

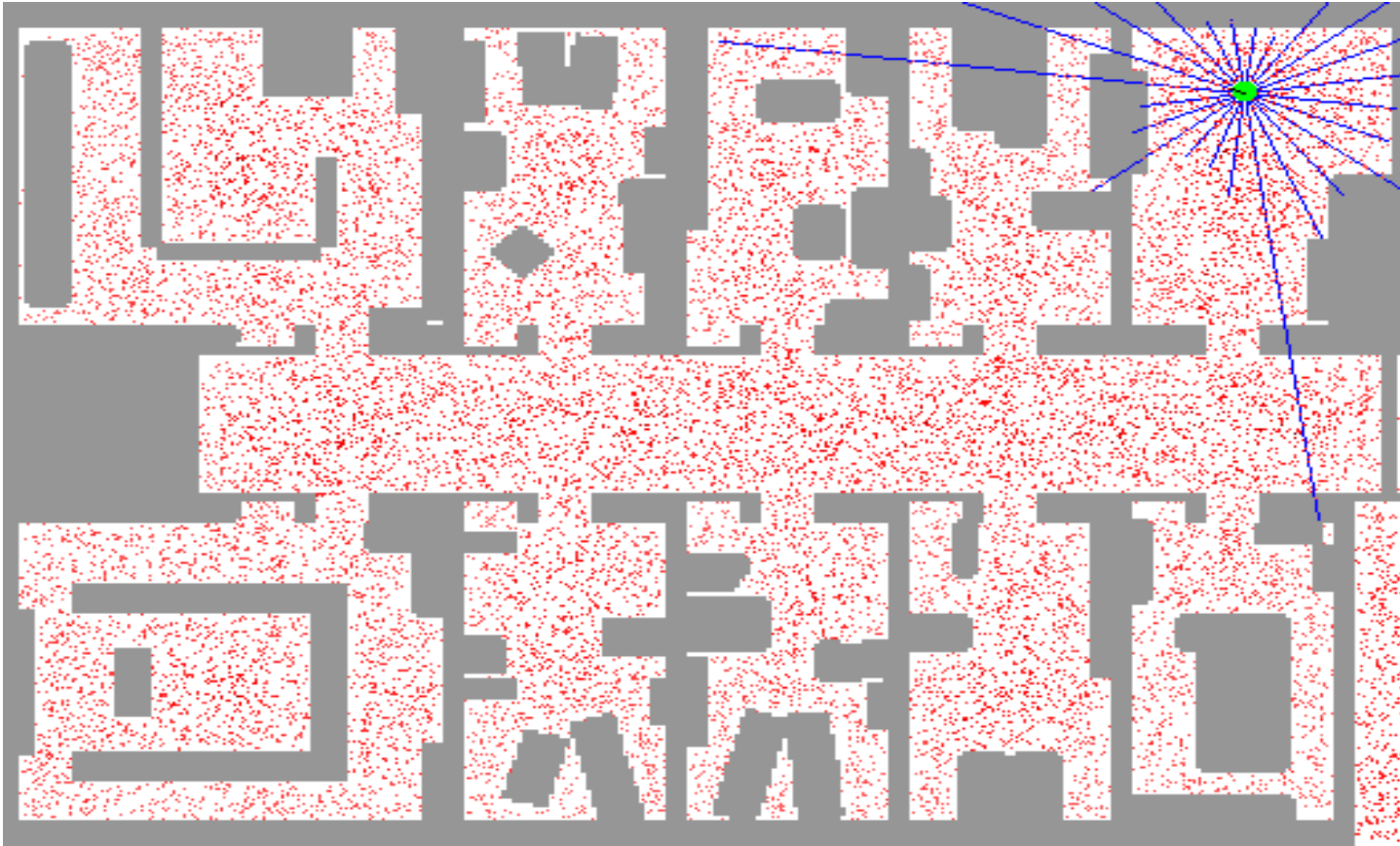
## Lecture 17

# Simultaneous Localization and Mapping

CS 3630



# Particle Filter Localization (using sonar)



We've seen how a robot can localize itself with respect to a known map.

But what if there's no map??

# The central question for SLAM

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Is it possible for a mobile robot to be placed at an **unknown location** in an **unknown environment** and for the robot to incrementally build a consistent map of this environment while simultaneously determining its location within this map?

# In other words...

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## Given:

- The robot's controls  $u_{1:T} = \{u_1, u_2, u_3 \dots, u_T\}$
- The robot's sensing observations  $z_{1:T} = \{z_1, z_2, z_3 \dots, z_T\}$

## Compute:

- Map of the environment  $m$
- Path of the robot  $x_{0:T} = \{x_0, x_1, x_2 \dots, x_T\}$

# A chicken and egg problem!

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- If the robot had a map, localization would be easier.
- If the robot could localize, mapping would be easier.
- ... But the robot has neither; it starts from a blank slate.
- Furthermore, the robot must also execute an exploration strategy.

# The basic algorithm

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## 1. The robot moves

- increases the uncertainty on robot pose
- need a mathematical model for the motion (*motion model*)

## 2. The robot discovers interesting features in the environment called *landmarks*

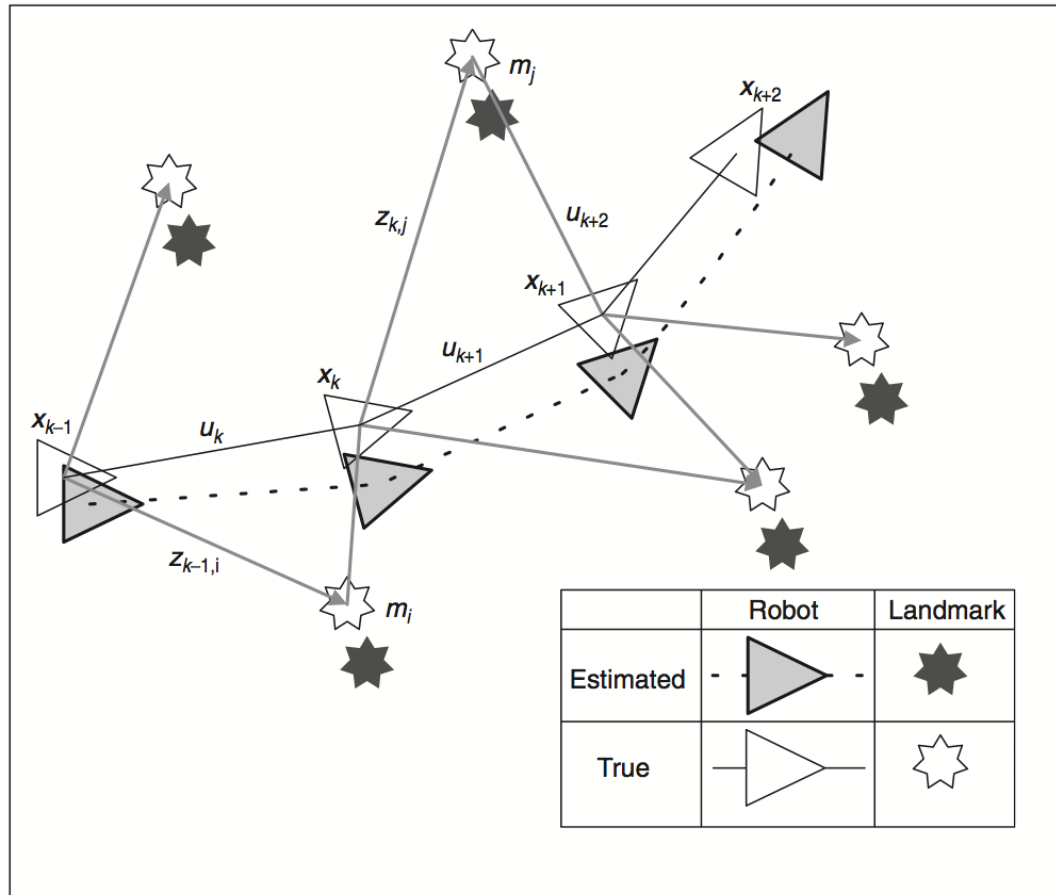
- need a mathematical model to determine the position of the landmarks from sensor data (*inverse observation model*)
- need to model uncertainty in the location of landmarks

## 3. The robot observes landmarks that had been previously mapped

- uses them to correct both self localization and the localization of all landmarks in space
- uncertainties for both decrease
- need a model to predict the measurement from predicted landmark location and robot localization (*direct observation model*)

# In pictures....

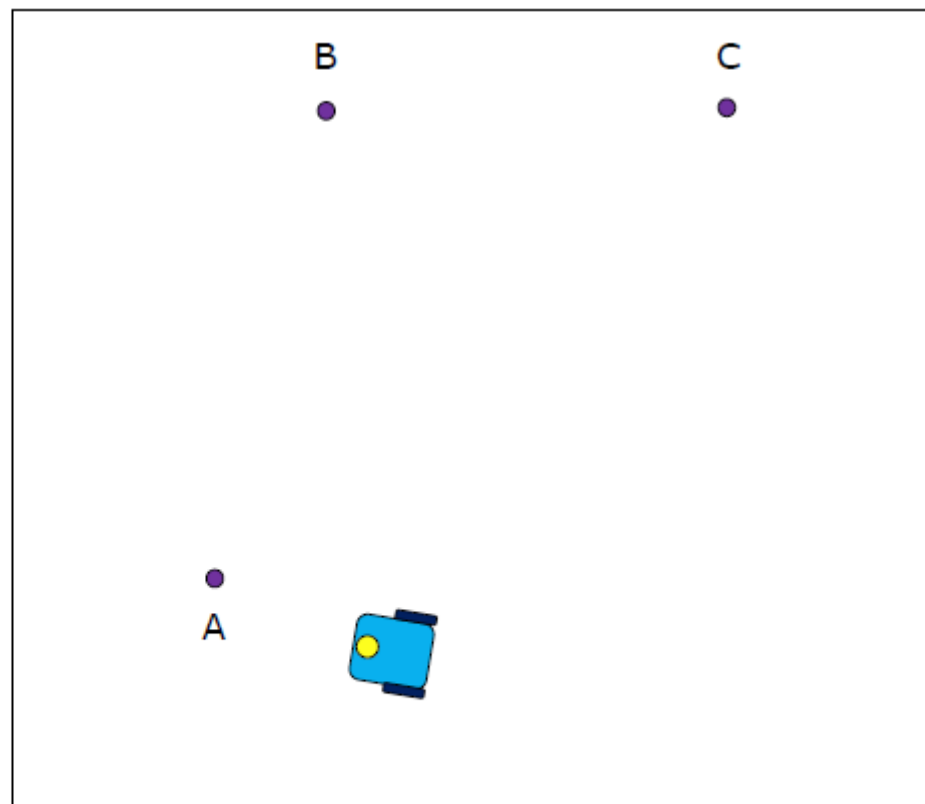
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# SLAM Example

