

Preparation before next session



You will need to present your research during the next session

LEARN

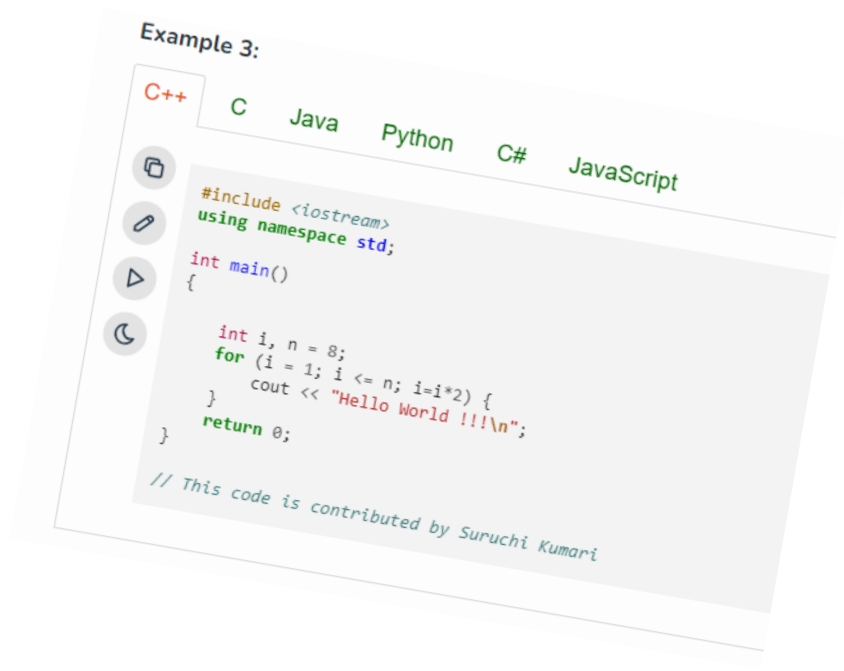
1. Read this [guide about algorithm complexity](#)

RESEARCH

Make researches on the following points

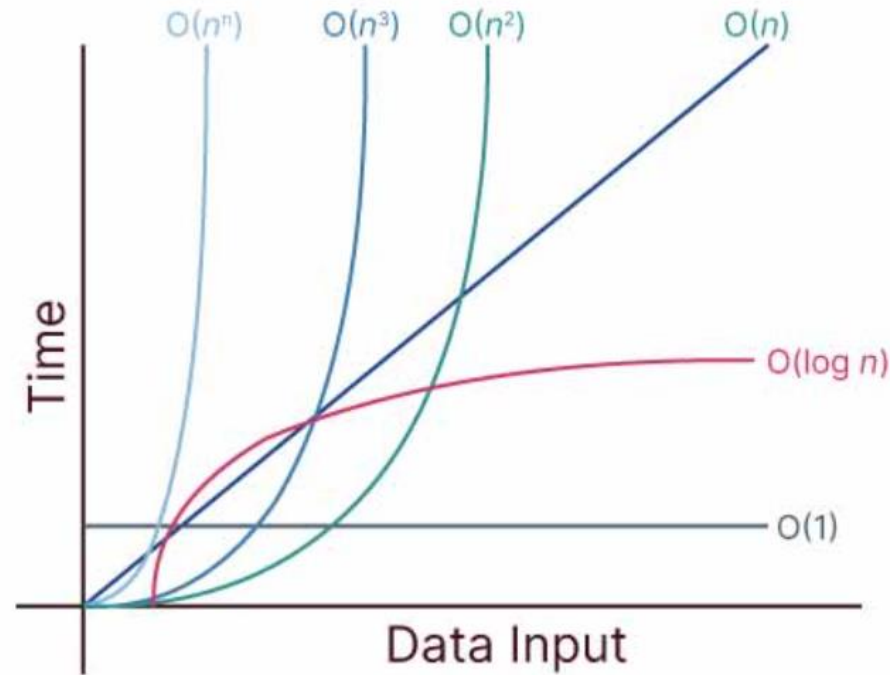
Q1 – What does the **time complexity** means ?

Q2 – What are the **impacts** of an algorithm with a **Hight complexity** ?



