L2/1

MEASUREMENT

SYSTEMATIC CHARACTERISTICS

a/ RANGE : input $I_{min} \dots I_{max}$

output $O_{min} \dots O_{max}$

EXAMPLE 1 $0 \dots 10^4$ Pa

$4 \dots 20$ mA

EX. 2 $100 \dots 250$ °C

$4 \dots 10$ mV

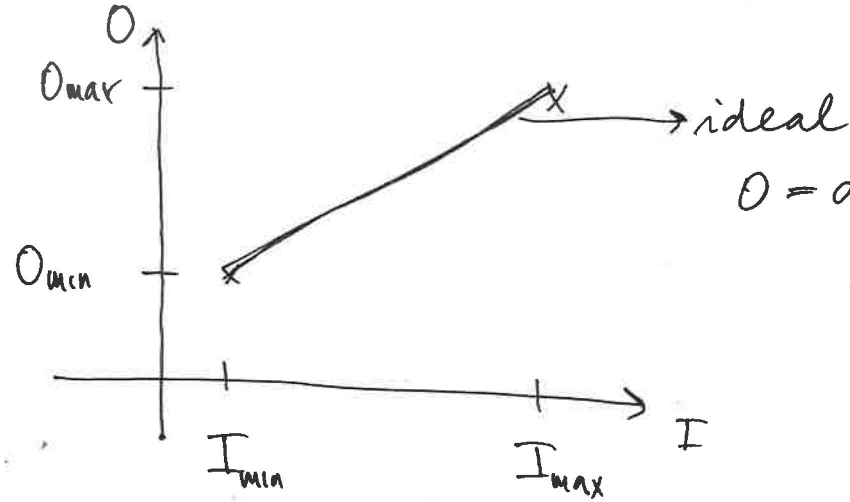
b/ SPAN: input $I_{max} - I_{min} = \Delta I$

output $O_{max} - O_{min} = \Delta O$

$\Delta I \rightarrow 10^4$ Pa   150 °C

$\Delta O \rightarrow 16$ mA   6 mV

IDEAL STRAIGHT LINE



$$O = a \cdot I + b$$

$$\boxed{a = \frac{\Delta O}{\Delta I} \qquad b = O_{min} - a \cdot I_{min}}$$

THE INPUT (I) FROM THE MEASURED OUTPUT (O) CAN BE CALCULATED AS:

$$I = \frac{O - b}{a}$$

c) SENSITIVITY

$$S = \frac{\Delta O}{\Delta I}$$ (equal with the slope or gradient of linear curve)

EXAMPLE 1

$$S = \frac{16\,mA}{10^4\,Pa} = 1{,}6\,\frac{mA}{Pa}$$

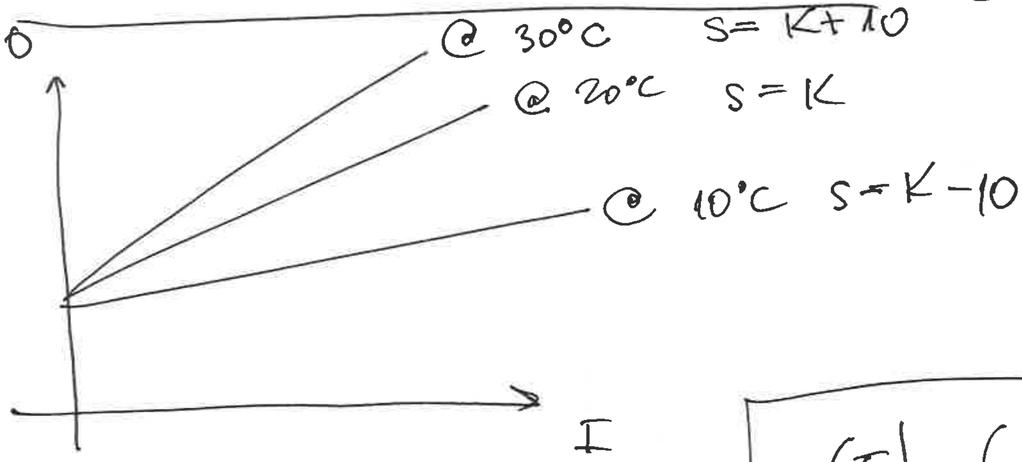
EXAMPLE 2

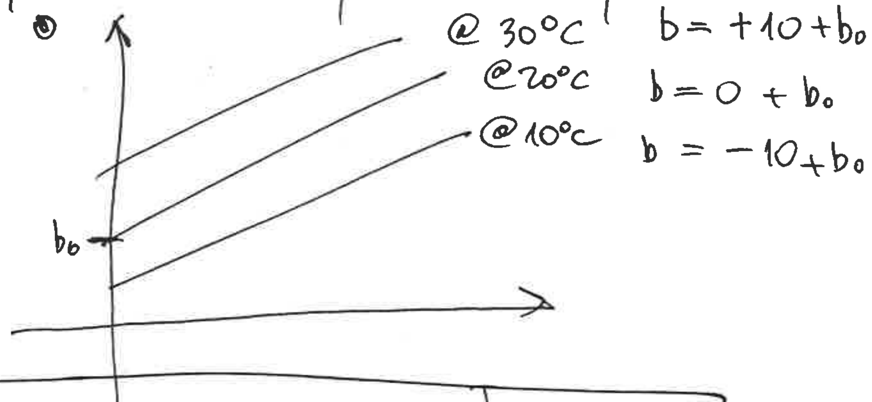
$$S = \frac{6\,mV}{150\,°C} = 0{,}04\,\frac{mV}{°C}$$

ENVIRONMENTAL EFFECTS

→ the output (O) depends not only on input (I) but on environmental inputs such as temperature, athmospheric pressure, relative humidity, supply voltage, ...etc.

