6.5 Restoring Force of a Spring; Hooke's Law

A body is said to be **elastic** if it suffers a deformation when a stretching or compressing force is applied to it and returns to its original shape when the force is removed. For example, suitable forces can stretch a coil spring or a rubber band and they can bend a flexible rod or a beam of metal or wood. Even bodies normally regarded as rigid, such as the balls of a ball bearing made of hardened steel, are somewhat elastic -- they will deform if a sufficiently large force is applied to them.

The force with which a body resists deformation is called its **restoring force.** If we stretch a spring by pulling with our hand, we can feel the restoring force opposing our pull. The restoring force and the force that produces the deformation are of equal magnitudes; they are an action-reaction pair.

Under static conditions, the restoring force with which an elastic body opposes whatever pulls on it often obeys a simple empirical law known as **Hooke's Law:**

The magnitude of the restoring force is directly proportional to the deformation.

This is not a general law of physics -- the *exact* restoring force produced by the deformation of an elastic body depends in a complicated way on the shape of the body and on the detailed properties of the material of the body. Hooke's Law is only an approximate, phenomenological description of the restoring force. However, it is often a quite good approximation, provided that the deformation is small.

As a special case, consider a coil spring. Figure 6.19a shows such a spring in its relaxed state; it is loosely coiled so that it can be compressed as well as stretched. Suppose that the left end of the spring is attached to a rigid support (wall), and we apply a stretching or compressing force to the right end. Under the influence of this force, the spring will settle into a new equilibrium configuration such that the restoring force exactly balances the externally applied force. We can measure the deformation of the spring by the displacement that the right end undergoes relative to its initial position. In Figure 6.19b this displacement is denoted by x. A positive value of x corresponds to an elongation of the spring, and a negative value corresponds to a compression. Clearly, x is nothing but the change in the length of the spring.

Hooke's Law for the value of the restoring force, then, is that the force is directly proportional to the change in the length of the spring,

$$\boxed{F = -kx} \tag{23}$$

The constant of proportionality k is the **spring constant;** it is a positive number characteristic of the spring. The spring constant is a measure of the stiffness of the spring -- a stiff spring has a high value of k, and a soft spring a low value of k. The unit for the spring constant is the newton per meter (N/m) or the pound-force per foot (lbf/ft). The negative sign in Eq. (23) indicates that the restoring force opposes the deformation: if the spring is elongated (positive x), then the restoring force is negative and opposes the external stretching force (Figure 6.20a); if the spring is compressed (negative x), then the restoring force is positive and opposes the external compressing force (Figure 6.20b).

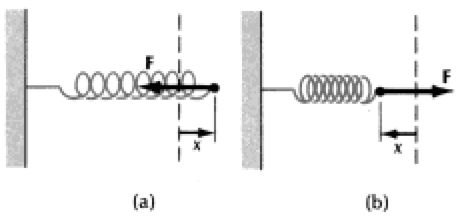

Fig. 6.20 (a) If the change x in the length of the spring is positive, then the restoring force F is negative. (b) If x is negative, then F is positive.

Coil springs usually obey Hooke's Law quite closely unless the deformation is excessively large. If a spring is stretched beyond its elastic limit, it will suffer a permanent deformation and not snap back to its original shape when released. Such damage to the spring destroys the simple proportionality given by Eq. (23). Furthermore, if a spring is stretched much beyond its elastic limit, it will break and then, of course, the restoring force disappears altogether. Figure 6.21 is a plot of the restoring force versus length for a small steel spring stretched beyond its elastic limit.

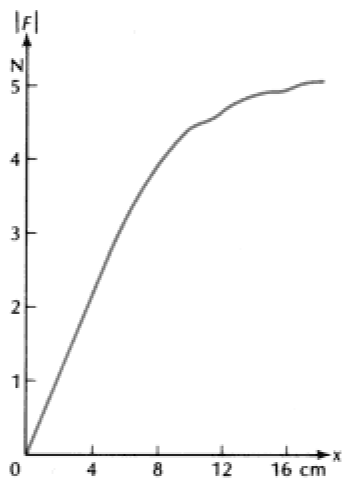

Fig. 6.21 Restoring force vs. length for a small steel spring. When stretched beyond 7 cm, the spring suffered a permanent deformation.

EXAMPLE 9. The manufacturer's specifications for the coil spring for the front suspension of a Triumph sports car call for a spring of 10 coils with a relaxed length of 0.316 m, and a length of

0.205 m when under a load of 399 kg. What is the spring constant?

SOLUTION: The restoring force that will balance the weight of 399 kg is F = 399 kg X 9.81 m/s^2 = 3.91 X 10^3 N. The corresponding change of length is x = 0.205 m - 0.316 m = -0.111 m. Hence,

$$k = -\frac{F}{x} = -\frac{3.91 \times 10^3\ \mathrm{N}}{-0.111\ \mathrm{m}} = 3.52 \times 10^4\ \mathrm{N/m}$$

EXAMPLE 10. If the spring described in the preceding example is cut into two equal pieces, what will be the spring constant of each piece?

SOLUTION: If the spring is cut into two equal pieces, the spring constant of each piece will be twice as large, i.e., k' = 7.04 X 10^4 N/m. This can be best understood by noting that if a 5-coil spring is to be compressed by, say, 10 cm, each coil must be deformed by 2 cm; whereas if a 10-coil spring is to be compressed by 10 cm, each coil must be deformed by only 1 cm. The force required to deform one coil by 2 cm is twice as large as the force required to deform one coil by 1 cm. Consequently, the spring constant of the 5-coil spring is twice as large as the spring constant of the 10-coil spring.

COMMENTS AND SUGGESTIONS: This example leads to a general rule: other things being equal, the spring constant is inversely proportional to the number of coils. With this rule we can understand how the suspension springs of an automobile can be stiffened by means of clamps that rigidly hold two adjacent coils so as to prevent their relative motion. Such a clamp makes the two coils act as one, that is, it effectively reduces the total number of coils.