Model development - qualitative

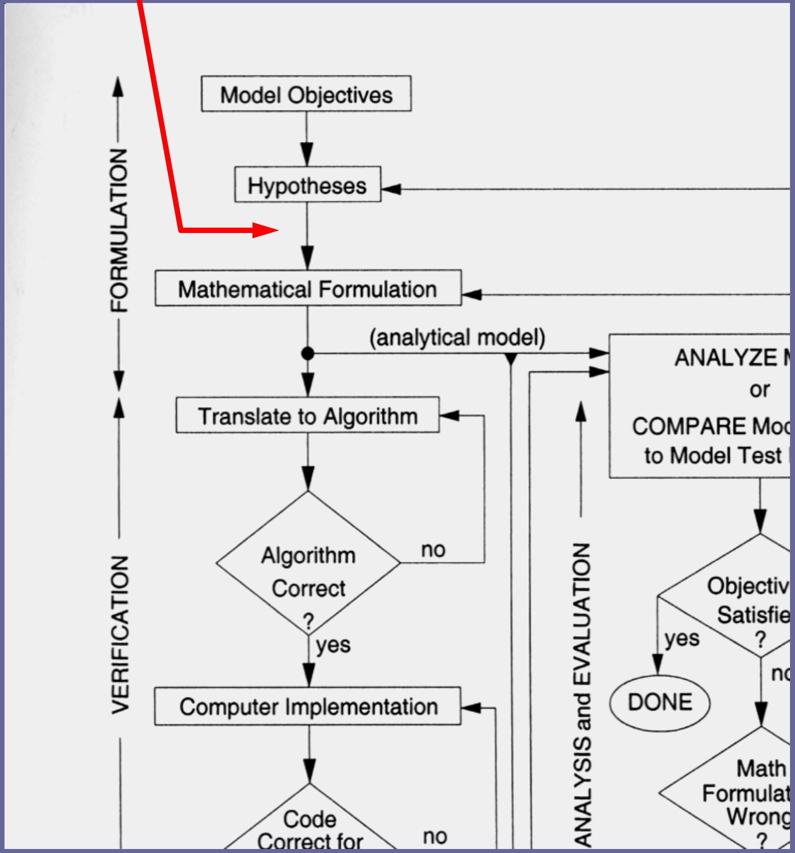
Haefner
Ch 3
pp 32-49

With a complex system, helpful to insert a conceptual phase – represent model as a diagram.

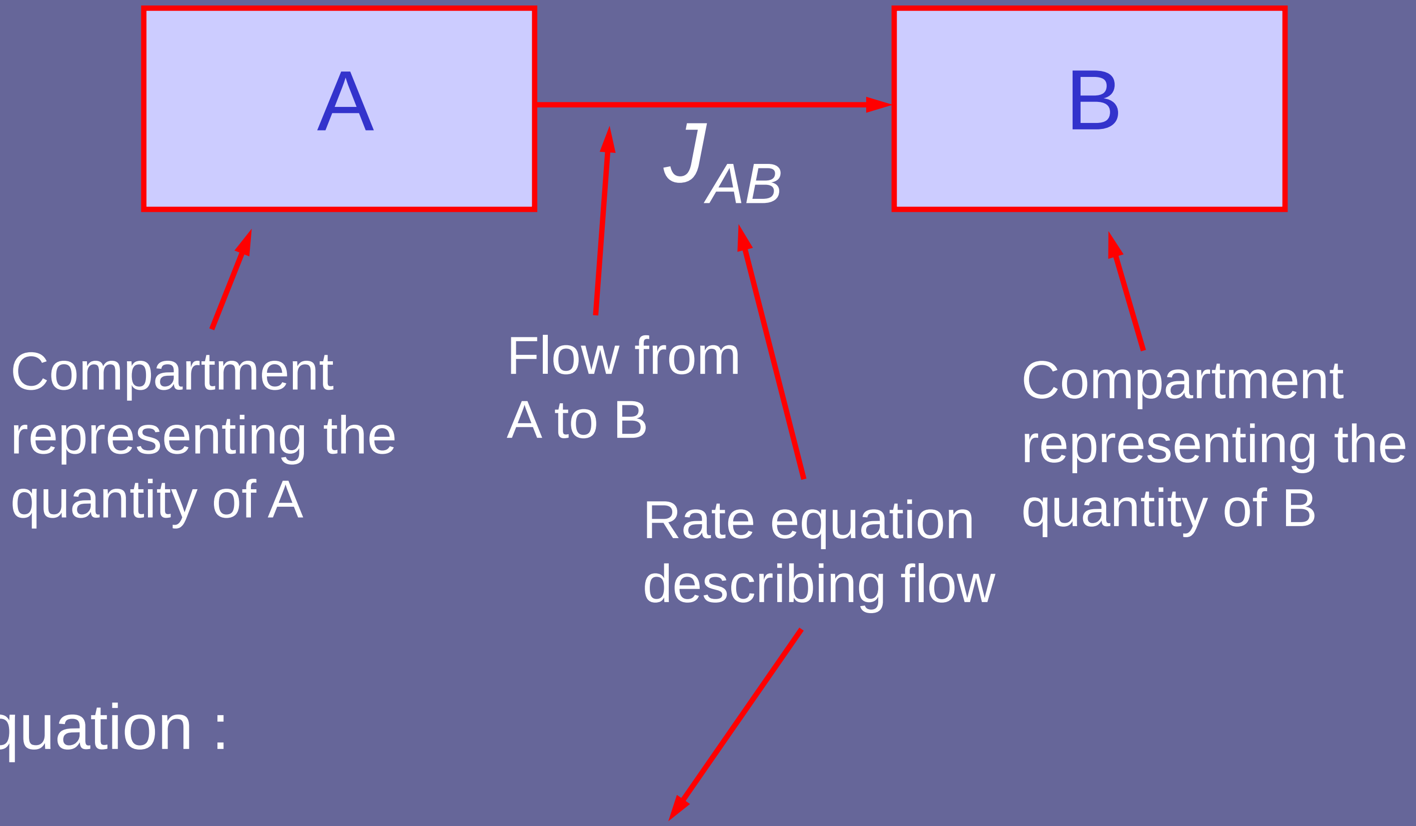
Types of diagrams:

