ECE 3723

Project 5

Tri Pham

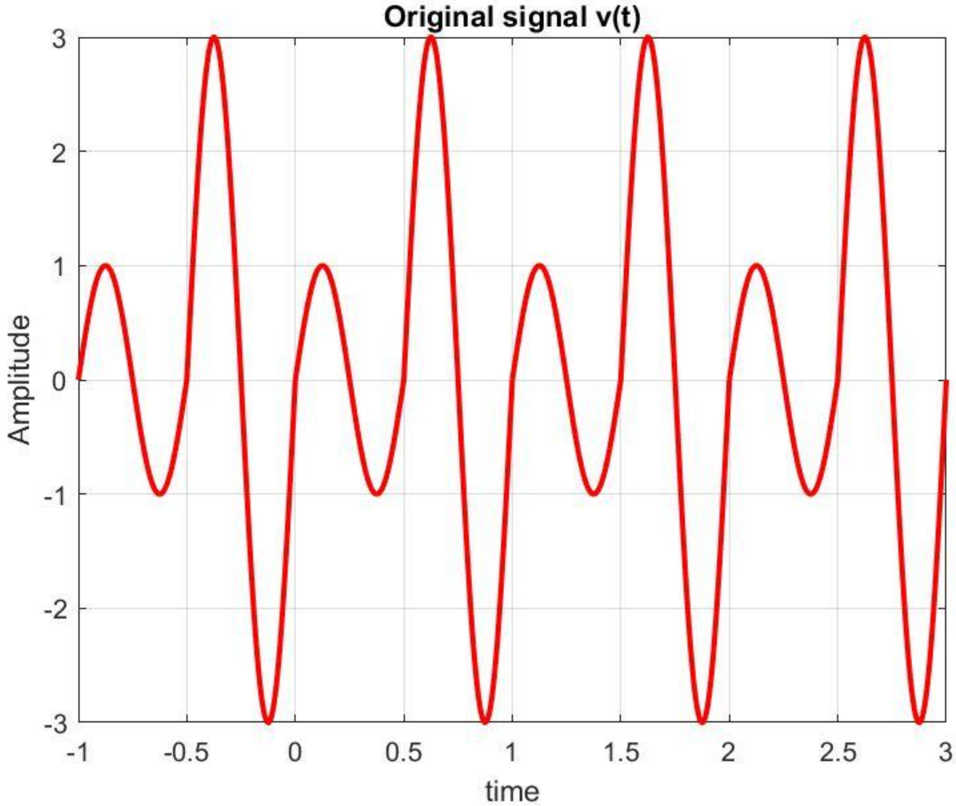
Nguyen vu

Dung Nguyen

Introduction

In this project, we will discover something that we never work on. Using the given function to make the plot for Fourier transformation function to represent the periodic function. Also see what are the different when we repeat many times.

Plot the function v(t) vs t from -T to 3T. (10 points)



What symmetries (if any) apply?

This function does not have any symmetry because f (0.7) = 1.7 and f (-0.7) = -0.58. Also, from 0 to T, there are two different functions so that it will give us the different answers.

Derive the Fourier series representation of the function. Show all steps of the derivation. Type all steps,
absolutely no credit will be given for handwritten answers

Av = $\frac{1}{T}\int_0^T f(t)dt$

$=\frac{1}{T}\int_0^{T/2} Vm * \sin\left(\frac{4\pi t}{T}\right)dt \ + \frac{1}{T}\int_{T/2}^{T} 3Vm * \sin\left(\frac{4\pi t}{T}\right)dt$

T = 1, Vm = 1.

$= \left[-\frac{\cos(4\pi t)}{4\pi}\right]_{0}^{1/2} + 3\left[\frac{-\cos(4\pi t)}{4\pi}\right]_{1/2}^{1}$

$= \left[-\frac{\cos\left(4\pi * \frac{1}{2}\right)}{4\pi} + \frac{\cos(0)}{4\pi}\right] + 3\left[-\frac{\cos(4\pi)}{4\pi} + \frac{\cos(2\pi)}{4\pi}\right]$

$= \frac{-1}{4\pi} + \frac{1}{4\pi} + 3\left(\frac{-1}{4\pi} + \frac{1}{4\pi}\right)$

Av = 0

Bk = $\frac{2}{T} f(t)\sin(kw_0 t)\,dt$

$$\frac{2}{T}\int_0^{T/2} Vmsin(4\pi t)\sin(kw_0 t)\,dt + \frac{2}{T}\int_{T/2}^{T} 3Vmsin(4\pi t)\sin(kw_0 t)\,dt$$

Vm = 1, T = 1.

