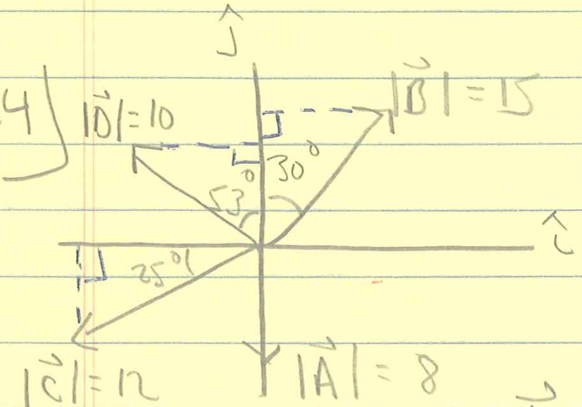
HW due 1/17 Selected Solutions (where I have value to add ☺)

1.24)



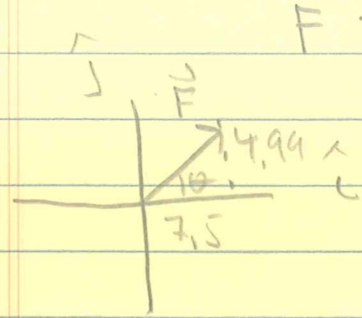
Cannot do the vector algebra until have vectors in $\hat{i}, \hat{j}, \hat{k}$ notation ("component form")

"adjacent means cosine"

$$\begin{aligned}
\vec{A} &= -8\hat{j} \\
\vec{B} &= +15\sin(30)\hat{i} + 15\cos(30)\hat{j} \\
\vec{C} &= -12\cos(25)\hat{i} - 12\sin(25)\hat{j} \\
\vec{D} &= -10\sin(53)\hat{i} + 10\cos(53)\hat{j}
\end{aligned}$$

a) $\vec{F} = \vec{A} + \vec{B} = [0 + 15\sin(30)]\hat{i} + [-8 + 15\cos(30)]\hat{j}$

$\vec{F} = 7.5\hat{i} + 4.99\hat{j}$ ☆ OR



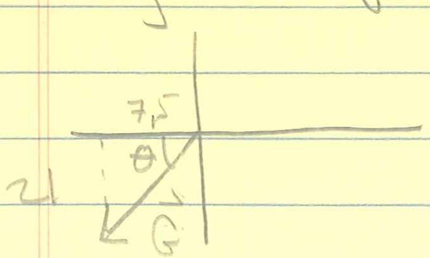
$|\vec{F}| = \sqrt{7.5^2 + 4.99^2} = 9$

$\theta = \text{Arctan}\left(\frac{4.99}{7.5}\right) = 33.6°$ ☆

b) $\vec{G} = \vec{A} - \vec{B} = [0 - 15\sin(30)]\hat{i} + [-8 - 15\cos(30)]\hat{j}$

$\vec{G} = -7.5\hat{i} - 21\hat{j}$ ☆

OR



$|\vec{G}| = \sqrt{7.5^2 + 21^2} = 22.3$

$\theta = \text{Arctan}\left(\frac{21}{7.5}\right) = 70.3°$ ☆

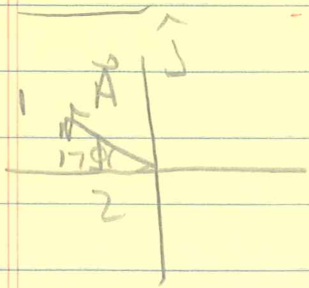
Could report as 250.3° or −109.7 if not shown.

This if angle is shown,

The rest of 1.24 is done exactly the same way ☺

1.28] θ is by convention. (+ counterclockwise from $\hat{i}$)
Find θ if $\vec{A} = -2\hat{i} + \hat{j}$

NOTE: The "1" is implied in front of the $\hat{j}$



$|\vec{A}| = \sqrt{1^2 + 2^2} = 2.24$

$\phi = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$

$\therefore$ The angle they asked for is $(180 - 26.6) = \underline{\underline{153.4^\circ}}$

Answer

Rest of problem done same way.