- Block diagrams
- Energy flow diagrams
- Forrester diagrams

System is represented as a collection of objects together with their interrelationships



Block diagrams (compartment models)



Transfer equation :

$$J_{AB} = k_{AB}\left(A - B\right)$$

Forrester Diagrams

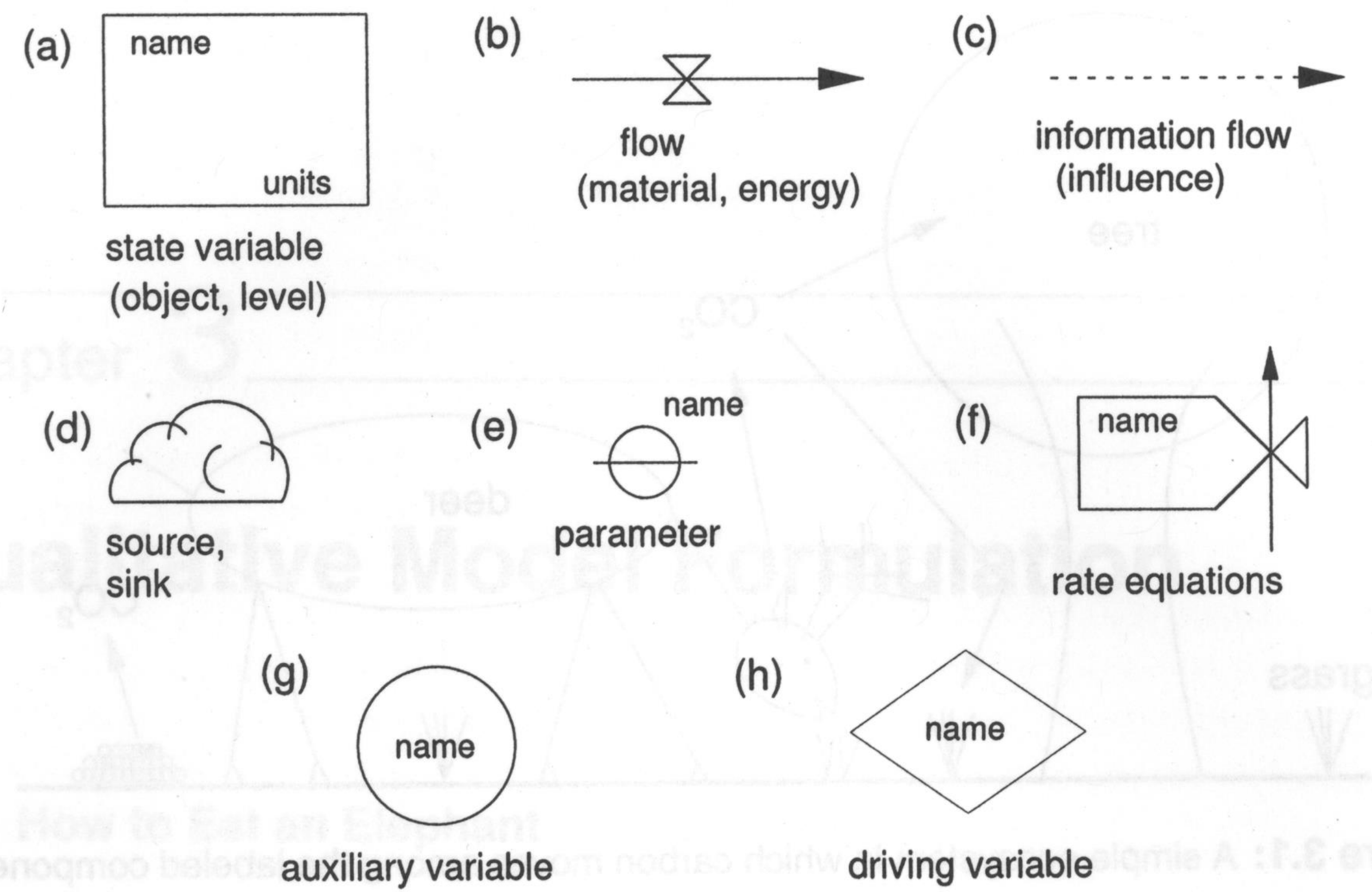
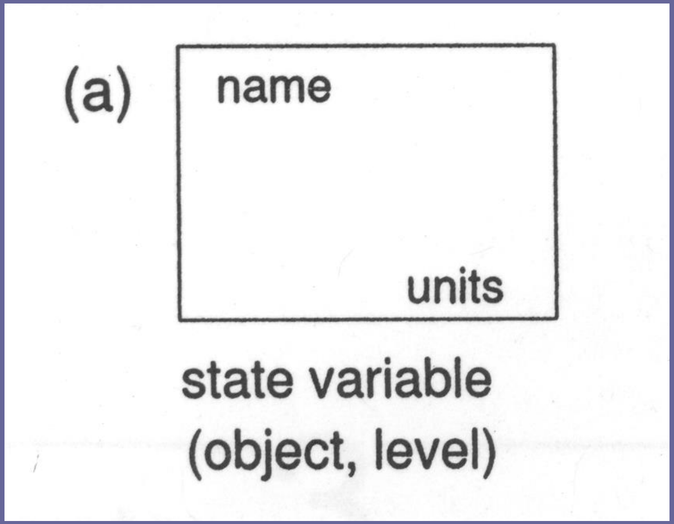
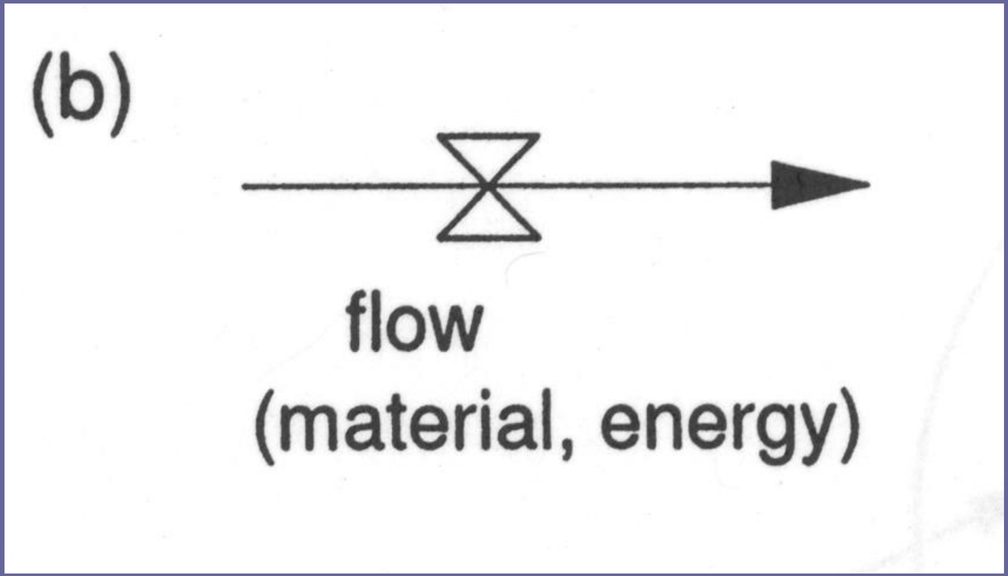