STANDARD MEASUREMENT : @ 20°C, 1000 mBar, 50% H, 10 V DC





$$\boxed{O(I) = (a + a_M)I + (b + b_M) + H(I)}$$

HYSTERESIS

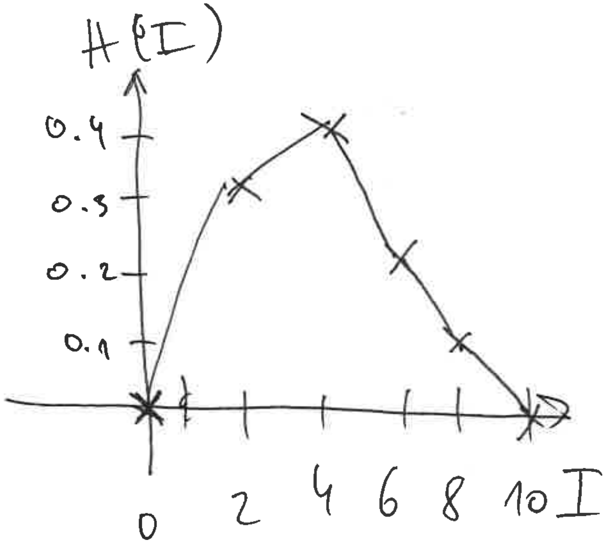
→ O may be different if I is decreasing or increasing

$$H(I) = O(I)_\downarrow - O(o)_\uparrow$$

EXAMPLE:

$I_\downarrow$	O
10	20
8	16,2
6	12,3
4	8,1
2	4,1
0	0

$I_\uparrow$	O
0	0
2	4,0
4	7,9
6	11,9
8	15,9
10	20



I	H(I)
0	20 - 20 = 0
2	16,2 - 15,9 = 0,3
4	12,3 - 11,9 = 0,4
6	8,1 - 7,9 = 0,2
8	4,1 - 4,0 = 0,1
10	0 - 0 = 0

OR

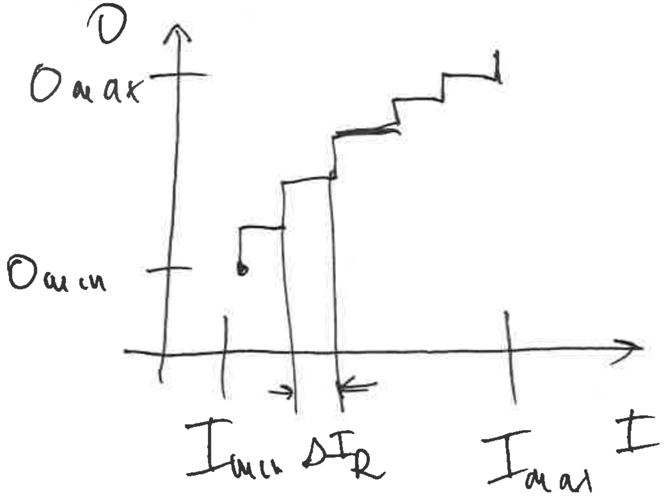
BACKLASH IN GEARS

L2/4 MEASUREMENTS

RESOLUTION

→ output increasing in discrete steps

EXAMPLES: POTENTIOMETER, ADC



$$R = \frac{\Delta I_R}{I_{max} - I_{min}} \cdot 100\,\%$$

EXAMPLE

$\Delta I_R = 5\,\Omega$

$\Delta I = 100\,\Omega$

$$R = \frac{5\,\Omega}{100\,\Omega} \cdot 100 = 5\,\%$$

SYSTEM OF UNITS

→ STANDARDIZATION: 1 FOOT = AVERAGE OF ~~12~~ FEET OF MEN

SI – SYSTEM INTERNATIONAL

→ BASE UNITS
TIME (s); LENGHT (m)
MASS (kg); CURRENT (A)
TEMP (K), SUBSTANCE AMOUNT (MOL)
LUMINOUS INTENSITY (CD)

→ DERIVED UNITS
(VOL, AREA, ACC., SPEED, ...

