If acceleration is constant ($a(t) = a$)

$$v(t) = \int a\,dt = at + \text{constant}$$

let $v(t=0) \equiv v_0$

$$\therefore v(t=0) = a(0) + \text{constant} \equiv v_0$$

$$\therefore \text{constant} = v_0$$

So we have:

$$v(t) = v_0 + at$$

$$x(t) = \int v(t)\,dt = \int (v_0 + at)\,dt$$

$$= v_0 t + \frac{at^2}{2} + \text{constant2}$$

let $x(t=0) \equiv x_0$

$$\therefore x(t=0) = v_0(0) + \frac{a(0)^2}{2} + \text{constant2} \equiv x_0$$

$$\therefore \text{constant2} = x_0$$

Eqtns. of Motion for an object undergoing constant acceleration 'a'

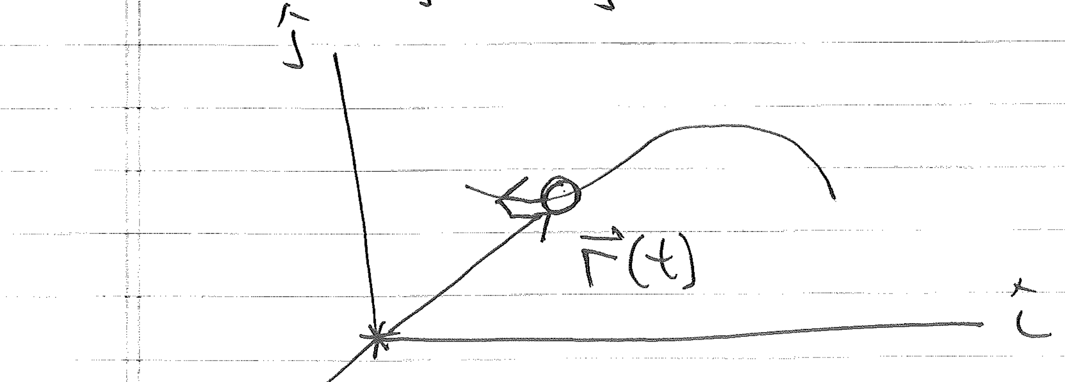
1. $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$
2. $v(t) = v_0 + at$
3. $a(t) = a$

Galileo ~1600

What about 3-d?

"Thou shall not mix x and y" ☺

Consider an object being tracked from the origin by the vector $\vec{r}(t)$



$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = \underbrace{v_x(t)}_{\frac{dx(t)}{dt}}\,\hat{i} + \underbrace{v_y(t)}_{\frac{dy(t)}{dt}}\,\hat{j} + v_z(t)\,\hat{k}$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \underbrace{a_x(t)}_{\frac{dv_x(t)}{dt}}\,\hat{i} + \underbrace{a_y(t)}_{\frac{dv_y(t)}{dt}}\,\hat{j} + a_z(t)\,\hat{k}$$

☆ If acceleration is constant, then we can write:

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$z(t) = z_0 + v_{0z}t + \frac{1}{2}a_z t^2$$

$$V_x(t) = V_{0x} + a_x t$$

$$V_y(t) = V_{0y} + a_y t$$

$$V_z(t) = V_{0z} + a_z t$$

$$\vec{a}(t) = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Write the equation for $\vec{V}(t)$.

$$\vec{V}(t) = (V_{0x} + a_x t)\hat{i} + (V_{0y} + a_y t)\hat{j} + (V_{0z} + a_z t)\hat{k}$$

Write $\underbrace{\vec{a}(t=0)}$, $\vec{V}(t=0)$

Initial accelration $\vec{a}_0 = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

Initial velocity $\vec{V}_0 = V_{0x} \hat{i} + V_{0y} \hat{j} + V_{0z} \hat{k}$

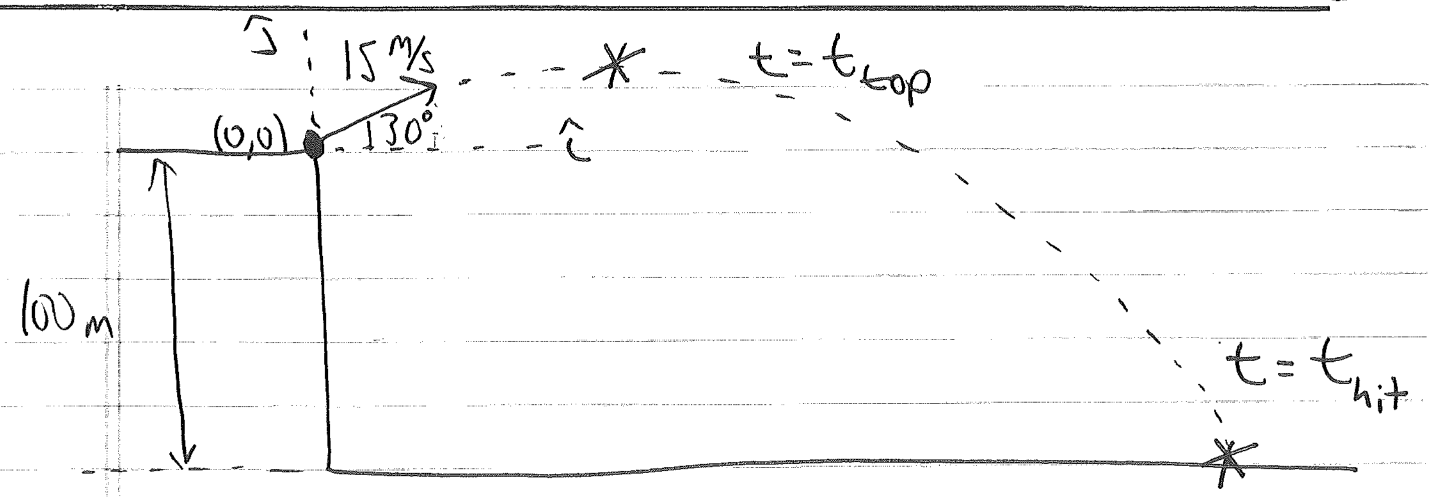
Process for solving motion w/ constant acceleration.

