Đáp Án Bài tập Số 5

Bài 1.1 [3 đ]

(1): $I = \int_0^2 \int_x^{2x} (x+y)^2 \, dy \, dx$

$= \int_0^2 dx \int_x^{2x} (x+y)^2 \, dy$

$= \int_0^2 dx \left[\frac{(x+y)^3}{3} \Big|_x^{2x} \right]$

$= \frac{1}{3} \int_0^2 dx \left[(3x)^3 - (2x)^3 \right]$ --> [1 đ]

$= \frac{19}{3} \int_0^2 dx \, x^3 = \frac{19}{3} \left(\frac{x^4}{4} \Big|_0^2 \right) =$

$= \frac{76}{3} \simeq 25,333$ [1,5 đ]

(2): $I = \int_0^3 \int_{-y}^{y} (x^2+y^2)\,dx\,dy$

$= \int_0^3 dy \int_{-y}^{y} dx\,(x^2+y^2)$

$= \int_0^3 dy \left[\left(\frac{x^3}{3} + y^2 \cdot x \right) \Big|_{-y}^{y} \right]$

$= \int_0^3 dy \left[\frac{y^3}{3} + y^3 + \frac{y^3}{3} + y^3 \right]$

$= \frac{8}{3} \int_0^3 dy\, y^3$ ----> [1 d]

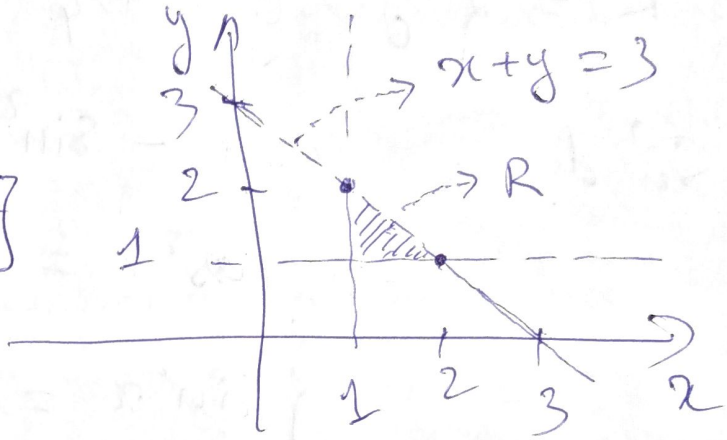
$= \frac{8}{3} \left(\frac{y^4}{4} \Big|_0^3 \right) = 54$ ---> [1,5 d]

Bài 1.2: [2đ]

$$I = \iint_R \frac{dx\,dy}{(x+y)^3}$$

$$R = \begin{cases} x \geq 1,\ y \geq 1 \\ x+y \leq 3 \end{cases}$$

Vẽ hình đúng ---> [0,5đ]



$$I = \int_1^2 dx \int_1^{3-x} dy\, \frac{1}{(x+y)^3} \quad \text{---> [1đ]}$$

$$= \int_1^2 dx \left[-\frac{1}{2}(x+y)^{-2} \right]_1^{3-x}$$

$$= \frac{1}{2}\int_1^2 dx \left\{ \frac{1}{3^2} - \frac{1}{(x+1)^2} \right\} \quad \text{---> [1,5đ]}$$

$$= \frac{1}{36} \approx 0{,}02778 \quad \text{---> [2đ]}$$

Bài 1.3 [2đ]

$$I = \iint_R (\cos^2 x + \sin^2 y)\, dx\, dy$$

$$R = \begin{cases} 0 \le x \le \pi/4 \\ 0 \le y \le \pi/4 \end{cases}$$

Sử dụng: $\cos^2 x - \sin^2 x = \cos(2x)$

$$\hookrightarrow \cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\begin{cases} \sin^2 x = \dfrac{1 - \cos(2x)}{2} \end{cases}$$

$$\Rightarrow \cos^2 x + \sin^2 y = \frac{1}{2}\left[2 + \cos(2x) - \cos(2y)\right] \dashrightarrow [0{,}5đ]$$

$$I = \frac{1}{2}\int_0^{\pi/4} dx \int_0^{\pi/4} dy \left[2 + \cos(2x) - \cos(2y)\right] \dashrightarrow [1{,}0đ]$$

$$= \frac{1}{2}\int_0^{\pi/4} dx \left[2y + y\cos(2x) - \frac{\sin(2y)}{2}\right]_0^{\pi/4}$$

