BT 3

Biểu điểm

Bài 1.17 → 1 đ

1.18 → 1 đ

1.19 → 1 đ

1.20 → 1 đ

1.21 → 1 đ

1.22 → 3 đ

1.23 → 2 đ

BT3

Bài 1.17

$$\sum_{x=0}^{\infty} f(x) = 1$$

$$\Rightarrow k \sum_{x=0}^{\infty} \frac{1}{2^x} = 1$$

Đặt $A = \sum_{x=0}^{\infty} \frac{1}{2^x} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots$

$$\Rightarrow 2A = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots = 2 + A$$

$$\Rightarrow A = 2$$

$$\Rightarrow k = \frac{1}{2}$$

$\cancel{P(x \geq 4) = 1 - P(x < 4) = 1 - P}$

$$P(x \geq 4) = k\left(\frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \cdots\right)$$

$$= \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \cdots = B$$

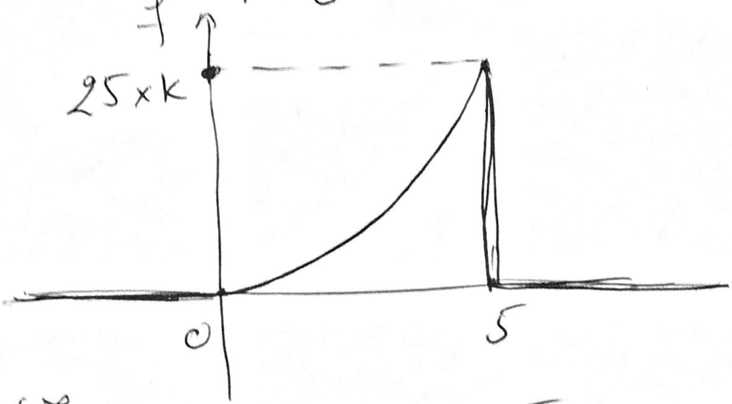
$$\rightarrow 2 \cdot B = \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} = \frac{1}{2^4} + B$$

$$\Rightarrow B = \frac{1}{2^4} = \frac{1}{16}$$

Vậy $P(x \geq 4) = \frac{1}{16}$

Bài 1.18

$$f(x) = \begin{cases} k x^2 & \text{nếu } 0 \le x \le 5 \\ 0 & x < 0 \text{ và } x > 5 \end{cases}$$



$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_0^5 k x^2\,dx = 1$$

$$\Rightarrow k \cdot \left(\frac{x^3}{3} \Big|_0^5 \right) = k \cdot \frac{5^3}{3} = 1$$

$$\Rightarrow k = \frac{3}{5^3} = \frac{3}{125}$$

Hàm phân bố tích luỹ $F(x_0)$:

$$F(x_0) = \int_{-\infty}^{x_0} f(x)\,dx$$

Nếu $x_0 \le 0 \Rightarrow F(x_0) = 0$

Nếu $0 < x_0 \le 5 \Rightarrow F(x_0) = \int_0^{x_0} k x^2\,dx$

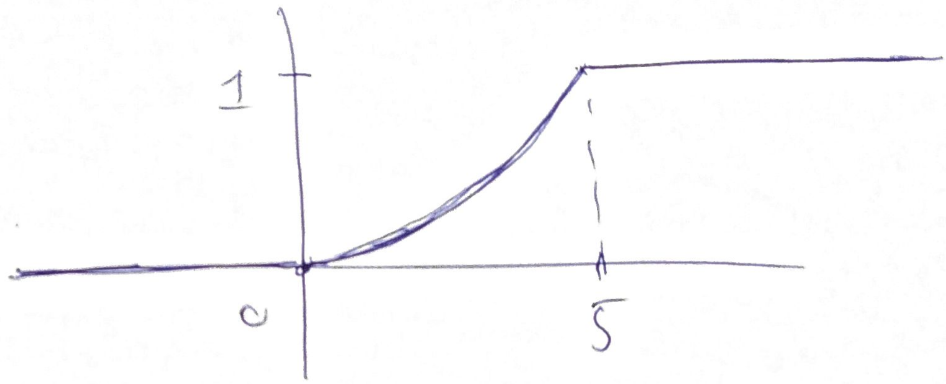
$$= k\left(\frac{x^3}{3} \Big|_0^{x_0}\right) = \frac{k}{3} \cdot x_0^3 = \frac{1}{125} x_0^3$$