- draw a picture and choose a coordinate system
- write the initial velocity and acceleration as vectors.
- write the general eqtns of motion and make them specific to the problem
- Choose a point in the picture where asked for or given more information
- Apply eqtns of motion to that point
- "Algebrate"
- Repeat as needed

Ex.) A rock is thrown with an intial speed of 15 m/s @ an angle of 30° from a cliff 100 meters tall.

What is its maximum height above base of cliff? When does it reach that height? how far from cliff is it @ that time.

what is its velocity when it hits the ground?



Ignore air

$\vec{a} = -\underbrace{9.8}\hat{j}$ || Galileo

acceleration due to gravity near Earth's surface (ONLY FORCE is gravity ⇒ Constant

$\vec{V}_0 = +15\cos(30)\hat{i} + 15\sin(30)\hat{j}$

$V_0 = 13\hat{i} + 7.5\hat{j}$

$$\begin{bmatrix} x(t) = x_0 + V_{0_x}t + \frac{1}{2}a_x t^2 \\ y(t) = y_0 + V_{0_y}t + \frac{1}{2}a_y t^2 \\ V_x(t) = V_{0_x} + a_x t \\ V_y(t) = V_{0_y} + a_y t \end{bmatrix} \Rightarrow \begin{bmatrix} x(t) = 13t & \boxed{1} \\ y(t) = 7.5t - 4.9t^2 & \boxed{2} \\ V_x(t) = 13 & \boxed{3} \\ V_y(t) = 7.5 - 9.8t & \boxed{4} \end{bmatrix}$$

Problem Solved!!

@ $t = t_{hit}$, $x = x_{hit}$, $y = y_{hit} = -100$

$\boxed{2}$ $y_{hit} = 7.5t_{hit} - 4.9t_{hit}^2 = -100$

$0 = 4.9t_{hit}^2 - 7.5t_{hit} - 100$

$$\therefore t_{hit} = \frac{+7.5 \pm \sqrt{(-7.5)^2 - 4(4.9)(-100)}}{9.8}$$

$$+5.35 \text{ seconds} \quad \boxed{\text{OR}} \quad -3.82 \text{ Seconds}$$

$\boxed{1}$ $X_{hit} = 13 t_{hit} = 69.55$ meters

$\boxed{3}$ $V_x(t = t_{hit}) = 13$

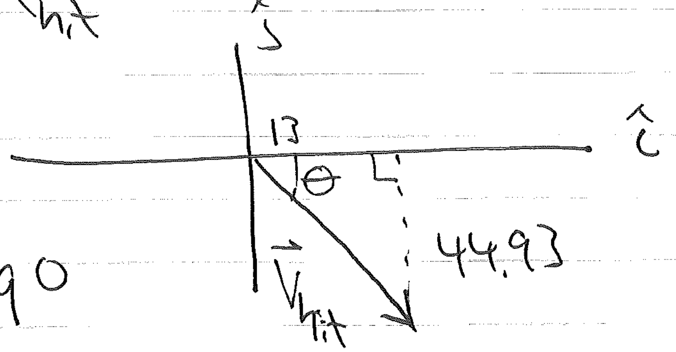
$V_x \big|_{t = t_{hit}}$

$\boxed{4}$ $V_y \big|_{t=t_{hit}} = 7.5 - 9.8 t_{hit} = -44.93$

$$\vec{V} \Big|_{t=t_{hit}} = 13\hat{i} - 44.93\hat{j}$$

$|\vec{V}_{hit}| = 46.77 \, m/s$

@ -73.9°



(a) $t = t_{top}$, $y = y_{top}$, $x = x_{top}$, $V_y\big|_{t=t_{top}} = 0$

$\boxed{4}$ $V_y\big|_{t=t_{top}} = 0 = 7.5 - 9.8\,t_{top}$

$\vdots$