IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 09 Issue 03, March 2022
ISSN (Online) 2348 – 7968 | Impact Factor – 6.72
www.ijiset.com

Triangular Statistical Arbitrage in Foreign Exchange Market

Naleli Jubert Matjelo[1]

[1] Department of Physics and Electronics, National University of Lesotho, Roma, 180, Lesotho

Abstract

This paper presents an application of statistical arbitrage to foreign exchange markets and specifically on instruments that obey a triangle rule/law. These instruments have a long-term cointegration relationship which leads to the mean-reversion behaviour evident in their data. Metatrader 4 is used to develop an indicator which in turn is used to gather and process price data and inform an investor on when and how to invest in a particular combination of three instruments obeying the triangle rule. That is, a simple strategy is outlined and its long-term profitability is demonstrated using expectation value under some mild assumption of stability of cointegration model parameters.

Keywords: *Statistical Arbitrage, Triangular Arbitrage, Cointegration, Stationarity, Unit Root, Mean-reversion.*

1. Introduction

The statistical description of the stock market is often based on stationary random processes [1]. Under this stationarity assumption, parameters of these random processes can be inferred from historical data. These parameters serve as a good statistical description of the future evolution of the market. Most practitioners rely on financial time-series that are stationary [1]. However, many time-series data in finance and economics have unit roots [2], and it is widely accepted that nonstationary processes cannot be forecasted [3] hence the reason for the invention of trading schemes such as pairs trading [4] and statistical arbitrage [5]. The pairs trading scheme was developed in several stages beginning with the research that was made by Tartaglia's group [5]. As the analysis techniques got more sophisticated and trading instruments increased beyond two, the idea of this scheme paved the way to a more general approach and the term "statistical arbitrage" started to be used in 1990 [5]. Cointegration tests and stationarity tests are central to both pairs trading and statistical arbitrage. There are various methods for testing stationarity and cointegration with some of the simplest being the Dickey-Fuller (DF) test [2], the Augmented Dickey-Fuller (ADF) test [3], and the Engle-Granger test [2]. This paper adopts the Dickey-Fuller test for stationarity test and/or Engle-Granger test for cointegration test.

Finding a combination of financial instruments with long-term cointegration can be challenging and even when found, there are factors (i.e. political, economic, natural, etc) that can disrupt the cointegration among instruments. One attractive arbitrage is the triangular arbitrage [6] which involves three instruments obeying a triangle rule; which states that one instrument is Mathematically equivalent to the product of the other two instruments. As such the instruments are locked in a long-term relationship bound by Mathematics, hence invariant to factors mentioned above. Attractive as it is, this arbitrage strategy is hard to execute for retail traders and can be considered as stealing the broker's money since it takes advantage of the broker's pricing inefficiencies happening at sub-second timescales. In this paper, we combine the triangle rule (from triangular arbitrage) with statistical arbitrage to form a statistical triangular arbitrage with one important feature that there is a guaranteed long-term cointegration dictated by the triangle rule.

The rest of this paper is organized as follows. Section II covers the theory and test for cointegration and stationarity of time series data. Section III outlines the details of combining triangular arbitrage with statistical arbitrage to get statistical triangular arbitrage. Section IV presents the implementation and testing of statistical triangular arbitrage on the Metatrader 4 platform. An indicator developed in this platform is shown and discussed. A link to the implemented indicator on www.mql5.com website is also given here. Section V concludes this paper by briefly stating the major findings in this work.

