2. LSTM parity function

input $x_t$ $h_{t-1}$

Output $Y_t = h_t$

Since in RNN, we apply XOR

$$Y_t = (\overline{x_t} \wedge Y_{t-1}) \vee (x_t \wedge \overline{Y_{t-1}})$$

$$= \overline{x_t} * Y_{t-1} + x_t * \overline{Y_{t-1}}$$

In LSTM, we have

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

comparing the two equations, we have

$C_t = Y_t = h_t$

$f_t = \overline{x_t}$

$i_t = x_t$

- So we set $C_t = h_t$, to find $W_f, b_f$.
- we solve.

$$f_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) = \overline{x_t}$$

$W_f = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$b_f = 1$

since when $x_t = 1$ $f_t = \sigma(0) = 0 = \overline{x_t}$

when $x_t = 0$ $f_t = \sigma(1) = 1 = \overline{x_t}$

- Solving $W_i, b_i$ we solve

$$i_t = \sigma(W_i [h_{t-1}, x_t] + b_i) = x_t$$

$W_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$b_i = 0$

Since $x_t = 1$ $\sigma(1) = 1 = x_t$

$x_t = 0$ $\sigma(0) = 0 = x_t$