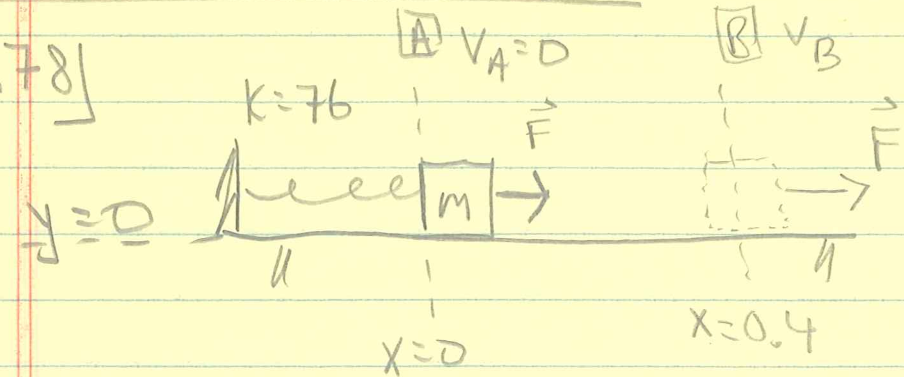
HW due 2/28 Selected Solutions

6.78



$m = 2$ kg

$|\vec{F}| = 54$

$+\hat{i}$

No friction

System: Spring, block, Earth

$$TE_{A} = \underbrace{KE_{A}}_{0} + \underbrace{PE_{g_A}}_{0} + \underbrace{PE_{s_A}}_{0} = 0$$

$$TE_{B} = KE_{B} + \underbrace{PE_{g_B}}_{0} + PE_{s_B} = \frac{1}{2} m V_B^2 + \frac{1}{2} k x_B^2$$

Cons. of Energy

$$TE_{A} + (\text{work done by } \vec{F}) = TE_{B}$$

$$\int \vec{F} \cdot d\vec{s}$$

$$54 \int d\vec{s}$$

$$54(0.4) = \frac{1}{2} m V_B^2 + \frac{1}{2} k x_B^2$$

$$3.94 \text{ m/s} = \sqrt{\frac{2(21.6 - 6.08)}{2}} = V_B$$

Answer (a)

(b) @ this instant, $\vec{F}_{spring} = -76(0.4)\hat{i} = -30.4\hat{i}$

while $\vec{F} = 54\hat{i}$

2^{nd} Law $(\hat{i})$ $\sum F_x = m a_x$

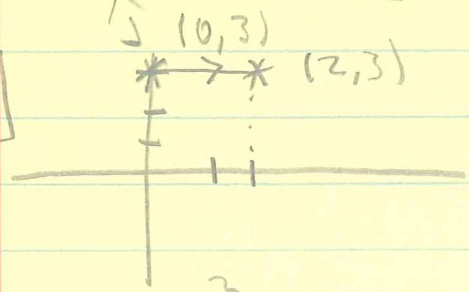
$$54 - 30.4 = 2a_x$$

$$+11.8\ m/s^2 = a_x$$ Answer.

to right

6.88] $\vec{F} = 2.5xy\,\hat{i}$

a.]



$$W = \int \vec{F}\cdot d\vec{s}$$

$d\vec{s} = dx\,\hat{i}$

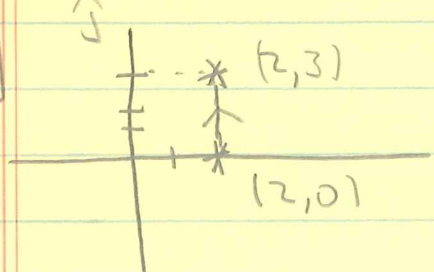
$$W = \int 2.5xy\,dx\,\underbrace{\hat{i}\cdot\hat{i}}_{1}$$

$$W = \int_0^2 2.5x(3)\,dx = 7.5\left(\frac{x^2}{2}\right)\Bigg|_0^2$$

constant along path

$W = +15\ J$ Answer.

b.]

