Jul, 2019

1 Mesh-independent study

Since we are studying the nonlinear non-convex optimization with obstacles, there may have multiple local solutions. There are many factors that influence to the local convergence of the gradient algorithm such as the volume penalization parameter η, the perimeter parameter κ, the internal length ℓ, or the ratio of the internal length ℓ and the minimum mesh size h, and the initial guess of the thickness field. We study how the mesh influence to the convergence of the active-set algorithm. We consider the Example 2 where we solve the optimization with one obstacle (details). The minimum mesh size h vary from 4.33×10^{-3} to 6.31×10^{-3} for the unstructured mesh, and from 0.005 to 0.01 for the structured mesh. We chose the follow meshes to study.

1.1 The meshes

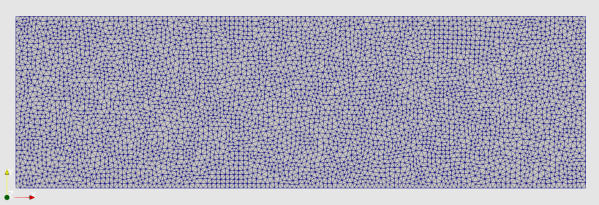

Figure 1: Mesh 00, $h_{min} = 6.31 \times 10^{-3}$, resolution 75, 9529 elements

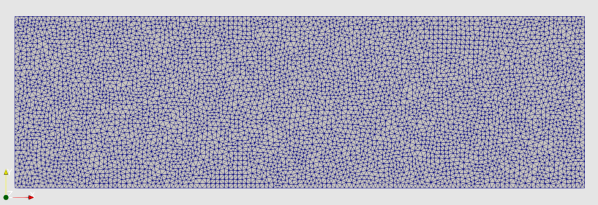

Figure 2: Mesh 01, $h_{min} = 6.01 \times 10^{-3}$, resolution 80, 10824 elements

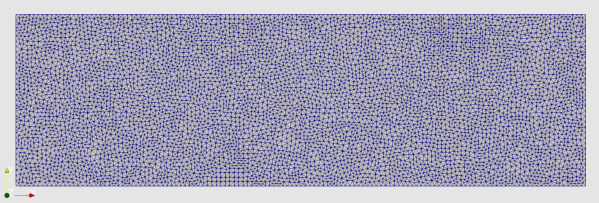

Figure 3: Mesh 02, $h_{min} = 5.66 \times 10^{-3}$, resolution 85, 12509 elements

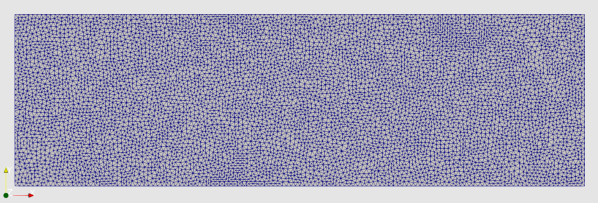

Figure 4: Mesh 03, $h_{min} = 5.25 \times 10^{-3}$, resolution 90, 13987 elements

The following meshes are the uniform crossed mesh that we use

Let us take a closer look on these two types of mesh

The mesh independent study can be summarized in the Fig 17

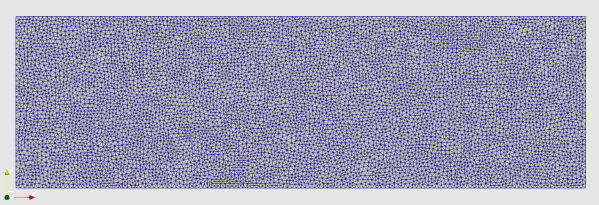

Figure 5: Mesh 04, $h_{min} = 5.11 \times 10^{-3}$, resolution 95, 15421 elements

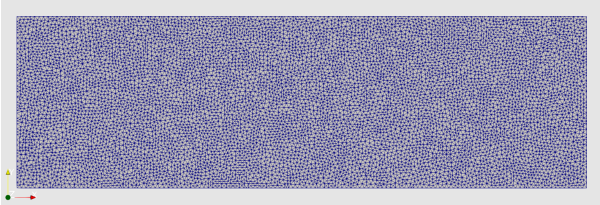

Figure 6: Mesh 05, $h_{min} = 4.79 \times 10^{-3}$, resolution 100, 17059 elements

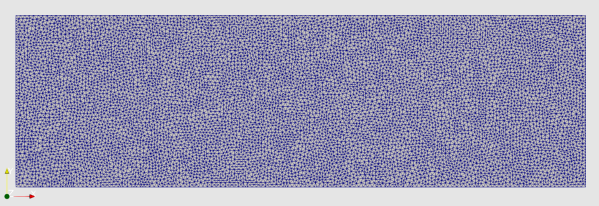

Figure 7: Mesh 06, $h_{min} = 4.54 \times 10^{-3}$, resolution 105, 18733 elements

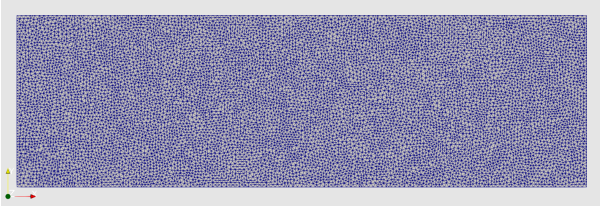

Figure 8: Mesh 07, $h_{min} = 4.33 \times 10^{-3}$, resolution 110, 20584 elements

1.2 The first initial guess results

These are the solutions of the Example 2 when we chose the initial guess $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{10\pi}{L_x})\sin(\frac{10\pi}{L_y})$ and the tolerance is 5×10^{-8}. The convergence of first attempt and the second attempt results is shown in the Fig 18.

