HW due 1/24 Selected Solutions

2.7) Given: $X(t) = 2.4t^2 - 0.12t^3$ $\boxed{1}$

and @ $t=0$, $X=0$, $v=0$.

$$V(t) = \frac{dx}{dt} = 4.8t - 0.36t^2 \quad \boxed{2}$$

$$a(t) = \frac{dv}{dt} = 4.8 - 0.72t \quad \boxed{3}$$

a) Find average velocity for time interval $t=0$ to $t=10$ seconds.

$$V_{AVG} = \frac{\Delta X}{\Delta t} = \frac{X(t=10) - X(t=0)}{10-0} = \frac{120}{10} = 12 \text{ m/s}$$

b) Instantaneous velocities

$$V(t=0) = 4.8(0) - 0.36(0)^2 = 0$$

$V|_{t=0}$

$$V|_{t=5} = 4.8(5) - 0.36(5)^2 = 15 \text{ m/s}$$

$$V|_{t=10} = 4.8(10) - 0.36(10)^2 = 12 \text{ m/s}$$

c) How long after starting from rest is car @ rest again?

Car is @ rest when $v(t) = 0$. Find the roots of eqtn. $\boxed{2}$ ☺

$\boxed{2} \Rightarrow v(t) = 4.8t - 0.36t^2 = 0$

$$\therefore t = \frac{4.8}{0.36} = \underline{13.33 \text{ seconds}}$$

2.2) Ball leaves hand @ 45 m/s. It is accelerated at a constant rate over a distance of 1.5 meters. Find the acceleration and the time over which it acted.

Two ways: $v(t) = \int a\,dt = at + \text{Constant}$

but starts from rest, so $v(t=0) \equiv 0$
Making this constant 0.

That leaves us with $v(t) = at$ $\boxed{1}$

$$x(t) = \int v(t)\,dt = \frac{at^2}{2} + \text{const.}$$

but starts @ $x = 0$, so $x(t=0) = 0$
Making this constant 0

That leaves us with $x(t) = \frac{at^2}{2}$ $\boxed{2}$

"leaves hand @ 45 m/s" after a time I will call t_{pitch}.

$$\boxed{1} \Rightarrow \quad V_{pitch} = a t_{pitch}$$

$$\frac{45}{a} = t_{pitch}$$

"over a distance of 1.5 meters" which I will call x_{pitch}

$$\boxed{2} \Rightarrow \quad x_{pitch} = \frac{a t_{pitch}^2}{2}$$

$$1.5 = \frac{a t_{pitch}^2}{2}$$

$$3 = a t_{pitch}^2$$

|| Substitute from above for t_{pitch}

$$3 = a \left(\frac{45}{a}\right)^2$$

$$3a = 2025$$

$$a = \underline{675 \text{ m/s}}$$

Answer.

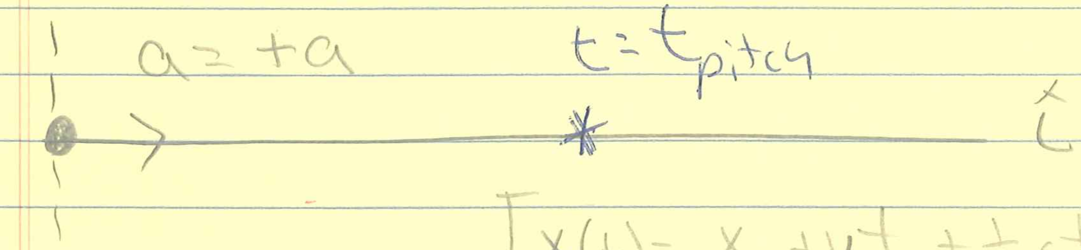
Using this result in our expression for t_{pitch} gives us:

$$t_{pitch} = \frac{45}{a} = \underline{0.067 \text{ Seconds}}$$

Answer.

THE OTHER WAY

Uses the process for solving problems w/ constant acceleration.



$X_0 = 0$
$t = 0$
$V_0 = 0$

$$\left[\begin{aligned} X(t) &= X_0 + V_0 t + \tfrac{1}{2} a t^2 \\ V(t) &= V_0 + a t \end{aligned}\right] \text{ General}$$

$$\left[\begin{aligned} X(t) &= \tfrac{1}{2} a t^2 \quad \boxed{1} \\ V(t) &= a t \quad \boxed{2} \end{aligned}\right] \text{ Specific to this Problem}$$

@ $t = t_{pitch}$, $X_{pitch} = 1.5$, $V_{pitch} = 45$

$\boxed{2} \Rightarrow \quad 45 = a t_{pitch}$

$\therefore \dfrac{45}{a} = t_{pitch}$

$\boxed{1} \Rightarrow \quad 1.5 = \tfrac{1}{2} a t_{pitch}^2$

$\therefore$ Two eqns., two unknowns.

Solved as before

2.51] Note that this is a 1^{-d} problem w/ "up" defined as $+\hat{j}$.

a.) For the 1^{st} 10 seconds

$$a = 2t \;*$$

So that:

$$v(t) = \int a\,dt = t^2 + \cancel{const.} \rightarrow 0\text{, starts from rest}$$

$$y(t) = \int v(t)\,dt = \frac{t^3}{3} + \cancel{const.} \rightarrow 0\text{, starts from origin.}$$

@ t = 10 seconds, we have

$$y(t=10) = \frac{10^3}{3} = \underline{\underline{333.3}} \text{ meters}$$

$$v(t=10) = 10^2 = \underline{\underline{100 \text{ m/s}}}$$

Answers

b.) Let t_1 be the specific time when the rocket is 325m above earth.

$$y(t=t_1) = 325 = \frac{t_1^3}{3}$$

$$\therefore t_1 = 9.92 \text{ sec}$$

and $v(t=t_1) = t_1^2 = \underline{\underline{98.4 \text{ m/s}}}$

Answer.

* NOTE that my acceleration is 2t. The book used 2.8t. Mastering may have given you a different #.