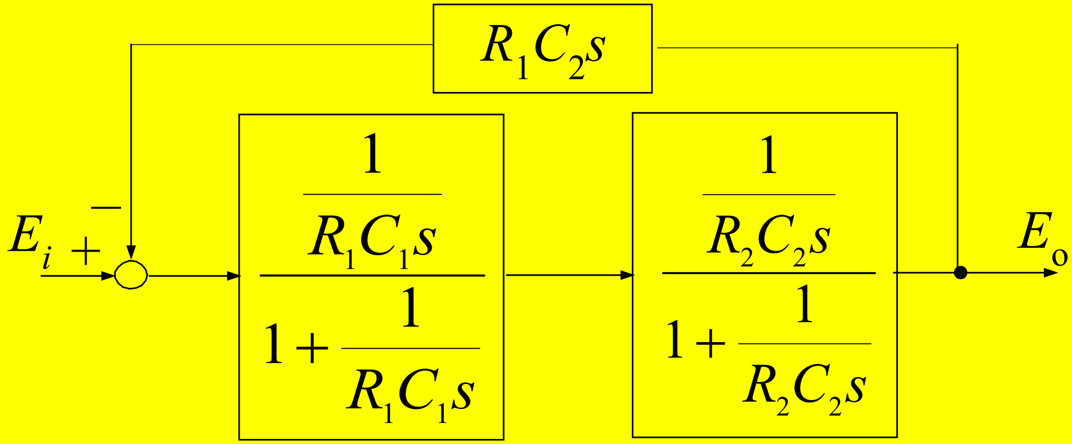
Q: Please find out the transfer function of the circuit on the right

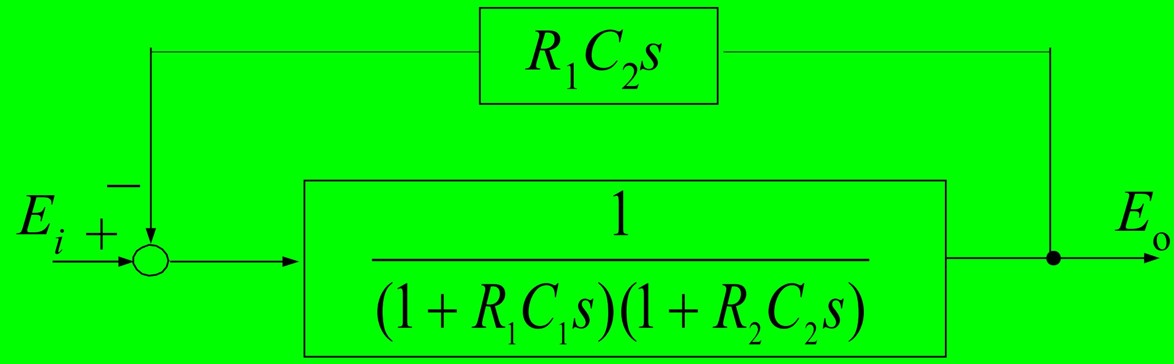







E_i
I_1
I
E
I_2
E_o
$\frac{1}{R_1}$
$\frac{1}{C_1 s}$
$\frac{1}{R_2}$
$\frac{1}{C_2 s}$
R_1
$C_2 s$
$R_1 C_2 s$
$\frac{1}{R_1 C_1 s}$
$\frac{1}{R_2 C_2 s}$


R_1C_2s
E_i
$+$
$-$
$\dfrac{\dfrac{1}{R_1C_1s}}{1+\dfrac{1}{R_1C_1s}}$
$\dfrac{\dfrac{1}{R_2C_2s}}{1+\dfrac{1}{R_2C_2s}}$
E_o


R_1C_2s
E_i
$+$
$-$
$\dfrac{1}{(1+R_1C_1s)(1+R_2C_2s)}$
E_o


E_i
$\dfrac{1}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$
E_o

Outlines

- **Block diagram of a linear system**
- **Block diagram transformation**
- **Signal-flow graph**
- **Gain formula (Mason Formula)**
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Signal-Flow Graphs

- Introduced for the cause-effect representation of linear systems that are modeled by algebraic equations

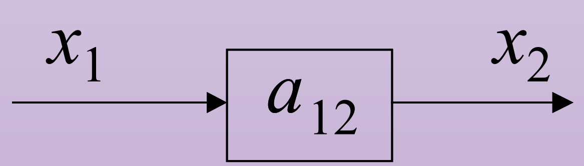
$$y_j = \sum_{k=1}^{N} a_{kj} x_k \qquad j = 1, 2, \cdots m$$

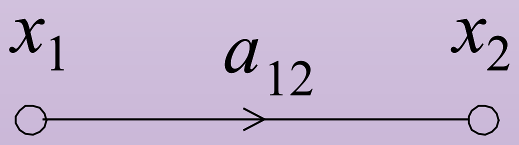
$$jth\ effect = \sum_{k=1}^{N} \left(gain\ \ from\ kth\ cause\ to\ \ jth\ \ effect \right) \times \left(kth\ cause \right)$$

- In the case when the system is represented by a set of integro-differential equations, Laplace transform must be performed first to convert the integrodifferential equation into an algebraic equation.
- May be regarded as a simplified version of a block diagram constrained by more rigid mathematical rules.

Signal-Flow Graphs

- Nodes represent variables. Normally, the nodes are arranged from left to right, from the input to the output, following a succession of cause-and-effect relations through the system.
- Branches with direction and gains represent cause-and-effect relationship
- Signals travel along branches only in the direction described by the arrows of the branches
- A signal x_1 traveling along a branch between x_1 and x_2 is multiplied by the gain of the branch, a_{12}, so that a signal $a_{12}x_1$ is delivered at x_2 .