ERRORS

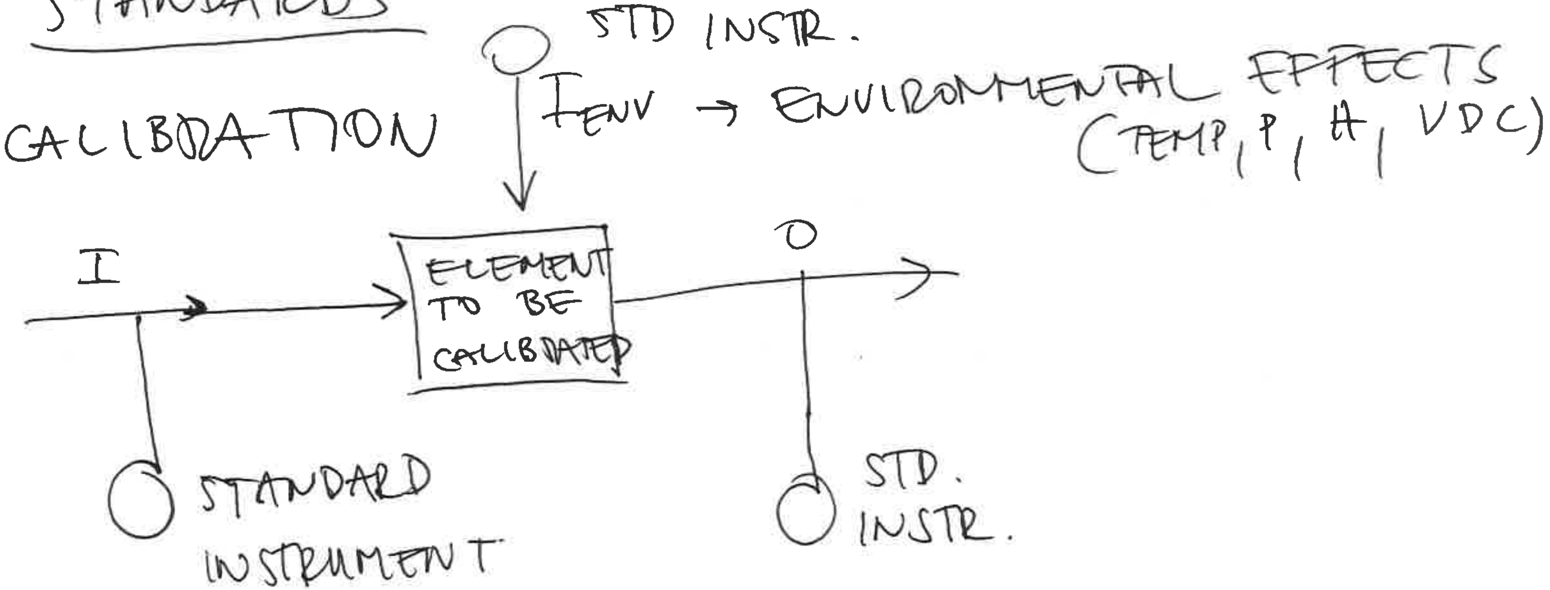
- SYSTEMATIC ERROR
- RANDOM ERROR

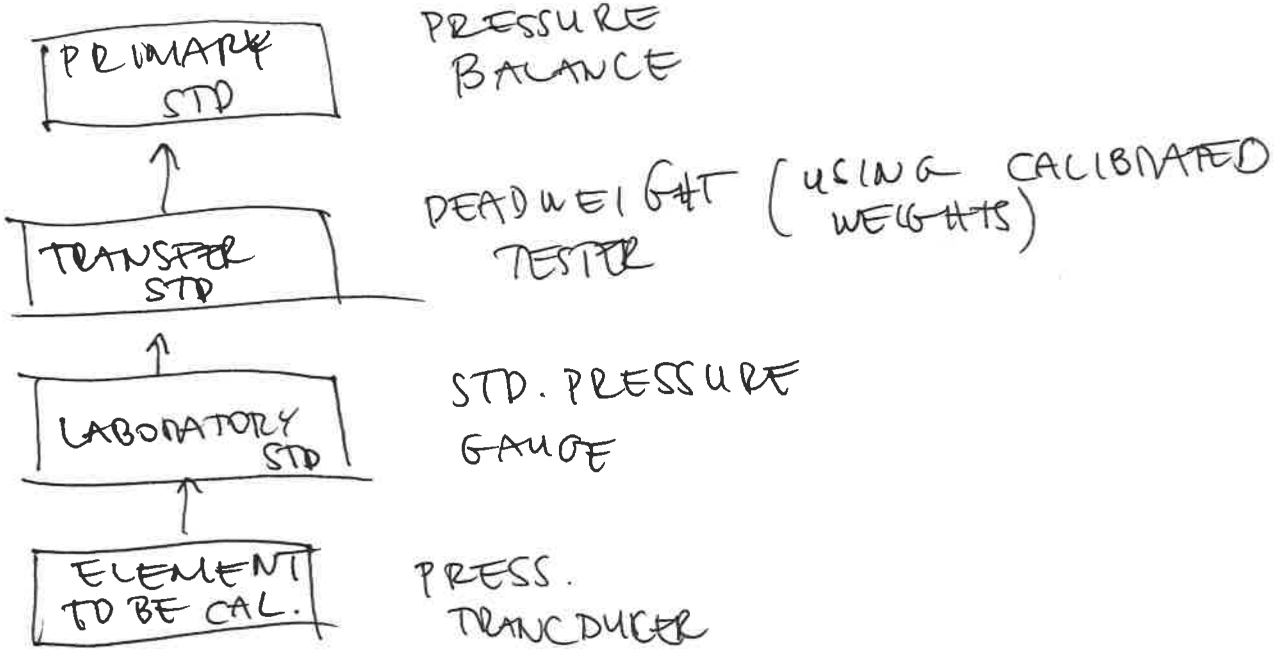
$$E_{MEAS} = E_{SYS} + E_{RAN}$$

L2 | 5 MEASUREMENT

STANDARDS

→ CALIBRATION





C 2/6 MEAS.

REPEATABILITY

→ measure a quantity (like voltage) multiple times
→ depends on the environmental effects + noise

QUESTIONS

- WHAT is the measured value, if I ~~measure~~ repeat the ~~measurement and~~ get different values?
- what is the error band? What is the error band, if I want to make a new measurement?
- what is the probability if the next measurement will be in a given range?
- is the probability different if I change the error band?

L 2/7 MEASUREMENT

→ we make a measurement with error $E = X_M - X_T$

→ we have to estimate the density function.
usually this is a normal distribution

(cumulative) distribution function:

$\underline{a}$ – random variable

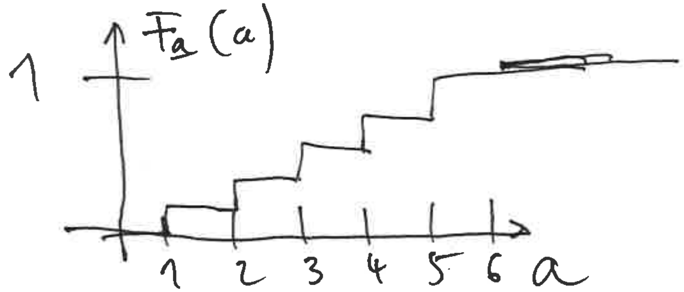
$$P(\underline{a} < a) = F_{\underline{a}}(a)$$

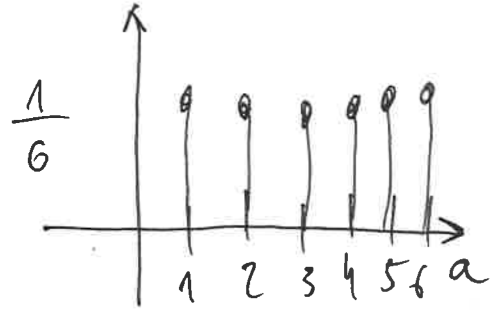
PROBABILITY DENSITY FUNCTION (PDF)

$$F_{\underline{a}}(a) = \int_{-\infty}^{a} f_{\underline{a}}(u)\, du$$

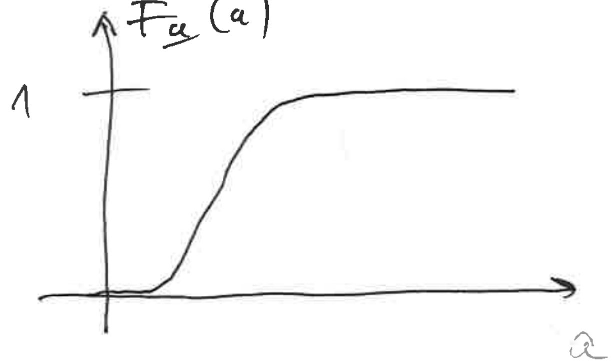
$$f_{\underline{a}}(a) = \frac{d}{da} F_{\underline{a}}(a)$$

