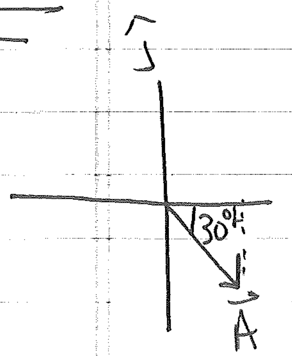
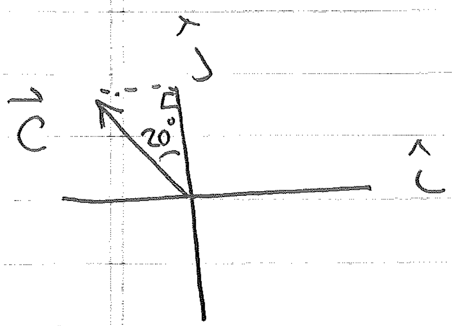
Ex: $\vec{A}$ is 15 units @ -30°
$\vec{C}$ is 30 units @ 110°

[1] If $\vec{C} = 2\vec{A} - \vec{B}$, what is the magnitude and direction of $\vec{B}$?



$$\vec{A} = +15\cos(30)\,\hat{i} - 15\sin(30)\,\hat{j}$$

$$\vec{A} = +13\hat{i} - 7.5\hat{j}$$



$$\vec{C} = -30\sin(20)\,\hat{i} + 30\cos(20)\,\hat{j}$$

$$\vec{C} = -10.3\hat{i} + 28.2\hat{j}$$

"Divide and Conquer"
"Thou shall not mix $\hat{i}$ and $\hat{j}$"

VECTOR Equations

[1] Three equations!!

$\hat{i}$: $C_x = 2A_x - B_x$

$$-10.3 = +26 - B_x$$

$\therefore B_x = 26 + 10.3 = 36.3$

$\boxed{\hat{j}}$ $C_y = 2A_y - B_y$

$28.2 = -15 - B_y$

$\therefore B_y = -15 - 28.2 = -43.2$

$\boxed{\hat{k}}$ $C_z = 2A_z - B_z$

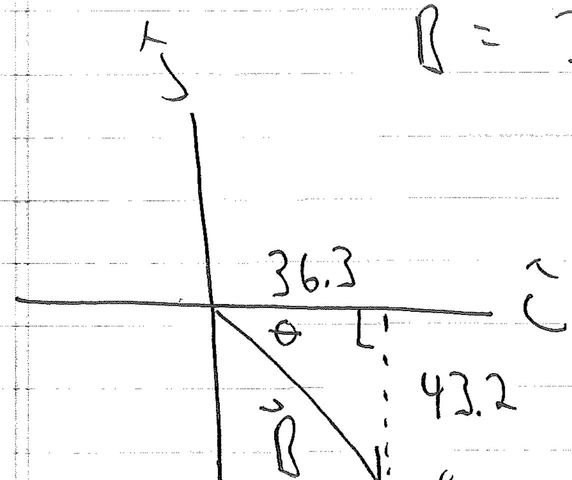
$0 = 0 - B_z$

$\therefore B_z = 0$

So we have $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

(a general expression)

$\vec{B} = 36.3\hat{i} - 43.2\hat{j}$

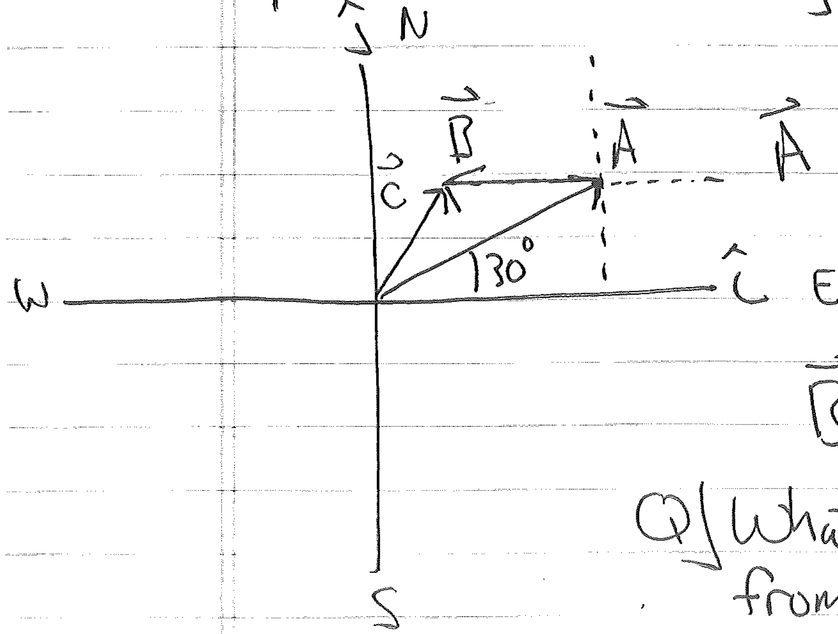


$|\vec{B}| = \sqrt{36.3^2 + 43.2^2} = \underline{\underline{56.4}}$

$\theta = \tan^{-1}\left(\frac{43.2}{36.3}\right) = \underline{\underline{50^\circ}}$

"$\vec{B}$ is 56.4 units @ -50°"

Key to vector HW problems: The word problem is describing a VECTOR equation.



$\vec{A}$ ["Walked 70 m in a direction 30° north of east."

$\vec{B}$ [Then walked 50 m due west

Q] What is my "displacement"* from my starting location?