Int 1

$$\int_0^{1/2}[\cos(4\pi - k2\pi)\,t - \cos(4\pi + k2\pi)\,t]dt$$

$$\left[\frac{\sin(4\pi - k2\pi)\,t}{4\pi - k2\pi}\right]\begin{matrix}1/2\\0\end{matrix} - \left[\frac{\sin(4\pi + k2\pi)\,t}{4\pi + k2\pi}\right]\begin{matrix}1/2\\0\end{matrix}$$

$$\left[\frac{\sin\,(2\pi - k\pi)}{4\pi - k2\pi} - \frac{\sin\,(2\pi + k\pi)}{4\pi + k2\pi}\right]$$

Int 2

$$3\int_{1/2}^{1}[\cos(4\pi - k2\pi)\,t - \cos(4\pi + k2\pi)\,t]dt$$

$$3\left[\frac{\sin(4\pi - k2\pi)\,t}{4\pi - k2\pi}\right]\begin{matrix}1\\1/2\end{matrix} - \left[\frac{\sin(4\pi + k2\pi)\,t}{4\pi + k2\pi}\right]\begin{matrix}1\\1/2\end{matrix}$$

$$3\left[\frac{\sin\,(4\pi-k2\pi)}{4\pi-k2\pi} - \frac{\sin(2\pi-k\pi)}{4\pi-k2\pi} - \left(\frac{\sin\,(4\pi+k2\pi)}{4\pi+k2\pi} - \frac{\sin\,(2\pi+k\pi)}{4\pi+k2\pi}\right)\right]$$

$$3\left[\frac{\sin(4\pi-k2\pi)-\sin\,(2\pi-k\pi)}{4\pi-k2\pi} - \frac{\sin(4\pi+k2\pi)+\sin\,(2\pi+k\pi)}{4\pi+k2\pi}\right]$$

Bk = Int 1 + Int 2

$$\left[\frac{\sin\,(2\pi-k\pi)}{4\pi-k2\pi} - \frac{\sin\,(2\pi+k\pi)}{4\pi+k2\pi}\right] + 3\left[\frac{\sin(4\pi-k2\pi)-\sin\,(2\pi-k\pi)}{4\pi-k2\pi} - \frac{\sin(4\pi+k2\pi)+\sin\,(2\pi+k\pi)}{4\pi+k2\pi}\right]$$

Bk = 0 for k is odd and even because sin(kπ) = 0. We have to use the L's Hopital Rule when k = 2 because the denominator will go to zero. Bk not equal to zero when k = 2

Ak = $\frac{2}{T}\int_0^{T/2} Vmsin(4\pi t)\cos(kw_0 t)\,dt +$
$\frac{2}{T}\int_{T/2}^{T} 3Vmsin(4\pi t)\cos(kw_0 t)\,dt$

Vm = 1, T = 1.

Int 1

$$\int_0^{1/2} \sin(4\pi t)\cos(k2\pi t)\,dt$$

$$\int_0^{1/2} [\sin(4\pi + k2\pi)\,t + \sin(4\pi - k2\pi)\,t]dt$$

$$\left[\frac{-\cos\,(4\pi + k2\pi)}{4\pi + k2\pi}\right]\begin{matrix}1/2\\0\end{matrix} - \left[\frac{\cos(4\pi - k2\pi)\,t}{4\pi - k2\pi}\right]\begin{matrix}1/2\\0\end{matrix}$$

$$\frac{-\cos\,(2\pi + k\pi)}{4\pi + k2\pi} + \frac{\cos\,(0)}{4\pi + k2\pi} - \left(\frac{\cos\,(2\pi - k2\pi}{4\pi - k2\pi} - \frac{\cos\,(0)}{4\pi - k2\pi}\right)$$

$$\frac{-\cos(2\pi + k\pi) + 1}{4\pi + k2\pi} + \frac{-\cos(2\pi - k\pi) + 1}{4\pi - k2\pi}$$

