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Topics: Modeling And Simulation Of Quadcopter Using PID Control

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Article

Modeling And Simulation Of Quadcopter Using PID Control

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Abstract: Nowadays, with the increasing popularity of quadcopter unmanned aerial vehicles (UAV) in several real-world applications, precise and reliable UAV control has become an urgent topic investigated in many studies. The paper presents a new model design method for the flight control of an autonomous quadcopter. The paper describes the controller architecture for the quadcopter as well and explains the developments of a PID (proportional-integral-derivative) control method to obtain stability in flying the Quad-copter flying object. The dynamic model of the quad-copter, which is an under actuated aircraft with fixed four-pitch angle copters was described. Simulation results are presented to verify the effectiveness of the proposed control system and also to compare with previous works.

Keywords: PID controller, quadcopter UAV, Unmanned Aerial Vehicle

I. Introduction

The terminology UAV (unmanned aircraft vehicle) refers not just to the aircraft, but for all fly machines controlled from the ground with the use of a controller with WIFI-connection and is small, without the need for any pilot, and is mostly called, a drone. UAVs are using new technologies of sensors, microcontrollers, Control software, communication hardware, and use interfaces. In researchers, UAVs can generally be divided into two types or categories fixed-wings and rotorcraft, this paper focuses on the quad-copter[1].

Unmanned aerials vehicles (UAVs) are gradually becoming more widely used across a wide range of real-world applications such as military operations, disaster relief, and exploration of hazardous remote areas. Multi-UAV systems have become more and more vital in the field of mission planning since wide variety of tasks need to be performed efficiently[1][3]. An important class of UAVs is quadcopters which have attracted great attention from researchers due to their superior properties such as high maneuverability, reliability, diverse applicability, and economy.

However, their usability still faces several control challenges such as ensuring stable, safe, and efficient operation in often extremely complicated working environments. From a theoretical perspective, it can be stated that: (i) mathematical models of quadcopters are nonlinear systems typically containing unstable open loops. Hence, the requirement of a controller must be sufficiently fast in response, and (ii) the parameters of the quadcopter's dynamic model such as moments of inertia and aerodynamic coefficients cannot be measured exactly[2]. Moreover, the operation of the vehicle is strongly affected by the aerodynamics of the multi-rotors which makes them more sensitive to external disturbances, especially wind, so that even the hovering becomes a non-trivial task. Thus, control of a nonlinear plant is a problem of both practical and theoretical interest[1].

Improved performance expected from the new generation of VTOL vehicles is possible through derivation and implementation of specific control techniques incorporating limitations related to sensors and actuators. The well-known approach to decoupling problem solution based on the Nonlinear Inverse Dynamics (NID) method may be used if the parameters of the plant model and external disturbances are exactly known. Usually, incomplete information about systems in real practical tasks take place. In this case adaptive control methods or control systems with sliding mode may be used for solving such control problem.

A way of the algorithmic solution of this problem under condition of incomplete information about varying parameters of the plant and unknown external disturbances is the application of the Dynamic Contraction Method (DCM)[7]. But the most problems of those approaches in real applications are: the high order of the controller equations and influence of measurement noise for control quality. Approximations of higher derivatives amplify the measurement noise and cause abrupt changes of control signal. Therefore in this paper the different structures of PID controllers, which can reduced the adverse effects are considered. In this paper we will present a very simplified study of quadcopter dynamics with design controllers PID. Then we will test our controllers with a numerical simulation of a quadcopter.

The paper is organized as follows: In Section 2, we describe the mathematical modeling with principal physical, kinematics, and dynamics flight mechanical of the quadcopter. In Section 3, we present the fundamentals of PID controller. In Section 4 we illustrate the essential simulation results of the present paper. In Section 5 concludes the paper.

II. Controller Design.

1. Mathematical modeling

1.1. Coordinate Frames

In order to create a flying controller, we should have a good understanding of the quadcopter movement, and its dynamics to extract the mathematical model, this understanding it's not just necessary for the creation of the controller but also needs to insure that the simulation behavior will be in good agreement with the reality of the applicable command.

The brief mathematic model of a quadcopter is presented in this section. The essential frames consist of an Earth frame, E, and body frame, B, of the vehicle shown in Figure 1.

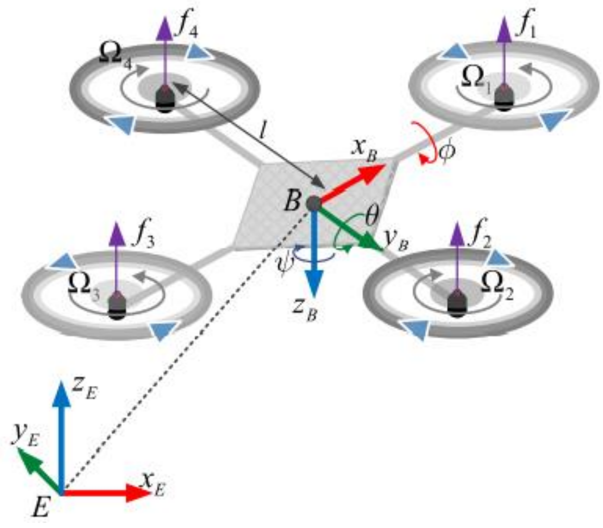

FIGURE 1. Configuration of an underactuated quadcopter

The Quadcopter is designed on the following assumptions:

- The structure is supposed to be rigid
- The Centre of Gravity and the body fixed frame origin are assumed to coincide
- Thrust and drag are proportional to the square of the propeller's speed
- The propellers are supposed to be rigid
- The structure is supposed to be axis symmetrical
- Rotation matrix defined to transform the coordinates from Body to Earth co-ordinates using Euler angles φ – roll angle, θ- pitch angle, ψ- yaw angle
- About by φ, by θ and by ψ

From the right-hand rule, the body of quadcopter fixed coordinates is defined as shown in (Fig. 1). To successfully drive our UAV, the fellow steps are:

- As a starting the mobile frame is coincided with that of the inertial frame, then the mobile frame start a rotation along x direction with a roll angle:

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$$

- A rotation along y direction with a pitch angle:

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

- A rotation along z direction with a yaw angle:

$$-\pi \leq \psi \leq \pi$$

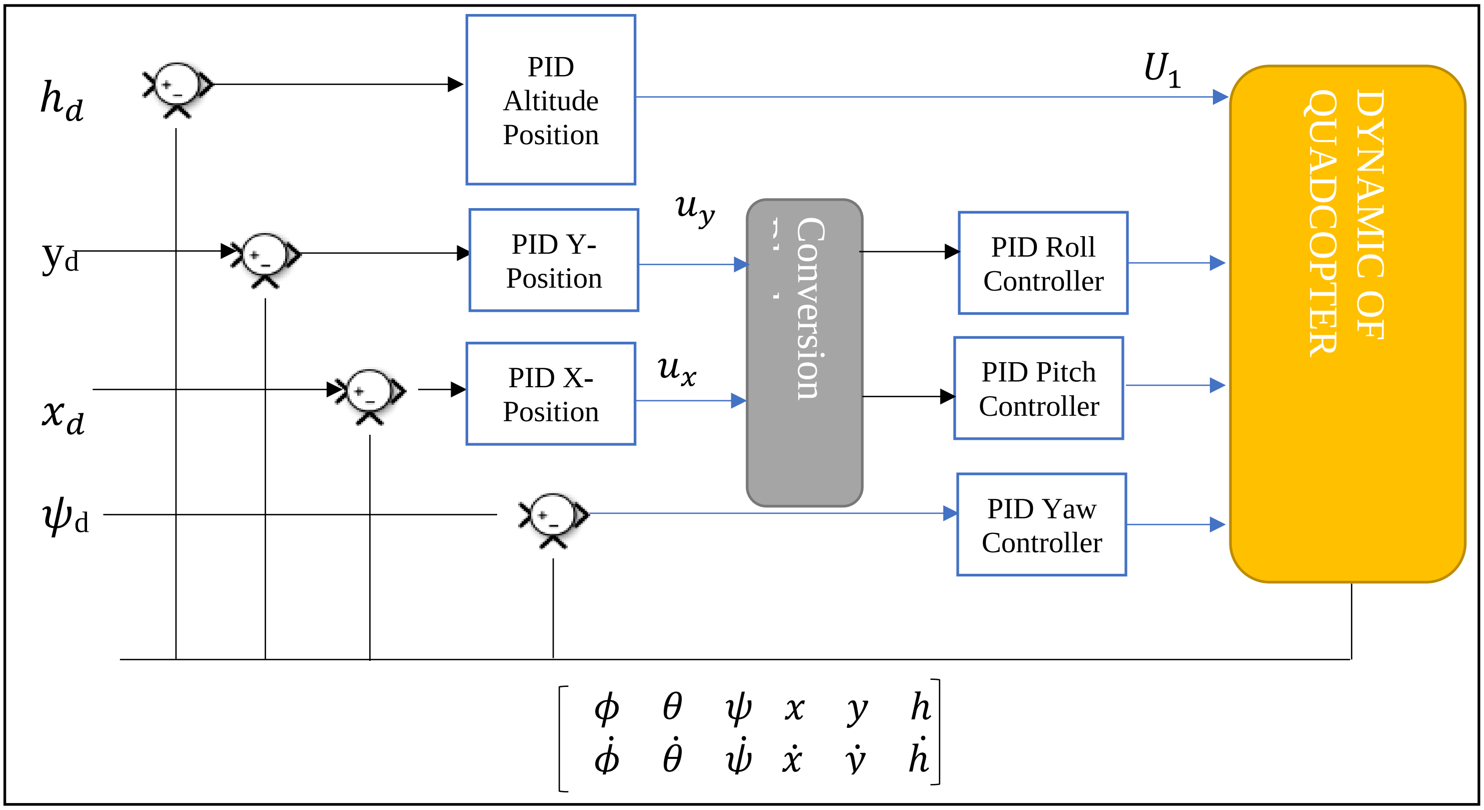

FIGURE 2. General Control Block Diagram

Table 1. System parameters of a quadcopter.

System Parameters	Descriptions
I_x, I_y, I_z (kgm^2)	Moments of inertia along there axes x, y, and z in the Earth frame
m(kg)	Total mass of a quadcopter
l (m)	Arm length of the quadcopter frame
b (Ns^2)	Thrust coefficient
d (Nms^2)	Drag coefficient
K_{th} (N)	Positive gain

1.2. Applied forces and torques

Taking into account the properties of the design, the quadcopter is only controlled by varying separately the speed of the four rotors (see Fig. 1). Let τ_i and F_i be the torque and thrust for i^{th} rotor respectively (i = 1 ... 4) (these values are normalized with the moment of inertia and mass, corresponding). Denoting L the distance between the rotor and center of mass, we can now start a set of four control inputs U_i, as functions of normalized character thrusts and torque as follows:

$$U_1 = F_1 + F_2 + F_3 + F_4 \tag{1}$$

U_1: This is the total thrust. The roll moment is obtained by unstable the left (motor 4) and right (motor 2) rotor speeds:

$$U_2 = L(F_4 - F_2) \tag{2}$$

A pitch moment is achieved by varying the ratio of the front (motor 1) and back (motor 3) rotor speeds.

