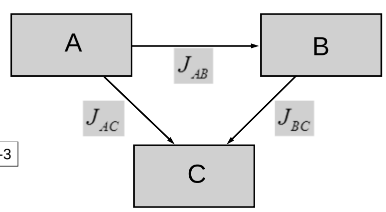
Compartment Models cont'd

Haefner
Ch 4

Model diagrams used in early stages of model development :

- Forrester Diagrams
- Simple compartments with arrows

1. Draw compartments, one per state variable
2. Draw arrows to show flows
3. Label all components
4. Mathematical formulation
   a) One DE per compartment
   b) Define flow equations (transfer equations)
5. Solve (quantities and flows *versus* time)



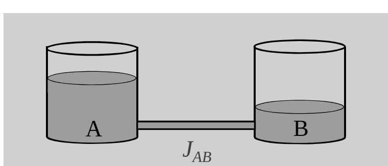
Steps 1-3

$$\frac{d}{dt} = inflows - outflows \Rightarrow \begin{aligned} \frac{dA}{dt} &= -J_{AB} - J_{AC} \\ \frac{dB}{dt} &= J_{AB} - J_{BC} \\ \frac{dC}{dt} &= J_{AC} + J_{BC} \end{aligned}$$

Step 4a

Step 4b

Flows between compartments



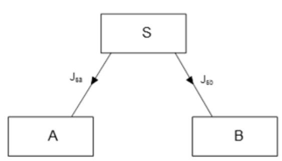
$$J_{AB} = k\left(A - B\right)$$

Step 4b – Write transfer equations

$$A \xrightarrow{J_{AB}} B$$

$J_{AB} = k$	constant flow
$J_{AB} = k_{AB} A$	donor dependent
$J_{AB} = k_{AB} B$	recipient dependent
$J_{AB} = k_{AB} (A - B)$	donor-recipient difference
$J_{AB} = k_{AB} AB$	donor-recipient product
$J_{AB} = k_{AB} A - l_{AB} A^2$	like logistic function
$J_{AB} = k_{AB} f(t)$	forcing function (time)

Model for Lab exercise 4



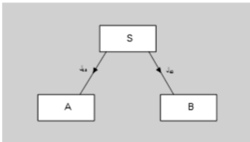
$$\frac{dS}{dt} = -J_{sa} - J_{sb}$$
$$\frac{dA}{dt} = J_{sa}$$
$$\frac{dB}{dt} = J_{sb}$$

$$J_{sa} = k_{sa}(S - A)$$
$$J_{sb} = k_{sb} S$$

Remember how we could solve the simple exponential population ODE model ?

$$\frac{dN}{dt} = rN$$
$$\frac{\Delta N}{\Delta t} \cong rN$$
$$\Delta N \cong rN\Delta t$$
$$\Delta N = N_{t+1} - N_t$$
$$N_{t+1} = N_t + \Delta N$$
$$N_{t+\Delta t} = N_t + rN_t\Delta t$$

Program for Lab 4 ?



$$\frac{dS}{dt} = -J_{sa} - J_{sb}$$
$$\frac{dA}{dt} = J_{sa}$$
$$\frac{dB}{dt} = J_{sb}$$

$$J_{sa} = k_{sa}(S - A)$$
$$J_{sb} = k_{sb}\,S$$

```
for( i=0, i<100, i++ )
      {
      Jsa = ksa * (S – A);
      Jsb = ksb * S;
      dS = (-Jsa – Jsb) * dt;
      dA = Jsa * dt;
      dB = Jsb * dt;
      S[i+1] = S[i] + dS;
      A[i+1] = A[i] + dA;
      B[i+1] = B[i] + dB;
      }
```