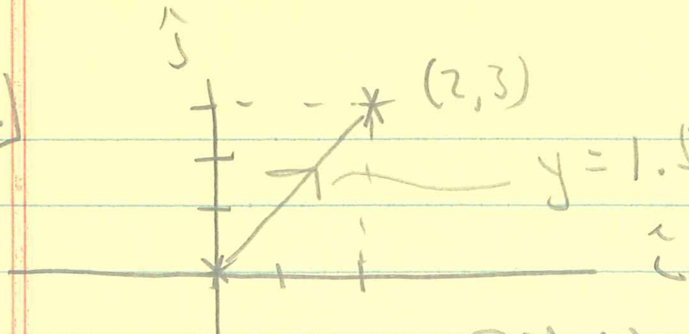


$$W = \int \vec{F}\cdot d\vec{s}$$

$d\vec{s} = dy\,\hat{j}$

$$W = \int 2.5xy\,dy\,\underbrace{\hat{i}\cdot\hat{j}}_{0} = 0$$ Answer.

c.)



$y = 1.5x \quad \therefore dy = 1.5\,dx$

$W = \int \vec{F} \cdot d\vec{s}$

use general form $d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$W = \int (2.5xy\,\hat{i}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

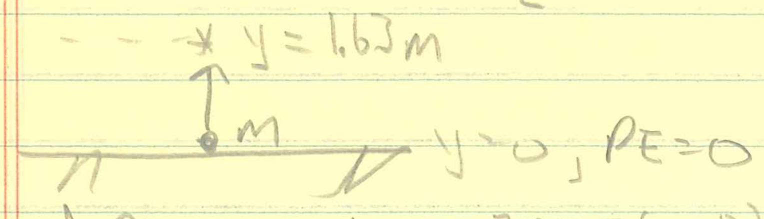
$W = \int 2.5xy\,dx = 3.75 \int_0^2 x^2\,dx = 3.75 \left(\frac{x^3}{3}\right)\Big|_0^2$

substitute $y = 1.5x$ to make this path specific

$W = 10$ joules Answer.

6.89) 1 liter $= 0.001\,m^3$

$M = 7500\,L \times \frac{0.001\,m^3}{L} \times 1.05 \times 10^3 \frac{kg}{m^3} = 7875\,kg$



$\Delta PE = mgh = 7875(9.8)(1.63) = 125.795 \times 10^3\,J$

Answer (a)

$1\,day = 24\,hr \times \frac{60\,min}{hr} \times \frac{60\,sec}{min} = 86400\,sec.$

$\therefore$ Power output $= \frac{125.795 \times 10^3}{86400} = 1.456$ watts

Answer (b)

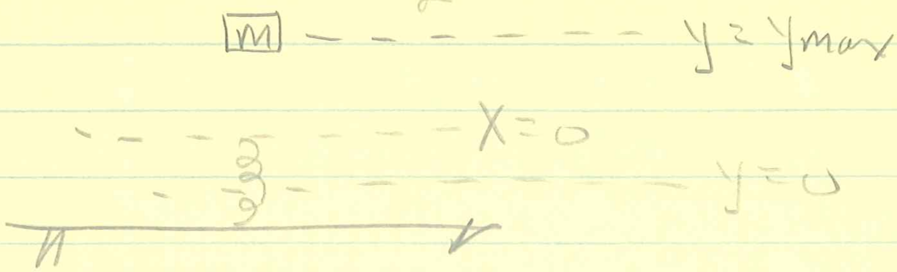
7.20 <u>System</u>: cheese, spring, Earth.