$$U_3 = L(F_1 - F_3) \tag{3}$$

Finally, a yaw moment is produced from the torque resulting from subtracting counterclockwise (front and back) from the clockwise (left and right) speeds.

$$U_4 = \tau_1 + \tau_3 - \tau_2 - \tau_4 \tag{4}$$

1.3. Rotational Dynamic

The torques acting on the quadcopter are:

- Roll torque:

$$\tau_x = \begin{bmatrix} lb(w_4^2 - w_2^2) \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

- Pitch torque:

$$\tau_y = \begin{bmatrix} 0 \\ lb(w_3^2 - w_1^2) \\ 0 \end{bmatrix} \tag{6}$$

- Yaw torque:

$$\tau_z = \begin{bmatrix} 0 \\ 0 \\ d(w_1^2 - w_2^2 + w_3^2 - w_4^2) \end{bmatrix} \tag{7}$$

By applying Euler's rotation equations, the equation of motion that governs the rotational motion of the quadcopter is given by:

$$\left.\begin{aligned} \ddot{\phi} &= \frac{lb}{I_x}(w_4^2 - w_2^2) + \frac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} \\ \ddot{\theta} &= \frac{lb}{I_y}(w_3^2 - w_1^2) + \frac{(I_z - I_x)}{I_y}\dot{\phi}\dot{\psi} \\ \ddot{\psi} &= \frac{d}{I_z}(w_1^2 - w_2^2 + w_3^2 - w_4^2) + \frac{(I_x - I_y)}{I_z}\dot{\phi}\dot{\theta} \end{aligned}\right\} \tag{8}$$

1.4. Translational dynamics

By neglecting the effects of body moments on the translational dynamics, the translational dynamics governing the quadrotor are given by:

$$\left.\begin{aligned} m\ddot{z} &= mg - (cos\psi cos\phi)\sum_{i=1}^{4} T_i \\ m\ddot{x} &= (sin\phi sin\psi + cos\psi sin\theta cos\phi)\textstyle\sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{xi} \\ m\ddot{y} &= (-sin\phi cos\psi + sin\psi sin\theta cos\phi)\sum_{i=1}^{4} T_i - \sum_{i=1}^{4} H_{yi} \end{aligned}\right\} \quad (9)$$

From this chosen modeling of angle, the rotation matrix R can be formulated by:

$$R = \begin{bmatrix} cos\psi cos\theta & sin\phi sin\theta cos\psi - sin\psi cos\phi & cos\phi sin\theta cos\psi + sin\psi sin\phi \\ sin\psi cos\theta & sin\phi sin\theta sin\psi + cos\psi cos\theta & cos\phi sin\theta sin\psi - sin\phi cos\psi \\ -sin\theta & sin\phi cos\theta & cos\phi cos\theta \end{bmatrix} \quad (10)$$

1.4. Total system model

The quadcopter's vertical momentum is achieved through the lift forces provided by the rotating propellers. Horizontal momentum is achieved through differences in the rotational angles such as roll, pitch, and yaw. Let x, y, z ∈ R represent the position of the vehicle in the Earth frame, E[3]. Then a nonlinear dynamical model for a quadcopter that accounts for external disturbances and uncertainties can be formulated as follows:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{-U_1(cos\phi.sin\theta.cos\psi + sin\phi.sin\psi}{m} \\ \frac{-U_1(cos\phi.sin\theta.cos\psi - sin\phi.sin\psi}{m} \\ g - \frac{U_1(cos\phi.cos\theta)}{m} \\ \frac{1}{I_x}U_2 + \frac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} \\ \frac{1}{I_y}U_3 + \frac{(I_z - I_x)}{I_y}\dot{\phi}\dot{\psi} \\ \frac{1}{I_z}U_4 + \frac{(I_x - I_y)}{I_z}\dot{\phi}\dot{\theta} \end{bmatrix} \quad (11)$$

where $U_i \in R$, i = 1, 2, 3, 4 denote the control inputs of a quadrotor, which are computed as follow:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} lb(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ lb(\Omega_4^2 - \Omega_2^2) \\ lb(\Omega_1^2 - \Omega_3^2) \\ d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{bmatrix} \quad (12)$$

Where Ω_i $(i = 1,2,3,4)$ is speed of motor i.

b is thrust coefficient.

d is drag coefficient.