Nếu $x_0 > 5 \Rightarrow F(x_0) = \int_0^5 k x^2\,dx = 1$

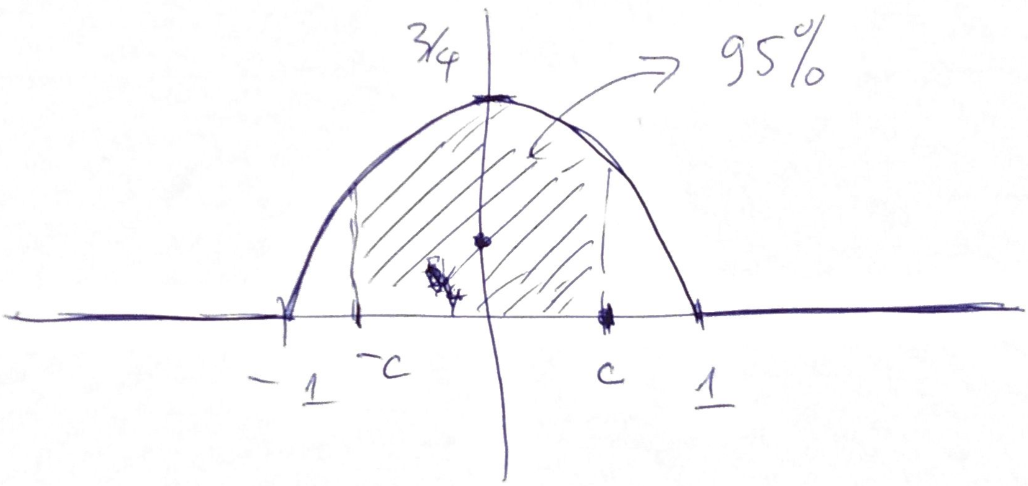
bt 3-3

Vậy $F(x) = \begin{cases} 0 & \text{nếu } x \le 0 \\ \dfrac{x^3}{125} & \text{nếu } 0 < x \le 5 \\ 1 & \text{nếu } x > 5 \end{cases}$

Vẽ hình



Bài 1.19



$L = 200 + x$

$200 - c \le L \le 200 + c \Rightarrow -c \le x \le c$

$\Rightarrow \int_{-c}^{c} f(x)\,dx = \frac{3}{4}\int_{-c}^{c}(1 - x^2)\,dx = 0,95$

$\Rightarrow \frac{3}{4}\left[\left(x - \frac{x^3}{3}\right)\Big|_{-c}^{c}\right] = 0,95$

$\Rightarrow 10c^3 - 30c + 19 = 0$

$\Rightarrow c = 0,811$

Bài 1.20 Bt 3-4

$$F(x) = \begin{cases} 0 & \text{nếu } x \le 0 \\ 1 - e^{-3x} & \text{nếu } x > 0 \end{cases}$$

$\Rightarrow f(x) = 0$ nếu $x \le 0$

Nếu $y > 0$ ta có:

$$F(y) = \int_0^y f(x)\,dx = G(y) - G(0)$$

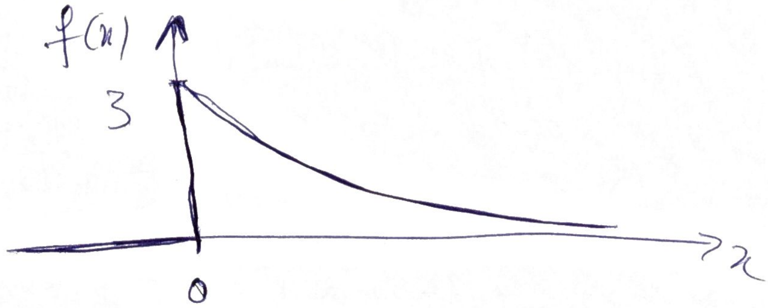
trong đó $G(x) = \int f(x)\,dx$: là nguyên hàm của $f(x)$