2. Stationarity and Cointegration

2.1 Cointegration Model

If the three nonstationary processes $x_0(t)$, $x_1(t)$ and $x_2(t)$ are cointegrated it is possible, then, to combine them such that their linear combination $u(t)$ is stationary. This is the principle behind pairs trading and statistical arbitrage. Consider the following linear combination of three processes,

$$x_0(t) = a_1 x_1(t) + a_2 x_2(t) + a_3 + u(t) \tag{1}$$

where a_1, a_2 and a_3 are constants and $u(t)$ is some noise process that must be stationary if the three processes cointegrate. The term a_3 defines a constant offset or intercept, the term a_2 defines the relationship of the instrument $x_2(t)$ with instrument $x_0(t)$ and lastly the term a_1 defines the relationship between the instrument $x_1(t)$ and instrument $x_0(t)$. These constants are usually unknown but can be estimated by fitting data to the model above. It is clear then that, if we want to test whether the three processes are cointegrated, all we have to do is estimate $u(t)$ (as residuals of ordinary least squares fit of data in our model above) and test if this estimated $u(t)$ is stationary or not. If $u(t)$ happens to be stationary, we can conclude that the three processes are cointegrated. Let us attempt to determine the constants using optimization of a linear regression cost function J as described below,

$$J = \frac{1}{2}\sum_{t=1}^{N} \left(a_1 x_1(t) + a_2 x_2(t) + a_3 - x_0(t)\right)^2 \tag{2}$$

with N as the sample size (i.e. total number of candlesticks in our data sample/horizon). Differentiating the regression cost function above with respect to the constants/coefficients and equating to zero yields a set of simultaneous equations which can be presented compactly in matrix form and the unknowns be solved by taking the matrix inversion as shown below,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum_{t=1}^{N} x_1(t)^2 & \sum_{t=1}^{N} x_1(t) x_2(t) & \sum_{t=1}^{N} x_1(t) \\ \sum_{t=1}^{N} x_1(t) x_2(t) & \sum_{t=1}^{N} x_2(t)^2 & \sum_{t=1}^{N} x_2(t) \\ \sum_{t=1}^{N} x_1(t) & \sum_{t=1}^{N} x_2(t) & N \end{bmatrix}^{-1} \begin{bmatrix} \sum_{t=1}^{N} x_0(t) x_1(t) \\ \sum_{t=1}^{N} x_0(t) x_2(t) \\ \sum_{t=1}^{N} x_0(t) \end{bmatrix} \tag{3}$$

Finding the inverse of a 3×3 matrix is pretty straightforward. There are many ways of finding the inverse of a matrix; one method being finding the determinant and adjoint of the matrix, another method is via using the Gauss-Jordan elimination method or even using Newton's iterative method.

2.2 Residual Model

Once we have estimated the constant coefficients, (a_0 , a_1 and a_2) we can obtain an estimate of the residual/noise process $u(t)$ as,

$$u(t) = x_0(t) - a_1 x_1(t) - a_2 x_2(t) - a_3 \tag{4}$$

To be able to use the Dickey-Fuller test to test if $u(t)$ is stationary, we need to first write it in differenced and autoregressive form as follows,

$$\Delta u(t) = c_0 u(t-1) + \sum_{i=1}^{k} c_i \Delta u(t-i) + n(t) \tag{5}$$

with c_i as constants, $\Delta u(t) = u(t) - u(t-1)$, $n(t)$ is some stationary (zero-mean) noise process and k is the number of lags in the autoregressive model. Since we have data for $u(t)$, we can fit it in equation (5) above and obtain estimates for the constants c_i. This is done by optimizing the linear regression cost function J closely related to the one presented in the previous section,

$$J = \frac{1}{2}\sum_{t=k+1}^{N} \left(c_0 u(t-1) + \sum_{i=1}^{k} c_i \Delta u(t-i) - \Delta u(t)\right)^2 \tag{6}$$

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 09 Issue 03, March 2022
ISSN (Online) 2348 – 7968 | Impact Factor – 6.72
www.ijiset.com