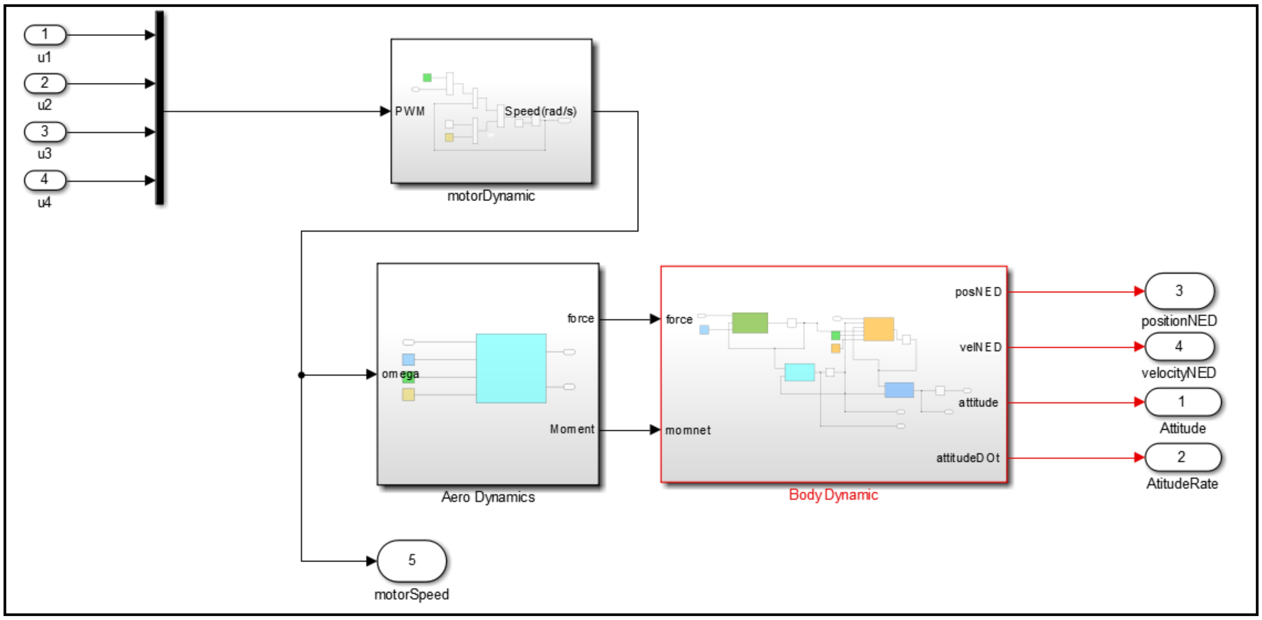

FIG 3. Simulation Quadcopter model

2.Attitude controller

For designing the attitude of quadcopter, a PID controller is also used. Let ϕ_d, θ_d and ψ_d be the desired states of roll, pitch, and yaw angles, respectively. Roll attitude is performed by increasing the arm's length of the motor (I_4) and decreasing the arm's length of the motor (I_2), while fixed the other two arm's length and the motors speed are also fixed[3]. In order to obtain the roll controller, U_2, the tracking errors of roll angle, are computed as:

$$e_\phi = \phi_d - \phi_{ref} \tag{13}$$

The PID controller equation for roll attitude is:

$$u_\phi = k_p e_\phi + k_i \int e_\phi dt + k_d \dot{e_\phi} \tag{14}$$

The equation of rotation for a quadcopter around x-axis include PID controller will be:

$$\ddot{\phi} = \frac{1}{I_x} u_2 = \frac{2K_{th} l}{I_x} \delta_{roll} \tag{15}$$

Similarly for other attitude angles (pitch and yaw) the error signal will be:

$$\begin{gathered} e_\theta = \theta_d - \theta_{ref} \\ e_\psi = \psi_d - \psi_{ref} \end{gathered} \tag{16}$$

The PID controller equations for pitch and yaw attitude are:

$$\begin{gathered} u_\theta = k_p e_\theta + k_i \int e_\theta dt + k_d \dot{e_\theta} \\ u_\psi = k_p e_\psi + k_i \int e_\psi dt + k_d \dot{e_\psi} \end{gathered} \tag{17}$$

The equations of pitch and yaw attitude include PID controller will be:

$$\ddot{\theta} = \frac{1}{I_y} u_3 = \frac{2K_{th}l}{I_y} \delta_{pitch}$$
$$\ddot{\psi} = \frac{1}{I_z} u_4 = \frac{4K_d}{I_z} \delta_{yaw} \quad (18)$$

By applying Laplace transform on Equation 15 and 18, we can calculate the PID parameters of Roll, Pitch ,Yaw controller:

- ✓ Roll control:

$$K_{p1(roll)} = \frac{I_x}{K_{th}l} \xi_{roll} w_{n(roll)}; \qquad K_{p2(roll)} = \frac{I_x}{2K_{th}lK_{p1(roll)}} w^2_{n(roll)}$$

- ✓ Pitch control:

$$K_{p1(pitch)} = \frac{I_y}{K_{th}l} \xi_{pitch} w_{n(pitch)} \qquad K_{p2(roll)} = \frac{I_y}{2K_{th}lK_{p1(pitch)}} w^2_{n(pitch)}$$

- ✓ Yaw control:

$$K_{p1(yaw)} = \frac{\xi_{yaw} w_{n(yaw)} I_z}{2K_d} \qquad K_{p2(yaw)} = \frac{w^2_{n(yaw)} I_z}{4K_{p1(yaw)} K_d}$$

