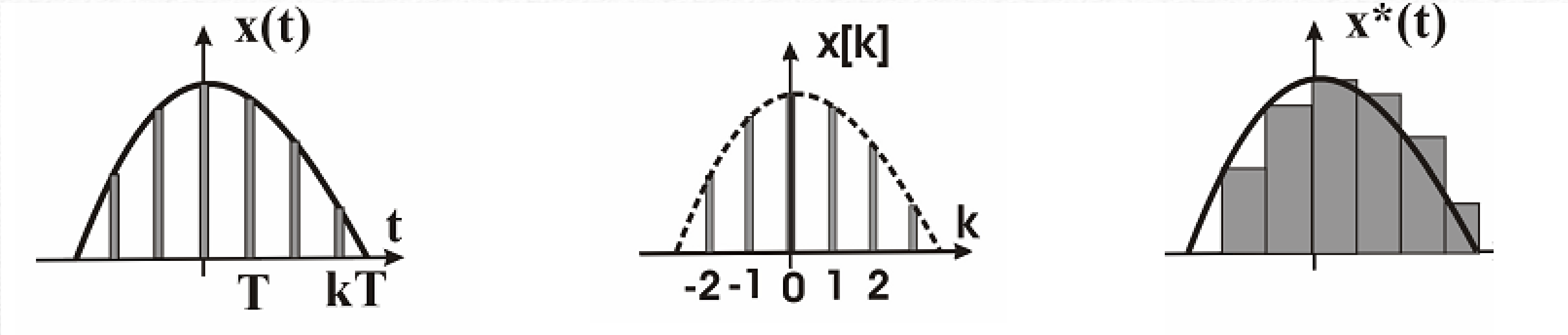
Measurement and Digital signal Processing

Sampling theory, digital signals



T_s-sampling time

f_s-sampling frequency

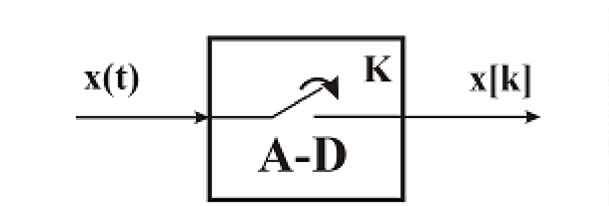
f_{max}-max frequency of the signal

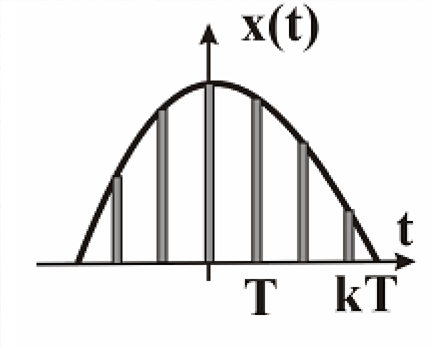
T_p-period time

n- number of the samples per one period

$f_s = 1/T_s$
$n = T_p/T_s$
$n = f_s/f_{signal}$

Sampling theory





K is closed if: kT<t<kT+dt

K is open if: kT+dt<t<(k+1)T

Sampling theory

- Example 1

T_p = 20ms; T_s=10ns. f_s=?

