$5^{10} \bmod 221 = [(5^{2} \bmod 221) \times (5^{8} \bmod 221)] \bmod 221$

$[(5^{2} \bmod 221)^{5}] \bmod 221 = 77$

$$18^{47} \bmod 221 = [(18^7 \bmod 221) \times (18^8 \bmod 221) \times (18^8 \bmod 221) \times (18^8 \bmod 221)$$

$$\times (18^8 \bmod 221) \times (18^8 \bmod 221)] \bmod 221$$

$$= [(18^7 \bmod 221) \times (18^8 \bmod 221)^5] \bmod 221$$

$$18^7 \bmod 221 = 86$$

$$18^8 \bmod 221 = 1$$

$$18^{47} \bmod 221 = (86 \times 1^5) \bmod 221 = 86$$

$$509^{253} \bmod 1147 = [(509^3 \bmod 1147) \times (509^2 \bmod 1147) \times (509^4 \bmod 1147)^{62}] \bmod 1147$$

$$509^3 \bmod 1147 = 492$$

$$509^2 \bmod 1147 = 1006$$

$$509^4 \bmod 1147 = 382$$

$$382^{62} \bmod 1147 = [(382^2 \bmod 1147) \times (382^5 \bmod 1147)^{12}] \bmod 1147$$

$$382^2 \bmod 1147 = 255$$

$$382^5 \bmod 1147 = 118$$

$$118^{12} \bmod 1147 = [(118^6 \bmod 1147)^2] \bmod 1147$$

$$118^6 \bmod 1147 = 63$$

$$118^{12} \bmod 1147 = 63^2 \bmod 1147 = 528$$

$$382^{62} \bmod 1147 = (255 \times 528) \bmod 1147 = 441$$

$$509^{253} \bmod 1147 = (492 \times 1006 \times 441) \bmod 1147 = 879$$