ALGORITHM ADVANCED

W2-S3 – Complexity & Big O notation





Session Objectives



Understand the concept of **time complexity**



Understand **5 Big O Notations**

✓ $O(1)$

Constant time complexity

✓ $O(\log n)$

Logarithmic time complexity

✓ $O(n)$

Linear time complexity

✓ $O(n \log n)$

Log-linear time complexity

✓ $O(n^2)$

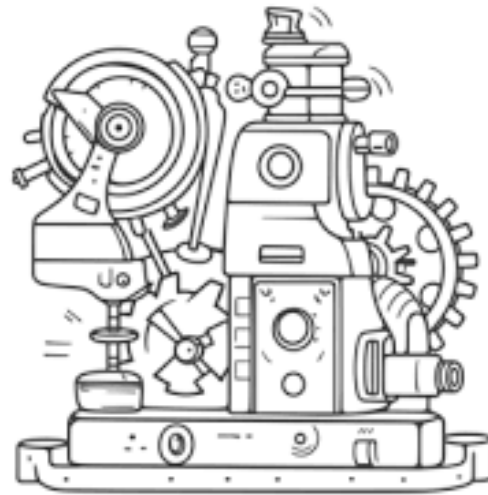
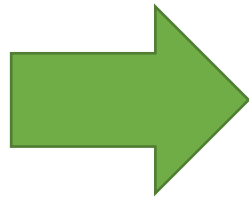
Quadratic time complexity



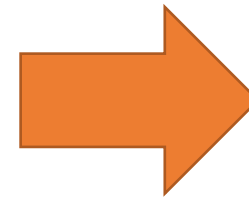
Calculate the time complexity in different use cases

