PHY 2048 HW due 3/27

9.10 Fan turned off @ $t = 0$.

$w_0 = 500 \frac{rev}{min} = 8.33$ rev/sec.

$\theta_0 = 0$

$$\left.\begin{array}{l}\theta(t) = \theta_0 + w_0 t + \frac{1}{2}\alpha t^2 \\ w(t) = w_0 + \alpha t\end{array}\right] \Rightarrow \left[\begin{array}{ll}\theta(t) = 8.33t + \frac{1}{2}\alpha t^2 & \boxed{1} \\ w(t) = 8.33 + \alpha t & \boxed{2}\end{array}\right.$$

@ $t = 4s$, $w = 200 \frac{rev}{min} = 3.33 \frac{rev}{sec}$, $\theta = ?$

$\boxed{2} \Rightarrow \quad 3.33 = 8.33 + \alpha(4)$

$\therefore \alpha = -1.25$ rev/s^2

Answer.

$\boxed{1} \Rightarrow \quad \theta(t=4) = 8.33(4) + \frac{1}{2}(-1.25)(4)^2 = 23.32$ rev.

Answer.

@ $t = t_{stop}$, $w_{stop} = 0$, $\theta_{stop} = ?$

$\boxed{2} \Rightarrow \quad 0 = 8.33 - 1.25 t_{stop}$

$\therefore t_{stop} = 6.584$ sec. ANSWER.

$\boxed{1} \Rightarrow \quad \theta_{stop} = 8.33 t_{stop} + \frac{1}{2}(-1.25) t_{stop}^2 = 27.75$ rev.

This is from $\theta_0 = 0$ @ $t = 0$

9.13] $\alpha = 2.25\ rad/s^2$

@ $t=0$, $\theta_0 = 0$ and ω_0 is not given.

$$\left.\begin{aligned}\theta(t) &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega(t) &= \omega_0 + \alpha t\end{aligned}\right\} \Rightarrow \left[\begin{aligned}\theta(t) &= \omega_0 t + 1.125t^2 \quad \boxed{1} \\ \omega(t) &= \omega_0 + 2.25t \quad \boxed{2}\end{aligned}\right.$$

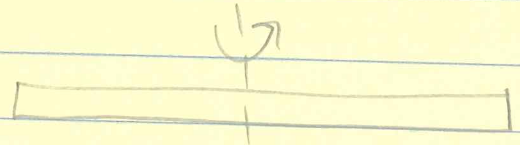
@ $t = 4\,sec$, $\theta = 30$ radians, $\omega = ?$

$\boxed{1} \Rightarrow \quad 30 = \omega_0(4) + 1.125(4)^2$

$\therefore\ \omega_0 = 3\ rad/s$ Answer.

$\boxed{2} \Rightarrow \quad \omega(t=4) = 3 + 2.25(4) = 12\ rad/s$

9.32] a.]

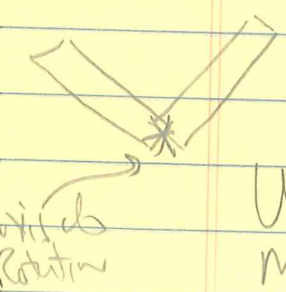


$L = 0.6\ m$

$M = 0.4\ kg$

TABLE 9.2 $\quad I = \frac{ML^2}{12} = 0.012\ kg\,m^2$

b.) Superposition! We now have two rods, each w/ length $\left(\frac{L}{2}\right)$ and Mass $\left(\frac{M}{2}\right)$. Each is rotating about an axis @ its end. $\left(I = \frac{ML^2}{3}\right)$ TABLE 9.2



Using appropriate values for Mass and length:

$$I = \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2}{3} + \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2}{3} = \frac{ML^2}{24} + \frac{ML^2}{24} = \frac{ML^2}{12} = 0.012\ kgm^2$$

FIRE the engineer who suggested it??