Figure 3.2: The basic components of a Forrester diagram.

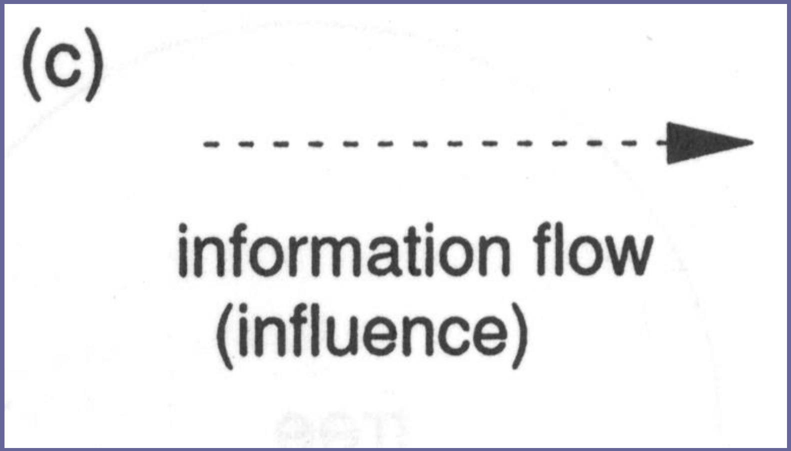


a) Object, state variable – primary components of the model, characterize condition or state of model

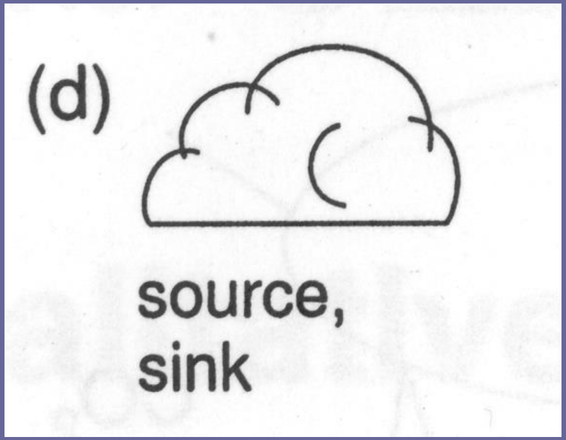
“Objects”
internal – state variables
external – sources or sinks that are not modeled