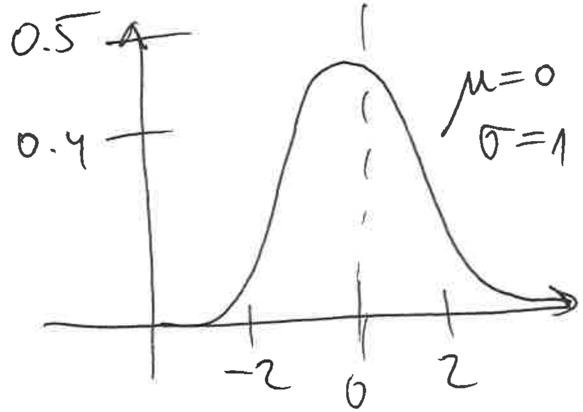
THROWING A DICE





NORMAL DISTRIBUTION





$$f_{\underline{a}}(a) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(a-\mu)^2}{2\sigma^2}\right)}$$

L2/8 MEASUREMENT

normal distribution μ – expected value (~~MEAN~~)
σ – standard deviation

if number of measurements (n) $n \to \infty$ then

$$E(x) = \mu \longrightarrow \overline{X}$$

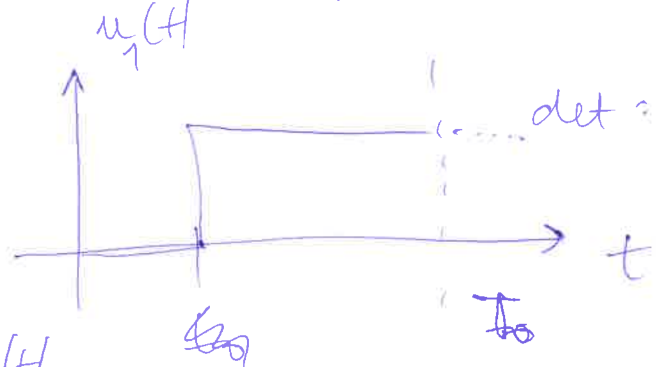
$$\overline{X} = \frac{1}{n} \sum_{k=1}^{n} X_k \longrightarrow \text{MEAN}$$

$$X_{MED} = X_{(n+1)/2} \longrightarrow \text{MEDIAN}$$

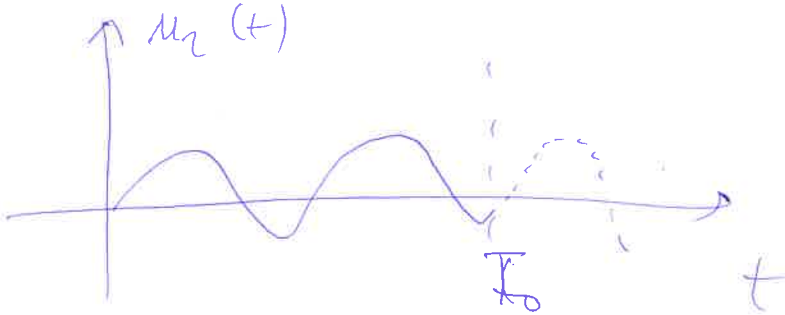
3/1 MEASUREMENT

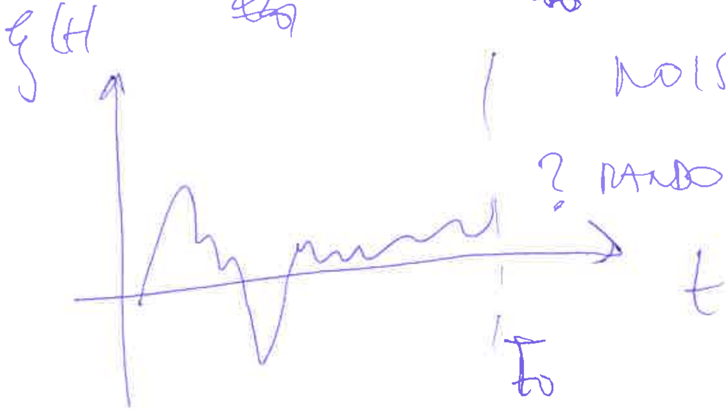
SIGNALS – deterministic : the values can be determined exactly : sine

– stochastic : –"– not : noise



det : $\varepsilon(t)$ step





NOISE

? RANDOM VALUES → STATISTICALLY REPRESENTATED

→ continuous, discrete $y(t)$ $y[n]$

3|2 MEASUREMENT

MEAN

$$\bar{y} = \frac{1}{T_0}\int_0^{T_0} y(t)\,dt \qquad \bar{y}[\cancel{N}] = \frac{1}{N}\sum_{i=1}^{N} y[i]$$

MEDIAN :

$$y_{MED} = y_{(n+1)/2} \qquad (y \text{ SHORTED}, \; y_i \le y_{i+1})$$

EXAMPLE 1

RESISTANCE : $R = [100{,}1 \quad 99{,}6 \quad 100{,}3 \quad 100{,}0 \quad 99{,}8 \quad 100{,}5 \quad 100{,}1 \quad 100{,}0 \quad 100{,}1 \quad 99{,}7]$

$$\bar{R} = 100{,}2$$

$$R_{MED} = R_{(10+1)/2} = (l_6 + l_5)/2 = 100{,}5$$

QUESTION: interval for error:

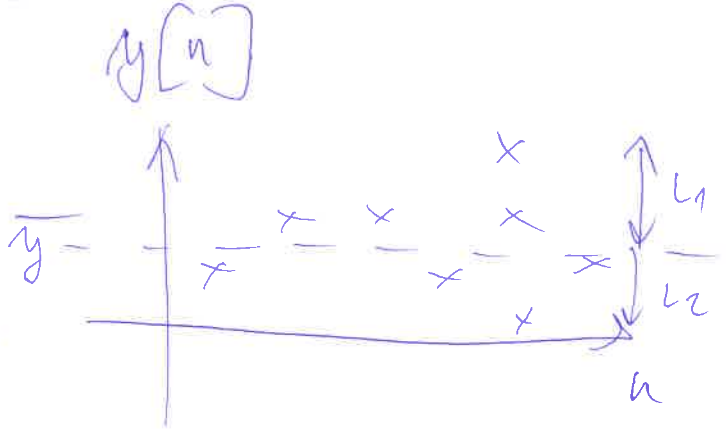
$$\boxed{y = \bar{y} \begin{array}{l} +L_1 \\ -L_2 \end{array}}$$

L3/3 MEASUREMENT

interval 1: RANGE y_{min} ; y_{max}

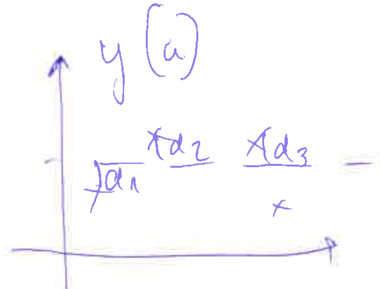
$$L_1 = y_{max} - \bar{y}$$

$$L_2 = \bar{y} - y_{min}$$



interval 2: average of errors:

$$d_i = y_k - \bar{y} \qquad \delta = \frac{1}{N}\sum_{i=1}^{N} |d_k|$$



$$\frac{|d_1| + \dots |d_N|}{N}$$

$$y = \bar{y} \pm \delta$$

interval 3 deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} d_i^2} \qquad \sigma^2 = \cdot$$