9.36] "constant acceleration"

@ $t=0$, $\theta_0 = 0$, $\omega_0 = 0$

This corresponds to a choice of coordinate systems

$$\left.\begin{array}{l}\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega(t) = \omega_0 + \alpha t\end{array}\right] \Rightarrow \left[\begin{array}{ll}\theta(t) = \frac{1}{2}\alpha t^2 & \boxed{1} \\ \omega(t) = \alpha t & \boxed{2}\end{array}\right.$$

@ $t = 12$ sec, $\theta = 8.2$ revolutions

$\boxed{1} \Rightarrow \quad 8.2 = \frac{1}{2}\alpha(12)^2$

$\therefore \alpha = 0.113\overline{8}\ \frac{\text{revolutions}}{s^2}$

$\boxed{2} \Rightarrow \omega(t=12) = \alpha(12) = 1.3\overline{66}\ \text{rev/sec.} \times \frac{2\pi\ \text{rad}}{1\ \text{rev}} = 8.58\ \frac{\text{rad}}{s}$

At $t=12$ sec, we are told KE = 36 joules. It is rotational KE, so:

$$36 = \frac{1}{2} I \omega^2$$

SI is joules

SI is $kg\,m^2$

has an angular unit that we MUST drop to have equation dimensionally correct!! ONLY one we can do that for is RADIANS!!!

$\therefore I = \frac{72}{(8.58)^2} = 0.978\ kg\,m^2$

Answer.

9.7) (missed it on page 1)

$\boxed{1}$ $\theta(t) = a + bt - ct^3$ GIVEN

So we have:

$\boxed{2}$ $\omega(t) = \frac{d\theta}{dt} = b - 3ct^2$

$\boxed{3}$ $\alpha(t) = \frac{d\omega}{dt} = -6ct$

GIVEN: @ $t=0$, $\theta = \pi/4$ and $\omega = 2$

$\therefore$ $\boxed{2} \Rightarrow b = 2$

$\boxed{1} \Rightarrow a = \pi/4$

ALSO GIVEN: @ $t = 1.5$, $\alpha = 1.25 \text{ rad/s}^2$

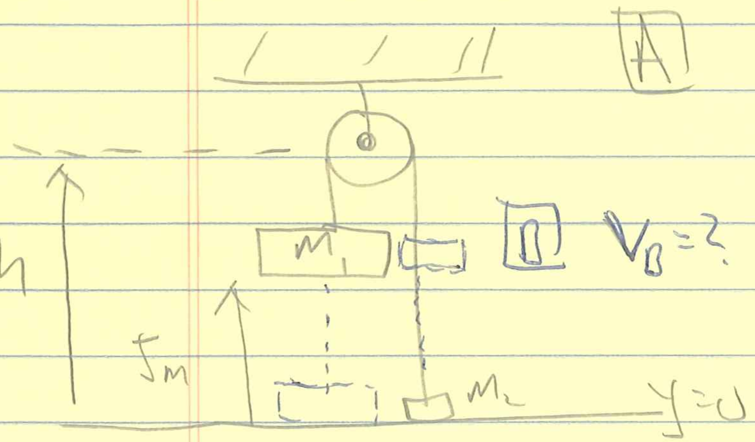
$\therefore$ $\boxed{3} \Rightarrow c = -0.139$

So our equations become:

$$\left[\begin{array}{l} \theta(t) = \pi/4 + 2t + 0.139t^3 \\ \omega(t) = 2 + 0.417t^2 \\ \alpha(t) = 0.834t \end{array}\right]$$

9.76) For pulley: $R = 0.160\,m$, $I = 0.380\,kg\,m^2$

System: Pulley, $m_1 = 4kg$, $m_2 = 2kg$, Earth



$$TE_{\boxed{A}} : \underbrace{KE_{\boxed{A}}}_{\to 0} + PE_{g\boxed{A}} = m_{pulley}\, g h + 5 m_1 g + 0$$

$$TE_{\boxed{B}} = KE_B + PE_{g\boxed{B}} = \tfrac{1}{2} m_1 V_B^2 + \tfrac{1}{2} m_2 V_B^2 + \tfrac{1}{2} I \omega_B^2 + m_{pulley}\, g h + 5 m_2 g + 0$$