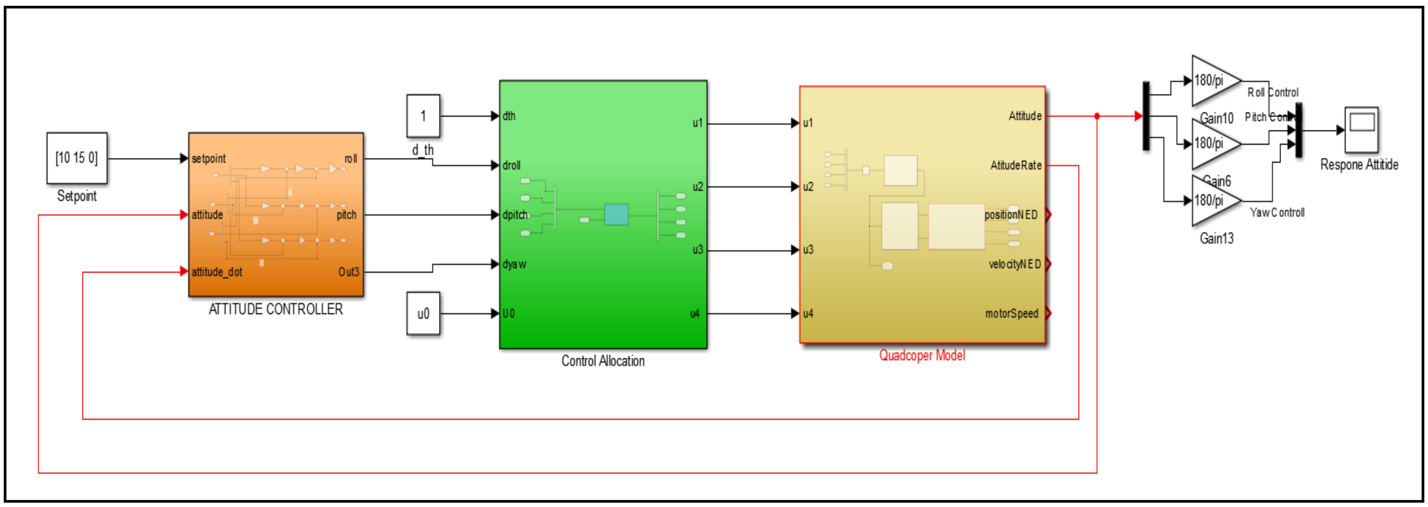

FIG 4. Simulation Attitude Controller

3.Altitude Controller Design

PID controller is used for controlling the altitude of the quadcopter by controlling the speed of four motors to reach the altitude required. The lift force required for takeoff must be greater than quadcopter full weight, where the motors speed for the hovering state is calculated as:

$$Mg = 4f_i$$
$$Mg = 4bw_i^2 \quad (19)$$

The error signal for this controller will be expressed as:

$$e_z = z_d - z_{ref} \quad (20)$$

Mathematics model of Altitude motion downward direction:

$$\ddot{z} = g - \frac{U_1(cos\phi cos\theta)}{m} \tag{21}$$

Therefore altitude motion upward direction is:

$$\ddot{h} = -g + \frac{U_1}{m}(cos\phi cos\theta) \tag{22}$$

By applying Laplace transform on Equation 22, we can calculate the PID parameters of altitude control as following:

$$K_p = \frac{2w_{n(altitide)}\xi_{altitude}}{4K_{th}}$$

Choose: $K_I = 0.04$

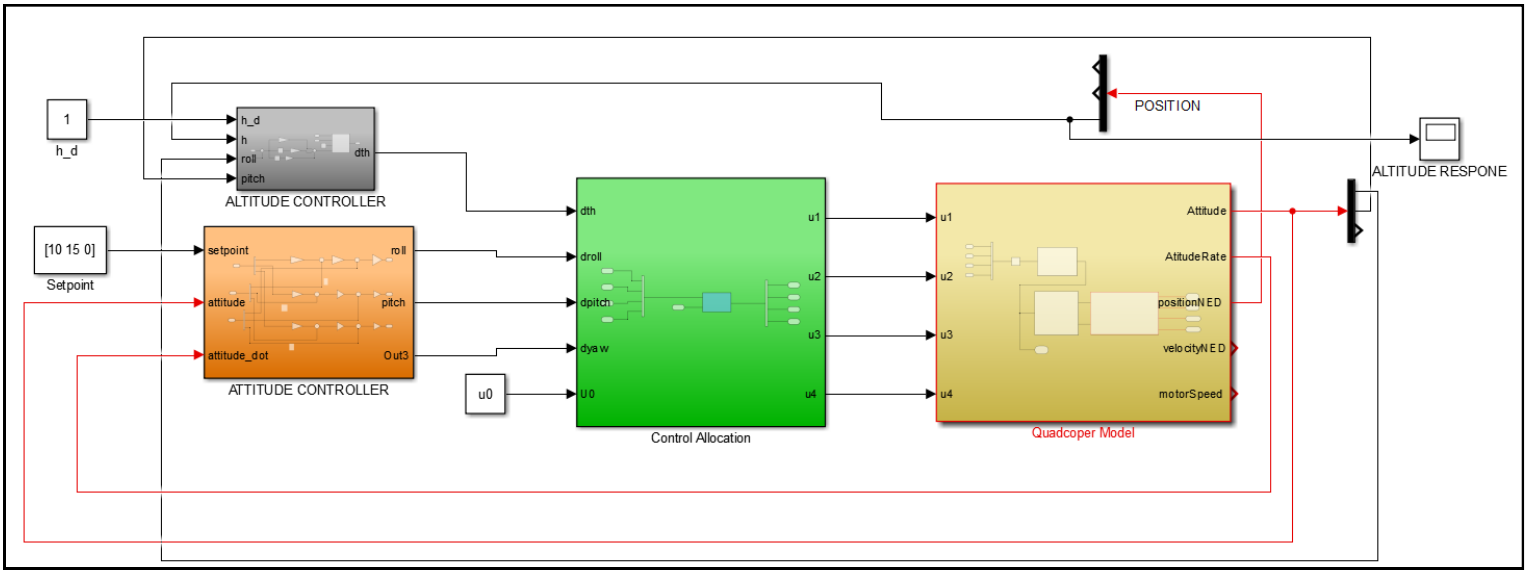

FIG 5. Simulation Altitude Controller Quadcopter

4.Position Controller Design

The horizontal motion in the x and y- axis can be accomplished by controlling the position of the quadrotor. Position control can be achieved by pitching or rolling the quadcopter, and tracking the desired waypoint from the x and y-axis. PID controller with saturation function is used for position control[7]. From the position controller calculating the desired acceleration $(\ddot{x}, \ddot{y})$:

$$\left.\begin{aligned} e_x &= x_d - x_{ref} \\ e_y &= y_d - y_{ref} \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} \ddot{x} &= k_p e_x + k_i \int e_x dt + k_d \dot{e_x} \\ \ddot{y} &= k_p e_y + k_i \int e_y dt + k_d \dot{e_y} \end{aligned}\right\} \quad (24)$$

