# SAT approach

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## Constraints

We define decision variables:

$$x_{ijr} \in \{0,1\}, \quad y_{ij} \in \{0,1\}.$$

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**Constraint (2): At most one placement per rectangle. MIP Form:**

$$\sum_{(i,j)\in A_r} x_{ijr} \ \le 1, \quad \forall r \in R$$

**SAT Form:**

$$\forall r \in R,\ \forall (i_1, j_1) \neq (i_2, j_2) \in A_r : \quad (\neg x_{i_1 j_1 r} \ \vee \ \neg x_{i_2 j_2 r})$$

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**Constraint (3): Positive reward cells ($g_{ij} > 0$). MIP Form:**

$$y_{ij} \ = \sum_{r\in R} \sum_{(u,v)\in B_{ijr}} x_{uvr}, \quad \forall i \in M, j \in N : g_{ij} > 0$$

**SAT Form:**

$$y_{ij} \ \leftrightarrow \bigvee_{r\in R} \ \bigvee_{(u,v)\in B_{ijr}} x_{uvr}$$

CNF expansion:
1. Forward:

$$(\neg y_{ij} \ \vee \bigvee_{r\in R} \ \bigvee_{(u,v)\in B_{ijr}} x_{uvr})$$

2. Backward (for each $(u, v, r) \in B_{ijr}$):

$$(\neg x_{uvr} \ \vee \ y_{ij})$$

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**Constraint (4): Negative reward cells ($g_{ij} < 0$).** We encode the pseudo-Boolean condition as a clause-like form with PB semantics:

$$(\neg y_{ij} \ \vee \bigvee_{r\in R} \ \bigvee_{(u,v)\in B_{ijr}} x_{uvr})$$

where the disjunction symbolically represents the inequality

$$|R| \cdot y_{ij} \ \le \ \sum_{r\in R} \sum_{(u,v)\in B_{ijr}} x_{uvr}.$$

*This is not a pure CNF clause but a pseudo-Boolean encoding; the coefficient $-R$ in front of $y_{ij}$ is implicit in this notation.*