Conservation of Energy: $TE_{\boxed{A}} = TE_{\boxed{B}}$

$$\cancel{m_{pulley}\, g h} + 5 m_1 g = \tfrac{1}{2} m_1 V_B^2 + \tfrac{1}{2} m_2 V_B^2 + \tfrac{1}{2} I \omega_B^2 + \cancel{m_{pulley}\, g h} + 5 m_2 g$$

Note: $V_B = R\omega_B$
$\therefore \omega_B = V_B / R$

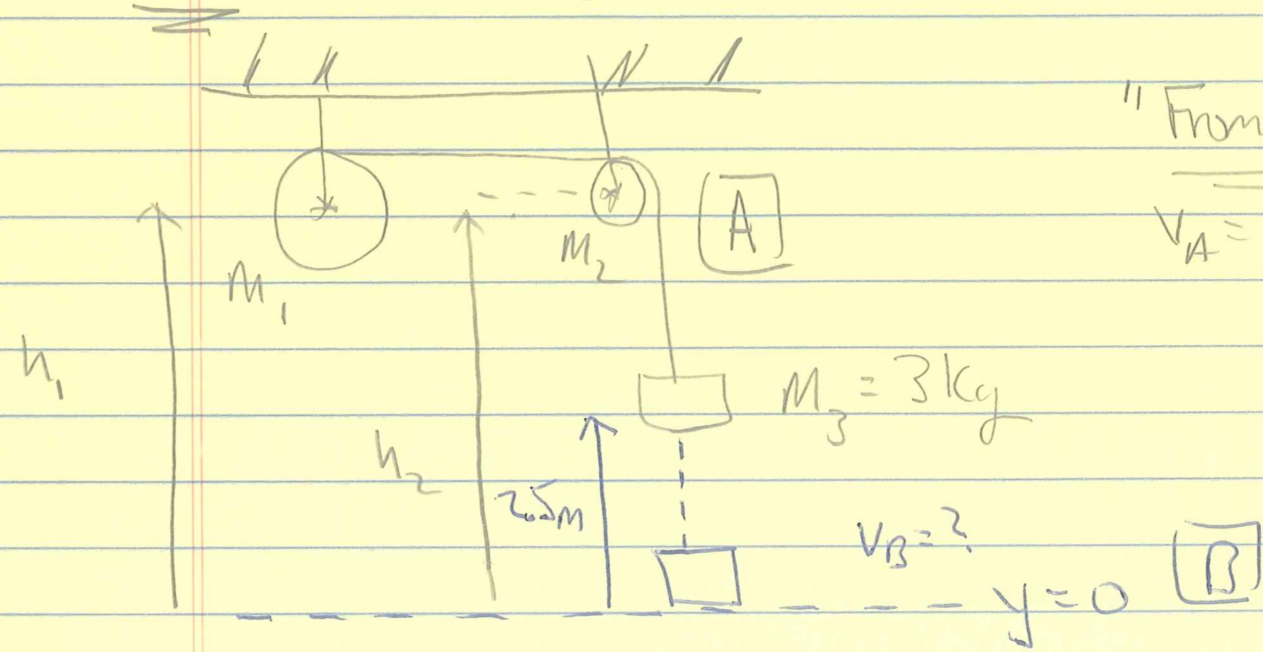
$$5(m_1 - m_2) g = \frac{V_B^2}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)$$

$$\therefore V_B = \sqrt{\frac{10(m_1 - m_2)}{\left(m_1 + m_2 + \frac{I}{R^2}\right)}} = \underline{\underline{0.98\ m/s}}$$

Answer.

9.80) $I_{cylinder} = \frac{M_1 R_1^2}{2}$ $\quad M_1 = 5\,kg,\ R_1 = 0.4\,m$

$I_{pulley} = \frac{M_2 R_2^2}{2}$ $\quad M_2 = 2\,kg,\ R_2 = 0.2\,m$



"From rest"

$V_A = 0$

System: Cylinder, pulley, box, EARTH

$TE_{\boxed{A}} = \underbrace{KE_{\boxed{A}}}_{0} + PE_{g\boxed{A}} = M_1 g h_1 + M_2 g h_2 + 2.5 M_3 g$