The unstructured mesh results are shown in the Fig 19 to Fig 34

The following are the results of the structured meshes: from Fig 35 to Fig 46

1.3 The second initial guess results

These are the solutions of the Example 2 when we chose the initial guess $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{8\pi}{L_x})\sin(\frac{8\pi}{L_y})$ and the tolerance is 5×10^{-8}. The convergence of the results are shown in the Fig 47.

The unstructured mesh results are in Fig 48 to Fig 55

The structured mesh results are in the Fig 56 to Fig 61.

1.4 The third initial guess results

These are the solutions of the Example 2 when we chose the initial guess $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{2\pi}{L_x})\sin(\frac{2\pi}{L_y})$ and the tolerance is 5×10^{-8}. The convergence of the results are shown in the Fig 62

The results of unstructured meshes are in Fig 63 to Fig 70.

The results of structured meshes are in Fig 71 to Fig 76.

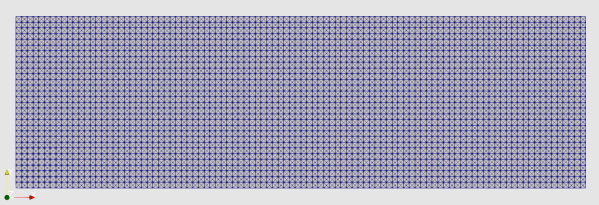

Figure 9: Mesh 10, $h_{min} = 0.01$, 12000 elements

