Critical Thinking Group 4 - HW3

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Overview

The purpose of this project is to predict if a neighborhood will be at risk for high crime levels using binary logistic regression models. Below is a short description of the variables in the dataset.

- zn: proportion of residential land zoned for large lots (over 25000 square feet)
- indus: proportion of non-retail business acres per suburb
- chas: a dummy var. for whether the suburb borders the Charles River (1) or not (0)
- nox: nitrogen oxides concentration (parts per 10 million)
- rm: average number of rooms per dwelling
- age: proportion of owner-occupied units built prior to 1940
- dis: weighted mean of distances to five Boston employment centers
- rad: index of accessibility to radial highways
- tax: full-value property-tax rate per \$10,000
- ptratio: pupil-teacher ratio by town
- black: $1000 (B_k 0.63)^2$ where Bk is the proportion of blacks by town
- lstat: lower status of the population (percent)
- medv: median value of owner-occupied homes in \$1000s
- target: whether the crime rate is above the median crime rate (1) or not (0) (response variable)

Dataset

Crime - Training data Crime - Evaluation Data

Data Exploration

The dataset contains 466 observations and 14 variables. The response variable is the **target** variable. Below is a glimpse of the data. A quick look indicates that chas and target might be classification variables.

```
## Observations: 466
## Variables: 14
## $ zn
            <dbl> 0, 0, 0, 30, 0, 0, 0, 0, 0, 80, 22, 0, 0, 22, 0, 0, 10...
            <dbl> 19.58, 19.58, 18.10, 4.93, 2.46, 8.56, 18.10, 18.10, 5...
## $ indus
## $ chas
            ## $ nox
            <dbl> 0.605, 0.871, 0.740, 0.428, 0.488, 0.520, 0.693, 0.693...
## $ rm
            <dbl> 7.929, 5.403, 6.485, 6.393, 7.155, 6.781, 5.453, 4.519...
## $ age
            <dbl> 96.2, 100.0, 100.0, 7.8, 92.2, 71.3, 100.0, 100.0, 38....
            <dbl> 2.0459, 1.3216, 1.9784, 7.0355, 2.7006, 2.8561, 1.4896...
## $ dis
## $ rad
            <int> 5, 5, 24, 6, 3, 5, 24, 24, 5, 1, 7, 5, 24, 7, 3, 3, 5,...
            <int> 403, 403, 666, 300, 193, 384, 666, 666, 224, 315, 330,...
## $ tax
## $ ptratio <dbl> 14.7, 14.7, 20.2, 16.6, 17.8, 20.9, 20.2, 20.2, 20.2, ...
## $ black
            <dbl> 369.30, 396.90, 386.73, 374.71, 394.12, 395.58, 396.90...
## $ 1stat
            <dbl> 3.70, 26.82, 18.85, 5.19, 4.82, 7.67, 30.59, 36.98, 5....
            <dbl> 50.0, 13.4, 15.4, 23.7, 37.9, 26.5, 5.0, 7.0, 22.2, 20...
## $ medv
## $ target <int> 1, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, ...
```

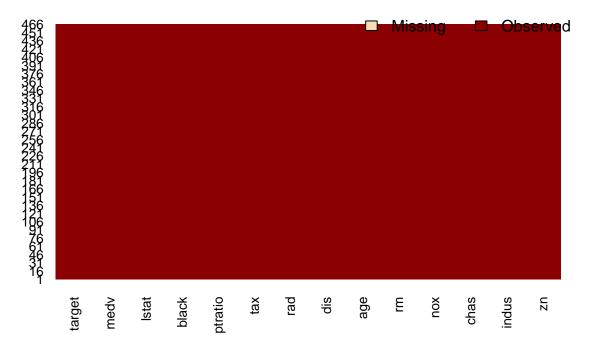
Taking a closer look at the data with summary statistics, we can see that two values (chas, target) should be converted to factors.

```
##
          zn
                          indus
                                              chas
                                                                 nox
##
           :
              0.00
                             : 0.460
                                                :0.00000
                                                                   :0.3890
    Min.
                      Min.
                                        Min.
                                                           Min.
    1st Qu.:
              0.00
                      1st Qu.: 5.145
                                        1st Qu.:0.00000
                                                           1st Qu.:0.4480
##
##
    Median: 0.00
                      Median: 9.690
                                        Median :0.00000
                                                           Median :0.5380
           : 11.58
                                               :0.07082
                                                                   :0.5543
##
    Mean
                      Mean
                             :11.105
                                        Mean
                                                           Mean
##
    3rd Qu.: 16.25
                      3rd Qu.:18.100
                                        3rd Qu.:0.00000
                                                           3rd Qu.:0.6240
           :100.00
##
    Max.
                      Max.
                              :27.740
                                        Max.
                                                :1.00000
                                                           Max.
                                                                   :0.8710
##
          rm
                          age
                                            dis
                                                               rad
##
    Min.
           :3.863
                                               : 1.130
                                                                 : 1.00
                     Min.
                            : 2.90
                                       Min.
                                                         Min.
                                       1st Qu.: 2.101
                                                         1st Qu.: 4.00
##
    1st Qu.:5.887
                     1st Qu.: 43.88
##
    Median :6.210
                     Median : 77.15
                                       Median : 3.191
                                                         Median: 5.00
##
    Mean
           :6.291
                     Mean : 68.37
                                       Mean
                                              : 3.796
                                                         Mean : 9.53
##
    3rd Qu.:6.630
                     3rd Qu.: 94.10
                                       3rd Qu.: 5.215
                                                         3rd Qu.:24.00
##
    Max.
           :8.780
                     Max.
                            :100.00
                                       Max.
                                               :12.127
                                                         Max.
                                                                 :24.00
##
         tax
                        ptratio
                                         black
                                                           lstat
##
    Min.
           :187.0
                            :12.6
                                            : 0.32
                                                               : 1.730
                     Min.
                                     Min.
                                                       Min.
##
    1st Qu.:281.0
                     1st Qu.:16.9
                                     1st Qu.:375.61
                                                       1st Qu.: 7.043
##
    Median :334.5
                     Median:18.9
                                     Median :391.34
                                                       Median :11.350
##
    Mean
           :409.5
                     Mean
                            :18.4
                                     Mean
                                            :357.12
                                                       Mean
                                                               :12.631
    3rd Qu.:666.0
                     3rd Qu.:20.2
                                     3rd Qu.:396.24
                                                       3rd Qu.:16.930
##
           :711.0
                            :22.0
##
    Max.
                     Max.
                                     Max.
                                             :396.90
                                                       Max.
                                                               :37.970
##
         medv
                         target
##
           : 5.00
                             :0.0000
    Min.
                     Min.
    1st Qu.:17.02
##
                     1st Qu.:0.0000
    Median :21.20
                     Median :0.0000
##
##
    Mean
           :22.59
                     Mean
                            :0.4914
    3rd Qu.:25.00
##
                     3rd Qu.:1.0000
##
    Max.
           :50.00
                     Max.
                            :1.0000
```

Visually assessing missing values:

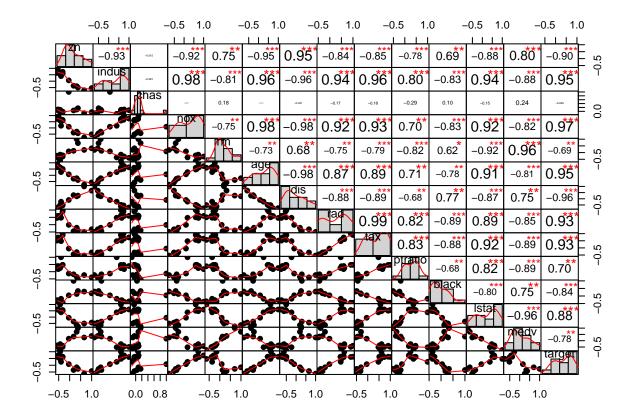
The Amelia package has a plotting function missmap() that will plot the dataset and highlight missing values:

Missing values vs observed



There are no missing values in the dataset. Lets plot the correlation between the variables.

