Project 6

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ECE3723

Introduction

In this project, we will learn about how to analyze the circuit by breaking it into basic elements. Then we will find 2-port network for each and combine them to get overall circuit. Hence, we will understand how useful two-port networks are for circuit analysis

Procedure

Two-Port Network

What is the center frequency of this filter (in Hz)?

$$Z(s) = \left(sL_2 \parallel \frac{1}{sC2}\right) + sL_1 + \frac{1}{sC1}$$

$$= \frac{(sL2)(\frac{1}{sC2})}{sL2+ \frac{1}{sC2}} + sL_1 + \frac{1}{sC1}$$

$$= \frac{sL2}{s^2L2C2+1} + sL_1 + \frac{1}{sC1}$$

$$= \frac{s^2\, L2C1+s^4\, L2C2L1C1+s^2L1C1+s^2L2C2+1}{S^3\, L2C2C1+sC1}$$

$$= \frac{s^4L2C2L1C1+s^2(L1C1+L2C2+L2C1)+1}{s^3L2C2C1+sC1}$$

$$Z(jW) = \frac{W^4L2C2L1C1-W^2(L1C1+L2C2+L2C1)+1}{-jW^3L2C2C1+jWC1}$$

$$= -j\left(\frac{W^4L2C2L1C1-W^2(L1C1+L2C2+L2C1)+1}{-W^3L2C2C1+WC1}\right)$$

At resonance, imaginary part to 0

$$\frac{W^4L2C2L1C1-W^2(L1C1+L2C2+L2C1)+1}{-W^3L2C2C1+WC1} = 0$$

$W^4L_2C_2L_1C_1 - W^2(L_1C_1 + L_2C_2 + L_2C_1) + 1 = 0$

Substitute

$L_1 = 112.54*10^{-9}$ H

$C_1 = 37.806*10^{-15}$ F

$L_2 = 94.514*10^{-12}$ H

$C2 = 45.016*10^{-12}$ F.

Then

$W^2 = 2.35*10^{20}$

$W = 1.53*10^{10}$ rad/s

fc = $\frac{W}{2\pi}$ = 2.44*10^9 Hz

Now split the circuit into four separate blocks (two-port networks containing one component each) and find the a-parameters for each. Hint: do it for a generic impedance or admittance to see the trend. (20 points)

I do this question first then I will find the a-parameters by using cascade.

Circuit 1: L_1

Open port 2: $I_2 = 0$

$a_{11} = \frac{V1}{V2}|_{I2\,=\,0} = 1$

$a_{21} = \frac{I1}{V2}\,|_{I2\,=0} = \frac{-I2}{V2} \quad = 0$

Short port 2: $V_2 = 0$

$a_{12} = \frac{-V1}{I2}|_{V2=0} = \frac{-V1}{-V1/sL1} = sL_1$

$a_{22} = \frac{-I1}{I2}|_{V2=0} = \frac{-I1}{-I1} = 1$

$[a_1] = \begin{bmatrix} 1 & sL1 \\ 0 & 1 \end{bmatrix}$

Circuit 2: C_1

Open port 2: $I_2 = 0$

$a_{11} = \frac{V1}{V2}|_{I2=0} = 1$

$a_{21} = \frac{I1}{V2}|_{I2=0} = \frac{-I2}{V2} = 0$

Short port 2: $V_2 = 0$

$a_{12} = \frac{-V1}{I2}|_{V2=0} = \frac{-V1}{-V1/(\frac{1}{sC1})} = \frac{1}{sC1}$

$a_{22} = \frac{-I1}{I2}|_{V2=0} = \frac{-I1}{-I1} = 1$

$[a_2] = \begin{bmatrix} 1 & \frac{1}{sC1} \\ 0 & 1 \end{bmatrix}$

Circuit 3: C_2

Open port 2: $I_2 = 0$

$a_{11} = \frac{V1}{V2}|_{I2=0} = 1$

$a_{21} = \frac{I1}{V2}|_{I2=0} = \frac{I1}{I1*sL2} = \frac{1}{sL2}$

Short port 2: $V_2 = 0$

$$a_{12} = \frac{-V1}{I2}\Big|_{V2\,=\,0} = \frac{-V2}{I2} = 0$$

$$a_{22} = \frac{-I1}{I2}\Big|_{V2\,=\,0} = \frac{-I1}{-I1} = 1$$

$$[a_3] = \begin{bmatrix} 1 & 0 \\ \frac{1}{sL2} & 1 \end{bmatrix}$$

Circuit 4: L_2

Open port 2: $I_2 = 0$

$$a_{11} = \frac{V1}{V2}\Big|_{I2\,=\,0} = 1$$

$$a_{21} = \frac{I1}{V2}\Big|_{I2\,=0} = \frac{I1}{I1*(\frac{1}{sc2})} = sC_2$$

Short port 2: $V_2 = 0$

$$a_{12} = \frac{-V1}{I2}\Big|_{V2\,=\,0} = \frac{-V2}{I2} = 0$$

$$a_{22} = \frac{-I1}{I2}\Big|_{V2\,=\,0} = \frac{-I1}{-I1} = 1$$

$$[a_4] = \begin{bmatrix} 1 & 0 \\ sC2 & 1 \end{bmatrix}$$

Find the a-parameters by hand. (15 points)