$TE_B = KE_{\boxed{B}} + PE_{g\boxed{B}} = \frac{1}{2} I_{cylinder}\,\omega_B^2 + \frac{1}{2} I_{pulley}\,\omega_B^2 + \frac{1}{2} M_3 V_B^2$
$\quad + M_1 g h_1 + M_2 g h_2 + 0$

Cons. of Energy $\quad TE_{\boxed{A}} \equiv TE_{\boxed{B}}$

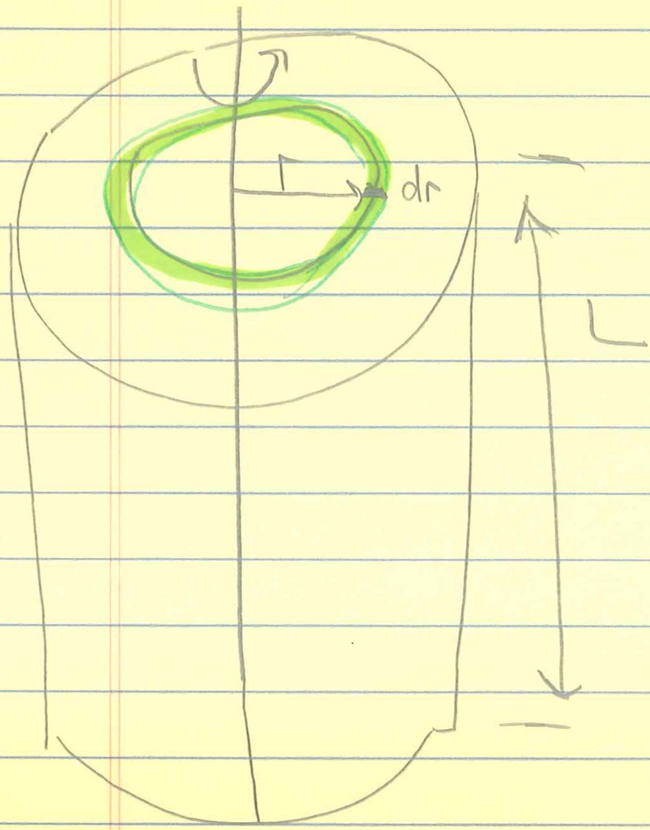
$2.5 M_3 g = \frac{1}{2}\left(\frac{M_1 R_1^2}{2}\right)\left(\frac{V_B^2}{R_1^2}\right) + \frac{1}{2}\left(\frac{M_2 R_2^2}{2}\right)\left(\frac{V_B^2}{R_2^2}\right) + \frac{1}{2} M_3 V_B^2$

$\underline{4.76 \frac{m}{s}} = \sqrt{\frac{5 M_3 g}{\left(\frac{M_1}{2} + \frac{M_2}{2} + M_3\right)}} = V_B$

Answer.

9.53) Disk of mass 'M' and radius 'R' rotating about axis through its center.

Picture is in example 9.10 on page 290. The difference is that we have a solid cylinder.



$$I = \int r^2 dM$$

dM ← a small piece of mass. ALL of that piece must be at the same distance 'r' from the axis of rotation

$$dM = \rho \, dV$$

Mass, density, volume

A definition of density

Here is how we imagine the small piece of volume dV:

dr is a small step along the radius

$(2\pi r) * dr$ is the area of the ring shown in green above.

$L * (2\pi r) * dr$ is the small piece of volume, dV. All of its mass is @ the same distance (r) from the axis of rotation!!

So $dV = 2\pi L r\,dr$

$$I = \int r^2\,dm = \int r^2 \rho\,dV = \int r^2 \rho\, 2\pi L r\,dr$$

$$= 2\pi \rho L \int_0^R r^3\,dr = \frac{2\pi\rho L R^4}{4} = \frac{\rho \pi L R^4}{2}$$

but the density of the cylinder can be written

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\pi R^2 L}$$

So we have:

$$I = \left(\frac{M}{\pi R^2 L}\right)\frac{\pi L R^4}{2} = \frac{MR^2}{2}$$

Which is exactly what TABLE 9.2 has

I do hope you spent time working this out instead of just entering the result from Table 9.2.