Maximum Ratio Transmission

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***Abstract*— This paper presents the concept, principles, and analysis of maximum ratio transmission for wireless communications, where multiple antennas are used for both transmission and reception. The principles and analysis are applicable to general cases, including maximum-ratio combining. Simulation results agree with the analysis. The analysis shows that the average overall signal-to-noise ratio (SNR) is proportional to the cross correlation between channel vectors and that error probability decreases inversely with the $(L \times K)$th power of the average SNR.**

***Index Terms*—Antenna arrays, diversity methods, radio communications.**

I. Introduction

THE MOST adverse propagation effect from which wireless communications systems suffer is the multipath fading. One of the common methods used by wireless communications engineers to combat multipath fading is the antenna diversity technique. A classical combining technique is maximum-ratio combining (MRC) [1], where the signals from the received antenna elements are weighted such that the signal-to-noise ratio (SNR) of their sum is maximized. The MRC technique so far has been exclusively for receiving applications. As there are more and more emerging wireless services, more and more applications may require diversity at the transmitter or at both transmitter and receiver to combat severe fading effects. Various transmit diversity techniques have been proposed in the open literature. For example, a delay transmit diversity scheme was proposed by Wittneben [2], [3]. A variation of the delay scheme was suggested by Seshadri and Winters [4], [5], where the replicas of the signal are transmitted through multiple antennas at different times. Another example of transmit diversity is a simple but effective scheme proposed by Alamouti [6], where a pair of symbols is transmitted using two antennas at first, and the transformed version of the pair is transmitted to obtain the MRC-like diversity. However, these transmit diversity techniques were built on objectives other than to maximize the SNR. That is, they are suboptimum in terms of SNR performance.

Accordingly, the frame work of maximum ratio transmission (MRT) will be established here in terms of concept and principles. It can be considered the generalization of the maximum ratio algorithm for multiple transmitting antennas and multiple receiving antennas. It also provides a reference for the optimum performance that a system may obtain using both transmit and receive diversity. Therefore, the focus of this document is on the analysis of the MRT scheme rather than on the implementation aspects. The rest of the document is organized as follows. The system model used in the study is described in Section II. In Section III, the MRT concept is presented. Discussions are given in terms of average SNR and the order of diversity in Section IV, which is followed by some numerical examples in Section V. Conclusions are given in Section VI.

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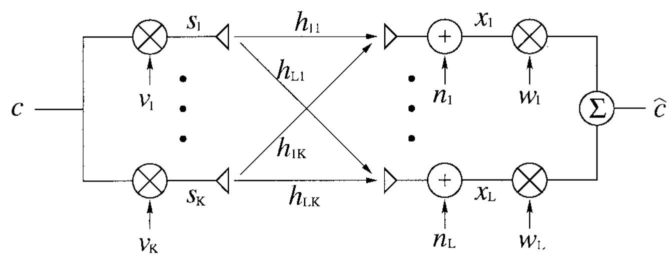

Fig. 1. The system model.

II. System Model

In this study, a system is considered, which consists of K antennas for transmission and L antennas for reception. The channel consists of $K \times L$ statistically independent coefficients, as shown in Fig. 1. It can be conveniently represented by a matrix

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & \cdots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{L1} & \cdots & h_{LK} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1 \\ \vdots \\ \boldsymbol{h}_L \end{bmatrix} \tag{1}$$

where the entry h_{pk} represents the channel coefficient for antenna k and antenna p. It is assumed that the channel coefficients are available to both the transmitter and receiver through some means. It should be pointed out that the feasibility and practicality for obtaining the channel coefficients are important implementation issues. However, these issues are out of the scope of this work; they will not be addressed here.

The system model shown in Fig. 1 is a simple baseband representation. The symbol c to be transmitted is weighted with a transmit weighting vector v to form the transmitted signal vector. The received signal vector is the product of the transmitted signal vector and the channel plus the noise; that is

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \tag{2}$$

where the transmitted signals $\boldsymbol{s}$ is given by

$$\boldsymbol{s} = [s_1 \cdots s_K]^T = c[v_1 \cdots v_K]^T. \tag{3}$$

The noise vector is expressed as

$$\boldsymbol{n} = [n_1 \cdots n_L]^T. \tag{4}$$

Noise is assumed to be white Gaussian and uncorrelated with the signals. The received signals are weighted and summed to produce the estimate of the symbol.