Example

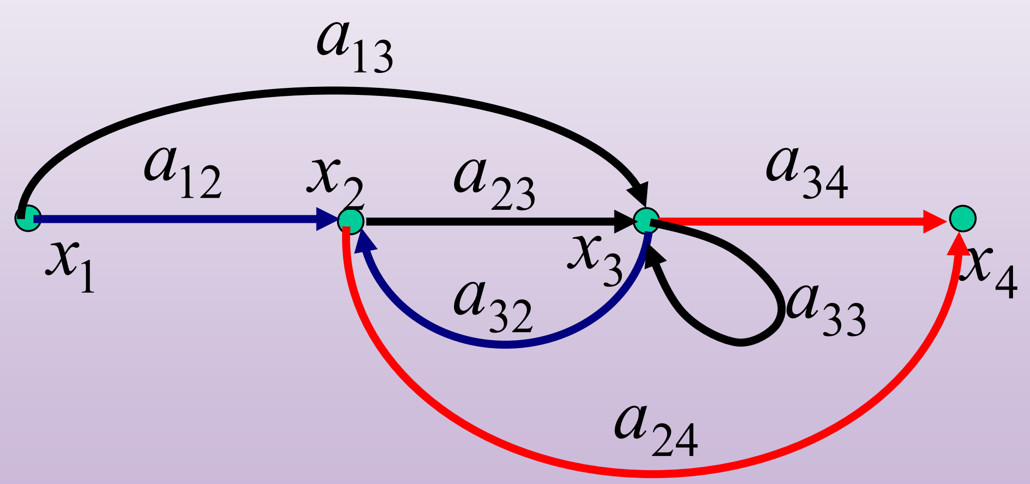
$$x_2 = a_{12}x_1 + a_{32}x_3$$

$$x_3 = a_{13}x_1 + a_{23}x_2 + a_{33}x_3$$

$$x_4 = a_{24}x_2 + a_{34}x_3$$

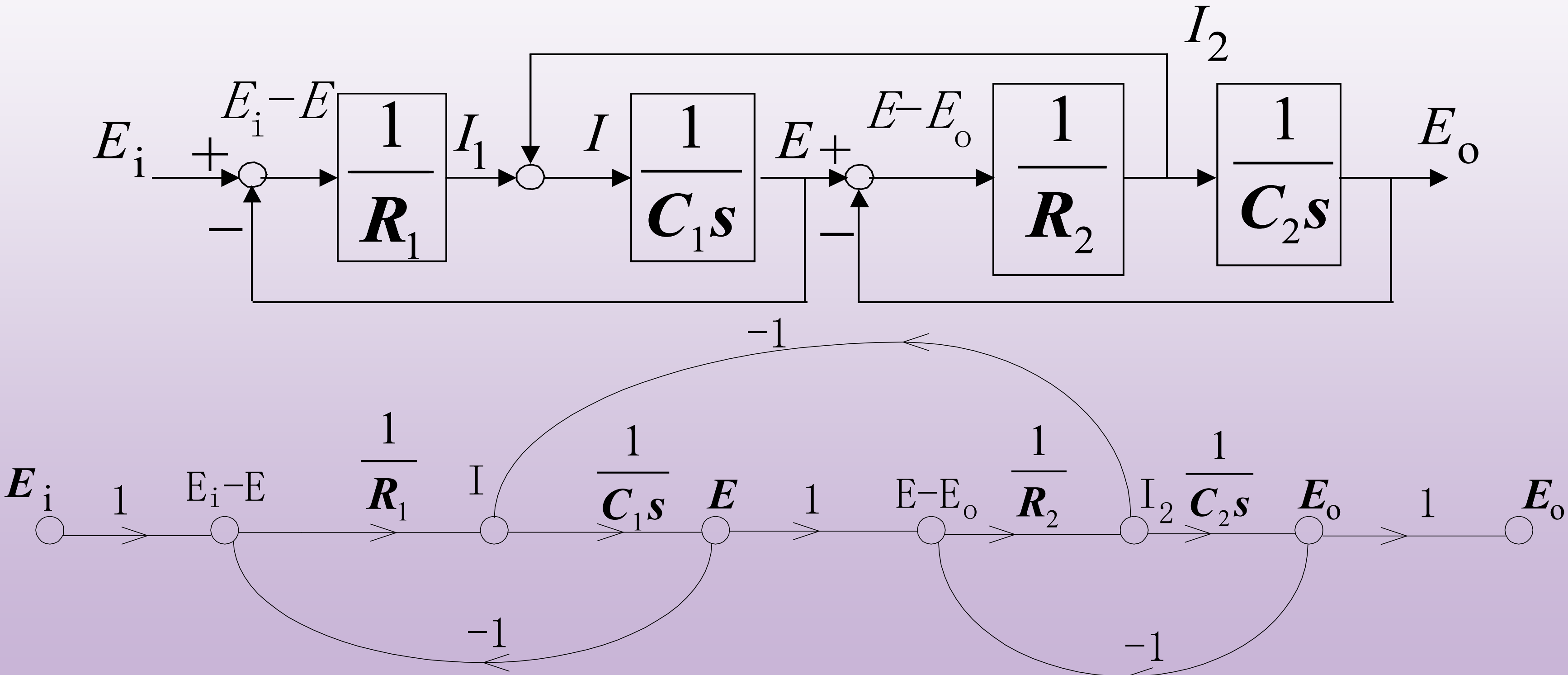
input: x_1

output: x_4



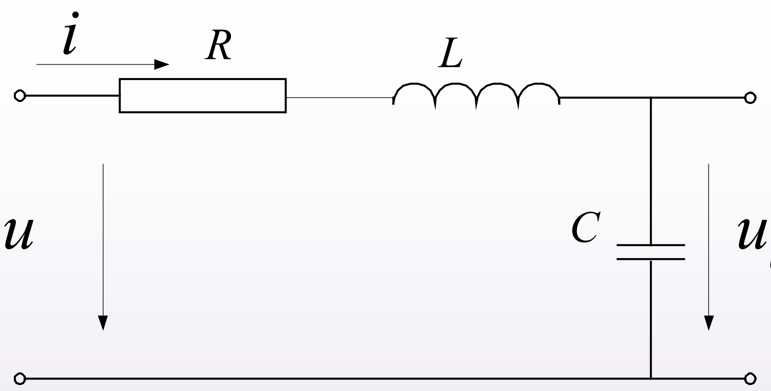
Tips

- Summing points don't exist anymore. A node is added to represent the variable after a summing point.
- A node need be added to represent the variable at a pickoff point.
- Special nodes are used to represent input and output variables.



Example

Q: Please draw the SFG of the circuit on the right. The input is u and the output is u_c.



A:

