$$W_{2} = \int_0^5 \left(-k_1 x\,dx - k_2 \left(\frac{x^2}{5}\right)\left(\frac{2x\,dx}{5}\right)\right)$$

FINISH

1

$$= -\frac{k_1 x^2}{2}\Bigg|_0^5 - \underbrace{\int_0^5 \frac{k_2 2x^3}{25}\,dx}_{\left[\frac{2k_2}{25}\frac{x^4}{4}\right]\Big|_0^5}$$

$$W_{2} = -12.5k_1 - 12.5k_2 \text{ joules}$$ ✱

✱ Hold that thought

??

SAME AS FOR PATH #1 !

Forces characterize interaction — VECTOR

$\boxed{\text{OK}}$

Connection ?

YES ✱

Energies characterize interaction — SCALAR

Conservation of Energy: "... cannot be created or destroyed..."

Kinds of Energy

Kinetic Energy (K.E.) $= \frac{1}{2} m v^2$

speed

Potential Energy due to gravity $(P.E._g) = m g y$

height above $y=0$

NOTE: • .. force

• .. arbitrary choice of location for ZERO energy.

Potential Energy stored in spring $(P.E._{elastic}) = \frac{1}{2} k x^2$

constant

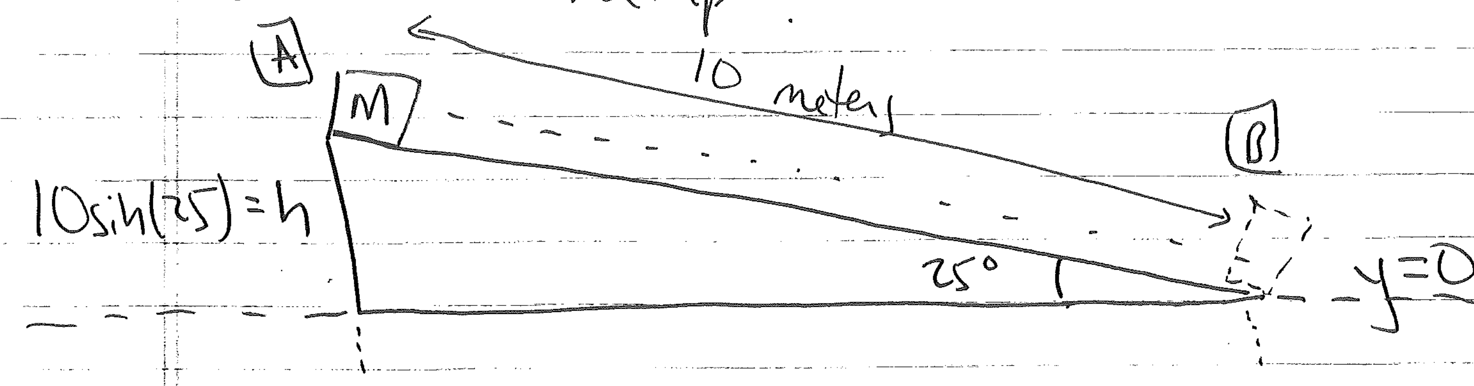
stretch or compression

Process for applying Conservation of Energy

- Define the "system" (all things for which you will track the energy

- Draw a picture and choose zero point(s) for PE,
- Identify configurations of interest and write expression for Total energy (T.E.) of each configuration
- Apply conservation of Energy
- Algebrate ☺

Ex.) A box of mass 'm' slides down a ramp, starting from rest. What is it's speed @ the bottom of the ramp?



System: Box, EARTH

$$TE_{\boxed{A}} = \underbrace{KE_{\boxed{A}}}_{\to 0 \text{ "rest"}} + PE_{g\boxed{A}} = mgh$$

$$TE_{\boxed{B}} = KE_{\boxed{B}} + PE_{g\boxed{B}} = \frac{1}{2}mv_B^2 + mg(0)$$

Cons. of Energy: $TE_{(A)} = TE_{(B)}$

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v_B^2$$

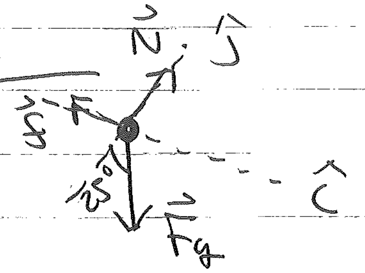
$$v_B = \sqrt{2gh} = \underline{\underline{9.1\,m/s}}$$

Suppose there is friction between the box and the ramp. $\mu_k = 0.01$

Find speed of box @ the bottom.