$m = 1.2$ kg
$k = 1800$ N/m

$\boxed{A}$

NOTE: "x" is measuring spring compression.
"y" is for PE_g

$\boxed{B}$



$$TE_{\boxed{A}} = \underbrace{\cancel{KE_{\boxed{A}}}}_{\to 0 \text{ "from rest"}} + \underbrace{\cancel{PE_{g\boxed{A}}}}_{\to 0,\ mg(0)} + PE_{s\boxed{A}} = \frac{1}{2}k\underbrace{x^2}_{0.15} = 20.25 \text{ J}$$

$$TE_{\boxed{B}} = \underbrace{\cancel{KE_{\boxed{B}}}}_{\to 0 \ @\ y_{max}} + PE_{g\boxed{B}} + \underbrace{\cancel{PE_{s\boxed{B}}}}_{\to 0,\ x=0} = mg\,y_{max}$$

Conservation of Energy: $TE_{\boxed{A}} = TE_{\boxed{B}}$

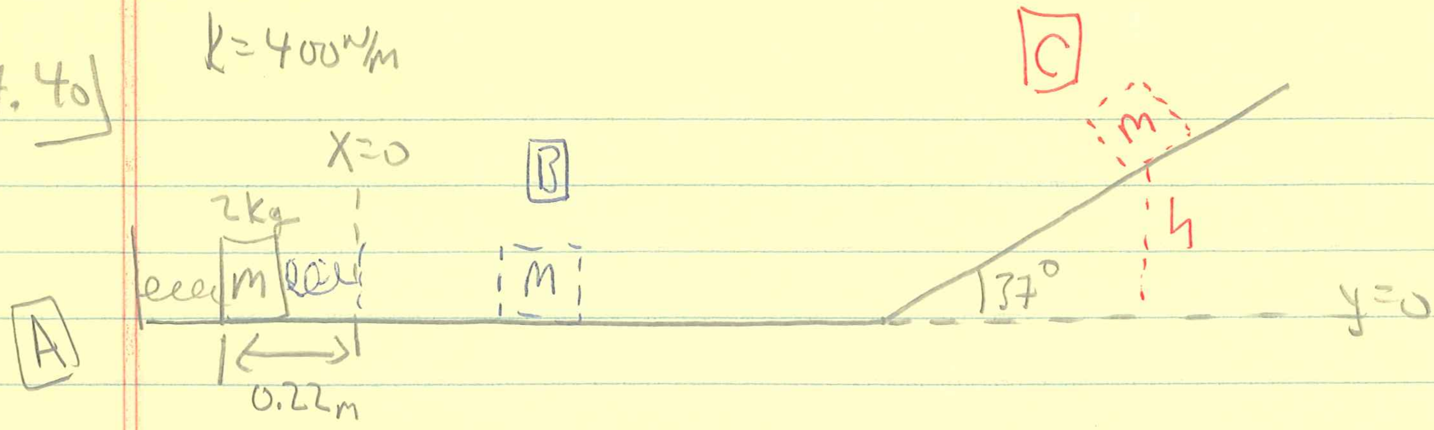
$$20.25 = mg\,y_{max}$$

$$y_{max} = \underline{\underline{1.72 \text{ meters}}}$$

Answer.

7.40)

$k = 400\ N/m$



System: Mass, Spring, Earth.

$$TE_{\boxed{A}} = \underbrace{\cancel{KE}_{\boxed{A}}}_{\to 0 \text{ "from rest"}} + \underbrace{\cancel{PE}_{g\boxed{A}}}_{\to 0\ y=0} + PE_{s\boxed{A}} = \frac{1}{2} \underset{400}{k} \underset{.22}{x}^2 = 9.68 \text{ joules}$$

$$TE_{\boxed{B}} = KE_{\boxed{B}} + \underbrace{\cancel{PE}_{g\boxed{B}}}_{\to 0\ y=0} + \underbrace{\cancel{PE}_{s\boxed{B}}}_{\to 0\ x=0} = \frac{1}{2} m v_B^2$$

$$TE_{\boxed{C}} = \underbrace{\cancel{KE}_{\boxed{C}}}_{\to 0 \text{ @turning point}} + PE_{g\boxed{C}} + \underbrace{\cancel{PE}_{s\boxed{C}}}_{\to 0\ x=0} = mgh$$

Conservation of Energy

$$TE_{\boxed{A}} = TE_{\boxed{B}}$$
$$9.68 = \frac{1}{2} m v_B^2$$
$$\therefore v_B = \sqrt{2(9.68)/2} = 3.11\ m/s$$

Answer.