Where (x_d, y_d) represent the waypoints desired, (x_{ref}, y_{ref}) represent the refrence positions and k_p, k_i, k_d represent the proportional, integral, and derivative controller gains respectively. The desired pitch and roll angles are calculated from the desired acceleration $\ddot{x}$ and $\ddot{y}$:

$$\left.\begin{aligned} \ddot{x_d} &= -\frac{1}{m} U_1 u_x \\ \ddot{y} &= -\frac{1}{m} U_1 u_y \end{aligned}\right\} \quad (25)$$

Where $$\left\{\begin{aligned} u_x &= cos\phi sin\theta cos\psi + sin\phi sin\psi \\ u_y &= \cos\phi sin\theta sin\psi - sin\phi cos\psi \end{aligned}\right. \quad (26)$$

By applying Laplace transform on Equation 25, we can calculate the PID parameters of altitude control as following:

$$K_{px} = K_{py} = \frac{2g}{w_n^2} \qquad\qquad K_{dx} = K_{dy} = \frac{4g}{\xi w_n}$$

As considering quadcopter model is under hover condition. Thus, $sin\phi_d = \phi_d$, $sin\theta_d = \theta_d$ and $cos\phi_d = cos\theta_d = 1$. Then, from Eq (25) we can find the desired roll (ϕ_d) and pitch (θ_d) angles as in:

$$\begin{bmatrix} \phi_d \\ \theta_d \end{bmatrix} = \begin{bmatrix} -sin\psi & -cos\psi \\ cos\psi & -sin\psi \end{bmatrix}^{-1} \frac{m}{u_1} \begin{bmatrix} \ddot{x_d} \\ \ddot{y_d} \end{bmatrix} \quad (27)$$

In the quadcopter model design, the system is divided into the altitude, attitude, and position subsystems as in Fig. 2. The complete control block diagram can be illustrated in the following.

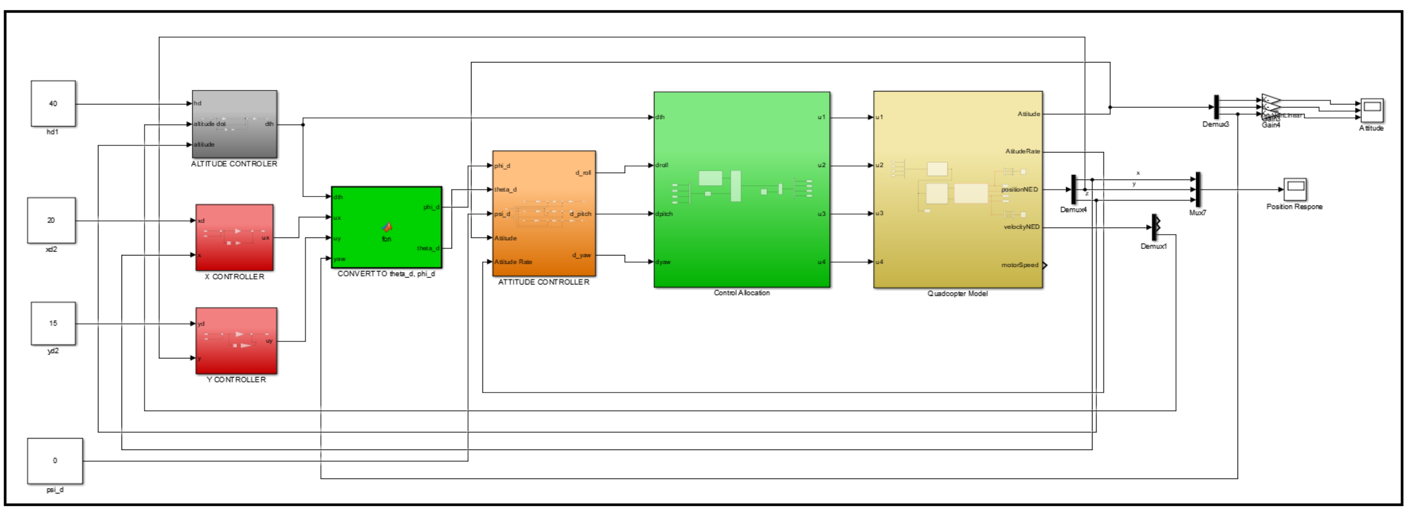

FIG 6. Simulation System Quadcopter

III. SIMULATION RESULTS AND DISCUSSIONS

The numerical simulation is performed to prove the effectiveness of the PID controller for the quadcopter system, presented in this section. The position controller inputs are the position measurements and the outputs are the desired roll and pitch angles to move the quadcopter to the desired x and y positions. The Altitude controller is implemented to stabilize the quadcopter hover[4]. The PID controller is used by the UAV model to control the angular, altitude, and translational positions base on parameters such as Table 2.

The angles and their time derivatives of the rotational subsystem do not depend on translation components, as equation (11) of the quadcopter has shown. However, the translations depend on the angles.

Table 2. System parameters of the quadcopter for simulation.

