# * Partial Derivatives

$P.E.(x,y,z,t) = 5x^2zt + 2yz + 4t$

"Multivariable function"

Find $\frac{\partial}{\partial x} P.E. = \frac{\partial}{\partial x}(5x^2zt + 2yz + 4t)$

*

$= \frac{\partial}{\partial x}(5x^2zt) + \frac{\partial}{\partial x}(2yz) + \frac{\partial}{\partial x}(4t)$

$= 5zt \underbrace{\frac{\partial}{\partial x}(x^2)}_{2x} + 2yz \underbrace{\frac{\partial}{\partial x}(1)}_{0} + 4t \underbrace{\frac{\partial}{\partial x}(1)}_{0}$

$\frac{\partial}{\partial x} P.E. = 10ztx$

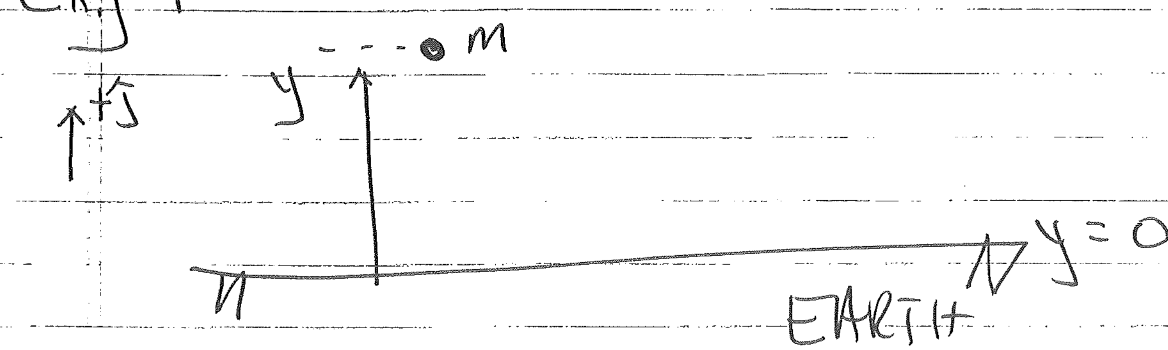
$\frac{\partial}{\partial y} P.E. = 2z$

$\frac{\partial}{\partial z} P.E. = 5x^2t + 2y$

$$\vec{Force} = \underset{\text{"downhill"}}{-} \underset{\text{"gradient"}}{\nabla} P.E.$$

$$\underbrace{\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]}_{\text{points in direction of maximum INCREASE}}$$

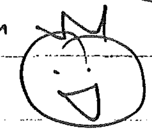
**Ex.) 1-d**




$$PE_g = mgy$$

$$\therefore \vec{F}_{gravity} = -\underbrace{\frac{\partial}{\partial x}PE_g}_{0}\hat{i} - \frac{\partial}{\partial y}PE_g\hat{j} - \underbrace{\frac{\partial}{\partial z}PE_g}_{0}\hat{k}$$

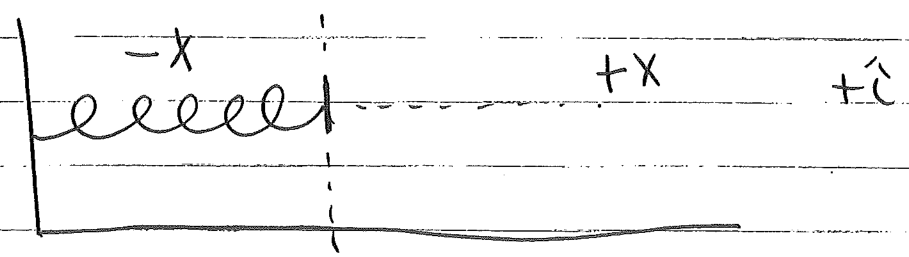
$$= -mg\,\hat{j} \quad \boxed{\checkmark}$$



**Aside:** Newton's Law of Gravity

$$|\vec{F}| = \left[\frac{GM}{r^2}\right]m \Rightarrow P.E. = \underbrace{-\frac{GMm}{r}}_{\text{Choice of P.E.}=0 \text{ @ } r\to\infty}$$

Robert Hooke (b.? older than Newton)