Differentiating with respect to the constants c_i, (similar to what we did in the previous section) we obtain a set of simultaneous equations which lead to the following matrix equation,

$$c = A^{-1}u \tag{7}$$

with a vector $c = [c_0 \quad c_1 \quad c_2]^T$, while A and u are given by

$$A = \begin{bmatrix} \sum_{i=1}^N u(t-1)u(t-1) & \sum_{i=1}^N \Delta u(t-1)u(t-1) & \sum_{i=1}^N \Delta u(t-1)u(t-2) & \dots & \sum_{i=1}^N \Delta u(t-1)u(t-k) \\ \sum_{i=1}^N u(t-1)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-1)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-1)\Delta u(t-2) & \dots & \sum_{i=1}^N \Delta u(t-1)\Delta u(t-k) \\ \sum_{i=1}^N u(t-2)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-2)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-2)\Delta u(t-2) & \dots & \sum_{i=1}^N \Delta u(t-2)\Delta u(t-k) \\ \vdots & \vdots & \vdots & & \vdots \\ \sum_{i=1}^N u(t-k)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-k)\Delta u(t-1) & \sum_{i=1}^N \Delta u(t-k)\Delta u(t-2) & \dots & \sum_{i=1}^N \Delta u(t-k)\Delta u(t-k) \end{bmatrix} \tag{8}$$

and

$$u = \begin{bmatrix} \sum_{i=1}^N u(t-1)\Delta u(t) \\ \sum_{i=1}^N \Delta u(t-1)\Delta u(t) \\ \sum_{i=1}^N \Delta u(t-2)\Delta u(t) \\ \vdots \\ \sum_{i=1}^N \Delta u(t-k)\Delta u(t) \end{bmatrix} \tag{9}$$

Just as it was shown in the previous section, the optimal estimates of constants can be obtained by solving the matrix equation above, with perhaps Gauss-Jordan elimination.

2.3 Computing Test Statistics

Having obtained the vector of constants c, we proceed to calculate the test statistic T_{H_0} as follows,

$$T_{H_0} = \frac{c_0}{SE(c_0)} = c_0 \sqrt{\frac{(N-2)\sum_{t=2}^{N}(u(t-1)-\bar{u})^2}{\sum_{t=k+1}^{N}(c_0 u(t-1) + \sum_{i=1}^{k} c_i \Delta u(t-i) - \Delta u(t))^2}} \tag{10}$$

where $SE(c_0)$ is the standard error corresponding to the estimated constant c_0, N is the sample size and $\bar{u}$ is the mean of $u(t)$. Under the Null hypothesis H_0, the test statistic T_{H_0} follows Dickey-Fuller distribution. We can then take the calculated value of T_{H_0} and compare it to the table of Dickey-Fuller to obtain the corresponding $pValue$. If the $pValue$ is less than the set significance level (say 5%) then we can confidently reject the null hypothesis otherwise we accept it. But what is our Null hypothesis in this case? The Null hypothesis, in this case, is the statement that $c_0 = 0$ which is true if, and only if, the $u(t)$ is nonstationary. Remember $u(t)$ not being stationary implies that our three processes are not cointegrated. With all this information, we can say our Null hypothesis H_0 is the statement that says "there is no cointegration". The alternative hypothesis H_A will then be the statement "there is cointegration".

3. Statistical Triangular Arbitrage

3.1 Triangle Rule In Foreign Exchange Market

In forex markets there exists the triangle rule as shown below with three instruments, $x_0(t)$, $x_1(t)$, and $x_2(t)$,

$$x_0(t) = x_1(t)x_2(t) \tag{11}$$

One example of instruments obeying this triangle rule is the triplet, $x_0 = eurusd$, $x_1 = eurgbp$, and $x_2 = gbpusd$. Some traders attempt to exploit and profit from the momentary violations of the triangle rule stated above. Such a practice is called triangular arbitrage. The basic idea behind triangular arbitrage practice is that we always buy deviation $u(t)$ when it is undervalued and sell it when it is overvalued. Here the deviation is given as,