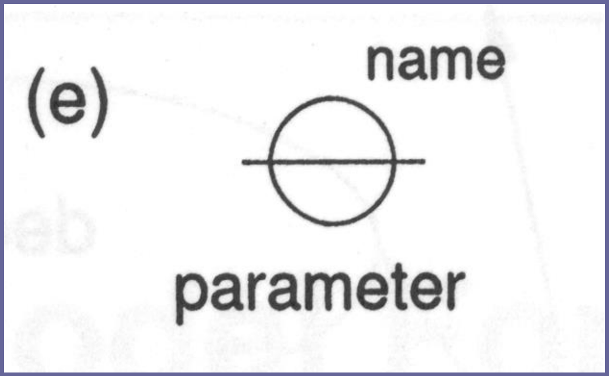
b) Flows – movement of material, rate is determined by state variables



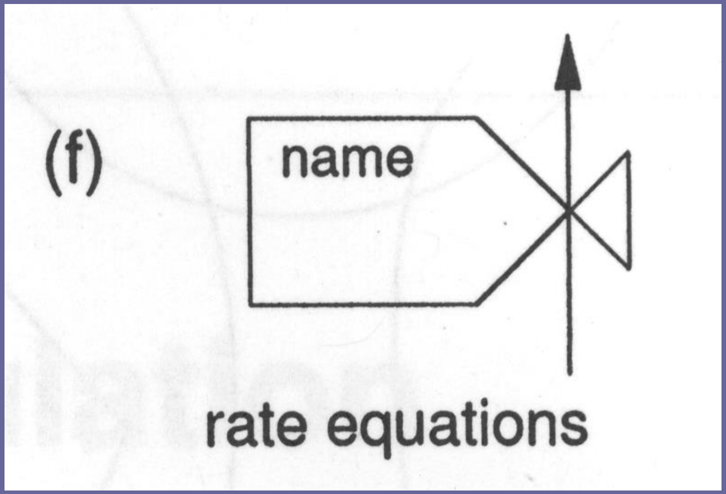
c) Information flow, influence – information transfer, effects of state variables on flows



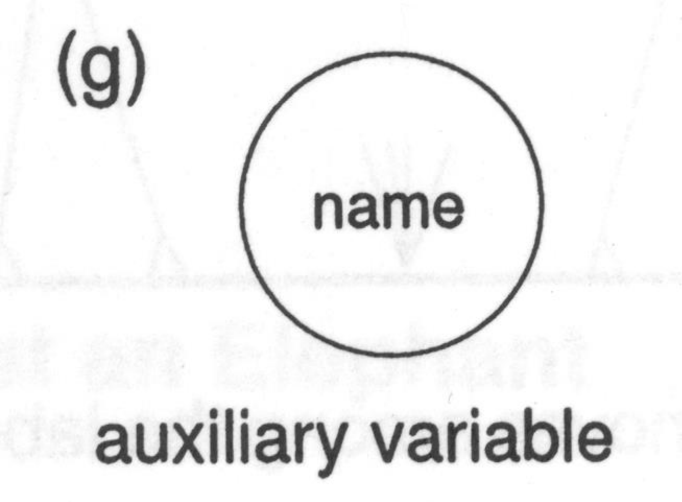
d) Sources, sinks – external objects, material flows to or from, may affect flows



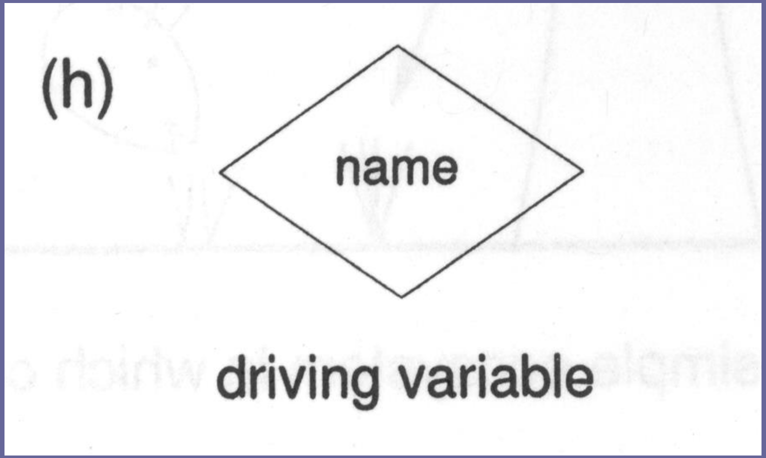
e) Parameters – constants that affect flows, part of flow equations



f) Rate equations – mathematical equations that describe (control) flows



g) Auxiliary variables and equations – sometimes added for clarity in writing of rate equations, may be parts of several flows



h) Driving variables – forcing functions (of time), external factors affecting flows

