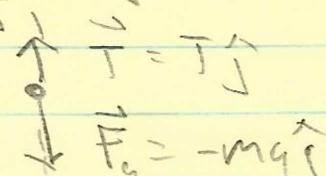
HW due 2/21 Selected Solutions

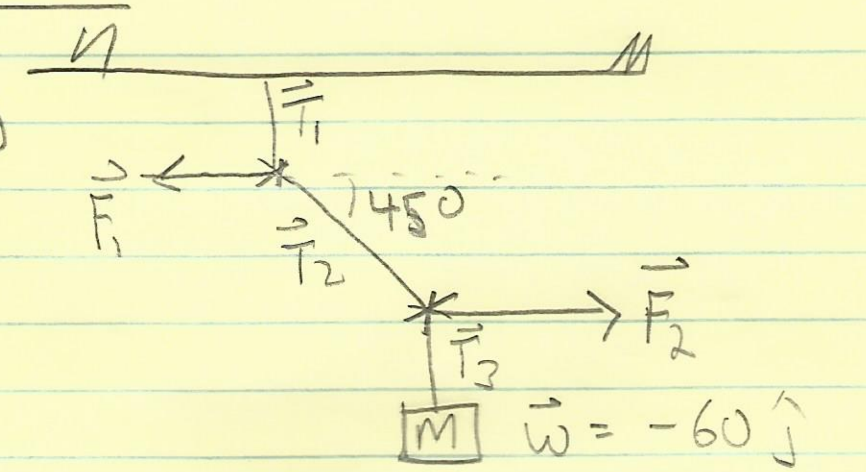
5.2) For EVERY case: Choose a mass to which you want to apply 2nd law. The freebody diagram will look like this:



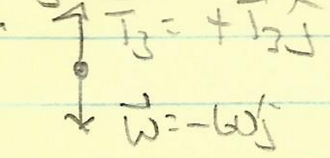
2nd $\sum F_y = m\cancel{a_y}^{\,0}$

$\therefore \underline{\underline{T = mg}}$

5.10)



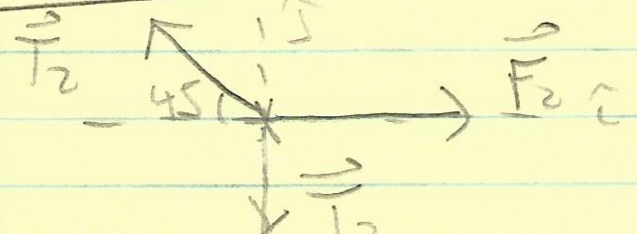
<u>for m</u>



2nd $\sum F_y = m\cancel{a_y}^{\,0}$

$\therefore \underline{T_3 = 60}$

<u>For the *point just above m</u>



$$\begin{cases} \vec{T}_3 = -60\hat{j} \\ \vec{F}_2 = F_2\hat{i} \\ \vec{T}_2 = -T_2\cos(45)\hat{i} + T_2\sin(45)\hat{j} \end{cases}$$

2nd $\sum \vec{F} = \cancel{m\vec{a}} \to 0$

?