Specification	Parameter	Unit	Value
Quadcopter mass	m	kg	0.016
Lateral moment arm	l	m	0.17
Thrust coefficient	b	Ns^2	5.9347e-06
Drag coefficient	d	Nms^2	1.2181e-07
Rolling moment of inertia	I_{xx}	kgm^2	0.0073
Pitching moment of inertia	I_{zz}	kgm^2	0.0073
Yawing moment of inertia	I_{zz}	kgm^2	0.0017
Roll angles	ϕ_0, ϕ_d	degree	10, 0
Pitch angles	θ_0, θ_d	degree	10, 0
Yaw angles	ψ_0, ψ_d	degree	5, 0
Altitude	h_0, h_d	m	0, 5
Position	x_d, y_d	m	10,15

Next, attitude controller simulation with a closed-loop system with a nonlinear control algorithm. The initial conditions are angles roll, pitch, and yaw as shown in table 2 with PID roll control ($K_{p1(roll)} = 0.0780, K_{p2(roll)} = 5.7143$), pitch control($K_{p1(pitch)} = 0.0780, K_{p2(pitch)} = 5.7143$) and yaw control($K_{p1(yaw)} = 0.5177, K_{p2(yaw)} = 5.7143$).

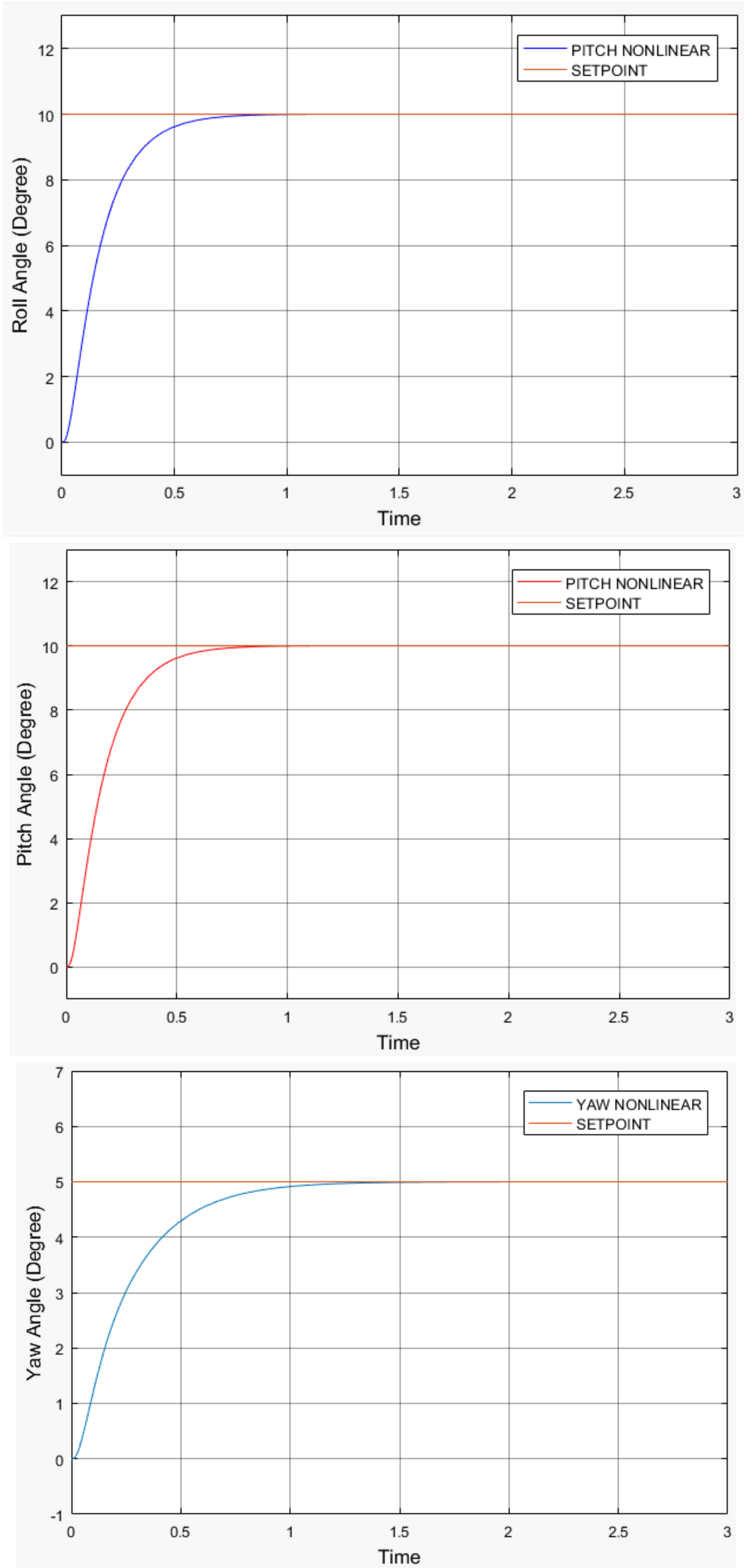

FIG 7. Attitude roll, pitch, and yaw response with PID controller (Attitude and Time)

Fig.7 shows the response the attitude of the nonlinear controller to stabilize the quadcopter during hovering. The simulation results in Fig.7 are acquired with a model inclusive of actuators' dynamics. Although the initial conditions are very strict, it can be seen from Fig.2 that the controller succeeded in controlling the roll, pitch, and yaw angles of the quadrotor in less than 1s.