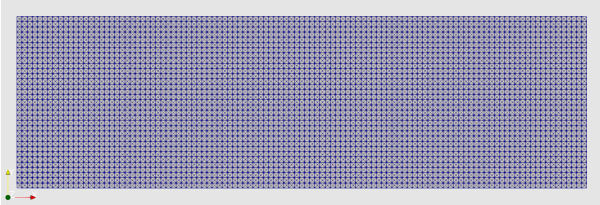

Figure 10: Mesh 11, $h_{min} = 0.009$, 14652 elements

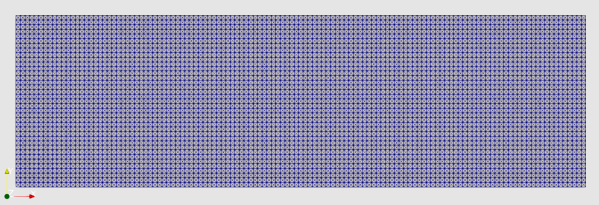

Figure 11: Mesh 12, $h_{min} = 0.008$, 18500 elements

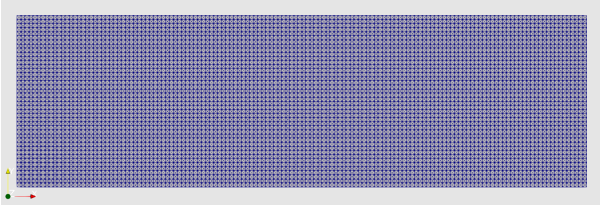

Figure 12: Mesh 13, $h_{min} = 0.007$, 23856 elements

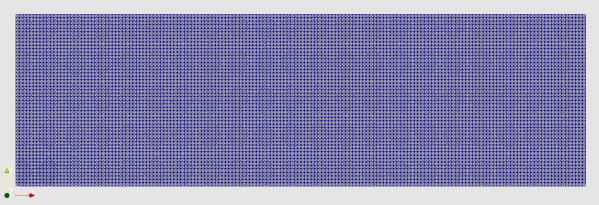

Figure 13: Mesh 14, $h_{min} = 0.006$, 33200 elements

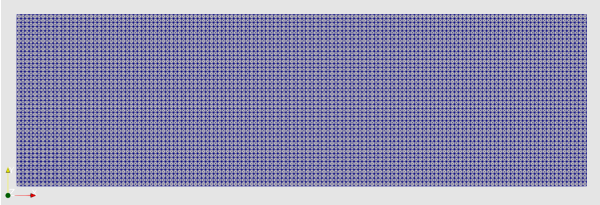

Figure 14: Mesh 15, $h_{min} = 0.005$, 48000 elements

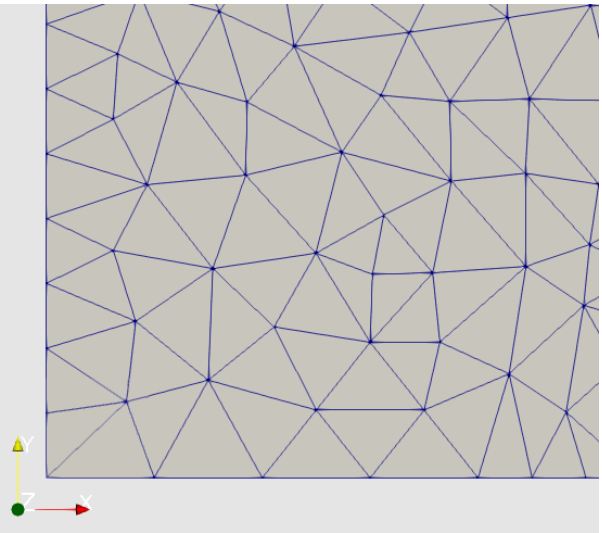

Figure 15: A corner of the Mesh 07

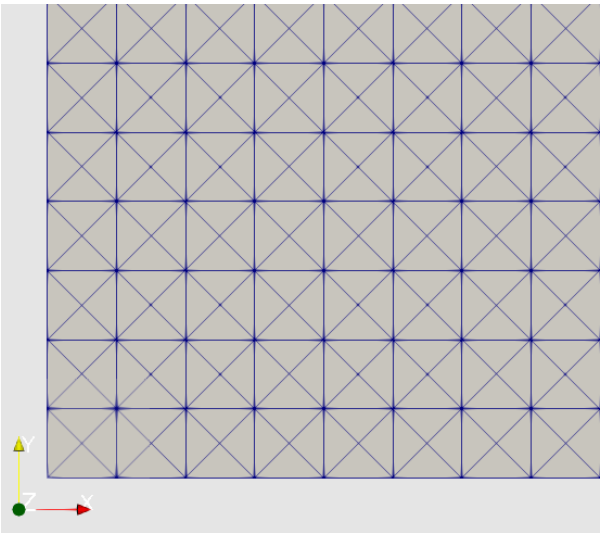

Figure 16: A corner of the Mesh 15

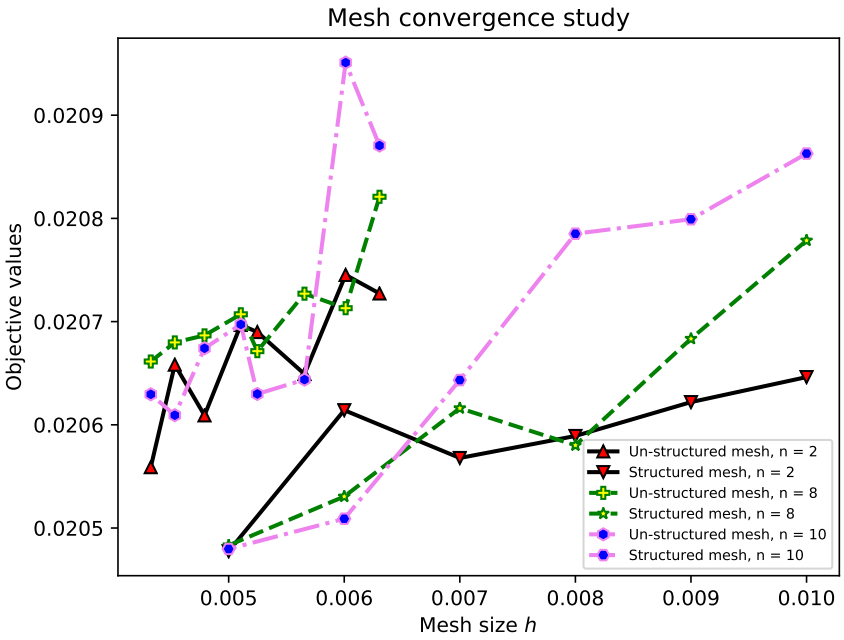

Figure 17: Mesh convergence with different initial guesses

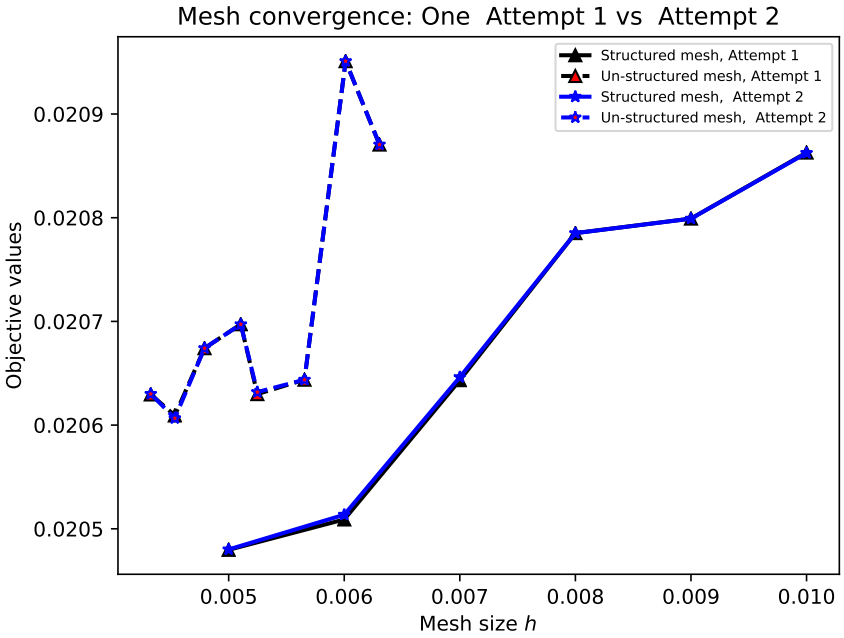

Figure 18: Mesh convergence with $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{10\pi}{L_x})\sin(\frac{10\pi}{L_y})$, the first attempt

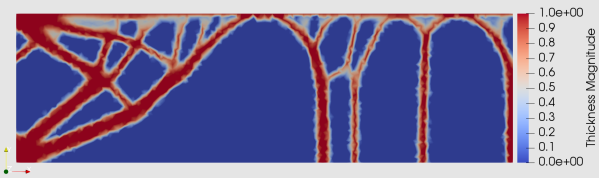

Figure 19: Mesh 00: the first attempt result, $tol = 5 \times 10^{-8}$.