$\hat{i}$ $\sum F_x = 0$

$\boxed{F_2 - T_2\cos(45) = 0}$ A

$\hat{j}$ $\sum F_y = 0$

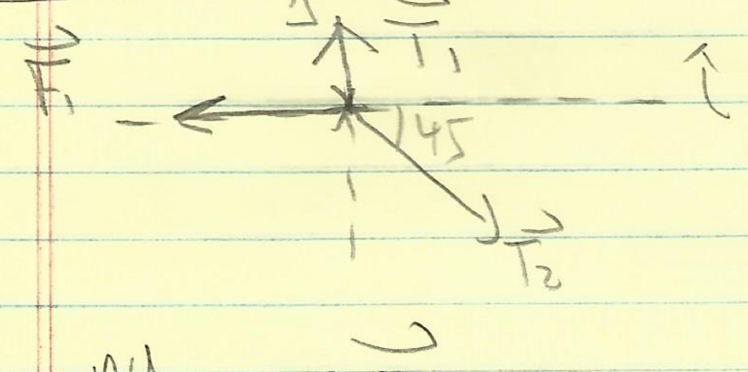
$-60 + T_2\sin(45) = 0$

$\therefore T_2 = \frac{60}{\sin(45)} = \underline{\underline{84.85\ N}}$

So from A, $F_2 = T_2\cos(45) = \underline{\underline{60\ N}}$

Answer

For the * point just above the previous * point



$\vec{T}_1 = T_1\hat{j}$

$\vec{F}_1 = -F_1\hat{i}$

$\vec{T}_2 = +60\cos(45)\hat{i} - 60\sin(45)\hat{j}$

2nd. $\sum \vec{F} = 0$

$\hat{i}$ $\sum F_x = 0$

$-F_1 + 60\cos(45) = 0$

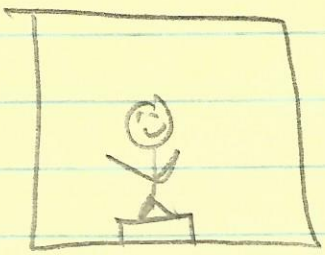
$\therefore F_1 = 60\cos(45) = \underline{\underline{42.4\,N}}$ Answer

$\hat{j}$ $\sum F_y = 0$

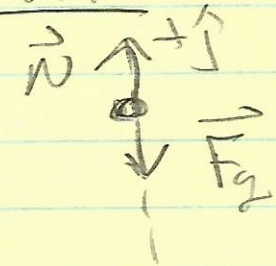
$T_1 - 60\sin(45) = 0$

$\therefore T_1 = 60\sin(45) = \underline{\underline{42.4\ N}}$ Answer

5.20) Scale reads the normal force applied to student.



for student



$\vec{N} = N\hat{j}$
$\vec{F}_g = -550\hat{j}$

2^{nd} $\sum F_y = m a_y$

note: $550 = mg$ from Law of gravity
$\therefore m = \frac{550}{g}$

a.) We are given $N = 450$ newtons

$2^{nd} \Rightarrow N - 550 = \left(\frac{550}{g}\right) a_y$

$\therefore a_y = \frac{(450-550)}{(550/g)} = -1.78\ m/s^2$

moving in $-\hat{j}$ direction.

Answer.

b.) We are given $N = 670N$.

$2^{nd} \Rightarrow a_y = \frac{(670-550)}{(550/g)} = +2.14\ m/s^2$

Answer

To Answer questions about the tension in the elevator cable, we have to apply 2^{nd} to the elevator (850 kg)

<u>for the elevator</u>

$+\hat{j}$

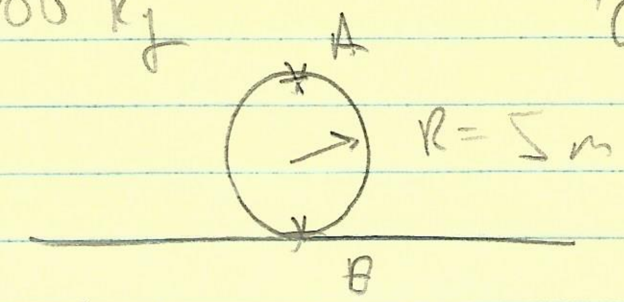
$\vec{T} = +T\hat{j}$

$\vec{F}_g = -850g\hat{j}$

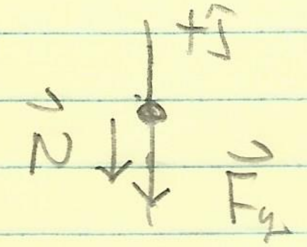
2^{nd} $\quad \Sigma F_y = (850)a_y$

do this for each of the accelerations we just found and find the corresponding tension

5.46 $M = 0.800\ kg$ "constant speed"



for car @ A



$$\begin{cases} \vec{N} = -6\hat{j} \\ \vec{F}_g = -mg\hat{j} \end{cases}$$

2nd $\sum F_y = m a_y$ ← $\left(-\frac{v_{tan}^2}{R}\right)$

my coordinates!!

$$-6 - mg = -\frac{m v_{tan}^2}{R}$$

$$\therefore V_{tan} = \sqrt{\frac{R(6+mg)}{M}} = \sqrt{\frac{5(6+0.8(9.8))}{0.8}}$$

$$V_{tan} = 86.5\ m/s$$

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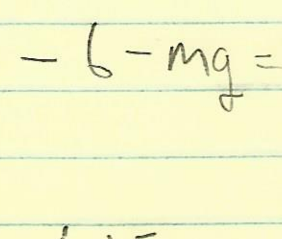
for car @ B



$$\begin{cases} \vec{N} = +N\hat{j} \\ \vec{F}_g = -mg\hat{j} \end{cases}$$

2nd Law: $\sum F_y = M a_y$ ← $\left(+\frac{v_{tan}^2}{R}\right)$

My coordinates!!

