

EECS 649: PROBLEM SET #7

Reading:

- R&N 12.1-5 [you can do PS#7 with this and the "Uncertainty" lecture]

Total Points: 100

Notes:

- Submitted electronically (via Gradescope)
 - This is a shorter homework in terms of the **number** of problems and the fact that there are not any programming problems.
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Problem 7.1 [25 points] *Marginalization and conditioning calculations from a joint distribution*

Given the full joint distribution shown in Figure 12.3, calculate the following **distributions**:

- a. $P(\text{toothache})$
- b. $P(\text{Cavity})$
- c. $P(\text{Toothache} \mid \text{cavity})$
- d. $P(\text{Cavity} \mid \text{toothache} \vee \text{catch})$

Problem 7.2 [25 points] *Good news, bad news from doctor*

Following your yearly checkup, the doctor shares some bad news and some good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

Formulate your answer in terms of the random variables T (the test is positive) and D (you have the disease).

Problem 7.3 [25 points] *Normalization calculations*

(R&N) In this exercise, you will complete the normalization calculation for the meningitis example. First, make up a suitable value for $P(s \mid \neg m)$, and use it to calculate unnormalized values for $P(m \mid s)$ and $P(\neg m \mid s)$ (i.e., ignoring the $P(s)$ term in the Bayes' rule expression, Equation (12.14)). Now normalize these values so that they add to 1.

In "making up a value" for $P(S \mid \text{not } M)$ aka $P(+s \mid \neg m)$, you can calculate it from the data given or choose the value 0.05.

Problem 7.4 [25 points] *Athenian taxis, or uncertain observations*

(Adapted from Pearl (1988).) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.

- a. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that it appears blue.)
- b. What if you know that 9 out of 10 Athenian taxis are green?

*The relevant aspects of the world to consider are the two random variables, B (the taxi **was** blue) and LB (the taxi **looked** blue). Since all taxis are either blue or green, you may consider "not B " to represent green.*