Ta có: $G(y) - G(0) = 1 - e^{-3y} = -e^{-3y} - (-e^{3\cdot 0})$

$$\Rightarrow G(x) = -e^{-3x}$$

$$\Rightarrow f(x) = G'(x) = -e^{-3x} \cdot (-3) = 3e^{-3x}$$

Vậy $f(x) = \begin{cases} 0 & \text{nếu } x \le 0 \\ 3e^{-3x} & \text{nếu } x > 0 \end{cases}$



$F(y) = 0.9$

$\Rightarrow 1 - e^{-3y} = 0.9$

$\Rightarrow e^{-3y} = 0.1$

$\Rightarrow -3y = \ln(0.1) \Rightarrow y = -\dfrac{\ln(0.1)}{3} = 0{,}768$

Bài 1.21

$$F(x) = \begin{cases} 0 & \text{nếu } x < 2 \\ (x^2-4)/5 & \text{nếu } 2 \le x < 3 \\ 1 & \text{nếu } x \geq 3 \end{cases}$$

$$P(2.5 \le x \le 5) = F(5) - F(2.5)$$

$$= 1 - \frac{2.5^2 - 4}{5} = 0,55$$

Bài 1.22

a) $$\mu = \sum_{i=1}^{6} x_i \cdot p_i = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \dots + 6 \cdot \frac{1}{6}$$

$$= (1+2+\dots+6) \cdot \frac{1}{6} = \frac{(1+6) \cdot 6}{2} \cdot \frac{1}{6} = \frac{7}{2} = 3,5$$

$$\sigma^2 = \sum_{i=1}^{6} x_i^2 p_i - \mu^2 = (1^2 + 2^2 + \dots + 6^2) \cdot \frac{1}{6} - (3,5)^2$$

$$= \frac{91}{6} - (3,5)^2 = 2,917$$

b) $$\mu = \sum_{i=0}^{3} x_i f(x_i) = 0 \times 0,512 + 1 \times 0,384 + 2 \times 0,096 + 3 \times 0,008 = 0,6$$

$$\sigma^2 = \sum_{i=0}^{3} x_i^2 f(x_i) - \mu^2 = 0^2 \times 0,512 + 1^2 \times 0,384 + 2^2 \times 0,096 + 3^2 \times 0,008 - (0,6)^2$$

$$= 0,48$$

c) $$\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 x \cdot 2x \cdot dx = 2 \cdot \frac{x^3}{3} \Big|_0^1 = \frac{2}{3}$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2 = \int_0^1 2x^3 dx - \frac{4}{9} = 2 \cdot \frac{1}{4} - \frac{4}{9} = \frac{1}{18}$$

Bài 1.23

$$X = x \cdot 10000 \text{ (ga lon)}$$

a,
$$\mu_x = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 x\, 6x(1-x)\, dx$$

$$= 6\int_0^1 (x^2 - x^3)\, dx = 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2}$$

$$\Rightarrow \mu_X = \frac{1}{2} \times 10000 = 5000 \text{ (ga lon)}$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu_x^2 = \int_0^1 6x^3(1-x)\, dx - \left(\frac{1}{2}\right)^2$$

$$= 6\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1 - \frac{1}{4} = \frac{1}{20}$$

$$\Rightarrow \sigma_X^2 = \frac{1}{20} \cdot 10000^2 = 5\,000\,000 \text{ (galon}^2\text{)}$$

Độ lệch chuẩn: $\sigma_X = 2\,236{,}068$ (ga lon)

b) Gọi V là thể tích của trạm xăng.

Xác suất bán hết là: $P(X \geq V) = P\left(x \geq \frac{V}{10000}\right)$

Đặt $\frac{V}{10000} = c$, ta có:
(Lưu ý: $0 < c < 1$)

$$\int_c^1 6x(1-x)\, dx = 0{,}95$$

$$\Rightarrow \int_0^c 6x(1-x)\, dx = 0{,}05 \Rightarrow 6\left(\frac{c^2}{2} - \frac{c^3}{3}\right) = 0{,}05$$

$$\Rightarrow c = 0{,}135 \Rightarrow V = 1350 \text{ (ga lon)}.$$