Forrester Diagram examples…

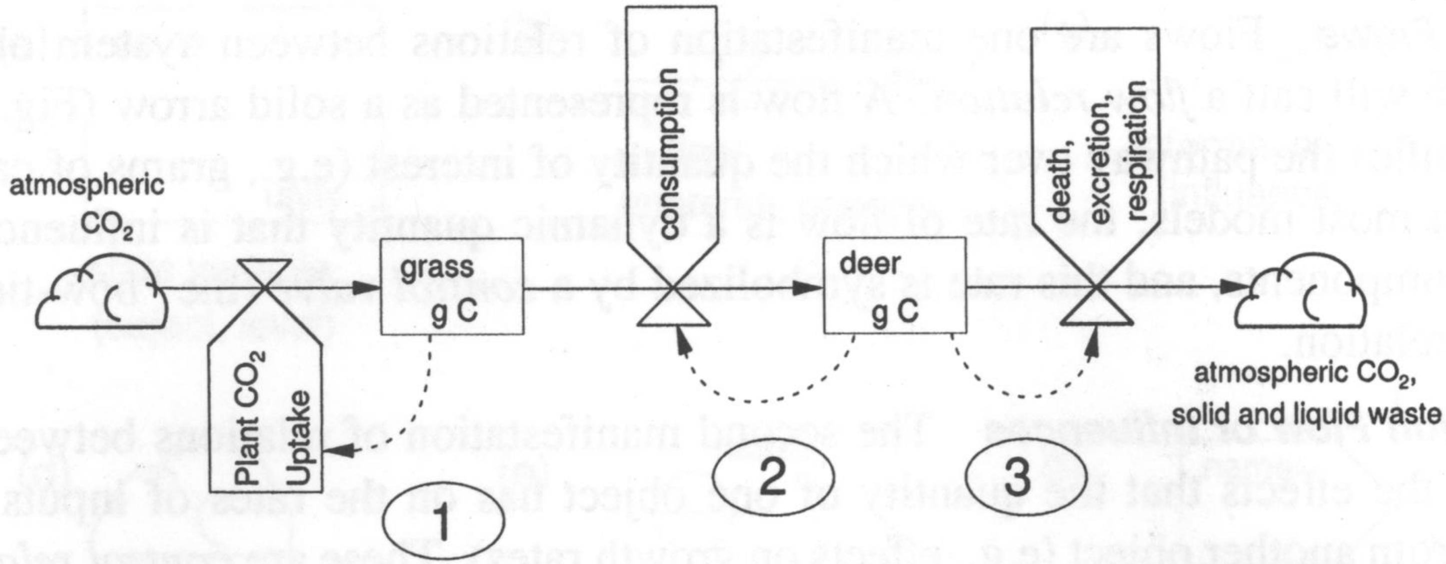


Figure 3.3: Forrester diagram for the grass–deer ecosystem. Solid arrows are pathways for C flow; dotted arrows represent relations between levels and input or output rates as hypothesized. (Numbered ellipses on information flows are not part of Forrester diagrams, but are used for explanatory purposes only.)

Density-independent population growth

$$N_{t+1} = N_t + bN_t - dN_t$$

births

deaths

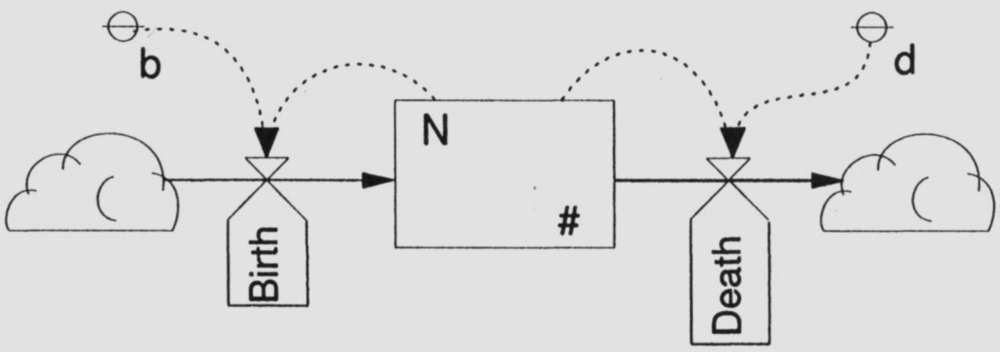


Figure 3.4: Forrester diagram for one form of the density-independent population growth model.