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- Assumption: Robot's uncertainty at starting position is zero



Start: robot has zero uncertainty

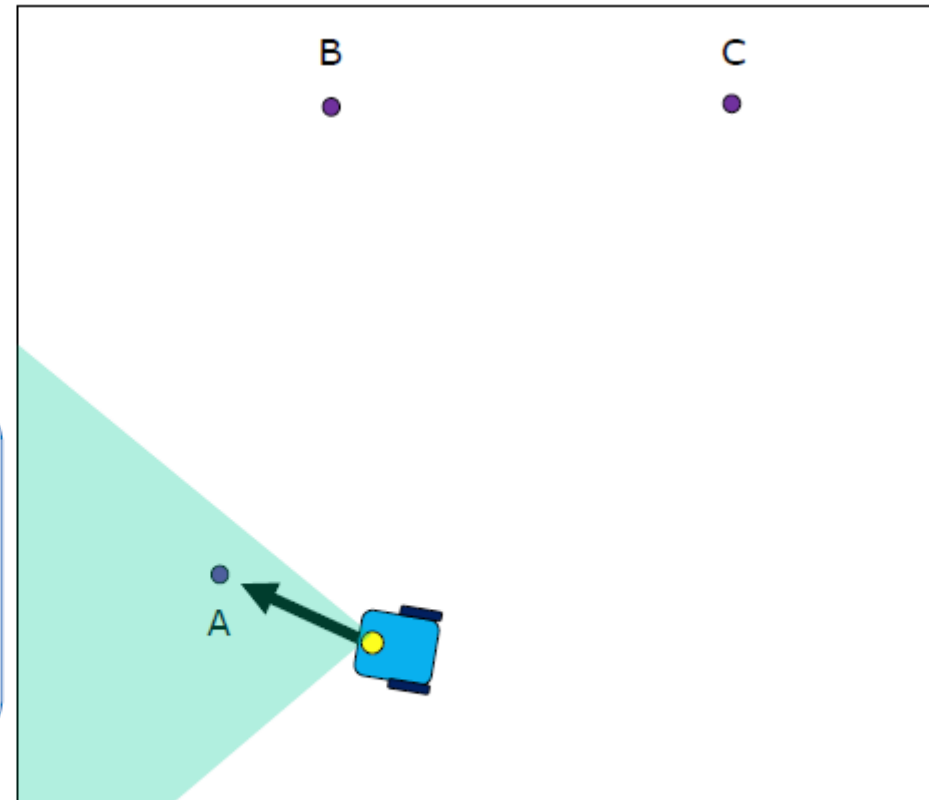


# SLAM Example

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On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



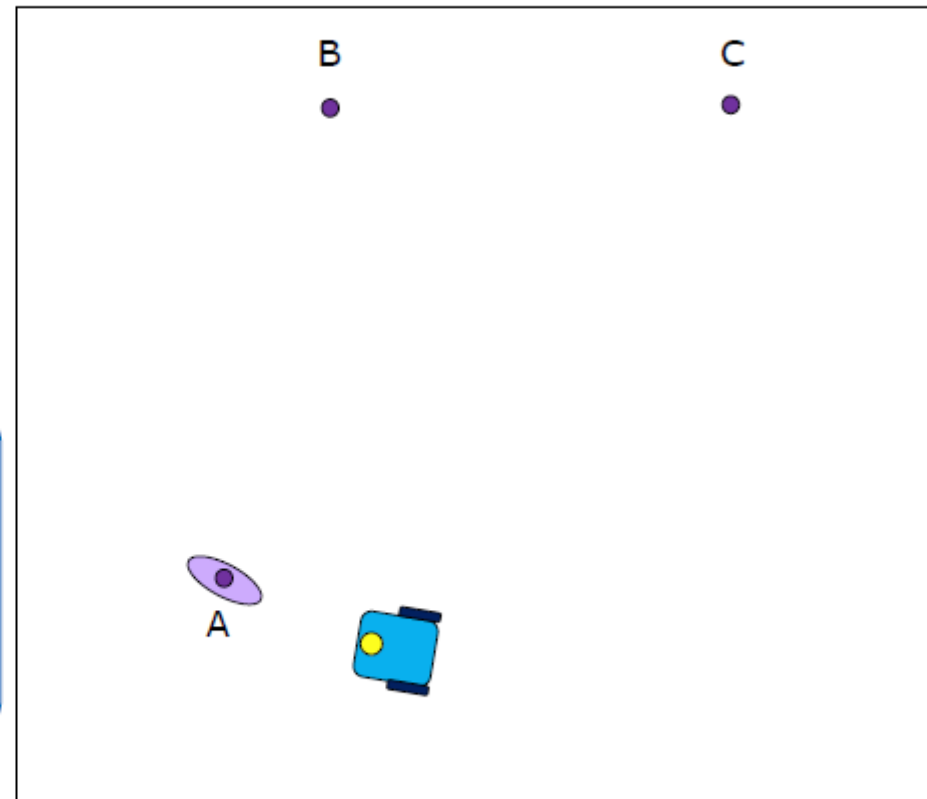
First measurement of feature A

# SLAM Example

- The robot observes a feature which is mapped with an uncertainty related to the **measurement model**

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations

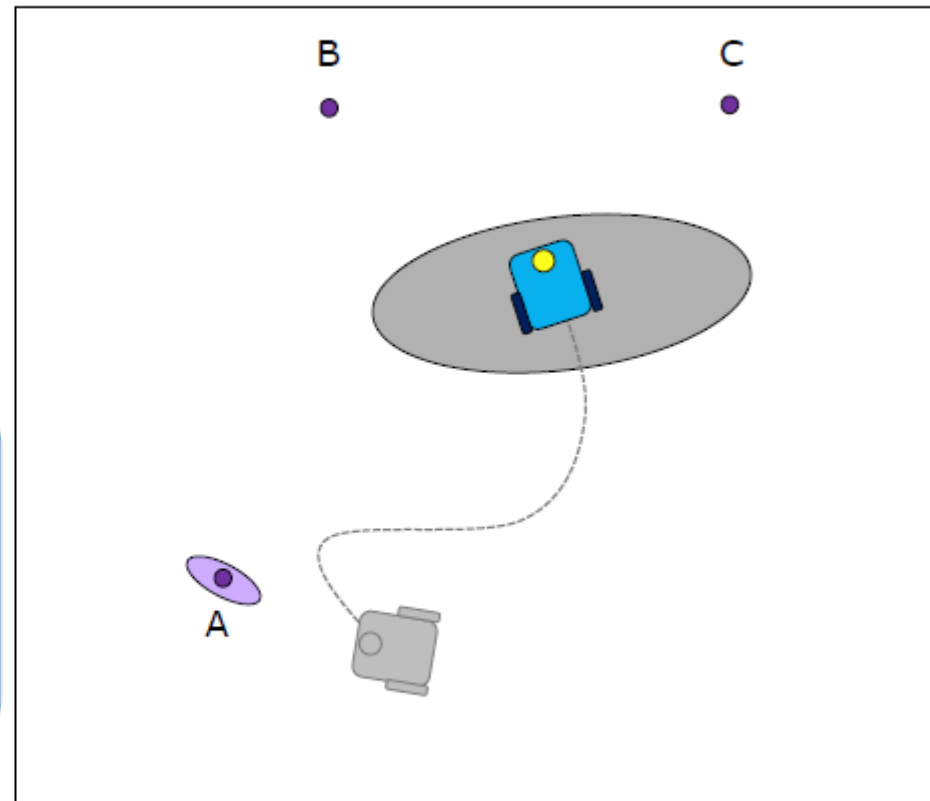


# SLAM Example

- As the robot moves, its pose uncertainty increases, obeying the robot's **motion model**.

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



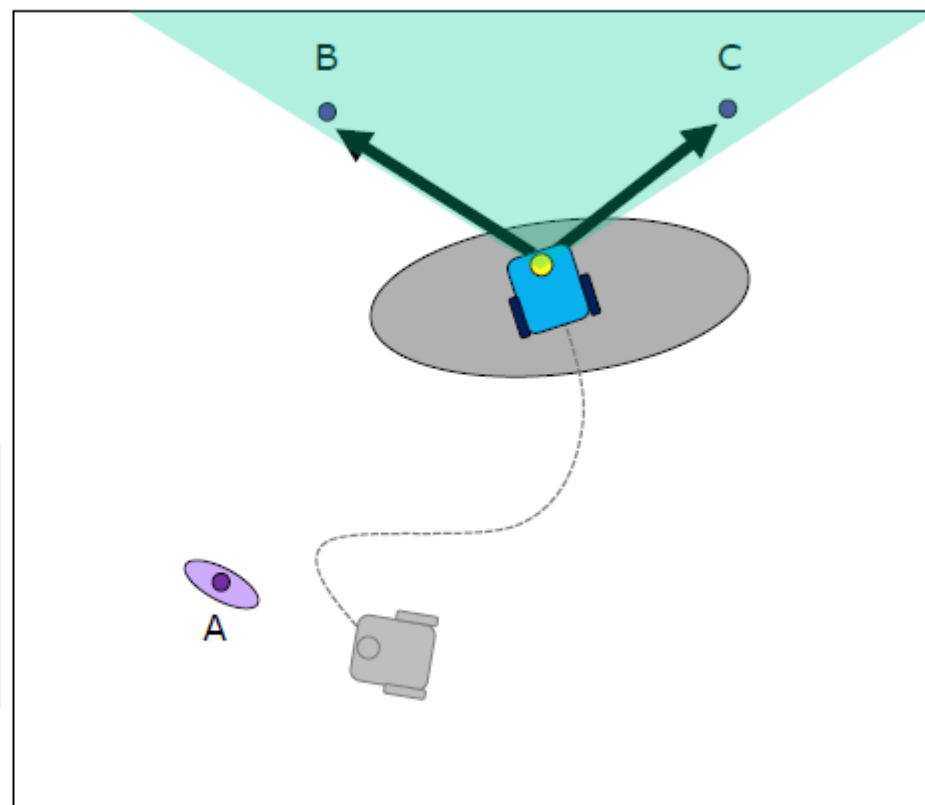
Robot moves forwards: uncertainty grows

# SLAM Example

- Robot observes two new features.