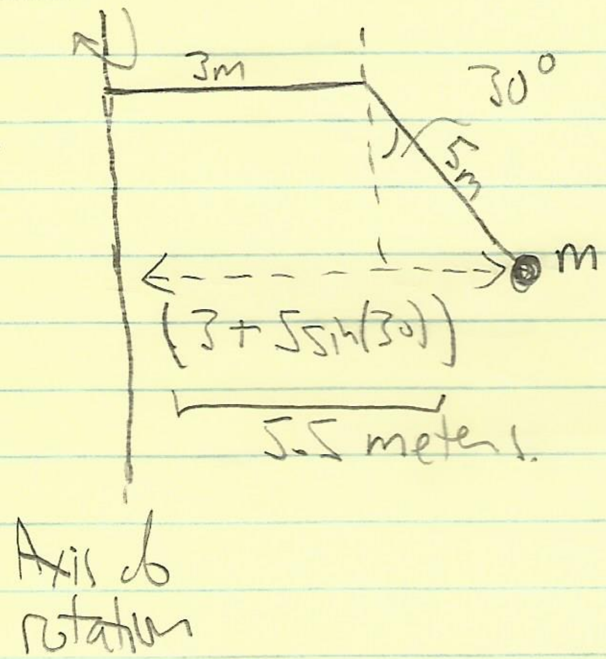
$$N - mg = \frac{m v_{tan}^2}{R}$$

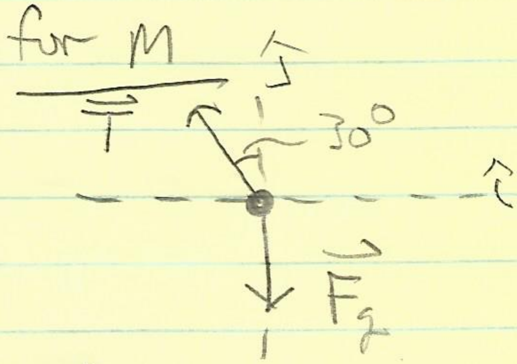
$$N = mg + \frac{m v_{tan}^2}{R} = m\left(g + \frac{v_{tan}^2}{R}\right)$$

$$= (0.8)\left(9.8 + \frac{(86.5)^2}{5}\right) = \underline{\underline{1205 \text{ newtons}}}$$

<u>Answer</u>

$\boxed{5.50}$





$$\vec{T} = -T\cos(30)\,\hat{i} + T\sin(30)\,\hat{j}$$

$$\vec{F}_g = -mg\,\hat{j}$$

2nd: $\boxed{\hat{j}}$ $\sum F_y = m\underbrace{a_y}_{\to 0}$

$$T\sin(30) - mg = 0$$

$$T = mg/\sin(30)$$

$\boxed{\hat{i}}$ $\sum F_x = m a_x \leftarrow$ <u>Centripetal</u> Required for circular mtn.

$$-T\cos(30) = m\left(\frac{-V_{tan}^2}{R}\right)$$

My coordinates!

We know R = 5.5 meters and can substitute for T from previous expression.