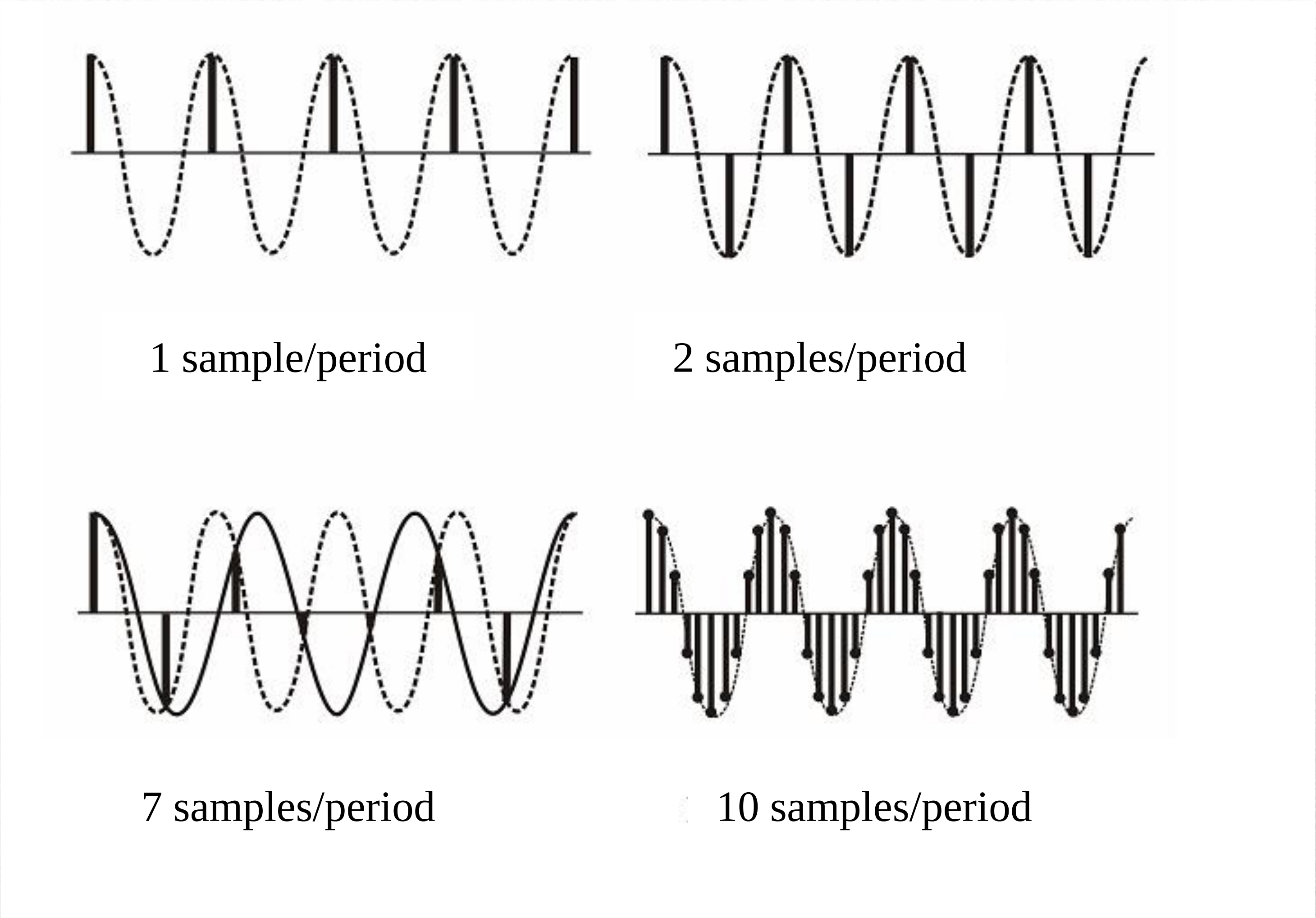
f_s=1/ T_s =10^8Hz = 100 MHz

- Example 2

T_p = 20ms; f_s=100 kHz; n=?

n= T_p / T_s= T_p f_s =2000 samples

Periodic signals



Sampling theory

- DC value

$$x_a^e = \frac{1}{T_p}\int_0^{T_p} x(t)\,dt, \quad x_d^e = \frac{1}{n_{s/p}}\sum_{k=0}^{n_{s/p}-1} x[k], \quad \left|\frac{x_a^e - x_d^e}{x_a^e}\right| \le \varepsilon$$

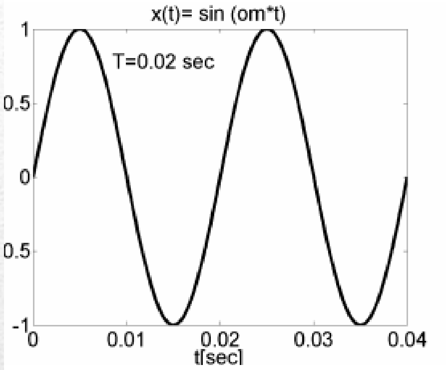
- Absolute Value

$$x_a^{abs} = \frac{1}{T_p}\int_0^{T_p} |x(t)|\,dt, \quad x_d^{abs} = \frac{1}{n_{s/p}}\sum_{k=0}^{n_{s/p}-1} |x[k]|, \quad \left|\frac{x_a^{abs} - x_d^{abs}}{x_a^{abs}}\right| \le \varepsilon$$

- RMS Value

$$x_a^{eff} = \sqrt{\frac{1}{T_p}\int_0^{T_p} x(t)^2\,dt}, \quad x_d^{eff} = \sqrt{\frac{1}{n_{s/p}}\sum_{k=0}^{n_{s/p}-1} x[k]^2}, \quad \left|\frac{x_a^{eff} - x_d^{eff}}{x_a^{eff}}\right| \le \varepsilon$$

MEAN



$$x_a(t) = \hat{X}\sin\omega t, \quad \omega = \frac{2\pi}{T_p}$$

$$x_a^e = \frac{1}{T_p}\int_0^{T_p} x(t)\,dt = \frac{1}{T_p}\int_0^{T_p} \hat{X}\sin\omega t\,dt = 0$$

$$x_a^{abs} = \frac{1}{T_p}\int_0^{T_p} |x(t)|\,dt = \frac{1}{T_p}\int_0^{T_p} \left|\hat{X}\sin\omega t\right| dt = 4\frac{\hat{X}}{T_p}\int_0^{T_p/4} \sin\omega t\,dt$$

$$= 4\frac{\hat{X}}{T_p}\left[\frac{-\cos\omega t}{\omega}\right]_0^{T_p/4} = 4\frac{\hat{X}}{T_p}\,\frac{1-\cos\left(\frac{2\pi}{T_p}\frac{T_p}{4}\right)}{2\pi/T_p} = \frac{2}{\pi}\hat{X}$$

$$x_a^{eff} = \sqrt{\frac{1}{T_p}\int_0^{T_p} x(t)^2\,dt} = \sqrt{\frac{1}{T_p}\int_0^{T_p} \hat{X}^2 \underbrace{\sin^2\omega t}_{\frac{1-\cos 2\omega t}{2}}\,dt} = \sqrt{\frac{1}{T_p}\hat{X}^2\frac{T_p}{2}} = \frac{\hat{X}}{\sqrt{2}}$$

example