EX. A circular saw (10 cm in diameter) starts from rest and is accelerated to 2500 rpm in 0.8 seconds @ a constant rate.

Find the angular acceleration.
Find the time @ which the ang. speed is 1000 rpm
Find the tangential speed @ the edge of the blade when $\omega = 2500$ rpm. $(r = 0.05\,m)$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega(t) = \omega_0 + \alpha t$$

At $t=0$, let $\theta_0 = 0$. "from rest" $\Rightarrow \omega_0 = 0$
$\alpha = ?$

$$\theta(t) = \frac{1}{2}\alpha t^2 \qquad \boxed{1}$$
$$\omega(t) = \alpha t \qquad \boxed{2}$$

@ $t = 0.8$, $\omega = 2500$, $\theta = ?$

$\boxed{2} \Rightarrow 2500 = \alpha(0.8)$

$3125 = \alpha$ X YIKES!!

r.p.m. X seconds

NO !!!

$$\omega = 2500 \frac{rev}{min} * \frac{1\,min}{60\,sec} = 41.7 \frac{rev}{sec}$$

YES

@ $t = 0.8\ sec$, $\omega = 41.7\ rev/sec$, $\theta = ?$

OK

$\boxed{2} \Rightarrow 41.7 = \alpha(0.8)$

$52.1\ \frac{rev.}{s^2} = \alpha$

$\boxed{1} \Rightarrow \theta_{0.8} = \frac{1}{2}\alpha(0.8)^2 = 16.7$ revolutions

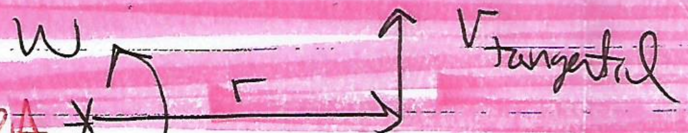
Aside: $1000\ \frac{rev.}{min} * \frac{1\ min}{60\ sec} = 16.7\ rev/sec$

@ $t = t_1$, $\omega_1 = 16.7\ rev/sec$, $\theta_1 = ?$

$\boxed{2} \Rightarrow \omega_1 = 52.1 t_1$

$0.32\ sec = \frac{16.7}{52.1} = t_1$

$\boxed{1} \Rightarrow \theta_1 = \frac{1}{2}(52.1)t_1^2 = 2.7$ revolutions




$V_{tan.} = \cancel{r\omega} = 0.66$ ??

SEE EXTRA PAGE ~~BELOW~~

? $\omega = 2500\ \frac{revolutions}{minute} * \frac{(2\pi \overset{0.05}{r})\ meters}{revolution} * \frac{1\ minute}{60\ seconds} = 13.1$ ??

# EXTRA PAGES

1 radian is the angle that is made by taking the radius of a circle and wrapping it around the arc. See the video link posted in an Announcement for an animation illustrating this. In any equation that mixes angular units with "regular" units (like $v = r\omega$), we need to us radians. (see page ③ of these pages)

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On page ②, we can reason an answer for $V_{tangential}$ @ $r = 0.05$ meters as follows:

$$2500 \frac{\text{revolutions}}{\text{minute}} * \frac{1 \text{ minute}}{60 \text{ seconds}} = 41.67 \frac{\text{revolutions}}{\text{second}}$$

So the time for 1 revolution is $\frac{1}{41.67 \frac{\text{rev.}}{\text{sec}}} = 0.024 \frac{\text{seconds}}{\text{revolution}}$

In that time, a point on the rim has completed one circle, moving a total distance of $2\pi r \Rightarrow 2\pi(0.05) \Rightarrow$ 0.314 meters

We can use translational definitions for velocity:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{0.314}{0.024} = 13.1\,\text{m/s}$$

This approach, OR the one below (bottom page 3) will give you correct answers.

Sorry for the post-spring break confusion

I believe you will be rock solid after the two H.W. problems. The process is same as before.

Angular units: degrees, revolutions, radians

degrees ↑ arbitrary

radians ↑ defined by geometry

$360^\circ = 1 \text{ revolution} = 2\pi \text{ radians}$

* RADIANS ⇒ ADD when need angular unit
TOSS when don't

SO:

$$\omega = 2500 \frac{\text{revolutions}}{\text{minute}} * \frac{2\pi \text{ radians}}{\text{revolution}} * \frac{1 \text{ minute}}{60 \text{ seconds}}$$

$$\omega = 261.7 \frac{\text{radians}}{\text{second}}$$

At $r = 0.05$ meters,

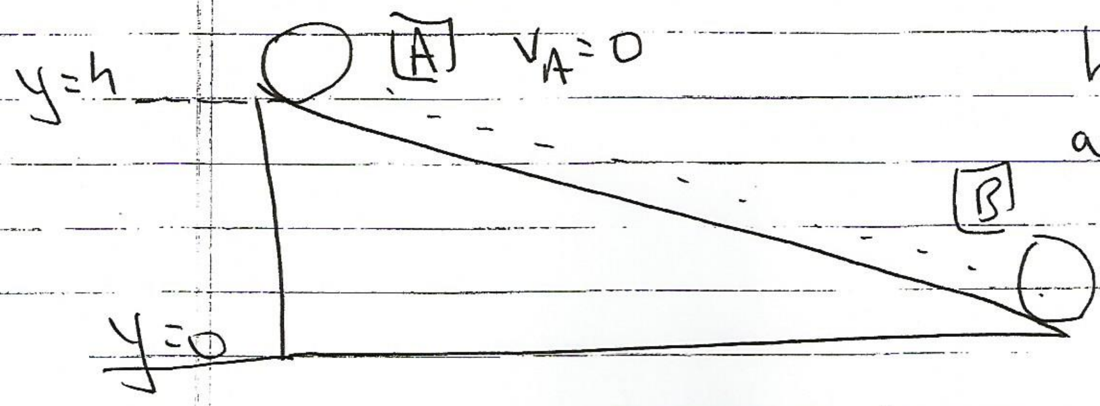
$$v_{\text{tangential}} = r\omega = 0.05\text{m}\left(261.7 \frac{\text{rad}}{\text{sec}}\right)$$