Density – dependent population growth

$$N_{t+1} = N_t + bN_t\underbrace{\left(1 - \frac{N_t}{K}\right)}_{R} - dN_t$$

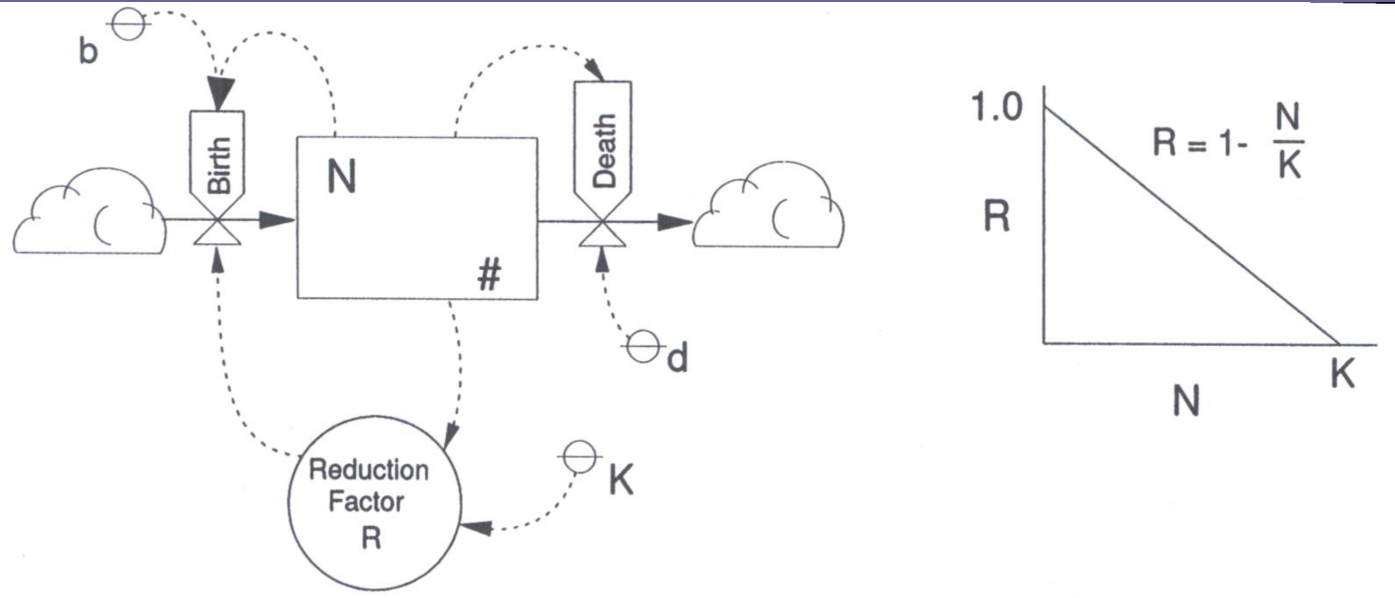


Figure 3.5: Forrester diagram for one form of density-dependent population growth model.

Predator-Prey Model

$$V_{t+1} = V_t + rV_t - aV_tP_t$$

$$P_{t+1} = P_t + abV_tP_t - dP_t$$

predation

predator death

predation

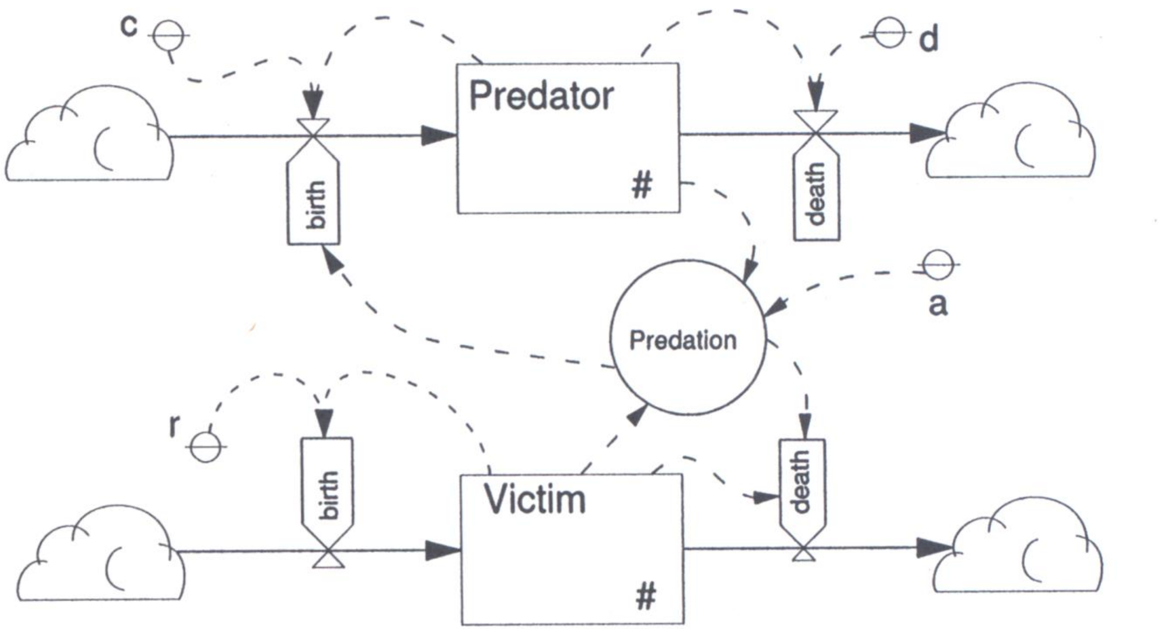


Figure 3.10: Simplified Forrester diagram for linked population models based on numbers of individuals.

Formulating Model Diagrams - general

- State variables?
- Flows between state variables?
- Controls on flow rates?
- Any auxiliary or driving variables?

This process helps us to think about the system being studied :

Model structure

Mathematical formulation

Species interactions

Romeo - R

(Feelings for Juliet)

$$R_{n+1} = a_R R_n$$

$$J_{n+1} = a_J J_n$$

Juliet - J

(Feelings for Romeo)

tomorrow

today

a – How much is tomorrow affected by today?

$a > 0$?

$a = 0$?

$a < 0$?

Love affairs - Interaction term

Feelings affected by perception of other's feelings

$$R_{n+1} = a_R R_n + p_R J_n$$

$$J_{n+1} = a_J J_n + p_J R_n$$

p_R – effect of Juliet's feelings on Romeo

p_J – effect of Romeo's feelings on Juliet

$p > 0$ means feelings of love from the other increase your own feelings towards them