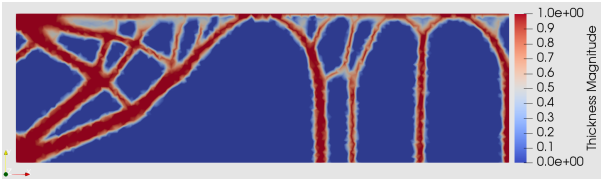

Figure 20: Mesh 00: the second attempt result, $tol = 5 \times 10^{-8}$.

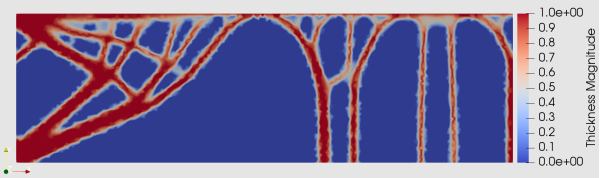

Figure 21: Mesh 01: the first attempt result, $tol = 5 \times 10^{-8}$.

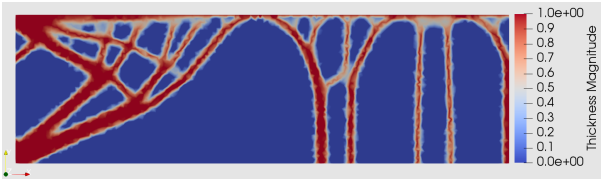

Figure 22: Mesh 01: the second attempt result, $tol = 5 \times 10^{-8}$.

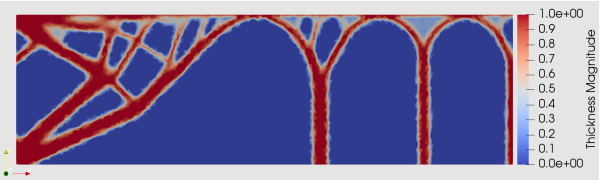

Figure 23: Mesh 02: the first attempt result, $tol = 5 \times 10^{-8}$.

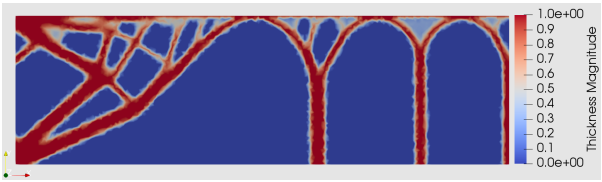

Figure 24: Mesh 02: the second attempt result, $tol = 5 \times 10^{-8}$.

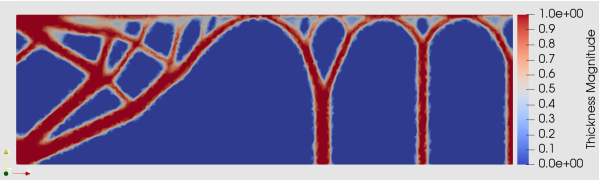

Figure 25: Mesh 03: the first attempt result, $tol = 5 \times 10^{-8}$.

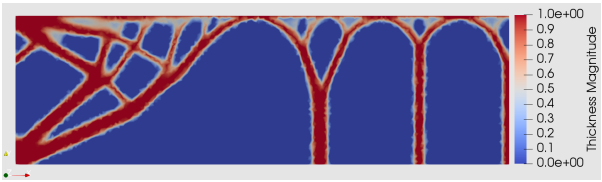

Figure 26: Mesh 03: the second attempt result, $tol = 5 \times 10^{-8}$.

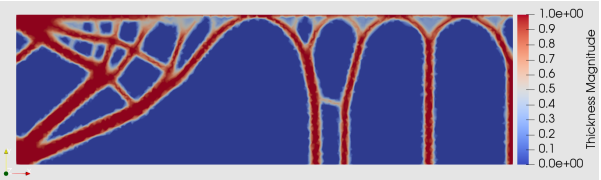

Figure 27: Mesh 04: the first attempt result, $tol = 5 \times 10^{-8}$.

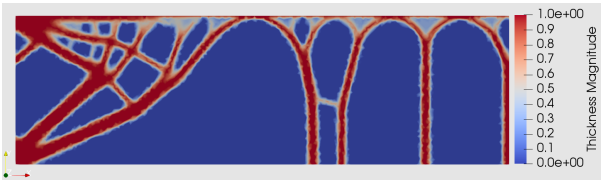

Figure 28: Mesh 04: the second attempt result, $tol = 5 \times 10^{-8}$.

Figure 29: Mesh 05: the first attempt result, $tol = 5 \times 10^{-8}$.

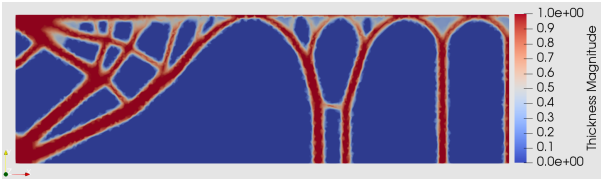

Figure 30: Mesh 05: the second attempt result, $tol = 5 \times 10^{-8}$.

Figure 31: Mesh 06: the first attempt result, $tol = 5 \times 10^{-8}$.

