PS7

Problem 7.1

a. $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

b. $P(Cavity) = \langle 0.2, 0.8 \rangle$

$P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$

$P(\neg cavity) = 0.016 + 0.064 + 0.144 + 0.576 = 0.8$

c. $P(Toothache \mid cavity) = \alpha P(Toothache, cavity)$

$= \alpha [P(Toothache, cavity, catch) + P(Toothache, cavity, \neg catch)]$

$= \alpha [\langle 0.108, 0.072 \rangle + \langle 0.012, 0.008 \rangle]$

$= \langle 0.120, 0.08 \rangle$

d. $P(Cavity \mid toothache \vee catch) = \alpha [P(cavity \mid toothache \vee catch), P(\neg cavity \mid toothache \vee catch)]$

$= \alpha \langle P(cavity \wedge (toothache \vee catch)), P(cavity \wedge (toothache \vee catch)) \rangle$

$= \alpha \langle 0.108 + 0.012 + 0.072, 0.016 + 0.064 + 0.144 \rangle$

$= \langle 0.192, 0.224 \rangle$

Problem 7.2:

- Giving the probability of the chance of someone has the disease as $P(D) = 1/10000 = 0.0001$ so the P of not have the disease is $P(\neg D) = 1 - P(D) = 0.999$

- The chance of the test is positive if you have the disease is $P(T \mid D) = 0.99$

- From $P(T \mid D)$ and what we know about the accurate of the test we can said that the chance of the test is positive and we don't have the disease is: $P(T \mid \neg D) = 0.01$

- Using this we can calculate the chance that the test is positive

$P(T) = P(T \mid D) P(D) - P(T \mid \neg D) P(\neg D)$

$= 0.99 \times 0.0001 + 0.01 \times 0.9999$

$= 0.010098$

- The chance that you have the disease if the test is positive is Using the Bayes'rule we have

$$P(D \mid T) = \frac{P(T \mid D) P(D)}{P(T)} = \frac{0.99 \times 0.0001}{0.010098} = \frac{1}{102} \approx 0.0098$$

Problem 7.3 Find $P(m|s)$ and $P(\neg m|s)$

Let $P(s|\neg m) = 0.05$ $P(m) = 1/50000$ $P(\neg m) = 0.99998$

The unnormalized equation is:

$$① \; P(\neg m|s) = P(s|\neg m)P(\neg m) = 0.05 \times 0.9998 = 0.049999$$

$$② \; P(m|s) = P(s|m)P(m) = 0.7 \times 0.00002 = 0.000014$$

$$P(s) = ① + ② = P(s|\neg m)P(\neg m) + P(s|m)P(m)$$

The normalized equation is:

$$P(\neg m|s) = \frac{①}{① + ②} \approx 0.99972$$

$$P(m|s) = \frac{②}{① + ②} \approx 0.0002799$$

Problem 7.4

a. No, It isn't possible because the information that the witness provide is not reliable

b. The probability of a blue taxis is

$P(B) = 0.1$ $P(\neg B) = 0.9$

Based on the witness info we have

$P(LB|B) = 0.75$

$P(LB|\neg B) = 1 - 0.75 = 0.25$

$$P(LB) = P(LB|B)P(B) + P(LB|\neg B)P(\neg B) = 0.3$$

- The chance of the taxis is blue or not blue if the taxis looked blue is:

$$P(B|LB) = \frac{P(LB|B)P(B)}{P(LB)} = \frac{0.75 \times 0.1}{0.3} = 0.25$$

$$P(\neg B|LB) = \frac{P(LB|\neg B)P(\neg B)}{P(LB)} = \frac{0.25 \times 0.9}{0.3} = 0.75 \quad (\text{Bigger})$$

So the most likely color of the taxis is green since the probability is higher