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6)

$$A = \begin{bmatrix} -1 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 5 \\ -3 & -1 \end{bmatrix}.$$

a, $A - X = -2B^T$.

$$\Leftrightarrow \begin{bmatrix} -1 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & -6 \\ 4 & 10 & -2 \end{bmatrix} = X.$$

$$\Leftrightarrow X = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 8 & -5 \end{bmatrix}$$

b, $Y^T - BA = 0$.

$$\Leftrightarrow Y^T = \begin{bmatrix} 1 & 2 \\ 0 & 5 \\ -3 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\Leftrightarrow Y^T = \begin{bmatrix} -1 & -1 & -2 \\ 0 & -10 & -15 \\ 3 & -7 & -9 \end{bmatrix}$$

$$\Leftrightarrow Y = \begin{bmatrix} -1 & 0 & 3 \\ -1 & -10 & -7 \\ -2 & -15 & -9 \end{bmatrix}.$$

10, a,

$$\begin{vmatrix} 1 & 3 & -6 \\ 1 & -1 & 2 \\ -8 & 5 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 \\ 5 & 4 \end{vmatrix} - 3 \times \begin{vmatrix} 1 & 2 \\ -8 & 4 \end{vmatrix} - 6 \times \begin{vmatrix} 1 & -1 \\ -8 & 5 \end{vmatrix}.$$

$$= -14 - 3 \times 20 - 6 \times (-3)$$

$$= -56$$

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10, 7,

$$\begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix}$$

$$\xrightarrow[h_4 - h_1]{h_3 - 2h_1} \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & -3 & 3 \\ 0 & -3 & 1 & 1 \end{vmatrix}$$

$$\xrightarrow[h_4 + 3h_2]{h_3 + 3h_2} \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -6 & 9 \\ 0 & 0 & -2 & 7 \end{vmatrix}$$

$$\xrightarrow{h_4 + h_3 \cdot \frac{1}{(-3)}} \begin{vmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -6 & 9 \\ 0 & 0 & 0 & 4 \end{vmatrix} = 24.$$

$\cancel{\Leftrightarrow 1 \quad 24}$

12,

a, $m = 1 \Rightarrow A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & -1 \\ 2 & 1 & 3 \end{bmatrix}$.

$$|A| = -2 \cdot \begin{vmatrix} -1 & -1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 4 - 5 + 3 = 2.$$

$$|5A^t| = 5^3 \cdot |A| = 125 \times 2 = 250.$$

$$|A^4| = |A|^4 = 2^4 = 16.$$



Vậy ~~$|A| = 2$~~ ,

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Vậy $\det A = 2$, $\det(5A^{-1}) = 250$, $\det(A^4) = 16$

12, b,

Có $|A| = 3$

$\Rightarrow |A^{-1}| = \dfrac{1}{3}$

$|2A^2| = 2^3 \cdot |A|^2 = 8 \cdot 9 = 72$.

Vậy $\det(A^{-1}) = \dfrac{1}{3}$, $\det(2A^2) = 72$.

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17, c

Ta có: $C^{-1} = \frac{1}{|C|}\cdot C^* = \cancel{\frac{[C_{ij}]^T}{|C|}},\ \frac{1}{|C|}\cdot [C_{ij}]^T$

Lại có $C_{ij} = (-1)^{(i+j)}\cdot M_{ij}$

$\Rightarrow C_{11} = \begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix} = 2.$

$C_{12} = -1\cdot\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1.$

. . . . . . . . .

$C_{33} = \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} = -5.$

$\Rightarrow [C_{ij}] = \begin{bmatrix} 2 & -1 & -1 \\ 10 & -2 & 4 \\ -14 & 1 & -5 \end{bmatrix}$

$\Rightarrow [C_{ij}]^T = \begin{pmatrix} 2 & 10 & -14 \\ -1 & -2 & 1 \\ -1 & 4 & 5 \end{pmatrix}$

$|C| = -1 \times \begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$

$= -2 - 1 - 3 = -6.$

$\Rightarrow C^{-1} = \frac{-1}{6}\cdot\begin{bmatrix} 2 & 10 & -14 \\ -1 & -2 & 1 \\ -1 & 4 & 5 \end{bmatrix}.$

Vậy $C^{-1} = \frac{-1}{6}\begin{bmatrix} 2 & 10 & -14 \\ -1 & -2 & 1 \\ -1 & 4 & 5 \end{bmatrix}.$

15,

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a, $|A| = 2 \cdot \begin{vmatrix} 3 & 0 \\ 1 & x \end{vmatrix} - \begin{vmatrix} 4 & 0 \\ 1 & x \end{vmatrix} - \begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix}$

$= 6x - 4x - 1 = 2x - 1.$

Có $|A| \cdot |A^{-1}| = 1$

mà $|A^{-1}| = 2 \Rightarrow |A| = \frac{1}{2}$

$\Leftrightarrow 2x - 1 = \frac{1}{2}.$

$\Leftrightarrow x = \frac{\frac{1}{2} + 1}{2} = \frac{3}{4}.$

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15, b,

$x = 2 \Rightarrow A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \cdot A^* = \frac{1}{|A|} \cdot [A_{ij}]^T$$

$$A_{11} = \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = 6$$

$$A_{12} = -\begin{vmatrix} 4 & 0 \\ 1 & 2 \end{vmatrix} = -8$$

.....