Correlation:

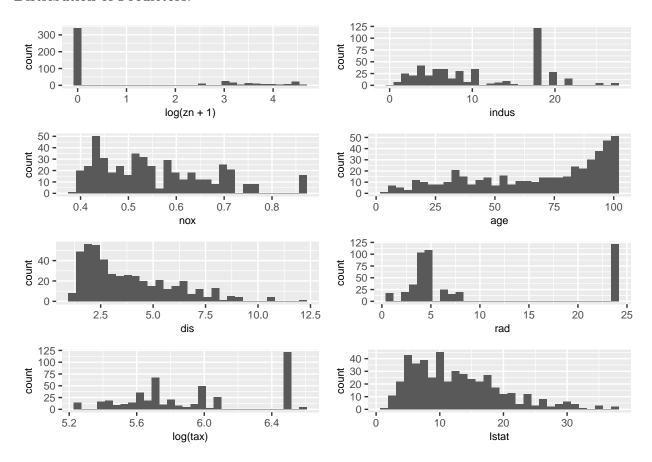


From the above correlation matrix , the ${f target}$ variable seems to have correlation with

- zn proportion of residential land zoned for large lots
- indus proportion of non-retail business acres per suburb
- nox nitrogen oxides concentration
- age proportion of owner-occupied units built prior to 1940
- dis weighted mean of distances to five Boston employment centers
- rad index of accessibility to radial highways
- tax full-value property-tax rate per \$10,000
- lstat lower status of the population

Lets look at each of the predictor variable's data:

Distribution of Predictors:



From the graphs above, the majority of the neighborhoods have no residential land zoned for large lots. Since log(0) is undefine, we performed a log(zn+1) graph to circumvent the undefined value.

We could not derive a specific categorization in the other predictors.

Data Preparation

Factorize Variables:

Convert the *chas* and *target* variables into factors:

```
crime.trn$chas <- as.factor(crime.trn$chas)
crime.trn$target <- as.factor(crime.trn$target)</pre>
```

For ZN variable, 339/466 are zeros. We are going to create a new variable \mathbf{zn} _ind as indicator for residential zones containing large lots (land size over 25,000 sq.ft as 1)

Check for Multicolinearity in the predictors:

Check for Multicolinearity among the predictor variables and remove those with excessive correlation among the explanatory variables.

Multicolinearity score
8.621044
6.114789
5.292859
4.736573
4.493409
3.971424
2.894468
2.768836
2.598383
2.582215
2.179424
1.898671
1.276748
1.090804

From the above table, we do not see multi-colinearity (with VIF > 10) among the predictors.

Split the dataset into training and test:

We will randomly split our dataset into training (80%) and test (20%).

```
set.seed(999)
s = sample(1:nrow(crime.trn), 0.8 * nrow(crime.trn))
crime.train = crime.trn[s, ]
crime.test = crime.trn[-s, ]
```

Number of observations in training dataset is 372

Number of observations in test dataset is 94

Build Models

Below are the different approaches we will use to build the models:

- 1. Stepwise Backward
- 2. Stepwise Forward
- 3. Manual
- 4. Bayesian

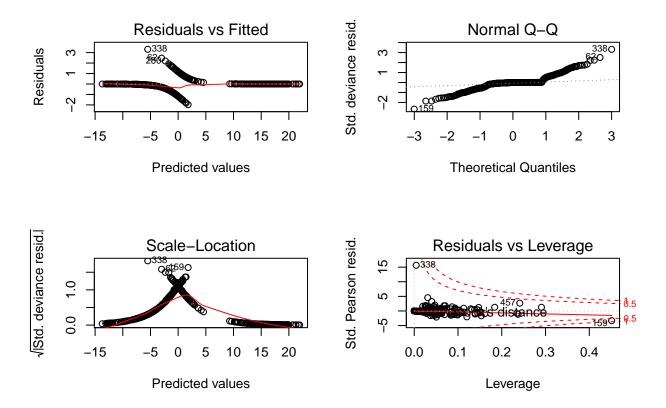
1. Backward elimination method

With backwards elimination, we start with full set of parameters and iteratively reduce the numbers of parameters using AIC.

```
##
## Call:
## stats::glm(formula = target ~ ., family = binomial(), data = crime.train)
## Deviance Residuals:
                    Median
##
      Min
           1Q
                                 3Q
                                         Max
## -2.0129 -0.1480 -0.0051
                             0.0029
                                      3.4512
##
## Coefficients:
##
                Estimate Std. Error z value Pr(>|z|)
## (Intercept) -38.402903
                          7.657669 -5.015 5.30e-07 ***
                          0.051713 -0.687 0.492073
## zn
               -0.035528
## indus
               -0.067372
                          0.055720 -1.209 0.226615
                          0.801451
## chas1
               1.147487
                                    1.432 0.152212
               47.077703
                          9.008344
                                    5.226 1.73e-07 ***
## nox
## rm
               -0.282639
                           0.820723 -0.344 0.730562
               0.032986
                          0.015177
                                     2.173 0.029746 *
## age
## dis
               0.841631
                          0.276398
                                     3.045 0.002327 **
                          0.177894
                                     3.469 0.000522 ***
## rad
               0.617164
## tax
               -0.006143
                          0.003312 -1.855 0.063605 .
## ptratio
               ## black
               -0.011410
                         0.006191 -1.843 0.065338 .
## 1stat
               0.112049
                          0.064401
                                    1.740 0.081883 .
## medv
                0.190297
                          0.077879
                                    2.444 0.014545 *
                          1.451991 -0.488 0.625834
## zn ind
               -0.707989
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 515.31 on 371 degrees of freedom
## Residual deviance: 151.82 on 357 degrees of freedom
## AIC: 181.82
##
## Number of Fisher Scoring iterations: 9
## target ~ zn + nox + age + dis + rad + tax + ptratio + black +
##
      1stat + medv
##
## Call:
## stats::glm(formula = target ~ zn + nox + age + dis + rad + tax +
##
      ptratio + black + lstat + medv, family = binomial(), data = crime.train)
##
## Deviance Residuals:
      Min
##
                1Q
                    Median
                                 3Q
                                         Max
## -1.9771 -0.1548 -0.0026
                            0.0028
                                      3.3163
##
## Coefficients:
##
                Estimate Std. Error z value Pr(>|z|)
## (Intercept) -34.933559
                          6.996318 -4.993 5.94e-07 ***
## zn
               -0.067617
                          0.036693 -1.843 0.065360 .
               39.658956
                          7.198565
                                     5.509 3.60e-08 ***
## nox
                          0.012501
                                     2.447 0.014414 *
                0.030587
## age
```

```
## dis
                  0.736719
                             0.246530
                                         2.988 0.002805 **
                             0.166956
## rad
                  0.695000
                                         4.163 3.14e-05 ***
##
  tax
                 -0.007973
                             0.003002
                                        -2.656 0.007912 **
                  0.358211
                             0.130070
                                         2.754 0.005887 **
##
  ptratio
##
  black
                 -0.010786
                             0.006052
                                        -1.782 0.074728
##
                                         2.170 0.030045
  lstat
                  0.117233
                             0.054037
                  0.167625
                                         3.361 0.000777 ***
##
  medv
                             0.049876
##
##
  Signif. codes:
                    0
                     '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
   (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 515.31
                               on 371
                                        degrees of freedom
                               on 361
  Residual deviance: 155.24
                                        degrees of freedom
   AIC: 177.24
##
## Number of Fisher Scoring iterations: 9
```

From the above table, *nox* and *rad* predictors shows low p-value. A unit increase in nitrogen oxides concentration increases the log odds by 47.07, while increase in rad increases the log odds by 0.61. The next significant predictors are *dis* and *ptratio*.