$$L\frac{di}{dt}+Ri=u-u_c$$

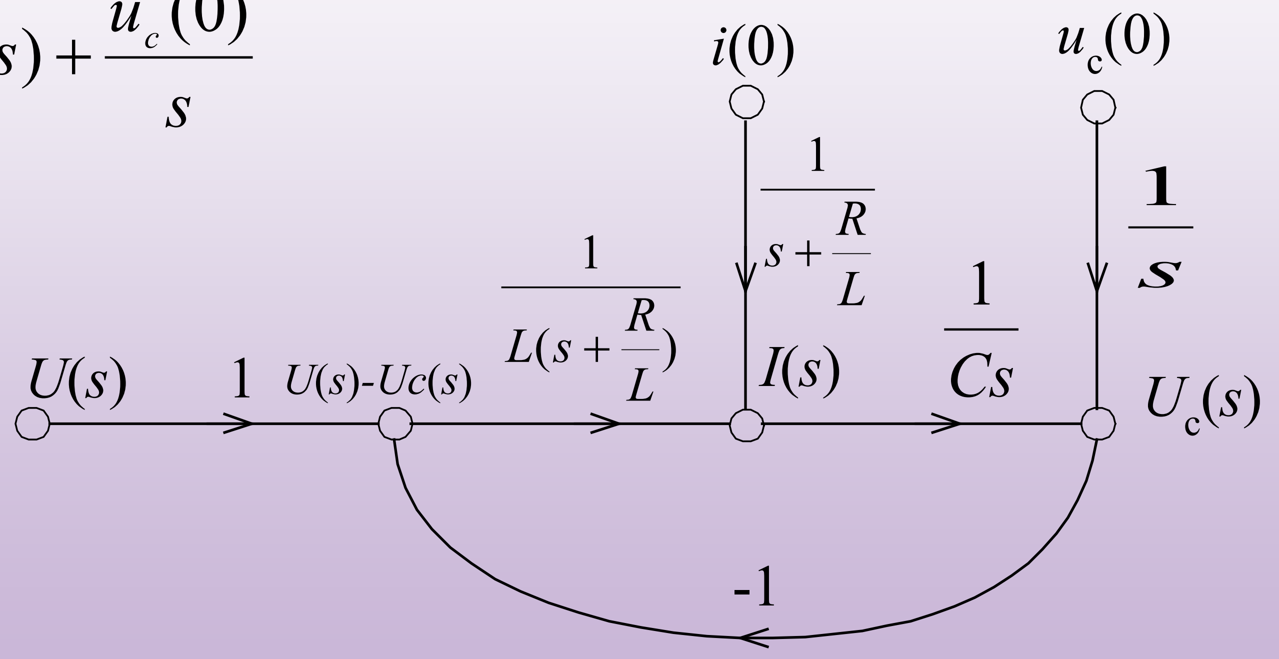
$$C\frac{du_c}{dt}=i$$

$$sLI(s)-Li(0)+RI(s)=U(s)-U_c(s)$$

$$sCU_c(s)-Cu_c(0)=I(s)$$

$$I(s)=\frac{U(s)-U_C(s)}{L(s+\frac{R}{L})}+\frac{i(0)}{s+\frac{R}{L}}$$

$$U_C(s)=\frac{1}{Cs}I(s)+\frac{u_c(0)}{s}$$



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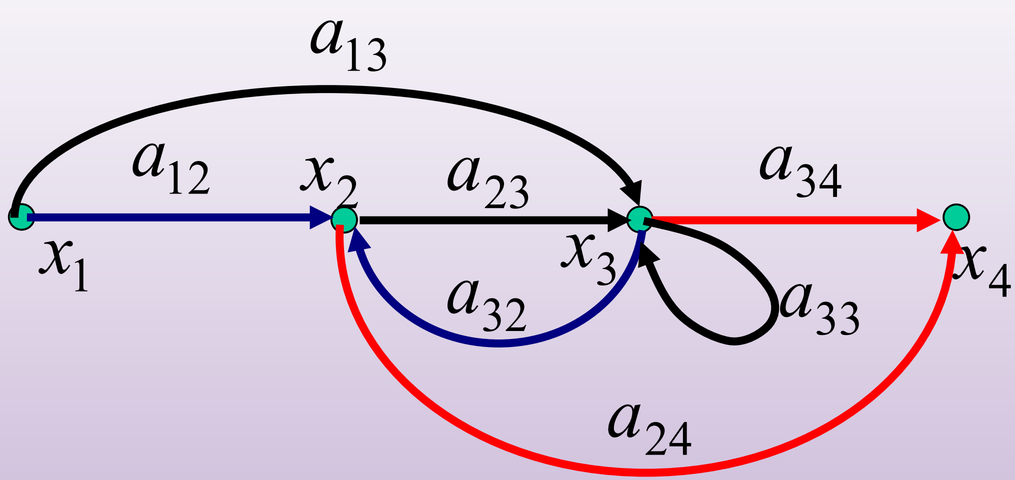
Gain Formula (Mason Formula)

- A gain formula is used to determine the input-output relations of a SFG by inspection
- Definition of SFG terms

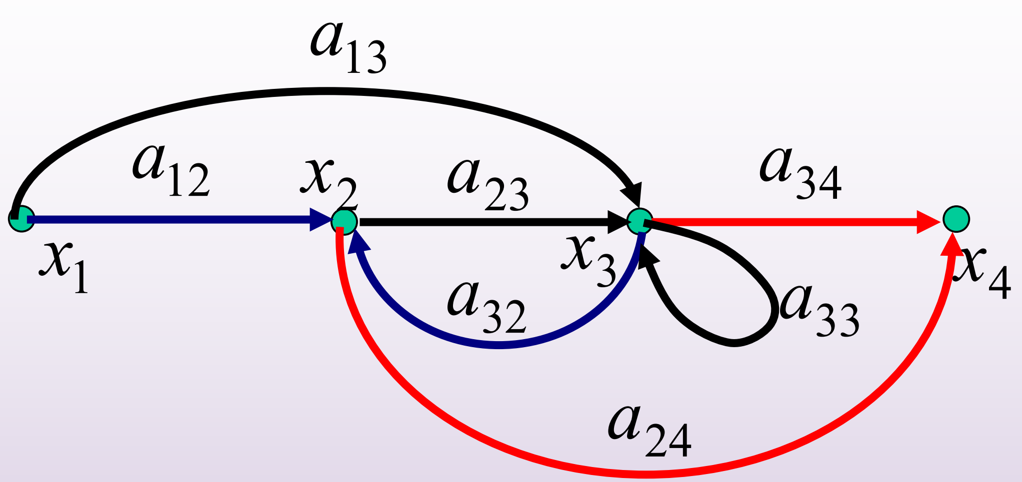


Input – node(s) that only has(have) outgoing branch(es).

Output – node(s) that only has(have) incoming branch(es).

Gain Formula (Mason Formula)

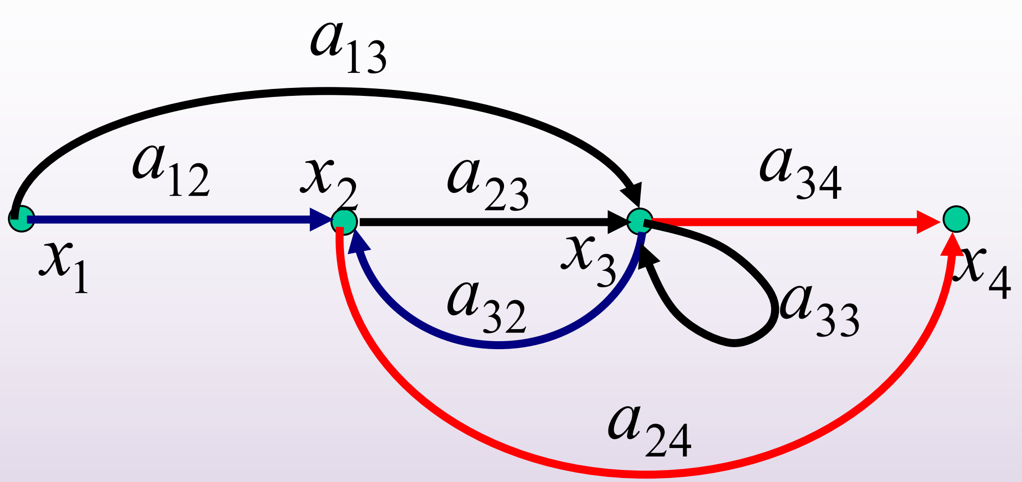
- Definition of SFG terms



  - Path: a branch or a continuous sequence of branches traversed from one signal node to another signal node.
  - Forward path: a path that starts at an input node and ends at an output node, and along which no node is traversed more than once.