$$A_{33} = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2.$$

$$\Rightarrow [A_{ij}] = \begin{bmatrix} 6 & 8 & 1 \\ -3 & 5 & -1 \\ 3 & -4 & 2 \end{bmatrix}.$$

$$\Rightarrow [A_{ij}]^T = \begin{bmatrix} 6 & -3 & 3 \\ 8 & 5 & -4 \\ 1 & -1 & 2 \end{bmatrix}.$$

$$|A| = 2 \times \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 4 & 0 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix}.$$

$$= 12 - 8 - 1 = 3.$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ 8/3 & 5/3 & -4/3 \\ 1/3 & -1/3 & 2/3 \end{bmatrix}$$

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26, c

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 3 & 1 & 3 & 1 \\ 3 & 5 & 3 & 5 & 3 \\ 7 & 9 & 7 & 9 & 7 \end{bmatrix}$$

$$\xrightarrow{h_1 + h_2} \begin{bmatrix} 1 & 4 & 1 & 4 & 1 \\ 1 & 3 & 1 & 3 & 1 \\ 3 & 5 & 3 & 5 & 3 \\ 7 & 9 & 7 & 9 & 7 \end{bmatrix}$$

$$\xrightarrow[h_4 - 7h_1]{h_2 - h_1,\ h_3 - 3h_1} \begin{bmatrix} 1 & 4 & 1 & 4 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & -7 & 0 & -7 & 0 \\ 0 & -19 & 0 & -19 & 0 \end{bmatrix}$$

$$\xrightarrow[h_4 - 19h_2]{h_3 - 7h_2} \begin{bmatrix} 1 & 4 & 1 & 4 & 1 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow r(C) = 2$

27, c,

$$C = \begin{bmatrix} 3 & 21 & 0 & 9 & 0 \\ 0 & 7 & -1 & -2 & -1 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow r(C) = 3$

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29,

$$A = \begin{bmatrix} 3 & 1 & 4 & 1 \\ m & 2 & 3 & 1 \\ 3 & -1 & 1 & 0 \\ 3 & 3 & 7 & 2 \end{bmatrix}$$

$$\xrightarrow{h_3 - h_1,\ h_4 - h_1} \begin{bmatrix} 3 & 1 & 4 & 1 \\ m & 2 & 3 & 1 \\ 0 & -2 & -3 & -1 \\ 0 & 2 & 3 & 1 \end{bmatrix}.$$

$$\xrightarrow[h_4 - h_2]{h_3 + h_2} \begin{bmatrix} 3 & 1 & 4 & 1 \\ m & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Để A có hạng = 2 => $m = 0$

33,

ý 3:

$$\begin{cases} 3x + 2y + 3z + 4t = 1 \\ x + y + z = -2. \\ 6x + 5y + 6z + 4t = -5. \\ 7x + 5y + 7z + 8t = 0. \end{cases}$$

$$A(bs) = \left[\begin{array}{cccc|c} 3 & 2 & 3 & 4 & 1 \\ 1 & 1 & 1 & 0 & -2 \\ 6 & 5 & 6 & 4 & -5 \\ 7 & 5 & 7 & 8 & 0 \end{array}\right].$$

hoán đổi $h_1$ và $h_2$

$$\Leftrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -2 \\ 3 & 2 & 3 & 4 & 1 \\ 6 & 5 & 6 & 4 & -5 \\ 7 & 5 & 7 & 8 & 0 \end{array}\right]$$

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$\xrightarrow[h_4 - 7h_1]{h_2 - 3h_1,\ h_3 - 6h_1} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -2 \\ 0 & -1 & 0 & 4 & 7 \\ 0 & -1 & 0 & 4 & 7 \\ 0 & -2 & 0 & 8 & 14 \end{array}\right]$

$\xrightarrow[h_4 - 2h_2]{h_3 - h_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -2 \\ 0 & -1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Ta thấy $r(A) = r(A^{bs}) = 2 <$ số ẩn $= 4$
=> hệ phương trình vô số nghiệm
Hệ phương trình trở thành

$\begin{cases} x + y + z = -2 \\ -y + 4t = 7 \end{cases}$

$\Leftrightarrow \begin{cases} x + y + z = -2 \\ y = -7 + 4t \end{cases}$

$\Leftrightarrow \begin{cases} x = \cancel{2} - z - 4t + \cancel{7}\,5 \\ y = -7 + 4t \end{cases}$

Vậy $\begin{cases} x = 5 - 4t - z \\ y = 4t - 7 \\ z, t \text{ tùy ý } \in \mathbb{R} \end{cases}$

35, a, $\begin{cases} x - 2y + z - t = -1 \\ 3x + y - 2z + t = 2 \\ x + 5y - 4z + mt = 5 \end{cases}$

$A^{bs} = \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 3 & 1 & -2 & 1 & 2 \\ 1 & 5 & -4 & m & 5 \end{array}\right]$

$\xrightarrow[h_3 - h_1]{h_2 - 3h_1} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 0 & 7 & -5 & 4 & 5 \\ 0 & 7 & -5 & m+1 & 6 \end{array}\right]$

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$$\xrightarrow{h_3 - h_2} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 1 \\ 0 & 7 & -5 & 4 & 5 \\ 0 & 0 & 0 & m-3 & 1 \end{array}\right]$$

Hệ phương trình có nghiệm $\Leftrightarrow r(A) = r(\bar{A}^{bs})$.

$\Leftrightarrow r(A) = 3$

$\Leftrightarrow m - 3 \neq 0$.

$\Leftrightarrow m \neq 3$.

Vậy $m \neq 3$ thì hệ phương trình có nghiệm.