III. MRT

In order to generate the $K \times 1$ transmission weight vector from the channel matrix, a linear transformation is required; that is

$$\boldsymbol{v} = \frac{1}{a}(\boldsymbol{gH})^H \tag{5}$$

where $\boldsymbol{g} = [g_1 \cdots g_L]$. The transmitted signal vector is then expressed as

$$\boldsymbol{s} = \frac{c}{a}(\boldsymbol{gH})^H. \tag{6}$$

The normalization factor a is required to be

$$a = |\boldsymbol{gH}| = \left(\sum_{p=1}^{L}\sum_{q=1}^{L} g_p g_q^* \sum_{k=1}^{K} h_{pk} h_{qk}^*\right)^{1/2}. \tag{7}$$

The received signal vector is, therefore, given by

$$\boldsymbol{x} = \frac{c}{a}\boldsymbol{H}(\boldsymbol{gH})^H + \boldsymbol{n}. \tag{8}$$

To estimate the transmitted symbol, the receive weight vector $\boldsymbol{w}$ has to be applied to the received signal vector $\boldsymbol{x}$. If $\boldsymbol{w}$ is set to be $\boldsymbol{g}$, the estimate of the symbol is given by

$$\tilde{c} = \boldsymbol{gx} = \frac{c}{a}\boldsymbol{gH}(\boldsymbol{gH})^H + \boldsymbol{gn} = ac + \boldsymbol{gn} \tag{9}$$

with the overall SNR given by

$$\gamma = \frac{a^2}{\boldsymbol{gg}^H}\gamma_0 = \frac{a^2\gamma_0}{\sum_{p=1}^{L}|g_p|^2} \tag{10}$$

where $\gamma_0 = (\sigma_c^2/\sigma_n^2)$ denotes the average SNR for the case of a single transmitting antenna, (i.e., without diversity). From (10), it can be observed that the overall SNR is a function of $\boldsymbol{g}$. Thus, it is possible to maximize the SNR by choosing the appropriate values for $\boldsymbol{g}$. Since h_{qk} are assumed to be statistically identical, the condition that $|g_1| = |g_2| = \cdots = |g_L|$ has to be satisfied for the maximum value of the SNR. Without changing the nature of the problem, one can set $|g_p| = 1$ for simplicity. Therefore, the overall SNR is rewritten as

$$\gamma = \frac{a^2}{L}\gamma_0 \tag{11}$$

which is maximized if a^2 is maximized. a^2 reaches the maximum value if we set

$$(g_p g_q^*)^* = \frac{\sum_{k=1}^{K} h_{pk} h_{qk}^*}{\left|\sum_{k=1}^{K} h_{pk} h_{qk}^*\right|}. \tag{12}$$

That is

$$a^2 = \sum_{p=1}^{L}\sum_{q=1}^{L}\left|\sum_{k=1}^{K} h_{pk} h_{qk}^*\right|. \tag{13}$$

IV. Discussion

A. Average SNR

The summation term with respect to K in (13) is actually the inner product of different pairs of channel vectors; namely

$$\left|\sum_{k=1}^{K} h_{pk} h_{qk}^*\right| = \left|\boldsymbol{h}_p \boldsymbol{h}_q^H\right|. \tag{14}$$