$$\Rightarrow \frac{\cancel{m}g}{\sin(30)}(\cos(30)) = \frac{\cancel{m}V_{tan}^2}{R}$$

$$\therefore V_{tan} = \sqrt{gR\cot(30)} = \underline{\underline{9.66\ m/s}}$$

NOTE: DOES NOT DEPEND ON m!! Otherwise they would have to weigh in the riders ☺

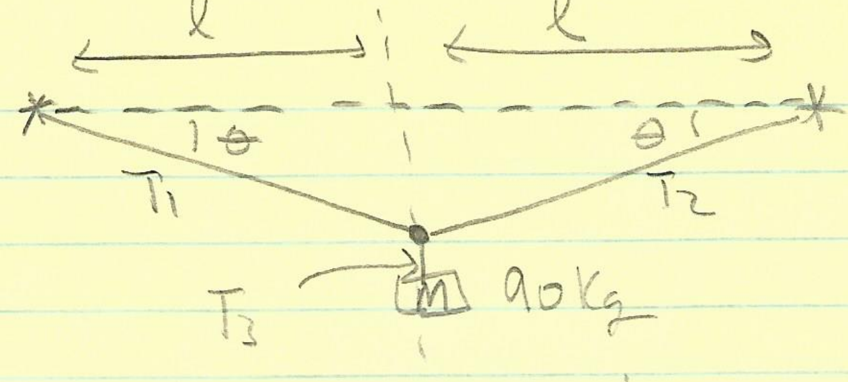
1 circle = $2\pi R$ = 34.54 meters

Distance = SPEED * TIME

$$\therefore \text{TIME} = \frac{34.54}{9.66} = \underline{\underline{3.58 \text{ seconds}}}$$

Answer.

5.60



Because of the symmetry, we can see $|\vec{T}_1| = |\vec{T}_2|$
Would NOT be true if mass hanging from a different point. I will write out the long way to illustrate process.

for m

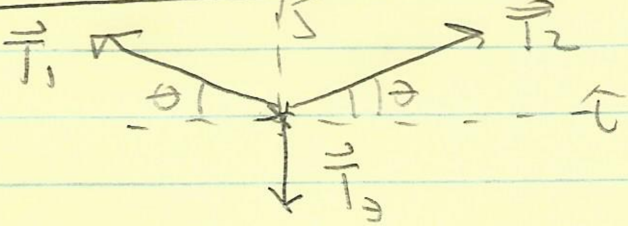
$\vec{T}_3 = +T_3\hat{j}$

$\vec{F}_g = -mg\hat{j}$

2nd $\sum F_y = m\cancel{a_y} \to 0$

$\therefore T_3 = mg$

for the point where the ropes attach



$\vec{T}_3 = -mg\hat{j}$

$\vec{T}_2 = T_2\cos\theta\,\hat{i} + T_2\sin\theta\,\hat{j}$

$\vec{T}_1 = -T_1\cos\theta\,\hat{i} + T_1\sin\theta\,\hat{j}$

2nd $\sum \vec{F} = m\vec{a} \to 0$

?

$\boxed{\hat{i}}$ $\sum F_x = 0$

$T_2\cos\theta - T_1\cos\theta = 0$

$\therefore T_2 = T_1$ ☺ Let's call it "T"