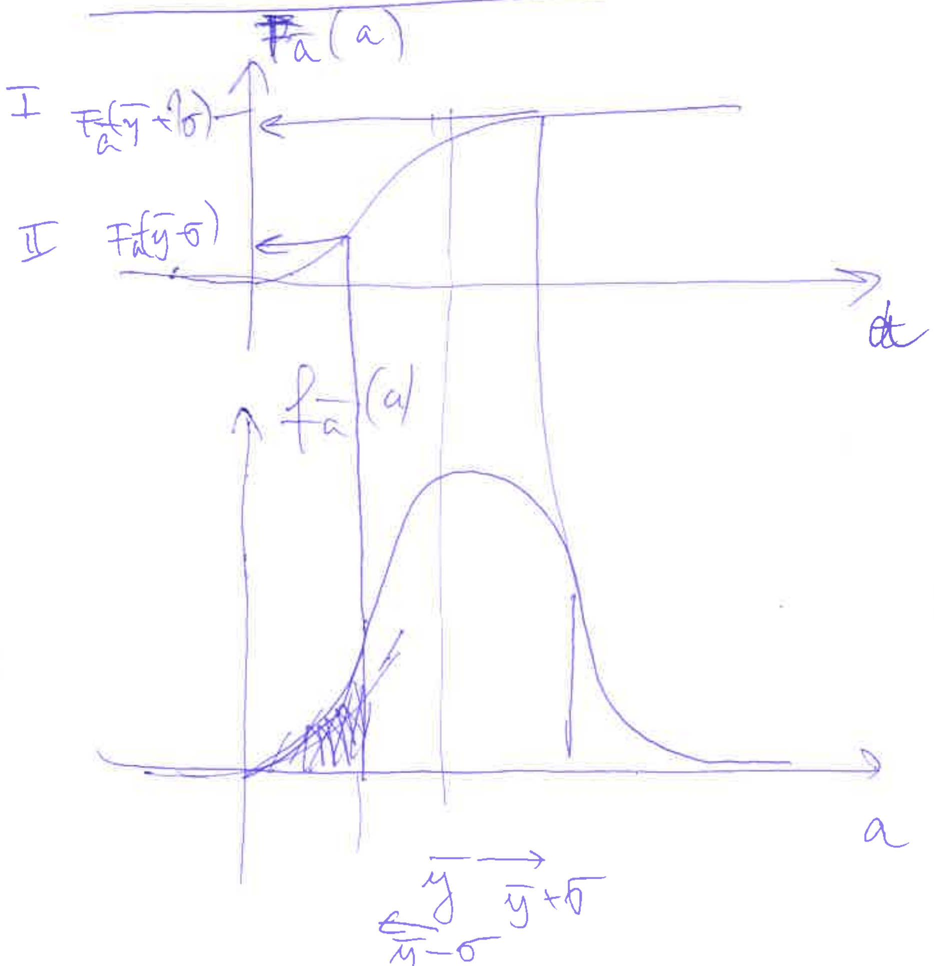
$$y = \bar{y} \pm \sigma$$

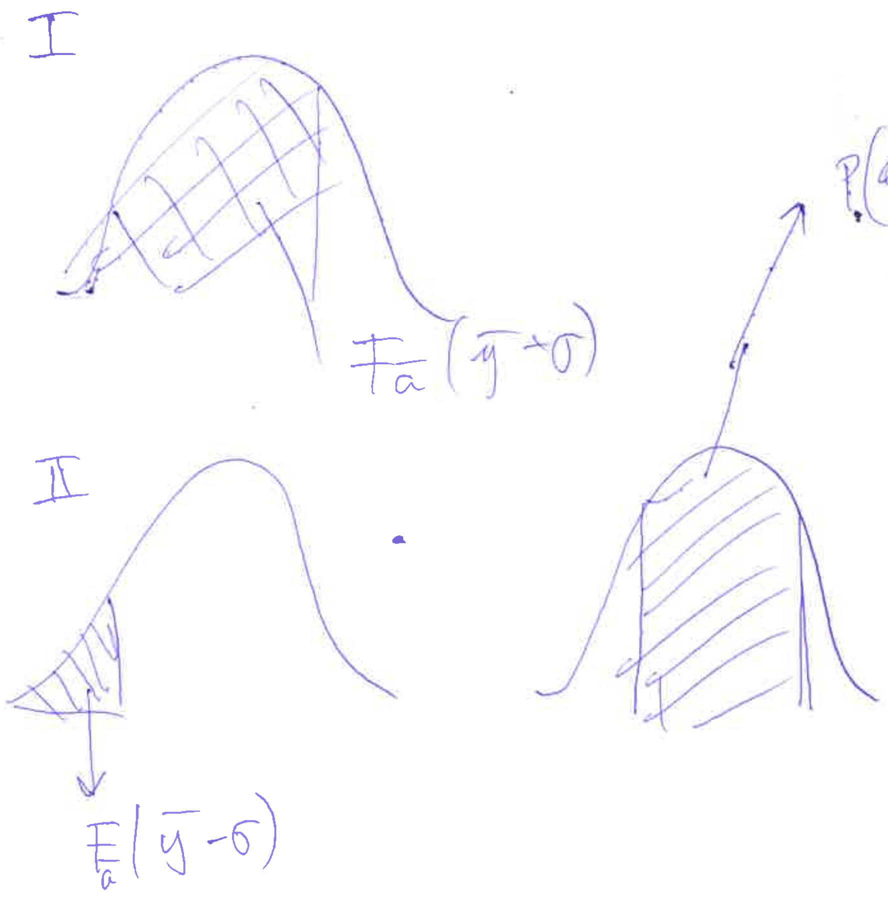
MEASUREMENT 3/4

interval 4 : probability interval : SHORT | drop upper + lower 25%

→ average ± interval

PROBABILITY?





$P(\bar{y}-\sigma \le a \le \bar{y}+\sigma)$

$= \int_{\bar{y}-\sigma}^{\bar{y}+\sigma} f_a(a)\,da$

$= F_a(\bar{y}+\sigma) - F_a(\bar{y}-\sigma)$

3/5 MEASUREMENT

TASK : —— Resistance | question : temperature dependency

→ production, manufactured | 10 pieces

1.) check the measurement | different methods
(pure wire | silver ending, tin)

$R_{W,1}$, ... $R_{W,5}$ → $\bar{R} \pm \frac{L_1}{L_2}$ P_W $(\bar{y} \pm \sigma)$

$R_{S,1}$, ... $R_{S,5}$ → $\bar{R} \pm \frac{L_1}{L_2}$ P_S $(\bar{y} \pm \sigma)$

$R_{T,1}$, ... , $R_{T,5}$ → $\bar{R} \pm \frac{L_1}{L_2}$ P_T $(\bar{y} \pm \sigma)$

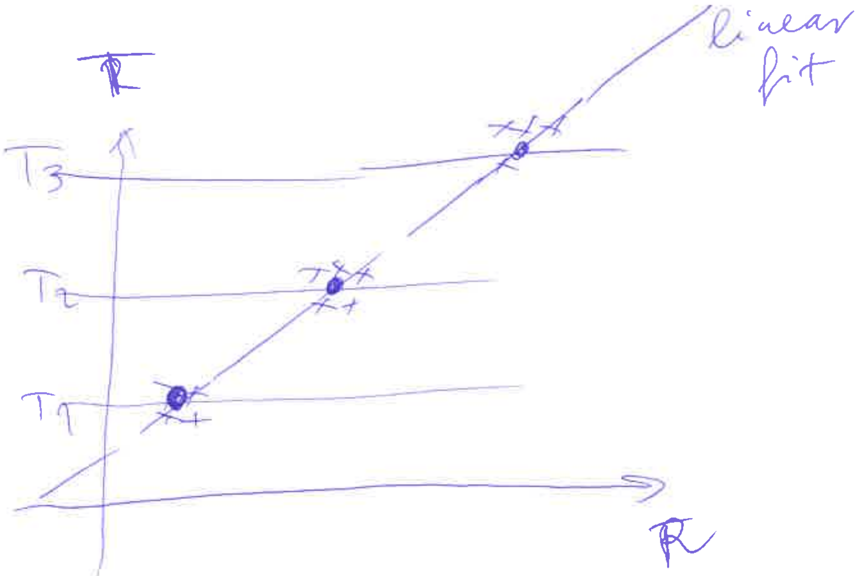
→ select the best (R_B)

temp. dependency

R_{B,T_1} 5 meas. check σ

R_{B,T_2} 5 meas check σ

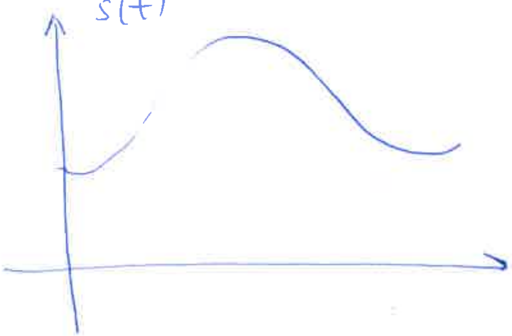
R_{B,T_3} 5 meas check σ

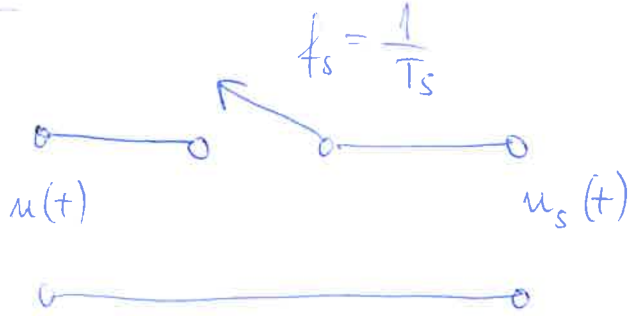


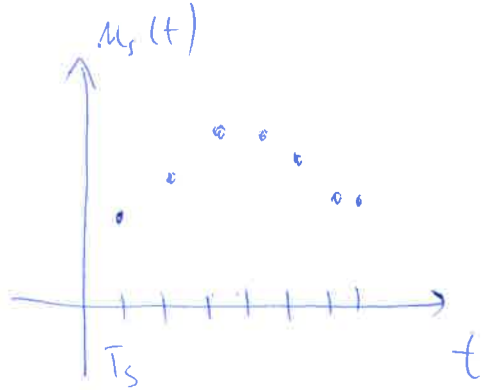
5/1 MEASUREMENT   ADC

1. SAMPLING THEORY

A)