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



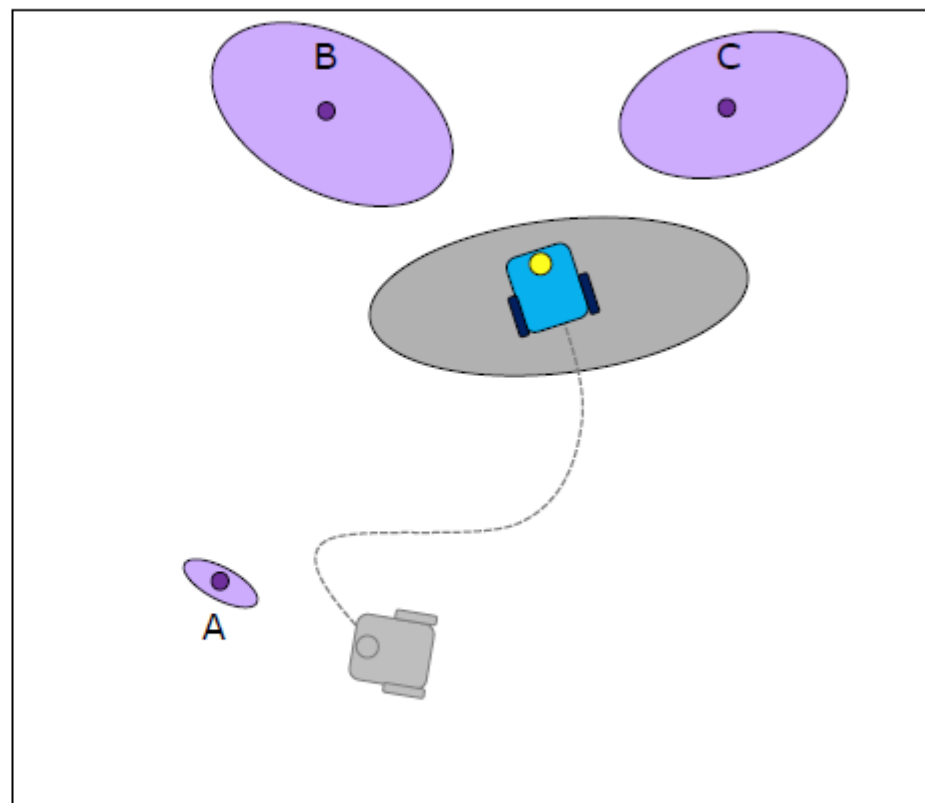
Robot makes first measurements of B & C

# SLAM Example

- Their position uncertainty results from the combination of the measurement error with the robot pose uncertainty.
- ⇒ map becomes correlated with the robot pose estimate.

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



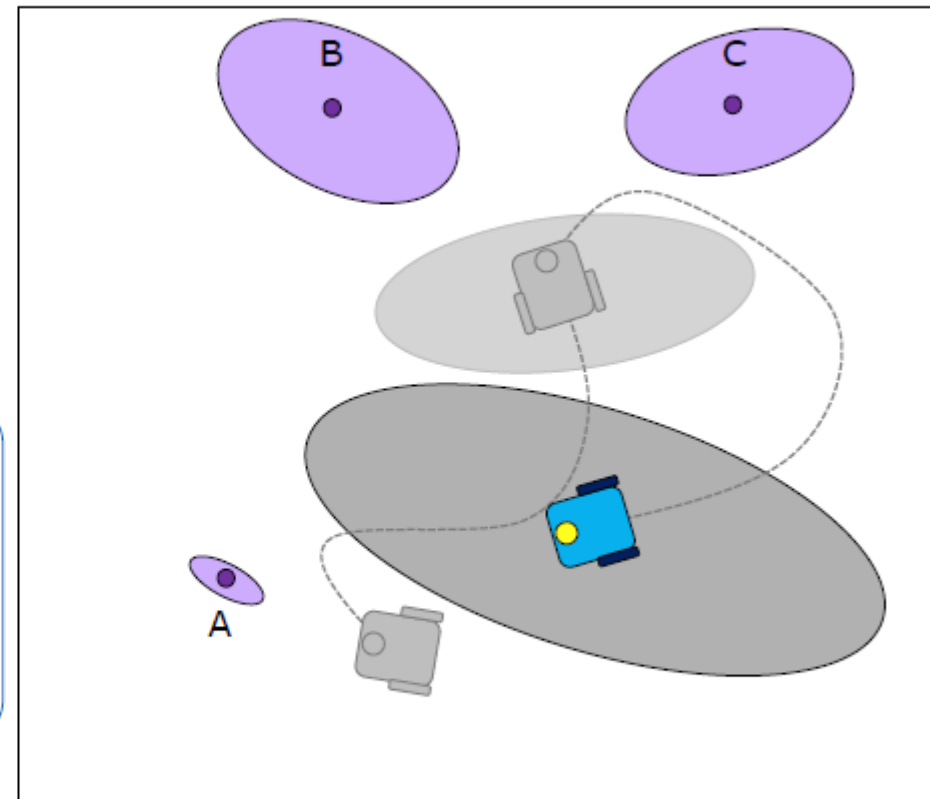
Robot makes first measurements of B & C

# SLAM Example

- Robot moves again and its uncertainty increases (motion model)

On every frame:

- Predict** how the robot has moved
- Measure**
- Update** the internal representations



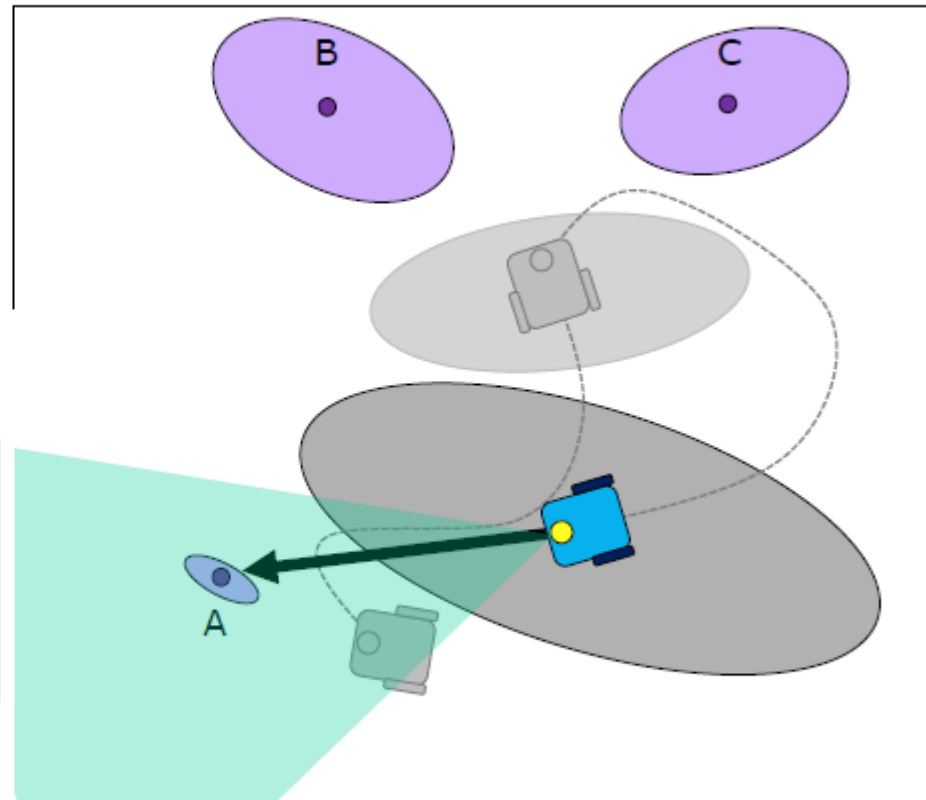
Robot moves again: uncertainty grows more

# SLAM Example

- Robot re-observes an old feature  
⇒ **Loop closure** detection

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations



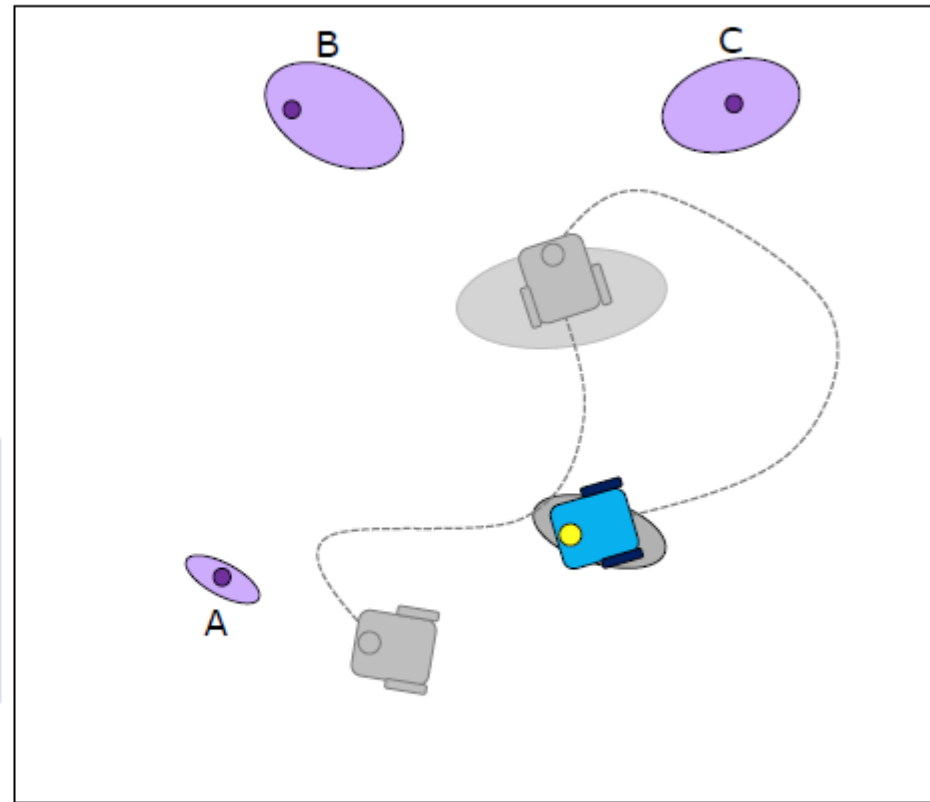
Robot re-measures A: "loop closure"

# SLAM Example

- Robot updates its position: the resulting position estimate becomes correlated with the feature location estimates.
- Robot's uncertainty shrinks and so does the uncertainty in the rest of the map

On every frame:

- **Predict** how the robot has moved
- **Measure**
- **Update** the internal representations