Figure 32: Mesh 06: the second attempt result, $tol = 5 \times 10^{-8}$.

Figure 33: Mesh 07: the first attempt result, $tol = 5 \times 10^{-8}$.

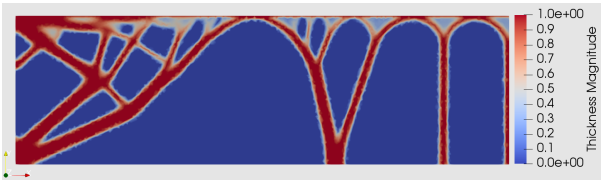

Figure 34: Mesh 07: the second attempt result, $tol = 5 \times 10^{-8}$.

Figure 35: Mesh 10: the first attempt result, $tol = 5 \times 10^{-8}$.

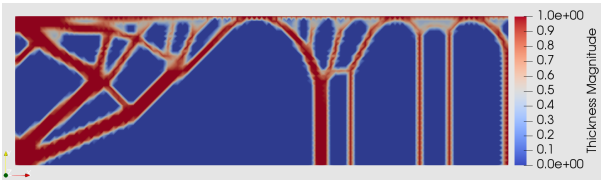

Figure 36: Mesh 10: the second attempt result, $tol = 5 \times 10^{-8}$.

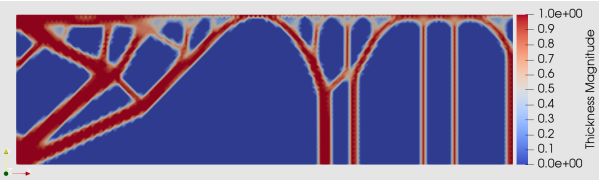

Figure 37: Mesh 11: the first attempt result, $tol = 5 \times 10^{-8}$.

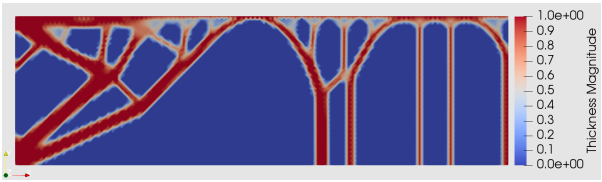

Figure 38: Mesh 11: the second attempt result, $tol = 5 \times 10^{-8}$.

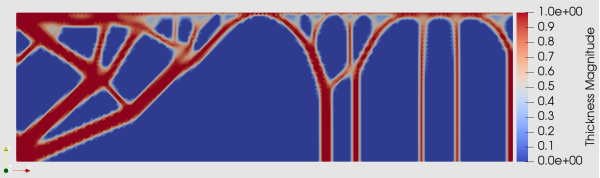

Figure 39: Mesh 12: the first attempt result, $tol = 5 \times 10^{-8}$.

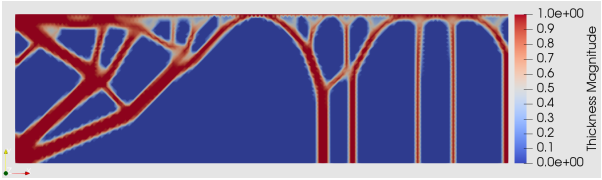

Figure 40: Mesh 12: the second attempt result, $tol = 5 \times 10^{-8}$.

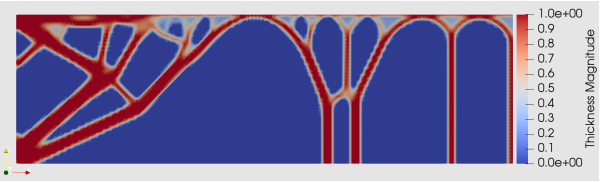

Figure 41: Mesh 13: the first attempt result, $tol = 5 \times 10^{-8}$.

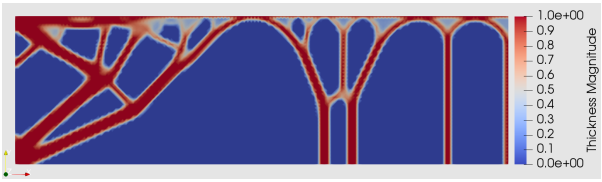

Figure 42: Mesh 13: the second attempt result, $tol = 5 \times 10^{-8}$.

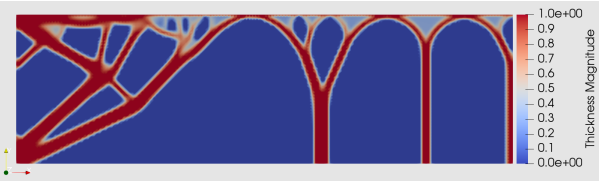

Figure 43: Mesh 14: the first attempt result, $tol = 5 \times 10^{-8}$.

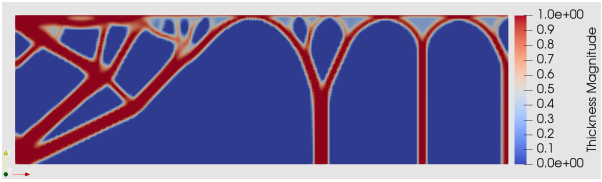

Figure 44: Mesh 14: the second attempt result, $tol = 5 \times 10^{-8}$.