In the residuals Vs Fitted graph, the red line is not flat, which indicates the linearity in residuals is not true. In the scale-location graph as well, the red line is not flat, which indicates that residual variance is not constant [homo scadasticity assumption]. The Normal Q-Q graph indicates that the most of the residuals are on the straight line. However, the Residual Vs Leverage plot has the redline not alligned with gray dotted line, this indicates that the assumption of standardized residuals centered around zero is NOT true here.

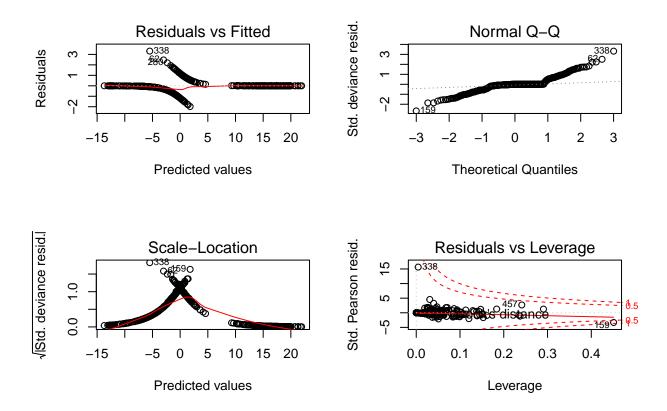
2. Forward elimination method

With Forward elimination, we start with an empty candidate set of parameters and iteratively add variables using AIC.

```
##
## Call:
## glm(formula = target ~ 1, family = binomial, data = crime.train,
      trace = FALSE)
##
## Deviance Residuals:
     Min
              1Q Median
                               3Q
                                      Max
## -1.150 -1.150 -1.150
                            1.205
                                    1.205
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.06454
                           0.10375 -0.622
##
## (Dispersion parameter for binomial family taken to be 1)
##
       Null deviance: 515.31 on 371 degrees of freedom
## Residual deviance: 515.31 on 371 degrees of freedom
## AIC: 517.31
## Number of Fisher Scoring iterations: 3
## target ~ nox + rad + tax + ptratio + age + black + medv + dis +
      zn + 1stat
##
##
## Call:
## glm(formula = target ~ nox + rad + tax + ptratio + age + black +
      medv + dis + zn + lstat, family = binomial, data = crime.train,
       trace = FALSE)
##
##
## Deviance Residuals:
      Min
                                           Max
                 1Q
                      Median
                                   3Q
## -1.9771 -0.1548 -0.0026
                               0.0028
                                        3.3163
##
## Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
                            6.996318 -4.993 5.94e-07 ***
## (Intercept) -34.933559
## nox
                39.658956
                            7.198565
                                       5.509 3.60e-08 ***
## rad
                            0.166956
                                       4.163 3.14e-05 ***
                 0.695000
                            0.003002 -2.656 0.007912 **
## tax
                -0.007973
## ptratio
                 0.358211
                            0.130070
                                       2.754 0.005887 **
## age
                 0.030587
                            0.012501
                                       2.447 0.014414 *
## black
                -0.010786
                            0.006052 -1.782 0.074728 .
## medv
                 0.167625
                            0.049876
                                       3.361 0.000777 ***
## dis
                           0.246530
                                      2.988 0.002805 **
                0.736719
## zn
                -0.067617
                            0.036693 -1.843 0.065360 .
                 0.117233
                                       2.170 0.030045 *
## 1stat
                            0.054037
## ---
```

```
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
##
   (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 515.31
                              on 371
                                      degrees of freedom
## Residual deviance: 155.24
                              on 361
                                      degrees of freedom
## AIC: 177.24
##
## Number of Fisher Scoring iterations: 9
```

From the above table, nox, rad, and medv predictors shows low p-value. A unit increase in nitrogen oxides concentration increases the log odds by 39.65, while increase in rad increases the log odds by 0.69 and a unit increase in mdev increases the log odds by 0.16.



In the residuals Vs Fitted graph, the red line is not flat, which indicates the linearity in residuals is not true. In the scale-location graph as well, the red line is not flat, which indicates that residual variance is not constant [homo scadasticity assumption]. The Normal Q-Q graph indicates that the most of the residuals are on the straight line. However, the Residual Vs Leverage plot has the redline not alligned with gray dotted line, this indicates that the assumption of standardized residuals centered around zero is NOT true here.

From the above two models we can see that zn & age are not statistically significant. As for the statistically significant variables, rad & nox have a strong positive association of crime rate while tax has a negative coefficient, suggests as all other variables being equal as tax increases crime rate decreases.

3. Manual

Both Forward and backward elimination models produced the same model. Using the model obtained from backwards/forwards elimination, we next remove variables of low significance. We will remove Zn and age from the above models.

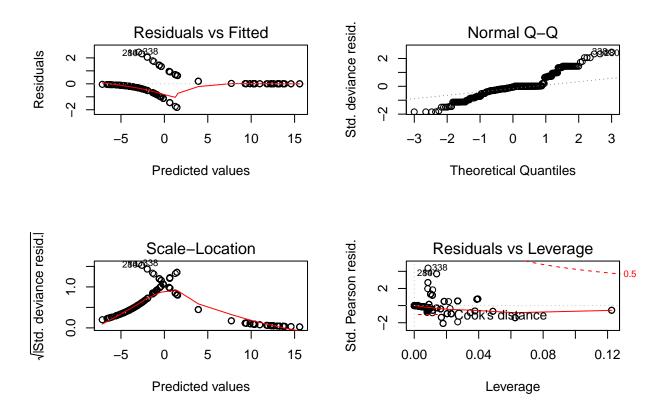
```
##
## Call:
## glm(formula = target ~ nox + rad + tax + dis, family = binomial(link = "logit"),
##
       data = crime.train, trace = FALSE)
##
## Deviance Residuals:
        Min
                                       3Q
                   1Q
                         Median
                                                Max
  -1.85375 -0.31225
                      -0.06564
                                  0.00749
                                            2.51549
##
## Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
##
                            3.652807
                                     -5.900 3.62e-09 ***
## (Intercept) -21.553260
                                       6.181 6.36e-10 ***
## nox
                37.708340
                            6.100531
                 0.562072
                            0.127164
                                       4.420 9.87e-06 ***
## rad
## tax
                -0.007533
                            0.002504
                                      -3.009 0.00262 **
                 0.209893
## dis
                            0.161309
                                       1.301 0.19319
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 515.31 on 371 degrees of freedom
## Residual deviance: 184.63 on 367 degrees of freedom
## AIC: 194.63
##
## Number of Fisher Scoring iterations: 8
```

We will remove distance from the above model since the p value is not significant. Now the new model:

```
##
## Call:
  glm(formula = target ~ nox + rad + tax, family = binomial(link = "logit"),
##
       data = crime.train, trace = FALSE)
##
## Deviance Residuals:
##
       Min
                   1Q
                         Median
                                       3Q
                                                Max
  -1.82233 -0.32010 -0.05947
                                  0.00843
                                            2.44822
##
## Coefficients:
                Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) -18.250944
                            2.447132 -7.458 8.78e-14 ***
                33.039818
                            4.740219
                                       6.970 3.17e-12 ***
## nox
                 0.562869
                            0.127143
                                       4.427 9.55e-06 ***
## rad
                            0.002517
                                     -3.053 0.00227 **
## tax
                -0.007685
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
```

```
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 515.31 on 371 degrees of freedom
## Residual deviance: 186.30 on 368 degrees of freedom
## AIC: 194.3
##
## Number of Fisher Scoring iterations: 8
```

A unit increase in index of accessibility to radial highways increases the log odds by 0.56. Also unit increase in nitrogen oxides concentration increases the log odds by 33.03, while increase in tax rate reduces the log odds by 0.008.



