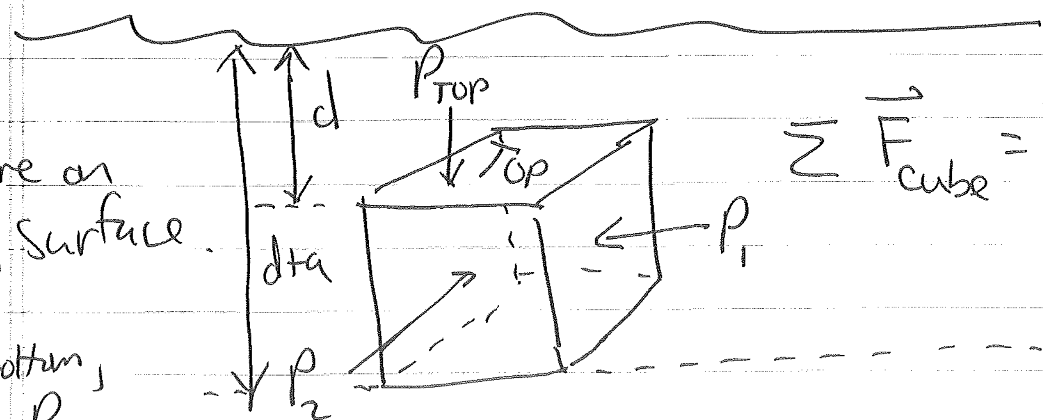
Archimedes proof?

Consider a cube (side length 'a') immersed to a depth 'd' in a fluid of density ρ.



$\Sigma \vec{F}_{cube} = ?$

$+\hat{j}$

Pressure on every surface.

P_{TOP}, P_{Bottom}, $\underbrace{P_1, P_2, P_3, P_4}_{\text{sides}}$

Symmetry $\Rightarrow$ NO net force horizontal

$$\left.\begin{aligned} F_1 &= P_1 * A_1 \\ F_2 &= P_2 * A_2 \\ &\vdots \end{aligned}\right]$$

definition

Cube $\Rightarrow$ all areas are same.

$\vec{F}_{buoyant}$

$$\vec{F}_{TOP} = P_{TOP} * A = -(\rho g d)(a^2)\hat{j}$$

"gauge"

$$\vec{F}_{bottom} = P_{bottom} * A = +(\rho g(d+a))(a^2)\hat{j}$$

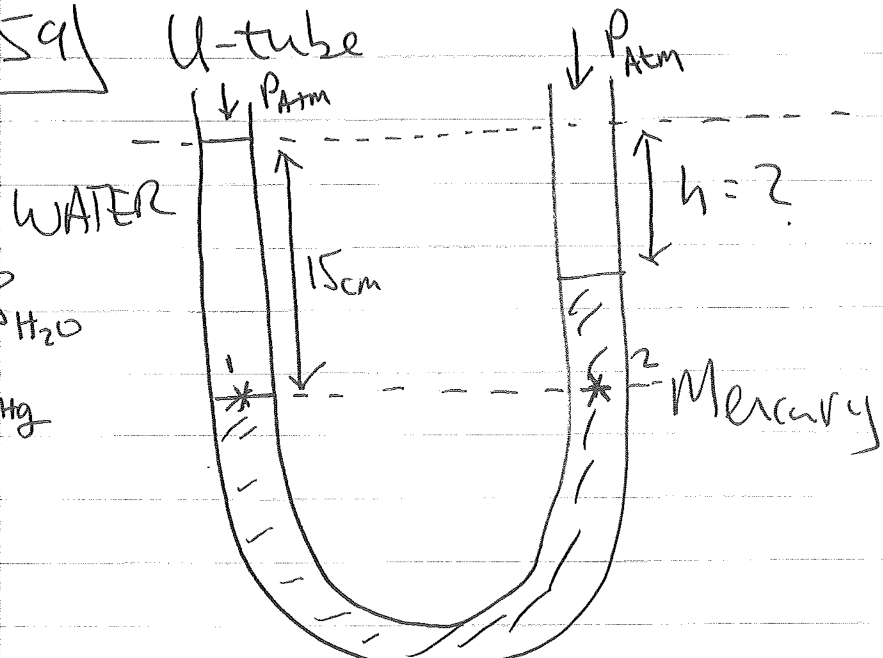
$$\vec{F}_{gravity} = -M_{cube}\, g\, \hat{j}$$

$$\vec{F}_{buoyant} = \vec{F}_{Top} + \vec{F}_{bottom} = -\rho g d a^2 + \rho g (d+a) a^2 \ \hat{j}$$

$$= \underbrace{+\rho g a^3}_{\text{weight of the fluid displaced}} \ \hat{j}$$

12.59) U-tube

Pg 394



given ρ_{H_2O}

ρ_{Hg}

Pressure @ 1 due to column of water:

$$P_1 = \rho_{H_2O}\, g (0.15) + P_{ATM}$$

Pressure @ 2 due to column of Hg:

$$P_2 = \rho_{Hg}\, g (0.15 - h) + P_{ATM}$$

$$P_1 \equiv P_2$$

$$\rho_{H_2O}\cancel{g}(0.15) + \cancel{P_{ATM}} = \rho_{Hg}\cancel{g}(0.15-h) + \cancel{P_{ATM}}$$