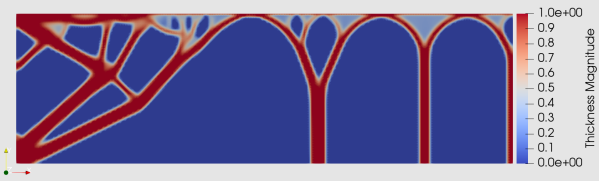

Figure 45: Mesh 15: the first attempt result, $tol = 5 \times 10^{-8}$.

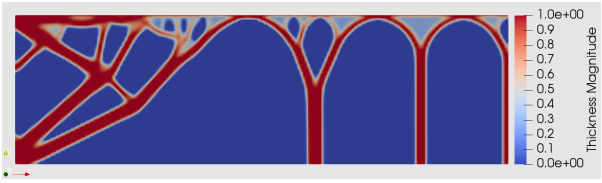

Figure 46: Mesh 15: the second attempt result, $tol = 5 \times 10^{-8}$.

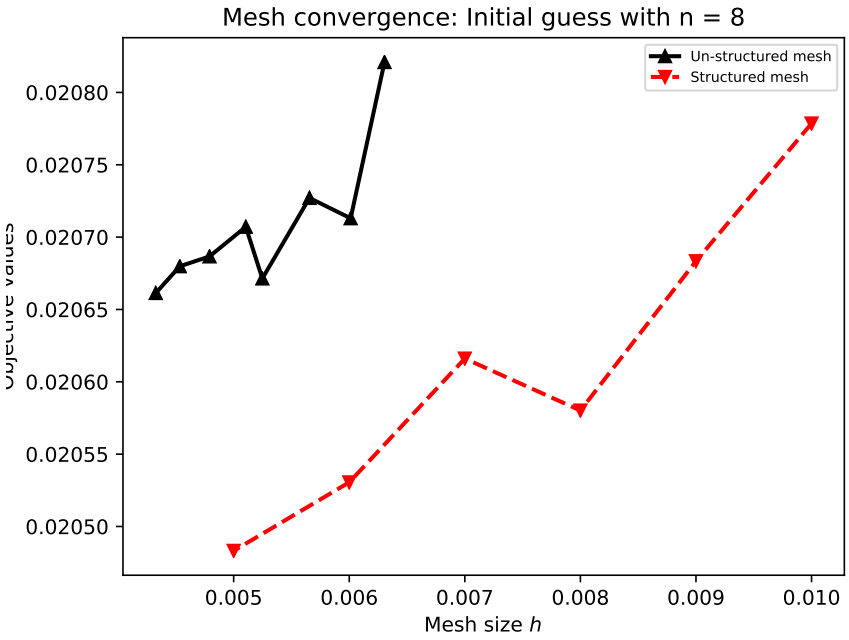

Figure 47: Mesh convergence with $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{8\pi}{L_x})\sin(\frac{8\pi}{L_y})$

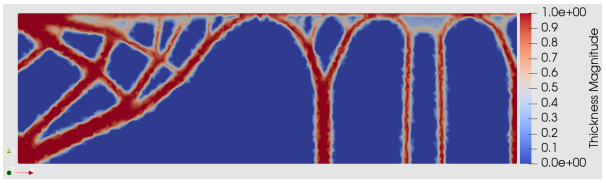

Figure 48: Mesh 00, $tol = 5 \times 10^{-8}$.

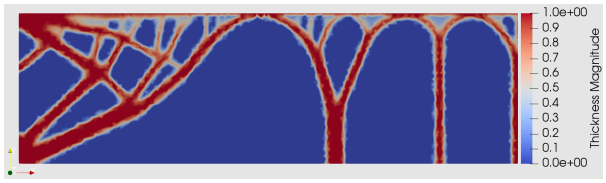

Figure 49: Mesh 01, $tol = 5 \times 10^{-8}$.

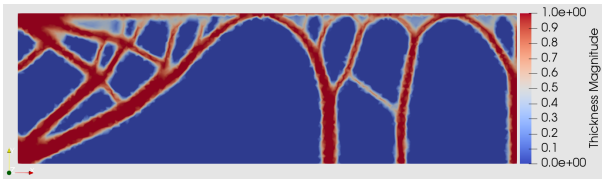

Figure 50: Mesh 02, $tol = 5 \times 10^{-8}$.

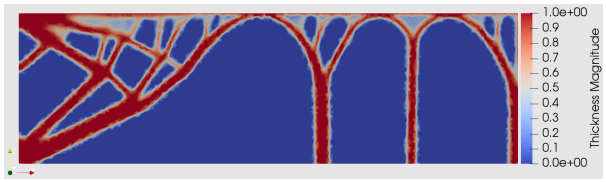

Figure 51: Mesh 03, $tol = 5 \times 10^{-8}$.

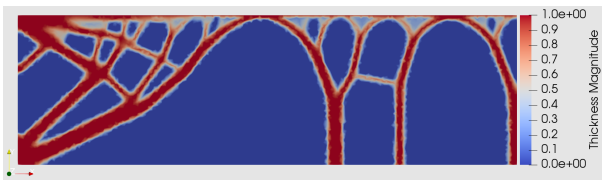

Figure 52: Mesh 04, $tol = 5 \times 10^{-8}$.