$\vec{C}$ [*A vector pointing from the start to the finish

$\vec{C} = \vec{A} + \vec{B}$

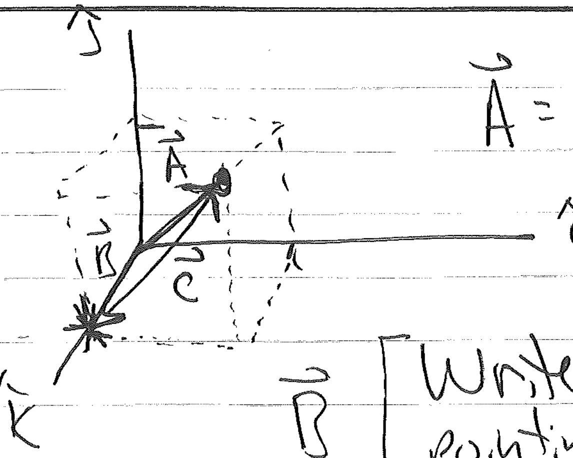
∴ Can do!

Vectors can be written to locate a point in space with respect to a chosen origin (coordinate system).

EX: Write the vector locating the point (1, 1, 1)

$$\underset{x}{1},\ \underset{y}{1},\ \underset{z}{1}$$

☆ (see later)



$\vec{A} = 1\hat{i} + 1\hat{j} + 1\hat{k}$

$\vec{B}$ [Write a vector pointing to (0,0,1)]

$\vec{B} = 0\hat{i} + 0\hat{j} + 1\hat{k} = \hat{k}$

☺ Write a vector ($\vec{C}$) pointing from (1,1,1) to (0,0,1)

Head-to-tail from diagram:

$$\vec{A} + \vec{C} = \vec{B}$$

$$\therefore \vec{C} = \vec{B} - \vec{A} = \underline{\underline{-\hat{i} - \hat{j}}}$$

☺ "displacement"

<u>NOTE</u>: $\vec{C} = \vec{B}_{\text{END}} - \vec{A}_{\text{START}}$ ☆

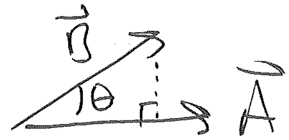
Vector Dot Product

☺ different A, B, C's

$$\vec{C} = \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad \| \; \underline{\underline{\text{Scalar!}}}$$

$\boxed{\text{OR}}$

$$\vec{C} = |\vec{A}||\vec{B}|\cos(\theta)$$



18 (later) ☺

Q] What is the angle between $\vec{A}$ and $\vec{B}$?

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$ | $|\vec{A}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\vec{B} = \hat{k}$ | $|\vec{B}| = \sqrt{0^2+0^2+1^2} = 1$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(\theta) = \sqrt{3}(1)\cos(\theta)$$

$\boxed{\text{OR}}$

$$\vec{A}\cdot\vec{B} = (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{k}) = \underbrace{\hat{i}\cdot\hat{k}}_{|\hat{i}||\hat{k}|\cos(90)\;=\;0} + \overbrace{\hat{j}\cdot\hat{k}}^{|\hat{j}||\hat{k}|\cos(90)} + \underbrace{\hat{k}\cdot\hat{k}}_{|\hat{k}||\hat{k}|\cos(0)\;=\;1\cdot1\cdot1} = 1$$

$$\therefore \sqrt{3}\cos(\theta) = 1$$

$$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \underline{\underline{54.7^0}}$$

END CHAPTER 1

Chapters 2 + 3: A Description of Motion

"Kinematics"

WARNING! WARNING! WARNING!

1-d $x(t)$ location SI unit is meters

$v(t) = \frac{dx}{dt}$ velocity (m/s) VECTORS ☆

$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ acceleration (m/s^2) ☆

Memorize: [Accelerations change Velocities
Velocities change Position.]

Ex.] An object starts from rest w/ an acceleration $a(t) = 5t$. (SI units)

Find x @ $t = 10$ seconds.
Find the velocity @ $t = 15$ seconds.
Find the average speed between $t = 10$ and $t = 15$.

1 $\boxed{a(t) = 5t}$

$\therefore\ v(t) = \int a(t)\,dt = \int 5t\,dt = \frac{5t^2}{2} + \text{Const.}$

$v(t) = 2.5t^2 + \text{constant}$

↑ Hmmm...

"starts from rest" $\Rightarrow$ $v(t=0) = 0 = 2.5(0)^2 + \text{constant}$

$\therefore$ constant $= 0$

$\therefore\ \boxed{v(t) = 2.5t^2}$ 2

Also, $x(t) = \int v(t)\,dt = \frac{2.5t^3}{3} + \text{const.}$

Assume @ $t=0$, $x(t=0) = 0$.
(Choosing an origin)

3 $\boxed{x(t) = 0.83t^3}$

[1, 2, 3 COMPLETELY describe the motion for all time ☺

@ $t = 10$ sec

1 $a(t=10) = 5(10) = +50\ m/s^2$

2 $v(t=10) = 2.5(10)^2 = +250\ m/s$

3 $x(t=10) = 0.83(10)^3 = +830$ meters

@ $t = 15$ sec

1. $a(t=15) = 75 \ m/s^2$
2. $v(t=15) = 562.5 \ m/s$
3. $x(t=15) = 2801 \ m$

[Instantaneous speed

$\boxed{\text{or}}$

Average Speed]

What if $a(t)$ is constant?

(not a function of time)

When would that happen?

Newton: Force $\propto$ acceleration

const. const.