# Project 3 report

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ECE-3723

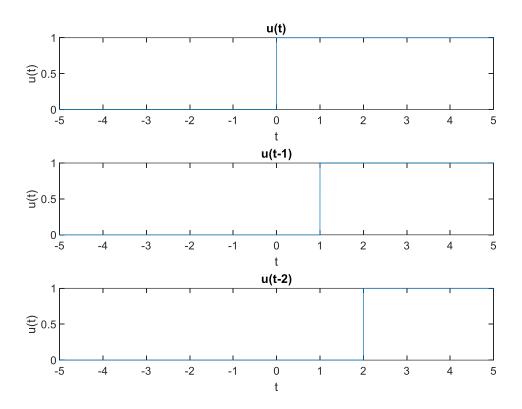
### Introduction

This project is to apply MATLAB to draw unit step function and sinusoidal signals. Besides, we also will learn about how to use functions such as tf function to create a transfer function or residue function to do partial fraction expansion for simplifying Laplace transform. Finally, we are able to use the results from these functions to write the function in either frequency-domain or time-domain.

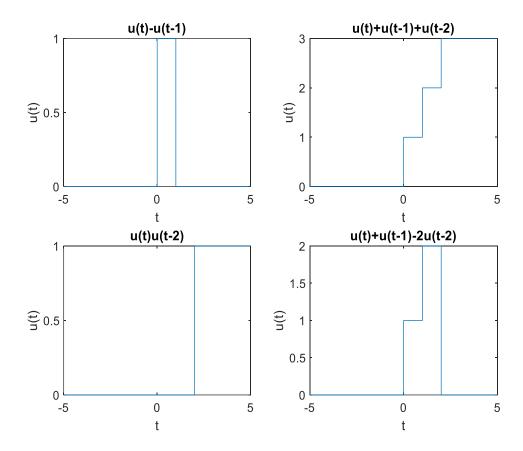
# Procedure

# 1. Unit Step Function

Show a figure that contains graphs of all three vectors as subplots in a 3x1 configuration (5 points).

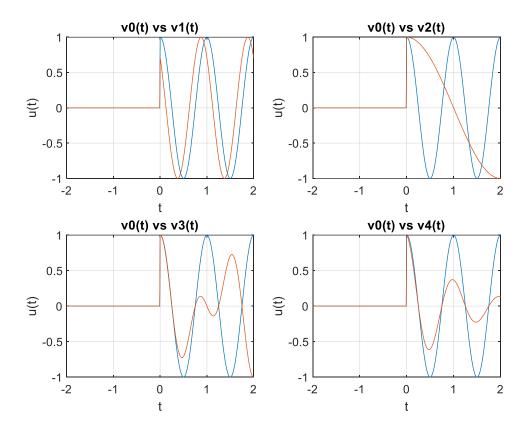


Plot these functions in a single figure with the graphs in a 2x2 configuration (10 points).



# 2 Signals in Time-Domain

Plot the following in a single plot, 2x2 configuration: 1)v0(t) and v1(t), 2)v0(t) and v2(t), 3)v0(t) and v3(t), and v3(t), and v3(t) and v3(t). There should be four graphs, each with two traces (10 points).



Blue line represents for v0(t)

Orange line represents for v1(t), v2(t), v3(t), v4(t)

#### 3. Transfer Functions

Show the new transfer function (5 points).

The new transfer function after multiplying  $H_1(s)$  and  $H_2(s)$  together

$$2 s^2 + 3 s + 2$$

$$s^4 + s^3 + 2 s^2 + 3 s + 1$$

#### 4. Partial Fraction Expansion and Laplace Transform

### 4.1 Basic polynomial functions

What is the value at s = 1(5points)?

• The value at s = 1 is 12

Show the factored polynomial (5 points).

• The factored polynomial is  $F1(s) = s^2 + 5s + 6 = (s+3)*(s+2)$ 

Show the resulting polynomial. Be sure to account for the two roots at s=0 (5 points).

• The resulting polynomial  $F2(s) = s^5 + 3 s^4 + 7 s^3 + 5 s^2$ 

Show the resulting polynomial. Also, verify by multiplying the two polynomials (F1 and F2) by hand. Show all your work (write it up on the computer, no handwritten solutions will be accepted) (10 points).

- The resulting polynomial  $H(s) = s^7 + 8 s^6 + 28 s^5 + 58 s^4 + 67 s^3 + 30 s^2$
- Verify

$$H(s) = F1(s)*F2(s)$$

$$= (s^2 + 5s + 6)*(s^5 + 3 s^4 + 7 s^3 + 5 s^2)$$

$$= s^7 + 3s^6 + 7s^5 + 5s^4 + 5s^6 + 15s^5 + 35s^4 + 25s^3 + 6s^5 + 18s^4 + 42s^3 + 30s^2$$

$$= s^7 + 8s^6 + 28s^5 + 58s^4 + 67s^3 + 30s^2$$

### 4.2 Partial fraction expansion for simplifying Laplace transform

Look at the transfer function by using the tf function. Show the output (5 points).

