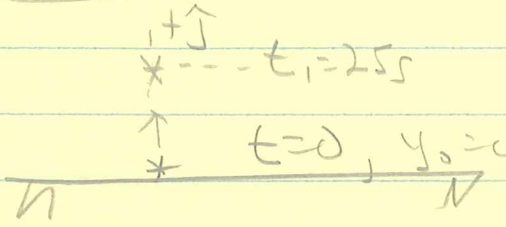
HW due 1/31

2.76 } STAGE I (1-d)



$$\left.\begin{aligned} y(t) &= y_0 + v_0 t + \tfrac{1}{2}at^2 \\ v(t) &= v_0 + at \end{aligned}\right] \Rightarrow \left[\begin{aligned} y(t) &= 1.75t^2 \quad (1) \\ v(t) &= 3.5t \quad (2) \end{aligned}\right.$$

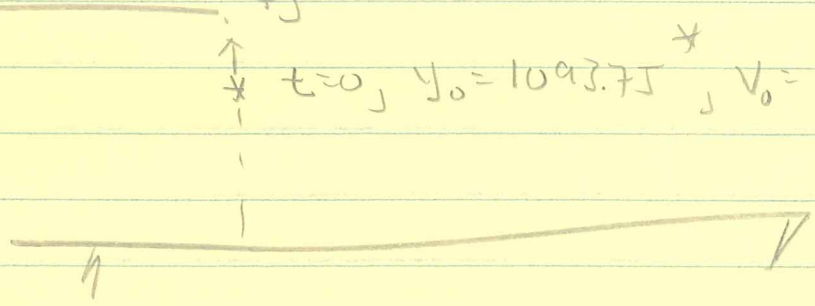
(3)

@ $t_1 = 25$, $y = y_1$, $v = v_1$

(1) $y_1 = 1.75(25)^2 = 1093.75$ m

(2) $v_1 = 3.5(25) = 87.5$ m/s

STAGE II $t = t_2 = (35-25) = 10$, $v_2 = 132.5$



$$\left.\begin{aligned} y(t) &= y_0 + v_0 t + \tfrac{1}{2}at^2 \\ v(t) &= v_0 + at \end{aligned}\right] \Rightarrow \left[\begin{aligned} y(t) &= 1093.75 + 87.5t + \tfrac{1}{2}at^2 \quad (1) \\ v(t) &= 87.5 + at \quad (2) \end{aligned}\right.$$

(3)

@ $t = t_2 = 10$, $V_2 = 132.5$, $y_2 = ?$

(2) $\Rightarrow$ $132.5 = 87.5 + a(10)$

$\therefore a = 4.5 \ m/s^2$

(1) $\Rightarrow$ $y_2 = 1093.75 + 87.5(10) + \frac{1}{2}(4.5)(10)^2 = 2193.75 \ m$

STAGE III

* $t = t_{top}$

↑

* t_0 , $y_0 = 2193.75$, $V_0 = 132.5$, $a = -9.8$

* $t = t_{hit}$, $y_{hit} = 0$

$y(t) = y_0 + V_0 t + \frac{1}{2}at^2$
$V(t) = V_0 + at$
$\Rightarrow$
$y(t) = 2193.75 + 132.5t - 4.9t^2$ (1)
$V(t) = 132.5 - 9.8t$ (2)

@ $t = t_{top}$, $y = y_{top}$, $V_{top} = 0$

(2) $\Rightarrow$ $0 = 132.5 - 9.8t_{top}$

$\therefore t_{top} = 13.52$ seconds

(1) $\Rightarrow$ $y_{top} = 2193.75 + 132.5t_{top} - 4.9t_{top}^2 = 3089.48 \ m$

Max. height

@ $t = t_{hit}$, $y_{hit} = 0$, $V_{hit} = ?$

(1) $\Rightarrow$ $0 = 2193.75 + 132.5t_{hit} - 4.9t_{hit}^2$

$4.9t_{hit}^2 - 132.5t_{hit} - 2193.75 = 0$

$$\therefore \quad t_{hit} = \frac{132.5 \pm \sqrt{132.5^2 + 4(4.9)(2143.75)}}{9.8}$$

$38.63\ s$ OR $-11.59\ s$

✓ ~~Answer~~

[2] $\Rightarrow v_{hit} = 132.5 - 9.8 t_{hit} = -246.07\ m/s$

~~Answer~~

NOTE: - Problem was broken into THREE, each w/ constant acceleration
- I reset $t=0$ for each problem. My choice. I think it makes things easier (:)

3.4 $\vec{r}(t) = (0.28t + 0.036t^2)\hat{i} + (0.019t^3)\hat{j}$ Given

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = (0.28 + 0.072t)\hat{i} + (0.057t^2)\hat{j}$$

Rewriting EXACTLY what is written above:

$x(t) = 0.28t + 0.036t^2$
$y(t) = 0.019t^3$
$v_x(t) = 0.28 + 0.072t$
$v_y(t) = 0.057t^2$