Gain Formula (Mason Formula)

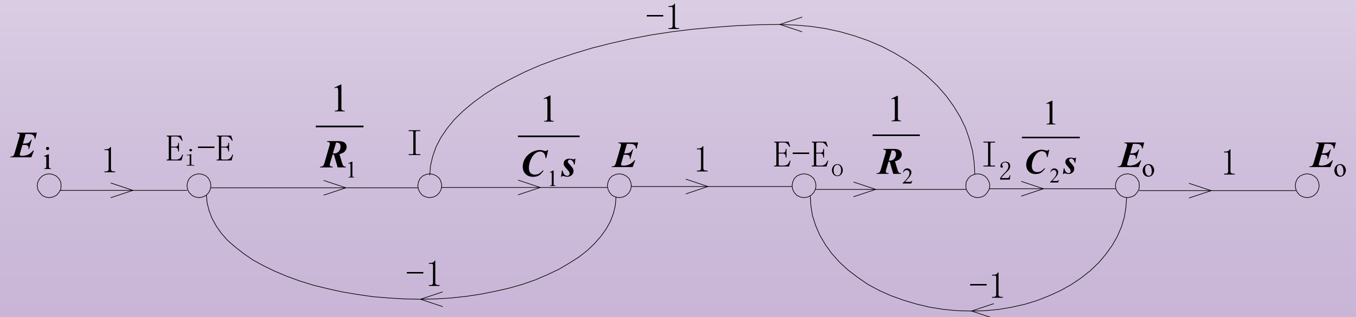
- Definition of SFG terms



  – Loop: a path that originates and terminates on the same node and along which no node is traversed more than once.

Gain Formula (Mason Formula)

- Definition of SFG terms (continued)
  - Path gain: the product of the branch gains encountered in traversing a path.
  - Forward-path gain: the path gain of a forward path.
  - Loop gain: the path gain of a loop.
  - Nontouching loops: two loops that they don’t share a common node



Gain Formula (Mason Formula)

$$G = \frac{\sum_k G_k \Delta_k}{\Delta}$$

G Gain between input and output

$$\Delta = 1 - \sum L_i + \sum L_a L_b - \sum L_\alpha L_\beta L_\gamma + \cdots$$

L_i Gain of an individual loop

$L_a L_b$ Gain product of two nontouching loops

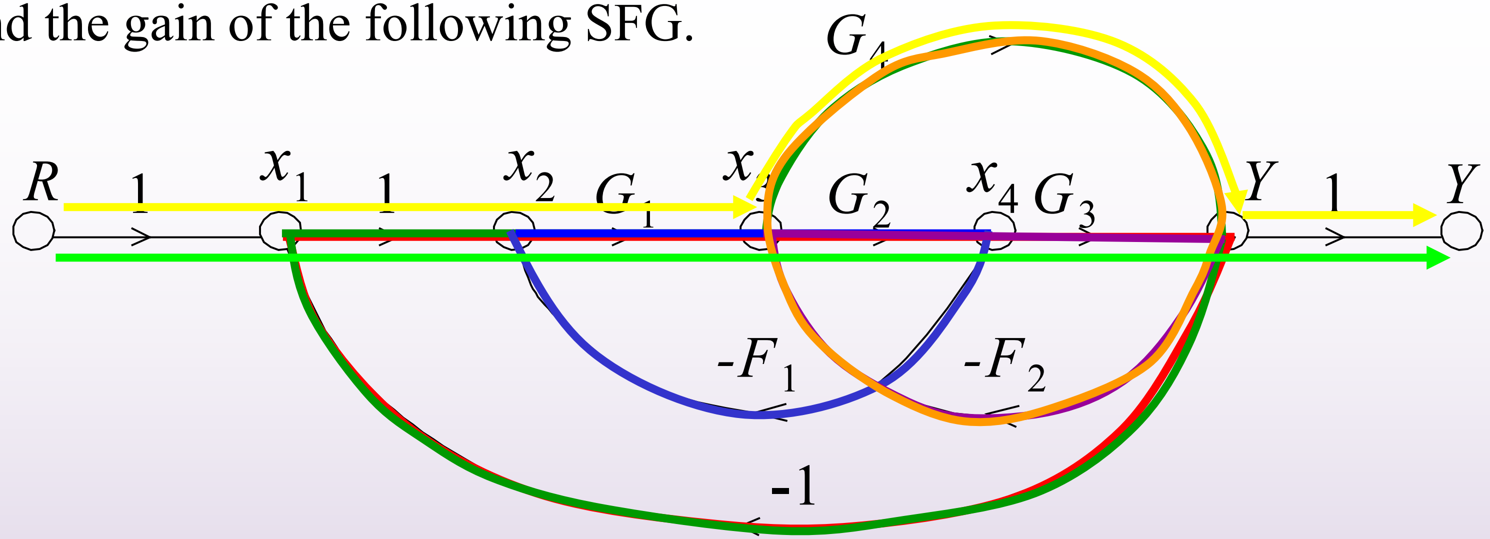
$L_\alpha L_\beta L_\gamma$ Gain product of any three nontouching loops

G_k Gain of the kth forward path between input and output

Δ_k The left part of Δ that is nontouching of the kth forward path

An Example

Q: Please find the gain of the following SFG.