$$= 13.1 \frac{\text{meters} \cdot \text{rad.}}{\text{s}} \text{ * TOSS}$$

$$= \underline{13.1 \text{ m/s}} \text{ Answer.}$$

## Conservation of Energy

$KE_{rotation} = \frac{1}{2} I \omega^2$




ball of radius R and mass M rolls down a ramp. Starts from rest. *

Find the translational speed @ bottom

System: Ball, Earth

$TE_{\boxed{A}} = KE_{\boxed{A}\,translational} + KE_{\boxed{A}\,rotational} + PE_{g\boxed{A}} = mgh$

$* \quad \cancel{\frac{1}{2} m v_A^2} \to 0 \qquad \cancel{\frac{1}{2} I \omega_A^2} \to 0 \qquad mgh$

$TE_{\boxed{B}} = KE_{\boxed{B}\,rotational} + KE_{\boxed{B}\,translational} + \cancel{PE_{g\boxed{B}}} \to 0$

$= \frac{1}{2} I \omega_B^2 + \frac{1}{2} m v_B^2$

Cons. of Energy: $TE_{\boxed{A}} = TE_{\boxed{B}}$

$mgh = \frac{1}{2} I \omega_B^2 + \frac{1}{2} m v_B^2$

Table 9.2 pg 286

$\frac{2}{5} MR^2$

$$mgh = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_B^2 + \frac{1}{2}mv_B^2$$

"cross-talk" $v_B = R\omega_B$

$$\therefore \omega_B = v_B/R$$

$$\cancel{m}gh = \frac{1}{5}\cancel{m}\cancel{R^2}\left(\frac{v_B^2}{\cancel{R^2}}\right) + \frac{1}{2}\cancel{m}v_B^2$$

$$gh = \frac{v_B^2}{5} + \frac{v_B^2}{2}$$

$$gh = \frac{7v_B^2}{10}$$

$$v_B = \sqrt{\frac{10gh}{7}}$$

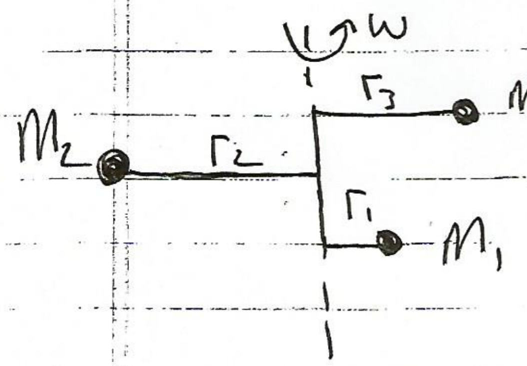
Answer.

$$\boxed{\begin{array}{l}\text{IF sliding} \\ KE_{rot.} = 0 \\ v_B = \sqrt{2gh}\end{array}}$$

Aside:

$$I = \int r^2\,dm = \sum_i r_i^2 m_i$$

$m_i$ is NOT squared (was in class typo)




Mass of connecting rods is small.

$$KE = \frac{1}{2}m_1\underset{\substack{\uparrow \\ \text{tangential}}}{v_1^2} + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$$

$$v_1 = r_1^2\omega_1^2$$

$$\therefore \quad KE = \frac{1}{2}m_1r_1^2\omega_1^2 + \frac{1}{2}m_2r_2^2\omega_2^2 + \frac{1}{2}m_3r_3^2\omega_3^2$$

Hmmm... aren't $\omega_1$, $\omega_2$, and $\omega_3$ the same?

YES! $\omega_1 = \omega_2 = \omega_3 = \omega$

$$\therefore \quad KE = \tfrac{1}{2} m_1 r_1^2 \omega^2 + \tfrac{1}{2} m_2 r_2^2 \omega^2 + \tfrac{1}{2} m_3 r_3^2 \omega^2$$

$$= \tfrac{1}{2} \underbrace{(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)}_{I} \omega^2$$

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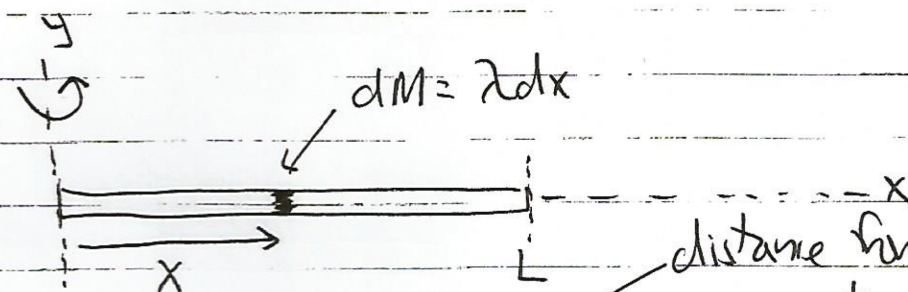
A rod of mass M and length L is spun about an axis ⊥ to one end.
Find I.

$$\lambda = \frac{M}{L}$$




distance from rot. axis to dM

$$I = \int r^2 \, dm$$

$$I = \int_0^L x^2 \lambda \, dx = \lambda \int_0^L x^2 = \lambda \frac{x^3}{3} \Big|_0^L$$

$$I = \frac{\lambda L^3}{3} = \frac{ML^2}{3}$$

Answer.