$$u(t) = x_0(t) - x_1(t)x_2(t) \tag{12}$$

Buying the deviation $u(t)$ here simply means buying $x_0(t)$ while also selling both $x_1(t)$ and $x_2(t)$ at the same time. This indicates that $u(t)$ becomes undervalued when $x_0(t)$ is undervalued and/or $x_1(t)x_2(t)$ is overvalued. The lot size for $x_0(t)$ should match the lot size for $x_1(t)x_2(t)$ hence if the lot size for $x_0(t)$ is L then the lot sizes for $x_1(t)$ and $x_2(t)$ shall be L and x_1L respectively. The opportunities for triangular arbitrage are rare and can be spotted with high-speed computations which are accessible to facilities, not individual retail traders. Also, this practice benefits from the broker's instantaneous inefficiencies, thus it may not be allowed by many brokers. In the next section, we outline the statistical approach to triangular arbitrage, which does not benefit from the broker's instantaneous inefficiencies but rather the market inefficiencies at the statistical level.

3.2 Statistical Arbitrage With Triangular Instruments

In this section, we build upon the cointegration theory, for statistical arbitrage, presented earlier and also on the triangular arbitrage outlined in the previous section. Any three instruments can be used in the cointegration test presented earlier. The test result will confirm whether there is cointegration or not thus informing a trader whether to proceed with investing in the triplet or choose a different triplet combination. This process of choosing the triplet combination which has long-term cointegration can be challenging. We propose here that one can always choose the triplet combination which obeys the triangle rule shown above so that long-term cointegration is guaranteed. Given that the chosen instruments, $x_0(t)$, $x_1(t)$, and $x_2(t)$, obey the triangle rule, the cointegration model will be presented as in equation (1). The triangle rule will ensure that there is always cointegration among the three instruments.

In general, the ratio of lot sizes for instruments $x_0(t)$, $x_1(t)$, and $x_2(t)$ is always given by the magnitudes of their coefficients which are 1, $|a_1|$, and $|a_2|$ respectively. Just as in the case of instantaneous triangular arbitrage we buy and sell the deviation $u(t)$ for the statistical triangular arbitrage too, however, the deviation, in this case, is given as the residual in equation (4). Because of the direct proportion between $x_0(t)$ and both $x_1(t)$ and $x_2(t)$ enforced by the triangle rule, then buying $x_0(t)$ necessarily means selling both $x_1(t)$ and $x_2(t)$ same time and vice versa. Without this direct proportion, one of the coefficients $|a_1|$ or $|a_2|$ would sometimes take negative values, in which case the anti-symmetry (in terms of buy vs sell) between $x_0(t)$ and both $x_1(t)$ and $x_2(t)$ would be broken. In the next section, we implement the statistical triangular arbitrage in Metatrader 4 as an indicator and show the plot of deviation $u(t)$ as well as how one could trade it using its mean-reversion feature.

4. Implementation And Testing

4.1 Indicator Implementation

We implemented our indicator in Metatrader 4 platform using Metaquotes language [7]. This indicator shows the model parameters on the main chart window and the graph of deviation $u(t)$ on the indicator window below the main chart window as shown in Fig. 1 below. The indicator recomputes everything on each new candlestick. In the case shown in Fig. 1 the data size was $N = 1024$ and each data point was a daily candlestick (with the last candle on the 3rd of June 2021) with the instrument triplet combination as $x_0 = eurusd$, $x_1 = eurgbp$, and $x_2 = gbpusd$. The values for parameters $a_1 = 1.30$, $a_2 = 0.88$ and $a_3 = -1.14$ are shown as coefficients in the cointegration model on the main chart screen with the goodness of fit at 99.95%. Statistical test results (i.e. $T_{H_0} = -5.3385$ and $pValue = 1.0\%$) are also shown below the model. The $pValue = 1.0\%$ means that the probability that we might have rejected the null hypothesis (i.e. no cointegration exists) by mistake is only 1.0%. The standard deviation of the residual/deviation $u(t)$ is also shown as 0.0012 (or 12 pips) on the main chart window.