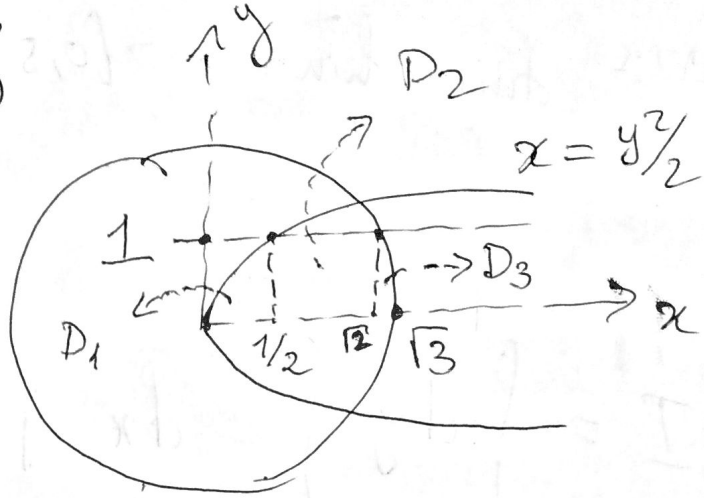
$$= \frac{1}{2}\int_0^{\pi/4} \left\{\frac{\pi}{2} + \frac{\pi}{4}\cos(2x) - \frac{1}{2}\sin\frac{\pi}{2}\right\} dx \dashrightarrow [1{,}5đ]$$

$$= \frac{\pi^2}{16} \approx 0,61685 \dashrightarrow [2đ]$$

Bài 1.4 [3đ]

a) $I = \int_0^1 dy \int_{y^2/2}^{\sqrt{3-y^2}} f(x,y)\,dx$ ---> [1đ]

Vẽ đúng hình ---> [0,5đ]



Miền lấy tích phân

$D = D_1 + D_2 + D_3$

$\to I = I_1 + I_2 + I_3$

$I_1 = \int_0^{1/2} dx \int_0^{\sqrt{2x}} dy\, f(x,y)$

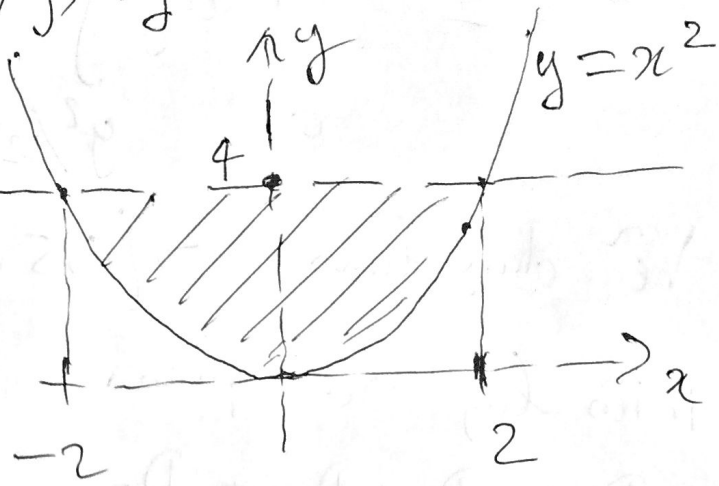
$I_2 = \int_{1/2}^{\sqrt{2}} dx \int_0^{1} dy\, f(x,y)$

$I_3 = \int_{\sqrt{2}}^{\sqrt{3}} dx \int_0^{\sqrt{3-x^2}} dy\, f(x,y)$

$\Rightarrow x^2 + y^2 = 3$

---> [1đ]

b) $I = \int_{-2}^{2} dx \int_{x^2}^{4} f(x,y)\,dy$ ----> [1đ]



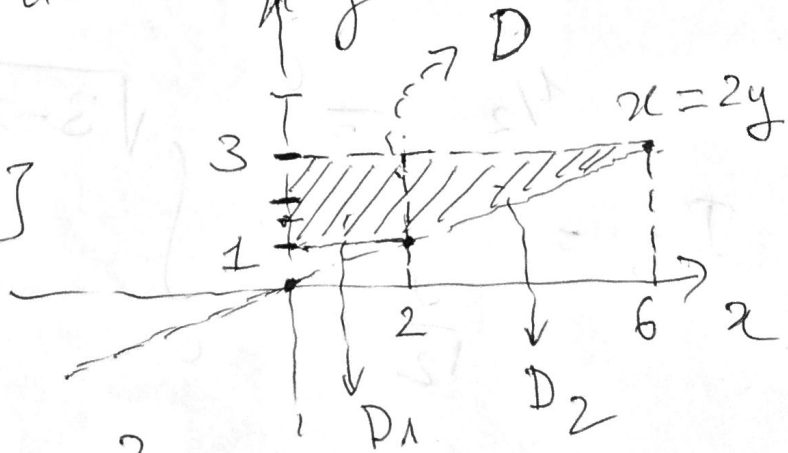
Vẽ được hình ---> [0,5đ]

$$I = \int_{0}^{4} dy \int_{-\sqrt{y}}^{\sqrt{y}} dx\, f(x,y)$$ ----> [1đ]

(Có thể tách thành 2 tích phân:
$\int_{-\sqrt{y}}^{\sqrt{y}} dx = \int_{-\sqrt{y}}^{0} dx + \int_{0}^{\sqrt{y}} dx$)

c)

$$I = \int_{1}^{3} dy \int_{0}^{2y} f(x,y)\,dx$$



Vẽ được hình ---> [0,5đ]

$$I = \int_{1}^{2} dx \int_{1}^{3} dy\, f(x,y) + \int_{2}^{6} dx \int_{x/2}^{3} dy\, f(x,y)$$: ---> [1đ]

Chia D thành 2 miền: $D = D_1 + D_2$.