In the residuals Vs Fitted graph, the red line is not flat, which indicates the linearity in residuals is not true. In the scale-location graph as well, the red line is not flat, which indicates that residual variance is not constant [homo scadasticity assumption]. The Normal Q-Q graph indicates that the most of the residuals are on the straight line. However, the Residual Vs Leverage plot has the redline not alligned with gray dotted line, this indicates that the assumption of standardized residuals centered around zero is NOT true here.

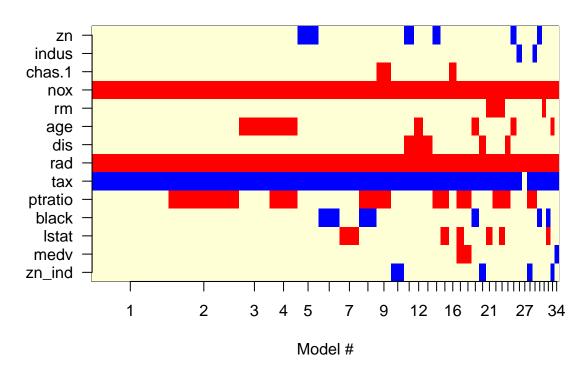
4. Bayesian Approach

```
##
## Call:
## bic.glm.formula(f = target ~ ., data = crime.train, glm.family = "binomial")
##
##
##
##
34 models were selected
```

```
## Best 5 models (cumulative posterior probability = 0.4857):
##
                                     model 1 model 2 model 3
##
          p!=0
                 EV
                            SD
## Intercept 100
                 -2.028e+01 5.000651 -1.825e+01 -2.354e+01 -1.719e+01
          10.6 -4.531e-03 0.016035
                                      .
           2.3 -8.786e-04 0.009366
## indus
            4.7
## chas
## .1
                  4.816e-02 0.266073
## nox 100.0 3.271e+01 5.748499 3.304e+01 3.443e+01 2.846e+01
           5.0 3.133e-02 0.182047
## rm
## age
           18.2 3.739e-03 0.009083
                                                          2.014e-02
            8.8 2.721e-02 0.106535
## dis
          100.0 6.143e-01 0.147026 5.629e-01 6.701e-01 5.779e-01
## rad
          98.9 -8.219e-03 0.002808 -7.685e-03 -8.698e-03 -8.246e-03
## tax
## ptratio 40.7 9.795e-02 0.135871
                                      •
                                               2.331e-01
         11.7 -1.080e-03 0.003581
## black
                                                 .
           11.1 9.526e-03 0.032120
## lstat
## medv
           4.1 2.170e-03 0.015109
## zn_ind
            6.4 -6.157e-02 0.295350
## nVar
                                      3
                                                4
## BIC
                                     -1.992e+03 -1.992e+03 -1.990e+03
                                     0.164 0.151 0.066
## post prob
           model 4 model 5
##
## Intercept -2.264e+01 -1.633e+01
             . -4.111e-02
## indus
## chas
## .1
          3.025e+01 2.940e+01
## nox
             .
## rm
          2.027e-02
## age
## dis
           6.906e-01 5.785e-01
## rad
## tax
           -9.503e-03 -7.441e-03
## ptratio 2.321e-01
## black
## lstat
## medv
## zn_ind
             5
## nVar
        -1.990e+03 -1.989e+03
## BIC
## post prob 0.059 0.046
## [1] 0.163696793 0.151392859 0.065592916 0.059231284 0.045806036
## [6] 0.045023458 0.041481075 0.037812642 0.030815856 0.028442165
## [11] 0.020566195 0.020098049 0.019496949 0.017847764 0.017546644
## [16] 0.016378431 0.016324291 0.015949196 0.015794791 0.015230272
## [21] 0.014103230 0.013617958 0.012737391 0.012497881 0.012380272
## [26] 0.011982998 0.011157320 0.011011428 0.010521782 0.009747272
## [31] 0.009067229 0.009066517 0.008929459 0.008651597
## [1] "nox,rad,tax"
                                  "nox,rad,tax,ptratio"
```

```
[3] "nox,age,rad,tax"
                                            "nox,age,rad,tax,ptratio"
##
       "zn,nox,rad,tax"
                                           "nox,rad,tax,black"
    [5]
    [7] "nox,rad,tax,lstat"
                                            "nox, rad, tax, ptratio, black"
    [9] "chas, nox, rad, tax, ptratio"
                                           "nox,rad,tax,zn_ind"
   [11]
       "zn,nox,dis,rad,tax"
                                            "nox,age,dis,rad,tax"
       "nox,dis,rad,tax"
                                           "zn,nox,rad,tax,ptratio"
##
  [13]
       "nox,rad,tax,ptratio,lstat"
                                            "chas, nox, rad, tax"
  [15]
        "nox,rad,tax,ptratio,lstat,medv"
  [17]
                                           "nox,rad,tax,ptratio,medv"
   [19]
       "nox,age,rad,tax,black"
                                            "nox,dis,rad,tax,zn_ind"
   [21]
       "nox,rm,rad,tax,lstat"
                                           "nox,rm,rad,tax,ptratio"
   [23]
       "nox,rm,rad,tax,ptratio,lstat"
                                            "nox,dis,rad,tax,ptratio"
                                           "indus,nox,rad,tax"
   [25] "zn,nox,age,rad,tax"
##
   [27]
       "nox,rad"
                                            "nox,rad,tax,ptratio,zn_ind"
   [29] "indus, nox, rad, tax, ptratio"
                                           "zn,nox,rad,tax,black"
  [31] "nox,rm,rad,tax"
                                            "nox,rad,tax,black,lstat"
   [33] "nox,age,rad,tax,zn_ind"
                                            "nox,rad,tax,medv"
    [1] "zn"
                   "indus"
                              "chas"
                                         "nox"
                                                   "rm"
                                                              "age"
                                                                         "dis"
    [8] "rad"
                              "ptratio" "black"
                   "tax"
                                                   "lstat"
                                                              "medv"
##
                                                                         "zn_ind"
##
        zn
             indus
                       chas
                                 nox
                                                           dis
                                                                   rad
                                                                            tax
                                          rm
                                                  age
##
      10.6
                2.3
                        4.7
                               100.0
                                         5.0
                                                 18.2
                                                           8.8
                                                                 100.0
                                                                           98.9
   ptratio
             black
                      lstat
                                medv
                                      zn ind
##
      40.7
               11.7
                       11.1
                                 4.1
                                         6.4
```

Models selected by BMA



(Intercept) zn indus chas1 nox

```
## -2.028043e+01 -4.530699e-03 -8.786346e-04 4.815851e-02 3.270613e+01
##
              rm
                                         dis
                                                        rad
                           age
                                                                      tax
                                2.721342e-02
                                              6.143394e-01 -8.219396e-03
##
   3.133168e-02 3.738830e-03
##
         ptratio
                         black
                                       lstat
                                                      medv
                                                                   zn_ind
   9.795350e-02 -1.079994e-03
                                9.526345e-03
                                              2.170472e-03 -6.156917e-02
```

From the above resuls it appears like nitrogen oxides concentration(nox), accessibility to radial highways(rad) and property-tax rate(tax) are the 3 variables contributing across all 5 best models selected out of 35 models prepared by the baysian approach. Hence, we consider those 3 predictors for our baysian model. (target \sim nox+rad+tax)

Select Models

Majority of the models provided us with the below model formula:

 $target \sim nox + rad + tax$

Let us try to apply the performance measurs to each of the above models and select the one with best possible accuracy.