Eqtns. of Motion (1)

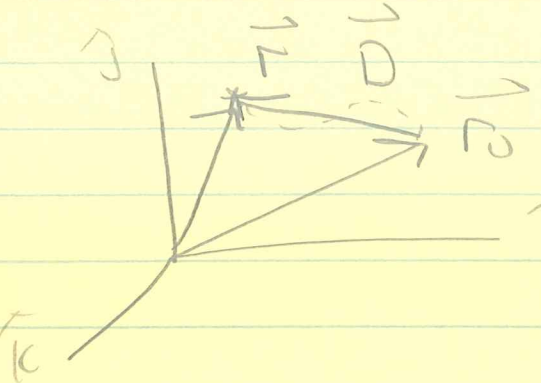
b) Initial position and velocity occurs @ $t=0$

$$\left.\begin{aligned} x(t=0) &= x_0 = 0 \\ y(t=0) &= y_0 = 0 \\ V_x(t=0) &= 0.28 \\ V_y(t=0) &= 0 \end{aligned}\right] \quad \begin{aligned} \vec{r}(t=0) &= 0\hat{i} + 0\hat{j} \\ \vec{V}(t=0) &= 0.28\hat{i} + 0\hat{j} \end{aligned}$$

@ $t=5$ we have:

$$\left.\begin{aligned} x(t=5) &= 2.3 \\ y(t=5) &= 2.375 \\ V_x(t=5) &= 0.64 \\ V_y(t=5) &= 1.425 \end{aligned}\right] \quad \begin{aligned} \vec{r}(t=5) &= 2.3\hat{i} + 2.375\hat{j} \\ \vec{V}(t=5) &= 0.64\hat{i} + 1.425\hat{j} \end{aligned}$$

Generally



$\vec{r}_0 \Rightarrow$ vector to initial position

$\vec{r} \Rightarrow$ vector to find position

$\vec{D} \Rightarrow$ "displacement" vector from initial to find

We can see from the picture that $\vec{r}_0 + \vec{D} = \vec{r}$

$\therefore \vec{D} = \vec{r} - \vec{r}_0$ || A general result ☺

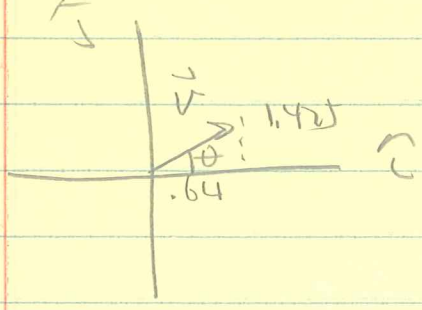
Using results from above: $\vec{D} = \vec{r}(t=5) - \vec{r}(t=0)$

$= 2.3\hat{i} + 2.375\hat{j}$

$|\vec{D}| = \sqrt{2.3^2 + 2.375^2} = 3.306$ Answer.

c.) From above:

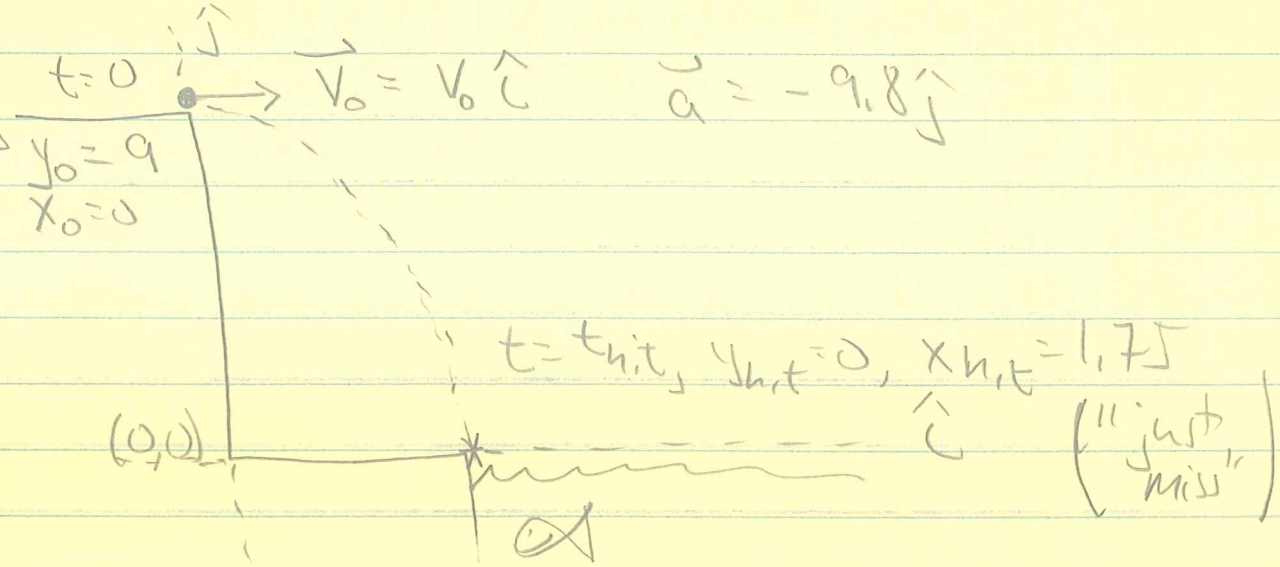
$\vec{V}(t=5) = 0.64\hat{i} + 1.425\hat{j}$



$|\vec{V}| = \sqrt{.64^2 + 1.425^2} = 1.56$ m/s Ans

$\theta = \tan^{-1}\left(\frac{1.425}{.64}\right) = 65.81^\circ$ Ans.