$\boxed{\hat{j}}$ $\sum F_y = 0$

$$-mg + T\sin\theta + T\sin\theta = 0$$

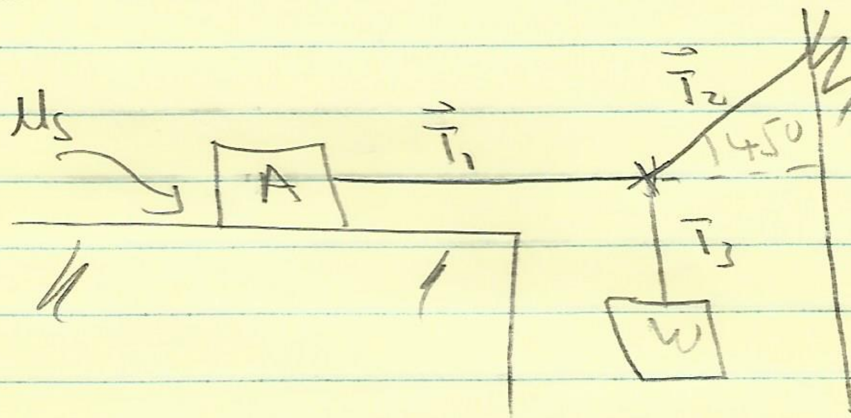
$$T = \frac{mg}{2\sin\theta}$$

Done: Pick θ, find T

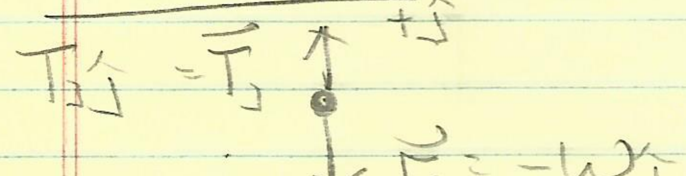
$\boxed{\text{OR}}$

Pick T, find θ

5.76



For w

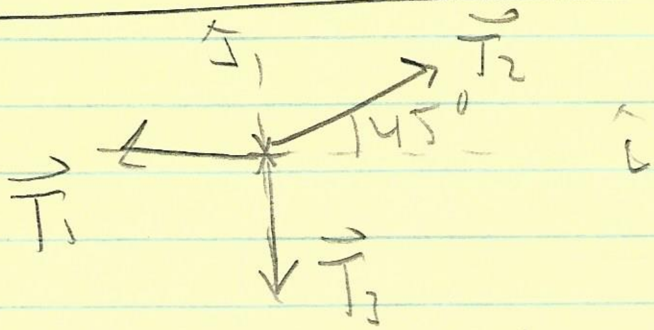


2nd $\sum F_y = 0$

$$T_3 - w = 0$$

$$\boxed{T_3 = w} \quad \boxed{\text{I}}$$

For point where ropes join



$\vec{T}_3 = -w\hat{j}$

$\vec{T}_1 = -T_1\hat{i}$

$\vec{T}_2 = T_2\cos(45)\hat{i} + T_2\sin(45)\hat{j}$

2nd $\sum \vec{F} = \underset{?}{m}\,\vec{a}$, $\vec{a} \to 0$

$\boxed{\hat{j}}$ $\sum F_y = 0$

$-w + T_2\sin(45) = 0$

$$\boxed{\therefore T_2 = \frac{w}{\sin(45)}} \quad \boxed{2}$$

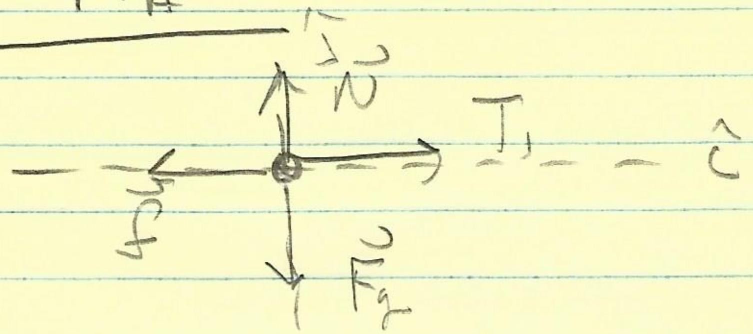
$\boxed{\hat{i}}$ $\sum F_x = 0$

$-T_1 + T_2\cos(45) = 0$

$\boxed{3}$ $\therefore T_1 = T_2\cos(45) = \frac{w\cos(45)}{\sin(45)}$

have used $\boxed{2}$

for m_A



$\vec{T}_1 = \frac{W\cos(45)}{\sin(45)}\hat{i}$

$\vec{N} = N\hat{j}$

$\vec{F}_g = -60\hat{j}$ ← given!