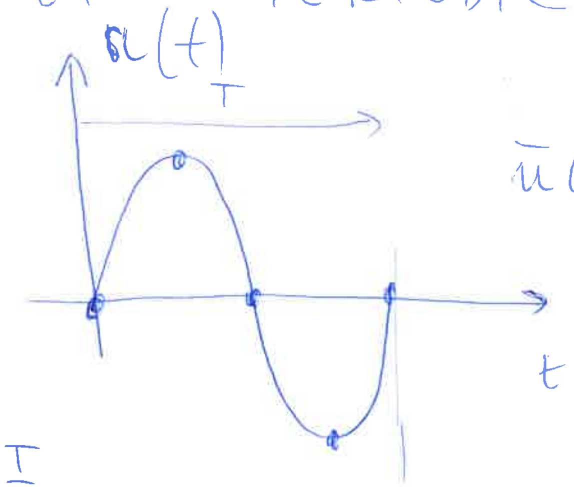


Sk: $u(kT_s) = u[k] = u_s(t)$

Shannon: $f_s > 2 \cdot f_{max}$ (CD: $f_s = 44100$ Hz, $f_{max} = 44100/2 = 22050$ Hz

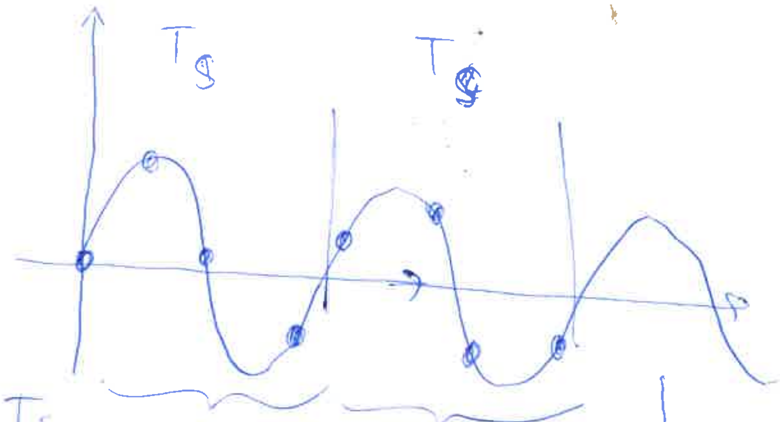
B) SAMPLING OF PERIODIC SIGNALS

problem



$\bar{u}(t) = 0$

$T_s = \frac{T}{4}$



$f_s = \frac{T_s}{3.8}$ $\quad \bar{u}_1(t) \neq \bar{u}_2(t)$

4/2 MEASUREMENT

Solution: $T = T_S \cdot n \longrightarrow n = \frac{T}{T_S} = \frac{f_s}{s}$

Ex 1: $T = 20ms \quad T_s = 10ns$

$f_s = ?$

$f_s = \frac{1}{T_s} = \frac{1}{10 \cdot 10^{-9} s} = 10^8 \frac{1}{s} = 1 MHz$

Ex 2: $T = 20ms \quad f_s = 100 kHz \quad n = ?$

$n = \frac{T}{T_s} = T \cdot f_s = 20 \cdot 10^{-3} \cdot 10^{2} \cdot 10^{3}$

$= 20 \cdot 10^{2} = \underline{\underline{2000}}$

Ex 3: $T = 6ms \quad T_s = 30ns \quad n = ?$

$n = \frac{T}{T_s} = \frac{6 \cdot 10^{-3}}{30 \cdot 10^{-9}} = 2 \cdot 10^{5} = 200000$

4|3 MEASUREMENT

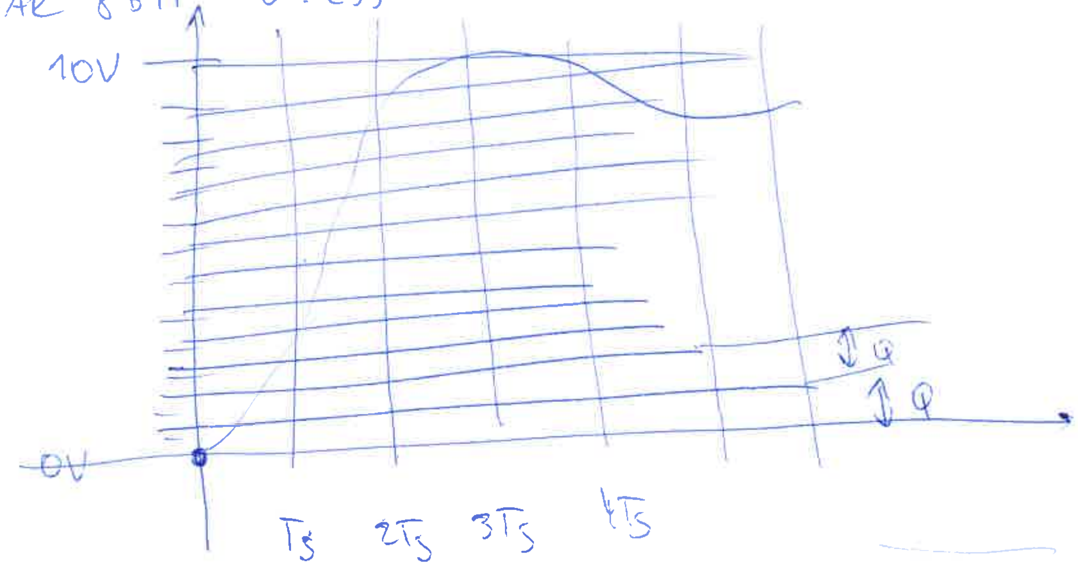
QUANTIZATION

UNIPOLAR $[0 .. U_{max}]$

BIPOLAR $[U_{min} .. U_{max}]$ SYMMETRIC TO Ø

UNIPOLAR 8BIT 0..255

B/W	DEC
11111111	255
10000000	128
	2
00000001	1
00000000	0



LSB = 00000001 $\quad U_{LSB} = ?$

MSB = 10000000 $\quad U_{MSB} = ?$

FS = 11111111 $\quad U_{FS} = ?$

$$Q = \frac{U_{max} - U_{min}}{2^k}$$

$$\rightarrow Q_8 = \frac{10V - 0V}{2^8} = \frac{10}{256} \sim 0{,}04V = 40mV$$

$$Q_{16} = \frac{10V - 0V}{2^{16}} = \frac{10}{65536} = 0{,}00015V = = 0{,}15mV$$