An algorithm get **inputs** and produce **results**

INPUTS
Number of input = N



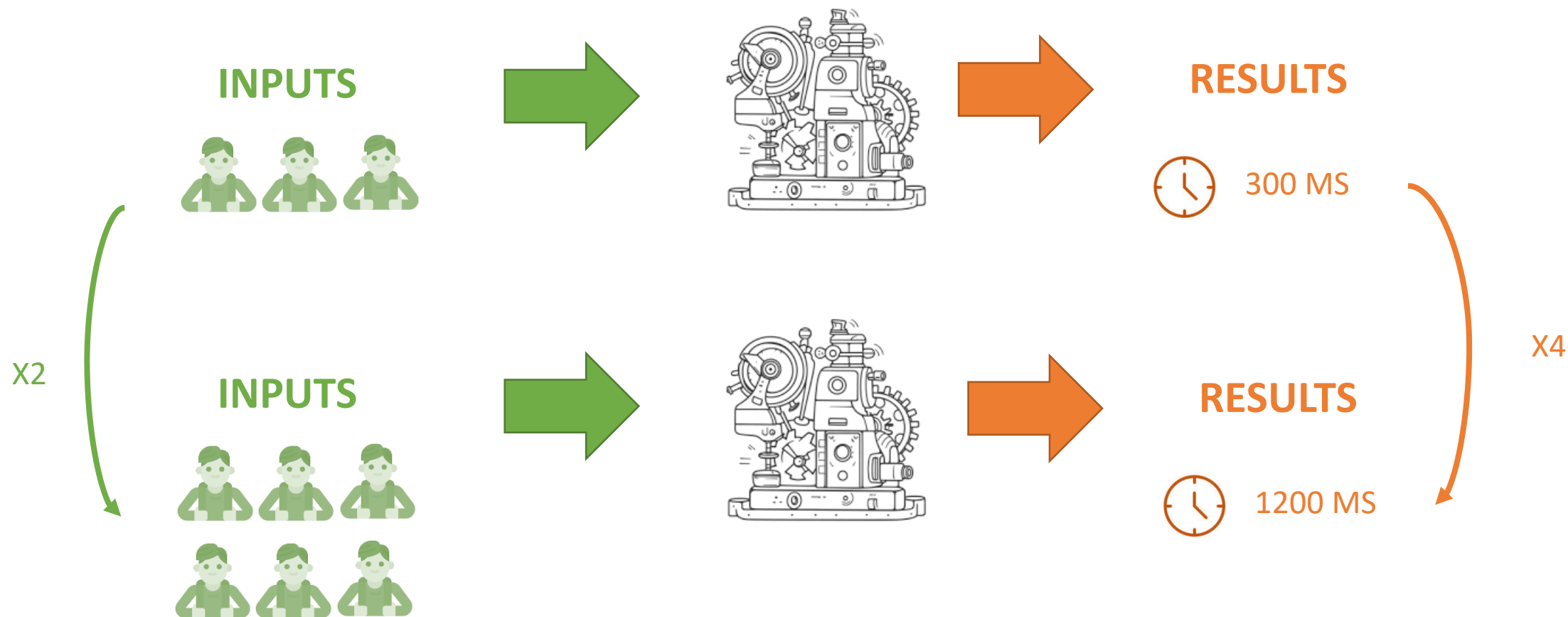
ALGORITHM



RESULTS

The Time complexity

Represents *how runtime grows* regarding the input size



Time is related to the number of **elementary operations** executed

- ✓ They refer to **atomic smallest** operations
- ✓ They always take the **same time to be executed** (for the same machine, same language)

```
print("Enter the value M:")
```

← 1 write operation

```
read(m)
```

← 1 assignment operation

```
s = 2 * 8
```

← 1 computation + 1 assignment

```
n = 2 + s + 3
```

← 2 computation + 1 assignment

7 EO for this algorithm

Big O - *An example*

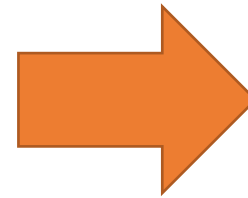
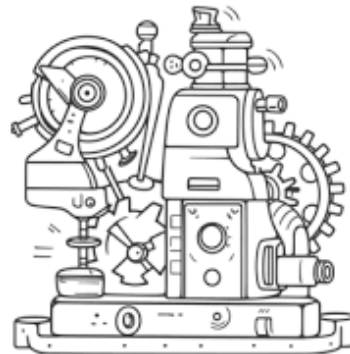
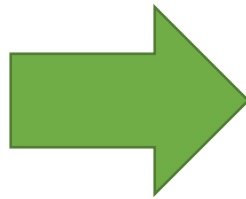


You lost
you pen !

- ✓ You gave your pen to a student, in a classroom of 100 students
- ✓ You have to find that pen without knowing to whom you gave it.
- ✓ *You need an algorithm to find you pen !*

INPUTS

*The 100 students
(N=100)*



RESULTS

The student name

Big O - *An example*



*Depending of **your strategy**, you might have different time complexities*

O order		Example in our case
$O(n^2)$	Quadratic time complexity	<ul style="list-style-type: none">✓ You go and ask the first person in the class if he has the pen.✓ Also, you ask this person about the other 99 people in the classroom if they have that pen and so on
$O(n)$	Linear time complexity	<ul style="list-style-type: none">✓ You go and ask each student individually
$O(\log n)$	Logarithmic time complexity	<ul style="list-style-type: none">✓ I divide the class into two groups, then ask: "Is it on the left side, or the right side of the classroom?"✓ I take that group and divide it into two and ask again, and so on.