Int 2

$$6\int_{1/2}^{1} \sin(4\pi t)\cos(k2\pi t)\,dt$$

$$3\int_{1/2}^{1} [\sin(4\pi + k2\pi)\,t + \sin(4\pi - k2\pi)\,t]dt$$

$$3\left[\left(\frac{-\cos\,(4\pi+k2\pi}{4\pi+k2\pi}\right)\,\begin{matrix}1\\1/2\end{matrix} - \left(\frac{\cos\,(4\pi-k2\pi}{4\pi-k2\pi}\right)\,\begin{matrix}1\\1/2\end{matrix}\right]$$

$$3\left[\left(\frac{-\cos\,(4\pi+k2\pi)}{4\pi+k2\pi}+\frac{\cos\,(2\pi+k\pi)}{4\pi+k2\pi}\right)-\left(\frac{\cos\,(4\pi-k2\pi)}{4\pi-k2\pi}-\frac{\cos\,(2\pi-k\pi)}{4\pi-k2\pi}\right)\right]$$

$$3\left[\frac{-\cos(4\pi+k2\pi)+\cos\,(2\pi+k\pi)}{4\pi+k2\pi}-\frac{\cos(4\pi-k2\pi)+\cos\,(2\pi-k\pi)}{4\pi-k2\pi}\right]$$

Ak = Int 1 + Int 2

$$\frac{-\cos(2\pi+k\pi)+1}{4\pi+k2\pi}+\frac{-\cos(2\pi-k\pi)+1}{4\pi-k2\pi}+3\left[\frac{-\cos(4\pi+k2\pi)+\cos\,(2\pi+k\pi)}{4\pi+k2\pi}-\frac{\cos(4\pi-k2\pi)+\cos\,(2\pi-k\pi)}{4\pi-k2\pi}\right]$$

For k is even

Ak = $\frac{-\cos(2\pi)+1}{4\pi}+\frac{-\cos(2\pi)+1}{4\pi}+3\left[\frac{-\cos(4\pi)+\cos\,(2\pi)}{4\pi}-\frac{\cos(4\pi)+\cos\,(2\pi)}{4\pi}\right]$