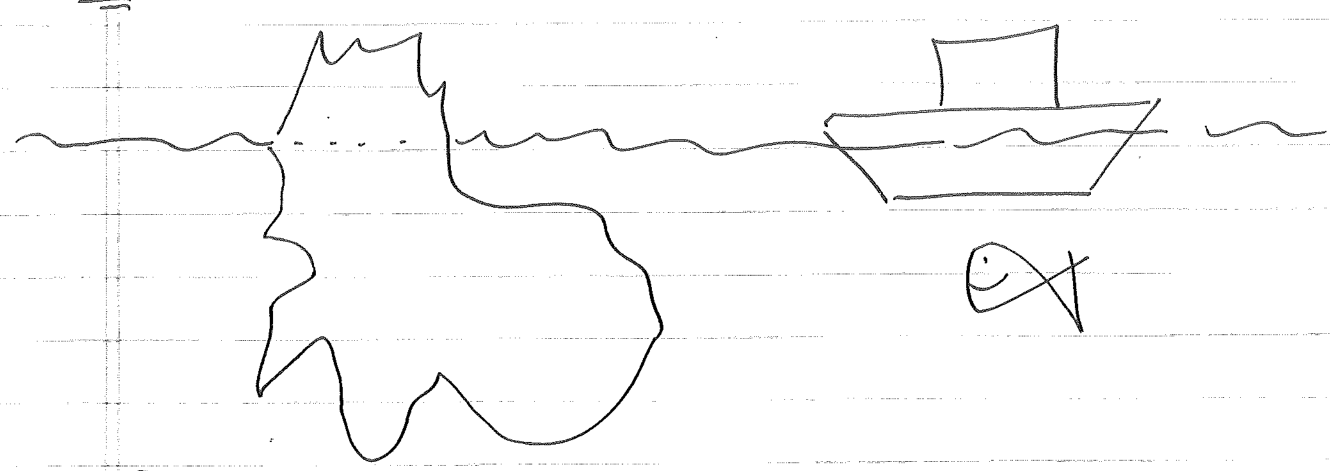
$$\rho_{Hg} h = \rho_{Hg}(0.15) - \rho_{H_2O}(0.15)$$

$$h = 0.15\left(1 - \frac{\rho_{H_2O}}{\rho_{Hg}}\right) \quad \boxed{\checkmark}$$

Ex.] What fraction of an icebergs volume is under water?

Iceberg is freshwater $\Rightarrow \rho_{H_2O}$

Seawater $\Rightarrow \rho_{SW}$ (greater than ρ_{H_2O})



for ICEBERG

$+m_{SW}g\hat{j} = \vec{F}_{buoyant}$ $\quad +\hat{j}$

$\vec{F}_g = -mg\hat{j}$

"Floating" (@ rest)