A:

Loops: $-G_1G_2G_3$ $\quad -G_1G_4$ $\quad -G_1G_2F_1$ $\quad -G_2G_3F_2$ $\quad -G_4F_2$

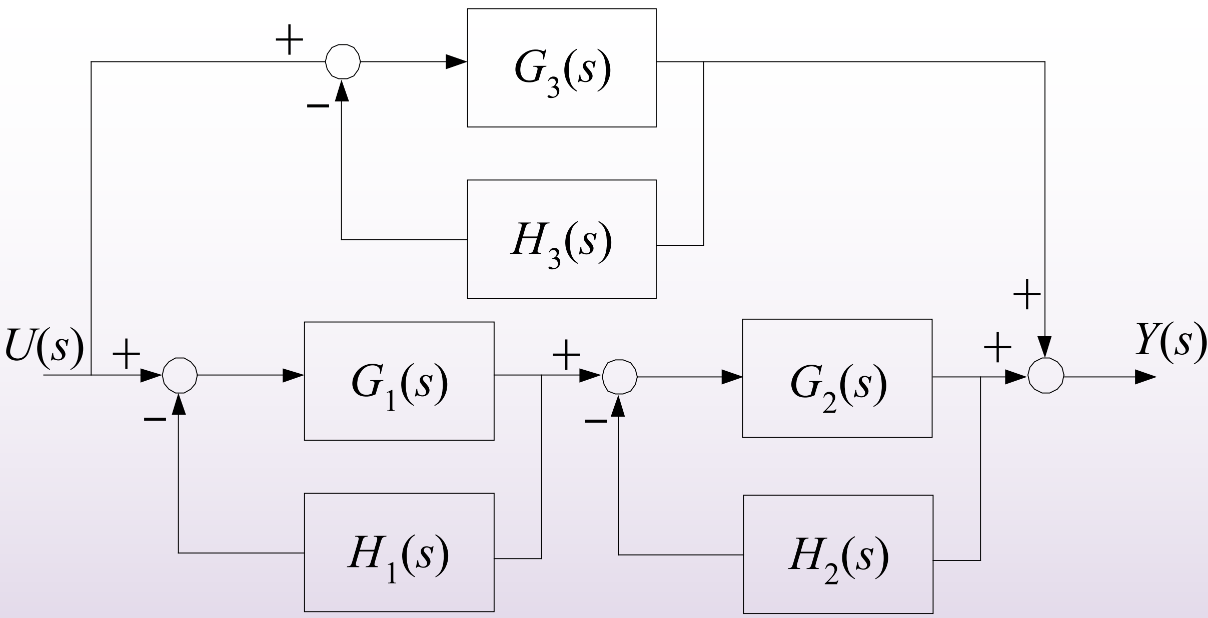
$$\Delta = 1 + G_1G_2G_3 + G_1G_4 + G_1G_2F_1 + G_2G_3F_2 + G_4F_2$$

Forward path: $G_1G_2G_3$ $\quad G_1G_4$ $\quad \Delta_1 = 1$ $\quad \Delta_2 = 1$

$$G = \frac{Y}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + G_1G_4 + G_1G_2F_1 + G_2G_3F_2 + G_4F_2}$$

Example 2.10

Q: Please find the gain from U(s) to Y(s).



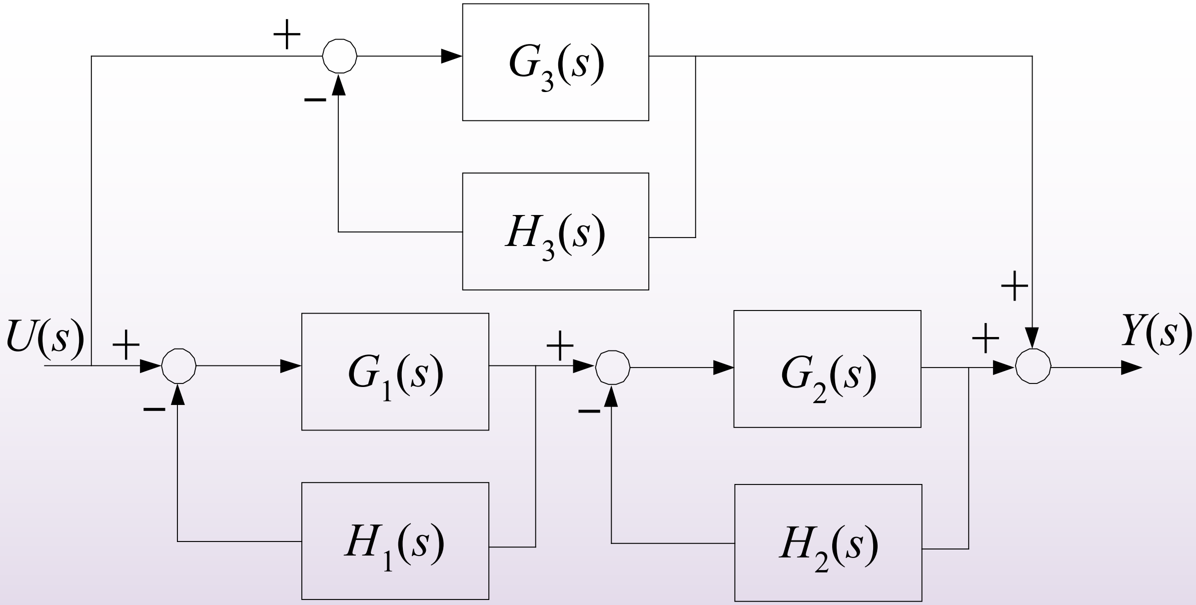
A: There are three loops and each of them is nontouching of other loops.

Loops: $-G_1H_1$ $\quad -G_2H_2$ $\quad -G_3H_3$

$$\Delta = 1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1H_1G_2H_2 + G_1H_1G_3H_3 + G_2H_2G_3H_3 + G_1H_1G_2H_2G_3H_3$$