By using cascade, I am able to find the a-parameters

$[a^T] = [a_1]\ [a_2]\ [a_3]\ [a_4]$

$$= \begin{bmatrix} 1 & sL1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{sC1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{sL2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{1}{sC1}+sL1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{sL2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (\frac{1}{sC1}+sL1)\frac{1}{sL2} & \frac{1}{sC1} + sL1 \\ \frac{1}{sL2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC2 & 1 \end{bmatrix}$$

$$[a^T] = \begin{bmatrix} \left(\frac{1}{sC1}+sL1\right)\left(\frac{1}{sL2} + sC2\right) + 1 & 0 \\ sC2 + \frac{1}{sL2} & 1 \end{bmatrix}$$

Find the transfer function and plot it in dB versus frequency. (20 points)

The transfer function by hands

$$H(s) = \frac{ZL}{(a11+a21*Zg)ZL+a12+a22*Zg}$$

Substitute

Z_L = 50 ohm

Zg = 50 ohm

$L_1 = 112.54*10^{-9}$ H

$C_1 = 37.806*10^{-15}$ F

$L_2 = 94.514*10^{-12}$ H

$C2 = 45.016*10^{-12}$ F.

$a_{11} = \left(\frac{1}{sC1}+sL1\right)\left(\frac{1}{sL2}+sC2\right)+1$

$= (\frac{1}{37.806*10^{-15}*s} + 112.54*10^{-9}*s)\ (\frac{1}{94.514*10^{-12}*s} + 45.016*10^{-12}*s) + 1$

$a_{12} = 0$

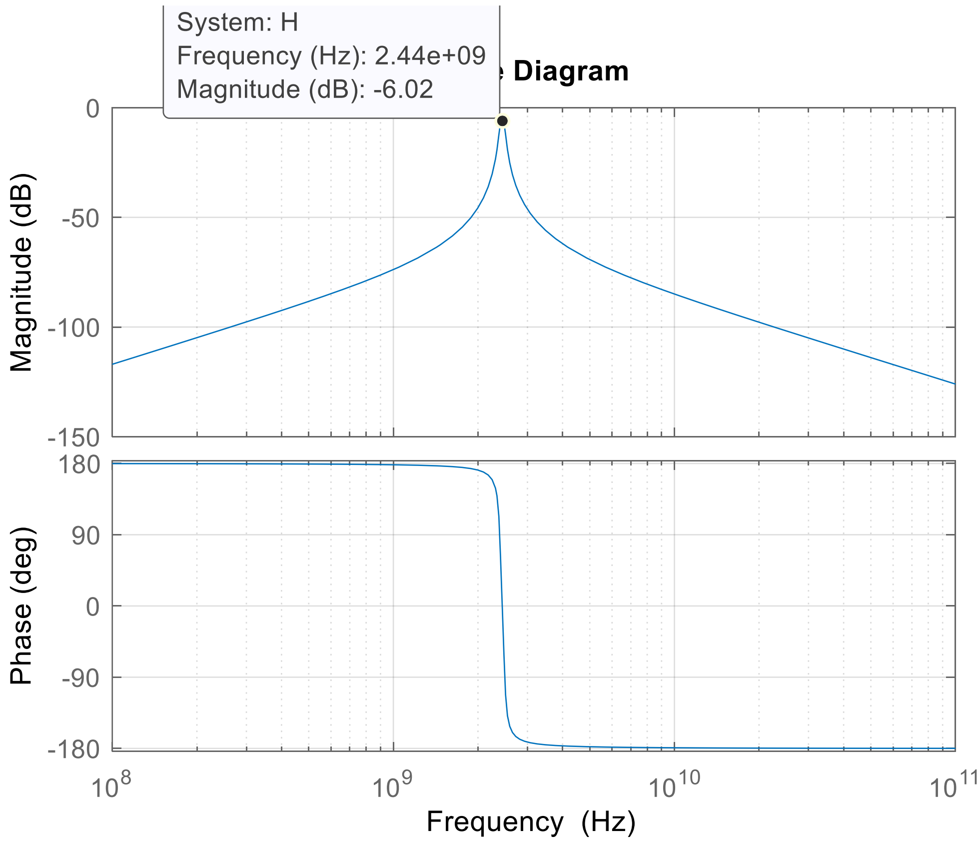
$a_{21} = \frac{1}{sL2} + sC2 = \frac{1}{94.514*10^{-12}*s} + 45.016*10^{-12}*s$