Multiples of the standard deviation (from the zero-mean line) are plotted on the indicator window to give a trader some reference to indicate when the deviation $u(t)$ has crossed the i^{th} multiple of the standard deviation. This helps guide a trader as to when to make entries and exits. One trading strategy example could be a trader making entry when the deviation crosses the first multiple (positive or negative) of standard deviation after touching the mean (the zero-mean line) and exiting when the deviation crosses either the mean (with profit) or the second multiple (positive or negative) of the standard deviation (with loss). The long-term profitability of this simple strategy is analyzed (under a mild assumption of no model change) in the next section.

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 09 Issue 03, March 2022
ISSN (Online) 2348 – 7968 | Impact Factor – 6.72
www.ijiset.com

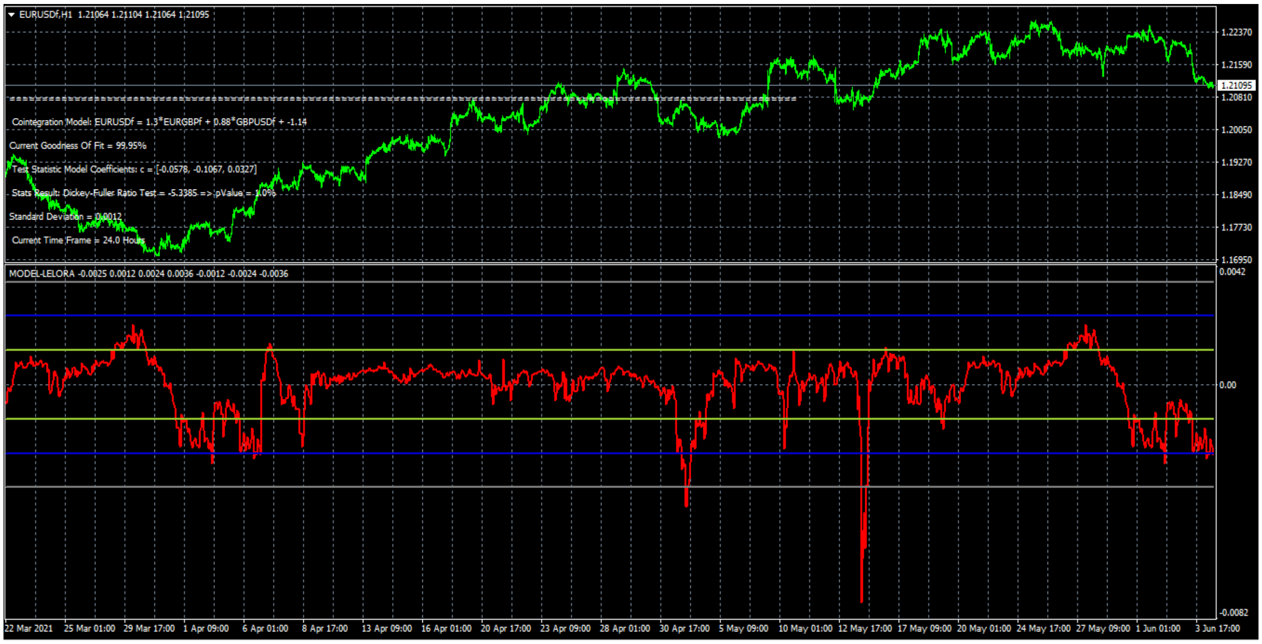

Fig. 1. Statistical Triangular Arbitrage Indicator in Metatrader 4.

4.2 Analysis and Expectation Value

It would be interesting to find out if the simple mean-reversion strategy hinted at in the previous section is theoretically and/or practically profitable or not in a long run. We first begin by plotting the histogram of deviations $u(t)$ from the mean and fit a Gaussian density function to this histogram as shown in Fig. 2 below.