Forward path: G_1G_2 $\quad G_3$

Example 2.10

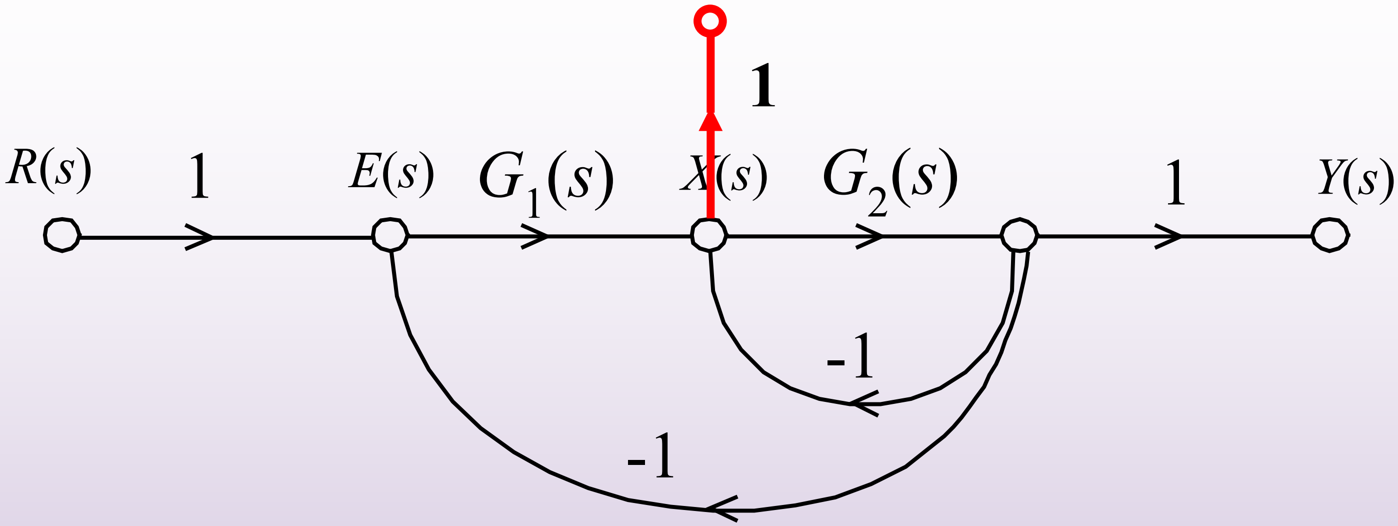


$$G(s)=\frac{G_1G_2(1+G_3H_3)+G_3(1+G_1H_1+G_2H_2+G_1H_1G_2H_2)}{1+G_1H_1+G_2H_2+G_3H_3+G_1H_1G_2H_2+G_1H_1G_3H_3+G_2H_2G_3H_3+G_1H_1G_2H_2G_3H_3}$$

$$G(s)=\frac{G_1(s)}{1+G_1(s)H_1(s)}\frac{G_2(s)}{1+G_2(s)H_2(s)}+\frac{G_3(s)}{1+G_3(s)H_3(s)}$$

Example 2.11

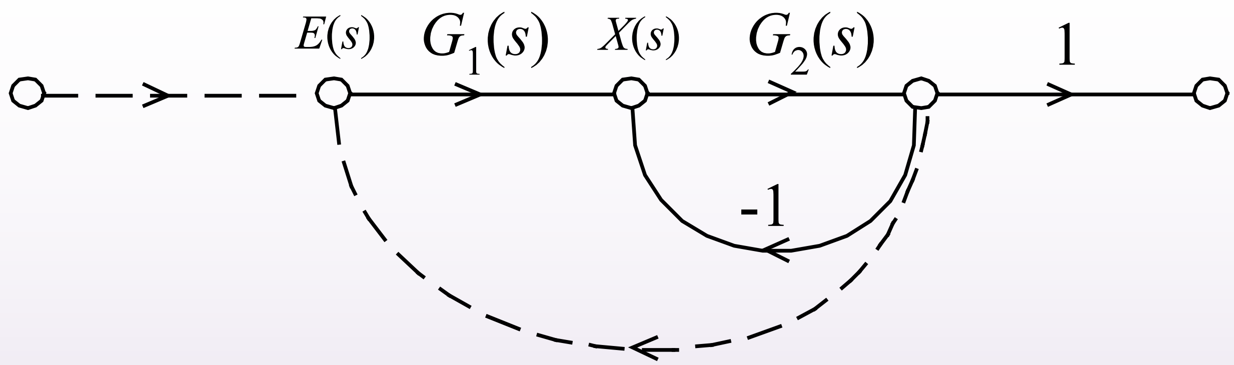
Q: Please find the transfer function of $X(s)/R(s)$, $X(s)/E(s)$



A: X(s) is a middle point. We can add an outgoing branch and make it an output node. Then use Mason formula to get the transfer function.

$$\frac{X(s)}{R(s)} = \frac{G_1(s)}{1 + G_2(s) + G_1(s)G_2(s)}$$

Example 2.11 (Continued)



E(s) is not an input node. Mason formula can not be applied directly. Incoming branches should be taken away before calculate the transfer function.

$$\frac{X(s)}{E(s)} = \frac{G_1(s)}{1+G_2(s)}$$

Discussions about SFG

- Advantages
- Disadvantages

- Advantages
  - Straight forward
  - Easy to be implemented by computers
- Disadvantages
  - Complex

Outlines

- Block diagram of a linear system
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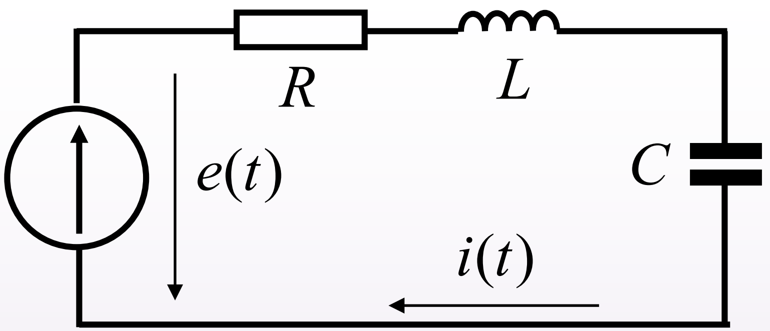
State Space Models

- Dynamic systems (LTI) are represented by ordinary differential equations (ODE) that can be high order ones;
- First-order ODEs is simpler than high-order ones to solve;
- High-order ODE can be decomposed into a set of first-order ODEs



$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = e(t)$$

$$x_1(t) = \int i(t)dt \qquad x_2(t) = \frac{dx_1(t)}{dt} = i(t)$$

$$\begin{cases} \dfrac{dx_1(t)}{dt} = i(t) \\ \dfrac{dx_2(t)}{dt} = -\dfrac{1}{LC}x_1(t) - \dfrac{R}{L}x_2(t) + \dfrac{1}{L}e(t) \end{cases}$$

Term Definition of State Space Models

- **State:** the state of a system refers to the past, present and future of the system;
- **State Variables:** a minimal set of variables such that knowledge of these variables at any time t_0 and information on the input excitation subsequently applied are sufficient to determine the state of the system at any time $t > t_0$;
- **State Equations:** a minimal set of first-order ordinary differential equations that completely define the future behavior of a dynamic system;
- **Output Variable:** a variable that can be measured;
- **Output Equation:** an expression of output variable which is an algebraic combination of the state variables;