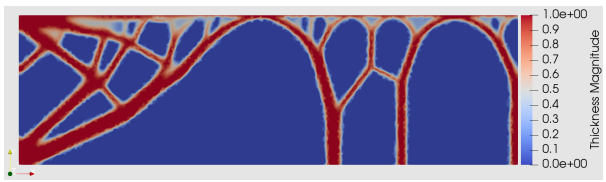

Figure 53: Mesh 05, $tol = 5 \times 10^{-8}$.

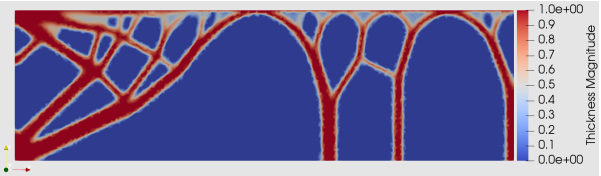

Figure 54: Mesh 06, $tol = 5 \times 10^{-8}$.

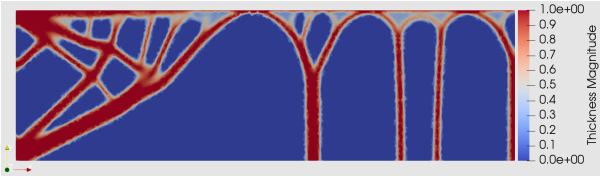

Figure 55: Mesh 07, $tol = 5 \times 10^{-8}$.

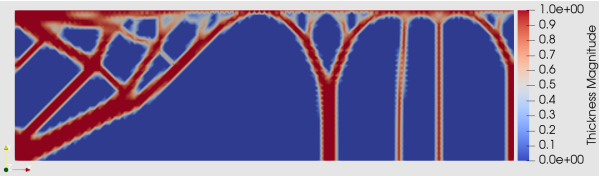

Figure 56: Mesh 10, $tol = 5 \times 10^{-8}$.

Figure 57: Mesh 11, $tol = 5 \times 10^{-8}$.

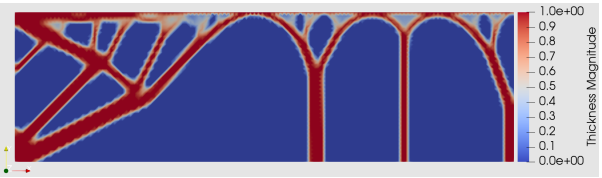

Figure 58: Mesh 12, $tol = 5 \times 10^{-8}$.

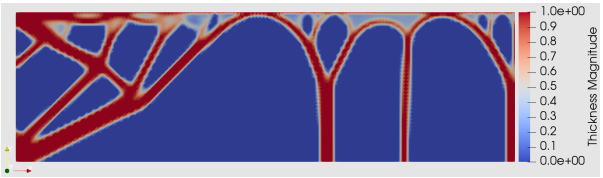

Figure 59: Mesh 13, $tol = 5 \times 10^{-8}$.

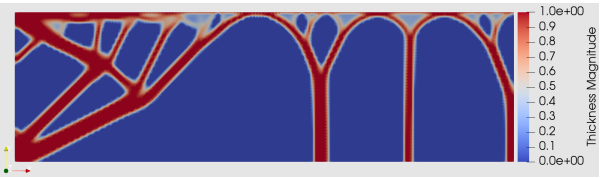

Figure 60: Mesh 14, $tol = 5 \times 10^{-8}$.

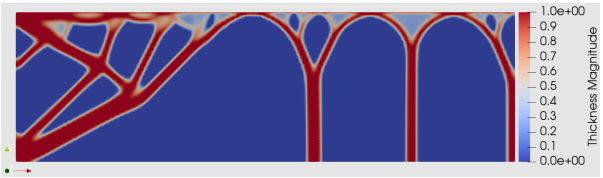

Figure 61: Mesh 15, $tol = 5 \times 10^{-8}$.

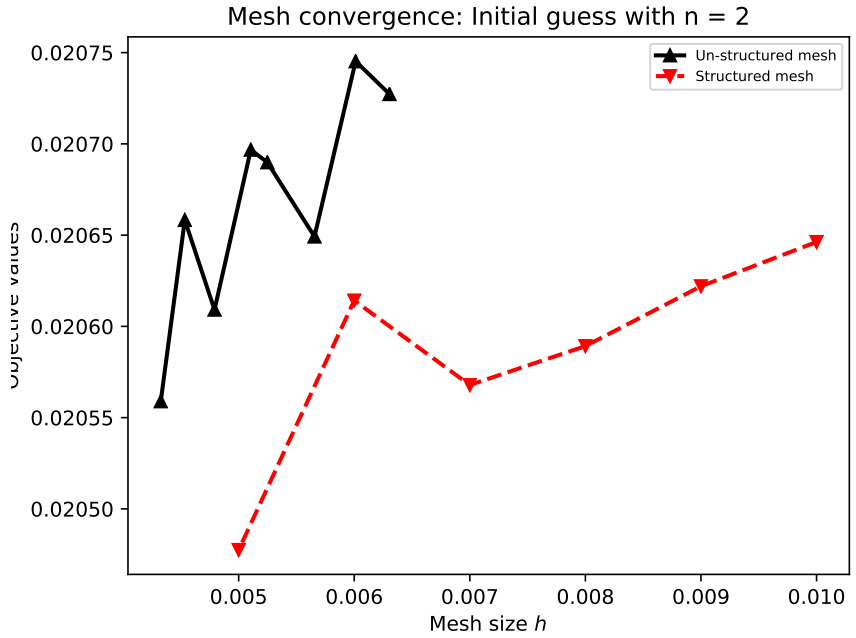

Figure 62: Mesh convergence with $\alpha_0 = \frac{1}{2} + \frac{1}{2}\sin(\frac{2\pi}{L_x})\sin(\frac{2\pi}{L_y})$

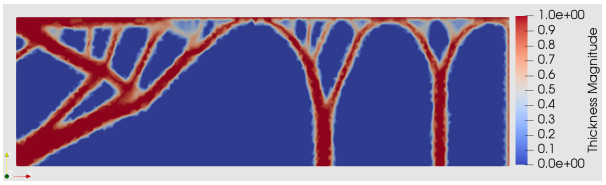

Figure 63: Mesh 00, $tol = 5 \times 10^{-8}$.