Ak = 0 for k is even

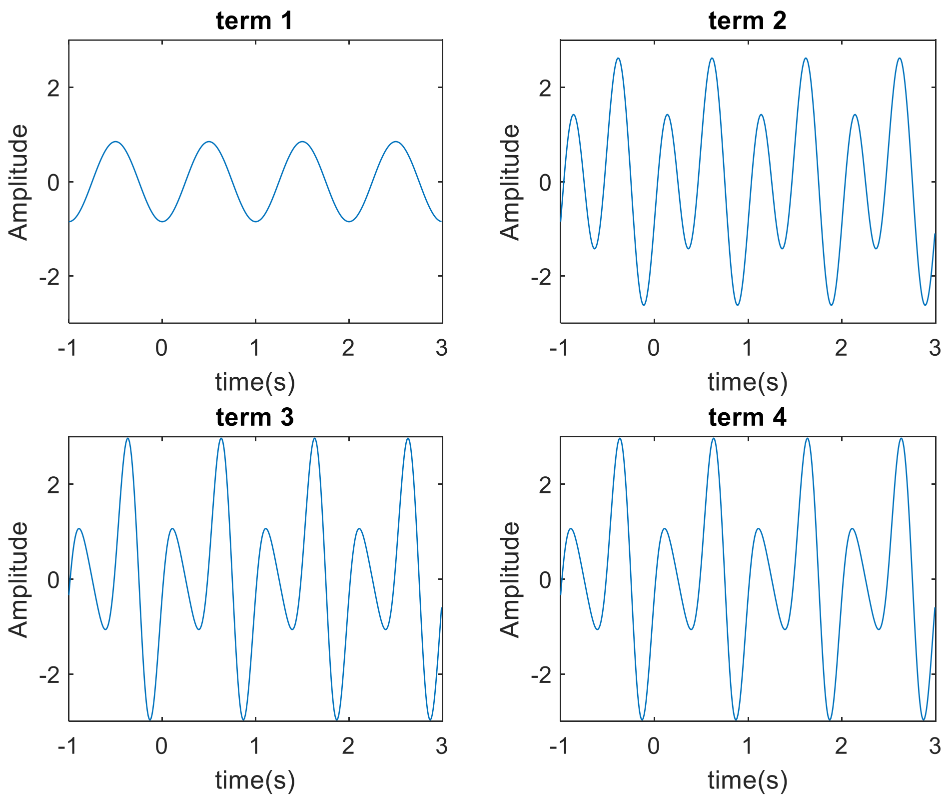
For k is odd

Ak = $\frac{-\cos(3\pi)+1}{6\pi}+\frac{-\cos(\pi)+1}{2\pi}+3\left[\frac{-\cos(6\pi)+\cos\,(3\pi)}{6\pi}-\frac{\cos(2\pi)+\cos\,(\pi)}{2\pi}\right]$

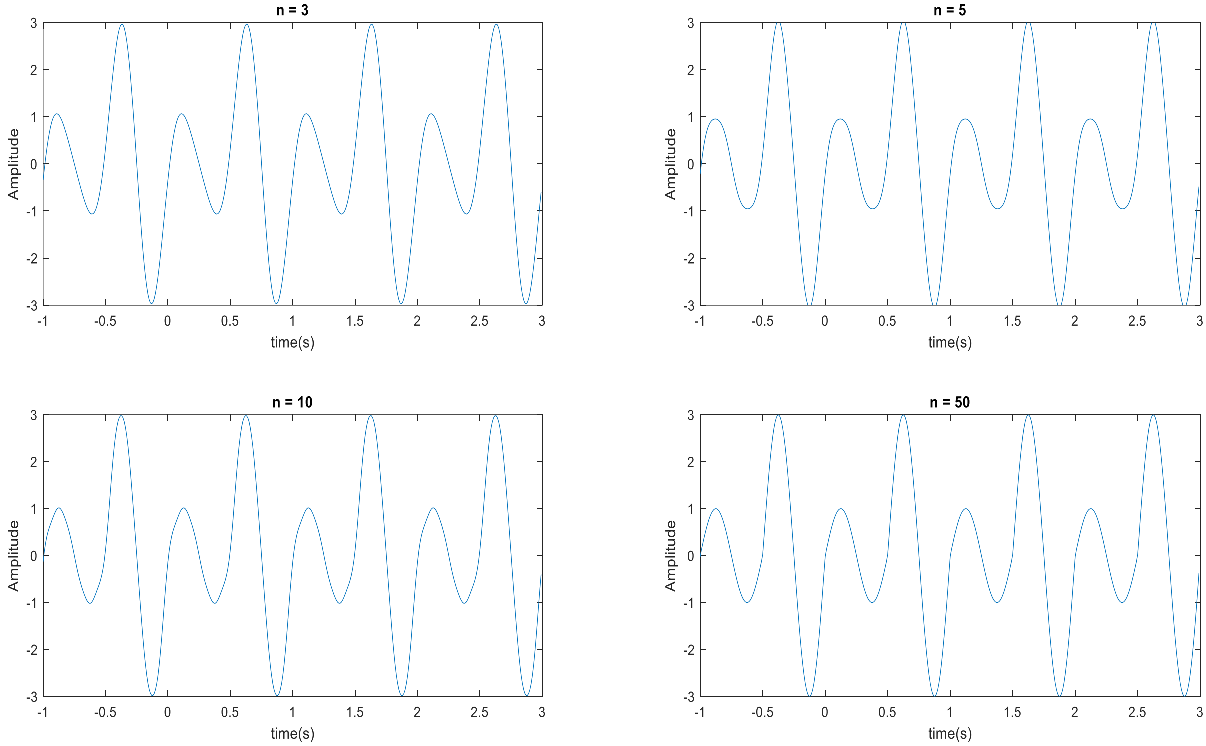
Ak = $\frac{8}{6\pi}+3(\frac{-2}{6\pi}-\frac{2}{2\pi})$