Performance measures:

Sensitivity is basically the ability of the model to capture all positives. And Specificity is the ability of the model to capture all negatives.

$$Sensitivity = \frac{TP}{TP + FN}$$

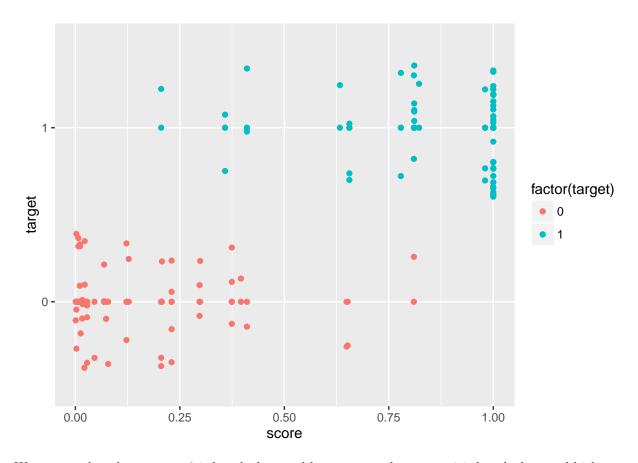
$$Specificity = \frac{TN}{TN + FP}$$

$$Accuracy = \frac{TP + TN}{(TP + FN) + (FP + TN)}$$

For an ideal model, the predictions will be perfect - meaning the 'accuracy, sensitivity and specificity' will all be 1 where as the mis-classification error will be zero. In practical scenarios we would like to have the sensitivity and specificity as close to 1 as possible.

Apply the performance measures on the Manual model

Score should be high when outcome is 1 and low when outcome is 0. Lets visualize how our binary response is behaving with respect to the score that we obtained.



We can see that the response 0 is bunched around low scores and response 1 is bunched around high scores. However, there is also overlap as well across some scores. We need to find a cutoff in the score so as to reach our target here.

Based on our previous homework, these are some properties of the cutoff/threshold:

All the predicted values above cutoff will be 1

All the predicted values below cutoff will be 0

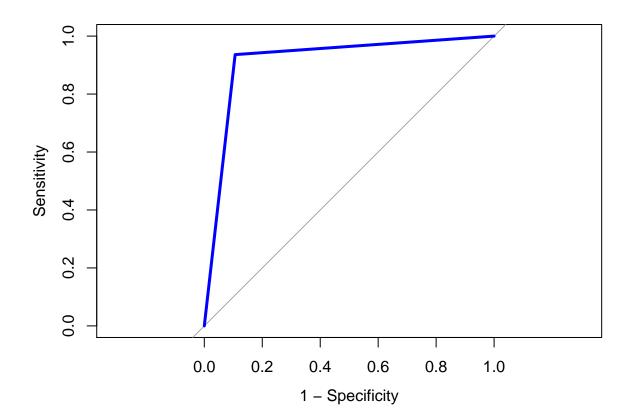
Response values above cutoff(predicted 1) which are 1 in reality will be noted as TP Response values above cutoff(predicted 1) which are 0 in reality will be noted as FP Response values below cutoff(predicted 0) which are 1 in reality will be noted as FN Response values below cutoff(predicted 0) which are 0 in reality will be noted as TN

Based on our visualization, it appears like 0.50 could be a better cutoff.

```
## Confusion Matrix and Statistics
##
##
             Reference
  Prediction
               0
##
                 1
##
            0 42 5
            1 3 44
##
##
##
                  Accuracy: 0.9149
                    95% CI : (0.8392, 0.9625)
##
##
       No Information Rate: 0.5213
       P-Value [Acc > NIR] : <2e-16
##
##
##
                     Kappa: 0.8298
```

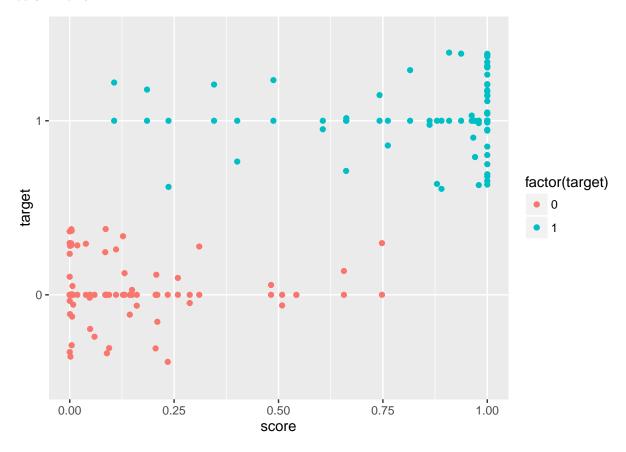
```
Mcnemar's Test P-Value: 0.7237
##
##
               Sensitivity: 0.8980
##
               Specificity: 0.9333
##
            Pos Pred Value : 0.9362
##
            Neg Pred Value: 0.8936
##
##
                Prevalence: 0.5213
            Detection Rate: 0.4681
##
##
      Detection Prevalence : 0.5000
         Balanced Accuracy: 0.9156
##
##
          'Positive' Class : 1
##
##
```

AUC for Manual model



##

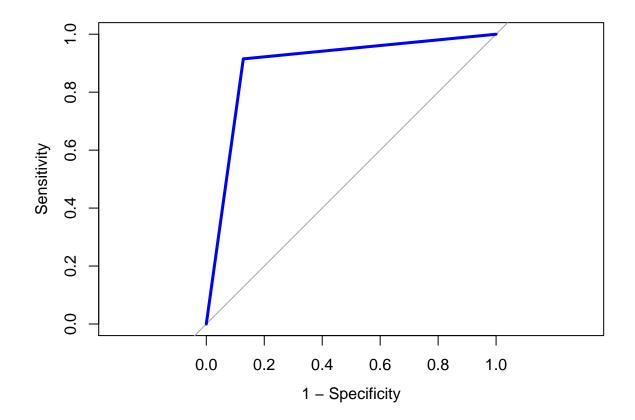
Apply the performance measures on the Forward Elimination model



```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction 0 1
            0 41 6
##
            1 4 43
##
##
##
                  Accuracy : 0.8936
##
                    95% CI : (0.813, 0.9478)
       No Information Rate: 0.5213
##
       P-Value [Acc > NIR] : 1.12e-14
##
##
##
                     Kappa : 0.7872
##
   Mcnemar's Test P-Value: 0.7518
##
```

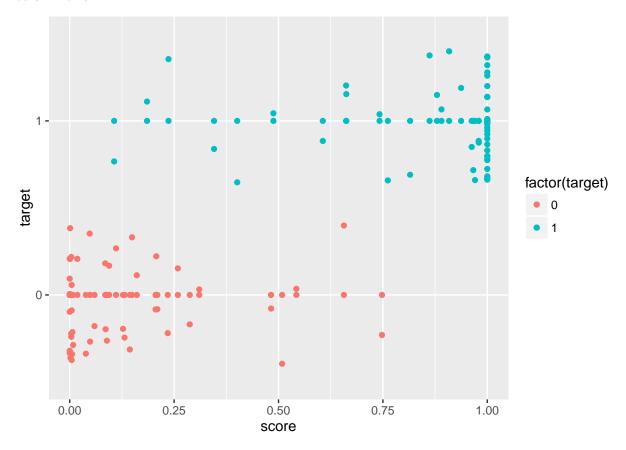
```
##
               Sensitivity: 0.8776
##
               Specificity: 0.9111
##
            Pos Pred Value: 0.9149
            Neg Pred Value: 0.8723
##
##
                Prevalence: 0.5213
            Detection Rate: 0.4574
##
##
      Detection Prevalence: 0.5000
         Balanced Accuracy: 0.8943
##
##
          'Positive' Class : 1
##
##
```