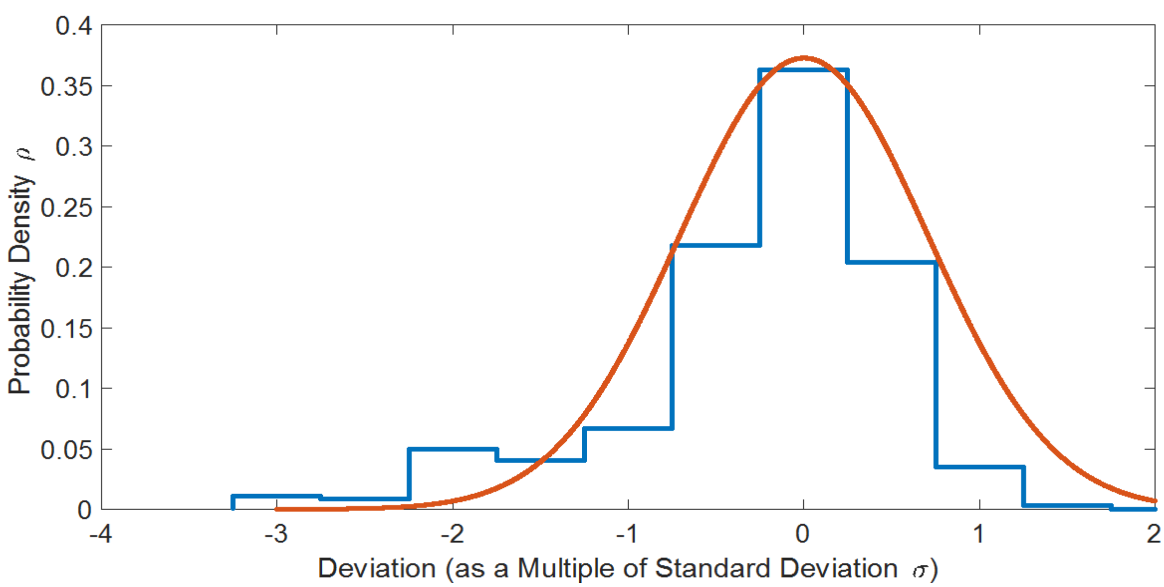

Fig. 2. Gaussian density profile for deviation $u(t)$.

The model fit in Fig. 2 above shows that the deviation $u(t)$ can be considered as a fairly Gaussian density. Based on the evidence of the Gaussian density profile inherent in the deviation, we present the generic Gaussian density profile in Fig. 3 below which shows the distribution of probabilities in various regions under the density profile.

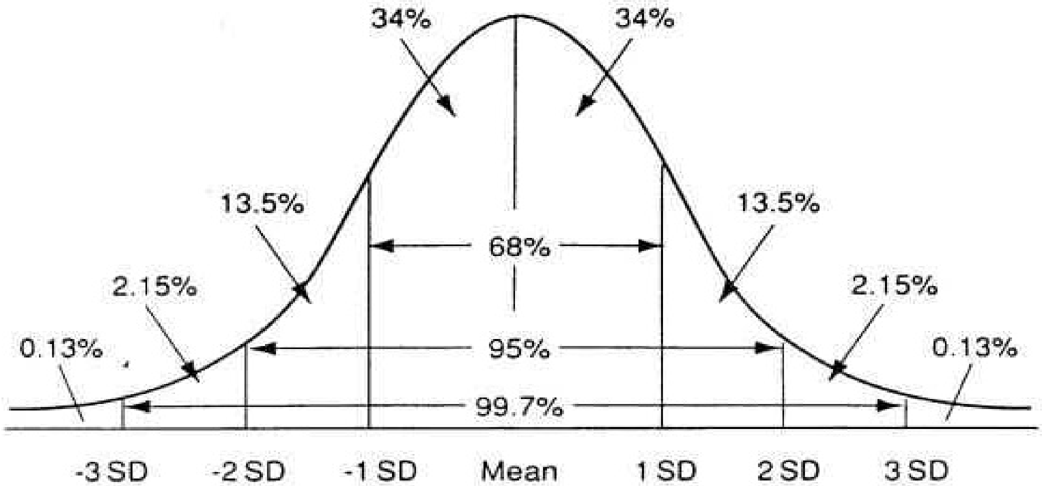

Fig. 3. Distribution of probabilities under a Gaussian density profile [8].