Ak = $\frac{-8}{3\pi}$

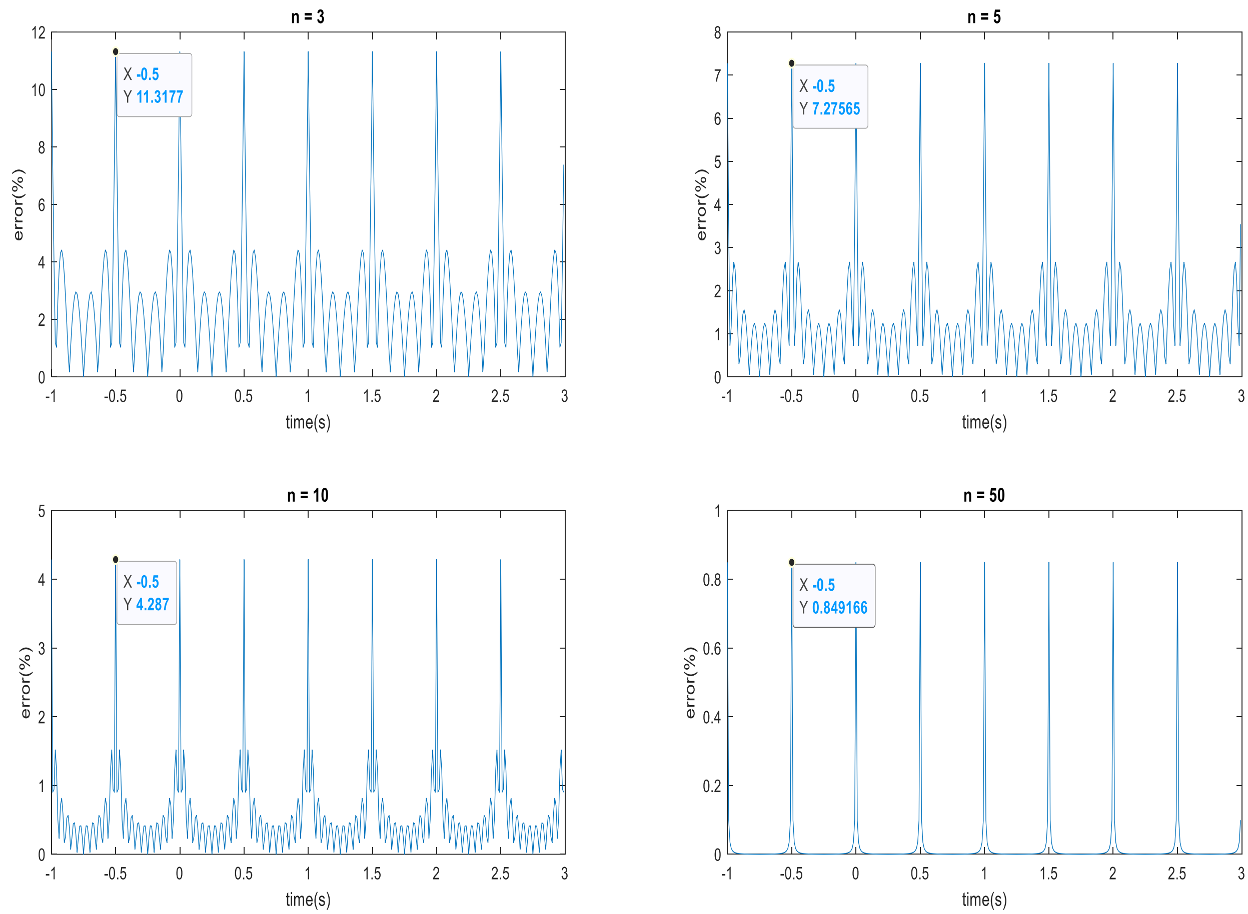
Plot each of the first four non-zero terms along with the actual function using subplot(2,2, n) to generate a single plot with all four cases shown in a 2×2 array configuration.



Plot v(t) for n = 3, n = 5, n = 10, and n = 50



Plot the error versus time for n = 3, n = 5, n = 10, and n = 50



What is the maximum error for each of those cases?