State Space Equation of SISO

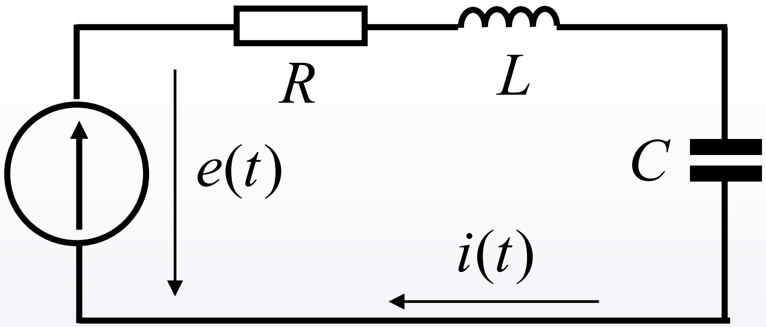
$$\dot{X} = AX + Bu$$

$$y = CX$$

$y, \quad u$ are scalars

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \qquad C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}^T \qquad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Example



$$\begin{cases} \dfrac{dx_1(t)}{dt} = i(t) \\ \dfrac{dx_2(t)}{dt} = -\dfrac{1}{LC}x_1(t) - \dfrac{R}{L}x_2(t) + \dfrac{1}{L}e(t) \end{cases}$$

$$x_1(t) = \int i(t)dt \qquad x_2(t) = \frac{dx_1(t)}{dt} = i(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{R}{L} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y = \begin{bmatrix} \dfrac{1}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Example

Question: find a state-space representation of the following system.

$$m_1\ddot{y}_1 + b\dot{y}_1 + k(y_1 - y_2) = 0$$

$$m_2\ddot{y}_2 + k(y_2 - y_1) = u$$

Solution: define state variables as

$$x_1 = y_1 \quad \Rightarrow \quad \dot{x}_1 = x_2$$

$$x_2 = \dot{y}_1 \quad \Rightarrow \quad \dot{x}_2 = \frac{1}{m_1}[-b\dot{y}_1 - k(y_1 - y_2)] = -\frac{k}{m_1}x_1 - \frac{b}{m_1}x_2 + \frac{k}{m_1}x_3$$

$$x_3 = y_2 \quad \Rightarrow \quad \dot{x}_3 = x_4$$

$$x_4 = \dot{y}_2 \quad \Rightarrow \quad \dot{x}_4 = \frac{1}{m_2}[-k(y_2 - y_1) + u] = -\frac{k}{m_2}x_1 - \frac{k}{m_2}x_3 + \frac{k}{m_2}u$$

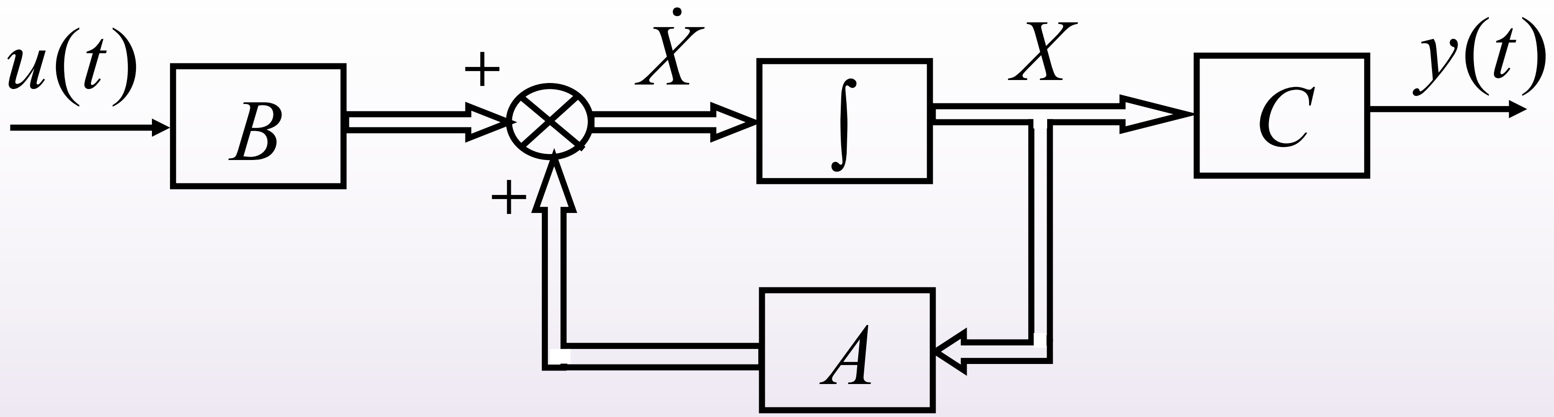
Example

Hence, the state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k}{m_1} & -\dfrac{b}{m_1} & \dfrac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{m_2} & 0 & -\dfrac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{m_2} \end{bmatrix} u$$

What's the output equation?

Block Diagrams of State Equations



Inertial

$$\dot{X} = AX + Bu$$
$$y = CX$$

Non-inertial

$$\dot{X} = AX + Bu$$
$$y = CX + Du$$

State Space Equation of MIMO

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

$U:\quad p\times 1$ vector

$Y:\quad q\times 1$ vector

$D:\quad q\times p$ vector

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix} \quad C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1q} \\ c_{21} & c_{22} & \cdots & c_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nq} \end{bmatrix}^T$$

Remarks about State Space Models

- All the state space models of systems have the same form, which is comprised of a state equation and an output equation, no matter the system is an open-loop control system or a closed-loop system, a subsystem or a whole system.
- Although the state variables of a system are the minimal set of variables that completely determine the dynamic behavior of the system, the choosing of state variables is not unique. But the number of state variables is constant.
- The number of state variables of a system depends on the number of independent dynamics of the system.

Question

- If a system can be described by different state equations, what is the relationship between the state equations?

Similarity Transformation

SISO:
$$\dot{X} = AX + Bu$$
$$y = CX + Du$$

Transformation: $X = PX'$ where P is a $n \times n$ nonsingular matrix

$$P\dot{X}' = APX' + Bu \qquad \dot{X}' = P^{-1}APX' + P^{-1}Bu$$
$$y = CPX' + Du \qquad y = CPX' + Du$$

If set $A' = P^{-1}AP$ $B' = P^{-1}B$ $C' = CP$ $D' = D$

$$\dot{X}' = A'X' + B'u$$
$$y = C'X' + D'u$$

Question

- Both transfer functions and state equations describe the dynamics of systems, what’s the relationship between them?

Outlines

- Block diagram of a linear system
- Block diagram transformation
- Signal-flow graph
- Gain formula (Mason Formula)
- State space model
- State equation versus transfer function – from SE to TF

Relationship between TF and SE

- **From SE to TF**

SISO:
$$\dot{X} = AX + Bu$$
$$y = CX$$

Laplace Transform:

$$sX(s) - X(0) = AX(s) + BU(s)$$
$$Y(s) = CX(s)$$
$$Y(s) = C(sI - A)^{-1}[X(0) + BU(s)]$$

Set $X(0) = 0$

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$

Relationship between TF and SE

For MIMO:

$$Y(s) = C(sI - A)^{-1}BU(s) = G(s)U(s)$$

$$U(s) = \begin{bmatrix} U_1(s) \\ \vdots \\ U_p(s) \end{bmatrix} \qquad Y(s) = \begin{bmatrix} Y_1(s) \\ \vdots \\ Y_q(s) \end{bmatrix} \qquad \cancel{G(s) = \frac{Y(s)}{U(s)}}$$

$$G(s) = C(sI - A)^{-1}B = \begin{bmatrix} G_{11}(s) & \cdots & G_{1p}(s) \\ \vdots & \vdots & \vdots \\ G_{q1}(s) & \cdots & G_{qp}(s) \end{bmatrix}_{q\times p}$$

$$G_{ij}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1}{s^n + a_n s^{n-1} + \cdots + a_1} \qquad (n \geq m)$$

Question

- A system may have different state equations but one transfer function for a certain input-output pair, will different state equations lead to different transfer function for the same input-output pair?