AUC for Forward Elimination



```
##
## Data: as.numeric(target) in 47 controls (factor(predicted) 0) < 47 cases (factor(predicted) 1).
## Area under the curve: 0.8936
## 95% CI: 0.8308-0.9565 (DeLong)
## [1] 0.8936</pre>
```

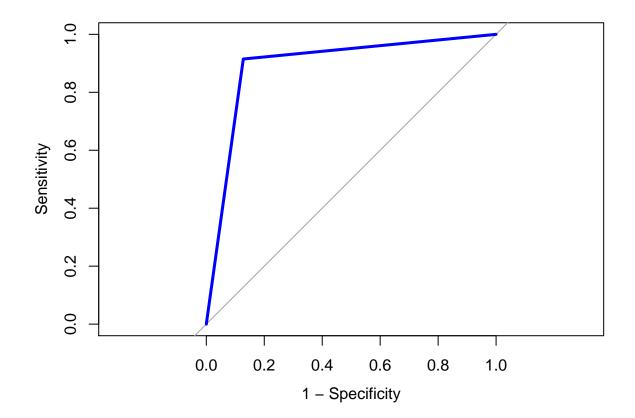
Apply the performance measures on the Backward Elimination model



```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction 0 1
##
            0 41 6
            1 4 43
##
##
##
                  Accuracy : 0.8936
                    95% CI : (0.813, 0.9478)
##
##
       No Information Rate: 0.5213
##
       P-Value [Acc > NIR] : 1.12e-14
##
##
                     Kappa: 0.7872
##
    Mcnemar's Test P-Value: 0.7518
##
##
               Sensitivity: 0.8776
               Specificity: 0.9111
##
```

```
##
            Pos Pred Value: 0.9149
##
            Neg Pred Value: 0.8723
                Prevalence: 0.5213
##
            Detection Rate: 0.4574
##
##
      Detection Prevalence : 0.5000
         Balanced Accuracy: 0.8943
##
##
          'Positive' Class : 1
##
##
```

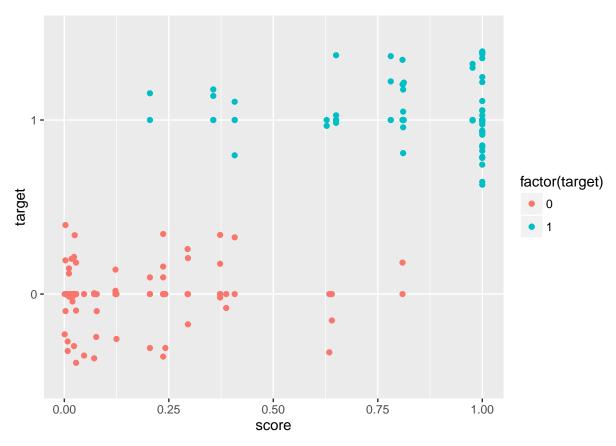
AUC for Backward Elimination



```
## Area under the curve: 0.8936
## 95% CI: 0.8308-0.9565 (DeLong)
```

[1] 0.8936

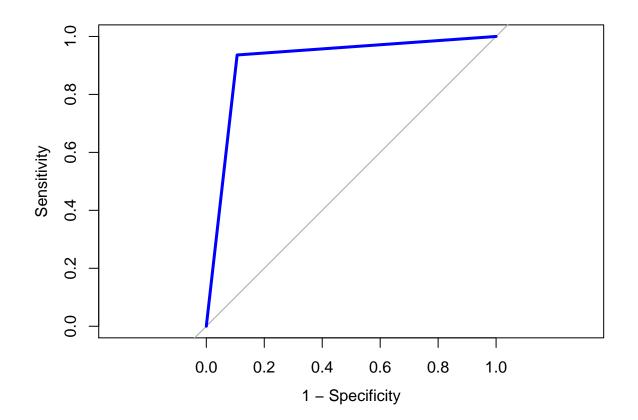
Apply the performance measures on the Bayesian model



```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction 0 1
##
            0 42 5
            1 3 44
##
##
                  Accuracy : 0.9149
##
##
                    95% CI: (0.8392, 0.9625)
##
       No Information Rate: 0.5213
       P-Value [Acc > NIR] : <2e-16
##
##
##
                     Kappa: 0.8298
##
    Mcnemar's Test P-Value : 0.7237
##
##
               Sensitivity: 0.8980
               Specificity: 0.9333
##
##
            Pos Pred Value: 0.9362
            Neg Pred Value: 0.8936
##
```

```
## Prevalence : 0.5213
## Detection Rate : 0.4681
## Detection Prevalence : 0.5000
## Balanced Accuracy : 0.9156
##
## 'Positive' Class : 1
```

AUC for Bayesian



[1] 0.9149

Compare Results:

Method	Sn	Sp	Accuracy	AUC
Manual	0.898	0.9333	0.9149	0.9149
Forward Elimination	0.8776	0.9111	0.8936	0.8936
Backward Elimination	0.8776	0.9111	0.8936	0.8936
Bayesian Model	0.898	0.9333	0.9149	0.9149

From the above, we select Manual/Bayesian models to predict the target variable of the given dataset.

Predictions

zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat	medv	predicted
0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7	0
0	8.14	0	0.538	6.096	84.5	4.4619	4	307	21.0	380.02	10.26	18.2	0
0	8.14	0	0.538	6.495	94.4	4.4547	4	307	21.0	387.94	12.80	18.4	0
0	8.14	0	0.538	5.950	82.0	3.9900	4	307	21.0	232.60	27.71	13.2	0
0	5.96	0	0.499	5.850	41.5	3.9342	5	279	19.2	396.90	8.77	21.0	0
25	5.13	0	0.453	5.741	66.2	7.2254	8	284	19.7	395.11	13.15	18.7	0
25	5.13	0	0.453	5.966	93.4	6.8185	8	284	19.7	378.08	14.44	16.0	0
0	4.49	0	0.449	6.630	56.1	4.4377	3	247	18.5	392.30	6.53	26.6	0
0	4.49	0	0.449	6.121	56.8	3.7476	3	247	18.5	395.15	8.44	22.2	0
0	2.89	0	0.445	6.163	69.6	3.4952	2	276	18.0	391.83	11.34	21.4	0
0	25.65	0	0.581	5.856	97.0	1.9444	2	188	19.1	370.31	25.41	17.3	1
0	25.65	0	0.581	5.613	95.6	1.7572	2	188	19.1	359.29	27.26	15.7	1
0	21.89	0	0.624	5.637	94.7	1.9799	4	437	21.2	396.90	18.34	14.3	1
0	19.58	0	0.605	6.101	93.0	2.2834	5	403	14.7	240.16	9.81	25.0	1
0	19.58	0	0.605	5.880	97.3	2.3887	5	403	14.7	348.13	12.03	19.1	1
0	10.59	1	0.489	5.960	92.1	3.8771	4	277	18.6	393.25	17.27	21.7	0
0	6.20	0	0.504	6.552	21.4	3.3751	8	307	17.4	380.34	3.76	31.5	1
0	6.20	0	0.507	8.247	70.4	3.6519	8	307	17.4	378.95	3.95	48.3	1
22	5.86	0	0.431	6.957	6.8	8.9067	7	330	19.1	386.09	3.53	29.6	0
90	2.97	0	0.400	7.088	20.8	7.3073	1	285	15.3	394.72	7.85	32.2	0
80	1.76	0	0.385	6.230	31.5	9.0892	1	241	18.2	341.60	12.93	20.1	0
33	2.18	0	0.472	6.616	58.1	3.3700	7	222	18.4	393.36	8.93	28.4	0
0	9.90	0	0.544	6.122	52.8	2.6403	4	304	18.4	396.90	5.98	22.1	0
0	7.38	0	0.493	6.415	40.1	4.7211	5	287	19.6	396.90	6.12	25.0	0
0	7.38	0	0.493	6.312	28.9	5.4159	5	287	19.6	396.90	6.15	23.0	0
0	5.19	0	0.515	5.895	59.6	5.6150	5	224	20.2	394.81	10.56	18.5	0
80	2.01	0	0.435	6.635	29.7	8.3440	4	280	17.0	390.94	5.99	24.5	0
0	18.10	0	0.718	3.561	87.9	1.6132	24	666	20.2	354.70	7.12	27.5	1
0	18.10	1	0.631	7.016	97.5	1.2024	24	666	20.2	392.05	2.96	50.0	1
0	18.10	0	0.584	6.348	86.1	2.0527	24	666	20.2	83.45	17.64	14.5	1
0	18.10	0	0.740	5.935	87.9	1.8206	24	666	20.2	68.95	34.02	8.4	1
0	18.10	0	0.740	5.627	93.9	1.8172	24	666	20.2	396.90	22.88	12.8	1
0	18.10	0	0.740	5.818	92.4	1.8662	24	666	20.2	391.45	22.11	10.5	1
0	18.10	0	0.740	6.219	100.0	2.0048	24	666	20.2	395.69	16.59	18.4	1
0	18.10	0	0.740	5.854	96.6	1.8956	24	666	20.2	240.52	23.79	10.8	1

zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat	medv	predicted
0	18.10	0	0.713	6.525	86.5	2.4358	24	666	20.2	50.92	18.13	14.1	1
0	18.10	0	0.713	6.376	88.4	2.5671	24	666	20.2	391.43	14.65	17.7	1
0	18.10	0	0.655	6.209	65.4	2.9634	24	666	20.2	396.90	13.22	21.4	1
0	9.69	0	0.585	5.794	70.6	2.8927	6	391	19.2	396.90	14.10	18.3	1
0	11.93	0	0.573	6.976	91.0	2.1675	1	273	21.0	396.90	5.64	23.9	0

Appendix

```
library(dplyr)
library(ggplot2)
library(gridExtra)
library(e1071)
library(car)
library(recommenderlab)
library(knitr)
library(Amelia)
library(PerformanceAnalytics)
library(robustbase)
library(BMA)
library(caret)
library(pROC)
crime.trn <- read.csv("https://raw.githubusercontent.com/Nguyver/DATA621-HW/master/HW3/crime-training-d</pre>
    header = TRUE, sep = ",", stringsAsFactors = FALSE)
crime.evl <- read.csv("https://raw.githubusercontent.com/Nguyver/DATA621-HW/master/HW3/crime-evaluation</pre>
    header = TRUE, sep = ",", stringsAsFactors = FALSE)
glimpse(crime.trn)
summary(crime.trn)
missmap(crime.trn, main = "Missing values vs observed")
cor.matrix <- cor(crime.trn[, 1:ncol(crime.trn)])</pre>
chart.Correlation(cor.matrix, histogram = TRUE, pch = 19)
g_zn <- ggplot(data = crime.trn) + geom_histogram(aes(x = log(zn))) + theme(axis.text = element_text(si.</pre>
    axis.title = element_text(size = 8))
g_indus <- ggplot(data = crime.trn) + geom_histogram(aes(x = indus)) + theme(axis.text = element_text(s</pre>
    axis.title = element_text(size = 8))
g_nox <- ggplot(data = crime.trn) + geom_histogram(aes(x = nox)) + theme(axis.text = element_text(size = nox))</pre>
    axis.title = element_text(size = 8))
g_age <- ggplot(data = crime.trn) + geom_histogram(aes(x = age)) + theme(axis.text = element_text(size = age))</pre>
    axis.title = element_text(size = 8))
g_dis <- ggplot(data = crime.trn) + geom_histogram(aes(x = dis)) + theme(axis.text = element_text(size = dis))</pre>
    axis.title = element_text(size = 8))
g_rad <- ggplot(data = crime.trn) + geom_histogram(aes(x = rad)) + theme(axis.text = element_text(size = rad))</pre>
    axis.title = element_text(size = 8))
g_tax <- ggplot(data = crime.trn) + geom_histogram(aes(x = log(tax))) + theme(axis.text = element_text(</pre>
```

```
axis.title = element_text(size = 8))
g_lstat <- ggplot(data = crime.trn) + geom_histogram(aes(x = lstat)) + theme(axis.text = element_text(s</pre>
    axis.title = element_text(size = 8))
grid.arrange(g_zn, g_indus, g_nox, g_age, g_dis, g_rad, g_tax, g_lstat, ncol = 2)
crime.trn$chas <- as.factor(crime.trn$chas)</pre>
crime.trn$target <- as.factor(crime.trn$target)</pre>
crime.trn$zn ind <- ifelse(crime.trn$zn > 0, 1, 0)
\# crime.trn$zn \leftarrow ifelse(crime.trn$zn > 0, 1, 0)
table(crime.trn$zn ind)
fit <- glm(target ~ ., data = crime.trn, family = binomial)</pre>
# Lets check for Multi-Collinearity - lets find vif value and drop those that has
vifFit1 <- vif(fit)</pre>
# sort by descending
vif.df <- as.data.frame(sort(vifFit1, decreasing = T))</pre>
names(vif.df) <- c("VIF")</pre>
kable(vif.df)
set.seed(999)
s = sample(1:nrow(crime.trn), 0.8 * nrow(crime.trn))
crime.train = crime.trn[s, ]
crime.test = crime.trn[-s, ]
fullmodel = stats::glm(target ~ ., family = binomial(), data = crime.train)
summary(fullmodel)
backwards.model = step(fullmodel, trace = FALSE) #Backwards selection is the default
backwards.formula <- formula(backwards.model)</pre>
backwards.formula
summary(backwards.model)
par(mfrow = c(2, 2))
graphics::plot(backwards.model)
forwards.model = step(nothing, scope = list(lower = formula(nothing), upper = formula(fullmodel)),
    direction = "forward", trace = FALSE)
forwards.formula <- formula(forwards.model)</pre>
forwards.formula
summary(forwards.model)
par(mfrow = c(2, 2))
graphics::plot(backwards.model)
manual.model <- glm(target ~ nox + rad + tax + dis, family = binomial(link = "logit"),</pre>
    data = crime.train, trace = FALSE)
summary(manual.model)
manual.final <- glm(target ~ nox + rad + tax, family = binomial(link = "logit"),</pre>
    data = crime.train, trace = FALSE)
summary(manual.final)
```

```
par(mfrow = c(2, 2))
graphics::plot(manual.final)
bayesian.model <- bic.glm(target ~ ., data = crime.train, glm.family = "binomial")</pre>
summary(bayesian.model)
# Posterior probability of each of 11 models (rest very small by comparison, so
# are omitted, change value of OR to see them)
bayesian.model$postprob
bayesian.model$label
# For each of 8 variables, probability they should be in the model
bayesian.model$names
bayesian.model$probne0
imageplot.bma(bayesian.model)
bayesian.model$postmean
bayesian.model.final <- bic.glm(target ~ nox + rad + tax, data = crime.train, glm.family = "binomial")
performance.results <- vector()</pre>
crime.test$score <- predict(manual.final, newdata = subset(crime.test, select = c(4,</pre>
    8, 9)), type = "response")
ggplot(crime.test, aes(y = target, x = score, color = factor(target))) + geom_point() +
    geom jitter()
cutoff = 0.5
crime.test$predicted = as.numeric(crime.test$score > cutoff)
TP = sum(crime.test$predicted == 1 & crime.test$target == 1)
FP = sum(crime.test$predicted == 1 & crime.test$target == 0)
FN = sum(crime.test$predicted == 0 & crime.test$target == 1)
TN = sum(crime.test$predicted == 0 & crime.test$target == 0)
# lets also calculate total number of real positives and negatives in the data
P = TP + FN
N = TN + FP
total = P + N
confusionMatrix(factor(crime.test$predicted), factor(crime.test$target), positive = "1")
sensitivity <- round(sensitivity(factor(crime.test$predicted), crime.test$target,</pre>
    positive = "1"), 4)
specificity <- round(specificity(factor(crime.test$predicted), crime.test$target,</pre>