Big O - *An example*



Choosing your algorithm also depend on the **context..**

O order		Example in our case
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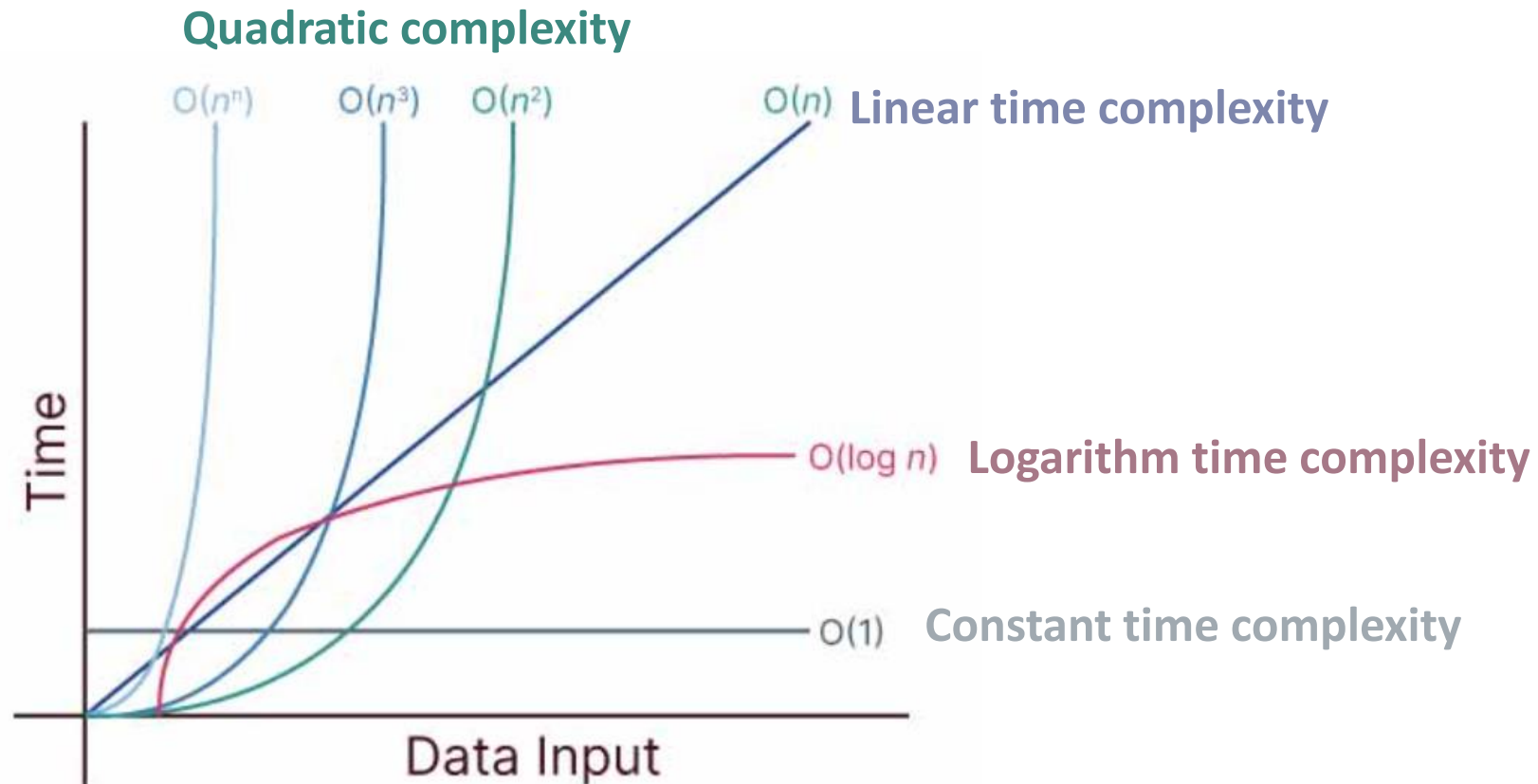
if only one student knows on which student the pen is hidden.