投票 最多可选1项

设置

Different state equations of a LTI system lead to different transfer functions for the same input-output pair

A True

B False

提交

Invariability of TF

- TF keeps invariant after similarity transform

$$\dot{X} = AX + Bu \qquad G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$
$$y = CX$$

$$\dot{X}' = A'X' + B'u \qquad X = PX' \qquad \boxed{M^{-1}N^{-1} = (NM)^{-1}}$$
$$y = C'X'$$

$$C' = CP \qquad A' = P^{-1}AP \qquad B' = P^{-1}B$$

$$G'(s) = C'(sI - A')^{-1}B' = CP \cdot \underline{(sI - P^{-1}AP)^{-1}} \cdot \underline{P^{-1}}B$$
$$= C \cdot \underline{(P^{-1})^{-1}} \cdot \underline{[P(sI - P^{-1}AP)]^{-1}} \cdot B$$
$$= C[P(sI - P^{-1}AP)P^{-1}]^{-1}B = C(sI - A)^{-1}B = G(s)$$

Wrap-up

- **Block diagram of a linear system**
- **Block diagram transformation**
- **Signal-flow graph**
- **Gain formula (Mason Formula)**
- **State space model**
- **State equation versus transfer function – from SE to TF**

Assignment

Page 34

- **1.**
- **2. (b)**
- **3. (c)**

Assignment

Page 35

- **5: (b)**
- **6: (c)**
- **9**

Discussion

- Autonomous system

In mathematics, an autonomous system or autonomous differential equation is a sytem of ordinary differential equations which does not explicitly depend on the independant variable. When the variable is the time, they are also named Time-invariant system

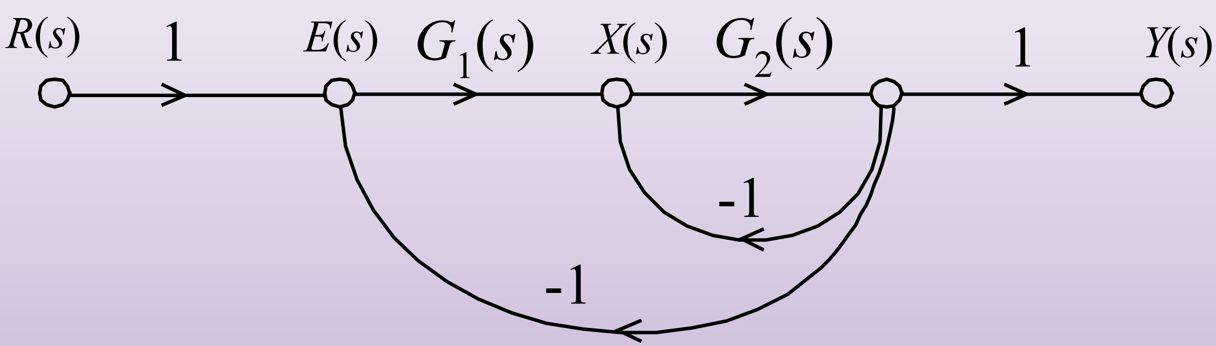
$$\frac{d}{dt}x(t) = f(x(t))$$

Discussions

- Can signal-flow graphs be applied to nonlinear systems?
- Can the gain formula be directly applied between any two nodes of a SFG?
- For an electric circuit with R, L and C, is the number of its state variables equal to the number of L and C?

Discussions

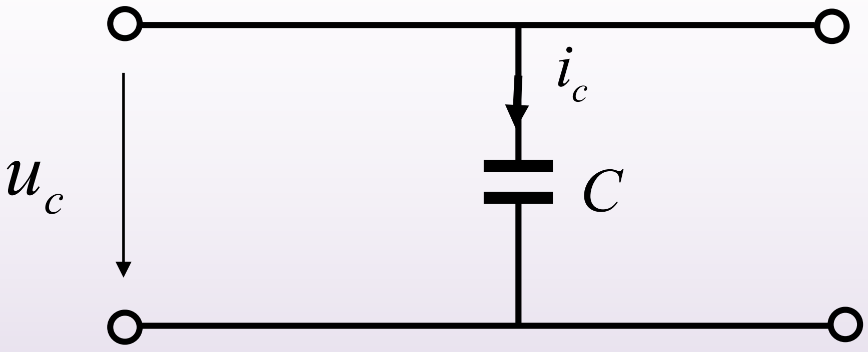
- For a multi-input multi-output LTI system, if the inputs and outputs have been determined, are the denominators of all the elements in the system transfer function matrix the same?



Discussions

- For a non-proper system, such as a pure derivative block, will it change to a proper one if we reverse its input and output?

Discussions

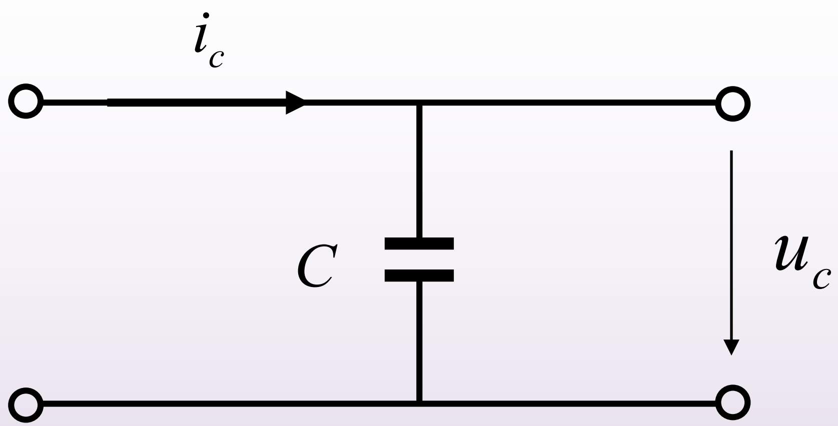


Transfer function from u_c to i_c :

$$G(s) = \frac{I(s)}{U(s)} = Cs$$

Non-proper system

Discussions



Transfer function from i_c to u_c : $G(s) = \dfrac{U(s)}{I(s)} = \dfrac{1}{Cs}$

Proper system