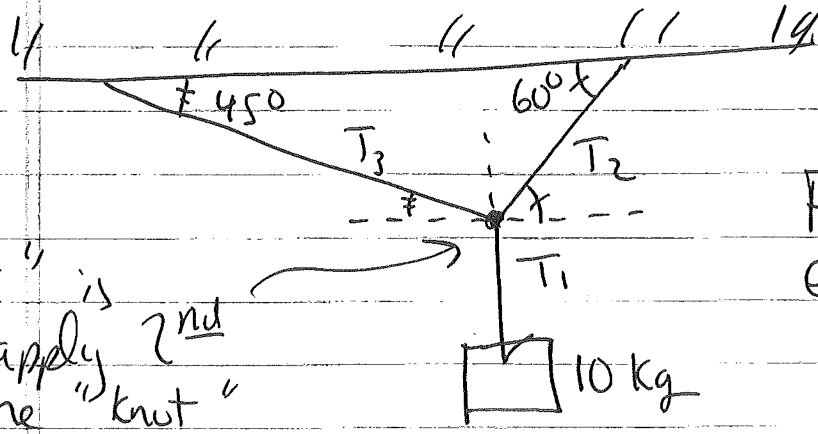
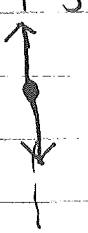
One last Newton example ☺



Find the tension in each rope (massless)

"Trick" is to apply 2nd to the "knot"

for 10 kg



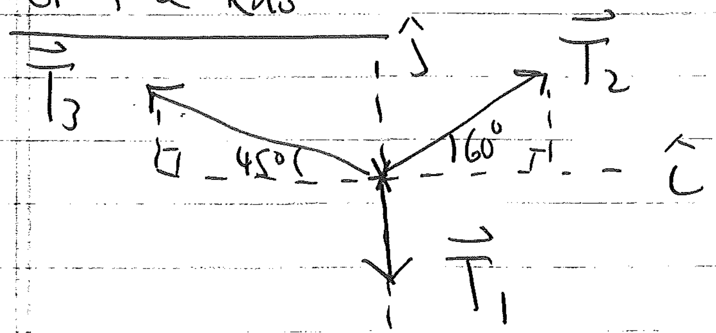
$\vec{T}_1 = +T_1\hat{j}$

$\vec{F}_g = -10*9.8\hat{j} = -98\hat{j}$

2nd $\boxed{\hat{j}}$ $\sum F_y = m\cancel{a_y} \to 0$

$\therefore$ $T_1 = 98$ newtons

for the knot



$$\vec{T}_1 = -98\hat{j}$$

$$\vec{T}_2 = +T_2\cos(60)\hat{i} + T_2\sin(60)\hat{j}$$

$$\vec{T}_3 = -T_3\cos(45)\hat{i} + T_3\sin(45)\hat{j}$$

2nd $\sum \vec{F} = m\underbrace{\vec{a}}_{\to 0}$

$\underset{?}{\uparrow}$

$\boxed{\hat{i}}$ $\sum F_x = 0$

A $\boxed{+T_2\cos(60) - T_3\cos(45) = 0}$

$\hat{j}$ $\quad \sum F_y = 0$

B $\boxed{-98 + T_2\sin(60) + T_3\sin(45) = 0}$

2 EQTNS, 2 UNKNOWNS | Algebrate ☺

Work and Energy (Ch. 6 + 7)

!! WARNING !! WARNING !! WARNING !!

Old Paradigm: Interactions are characterized
~1666 (NEWTON) by Forces. VECTORS

New Paradigm: Interactions are characterized
~1800's by Energy. SCALARS

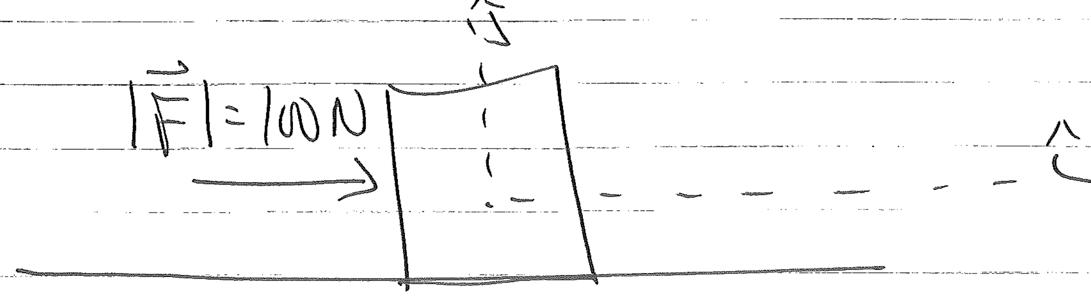
Words mean Things ☺

SI	
newtons	Force ⇒ push or pull
(J) joules	WORK ⇒ $W = \int \vec{F} \cdot d\vec{s}$ ‖ "Force acting through a distance"
(J) joules	ENERGY ⇒ Ability to do work ☺
(W) watt	Power ⇒ $\frac{\text{Amt. of Work done OR Energy delivered}}{\text{time interval}}$

☆ Forces do work to transform and/or transfer energy from one kind to another and/or one object to another.