if one student had the pen and only they knew it

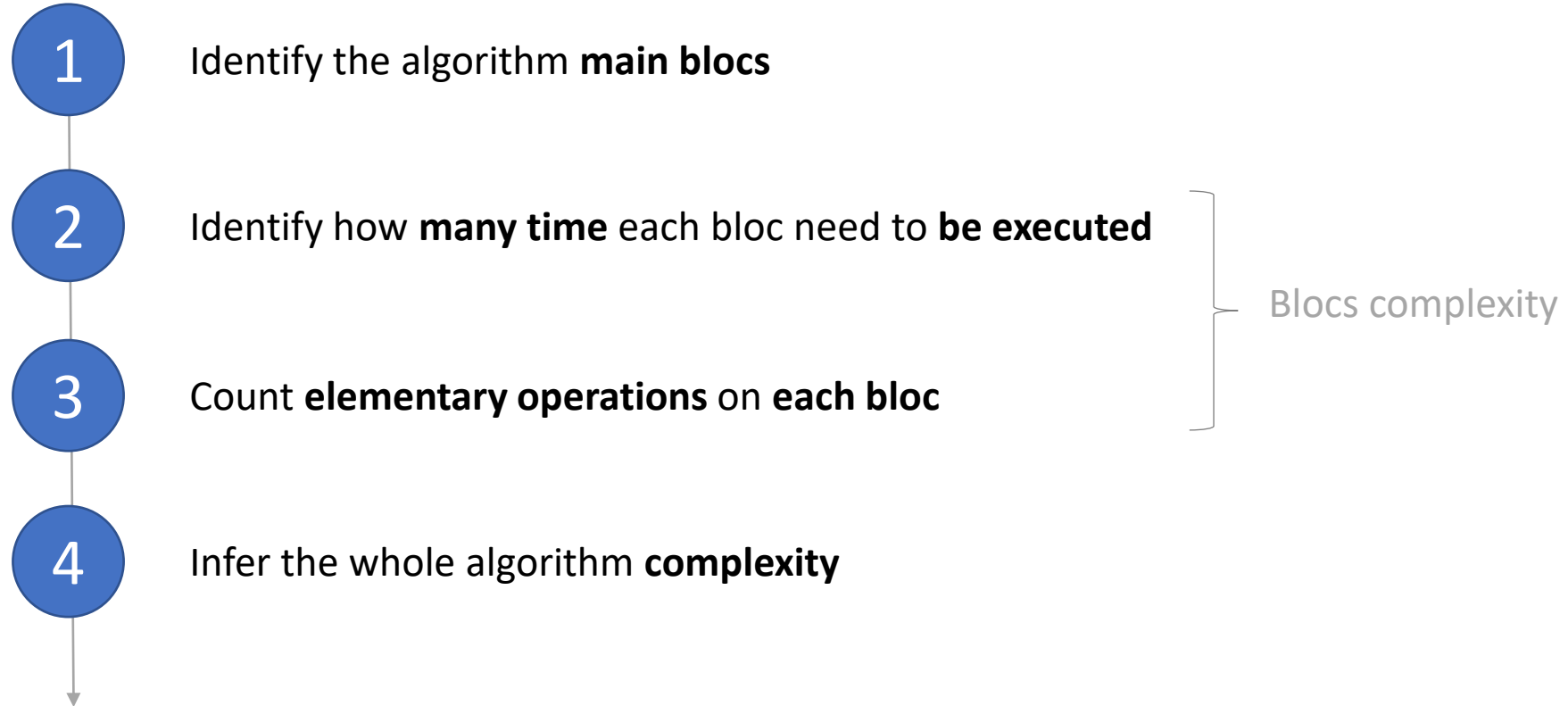
If all the students knew, but would only tell me if I guessed the right side.

Big O notations

We will explore 5 types of complexity today



How to **compute** complexity ?



An example of workflow ...

Identify the algorithm **main blocs**

```
print("Enter the value N:")  
read(n)
```

```
print("Enter the value M:")  
read(m)
```

```
s = n  
n = m  
m = s
```

```
print("N value is: ", N)  
print("M value is: ", M)
```

How many main blocs
This algorithm is composed of?

Can you identify what
Does this algorithm perform?

1 Identify the **main blocs**

Only 1 bloc, as this algorithm as not specific branching here

MAIN BLOC

```
print("Enter the value N:")  
read(n)
```

```
print("Enter the value M:")  
read(m)
```

```
s = n  
n = m  
m = s
```

```
print("N value is: ", N)  
print("M value is: ", M)
```

This algorithm just **swap** the 2 values, n and m

MAIN BLOC

2 EO

```
print("Enter the value N:")  
read(n)
```

2 EO

```
print("Enter the value M:")  
read(m)
```

3 EO

```
s = n  
n = m  
m = s
```

2 EO

```
print("N value is: ", N)  
print("M value is: ", M)
```

9 EO

DEMO

3

Identify how **many times** the bloc need to **be executed**
– *depending on the input (here n, m)*

INIT BLOC

```
print("Enter the value N:")  
read(n)
```

```
print("Enter the value M:")  
read(m)
```

```
s = n  
n = m  
m = s
```

```
print("N value is: ", N)  
print("M value is: ", M)
```

EO
9

iteration

X 1

Whatever the inputs, the
bloc runs **ONLY 1 time** !