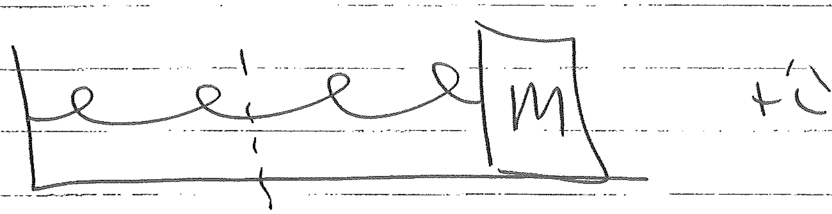



X=0
unstretched

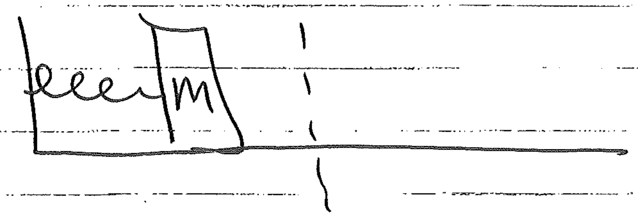
displacement

$$\vec{F}_{spring} = -kx\hat{i}$$

spring constant




X=0




X=0

Recall: $PE_{\substack{spring\\ elastic}} = \underline{\frac{1}{2}kx^2}$

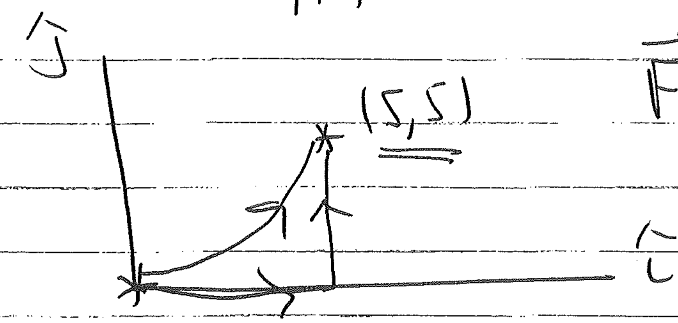
$$\vec{F} = -\nabla PE$$

$$\vec{F}_{spring} = -\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)\hat{i}$$

$$= \underline{-kx\hat{i}} \quad \boxed{\checkmark}$$

Go back and look @ "These are not

"notes" from 2/17




$\vec{F} = -k_1 x \hat{i} - k_2 y \hat{j}$

$$\text{Work}_{(0,0)\to(5,5)} = \underbrace{-12.5k_1}_{\Delta PE_{spring,1}} - 12.5k_2$$

$\Delta PE_{spring_1}$

$\underbrace{\frac{1}{2}k_1(5)^2 - \frac{1}{2}k_1(0)^2}$

$\to -12.5k_1$

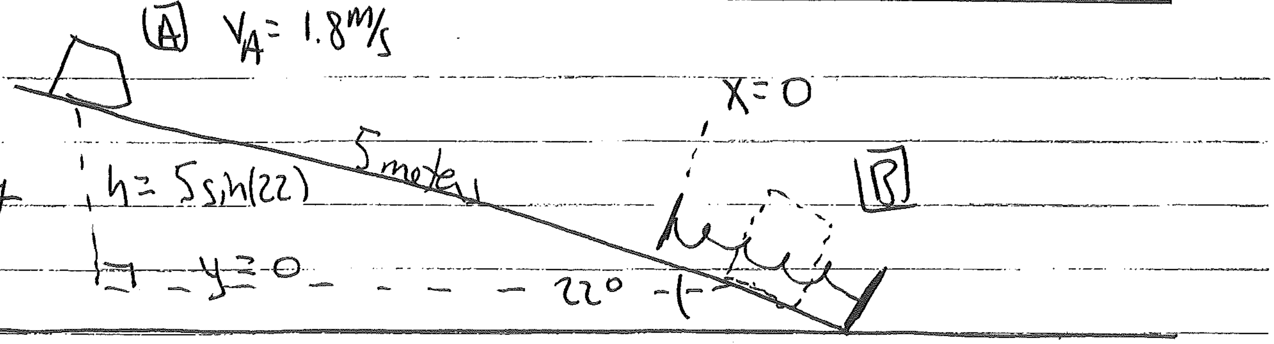
Energy 'sucked' away from moving mass.

Consistent.... hmm... Yes!

As mass moved from (0,0) to (5,5) this $\vec{F}$ did work to 'suck' this amount of energy away from the mass.