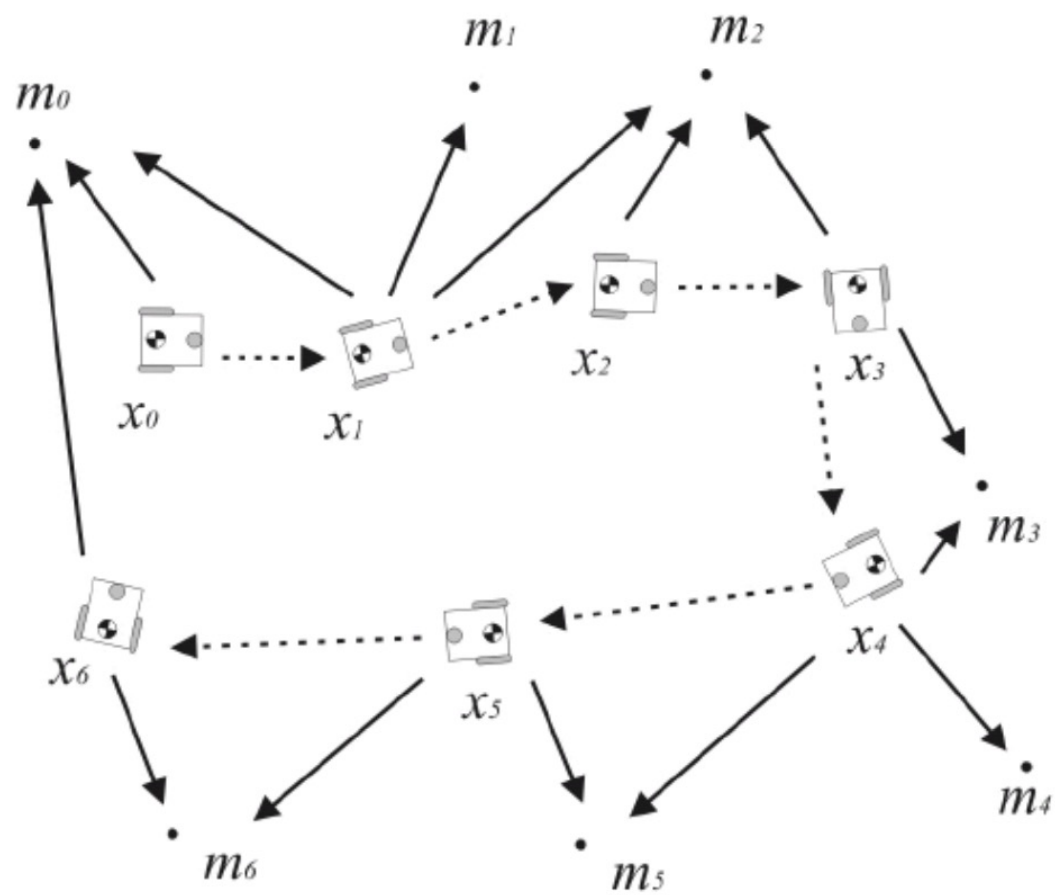


Robot re-measures A: "loop closure"  
uncertainty shrinks



# SLAM

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# SLAM In Action

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# SLAM Formalisms

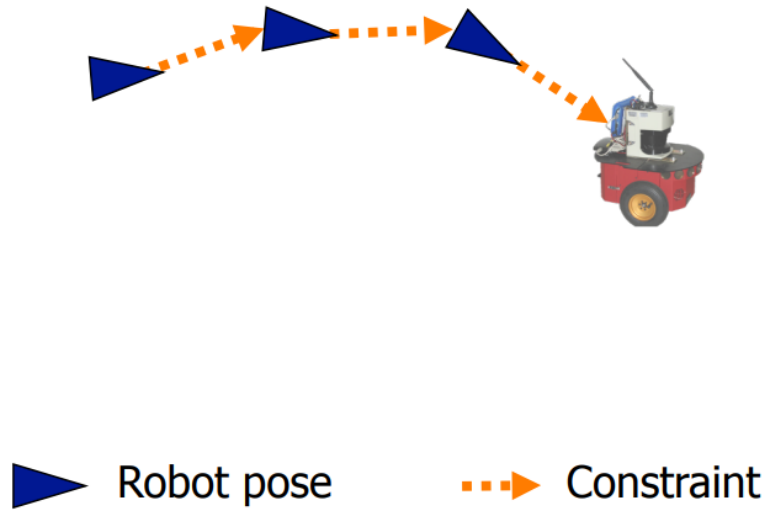
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- Some of the most influential approaches to SLAM:
  - GraphSLAM (Factor Graphs) --- We'll see this in more detail next time
  - Extended Kalman Filter SLAM (EKF SLAM)
  - Particle Filter SLAM (FAST SLAM)
  - ... many variants

# Graph SLAM

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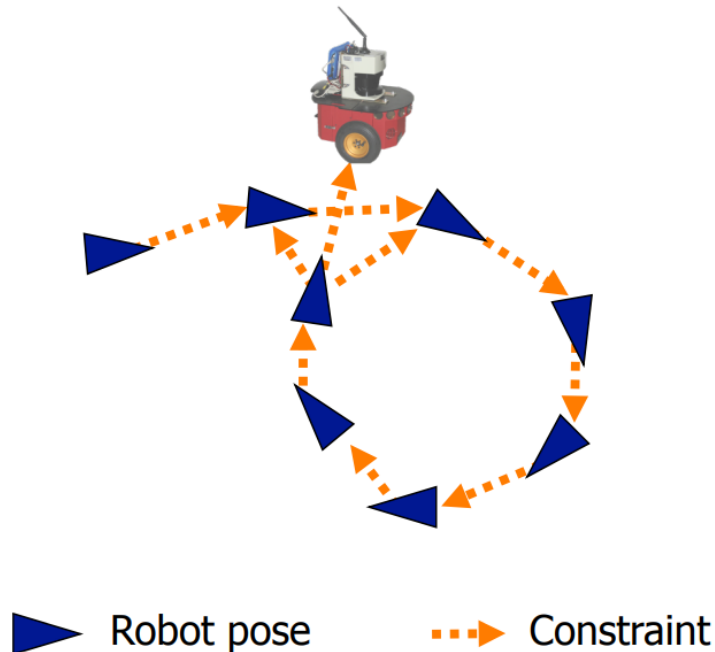
- Constraints connect the poses of the robot while it is moving
- Constraints are inherently uncertain



# Graph SLAM

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- Observing previously seen areas generates constraints between non-successive poses
- Constraints are inherently uncertain



# Basic idea behind Graph SLAM

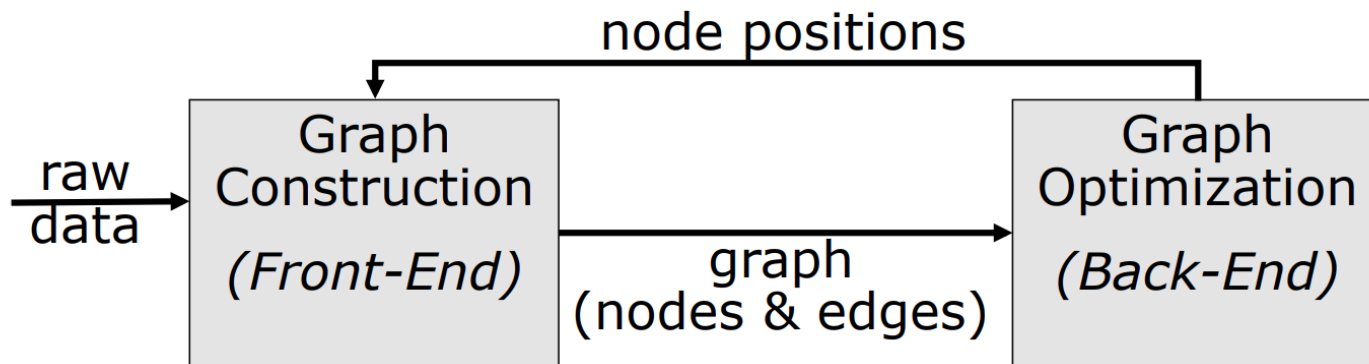
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- Use a **graph** to represent the problem
- Every **node** in the graph corresponds to a pose of the robot during mapping
- Every **edge** between two nodes corresponds to a spatial constraint between them
- **The method**: Build the graph and find a node configuration that minimizes the error introduced by the constraints

# Graph SLAM: a system view

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- Interplay of front-end and back-end
- A consistent map helps to determine new constraints by reducing the search space
- Optimize the graph by minimizing squared errors (i.e., least squares)



# EKF SLAM

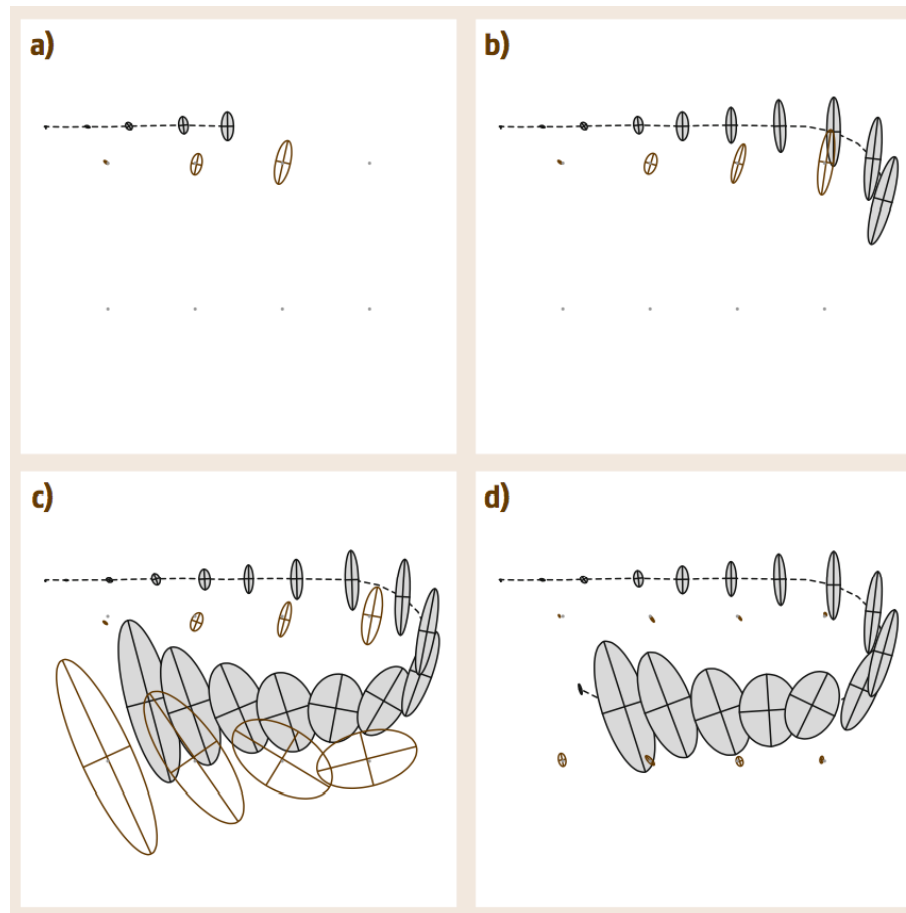
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- Extended Kalman Filter
  - Non-linear version of the Kalman Filter
- EKF SLAM
  - The “map” is a large vector that combines the robot’s position and landmark states, modeled by a Gaussian
  - Motion and sensing update the vector, adding new dimensions when new landmarks are found