1.40] More practice of same stuff. By now you should be able to:

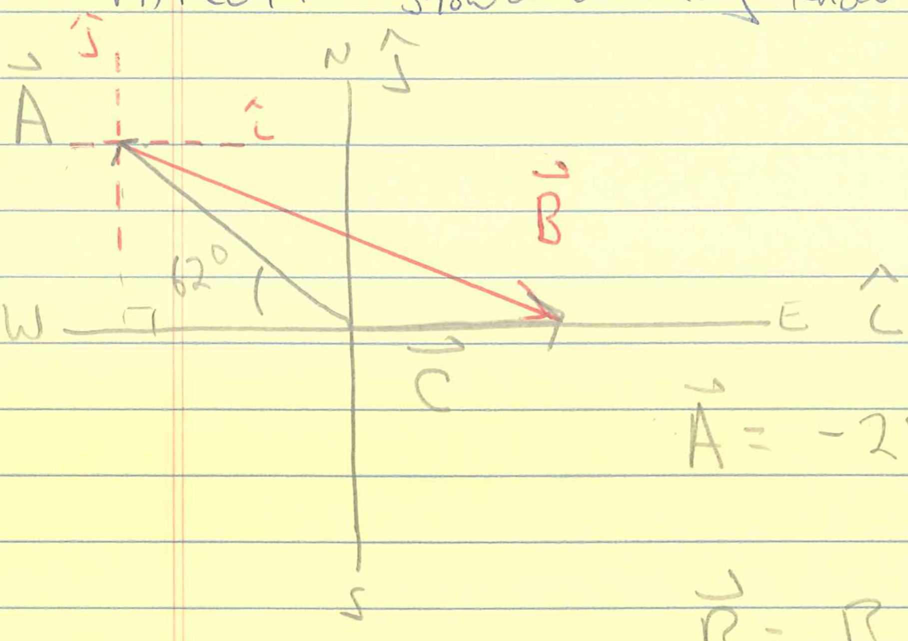
- Given a magnitude and direction, represent a vector in $\hat{i}, \hat{j}, \hat{k}$ notation.
- Given a vector in $\hat{i}, \hat{j}, \hat{k}$, represent it as a magnitude and direction
- Recognize that ALL vector algebra requires $\hat{i}, \hat{j}, \hat{k}$ notations.

1.70) "Ship sails 285 km @ 62° north of west."
Let this be the vector $\vec{A}$

"In which direction must it sail..."
Let this be the vector $\vec{B}$

"So that its resultant * displacement will be 115 km directly east"
Let this be the vector $\vec{C}$

*PHY2048 - See notes for this term.
PHY2049 - Should already know.



$\vec{A} = -285\cos(62)\hat{i} + 285\sin(62)\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

$\vec{C} = 115\hat{i} + \underbrace{0\hat{j}}_{\text{usually do not write the zeros}}$

We are told that $\vec{A} + \vec{B} = \vec{C}$

That means:

$\boxed{\hat{i}}$ $A_x + B_x = C_x$

$-285\cos(62) + B_x = 115$

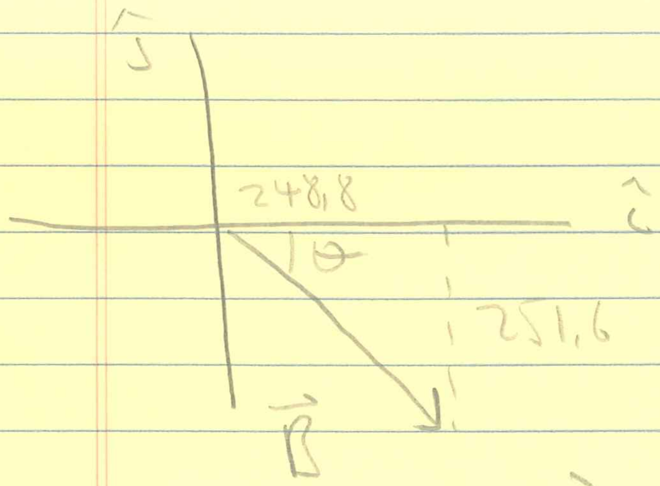
$B_x = 248.8$

$\boxed{\hat{j}}$ $A_y + B_y = C_y$

$285\sin(62) + B_y = 0$

$B_y = -251.6$

$\therefore \vec{B} = 248.8\hat{i} - 251.6\hat{j}$



$|\vec{B}| = \sqrt{248.8^2 + 251.6^2} = 353.8$ km

$\theta = \tan^{-1}\left(\frac{251.6}{248.8}\right) = 45.3^\circ$

$\vec{B}$ is 353.8 km @ (-45.3°)

45.3° south of East

1.78] We are given two vectors that point from the carbon atom (at the origin) to hydrogen atoms.

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} - \hat{j} - \hat{k}$

Notice that these are NOT unit vectors.

$|\vec{A}| = \sqrt{3}$!!

This question expects that you use both definitions of the vector dot product.

1st definition: $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$

$\uparrow\ \sqrt{3}$ $\quad \uparrow\ \sqrt{3}$

$$\vec{A}\cdot\vec{B} = 3\cos\theta \qquad \boxed{1}$$

2nd definition: $\vec{A}\cdot\vec{B} = (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})$

$= \hat{i}\cdot\hat{i} + \hat{i}\cdot(-\hat{j}) + \hat{i}\cdot(-\hat{k}) + \hat{j}\cdot\hat{i} + \ldots$ on other terms

For each term, 1st def. can be applied.

$\Rightarrow \hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos(0) = 1$

$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos(90) = 0$

leaving us with $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = 1 - 1 - 1 = -1$

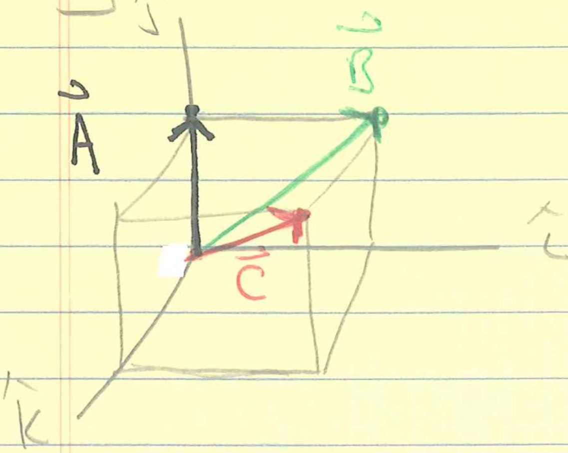
$\boxed{2}$

Both of the definitions for $\vec{A}\cdot\vec{B}$ must produce the same result.

$\therefore \quad 3\cos(\theta) = -1$

$\theta = 109.5^\circ$ Answer

1.80) Let cube have sides of length 1.



$\vec{A} = +1\hat{j}$
$\vec{B} = +1\hat{i} + 1\hat{j}$
$\vec{C} = +1\hat{i} + 1\hat{j} + 1\hat{k}$

Think of these as GPS directions from the tail of the vector to the head of the vector. That is the easiest way to write out these vectors!

Note: $|\vec{A}| = 1$
$|\vec{B}| = \sqrt{2}$
$|\vec{C}| = \sqrt{3}$

To get angle between $\vec{A}$ and $\vec{C}$:

$\vec{A}\cdot\vec{C} = |\vec{A}||\vec{C}|\cos(\theta) = 1(\sqrt{3})\cos\theta$

also $\vec{A}\cdot\vec{C} = 1$

$\therefore \quad 1(\sqrt{3})\cos(\theta) = 1$

$\theta = 54.7^\circ$ Answer

Similar calculation for angle between $\vec{B}$ and $\vec{C}$