$p < 0$ means your feelings are reduced by positive feelings from the other

Love affairs model - results

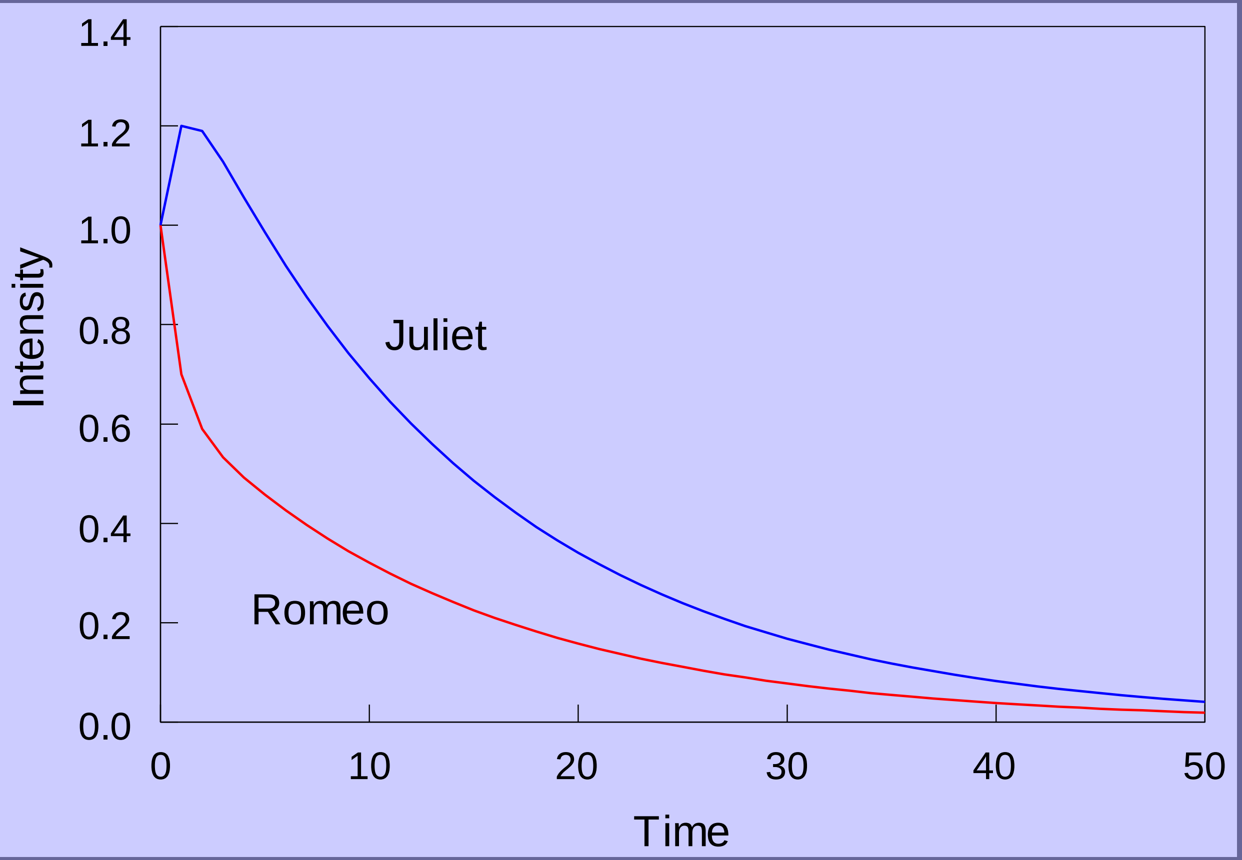
Consider two case studies

$$R_{n+1} = a_R R_n + p_R J_n$$
$$J_{n+1} = a_J J_n + p_J R_n$$

$R_0 = J_0 = 1$

Case 1 :

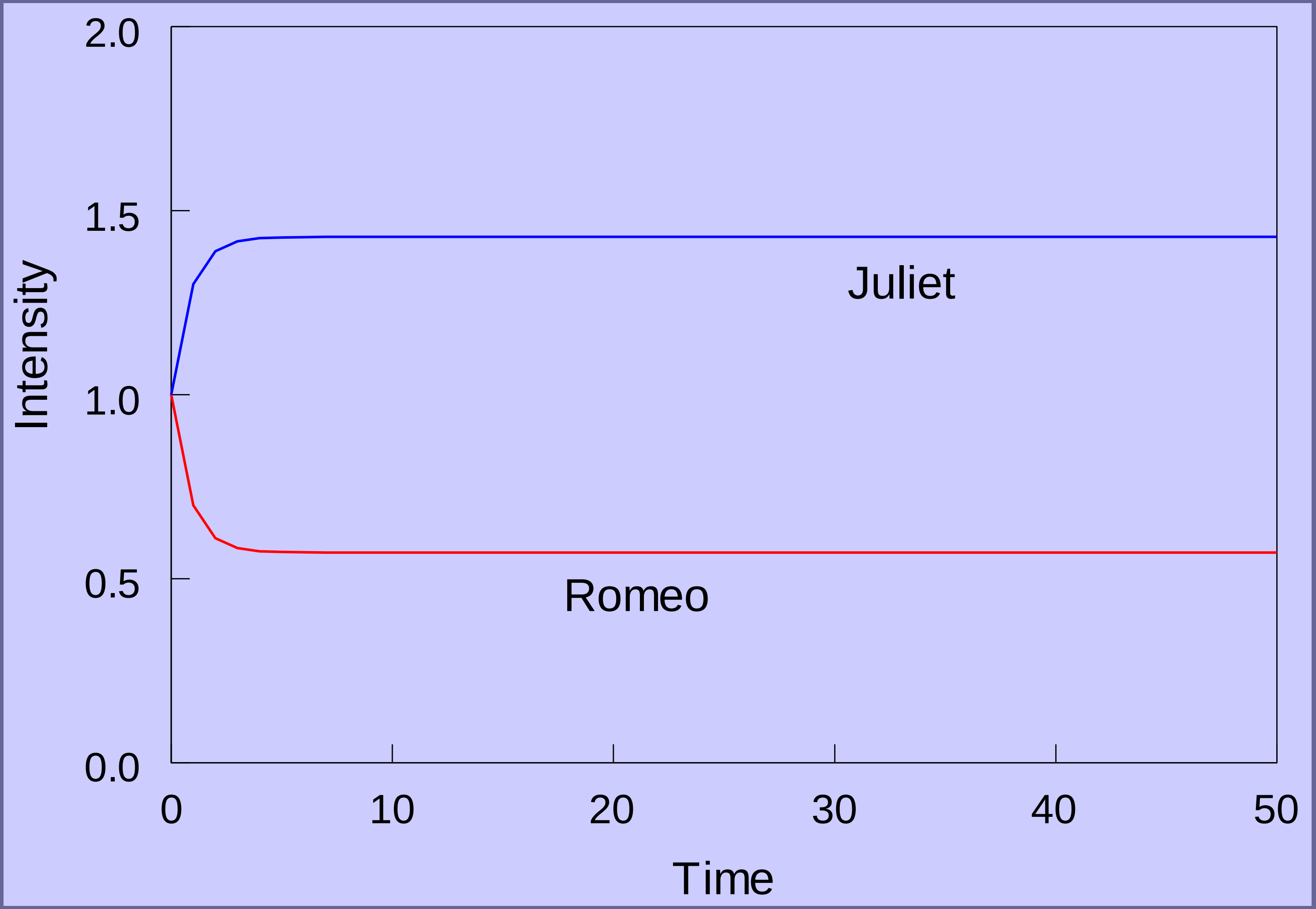
$a_R = 0.5$
$a_J = 0.7$
$p_R = 0.2$
$p_J = 0.5$