In our case probability under one region translates to the likelihood of the deviation $u(t)$ to be found in that region of space. With that, it can already be seen that the likelihood of finding the deviation far away from the mean gets smaller and smaller hence the idea of mean-reversion is evident here. To determine whether the simple strategy stated above will be profitable (provided the model parameters do not change for the duration of the trades opened), we need to calculate the expectation value EV as follows,

$$EV = P_w A_w + P_l A_l \tag{13}$$

where P_w and P_l are the probabilities for the strategy closing a trade with profit and with loss respectively. The parameters A_w and A_l are the amounts won and lost per trade respectively. In this case of a simple strategy $A_w = -Al = \sigma L$, with L as the lot size and σ as the standard deviation. From Fig. 3, the probability for the deviation to get back to the mean (and beyond it) is roughly $P_w \approx 50\%$ while the probability for the deviation to go further away from the mean and get to the second deviation (and beyond it) is roughly $P_l \approx 2.50\%$. Substituting these values in equation (13) we get the expected value estimate of $EV \approx 0.475\sigma L > 0$. Since the expected value is positive, that means on average we gain $0.0.475\sigma L$ per trade in the long run.

4. Conclusions

In this work, we have shown how triangle rule in foreign exchange markets can be exploited for statistical arbitrage investment strategy. The triangle rule ensures the long-term stability of cointegration among the traded instruments. It was demonstrated with the example of the triplet $eurusd$, $eurgbp$, and $gbpusd$ that mean-reversion approach is justified and that it will lead to profitable trading, with a gain of $0.0.475\sigma L$ per trade, provided the model doesn't change drastically in the lifetime of the running trades. The same approach has been extended to four instrument combination (to be reported separately) which obeys the relation $x_0(t)x_1(t) = x_2(t)x_3(t)$ such as $eurnzd$, $nzdusd$, $eurgbp$, and $gbpusd$.

Acknowledgments

I declare here that the content in this manuscript is my own unaided contribution/work and the sources used have been well-referenced. There is no one else who contributed to this work. This work is not associated with any funding agency/institution.

References

[1] R. Guhr, T. Schafer, Local normalization: Uncovering correlations in non-stationary financial time series. Physica A: Statistical Mechanics and its Applications, Elsevier, 389(18):3856–3865, 2010.
[2] E. Herranz, Unit Root Tests: Computational Statistics. 2017.

[3] F. Baum, Test for Stationarity and Stability in Time Series Data. 2009.
[4] R. J. van de Hoek, J. Malcolm, W. P. Elliott, Pairs Trading. Quantitative Finance, 5(3):271–276, June 2005.
[5] A. Pole, Statistical Arbitrage: Algorithmic Trading Insights and Techniques. John Wiley & Sons, Inc, 2007.
[6] N. Hatano, H. Takayasu, K. Marumo, T. Shimizu, Y. Aiba, Triangular Arbitrage in the Foreign Exchange Market. The Application of Econophysics. Springer, pages 18–23, 2004.
[7] www.mql5.com (Accessed 03/12/2021).
[8] https://lesn.appstate.edu/olson/normal_curve.htm (Accessed 03/12/2021).

First Author: Currently working as a lecturer at the National University of Lesotho (NUL). Obtained Bachelor of Engineering in Electronics degree at NUL in 2011, then MSc in Electrical Engineering degree at University of Cape Town in 2014 and lastly PhD in Physics at Stellenbosch University in 2020. Research interests include Control Systems, Computer Vision, Atomic & Laser Physics, Theoretical Physics, Econophysics and Quantum Finance.