• The output is:

H3 =

$$s^2 + 4 s$$

Calculate by hand the inverse Laplace transform and show in the report (5 points).

$$H3(s) = 3/(4*(s+4)) + 1/(4*s)$$

Calculate by hand the inverse Laplace transform of H3(s) is

•  $h3(t) = [3*(e^{(-4t)}) + \frac{1}{4}] u(t)$ 

Show both the function in frequency-domain and time-domain (5 points).

• H4 in frequency -domain

H4 =

$$H4(s) = (5/(4*(s+4))) - (1/(4*s)) + (1/(s^2))$$

• H4 in time-domain

By hand

$$h4(t) = [(5/4)*(e^{(-4t)}) - (1/4) + t] u(t)$$

By MATLAB

ans =

$$t + (5*exp(-4*t))/4 - \frac{1}{4}$$

• H5 in frequency- domain H5 =

$$\begin{array}{l} H5(s) = 0.1549/(s+3) + 0.3199/(s+3)^2 + (-0.0847 - 0.0993i)/(s-1.4142i) \\ + (-0.0847 + 0.0993i)/(s+1.4142i) + 0.0185/s - 0.1481/s^2 + 0.2222/s^3 \end{array}$$

• H5 in time- domain

By MATLAB

ans =

$$(493*exp(-3*t))/3267 - (4*t)/27 - (41*cos(2^(1/2)*t))/242 + (95*t*exp(-3*t))/297 + (17*2^(1/2)*sin(2^(1/2)*t))/121 + t^2/9 + 1/54$$

#### Conclusion

This project helped us know how to create the unit step function and sinusoidal signals by using the functions of MATLAB. Then we are able to plot the graph based on the created functions. From there, we can observe the phase shift between different functions. We are able to use subplot function to separate the graphs as well. Through this project, we are also able apply functions such as poly2str, tf, poly, root function to create the transfer function. Then we can use conv function to multiply two polynomials. Finally, we can use residue function to find the to do partial fraction. Hence, we can write the function in either frequency-domain or time-domain based on that.

#### **Appendix**

Show a figure that contains graphs of the all three vectors as subplots in a 3x1 configuration (5 points).

```
%part1
%create time vector interval
t = (-5:0.001:5);
u1 = t > = 0 \% u(t)
% 3 rows, 1 column and figure 1
subplot(3,1,1)
plot(t,u1)
title('u(t)')
xlabel('t')
ylabel('u(t)')
u2 = t > = 1 \% u(t-1)
% 3 rows, 1 comlumn and figure 2
subplot(312)
plot(t,u2)
title('u(t-1)')
xlabel('t')
ylabel('u(t)')
u3 = t > = 2 \% u(t-2)
% 3 rows, 1 comlumn and figure 3
subplot(313)
plot(t,u3)
title('u(t-2)')
xlabel('t')
ylabel('u(t)')
```

#### Plot these functions in a single figure with the graphs in a 2x2 configuration (10 points).

```
%Part 2
%create time vector interval
t = (-5:0.001:5);
 u(t)-u(t-1) 
y1=(t>=0)-(t>=1);
subplot(221)
plot(t,y1)
title('u(t)-u(t-1)')
xlabel('t')
ylabel('u(t)')
%u(t) + u(t-1) + u(t-2)
y2 = (t>=0)+(t>=1)+(t>=2);
subplot(222)
plot(t,y2)
title('u(t)+u(t-1)+u(t-2)')
xlabel('t')
ylabel('u(t)')
%u(t)u(t-2)
y3 = (t>=0).*(t>=2);
subplot(223)
```

```
plot(t,y3)
title('u(t)u(t-2)')
xlabel('t')
ylabel('u(t)')

%u(t) +u(t-1)-2u(t-2)
y4 = (t>=0) + (t>=1) - 2*(t>=2);
subplot(224)
plot(t,y4)
title('u(t)+u(t-1)-2u(t-2)')
xlabel('t')
ylabel('u(t)')
```