☆ The amount of work done is exactly equal to the amount of energy transferred and/or transformed.

EX.



$\vec{F}$ pushes a box 10 meters across the floor. How much work was done on the box?

$$\vec{F} = 100\ \hat{i}$$

$$W = \int \vec{F} \cdot d\vec{s} = \int (100\hat{i}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$d\vec{s}$: [a step in a particular direction
$3^d \Rightarrow \underline{d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}}$]

$$= \int \Big[100\,dx\,\underbrace{\hat{i}\cdot\hat{i}}_{\substack{|\hat{i}||\hat{i}|\cos(0) \\ 1}} + 100\,dy\,\underbrace{\hat{i}\cdot\hat{j}}_{\substack{|\hat{i}||\hat{j}|\cos(90) \\ 0}} + 100\,dz\,\underbrace{\hat{i}\cdot\hat{k}}_{0} \Big]$$

$$W = \int_0^{10} 100\,dx = 100x\Big|_0^{10} = \underline{\underline{1000 \text{ Joules}}}$$

Suppose a frictional force of $\vec{f} = -10\hat{i}$ were also acting on the box. What is the work done by friction on the box?

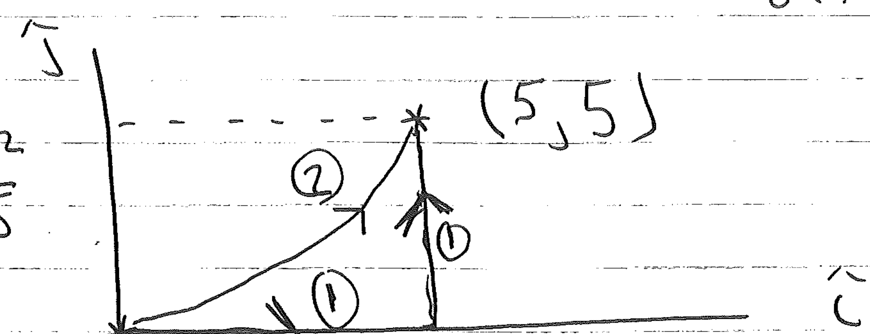
$$W_{friction} = \int \vec{f}\cdot d\vec{s} = \int_0^{10} -10\,dx = -10(x)\Big|_0^{10} = \underline{\underline{-100\text{ J}}}$$

?

Box has gained 900 J of Energy!

GOOD STUFF HERE (Put those seat belts on!)

② $y = \frac{x^2}{5}$



A force $\vec{F} = -K_1 x\hat{i} - K_2 y\hat{j}$ is acting on a mass as the mass is moved from the origin to the point $x=5, y=5$. What is the work done by this force on the mass?

$$W = \int \vec{F} \cdot d\vec{s}$$

$d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Hmmm.....

Choose a path for the mass ☺

for path ①

$$W_{①} = \int_{path\ ①} \vec{F} \cdot d\vec{s} = \int_0^5 (-k_1 x\hat{i} - k_2 y\hat{j}) \cdot (dx\,\hat{i}) + \int_0^5 (-k_1 x\hat{i} - k_2 y\hat{j}) \cdot (dy\,\hat{j})$$

$$= \int_0^5 -k_1 x\,dx + \int_0^5 -k_2 y\,dy$$

$$= -k_1 \frac{x^2}{2}\Big|_0^5 - k_2 \frac{y^2}{2}\Big|_0^5 = -12.5k_1 - 12.5k_2$$

$$= -12.5(k_1 + k_2) \text{ joules}$$

"Path" or "line" Integral

Work along path ②:

$$W_{②} = \int_{②} \vec{F} \cdot d\vec{s} = \int (-k_1 x\,dx - k_2 y\,dy)$$

How to make this along path ②?

Choose y, dy

$$\begin{bmatrix} y = \frac{x^2}{5} \\ dy = \frac{2x\,dx}{5} \end{bmatrix} \text{ TRUE}$$

$$W_{②} = \int \left(-k_1 x\,dx - k_2 \left(\frac{x^2}{5} \right) \left(\frac{2x\,dx}{5} \right) \right)$$