

Bài tập chương 1.

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1. $f(x) = (x-1)(x-2)^2(x-3)^3$, tính $f'(1)$, $f'(2)$, $f'(3)$

$$f'(x) = 6x^5 - 70x^4 + 320x^3 - 714x^2 + 774x - 324$$

$$\Rightarrow f'(1) = 1.2324, f'(2) = 2, f'(3) = 3$$

2. $f(x) = x + (x+1)\arcsin \frac{x}{x+1}$, tính $f'(1)$

$$f'(x) = x + x\arcsin \left(\frac{\sqrt{x+1}}{x+1} \right) - \arcsin \left(\frac{\sqrt{x+1}}{x+1} \right)$$

$$\Rightarrow f'(1) = 1.$$

3. Tính đạo hàm

a) $y' = x + \sqrt{x} + \sqrt[3]{x}$ $\text{đ/c } y = \frac{\sin^2 x}{\sin x^2}$

$$\Rightarrow y' = 1 + \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}} \quad \rightarrow \quad y' = \frac{\cos(x^2)}{\sin^2(x^2)}$$

b) $y = \frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$ $\text{đ/c } y = \tan \frac{x}{2} - \cot \frac{x}{2}$

$$\Rightarrow y' = -\frac{1}{x^2} - \frac{1}{2x^{\frac{3}{2}}} - \frac{1}{3x^{\frac{4}{3}}} \quad \rightarrow \cancel{y'} =$$

$$\text{đ/c } y = \sqrt[3]{\frac{1+x^3}{1-x^3}}$$

$$\Rightarrow y' = \frac{2x^2}{(1-x^3)^{\frac{4}{3}}(1+x^3)^{\frac{2}{3}}}$$

$$\text{đ/c } y = \ln(x + \sqrt{1+x^2})$$

$$y' = \frac{(1+x^2)^{\frac{1}{2}} + x}{x^2 + x(1+x^2)^{\frac{1}{2}} + 1}$$

i) $y = e^x \ln(\sin x)$

$$y' = e^x \cot(x) + e^x \ln(\sin(x))$$

j) $y = e^{\arctan x} \rightarrow y' = \frac{e^{\arctan x}}{x^2 + 1}$

4. Viết phương trình tiếp tuyến với đường cong

$$y = 2x^3 - 3x^2 - x + 5 \text{ tại } A(3, 2)$$

$$y' = 3x^2 - 6x - 1 \Rightarrow k = y'(3) = 8$$

phương trình tiếp tuyến tại điểm $M(3; 3)$ là

$$\begin{aligned} d: y &= y_0(x - x_0) + y_0 \\ &= 8(x - 3) + 2 = 8x - 22. \end{aligned}$$

5. Tìm vi phân các hàm số sau

a) $y = \frac{1}{x} \rightarrow dy = \frac{-1}{x^2} dx$

b) $y = \frac{1}{a} \arctan\left(\frac{x}{a}\right) \text{ với } a \neq 0$

$$\rightarrow dy = \frac{1}{a^2 + x^2} dx$$

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c) $y = \frac{1}{2a} \ln(1 + \sqrt{x^2 + a^2}) \text{ với } a \neq 0$

$$\rightarrow dy = \frac{x}{2a(x^2 + a^2)^{\frac{1}{2}} (1 + (x^2 + a^2)^{\frac{1}{2}})} dx$$

d) $y = \arcsin \frac{x}{a} \text{ với } a \neq 0$

$$\rightarrow dy = \frac{1}{a\sqrt{1 - \frac{x^2}{a^2}}} dx$$

e) $y = xe^x \rightarrow dy = xe^x + e^x$

$$f) y = \sqrt{a^2 + x^2} \rightarrow dy = \frac{x}{(a^2 + x^2)^{\frac{1}{2}}} dx$$

$$g) y = \frac{x}{\sqrt{1-x^2}} \rightarrow dy = \frac{1}{(-x^2+1)^{\frac{3}{2}}} dx$$

$$h) y = \ln(1-x^2) \rightarrow dy = -\frac{2x}{1-x^2} dx.$$

6.