$$\sum F_y = 0$$

$$m_{SW}g - mg = 0$$

$$m_{SW} = m$$

$$\rho_{SW} V_{SW\ \text{displaced}} = \rho_{H_2O} V_{ICEBERG}$$

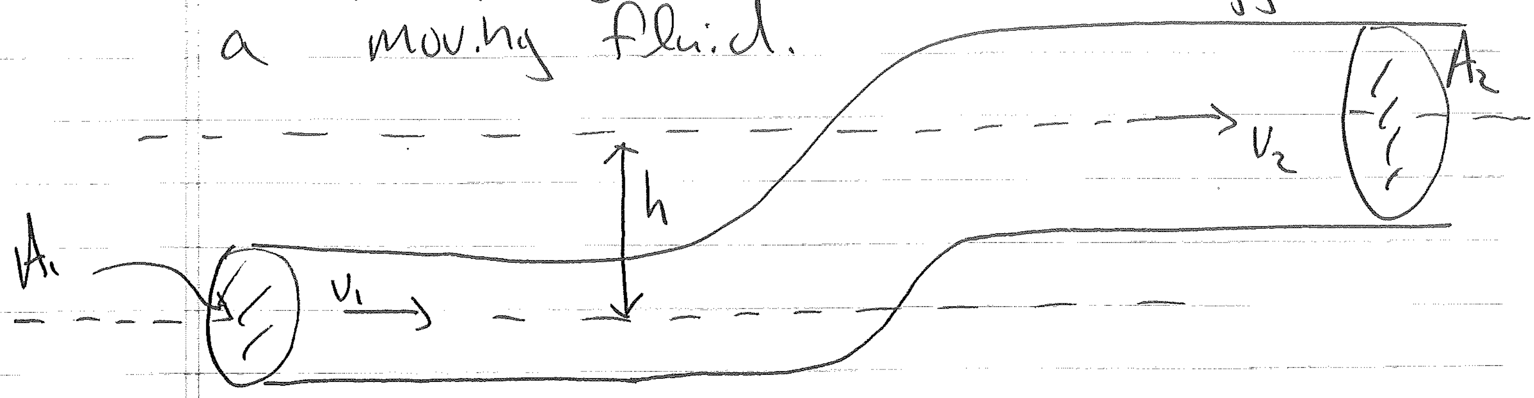
Archimedes $\Rightarrow$ $V_{SW \text{ displaced}} = V_{\text{ICEBERG submerged}}$:)

$\therefore \dfrac{V_{\text{submerged}}}{V_{\text{Iceberg}}} = \dfrac{\rho_{H_2O}}{\rho_{SW}} = \underbrace{0.89}_{89\%}$

NOTE: $\rho_{H_2O} \Rightarrow$ ICE

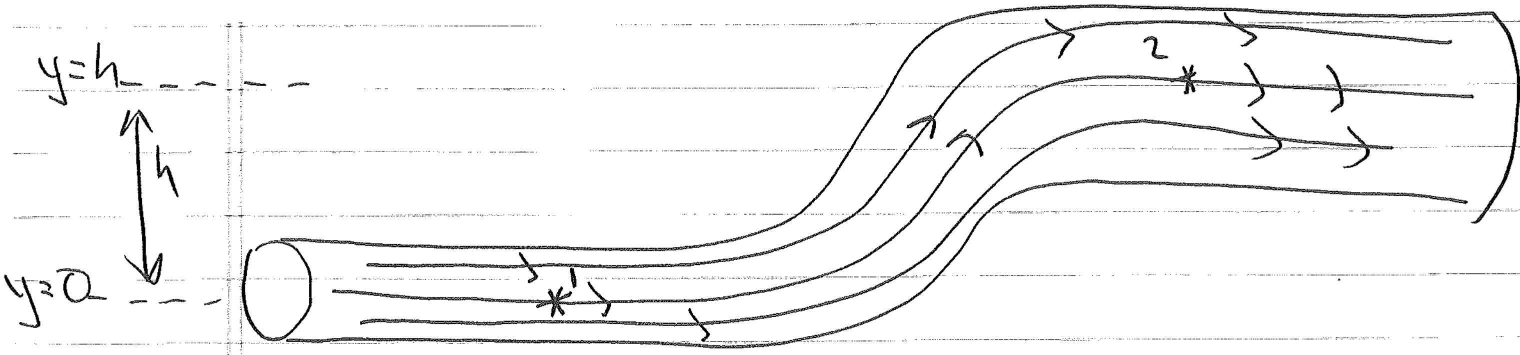
* Bernoulli $\Rightarrow$ Pressure is lower in a moving fluid

* A statement of conservation of energy for a moving fluid.



$$\boxed{A_1 v_1 = A_2 v_2}$$

* $P + \frac{1}{2}\rho v^2 + \rho g h \equiv$ Constant along any streamline in a non-viscous fluid (w\ laminar flow).
Smooth

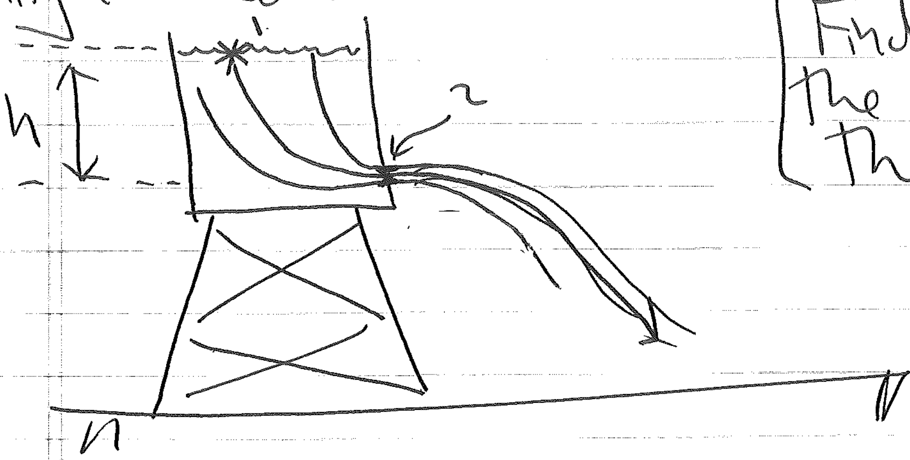


For point #1 : $P_1 + \frac{1}{2}\rho V_1^2 + \rho g(0) \equiv$ Constant

For point #2 : $P_2 + \frac{1}{2}\rho V_2^2 + \rho g h \equiv$ " "