# EKF-SLAM

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# EKF SLAM state representation

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State representation for  $n$  landmarks

$$x_t = \left( \underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1,x}, m_{1,y}}_{\text{landmark 1}}, \dots, \underbrace{m_{n,x}, m_{n,y}}_{\text{landmark n}} \right)^T$$

# EKF SLAM state representation

- Map with  $n$  landmarks results in a  $(3+2n)$ -dimensional Gaussian
- Belief is represented by

$$\underbrace{\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix}}_{\mu} \underbrace{\begin{pmatrix} \begin{array}{ccc} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} \end{array} & \begin{array}{ccccc} \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \dots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \dots & \sigma_{ym_{n,x}} & \sigma_{ym_{n,y}} \\ \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \dots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \end{array} \\ \begin{array}{ccc} \sigma_{m_{1,x}x} & \sigma_{m_{1,x}y} & \sigma_{\theta} \\ \sigma_{m_{1,y}x} & \sigma_{m_{1,y}y} & \sigma_{\theta} \\ \vdots & \vdots & \vdots \\ \sigma_{m_{n,x}x} & \sigma_{m_{n,x}y} & \sigma_{\theta} \\ \sigma_{m_{n,y}x} & \sigma_{m_{n,y}y} & \sigma_{\theta} \end{array} & \begin{array}{ccccc} \sigma_{m_{1,x}m_{1,x}} & \sigma_{m_{1,x}m_{1,y}} & \dots & \sigma_{m_{1,x}m_{n,x}} & \sigma_{m_{1,x}m_{n,y}} \\ \sigma_{m_{1,y}m_{1,x}} & \sigma_{m_{1,y}m_{1,y}} & \dots & \sigma_{m_{1,y}m_{n,x}} & \sigma_{m_{1,y}m_{n,y}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{m_{n,x}m_{1,x}} & \sigma_{m_{n,x}m_{1,y}} & \dots & \sigma_{m_{n,x}m_{n,x}} & \sigma_{m_{n,x}m_{n,y}} \\ \sigma_{m_{n,y}m_{1,x}} & \sigma_{m_{n,y}m_{1,y}} & \dots & \sigma_{m_{n,y}m_{n,x}} & \sigma_{m_{n,y}m_{n,y}} \end{array} \end{pmatrix}}_{\Sigma}$$

# EKF SLAM state representation

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- More compactly

$$\underbrace{\begin{pmatrix} \boxed{x_R} \\ \boxed{m_1} \\ \vdots \\ \boxed{m_n} \end{pmatrix}}_{\mu} \quad \underbrace{\begin{pmatrix} \boxed{\Sigma x_R x_R} & \boxed{\Sigma x_R m_1} & \cdots & \boxed{\Sigma x_R m_n} \\ \boxed{\Sigma m_1 x_R} & \boxed{\Sigma m_1 m_1} & \cdots & \boxed{\Sigma m_1 m_n} \\ \vdots & \vdots & \ddots & \vdots \\ \boxed{\Sigma m_n x_R} & \boxed{\Sigma m_n m_1} & \cdots & \boxed{\Sigma m_n m_n} \end{pmatrix}}_{\Sigma}$$

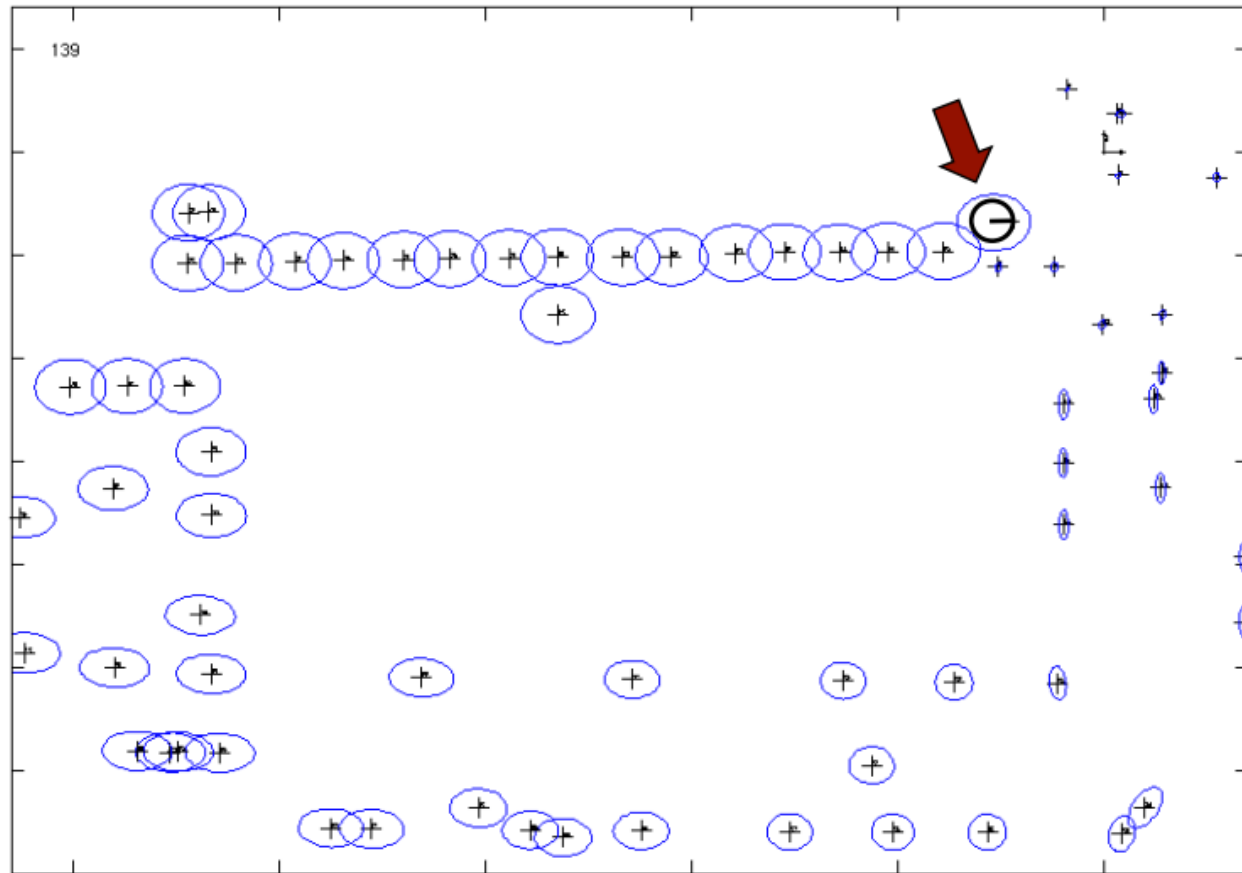
# Loop closure

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- **Recognizing an already mapped area**
- Data association with
  - high ambiguity
  - possible environment symmetries
- Uncertainties collapse after a loop closure (whether the closure was correct or not!)

# Example: before loop closure

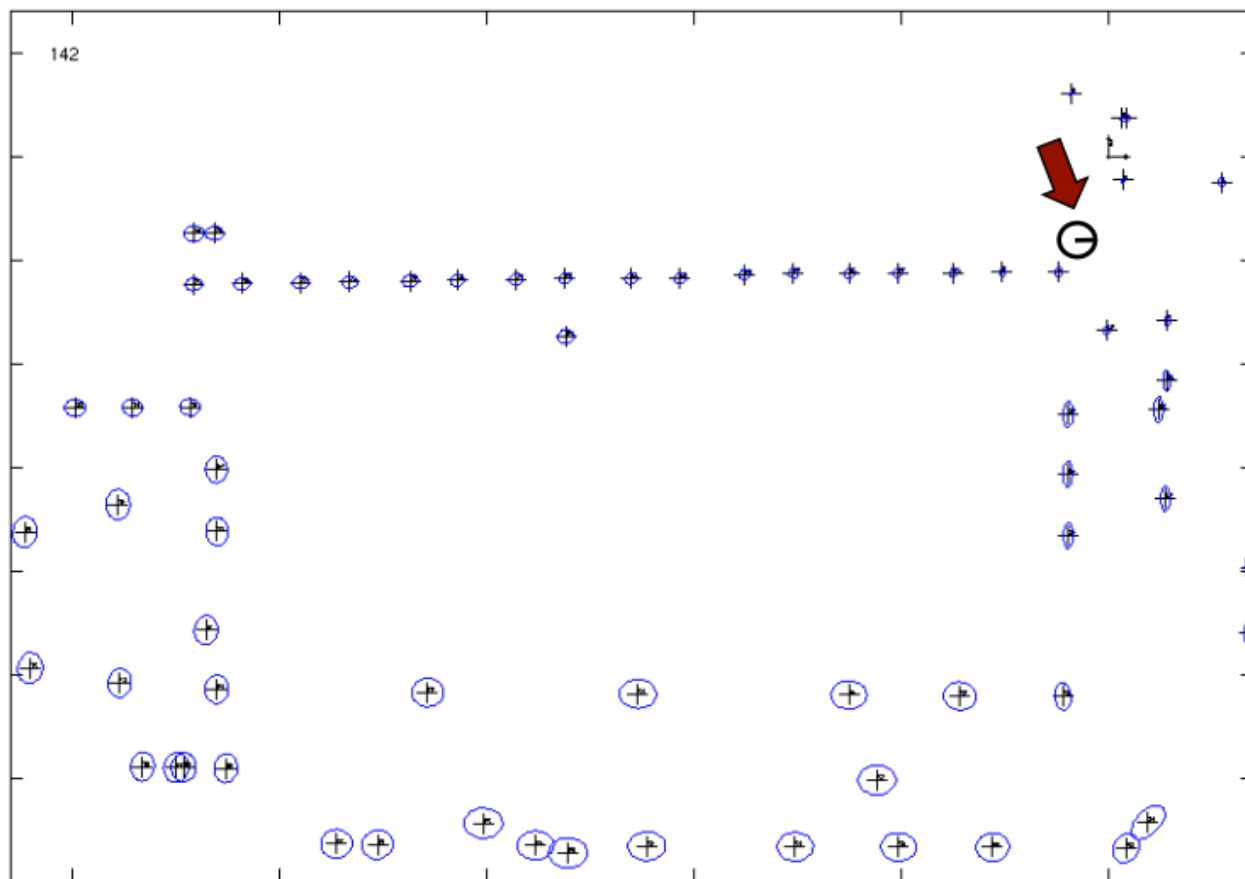
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Courtesy of K. Arras

# Example: after loop closure

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Courtesy of K. Arras

# Loop Closure

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- Loop closing reduces the uncertainty in robot and landmark estimates
- This can be exploited when exploring an environment for the sake of better (e.g. more accurate) maps
- Wrong loop closures lead to filter divergence!