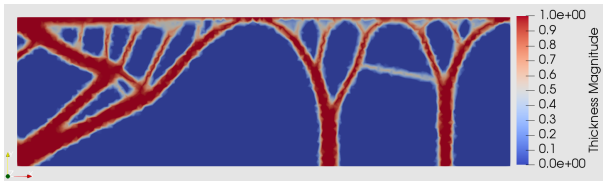

Figure 64: Mesh 01, $tol = 5 \times 10^{-8}$.

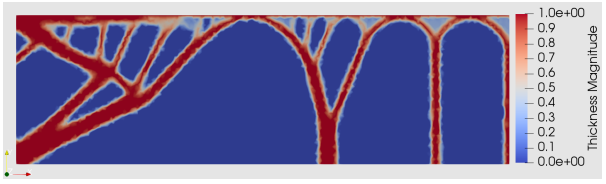

Figure 65: Mesh 02, $tol = 5 \times 10^{-8}$.

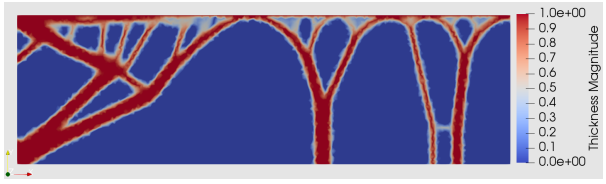

Figure 66: Mesh 03, $tol = 5 \times 10^{-8}$.

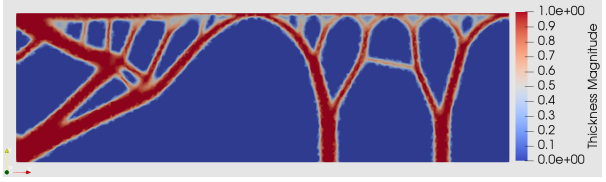

Figure 67: Mesh 04, $tol = 5 \times 10^{-8}$.

Figure 68: Mesh 05, $tol = 5 \times 10^{-8}$.

Figure 69: Mesh 06, $tol = 5 \times 10^{-8}$.

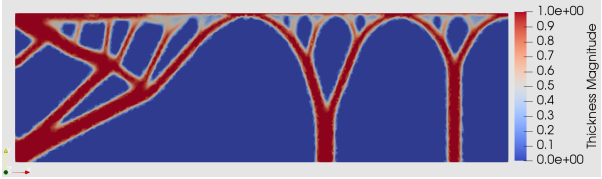

Figure 70: Mesh 07, $tol = 5 \times 10^{-8}$.

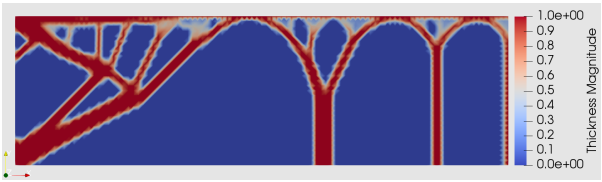

Figure 71: Mesh 10, $tol = 5 \times 10^{-8}$.

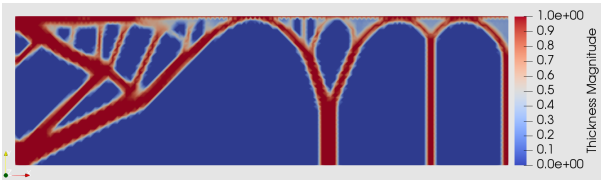

Figure 72: Mesh 01, $tol = 5 \times 10^{-8}$.

Figure 73: Mesh 12, $tol = 5 \times 10^{-8}$.

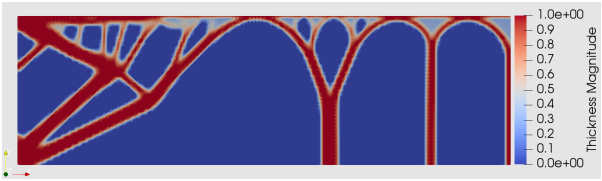

Figure 74: Mesh 13, $tol = 5 \times 10^{-8}$.

Figure 75: Mesh 14, $tol = 5 \times 10^{-8}$.

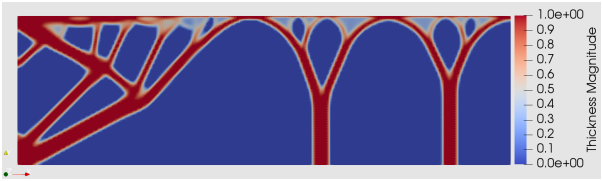

Figure 76: Mesh 15, $tol = 5 \times 10^{-8}$.