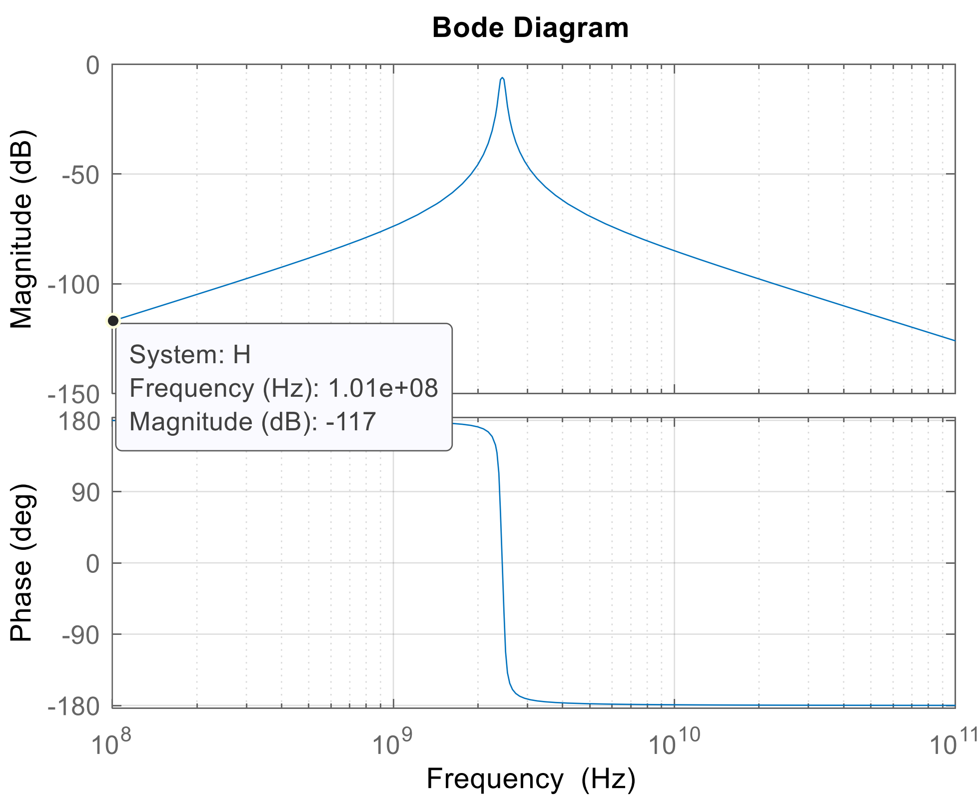
$a_{22} = 1$

$$H(s) = \frac{1.97*10^{17}*s^2}{s^4+8.89*10^{8}*s^3+4.71*10^{20}*s^2+2.09*10^{29}*s+5.52*10^{40}}$$

The transfer function from MATLAB

$$H(s) = \frac{1.974e17\ s^2}{s^4 + 8.886e08\ s^3 + 4.705e20\ s^2 + 2.088e29\ s + 5.524e40}$$


System: H
Frequency (Hz): 2.44e+09
Magnitude (dB): -6.02
Diagram
Magnitude (dB)
Phase (deg)
0
-50
-100
-150
180
90
0
-90
-180
10^8
10^9
10^{10}
10^{11}
Frequency (Hz)


Bode Diagram
Magnitude (dB)
Phase (deg)
0
-50
-100
-150
System: H
Frequency (Hz): 1.01e+08
Magnitude (dB): -117
180
90
0
-90
-180
10^8
10^9
10^{10}
10^{11}
Frequency (Hz)

What is the maximum amplitude? Explain why the amplitude is what it is. (10 points)

We can observe that peak response at $2.44*10^9$ Hz.
The maximum amplitude response of this type of filter in the circuit so that the minimum value starts at -117 dB and stop at -6.02dB. Then we can get the maximum amplitude
$|-117\text{dB} - (-6.02\text{dB})|$ = 110.98dB
Because w_0 is the frequency, which is 2.44e+09, at which the maximum amplitude occur.

Conclusion

Through this project, I can understand the useful of two port networks for circuit analysis. I can find the center frequency without looking for the cut off frequency, and I can also use the MATLAB to find the center frequency through bode plot function. Finally, I can find out the transfer function quickly by using MATLAB.

Appendix

```
clc;
clear;
close all;
% Assign the value of L1,C1,L2,C2
L1 = 112.54e-9;
C1 = 37.806e-15;
L2 = 94.514e-12;
C2 = 45.016e-12;
% Source is 50 Ohm and Load is 50 Ohm
RS = 50;
RL = 50;
% Continuous-time transfer function.
s = tf('s');
% Create frequency domain function
Z1 = s*L1 + (1/(s*C1)) + RS;
Z2 = 1/(1/(s*L2) + s*C2 + (1/RL));
% Transfer function
H = Z2/(Z1 + Z2)
% Use bodeplot function to plot the graph
h = bodeplot(H);
% Transfer the frequency from rad/s to Hz
p = getoptions(h);
p.FreqUnits = 'Hz';
setoptions(h,p);
grid on;
```