## SLAM simulation

SLAM simulation by Sjoerd de Jong  
under supervision of Gert Kootstra.  
The Kalman functionality is partly  
based on the EKF SLAM simulation  
of Tim Bailey. [\*www-personal.acfr.usyd.edu.au/tbailey\*](http://www-personal.acfr.usyd.edu.au/tbailey)

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Department of Artificial Intelligence  
University of Groningen

# FastSLAM

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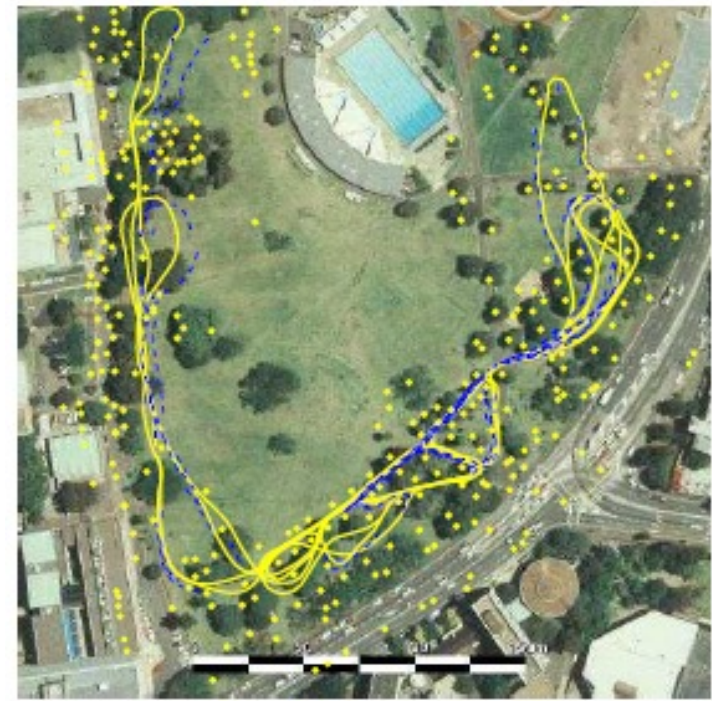
- Use particle filter to estimate the state of the robot
- Use Extended Kalman Filter (EKF) to estimate the state of the landmarks
- Each particle keeps track of its own estimate of the map (i.e., the landmark positions)
- Each particle does its own data association (i.e., matching landmarks between image frames)
- If a particle is lost during resampling, its entire map estimate is also lost.

Algorithm FastSLAM 1.0( $z_t, u_t, S_{t-1}$ ):

```

for  $m = 1$  to  $M$  do                                     // loop over all particles
    retrieve  $\left\langle s_{t-1}^{[m]}, N_{t-1}^{[m]}, \left\langle \mu_{1,t-1}^{[m]}, \Sigma_{1,t-1}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_{t-1}^{[m]},t-1}^{[m]}, \Sigma_{N_{t-1}^{[m]},t-1}^{[m]}, i_{N_{t-1}^{[m]}}^{[m]} \right\rangle \right\rangle$  from  $S_{t-1}$ 
     $s_t^{[m]} \sim p(s_t | s_{t-1}^{[m]}, u_t)$                        // sample new pose
    for  $n = 1$  to  $N_{t-1}^{[m]}$  do                               // calculate measurement likelihoods
         $\hat{z}_n = g(\mu_{n,t-1}^{[m]}, s_t^{[m]})$                  // measurement prediction
         $G_n = g'(\mu_{n,t-1}^{[m]}, s_t^{[m]})$                // calculate Jacobian
         $Q_n = G_n^T \Sigma_{n,t-1}^{[m]} G_n + R_t$            // measurement covariance
         $w_n = |2\pi Q_n|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (z_t - \hat{z}_n)^T Q_n^{-1} (z_t - \hat{z}_n) \right\}$  // likelihood of correspondence
    endfor
     $w_{N_{t-1}^{[m]}+1} = p_0$                                // importance factor of new landmark
     $\hat{n} = \underset{n=1, \dots, N_{t-1}^{[m]}+1}{\operatorname{argmax}} w_n$  // max likelihood correspondence
     $N_t^{[m]} = \max\{N_{t-1}^{[m]}, \hat{n}\}$                      // new number of features in map
    for  $n = 0$  to  $N_t^{[m]}$  do                                // update Kalman filters
        if  $n = N_{t-1}^{[m]} + 1$  then                       // is new feature?
             $\mu_{n,t}^{[m]} = g^{-1}(z_t, s_t^{[m]})$          // initialize mean
             $\Sigma_{n,t}^{[m]} = G_{\hat{n}}^{-1} R_t (G_{\hat{n}}^{-1})^T$  // initialize covariance
             $i_{n,t}^{[m]} = 1$                              // initialize counter
        else if  $n = \hat{n}$  then                             // is observed feature?
             $K = \Sigma_{n,t-1}^{[m]} G_{\hat{n}} Q_{\hat{n}}^{-1}$        // calculate Kalman gain
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]} + K(z_t - \hat{z}_{\hat{n}})^T$  // update mean
             $\Sigma_{n,t}^{[m]} = (I - K G_{\hat{n}}^T) \Sigma_{n,t-1}^{[m]}$  // update covariance
             $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} + 1$            // increment counter
        else                                               // all other features
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]}$                // copy old mean
             $\Sigma_{n,t}^{[m]} = \Sigma_{n,t-1}^{[m]}$          // copy old covariance
            if  $\mu_{n,t-1}^{[m]}$  outside perceptual range of  $s_t^{[m]}$  then // should feature have been observed?
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]}$            // no, do not change
            else                                           // yes, decrement counter
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} - 1$ 
                if  $i_{n,t-1}^{[m]} < 0$  then discard feature  $n$  endif // discard dubious features
            endif
        endif
    endfor
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_{\text{aux}}$ 
endfor
 $S_t = \emptyset$                                            // construct new particle set
for  $m' = 1$  to  $M$  do                                     // resample M particles
    draw random index  $m$  with probability  $\propto w_t^{[m]}$  // resample
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_t$ 
end for
return  $S_t$ 
end algorithm

```



## ***FastSLAM: An Efficient Solution to the Simultaneous Localization And Mapping Problem with Unknown Data Association***

Sebastian Thrun, Michael Montemerlo, Daphne Koller, Ben Wegbreit, Juan Nieto, and Eduardo Nebot

May 2004, Journal of Machine Learning Research 4(3)

```

for  $m = 1$  to  $M$  do                                     // loop over all particles
    retrieve  $\left\langle s_{t-1}^{[m]}, N_{t-1}^{[m]}, \left\langle \mu_{1,t-1}^{[m]}, \Sigma_{1,t-1}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_{t-1}^{[m]},t-1}^{[m]}, \Sigma_{N_{t-1}^{[m]},t-1}^{[m]}, i_{N_{t-1}^{[m]}}^{[m]} \right\rangle \right\rangle$  from  $S_{t-1}$ 
     $s_t^{[m]} \sim p(s_t | s_{t-1}^{[m]}, u_t)$                      // sample new pose
    for  $n = 1$  to  $N_{t-1}^{[m]}$  do                               // calculate measurement likelihoods
         $\hat{z}_n = g(\mu_{n,t-1}^{[m]}, s_t^{[m]})$                  // measurement prediction
         $G_n = g'(\mu_{n,t-1}^{[m]}, \mu_{n,t-1}^{[m]})$            // calculate Jacobian
         $Q_n = G_n^T \Sigma_{n,t-1}^{[m]} G_n + R_t$            // measurement covariance
         $w_n = |2\pi Q_n|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (z_t - \hat{z}_n)^T Q_n^{-1} (z_t - \hat{z}_n) \right\}$  // likelihood of correspondence
    endfor
     $w_{N_{t-1}^{[m]}+1} = p_0$                                // importance factor of new landmark
     $\hat{n} = \underset{n=1, \dots, N_{t-1}^{[m]}+1}{\operatorname{argmax}} w_n$  // max likelihood correspondence
     $N_t^{[m]} = \max\{N_{t-1}^{[m]}, \hat{n}\}$                      // new number of features in map
    for  $n = 0$  to  $N_t^{[m]}$  do                               // update Kalman filters
        if  $n = N_{t-1}^{[m]} + 1$  then                       // is new feature?
             $\mu_{n,t}^{[m]} = g^{-1}(z_t, s_t^{[m]})$ 
             $\Sigma_{n,t}^{[m]} = G_{\hat{n}}^{-1} R_t (G_{\hat{n}}^{-1})^T$ 
             $i_{n,t}^{[m]} = 1$ 
        else if  $n = \hat{n}$  then
             $K = \Sigma_{n,t-1}^{[m]} G_{\hat{n}} Q_{\hat{n}}^{-1}$ 
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]} + K(z_t - \hat{z}_n)$ 
             $\Sigma_{n,t}^{[m]} = (I - K G_{\hat{n}}^T) \Sigma_{n,t-1}^{[m]}$ 
             $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} + 1$ 
        else
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]}$ 
             $\Sigma_{n,t}^{[m]} = \Sigma_{n,t-1}^{[m]}$ 
            if  $\mu_{n,t-1}^{[m]}$  outside percept
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]}$ 
            else
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} - 1$  // yes, decrement counter
                if  $i_{n,t-1}^{[m]} < 0$  then discard feature  $n$  endif // discard dubious features
            endif
        endif
    endfor
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_{\text{aux}}$ 
endfor
 $S_t = \emptyset$  // construct new particle set
for  $m' = 1$  to  $M$  do // resample M particles
    draw random index  $m$  with probability  $\propto w_t^{[m]}$  // resample
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_t$ 
end for
return  $S_t$ 
end algorithm

```

1. Execute for every particle
2. Retrieve previous info for this particle
3. Particle filter:
  1. Sample pose
  2. Update measurement likelihoods
  3. Compute particle weights
4. Resample

Algorithm FastSLAM 1.0( $z_t, u_t, S_{t-1}$ ):

