

4. Exploding Gradient.

$$h_t = W^T h_{t-1}$$

$$h_{t-1} = W^T h_{t-2}$$

$$\vdots$$
$$h_1 = W^T h_0$$

...

$$\text{Then } h_t = (W^T)^t h_0$$

W could be decomposed as

$$W = P D P^{-1}$$

where D is diagonal matrix of eigenvalues.

P is the eigen vector of W

Then we can write

$$h_t = ((P D P^{-1})^T)^t h_0$$

$$= ((P^{-1})^T D^T P^T)^t h_0$$

$$= ((P^{-1})^T D P^T)^t h_0$$

$$= (P^{-1})^T D P^T (P^{-1})^T D P^T \dots (P^{-1})^T D P^T h_0$$

$$\text{as } (P^{-1})^T \cdot P^T = I$$

$$= (P^{-1})^T D^t P^T h_0$$

$$\frac{dh_t}{dh_0} = (P^{-1})^T D^t P^T$$

when $t \gg 0$, if a eigenvalues in $D < 1$, the value in $D^t \rightarrow 0$ thus vanishing the gradient

if a eigenvalue in $D > 1$, the value in $D^t \rightarrow \infty$, thus exploding the gradient