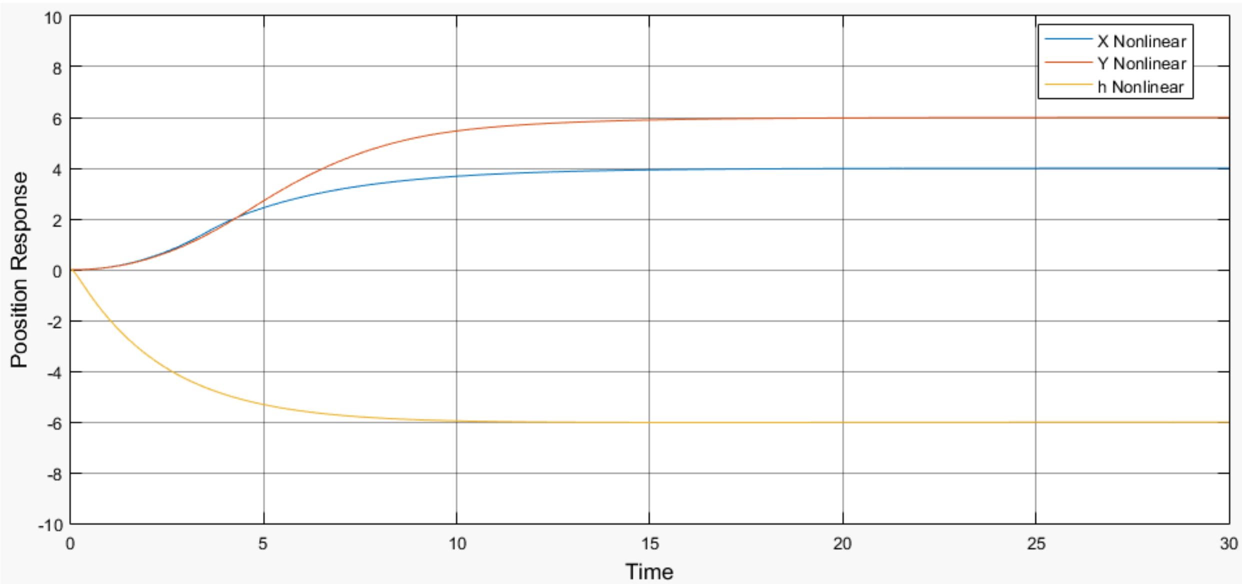

FIG 8. The Response Position And Altitude of system UAV

The altitude and position response of the quadcopter are shown in Fig.8. Results from Fig.8 indicate that the position controller effectively makes the attitude controller keep the quadcopter at a given point. However, the setup time is quite slow.

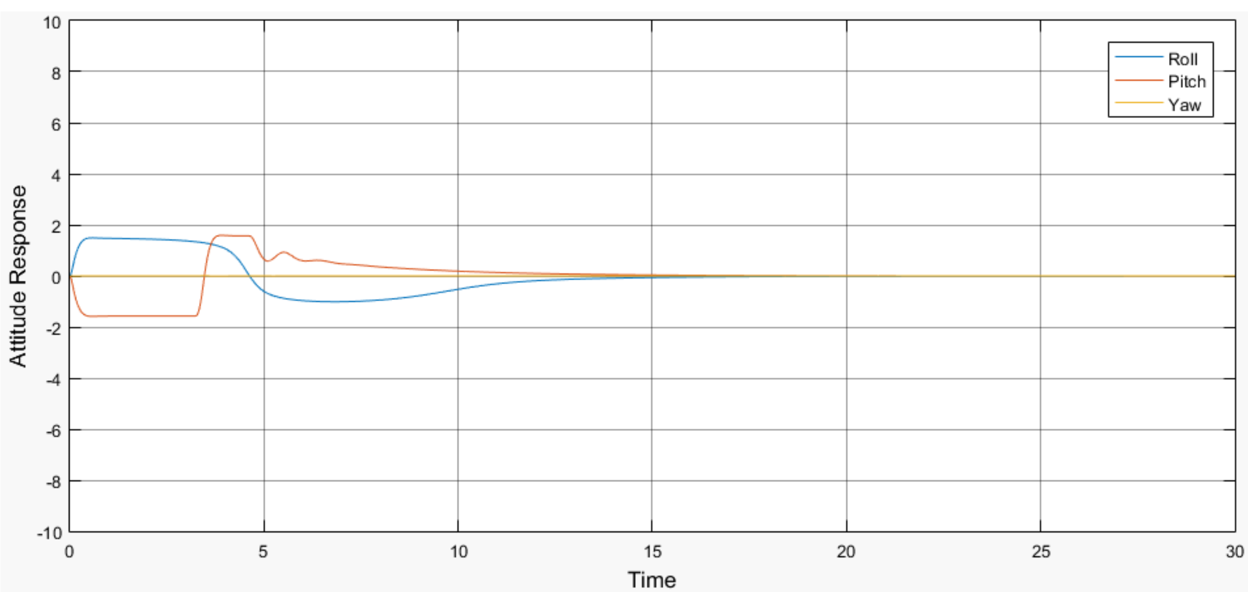

FIG 9. The Response Attitude Of system UAV

Position control keeps the 3D orientation of the quadcopter in the desired state. Roll and pitch angles are usually made to be zero to realize hovering. The rotational controller is responsible for compensating for the initial errors, stabilizing roll, pitch, and yaw angles, and maintaining them at zero as Fig.9.

IV. CONCLUSION.

This paper investigates the mathematical modeling, stabilization, and control of a small quadcopter UAV. The obtained model is coupled with a PID control algorithm. The resulting system is converted to a Matlab algorithm to study the performance of the present model[4]. The simulation results prove that the adopted process of modeling and control is uncomplicated, prompt, and effective. The error between the desired and simulation trajectory is very low for the three control attitude, and the position which proves the robustness towards stability and tracking of the proposed model.

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