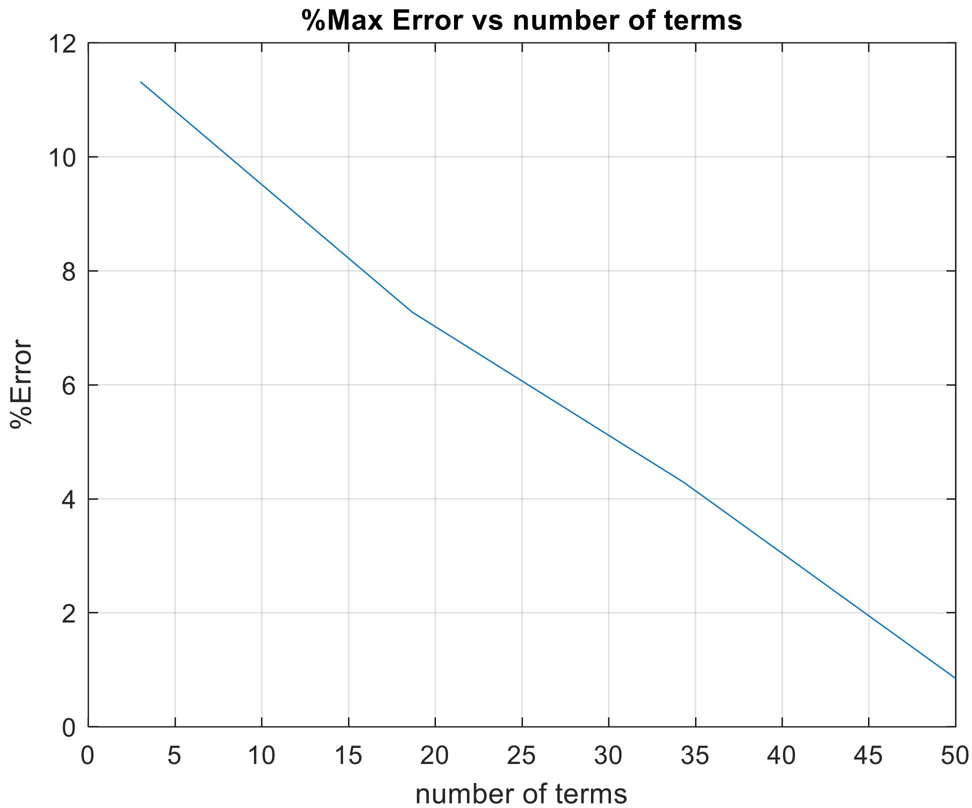
For n = 3, maximum error is 11.31

For n = 5, maximum error is 7.27

For n = 10, maximum error is 4.3

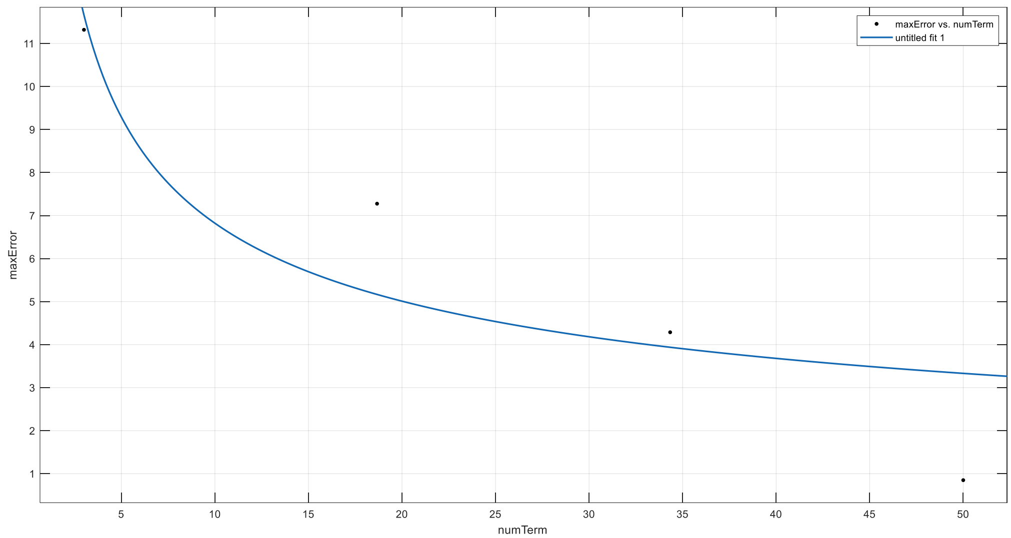
For n = 50, maximum error is 0.85

How many terms (n) are needed for the error to be less than 5%?



Base on this plot, there are approximately about 20 terms from n = 30 to n = 50 that has the percent error less than 5 %.