$\vec{f} = -\mu_s N\hat{i}$

2nd $\sum \vec{F} = m_A \cancel{\vec{a}} \to 0$

$\boxed{\hat{j}}$ $\sum F_y = 0$

$-60 + N = 0$

$\boxed{N = +60}$ $\boxed{4}$

$\boxed{\hat{i}}$ $\sum \vec{F}_x = 0$

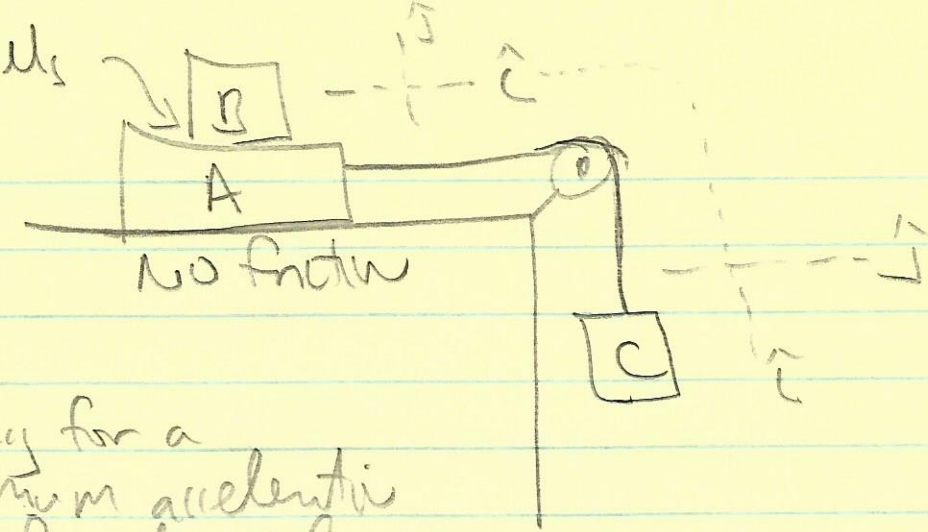
$\frac{W\cos(45)}{\sin(45)} - \mu_s N = 0$ use N from $\boxed{4}$

to get ⇒ $\boxed{\frac{W\cos(45)}{\sin(45)} - \mu_s 60 = 0}$ $\boxed{5}$

14 Done: $f = \mu_s N = 60(0.25) = 15$ Newtons

from $\boxed{5}$, $W = \frac{\mu_s 60 \sin(45)}{\cos(45)} = 15$ kg

5.92



SEE NOTES ON "Consistency"

looking for a maximum acceleration (last line of problem statement)

For M_c

$$\vec{T} = -T\hat{i}$$

$$\vec{F}_g = +m_c g\,\hat{i}$$

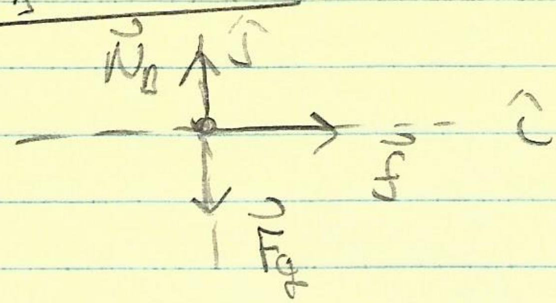
$+\hat{i}$

2nd $\boxed{\hat{i}}$ $\sum F_x = m_c a_x$

$$-T + m_c g = m_c a_x$$

(1) $$\boxed{T = -m_c a_x + m_c g}$$

for M_B



$$\vec{F}_g = -M_B g\,\hat{j}$$
$$\vec{N}_B = N_B\,\hat{j}$$
$$\vec{f} = \mu_s N_B\,\hat{i} \quad ☆$$

2nd $\boxed{\hat{j}}$ $\sum F_y = \cancel{M_B a_y} \to 0$

$$N_B - M_B g = 0$$
$$\underline{N_B = M_B g} \quad *$$

$\boxed{\hat{i}}$ $\sum F_x = M_B a_x$

$\mu_s N_B = M_B a_x$ substitute from above to get

$$\Rightarrow \quad \mu_s M_B g = M_B a_x$$

$$\boxed{a_x = \mu_s g} \quad \boxed{2}$$

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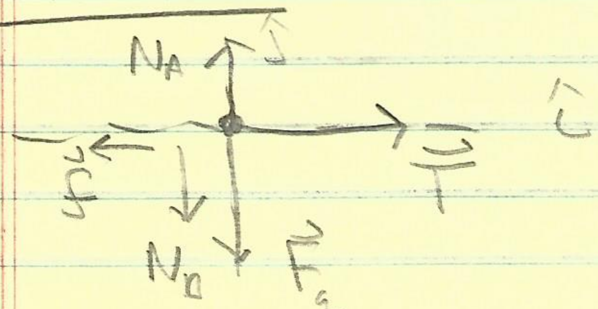
for M_A



$$\vec{N}_A = N_A\,\hat{j}$$
$$\vec{T} = T\,\hat{i}$$
$$\vec{F}_g = -M_A g\,\hat{j}$$
$$\vec{N}_B = -M_B g\,\hat{j} \quad *$$
$$\vec{f} = -\mu_s M_B g\,\hat{i} \quad ☆$$

2^{nd} $\boxed{\hat{j}}$ $\sum F_y = m_A \cancel{a_y}^{=0}$

$$N_A - m_A g - m_B g = 0$$

$$N_A = m_A g + m_B g$$

would have been important if friction on lower surface.