Find work done by friction as box moves 10 meters down ramp.

for m



$\ldots\ |\vec{f}| = \mu_k mg\cos(25)$

$$W_{\substack{\text{friction}\\ \text{on box}}} = \int \vec{f}\cdot d\vec{s} = \int |\vec{f}|\,|d\vec{s}|\cos(180)$$

$$= -|\vec{f}|\underbrace{\int |d\vec{s}|}_{10\text{ meters}} = -10\mu_k mg\cos(25)$$

Significance?

- ~
- ~

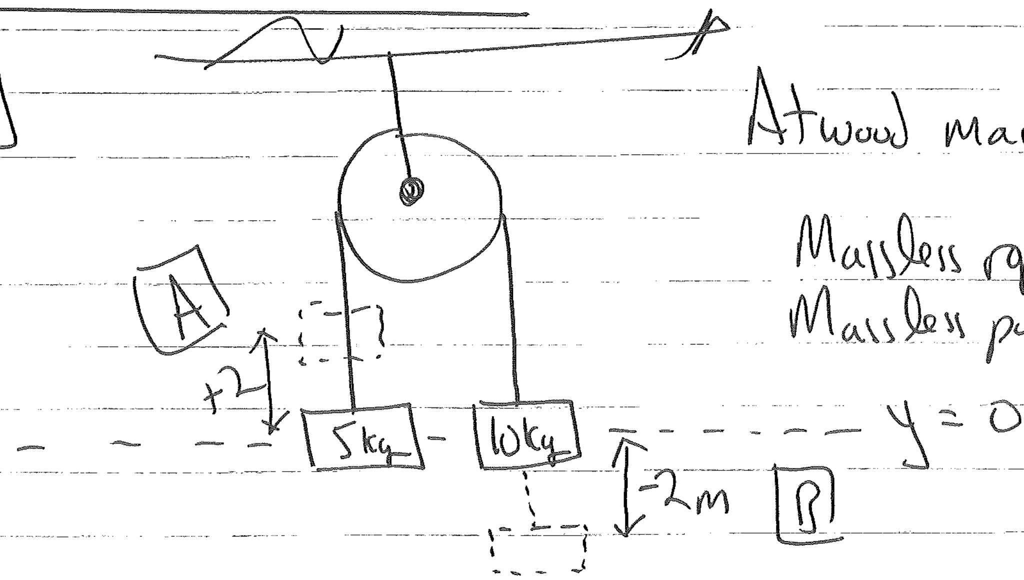
Apply Conservation of Energy

$$TE_{\boxed{A}} + (\text{WORK DONE}) = TE_{\boxed{B}}$$

$$\cancel{m}gh - 10\mu_k \cancel{m}g\cos(25) = \frac{1}{2}\cancel{m}v_B^2$$

$$\underline{\underline{9\ m/s}} = \sqrt{2gh - 20\mu_k\cos(25)g} = v_B$$

Ex.



Atwood machine

Massless rope
Massless pulley

Find the speed of 10 kg block if it starts from rest and has fallen 2 meters.

System: 5kg, 10kg, EARTH

$$TE_{\boxed{A}} = KE_{\boxed{A}} + PE_{g\boxed{A}} = 0$$

$KE_{\boxed{A}} \to 0$ "rest"

$PE_{g\boxed{A}} \to 0$: $5g(0) + 10g(0)$

$$TE_{\boxed{B}} = KE_{\boxed{B}} + PE_{g\boxed{B}}$$

$$= \frac{1}{2}(5)v_B^2 + \frac{1}{2}(10)v_B^2 + 5g(+2) + 10g(-2)$$

Conservation of Energy

$$TE_{(A)} = TE_{(B)}$$

$$0 = v_B^2 (2.5 + 5) - 10g$$

$$3.6 \, m/s \approx \sqrt{\frac{10(9.8)}{7.5}} = v_B$$

Back to "Hold This"

"Conservative" Forces are forces that are associated w/ Potential energies.

There is a connection between the vector force and the associated scalar energy.

"gradient"

$$\vec{F} = -\nabla(P.E.)$$

in chapter!!

$$\vec{F}(x,y,z,t) = -\frac{\partial}{\partial x} PE(x,y,z,t)\,\hat{i}$$

time

$$-\frac{\partial}{\partial y} PE(x,y,z,t)\,\hat{j}$$

$$-\frac{\partial}{\partial z} PE(x,y,z,t)\,\hat{k}$$