$$\therefore \boxed{P_1 + \frac{1}{2}\rho v_1^2 \equiv P_2 + \frac{1}{2}\rho V_2^2 + \rho g h}$$

Ex. (NOT so obvious ☺)



Find speed @ which the water exits the The hole

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h \equiv P_2 + \frac{1}{2}\rho V_2^2 + \rho g(0)$$

$P_{ATMOSPHERE}$

large tank w/ small hole $V_1 \approx 0$

$P_{ATMOSPHERE}$

$$\therefore \rho g h = \frac{1}{2}\rho V_2^2$$

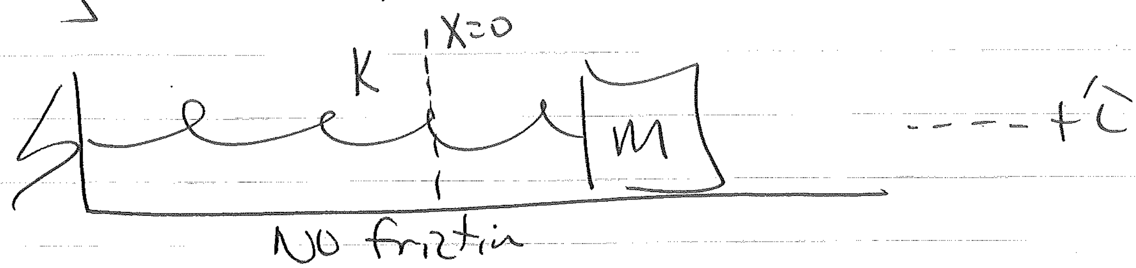
$$\sqrt{2gh} = V_2$$

Toricelli's Law

END EXAM 3 CONTENT

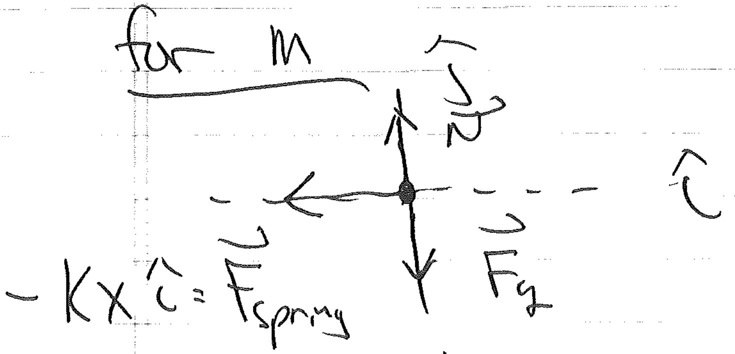
"NEW" Stuff (Ch.14)

Consider a mass 'm' on a spring w/ spring constant 'K'



Write the eqtns. of motion. $x(t)$, $v(t)$, $a(t)$

for m



NOT CONSTANT !! $\therefore$ a is Not constant

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

NO, NO, NO, NO,

Use Newton!

$\hat{i}$ $\sum F_x = m a_x$

$$\boxed{-kx = m\frac{d^2x}{dt^2}} \quad [1]$$

2nd order, linear differential equation

Solutions: sines and cosines.

let $x(t) = A\sin(\omega t)$

ω "angular frequency"

$x'(t) = A\omega\cos(\omega t)$

$x''(t) = -A\omega^2\sin(\omega t)$

$\omega = 2\pi f$

f: # reps / second

[1] $\Rightarrow$ $-kA\sin(\omega t) = m(-A\omega^2\sin(\omega t))$

$$k = m\omega^2$$

$$\sqrt{\frac{k}{m}} = \omega$$

"natural frequency"
"resonant frequency"

"All objects tend to vibrate @ preferred (natural, resonant) frequencies which are determined by their physical structure and composition".

Periodic Motion ⇒ repeating motion

Simple Harmonic Motion ⇒ repeating motion described by sines and cosines.

(f) frequency = $\frac{\#\text{repetitions}}{\text{second}}$ hertz (Hz)

Period ⇒ time over which the motion repeats
(T)

$$T = \frac{1}{f}$$