$TE_{\boxed{A}} = TE_{\boxed{C}}$

$9.68 = mgh$

$\therefore h = \dfrac{9.68}{2(9.8)} = 0.4939$

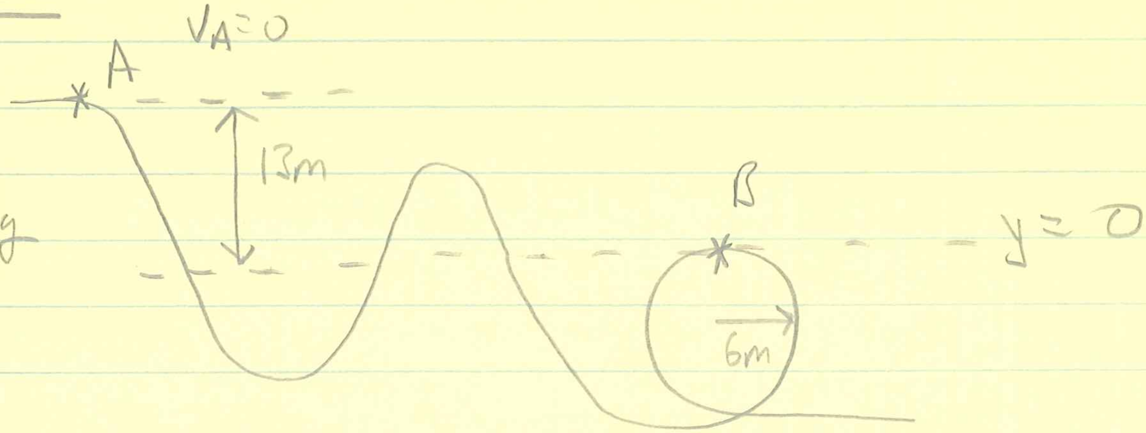
L, 37°, h

$h = L\sin(37)$

$\therefore L = h/\sin(37) = \underline{\underline{0.821 \text{ meters}}}$ Answer

7.4

$M = 350\text{kg}$



System: Car, Earth

$TE_{\boxed{A}} = \underbrace{KE_{\boxed{A}}}_{\to 0 \text{ "from rest"}} + PE_{g\boxed{A}} = 13mg$

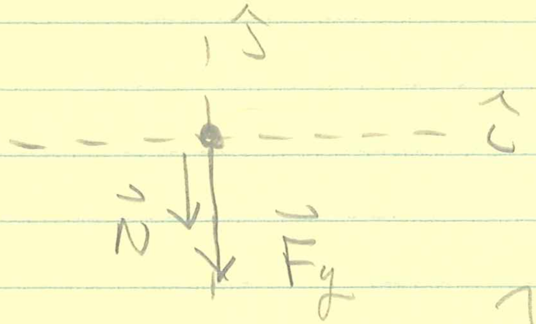
$TE_{\boxed{B}} = KE_{\boxed{B}} + \underbrace{PE_{g\boxed{B}}}_{\to y=0} = \frac{1}{2}mv_B^2$

Conservation of Energy: $TE_{\boxed{A}} = TE_{\boxed{B}}$

$13mg = \frac{1}{2}mv_B^2$

$\therefore v_B = \sqrt{26g} = \underline{\underline{15.96 \text{ m/s}}}$ Ans.

for m @ location B



$\vec{N} = -N\hat{j}$

$\vec{F}_y = -mg\hat{j}$

2^{nd}: $\boxed{\hat{j}}$ $\sum F_y = m a_y$

centripetal

$\left(-\frac{v_B^2}{R}\right)$

my coordinates

$$2^{nd} \Rightarrow -N - mg = -\frac{m v_B^2}{R}$$

$$\therefore N = \frac{m v_B^2}{R} - mg = \frac{350(15.96)^2}{6} - 350(9.8)$$

$$\underline{\underline{N = 11429 \text{ newtons}}}$$

Answer.

