ECE 3723 - Electric Circuits II - Fall 2021
Project 5: Fourier Series

Due: 11/16/21 on Canvas

Introduction

Fourier series are very important to engineers for representing period functions and they can be expanded to Fourier transforms for representing aperiodic signals as well. The purpose of this project is to help better visualize the Fourier representation of a few common signals.

In this project, a Fourier series representation of a given function will be found. This series will then be plotted along with the actual function and the error quantified. By increasing the number of terms the series representation will start to closer match the actual function.

1 Fourier Series

You are given a periodic function $v(t)$ as:

$$v(t) = \begin{cases} V_m \sin\left(\frac{4\pi t}{T}\right) & \text{for } 0 \leq t \leq \frac{T}{2} \\ \\ 3V_m \sin\left(\frac{4\pi t}{T}\right) & \text{for } \frac{T}{2} \leq t \leq T \end{cases}$$

- Assume $V_m = 1$ V and $T = 1$ second. **Plot the function v(t) vs t from -T to 3T.** (10 points)
- **What symmetries (if any) apply?** Justify the answer using math. (5 points)
- Derive the Fourier series representation of the function. **Show all steps of the derivation. Type all steps, absolutely no credit will be given for handwritten answers.** (10 points)
- **Plot each of the first four non-zero terms along with the actual function using subplot(2,2,n) to generate a single plot with all four cases shown in a 2×2 array configuration.** The title of each subplot should indicate which term is being plotted. (10 points)

- Now add the four first non-zero terms together to create v(t), and then plot the resulting waveform.
- The code created for this should allow for simply changing the number of desired harmonics.
  Plot v(t) for n = 3, n = 5, n = 10, and n = 50. (10 points)
- Find the error as a function of n, where the error is defined as the difference between two the voltage from the Fourier series $(v_F(t))$ and the value from the ideal function $(v(t))$, normalized to the maximum magnitude (V_m):

$$\%error = \left| \frac{v_F(t) - v(t)}{V_m} \right| \times 100\%$$

- **Plot the error versus time for n = 3, n = 5, n = 10, and n = 50.** (10 points)
- **Question: What is the maximum error for each of those cases?** (5 points)
- Create a vector that contains the maximum error as a function of n.

  Finally, use a power series (an^b) to fit the maximum error and analytically solve for the number of terms needed to achieve less than 5% of error. Hint: the function `fit` in MATLAB®with `'power1'` is useful for fitting power series (for some reason that is not showing as a default `fittype` for `fit`, but it still works).
  How many terms (n) are needed for the error to be less than 5%? (5 points)

  Plot the maximum-error vector as a function of n. Also on the same plot, plot the fitted function.
  (10 points)

2 Deliverables

This project should be written up in a neatly organized report that includes the results from all the sections, answers to all the questions, and all plots that are generated. All calculations and derivations must be typed up, no credit will be given for handwritten solutions.

3 Grading

- The report itself is worth 15 points, which will be given based on the structure of the report, grammar, clarity, and formatting.
- Answering the questions listed in the handout in the main body of the report is worth 75 points.
- Turning in the code in an appendix is worth 10 points.