3.10



$$x(t) = x_0 + V_{0x}t + \tfrac{1}{2}a_x t^2$$
$$y(t) = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$$
$$V_x(t) = V_{0x} + a_x t$$
$$V_y(t) = V_{0y} + a_y t$$

$\Rightarrow$

$$x(t) = V_0 t \quad [1]$$
$$y(t) = 9 - 4.9t^2 \quad [2]$$
$$V_x(t) = V_0 \quad [3]$$
$$V_y(t) = -9.8t \quad [4]$$

(4)

(a) $t = t_{hit}$, $y_{hit} = 0$, $x_{hit} = 1.75$

[2] $\Rightarrow$ $0 = 9 - 4.9t_{hit}^2$

$\therefore t_{hit} = 1.355$ sec

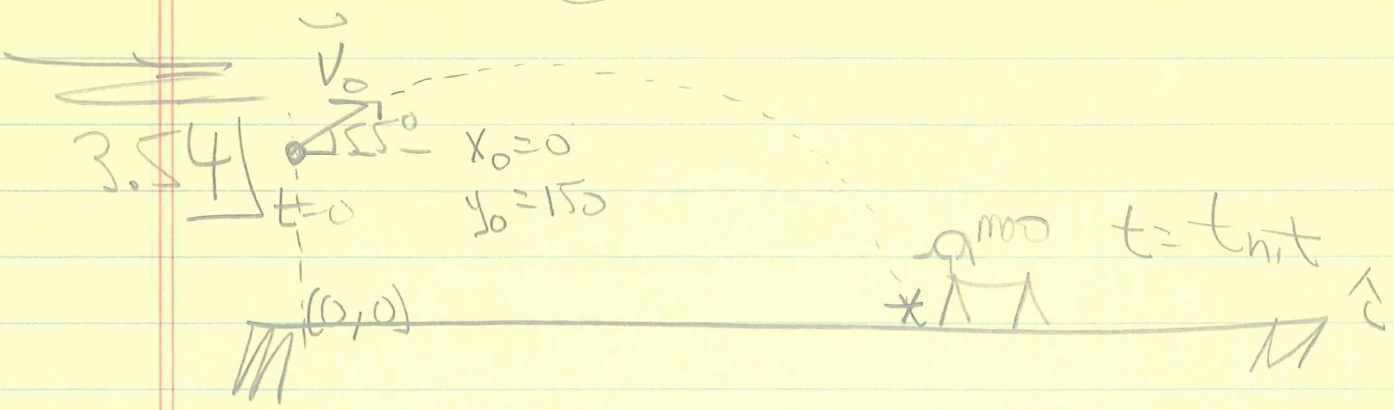
(1) $\Rightarrow X_{hit} = 1.75 = V_0(1.355)$

$V_0 = \underline{1.29 \text{ m/s}}$ (Ans.

⋮

We could go on, but this is all they asked for (4)

3.5 [4]



$\vec{V}_0 = 75\cos(55)\hat{i} + 75\sin(55)\hat{j} = 43.02\hat{i} + 61.44\hat{j}$

$\vec{a} = -9.8\,\hat{j}$

$$\left.\begin{aligned} x(t) &= x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \\ y(t) &= y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \\ v_x(t) &= v_{0x} + a_x t \\ v_y(t) &= v_{0y} + a_y t \end{aligned}\right] \Rightarrow \left[\begin{aligned} x(t) &= 43.02t && (1) \\ y(t) &= 150 + 61.44t - 4.9t^2 && (2) \\ v_x(t) &= 43.02 && (3) \\ v_y(t) &= 61.44 - 9.8t && (4) \end{aligned}\right]$$

(4)

@ $t = t_{hit}$, $X = X_{hit}$, $Y_{hit} = 0$

(2) $\Rightarrow$ $0 = 150 + 61.44 t_{hit} - 4.9 t_{hit}^2$

$4.9 t_{hit}^2 - 61.44 t_{hit} - 150 = 0$

$$t_{hit} = \frac{61.44 \pm \sqrt{61.44^2 + 4(4.9)(150)}}{9.8}$$

14.63 sec (or) -2.09

(1) $\Rightarrow$ $X_{hit} = 43.02 t_{hit} = 629.38$ m

Answer

∴

Could do more, but Ray did not ask ☺