Plot the following in a single plot, 2x2 configuration: 1)v0(t) and v1(t), 2)v0(t) and v2(t), 3)v0(t) and v3(t), and 4)v0(t) and v4(t). There should be four graphs, each with two traces (10 points).

```
%part 3
t = (-2:0.01:2)
v0 = (t \ge 0).*cos(2*pi*t);
v1 = (t \ge 0).*cos(2*pi*t + pi/4);
v2 = (t \ge 0).*cos(0.5*pi*t);
v3 = v0.*v2
v4 = v0.*(exp(-abs(t)))
subplot(2,2,1)
plot(t,v0)
hold on
plot(t,v1)
hold off
grid on
title('v0(t) vs v1(t)')
xlabel('t')
ylabel('u(t)')
subplot(222)
plot(t,v0)
hold on
plot(t,v2)
hold off
grid on
title('v0(t) vs v2(t)')
xlabel('t')
ylabel('u(t)')
subplot(223)
plot(t,v0)
hold on
plot(t,v3)
hold off
grid on
title('v0(t) vs v3(t)')
xlabel('t')
ylabel('u(t)')
```

```
subplot(224)
plot (t,v0)
hold on
plot(t,v4)
hold off
grid on
title('v0(t) vs v4(t)')
xlabel('t')
ylabel('u(t)')
Show the new transfer function (5 points).
%part4
s = tf('s')
H1 = 1/(s+1);
H2 = (2*s^2+3*s+2)/(s^3+2*s+1);
H = H1*H2
H =
     2 s^2 + 3 s + 2
 s^4 + s^3 + 2 s^2 + 3 s + 1
What is the value ats = 1 (5points)?
F1 = [1 5 6];
poly2str(F1,'s');
s = 1;
polyval(F1,s)
ans =
  12
Show the factored polynomial (5 points).
roots(F1)
ans =
 -3.0000
 -2.0000
Show the resulting polynomial. Be sure to account for the two roots at s=0 (5 points).
x=[-1+2j-1-2j-1\ 0\ 0\ ];
F2 = poly(x);
poly2str(F2,'s')
ans =
  s^5 + 3 s^4 + 7 s^3 + 5 s^2
```

Show the resulting polynomial. Also, verify by multiplying the two polynomials (F1 and F2) by hand. Show all your work (write it up on the computer, no handwritten solutions will be accepted) (10 points).

```
H = conv(F1,F2)
poly2str(H,'s')
ans =
' s^7 + 8 s^6 + 28 s^5 + 58 s^4 + 67 s^3 + 30 s^2'
```

Look at the transfer function by using thetffunction. Show the output (5 points).

```
s = tf('s')
n = [1,1]
d = [1,4,0]
H3 = tf(n,d)
H3 =
s+1
------
s^2 + 4 s
```

Calculate by hand the inverse Laplace transform and show in the report (5 points).

```
\begin{split} &[r,p,k] = residue(n,d) \\ &H3 = (r(1)/(s-p(1))) + (r(2)/(s-p(2))) \\ &r = \\ &0.7500 \\ &0.2500 \\ \end{split} p = \\ &-4 \\ &0 \\ \end{split} k = \\ &[] H3 = \\ &s+1
```

```
-----
 s^2 + 4 s
syms s;
ilaplace((s+1)/(s^2+4*s))
ans =
(3*exp(-4*t))/4 + 1/4
Show both the function in frequency-domain and time-domain (5 points).
s = tf('s')
num = [1,0,4]
den = [1,4,0,0]
H4 = tf(num,den)
[r,p,k] = residue(num,den)
syms s;
ilaplace((s^2+4)/(s^3+4*s^2))
H4 =
  s^2 + 4
 s^3 + 4 s^2
\mathbf{r} =
  1.2500
  -0.2500
  1.0000
p =
  -4
   0
```

0

 $\mathbf{k} =$ 

ans =

 $t + (5*exp(-4*t))/4 - \frac{1}{4}$ 

$$\begin{split} n &= [4,1,0,4] \\ \% \, Use \,\, "conv" \,\, to \,\, multiply \,\, polynomial \\ d &= conv([1\,\,0\,\,0\,\,0], conv \,\, ([1\,\,3], conv \,\, ([1\,\,0\,\,2], [1\,\,3]))); \\ H5 &= tf(n,d) \\ [r,p,k] &= residue(n,d) \\ syms \,\, s; \\ ilaplace((4*s^3+s^2+4)/(s^3*(s+3)^2*(s^2+2))) \end{split}$$

4 1 0 4

H5 =

n =

Continuous-time transfer function.

 $\mathbf{r} =$ 

0.1509 + 0.0000i 0.3199 + 0.0000i -0.0847 - 0.0993i -0.0847 + 0.0993i 0.0185 + 0.0000i -0.1481 + 0.0000i 0.2222 + 0.0000i

p =

-3.0000 + 0.0000i -3.0000 + 0.0000i -0.0000 + 1.4142i -0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i

 $\mathbf{k} =$ 

[]

ans =

 $(493*exp(-3*t))/3267 - (4*t)/27 - (41*cos(2^(1/2)*t))/242 + (95*t*exp(-3*t))/297 + (17*2^(1/2)*sin(2^(1/2)*t))/121 + t^2/9 + 1/54$