**7.48**

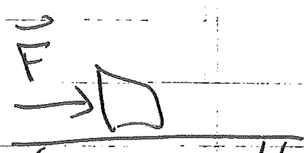
$m = 150\,kg$

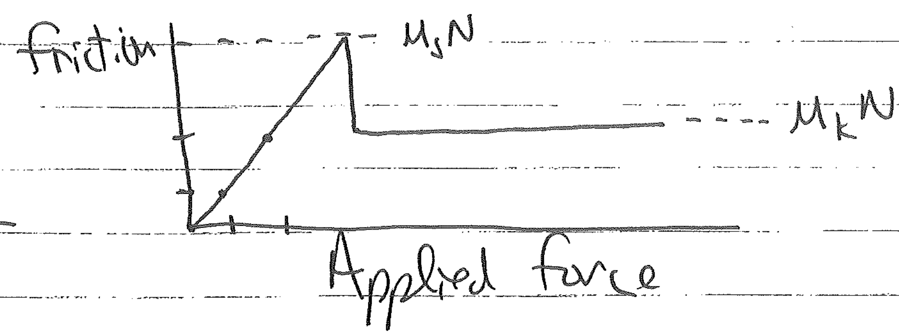



Slides down ramp a total distance of 5 meters. Compresses spring and come to rest. ☆
$f_k = 515$ newtons as box slides down ramp.

$\boxed{\text{AND}}$ This is also the maximum size of the force of static friction $(f_s)$ ☹

☹

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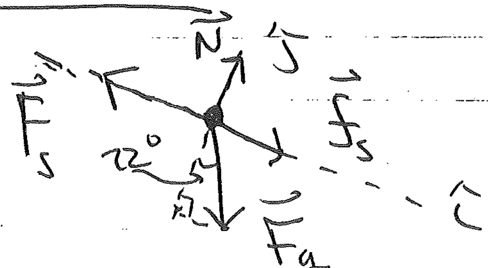
System: Box, Earth, Spring (massless)

FIND

☆ Require the $2^{nd}$ law (Spring constant $k$ is unknown / Amt. of compression is unknown)

for box @ B @ rest




$\vec{N} = N\hat{j}$
$\vec{F}_s = -kx\,\hat{i}$
$\vec{f}_s = +515\,\hat{i}$
$\vec{F}_g = mg\sin(22)\,\hat{i} - mg\cos(22)\,\hat{j}$

2nd $\sum \vec{F} = m\vec{a}$

$\hat{j}$ $\sum F_y = m\cancel{a_y}^{0}$

$N - mg\cos(22) = 0$

$\underline{N = mg\cos(22)}$

$\hat{i}$ $\sum F_x = m\underbrace{a_x}_{\to 0}$

$-kx + 515 + mg\sin(22) = 0$

$\boxed{1}$ $\underline{kx = 515 + mg\sin(22) = 1065.67}$

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$TE_{\boxed{A}} = KE_{\boxed{A}} + PE_{g\boxed{A}} + \underbrace{PE_{s\boxed{A}}}_{\to 0}$

$= \frac{1}{2}mv_A^2 + mgh$

$TE_{\boxed{B}} = \underbrace{KE_{\boxed{B}}}_{\to 0} + \underbrace{PE_{g\boxed{B}}}_{\to 0} + PE_{s\boxed{B}} = \frac{1}{2}kx^2$

Conservation: $TE_{\boxed{A}} + (\text{Work}) = TE_{\boxed{B}}$

$\int \vec{f} \cdot d\vec{s}$

$\int 515 |d\vec{s}| \cos(180)$

$\underbrace{-515 \underbrace{\int |d\vec{s}|}_{5}}_{-2575}$

$\boxed{2}$ $\therefore$ $\frac{1}{2}mv_A^2 + mgh - 2575 = \frac{1}{2}\underbrace{kx^2}_{\text{only unknowns!}}$

: ☺

Newton → 1666 (~)

Energy-Work ≈ 1800's

Consider 2nd Law: $\vec{F} = \frac{d}{dt}(m\vec{v})$

1-d

$$F_x = m\frac{dv_x}{dt} = m\frac{dv_x}{dx}\underbrace{\left(\frac{dx}{dt}\right)}_{v_x}$$

$$F_x = m v_x \frac{dv_x}{dx}$$

WORK-K.E. Therom

150 yrs.

$$\int_{x_i}^{x_f} F_x\, dx = \int_{v_i}^{v_f} m v_x\, dv_x$$

$$F_x x \Big|_{x_i}^{x_f} = m\frac{v_x^2}{2}\Big|_{v_i}^{v_f}$$

$$\underbrace{F_x \Delta x}_{\text{TOTAL WORK DONE}} = \underbrace{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}_{\text{Change in K.E.}}$$