

Find square root:

Before the  $n$ th iteration, the  $(n-1)^2 == x$  and loops terminates or it does not equal  $x$  and the  $n$ th iteration runs.

Initialization:

The loop invariant is correct before the first iteration.

case 1:  $x == 0$ ,  $(1-1)^2 == x$

case 2:  $x == \text{some number}$ , and  $(1-1)^2 != x$

Brain kinda stopped here not sure if I can do this lol. Or I have to use words instead.

Maintenance:

Is\_prime():

Before the  $n$ th iteration  $x$  is found to not be a prime number or it is unknown if  $x$  is a prime number.

Product by addition:

A bound function for Product by addition( $a, b$ ) can be  $f(a, b) = a$

1, for base case  $f(a, b) \leq 0$

2, for the recursive call  $f(a, b) > 0$

3,  $f(a-1, b) < f(a, b)$ , so product by addition will always terminate.

Claim: product by addition will always return  $b * a$ .

Proof:

Let  $P(a, b)$  represent the function pba.

Base case:

When  $a = 0$ , 0 is returned as required since anything multiplied by 0 = 0.

Inductive hypothesis:

Assume that  $P(k, b)$  correctly returns  $b * k$ .  $K > 0$ .

Inductive claim:

We want to show that  $P(k+1, b)$  correctly returns  $b * k+1$ .

Inductive proof:

$P(k+1, b)$  returns  $b + P(k+1-1, b) = b + P(k, b)$

$P(k, b)$  returns  $b * k$  by IH.

$b + b \cdot k = b \cdot (k+1)$  is returned by  $P(k+1, b)$  as needed

Therefore,  $P(a, b)$  calculates  $b \cdot a$  correctly.

Since  $P(a, b)$  always terminates and calculates  $b \cdot a$  correctly. It is correct for any integer input. Or w/e it is in python.