Both end up not caring about the other !!

Case 2 :

$a_R = 0.5$
$a_J = 0.8$
$p_R = 0.2$
$p_J = 0.5$



Stable solution – What does “stable” mean ?

Predator – Prey Model

$$V_{t+1} = V_t + rV_t - aV_tP_t$$

$$P_{t+1} = P_t + abV_tP_t - dP_t$$

$r = 0.1$ $d = 0.2$
$a = 0.002$ $b = 0.1$

$V_0 = 1500$
$P_0 = 50$

Present results as :

- Plot of V and P versus time
- Plot of V versus P

Predator-prey model solution…

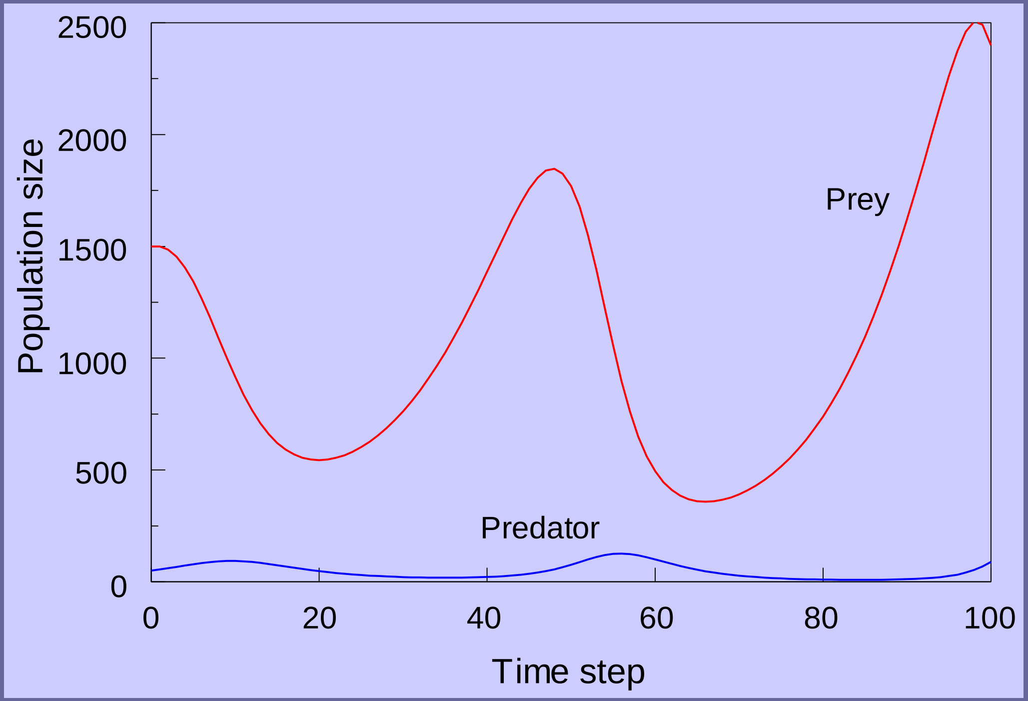
$$V_{t+1} = V_t + rV_t - aV_tP_t$$

$$P_{t+1} = P_t + abV_tP_t - dP_t$$

```
/* Exponential growth model
                    - array method
*/

#include <stdheaders.h>

int main( void )
{
          int i;
          float r;
          float N[11], t[11];

          r = 0.4;
          t[0]= 0;
          N[0] = 2;

          for( i=0; i<10; i++ )          //This loop does the calculating
                    {
                              N[i+1] = N[i] + r * N[i];
                              t[i+1] = t[i] + 1;
                    }
          for( i=0; i<=10; i++ )         //This loop does the printing
                    {
                              printf( "At time %4.1f, N = %7.4f\n", t[i], N[i] );
                    }
          return 0;
}
```

need more steps!!

Declare & define new variables



Predator and Prey *versus* time

Predator *versus* Prey

Phase-space diagram