7.48] From classnotes on 2/24 we had worked down to two equations:

$\boxed{1}$ $Kx = 1065.67$

$\boxed{2}$ $\frac{1}{2}mv_A^2 + mgh - 2575 = \frac{1}{2}Kx^2$

Lets go ahead and substitute values into $\boxed{2}$

$$\boxed{2} \Rightarrow \frac{1}{2}(150)(1.8)^2 + 150(9.8)(5\sin(22)) - 2575 = \frac{1}{2}kx^2$$

$$842.7169 = kx^2$$

From $\boxed{1}$, $x = \frac{1065.67}{k}$. Use this above

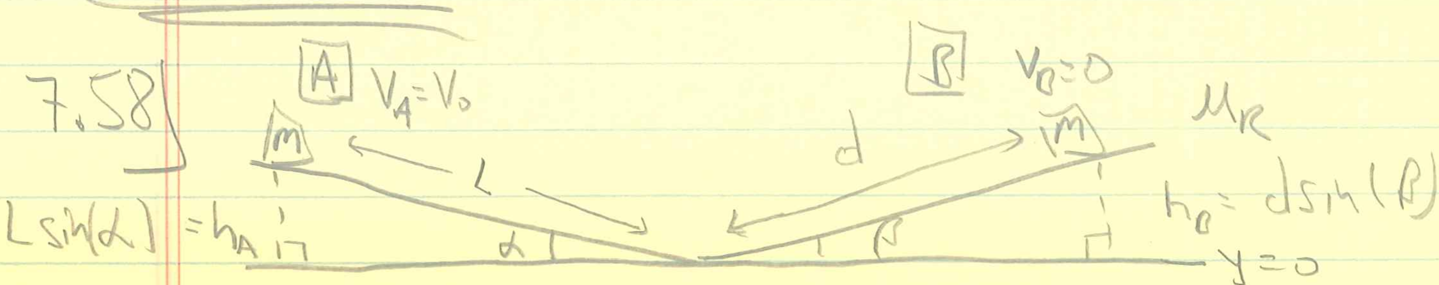
to get:

$$842.7169 = \frac{1065.67^2}{k}$$

$$k = 1347.6 \text{ N/m}$$

Answer.

7.58



System: Truck, Earth

$$TE_{\boxed{A}} = KE_{\boxed{A}} + PE_{g\boxed{A}} = \frac{1}{2}mv_0^2 + mgL\sin(\alpha)$$

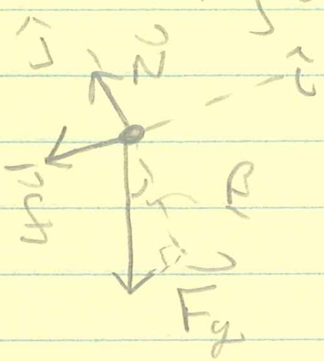
$$TE_{\boxed{B}} = \underbrace{KE_{\boxed{B}}}_{0} + PE_{g\boxed{B}} = mgd\sin(\beta)$$

Cons. of Energy

$$TE_{\boxed{A}} + \left(\text{Work done by friction}\right) = TE_{\boxed{B}}$$

Need to find work done by friction.

For truck moving up ramp:



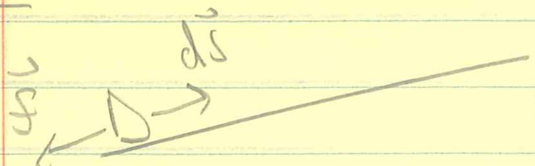
$$\vec{N} = N\hat{j}$$
$$\vec{f} = -\mu_R N \hat{i}$$
$$\vec{F}_g = -mg\sin(\beta)\hat{i} - mg\cos(\beta)\hat{j}$$

2nd $\boxed{\hat{j}}$ $\sum F_y = m\underbrace{a_y}_{\to 0}$

$$N - mg\cos(\beta) = 0$$
$$\underline{N = mg\cos(\beta)}$$

This is as far as we need to go because we can now write an expression for f:

$$\vec{f} = -\mu_R Mg\cos(\beta)\hat{i}$$



$$W_{\vec{f}} = \int \vec{f}\cdot d\vec{s} = \int |\vec{f}||d\vec{s}|\cos(180)$$

$$W_{\vec{f}} = -\int \mu_R mg\cos(\beta)|d\vec{s}|$$

$$= -\mu_R Mg\cos(\beta)\underbrace{\int |d\vec{s}|}$$

Distance traveled up ramp!