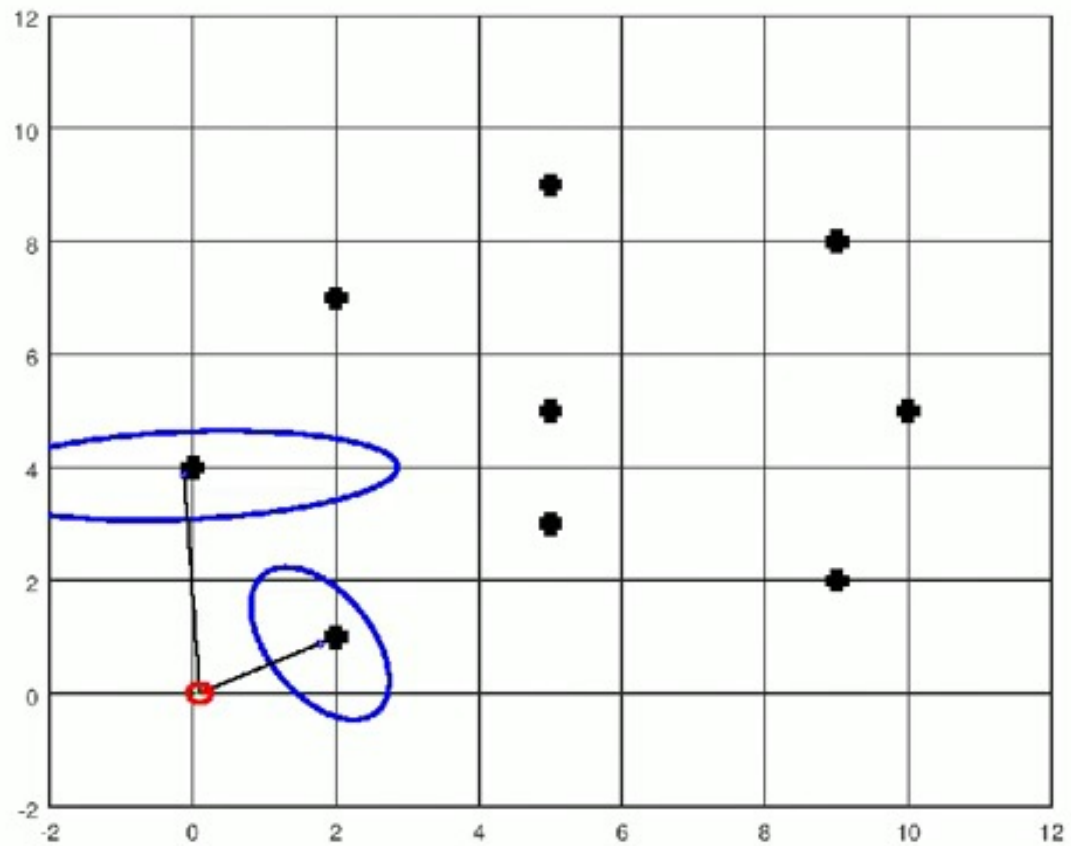
```

for  $m = 1$  to  $M$  do
    // loop over all particles
    retrieve  $\left\langle s_{t-1}^{[m]}, N_{t-1}^{[m]}, \left\langle \mu_{1,t-1}^{[m]}, \Sigma_{1,t-1}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_{t-1}^{[m]},t-1}^{[m]}, \Sigma_{N_{t-1}^{[m]},t-1}^{[m]}, i_{N_{t-1}^{[m]}}^{[m]} \right\rangle \right\rangle$  from  $S_{t-1}$ 
     $s_t^{[m]} \sim p(s_t | s_{t-1}^{[m]}, u_t)$  // sample new pose
    for  $n = 1$  to  $N_{t-1}^{[m]}$  do
        // calculate measurement likelihoods
         $\hat{z}_n = g(\mu_{n,t-1}^{[m]}, s_t^{[m]})$  // measurement prediction
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         $Q_n = G_n^T \Sigma_{n,t-1}^{[m]} G_n + R_t$  // measurement covariance
         $w_n = |2\pi Q_n|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (z_t - \hat{z}_n)^T Q_n^{-1} (z_t - \hat{z}_n) \right\}$  // likelihood of correspondence
    endfor
     $w_{N_{t-1}^{[m]}+1} = p_0$  // importance factor of new landmark
     $\hat{n} = \underset{n=1, \dots, N_{t-1}^{[m]}+1}{\operatorname{argmax}} w_n$  // max likelihood correspondence
     $N_t^{[m]} = \max\{N_{t-1}^{[m]}, \hat{n}\}$  // new number of features in map
    for  $n = 0$  to  $N_t^{[m]}$  do
        // update Kalman filters
        if  $n = N_{t-1}^{[m]} + 1$  then
            // is new feature?
             $\mu_{n,t}^{[m]} = g^{-1}(z_t, s_t^{[m]})$  // initialize mean
             $\Sigma_{n,t}^{[m]} = G_n^{-1} R_t (G_n^{-1})^T$  // initialize covariance
             $i_{n,t}^{[m]} = 1$  // initialize counter
        else if  $n = \hat{n}$  then
            // is observed feature?
             $K = \Sigma_{n,t-1}^{[m]} G_n Q_n^{-1}$  // calculate Kalman gain
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]} + K(z_t - \hat{z}_n)^T$  // update mean
             $\Sigma_{n,t}^{[m]} = (I - K G_n^T) \Sigma_{n,t-1}^{[m]}$  // update covariance
             $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} + 1$  // increment counter
        else
            // all other features
             $\mu_{n,t}^{[m]} = \mu_{n,t-1}^{[m]}$  // copy old mean
             $\Sigma_{n,t}^{[m]} = \Sigma_{n,t-1}^{[m]}$  // copy old covariance
            if  $\mu_{n,t-1}^{[m]}$  outside perceptual range of  $s_t^{[m]}$  then
                // should feature have been observed?
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]}$  // no, do not change
            else
                // yes, decrement counter
                 $i_{n,t}^{[m]} = i_{n,t-1}^{[m]} - 1$ 
                if  $i_{n,t-1}^{[m]} < 0$  then discard feature  $n$  endif
            endif
        endif
    endfor
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_{\text{aux}}$ 
endfor
 $S_t = \emptyset$  // construct new particle set
for  $m' = 1$  to  $M$  do
    // resample M particles
    draw random index  $m$  with probability  $\propto w_t^{[m]}$  // resample
    add  $\left\langle s_t^{[m]}, N_t^{[m]}, \left\langle \mu_{1,t}^{[m]}, \Sigma_{1,t}^{[m]}, i_1^{[m]} \right\rangle, \dots, \left\langle \mu_{N_t^{[m]},t}^{[m]}, \Sigma_{N_t^{[m]},t}^{[m]}, i_{N_t^{[m]}}^{[m]} \right\rangle \right\rangle$  to  $S_t$ 
endfor
return  $S_t$ 
end algorithm

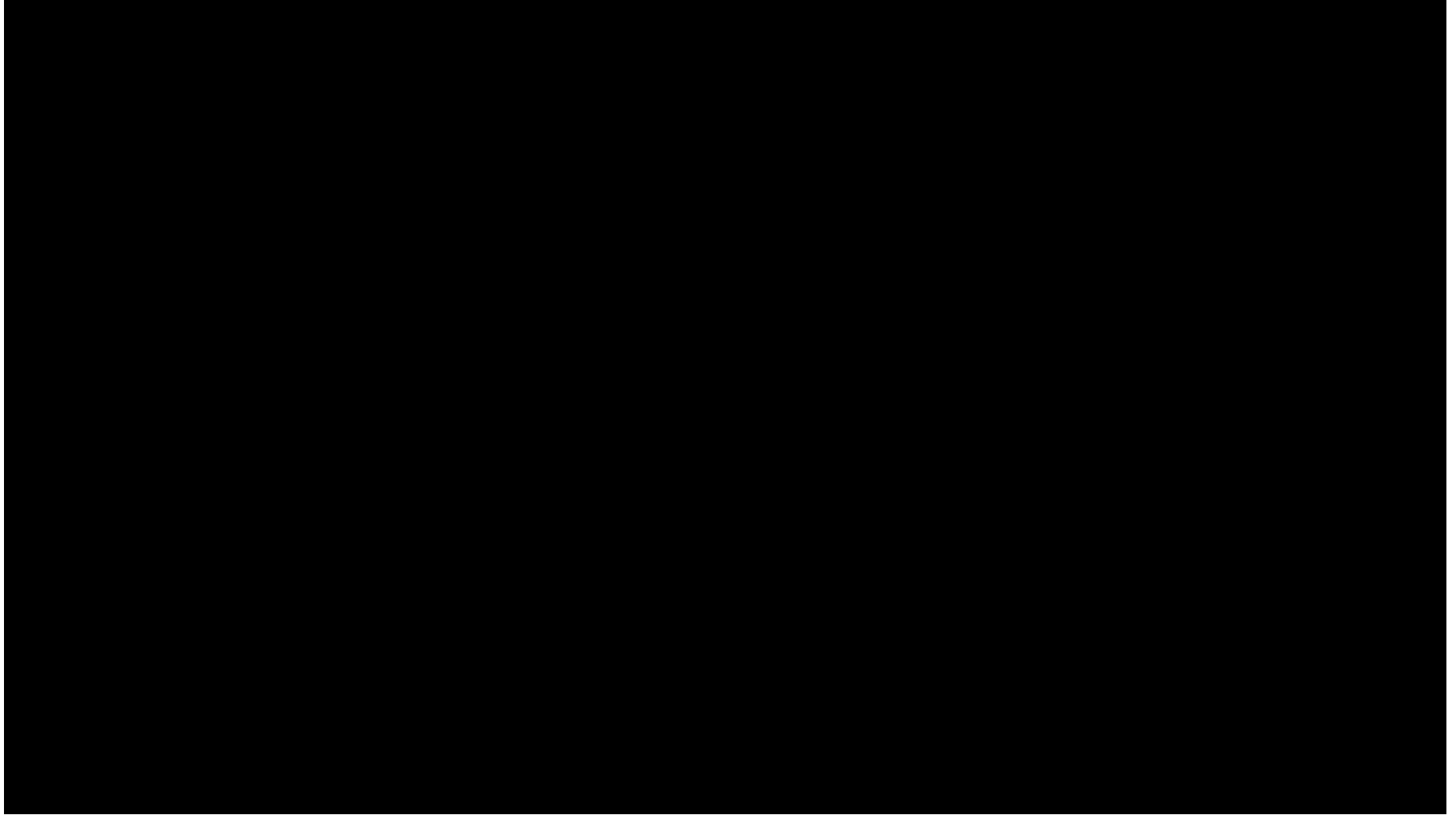
```

Update the estimates for each particle using the EKF





<https://www.youtube.com/watch?v=viMDOaV9bK8>



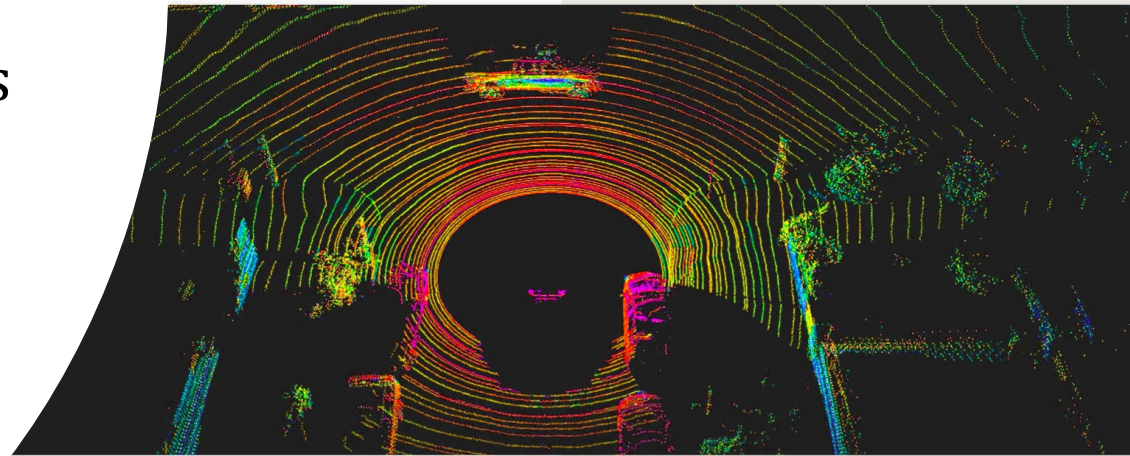
<https://www.youtube.com/watch?v=kuxl8two5GA>