Quantization error $\quad H_Q = \frac{\pm Q}{2}$

Quantization relative error (U_n known) $\rightarrow h_Q = \frac{H_Q}{U_n} \cdot 100\,(\%)$

EXAMPLE

4/4 MEASUREMENT

EXAMPLE

ADC : 12 bits, unipolar : 0 – 10V

$u_{FS} = 10V$

$u_{LSB} = 10/2^{12} = 2{,}44\,mV$

$u_{MSB} = \frac{10}{2} = 5V$

$H_q = \pm\frac{2{,}44\,mV}{2} = \pm 1{,}22\,mV$

$h_q = ?$ $u_m = 8V$ $h_q = \frac{\pm 1.22\cdot 10^{-3}}{8V}\cdot 100 = \pm 0{,}015\,\%$

$h_q = ?$ u_{FS} $h_q = \frac{\pm 1.22\cdot 10^{-3}}{10V}\cdot 100 = \pm 0{,}0122\,\%$

$h_q = ?$ $u_m = 50\,mV$ $h_q = \frac{\pm 1{,}22\cdot 10^{-3}}{0{,}05V}\cdot 100 = \pm 2{,}44\,\%$

ADC : 16 bits

$H_Q = u_{LSB} = \left(\frac{10}{65535}\right)/2 = \pm 76\,\mu V$

$h_q = ?$ u_{FS} $h_q = \frac{\pm \cancel{76\cdot 10^{-2}}\ 76\cdot 10^{-6}}{10V}\cdot 100 = \pm 7{,}6\cdot 10^{-4}\,\%$ (!)

$h_q = ?$ $u_m = 50\,mV$ $h_q = \pm 0{,}152\,\%$

4/5 MEASUREMENT

SIGNAL TO NOISE RATIO (SNR)

$$SNR = \frac{U_{SIGNAL}}{U_{NOISE}} \qquad SNR\,(dB) = 20\,lg\,\frac{U_{SIG}}{U_{NOISE}}$$

if $A_Q = 1{,}22\,mV$ in $10\,V$

$$\cancel{SNR = \frac{0{,}00122}{10} = 1{,}22 \cdot 10^{-4}} \qquad SNR\,[dB] =$$

MEDIUM NOISE ↑

$$SNR = \frac{10\,V}{0{,}00122} = 8196 \qquad SNR\,(dB) = 78{,}27\,dB$$

$A_Q = 76\,\mu V$ in $10\,V$

$$SNR = \frac{10\,V}{76 \cdot 10^{-6}} = 131578 \qquad SNR\,[dB] = 102\,dB$$

↓ SMALL NOISE

QUESTION: we need 65 dB SNR ~~with~~ according to quantum noise @ 10V DC
number of bits? →

$$20\,lg\,\frac{10}{\left(\frac{10}{2^k}\right):2} > 65$$

$lg(\,) > 3{,}25$

$(\,) > 1778$ $\cancel{k}$ / :10

$\frac{1}{(\,):2} > 177{,}8$

$\frac{10}{2^k}:2 > 0{,}0056234$

$\frac{10}{2^k} > 0{,}011246$

$2^k > 889$

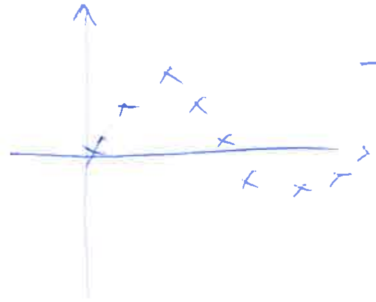
$\boxed{10\ BIT}$

4/6 MEASUREMENT

NUMBER OF DISCRETE ELEMENTS / PERIOD

QUESTION: How many sampled values are required?

→ check with different mean values



→ ∅ with 4 and 16 points!

DISCRETE

i) simple mean (DC)

$$\bar{u}_{S,D} = \frac{1}{n}\sum_{i=0}^{n-1} u[i]$$

ii) absolute mean

$$\bar{u}_{a,D} = \frac{1}{n}\sum_{i=0}^{n-1} |u[i]|$$

iii) RMS (Root Mean Square)

$$\bar{u}_{RMS,D} = \sqrt{\frac{1}{n}\sum_{i=0}^{n-1} u^2[i]}$$

GENERAL REQUIREMENT

$$\left| \frac{\bar{u}_{x,ANAL} - \bar{u}_{x,DIG}}{\bar{u}_{x,ANAL}} \right| \le \varepsilon$$

ANALOG VALUES (SINE)

$$\bar{u}_{S,A} = 0 \qquad \bar{u}_{a,A} = \frac{2}{\pi} \cdot \hat{u}$$

$$u_{RMS,A} = \frac{\hat{u}}{\sqrt{2}} = \hat{u} \cdot 0{,}71$$

(supply voltage 230 V, $\hat{u} = ?$)

4/7 MEAS.

DATA AMOUNT

EX 1 : $f_s = 10$ kHz @ 12 bit ; 100 secs ; 8 channels

1 sec (bandwidth): $10000 \cdot 1{,}5$ Byte $\cdot 8 = 120000$ Byte $\cong 120$ k Byte

100 secs $\cdot$ 12000 k Byte $\cong$ 12 M Byte

EX 2 : $f_s = 15$ MHz @ 16 bit ; 30 mins, 16 ch

1 sec (bw) : $15 \cdot 10^6 \cdot 2$ Byte $\cdot 16 = 480$ MByte /sec

~~STORIN~~ STORAGE ?

30 mins : 864000 MByte = 864 GByte = 0,864 TByte

GYAK (PRACTICE)

1. DAY HW
2. TASK → + WIRING.
3. identify CARDS → MEX
4. MEX AI – AO
5. MEX AO — LW AI
6. MEX AI — LW AO