At one extreme, if $\boldsymbol{h}_p$ and $\boldsymbol{h}_q$ are mutually orthogonal (i.e., $\boldsymbol{h}_p \boldsymbol{h}_q^H = 0$), a^2 takes on the smallest value; that is

$$a^2 = \sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2 \tag{15}$$

and

$$E[a^2] = LK\overline{r^2}. \tag{16}$$

At the other extreme, if $\boldsymbol{h}_p$ and $\boldsymbol{h}_q$ are fully correlated (i.e., $\boldsymbol{h}_p \boldsymbol{h}_q^H = |\boldsymbol{h}_q|^2$), a^2 takes on the largest value; that is

$$a^2 = \sum_{p=1}^{L}\sum_{q=1}^{L}\sum_{k=1}^{K}|h_{qk}|^2 \tag{17}$$

and

$$E[a^2] = L^2K\overline{r^2}. \tag{18}$$

Therefore, the average overall SNR is bounded by

$$K\overline{r^2}\gamma_0 \le \overline{\gamma} \le LK\overline{r^2}\gamma_0. \tag{19}$$

B. Order of Diversity

For a system consisting of $K \times L$ antennas, it is expected that the order of diversity be $K \times L$; that is, the probability of error should decrease inversely with the $(K \times L)$th power of the average SNR. To see this, one may consider the following example.

In the example, a system with binary phase-shift keying modulation is assumed. The channel coefficients are complex Gaussian and mutually, statistically independent. From the above analysis, the following inequality holds

$$a^2 \ge \sum_{p=1}^{L}\sum_{k=1}^{K}|h_{pk}|^2. \tag{20}$$

Thus, the worst error probability P is the one evaluated under the equal condition. To determine P, the probability of error conditioned on a set of channel coefficients $\{h_{pk}\}$ must be obtained first. Then, the conditional error probability is averaged over the probability density (pdf) function of $\{h_{pk}\}$. For Gaussian noise, the conditional error probability is expressed as

$$P(\gamma) = Q(\sqrt{2\gamma}). \tag{21}$$

The pdf $p(\gamma)$ can be determined via the characteristic function of γ, which turns out to be the characteristic function of a

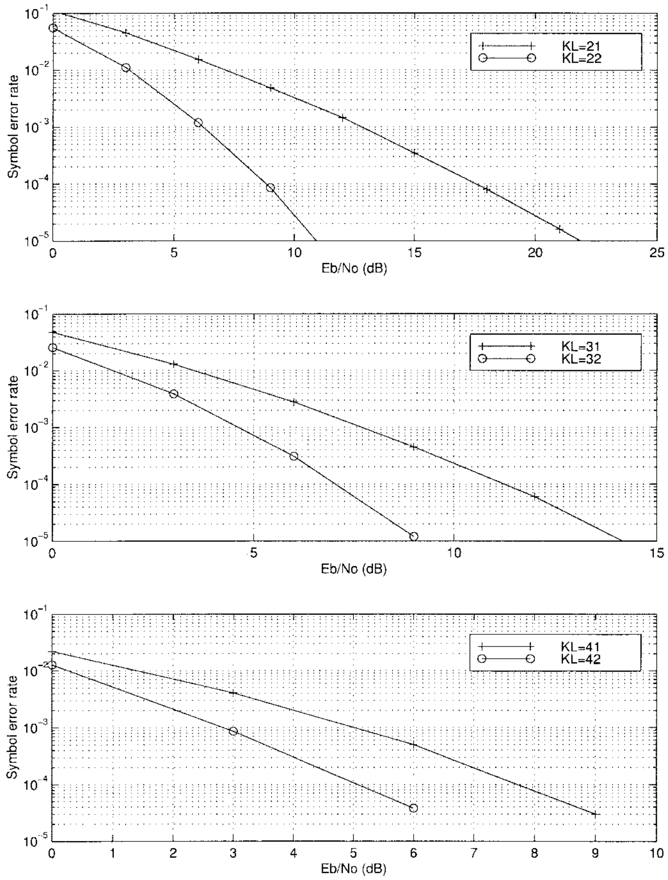

Fig. 2. The comparison of MRT performance curves (SER versus SNR) showing the effect of adding the second receiving antenna.