$\boxed{\hat{i}}$ $\sum F_x = m_A a_x$

$$\boxed{T - \mu_s m_B g = m_A a_x} \quad \boxed{3}$$

W/ eqns $\boxed{1}$, $\boxed{2}$, and $\boxed{3}$ we can answer the questions ☺

Substitute $\boxed{1}$ and $\boxed{2}$ into $\boxed{3}$:

$$\boxed{3} \Rightarrow -m_c a_x + m_c g - \mu_s m_B g = m_A a_x$$

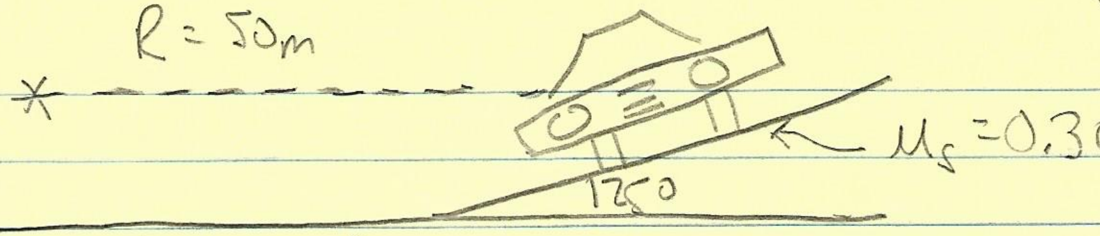
$$-m_c \mu_s g + m_c g - \mu_s m_B g = m_A \mu_s g$$

$$m_c(-\mu_s + 1) = m_A \mu_s + m_B \mu_s$$

ANSWER ☺ !

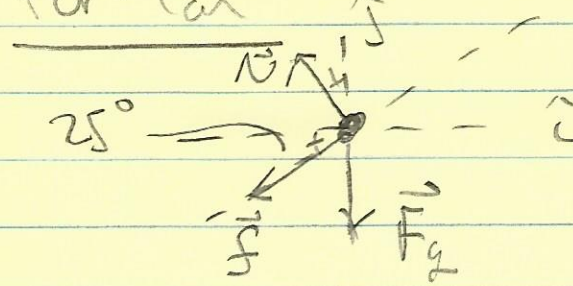
$$m_c = \frac{\mu_s (m_A + m_B)}{(1 - \mu_s)}$$

5.100



[Friction prevents car from sliding up ramp:

for car



$$\vec{N} = -N\sin(25)\,\hat{i} + N\cos(25)\,\hat{j}$$

$$\vec{f} = -\mu_s N\cos(25)\,\hat{i} - \mu_s N\sin(25)\,\hat{j}$$

$$\vec{F}_g = -mg\,\hat{j}$$

2nd $\boxed{\hat{j}}$ $\sum F_y = \cancel{ma_y}^{\,0}$

$$N\cos(25) - \mu_s N\sin(25) - mg = 0$$

$$\therefore N = \frac{mg}{\cos(25) - \mu_s \sin(25)}$$

$\boxed{\hat{i}}$ $\sum F_x = ma_x \leftarrow \left(-\frac{v^2}{R}\right)$

my coordinate system

$$-N\sin(25) - \mu_s N\cos(25) = -\frac{mv^2}{R}$$

$$N\left(\sin(25) + \mu_s\cos(25)\right) = \frac{mv^2}{R}$$

Substitute for N to get:

$$\frac{\cancel{M} g}{\left(\cos(25) - \mu_s \sin(25)\right)} \left(\sin(25) + \mu_s \cos(25)\right) = \cancel{M} \frac{v^2}{R}$$

$$\therefore \; v = \sqrt{\frac{Rg\left(\sin(25) + \mu_s \cos(25)\right)}{\left(\cos(25) - \mu_s \sin(25)\right)}} = 20.8 \, m/s$$

This is a maximum speed (because of my choice for direction of friction).

To get minimum speed, go back to free body diagram and chose friction to be in the opposite direction