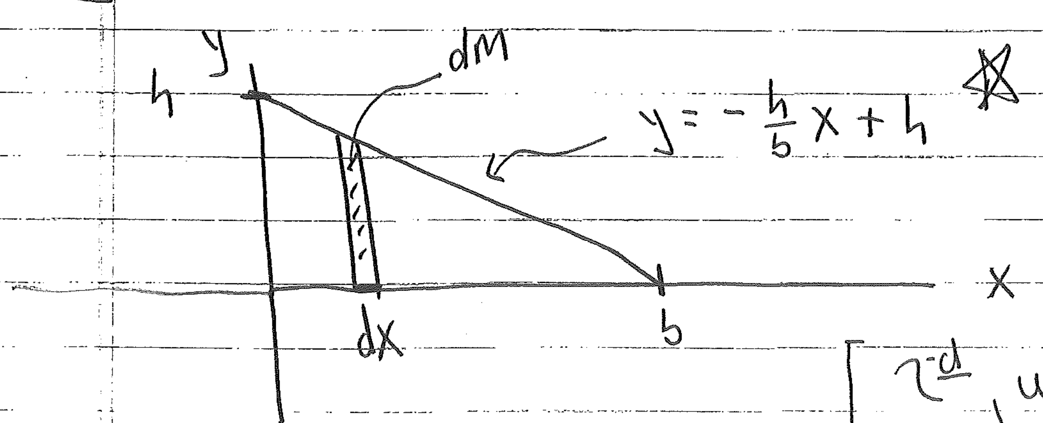
EX] A triangular plate has a mass M. The length of its base is 'b'. Its height is 'h'.

Find the location of its CM (COM).




$$X_{cm} = \frac{\int x\, dM}{[\int dM]}$$ This is M!!!

[2-d, uniform distribution of mass
$\sigma = \frac{M}{\frac{1}{2}bh} = \frac{2M}{bh}$ ]

dM ⇒ a small piece of mass
(density) * Area in 2-d

$$X_{cm} = \frac{1}{M}\int x\, dm$$

$$dM = \sigma \underbrace{y\, dx}_{\text{area}}$$

$$= \frac{1}{M}\int x \sigma y\, dx$$

$$= \frac{1}{M}\int_0^b x\sigma\left(-\frac{h}{b}x + h\right)dx$$

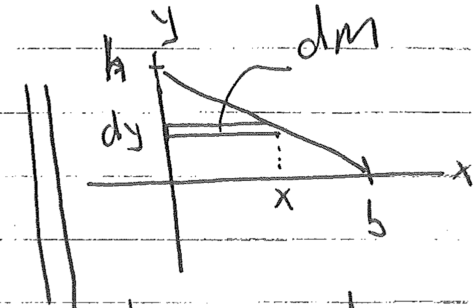
$$= \frac{\sigma}{M}\left[-\frac{h}{b}\int_0^b x^2 dx + h\int_0^b x\, dx\right]$$

$$= \frac{\sigma}{M}\left[ -\frac{h}{b}\frac{x^3}{3}\Big|_0^b + \frac{h x^2}{2}\Big|_0^b \right]$$

$$= \frac{\sigma}{M}\left[ -\frac{hb^3}{3b} + \frac{hb^2}{2} \right] = \frac{\sigma}{M} hb^2 \left[ -\frac{1}{3} + \frac{1}{2} \right]$$

$$= \frac{\sigma h b^2}{6M} = \frac{2M}{bh}\frac{hb^2}{6M} = \frac{b}{3} \quad \boxed{\checkmark} \; X_{cm}$$

$$y_{cm} = \frac{\int y\, dm}{\int dm} = \frac{1}{M}\int y\, dm$$



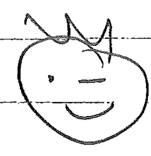

$$dm = \sigma x\, dy$$

$$= \frac{1}{M}\int y \sigma x\, dy$$

☆

$$x = \frac{b}{h}(h-y)$$

$$= \frac{\sigma}{M}\int_0^h y \frac{b}{h}(h-y)\, dy$$



$\vdots$

$$y_{cm} = h/3$$

# Rotational Motion (Chapters 9, 10, 11)

## NO NEW PROBLEM SOLVING TECHNIQUES

| Translational | Rotational |
| --- | --- |
| position (x) meters | → angle ($\theta$) radians, degrees, rotations |
| velocity $= \frac{dx}{dt}$ m/s | → angular velocity $\omega = \frac{d\theta}{dt}$ $\frac{\text{radians}}{s}$, $\frac{{}^\circ}{\text{sec}}$, ... |
| acceleration $= \frac{dv}{dt}$ m/s² | → Angular acceleration $\alpha = \frac{d\omega}{dt}$ radians/s² |
| for constant acceleration | for constant $\alpha$ |
| $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$ | $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $v(t) = v_0 + at$ | $\omega(t) = \omega_0 + \alpha t$ |
| Mass (m) kg | → Moment of Inertia (I) $I = \sum_i m_i r_i^2$ OR $= \int r^2 dm$ |
| Force $\vec{F}$ | Torque, $\vec{\tau}$ |
| 2nd $\sum \vec{F} = m\vec{a}$ | → $\sum \vec{\tau} = I\vec{\alpha}$ |

$\vec{\tau} = \vec{r} \times \vec{F}$

$KE = \frac{1}{2} m v^2 \longrightarrow KE_{rotation} = \frac{1}{2} I \omega^2$

$\vec{p} = m\vec{v}$

Conservation of $\vec{p}$ $\longrightarrow$ Angular momentum $(\vec{L})$

$\vec{L} = I\vec{\omega}$

$\boxed{OR}$ $\vec{L} = \vec{r} \times \vec{p}$

$$\boxed{\begin{aligned} v &= r\omega \\ a &= r\alpha \end{aligned}}$$

"cross-talk" :-?