$$a) \frac{d(x^3 - 2x^6 - x^9)}{d(x^3)} = \frac{(3x^2 - 12x^5 - 9x^8)dx}{3x^2 dx} = 1 - 4x^3 - 3x^6$$

$$c) \frac{d(\sin x)}{d(\cos x)} = \frac{\cos x dx}{-\sin x dx} = -\cot x$$

$$b) \frac{d}{d(x^2)} \left(\frac{\sin x}{x} \right) = \frac{x \cos(x) - 2 \sin(x)}{x^3}$$

7. Tim y''

$$a) y = x\sqrt{1+x^2} \rightarrow y'' = \frac{x(2x^2+3)}{(x^2+1)^{\frac{3}{2}}}$$

$$b) y = \frac{x}{\sqrt{x-x^2}} \rightarrow y' = \frac{-3x(x-x^2)^{\frac{1}{2}} + 6x^2(x-x^2)^{\frac{1}{2}}}{4(x-x^2)^{\frac{3}{2}}} \\ \rightarrow y'' = \frac{-3x(x-x^2)^{\frac{1}{2}} + 6x^2(x-x^2)^{\frac{1}{2}} + 2(x-x^2)^{\frac{3}{2}}}{4(x-x^2)^{\frac{5}{2}}}.$$

$$c) y = e^{-x^2} \rightarrow y'' = 4x^2 e^{-x^2} - 2e^{-x^2}$$

$$d) y = \ln(f(x)) \rightarrow y'' = 0$$

8. Tìm $y'(x)$, $y''(x)$ của hàm số $y = f(x)$ cho dưới dạng tham số

a) $x = 2t - t^2$, $y = 3t - t^3$

$$\begin{aligned} \text{)} \quad & x'(t) = 2 - 2t \\ \text{)} \quad & y'(t) = 3 - 3t^2 \end{aligned}$$

$$\rightarrow x''(t) = -2$$

$$y''(t) = -6t$$

taco': $y'(x) = \frac{y'(t)}{x'(t)}$
 $y''(x) = \frac{y''(t)x'(t) - y'(t)x''(t)}{(x'(t))^3} \cdot y(t)$

ICL: $y'(x) = \frac{y'(t)}{x'(t)} = \frac{3 - 3t^2}{2 - 2t} = \frac{3}{2}(1+t)$

$$y''(x) = \frac{-6t(2-2t) - (-2)(3-3t^2)}{(2-2t)^3}$$

$$= \frac{-12t + 12t^2 + 6 - 6t^2}{8(1-t)^3}$$

$$= \frac{12t^2 - 12t + 6}{8(1-t)^3}$$

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b) $x = a \cot(t)$, $y = a \sin(t)$

$$x'(t) = -a \csc^2(t)$$

$$\text{)} \quad y'(t) = a \cos(t)$$

$$\rightarrow x''(t) = 2a \csc^2(t) \cot(t)$$

$$y''(t) = -a \sin(t)$$

ICL: $y'(x) = \frac{y'(t)}{x'(t)} = \frac{-a \csc^2(t)}{a \cos(t)}$

$$= -\sec(t) \csc^2(t)$$

$$y''(x) = \frac{-a \sin(t) \cdot (-a) \csc^2(t) - 2a \csc^2(t) \cot(t) \cdot a \cos(t)}{(-a \csc^2(t))^3}$$



$$= -\frac{\sin^5(t)}{a} + \frac{2\cos^2(t)\sin^3(t)}{a^2}$$

$$\text{g, } u = a(t - \sin t), y = a(1 - \cos t)$$

$$\left\{ \begin{array}{l} x(t) = a - a \cos(t) \Rightarrow x'(t) = a \sin(t) \\ y(t) = a \sin(t) \quad \quad \quad y''(t) = a \cos(t) \\ y'(x) = \frac{y'(t)}{x'(t)} = \frac{a - a \cos(t)}{a \sin(t)} = \frac{1 - \cos(t)}{\sin(t)} \end{array} \right.$$

$$y''(x) = \frac{a \cos(t) \cdot a - a \cos(t) - a \sin(t) \cdot a \sin(t)}{(a - a \cos(t))^3}$$

$$= \frac{\cos(t) - \frac{\cos(t)}{a} - \sin^2(t)}{a(1 - \cos(t))^3}$$

$$a(1 - \cos(t))^3$$

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