χ^2-distributed random variable with $2 \times L \times K$ degrees of freedom. It follows that $p(\gamma)$ is given by

$$p(\gamma) = \frac{\gamma_b^{LK-1} e^{-\gamma/\overline{\gamma}_a}}{(LK-1)! \overline{\gamma}_a^{LK}} \tag{22}$$

where

$$\overline{\gamma}_a = \gamma_0 E\left[|h_{pk}|^2\right] = \gamma_0 \overline{r^2}. \tag{23}$$

The error probability is then given by the following integral

$$P = \int_0^\infty P(\gamma) p(\gamma)\, d\gamma. \tag{24}$$

For $\overline{\gamma}_a \gg 1$

$$P \approx \left(\frac{1}{4\overline{\gamma}_a}\right)^{LK} \frac{(2LK-1)!}{(LK)!(LK-1)!} \tag{25}$$

which indicates that the error probability decreases inversely with the $(L \times K)$th power of the average SNR γ_0.

V. Numerical Results

The simulations are carried out on the discrete-event (i.e., symbol-by-symbol) basis. Furthermore, they are performed only at the level of baseband processing. That is, the effects of radio frequency and intermediate frequency components are not considered here. Without loss of generality, quantenary phase-shift keying is used in the simulation for simplicity for most cases. Each value of symbol-error rate (SER) is obtained by transmitting one million symbols. A simple channel model is used, where the fading channel coefficients are complex Gaussian, that is, a random variable with a complex Gaussian distribution. Because the objective of carrying out the simulations is to evaluate the performance, it is assumed that perfect knowledge of channel fading coefficients are available to both transmitting and receiving stations.

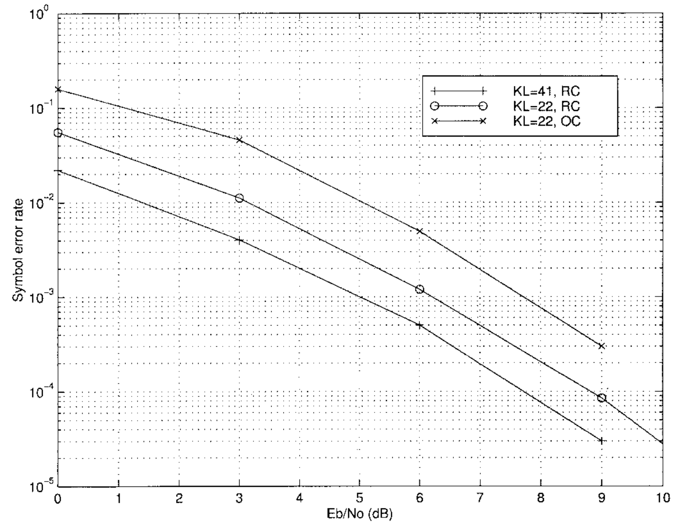

Fig. 3. The comparison of MRT performance curves (SER versus SNR) for the fourth-order diversity.

The SER performance curves plotted in Fig. 2 show the results using two receiving antennas. Two characteristics that are particularly associated with diversity can be observed here.

1) The improvement becomes greater as SNR increases.
2) The incremental improvement becomes smaller as the diversity order increases.

In Fig. 3, the performance curves for different cases of the fourth-order diversity (i.e., $K \times L = 4$) are given. The results validate the observations made earlier in the previous section. Comparing the curve corresponding to $KL = 41$ with that corresponding to $KL = 22$ and mutually orthogonal channels (OC), one may observe the 3-dB difference in SNR for the same error rate. Furthermore, the performance for $KL = 22$ with random channel (RC) coefficients is somewhere in between, as predicted.

VI. Conclusion

In this paper, the concept of MRT has been presented. It has been shown how the maximum SNR can be obtained in wireless communications where multiple antennas are used for both transmission and reception. The principles and analysis are applicable to general cases, including MRC. Simulation results agree with what has been predicted in the analysis. It has been shown that the average overall SNR is proportional to the cross correlation between channel vectors. It is also observed that the average gain in SNR in the OC case will be $10 \log L$ dB less than the $K \times L$ transmitting antennas and one receiving antenna. Finally, the analysis also shows that error probability decreases inversely with the $(L \times K)$th power of the average SNR.

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