Plot the maximum-error vector as a function of n. Also on the same plot, plot the fitted function.



Conclusion

Based on this project, we have learned that how to use the repeated function for periodic function, representing the Fourier Transform, calculating the percent error of the Fourier transformation function and how to plot it.

Appendix

```
clc;
close all;
clear all;

%Part 1
% Define the signal parameters
Vm = 1;
T = 1;
w0 = 2*pi/T;

% Define the symbolic variables
syms n t;

% Define the signal
v1 = Vm*sin(4*pi*t/T);
v2 = 3*Vm*sin(4*pi*t/T);

% Evaluate the fourier series integral
ak = 2/T*int(v1*cos(n*w0*t),0,T/2) + 2/T*int(v2*cos(n*w0*t),T/2,T);
bk = 2/T*int(v1*sin(n*w0*t),0,T/2) + 2/T*int(v2*sin(n*w0*t),T/2,T);
av = 1/T*int(v1,0,T/2) + 1/T*int(v2,T/2,T);

% Declare the range of terms
nMax = 100;
n = 1:nMax;
a = subs(ak);
b = subs(bk);

% define the time vector
ts = 1e-2; % ts is sampling the
t = -1:ts:3*T-ts;

% directly plot the signal x(t)
t1 = -1:ts:0-ts;
v1 = Vm*sin(4*pi*t1/T).*(t1<=-T/2);
v2 = 3*Vm*sin(4*pi*t1/T).*(t1>-T/2).*(t1<0);
v = v1+v2;
x = repmat(v,1,4);


plot(t,x,'r','linewidth',2);
grid on;
title('Original signal v(t)')
xlabel('time(s)');
ylabel('Amplitude');
ylim([-3 3]);


%Part2
%First 4 terms Harmonic n = 1,2,3,4
for i = 1:length(t)
for k = 1
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
```

```
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,1)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 1')

for i = 1:length(t)
for k = 1:2
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,2)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 2')
for i = 1:length(t)
for k = 1:3
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,3)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 3')

for i = 1:length(t)
for k = 1:4
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,4)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 4')


%Part 3
%Harmonic n = 3,5,10,50
for i = 1:length(t)
for k = 1:3
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,1)
plot(t,y)
```

```
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 3')

for i = 1:length(t)
for k = 1:5
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,2)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 5')
for i = 1:length(t)
for k = 1:10
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,3)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 10')

for i = 1:length(t)
for k = 1:50
x(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(x(:,i)); % Add DC term
end
subplot(2,2,4)
plot(t,y)
xlabel('time(s)');ylabel('Amplitude');
ylim([-3 3]);
title('n = 50')


%Part 4
% Error vs Time
for i = 1:length(t)
for k = 1:3
%p preresents for Vf
p(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end

y(i) = av+sum(p(:,i));
end
% fnc preresent for error
fnc = ((abs(y-x))/3)*100;
subplot(221)
plot(t,fnc)
title('n = 3')
```

```
xlabel('time(s)')
ylabel('error(%)')

for i = 1:length(t)
for k = 1:5
p(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(p(:,i));
end
fnc = ((abs(y-x))/3)*100;
subplot(222)
plot(t,fnc)
title('n = 5')
xlabel('time(s)')
ylabel('error(%)')

for i = 1:length(t)
for k = 1:10
p(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(p(:,i));
end
fnc = ((abs(y-x))/3)*100;
subplot(223)
plot(t,fnc)
title ('n = 10')
xlabel('time(s)')
ylabel('error(%)')

for i = 1:length(t)
for k = 1:50
p(k,i) = a(k)*cos(k*w0*t(i)) + b(k)*sin(k*w0*t(i));
end
y(i) = av+sum(p(:,i));
end
fnc = ((abs(y-x))/3)*100;
subplot(224)
plot(t,fnc)
title ('n = 50')
xlabel('time(s)')
ylabel('error(%)')


%Part 5
% Create the vector that contain maximum error 11.3177, 7.27565, 4.287,
% 0.849166
maxError = [11.3177 7.27565 4.287 0.849166];
numTerm = linspace(3,50,4.1);
plot(numTerm, maxError);
grid on;
xlabel('number of terms');
ylabel('%Error');
title('%Max Error vs number of terms');
```