No.
Date

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1.1.(a) $I = \int_0^2 \int_x^{2x} (x+y)^2 \, dy \, dx$.

Đặt $\int_x^{2x} (x+y)^2 \, dy = I_1$.

$\Rightarrow I_1 = \int_x^{2x} (x^2 + y^2 + 2x) \, dy$

$= x^2 y + \frac{y^3}{3} + 2xy \Big|_x^{2x}$.

$= x^3 + 2x^2 + \frac{8x^3}{3} - \frac{x^3}{3}$.

$= \frac{10x^3}{3} + 2x^2$.

$\Rightarrow I = \int_0^2 \frac{10x^3}{3} + 2x^2 \, dx$.

$= \frac{10x^4}{12} + \frac{2x^3}{3} \Big|_0^2$.

$= \frac{5 \cdot 2^4}{6} + \frac{2 \cdot 2^3}{3}$

$= \frac{56}{3}$.

(2) $I = \int_0^3 \int_{-y}^{y} (x^2 + y^2) \, dx \, dy$.

Xét $\int_{-y}^{y} (x^2 + y^2) \, dx = \frac{x^3}{3} + y^2 x \Big|_{-y}^{y}$.

$= \frac{y^3}{3} + y^3 - \frac{y^3}{3} - y^3$

$= 0$.

$\Rightarrow I = \int_0^3 0 \, dy$

$= 3 \cdot 0 = 3 \cdot 0 = 0$.

1.2, Vì $x \geq 1, y \geq 1, x+y \leq 3$

$\Leftrightarrow 1 \leq x \leq 3-y$.

$\Leftrightarrow 1 \leq 3-y$

$\Leftrightarrow y \leq 2$.

$\Leftrightarrow 1 \leq y \leq 2$.

Tương tự, $\Rightarrow 1 \leq x \leq 2$.

Do đó: $I = \int_1^2 \int_1^2 \frac{1}{(x+y)^3} \, dx \, dy$.

Xét $\int_1^2 \frac{1}{(x+y)^3} \, dx = \frac{1}{-2(x+y)^2} \Big|_1^2$.

$= \frac{1}{-2(2+y)^2} + \frac{1}{2(1+y)^2}$

$\Rightarrow I = \int_1^2 \left(\frac{1}{-2(2+y)^2} + \frac{1}{2(1+y)^2} \right) dy$.

$= \frac{1}{24}$.

1.3 $I = \int_0^{\pi/4} \int_0^{\pi/4} (\cos^2 x + \sin^2 y)\, dx\, dy$.

Xét $\int_0^{\pi/4} (\cos^2 x + \sin^2 y)\, dx$

$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + x\sin^2 y \Big|_0^{\pi/4}$.

$= \frac{\pi}{8} + \frac{1}{4} + \frac{\pi}{4}\sin^2 y$

$\Rightarrow I = \int_0^{\pi/4} \left(\frac{\pi}{8} + \frac{1}{4} + \frac{\pi}{4}\sin^2 y\right) dy$

$= \frac{\pi y}{8} + \frac{y}{4} + \frac{\pi y}{8} - \frac{\pi}{16}\sin(2y) \Big|_0^{\pi/4}$.