[Smart Mobility Research Team@AIST](#)

# LIDAR

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- Superpowers:
  - 360 Visibility
  - Accurate depth!
- Almost all AV prototypes have them (not all 360)



[Images and exposition take from excellent Voyage Blog post](https://news.voyage.auto/an-introduction-to-lidar-the-key-self-driving-car-sensor-a7e405590cf)

<https://news.voyage.auto/an-introduction-to-lidar-the-key-self-driving-car-sensor-a7e405590cf>



# Localization with LIDAR

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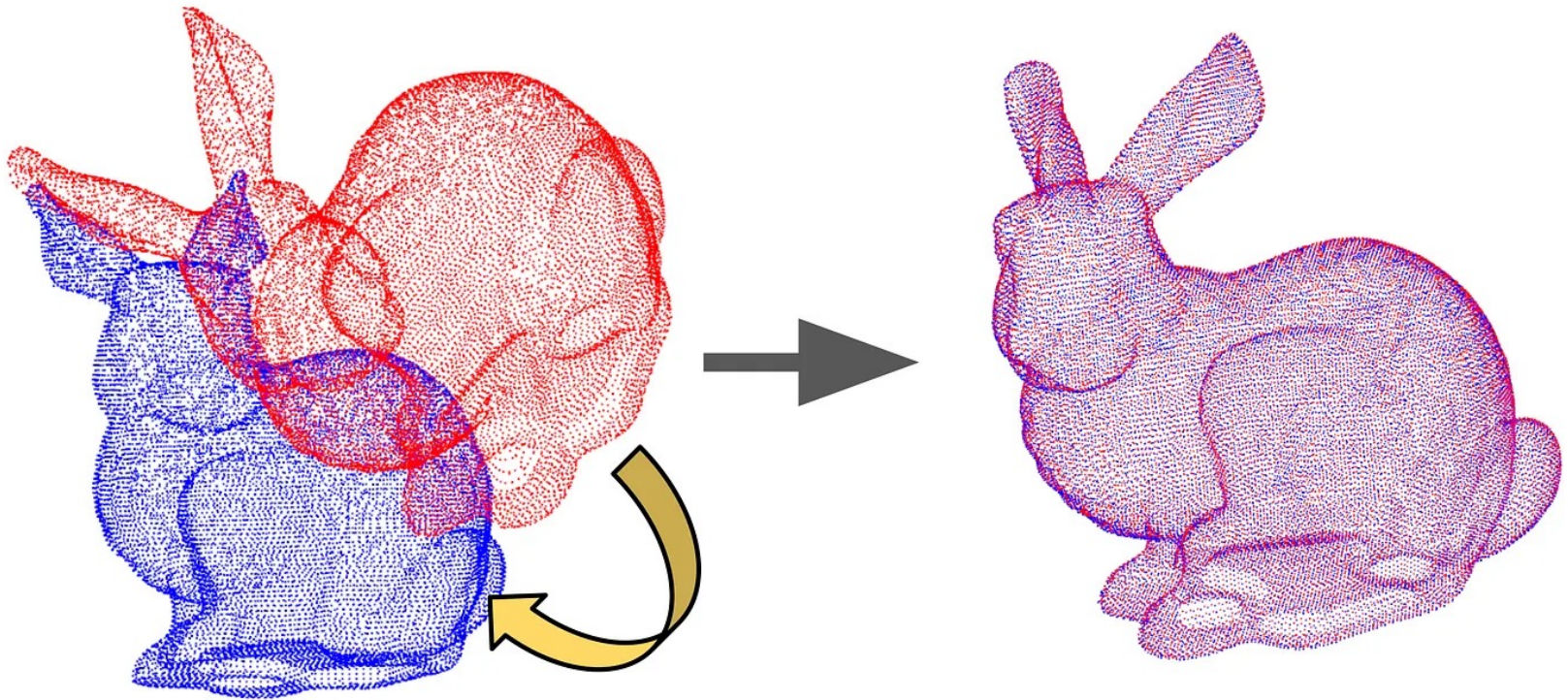
- ICP = **Iterated Closest Points**:
- Call current scan  $S$ , map  $M$
- Predict pose from motion model:  
use other sensors if available
- Iterate:
  - For every point  $s$ : find closest  $m$
  - Re-estimate pose
- In practice:
  - outlier rejection to account for moving objects, unmodeled structures, parked cars etc...



**Image Credits:** Innoviz

# Aligning two point clouds using Iterative Closest Point Algorithm

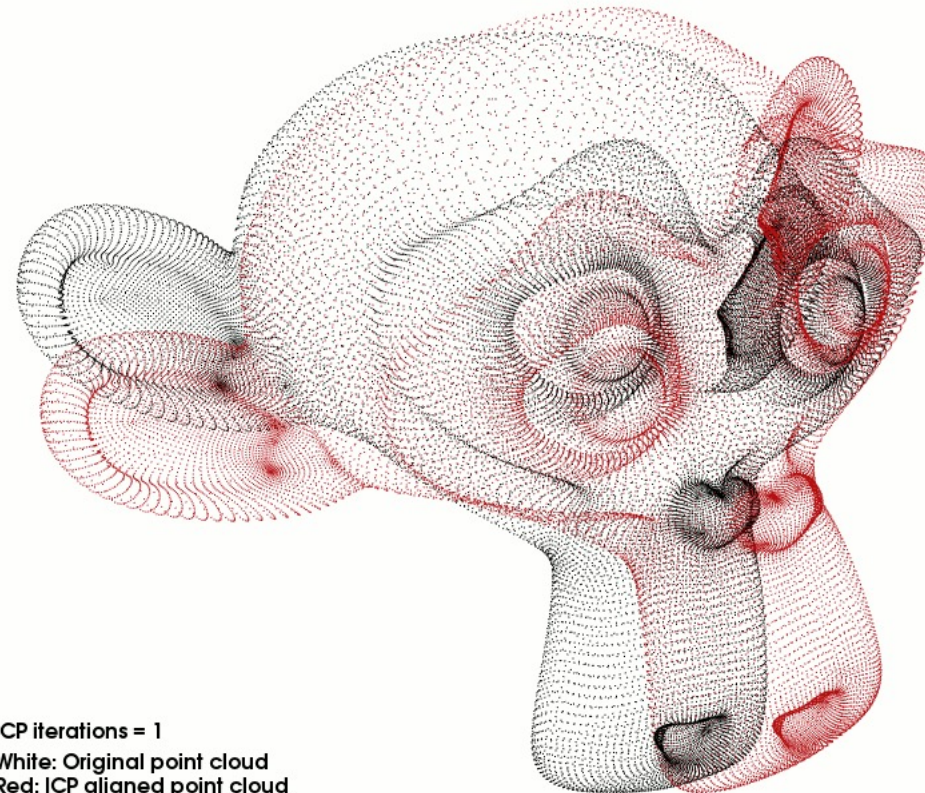
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Source: [Biorobotics Lab at Carnegie Mellon University](#).

# Aligning two point clouds using Iterative Closest Point Algorithm

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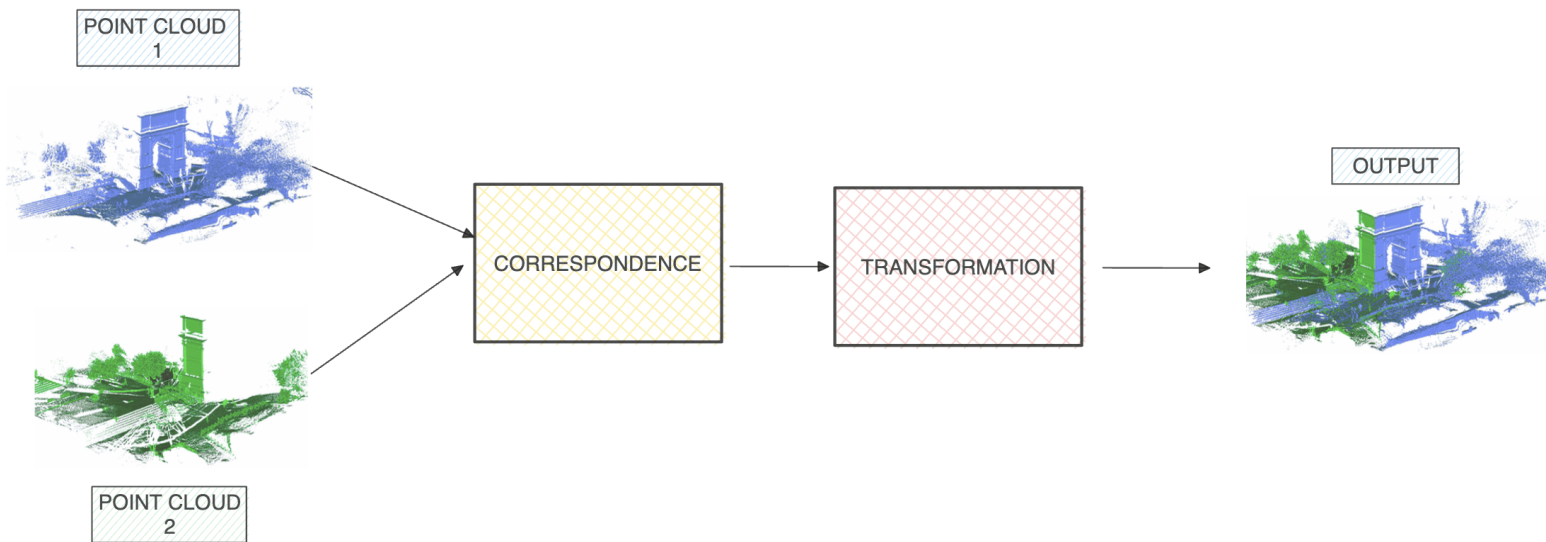


ICP iterations = 1  
White: Original point cloud  
Red: ICP aligned point cloud

Source: [Point Cloud Library Documentation](#)

# Two key elements: Correspondence and Transformation

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More on this next time!