So $W_{\vec{f}} = -\mu_R m g d \cos(\beta)$

Back to conservation of energy:

$$TE_{\boxed{A}} + (W_{\vec{f}}) = TE_{\boxed{B}}$$

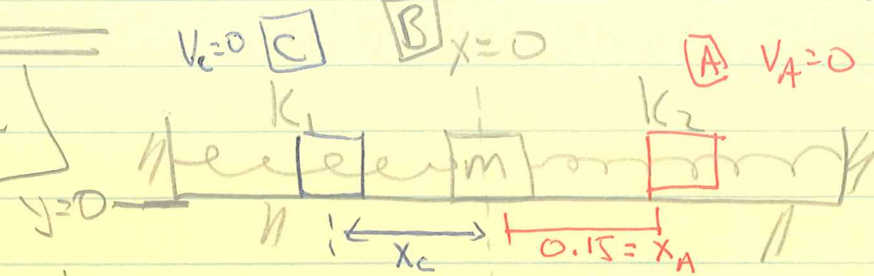
$$\frac{1}{2}\cancel{m}V_0^2 + \cancel{m}gL\sin(\alpha) - \mu_R \cancel{m} g d\cos(\beta) = \cancel{m}gd\sin(\beta)$$

$$\frac{V_0^2}{2} + gL\sin(\alpha) = dg(\sin(\beta) + \mu_R\cos(\beta))$$

$$\frac{\frac{V_0^2}{2} + gL\sin(\alpha)}{g(\sin(\beta)+\mu_R\cos(\beta))} = d$$

Answer.

7.62



$m = 3\,kg$
$k_1 = 2500\,N/m$
$k_2 = 2000\,N/m$

System: Springs, Mass, EARTH

$$TE_{\boxed{A}} = \underbrace{\cancel{KE}_{\boxed{A}}}_{\to 0} + \underbrace{\cancel{PE_g}_{\boxed{A}}}_{\to 0} + PE_{s1\boxed{A}} + PE_{s2\boxed{A}} = \frac{1}{2}k_1 x_A^2 + \frac{1}{2}k_2 x_A^2$$

"from rest" $\quad y=0$

$$TE_{\boxed{B}} = KE_{\boxed{B}} + \underbrace{\cancel{PE_g}_{\boxed{B}}}_{\to 0} + \underbrace{PE_{s1\boxed{B}}}_{\to 0} + \underbrace{\cancel{PE}_{s2\boxed{B}}}_{\to 0} = \frac{1}{2}mV_B^2$$

$$TE_{\boxed{C}} = \underbrace{KE_{\boxed{C}}}_{\to 0} + \underbrace{\cancel{PE_g}_{\boxed{C}}}_{\to 0} + PE_{s1\boxed{C}} + PE_{s2\boxed{C}} = \frac{1}{2}k_1 x_c^2 + \frac{1}{2}k_2 x_c^2$$

turning point

Apply Conservation of Energy:

$$TE_{\boxed{A}} = TE_{\boxed{B}}$$

$$\frac{1}{2}k_1 x_A^2 + \frac{1}{2}k_2 x_A^2 = \frac{1}{2} m v_B^2$$

$$\frac{x_A^2 (k_1 + k_2)}{m} = v_B^2$$

$$\underline{\underline{5.81\,m/s}} = x_A \sqrt{\frac{(k_1+k_2)}{m}} = v_B$$

↑ This is a max. value because configuration $\boxed{B}$ is the only one where ALL of the energy is K.E..

Continue w/ Conservation of Energy

$$TE_{\boxed{A}} = TE_{\boxed{C}}$$

$$\frac{1}{2}k_1 x_A^2 + \frac{1}{2}k_2 x_A^2 = \frac{1}{2}k_1 x_C^2 + \frac{1}{2}k_2 x_C^2$$

⋮

x_C must equal x_A

$x_C = 0.15$ meters Answer.