$$= \frac{y(\pi+1)}{4} - \frac{\pi}{16}\sin(2y)\Big|_0^{\pi/4}$$

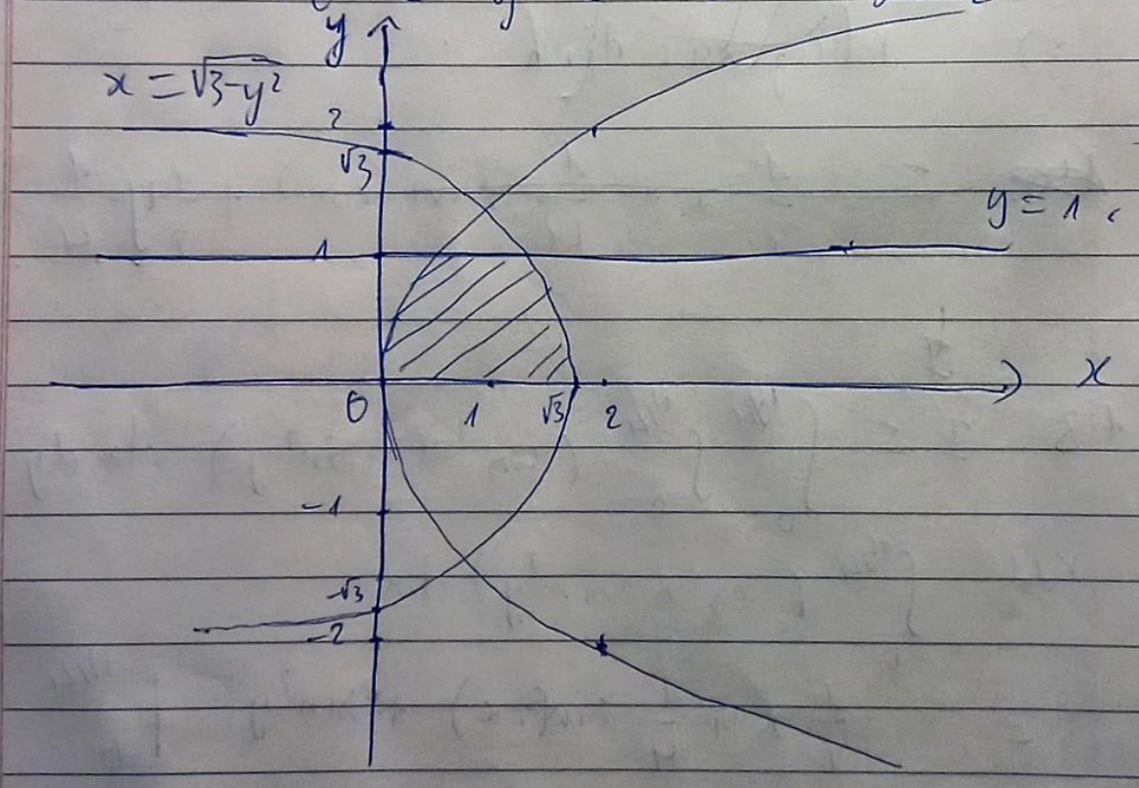
$$= \frac{\pi(\pi+1)}{16} - \frac{\pi}{16}$$

$$= \frac{\pi^2}{16}.$$

1.4.

(5) $$I = \int_0^1 dy \int_{y^2/2}^{\sqrt{3-y^2}} f(x,y)\,dx.$$

Ta có: $R\begin{cases} y^2/2 \le x \le \sqrt{3-y^2} \\ 0 \le y \le 1. \end{cases}$



Thay đổi thứ tự tích phân

$$\Rightarrow R\begin{cases} 0 \le x \le \sqrt{3} \\ \sqrt{2x} \le y \le \sqrt{3-x^2}. \end{cases}$$

$$\Rightarrow I = \int_0^{\sqrt{3}} \int_{\sqrt{2x}}^{\sqrt{3-x^2}} f(x,y)\,dy\,dx.$$

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(6) $I = \int_{-2}^{2} dx \int_{x^2}^{4} f(x,y)\,dy$.

Ta có: $R \begin{cases} -2 \le x \le 2 \\ x^2 \le y \le 4. \end{cases}$

Vẽ hình (c) xuống!

⇒ Phạy đổi thứ tự

~~$R \begin{cases} 2 \le y \le 4. \end{cases}$~~

$\Rightarrow R \begin{cases} 0 \le y \le 4. \\ \sqrt{y} \le x \le 4. \end{cases}$

~~(7) $I = \int_{1}^{3}$~~ $\Rightarrow I = \int_0^4 dy \int_{\sqrt{y}}^{4} f(x,y)\,dx$

(7) $I = \int_1^3 dy \int_0^{2y} f(x,y)\,dx$

Ta có $R \begin{cases} 1 \le y \le 3 \\ 0 \le x \le 2y. \end{cases}$

⇒ đổi thứ tự:

$\Rightarrow R \begin{cases} 0 \le x \le 6 \\ 1 \le y \le 3. \end{cases}$

$\Rightarrow I = \int_0^6 dx \int_1^3 f(x,y)\,dy$

$\Rightarrow R \begin{cases} 0 \le x \le 1 \\ 1 \le y \le 3 \end{cases}$ và $\begin{cases} 2 \le x \le 6 \\ x/2 \le y \le 3 \end{cases}$

$\Rightarrow I = \int_0^2 dx \int_1^3 f(x,y)\,dy + \int_2^6 dx \int_{x/2}^{3} f(x,y)\,dy$