    negative = "0"), 4)
\# accuracy = (TP+TN)/(P+N)
accuracy \leftarrow round(((TP + TN)/(P + N)), 4)
cnfMtx <- confusionMatrix(crime.test$predicted, crime.test$target, positive = "1")</pre>
(roc <- roc(factor(predicted) ~ as.numeric(target), data = crime.test, plot = FALSE,</pre>
```

```
ci = TRUE))
graphics::plot(roc, legacy.axes = TRUE, col = "blue", lwd = 3)
(auc <- round(auc(factor(predicted) ~ as.numeric(target), crime.test), 4))</pre>
performance.results <- rbind(performance.results, c("Manual", sensitivity, specificity,
    accuracy, auc))
crime.test$score <- predict(forwards.model, newdata = subset(crime.test, select = c(nox,</pre>
    rad, tax, ptratio, age, black, medv, dis, zn, lstat)), type = "response")
ggplot(crime.test, aes(y = target, x = score, color = factor(target))) + geom_point() +
    geom_jitter()
cutoff = 0.5
crime.test$predicted = as.numeric(crime.test$score > cutoff)
TP = sum(crime.test$predicted == 1 & crime.test$target == 1)
FP = sum(crime.test$predicted == 1 & crime.test$target == 0)
FN = sum(crime.test$predicted == 0 & crime.test$target == 1)
TN = sum(crime.test$predicted == 0 & crime.test$target == 0)
# lets also calculate total number of real positives and negatives in the data
P = TP + FN
N = TN + FP
total = P + N
confusionMatrix(factor(crime.test$predicted), factor(crime.test$target), positive = "1")
sensitivity <- round(sensitivity(factor(crime.test$predicted), crime.test$target,</pre>
    positive = "1"), 4)
specificity <- round(specificity(factor(crime.test$predicted), crime.test$target,</pre>
    negative = "0"), 4)
\# accuracy = (TP+TN)/(P+N)
accuracy \leftarrow round(((TP + TN)/(P + N)), 4)
cnfMtx <- confusionMatrix(crime.test$predicted, crime.test$target, positive = "1")</pre>
(roc <- roc(factor(predicted) ~ as.numeric(target), data = crime.test, plot = FALSE,</pre>
    ci = TRUE))
graphics::plot(roc, legacy.axes = TRUE, col = "blue", lwd = 3)
(auc <- round(auc(factor(predicted) ~ as.numeric(target), crime.test), 4))</pre>
performance.results <- rbind(performance.results, c("Forward Elimination", sensitivity,
    specificity, accuracy, auc))
crime.test$score <- predict(backwards.model, newdata = subset(crime.test, select = c(zn,</pre>
    nox, age, dis, rad, tax, ptratio, black, lstat, medv)), type = "response")
ggplot(crime.test, aes(y = target, x = score, color = factor(target))) + geom_point() +
    geom_jitter()
cutoff = 0.5
```

```
crime.test$predicted = as.numeric(crime.test$score > cutoff)
TP = sum(crime.test$predicted == 1 & crime.test$target == 1)
FP = sum(crime.test$predicted == 1 & crime.test$target == 0)
FN = sum(crime.test$predicted == 0 & crime.test$target == 1)
TN = sum(crime.test$predicted == 0 & crime.test$target == 0)
# lets also calculate total number of real positives and negatives in the data
P = TP + FN
N = TN + FP
total = P + N
confusionMatrix(factor(crime.test$predicted), factor(crime.test$target), positive = "1")
sensitivity <- round(sensitivity(factor(crime.test$predicted), crime.test$target,</pre>
    positive = "1"), 4)
specificity <- round(specificity(factor(crime.test$predicted), crime.test$target,</pre>
    negative = "0"), 4)
\# accuracy = (TP+TN)/(P+N)
accuracy \leftarrow round(((TP + TN)/(P + N)), 4)
cnfMtx <- confusionMatrix(crime.test$predicted, crime.test$target, positive = "1")</pre>
(roc <- roc(factor(predicted) ~ as.numeric(target), data = crime.test, plot = FALSE,</pre>
    ci = TRUE)
graphics::plot(roc, legacy.axes = TRUE, col = "blue", lwd = 3)
(auc <- round(auc(factor(predicted) ~ as.numeric(target), crime.test), 4))</pre>
performance.results <- rbind(performance.results, c("Backward Elimination", sensitivity,
    specificity, accuracy, auc))
crime.test$score <- predict(bayesian.model.final, newdata = subset(crime.test, select = c(nox,</pre>
    rad, tax, target)), type = "response")
ggplot(crime.test, aes(y = target, x = score, color = factor(target))) + geom_point() +
    geom_jitter()
cutoff = 0.5
crime.test$predicted = as.numeric(crime.test$score > cutoff)
TP = sum(crime.test$predicted == 1 & crime.test$target == 1)
FP = sum(crime.test$predicted == 1 & crime.test$target == 0)
FN = sum(crime.test$predicted == 0 & crime.test$target == 1)
TN = sum(crime.test$predicted == 0 & crime.test$target == 0)
# lets also calculate total number of real positives and negatives in the data
P = TP + FN
N = TN + FP
total = P + N
confusionMatrix(factor(crime.test$predicted), factor(crime.test$target), positive = "1")
sensitivity <- round(sensitivity(factor(crime.test$predicted), crime.test$target,</pre>
```

```
positive = "1"), 4)
specificity <- round(specificity(factor(crime.test$predicted), crime.test$target,</pre>
    negative = "0"), 4)
\# \ accuracy = (TP+TN)/(P+N)
accuracy <- round(((TP + TN)/(P + N)), 4)
cnfMtx <- confusionMatrix(crime.test$predicted, crime.test$target, positive = "1")</pre>
(roc <- roc(factor(predicted) ~ as.numeric(target), data = crime.test, plot = FALSE,</pre>
    ci = TRUE))
graphics::plot(roc, legacy.axes = TRUE, col = "blue", lwd = 3)
(auc <- round(auc(factor(predicted) ~ as.numeric(target), crime.test), 4))</pre>
performance.results <- rbind(performance.results, c("Bayesian Model", sensitivity,
    specificity, accuracy, auc))
results <- as.data.frame(performance.results)</pre>
colnames(results) <- c("Method", "Sn", "Sp", "Accuracy", "AUC")</pre>
kable(results)
crime.evl$chas <- as.factor(crime.evl$chas)</pre>
crime.prd <- predict(manual.final, newdata = subset(crime.evl, select = c(4, 8, 9)),</pre>
    type = "response")
crime.prd <- ifelse(crime.prd > 0.5, 1, 0)
crime.evl$predicted <- crime.prd</pre>
crime.evl$predicted <- factor(crime.evl$predicted)</pre>
kable(crime.evl)
```