Infer the **whole algorithm complexity**

INIT BLOC

```
print("Enter the value N:")  
read(n)
```

```
print("Enter the value M:")  
read(m)
```

```
s = n  
n = m  
m = s
```

```
print("N value is: ", N)  
print("M value is: ", M)
```

EO

9

iteration

x 1

9 x (1)

O (9)

Constants can be **ignored**
in the complexity computation

O (1)

Constant complexity

NOW YOU KNOW

$O(1)$

Constant time complexity

*Algorithm execution time **does not change** when inputs grow !*



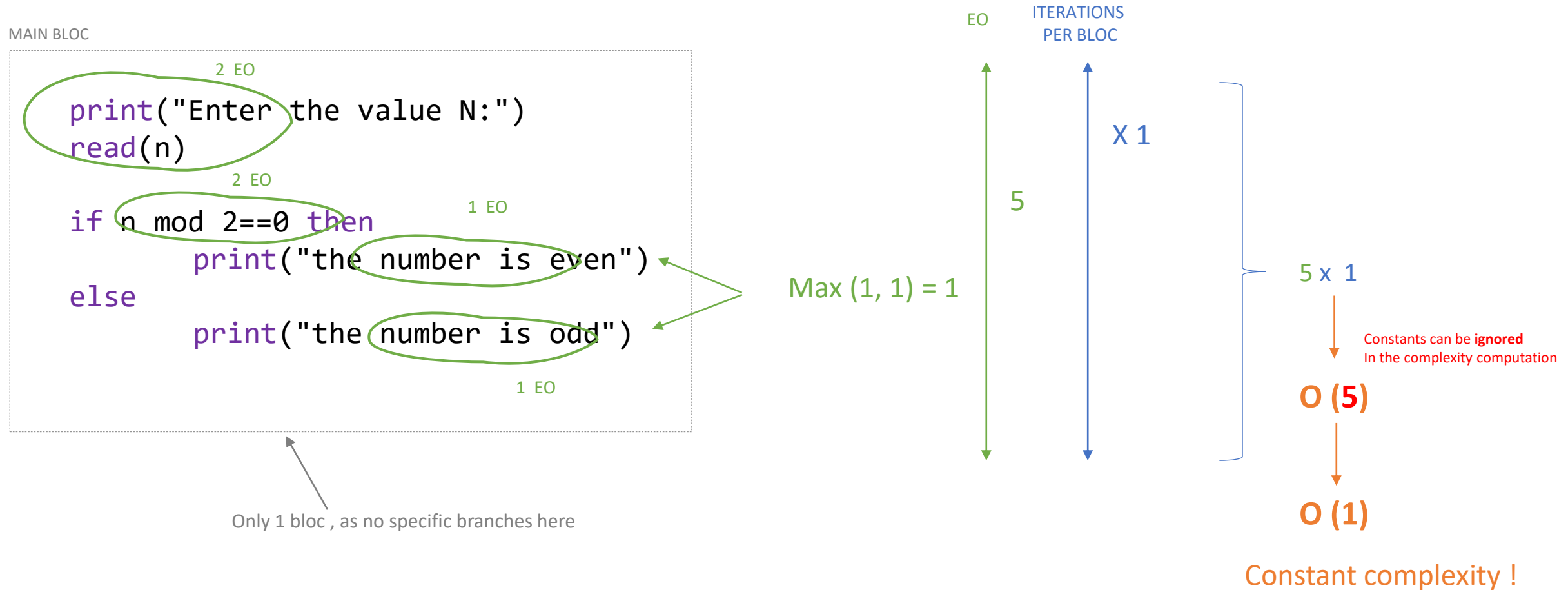
Activity 1

Analyze the complexity of this algorithm following the workflow

```
print("Enter the value N:")  
read(n)  
  
if n mod 2==0 then  
    print("the number is even")  
else  
    print("the number is odd")
```

Conditional branches

For conditional branch, **we compute the max** btw the different branch complexities





Activity 2

Analyze the complexity of this algorithm following the 4 steps

```
function sum(t: int[]) : int
```

```
    n = 20
```

```
    s = 0
```

```
    for i from 0 to n, do
```

```
        s = s + t[i]
```

```
    return s
```

A loop, but with a **constant number of iterations!**

You might have put $O(n)$ for the loop bloc, but n here **is NOT an input !**

```
function sum(t: int[]) : int
```

INIT BLOC

```
n = 20  
s = 0
```

2 EO

2

LOOP 1

```
for i from 0 to n, do  
    s = s + t[i]
```

3

3 EO

RETURN BLOC

```
return s
```

1

EO

ITERATIONS
PER BLOC

X 1

X 21



n is a constant
Not an input

X 1

Constants can be **ignored**
In the complexity computation

$O(66)$

$O(1)$

Constant complexity !



Activity 3

Analyze the complexity of this algorithm following the 4 steps

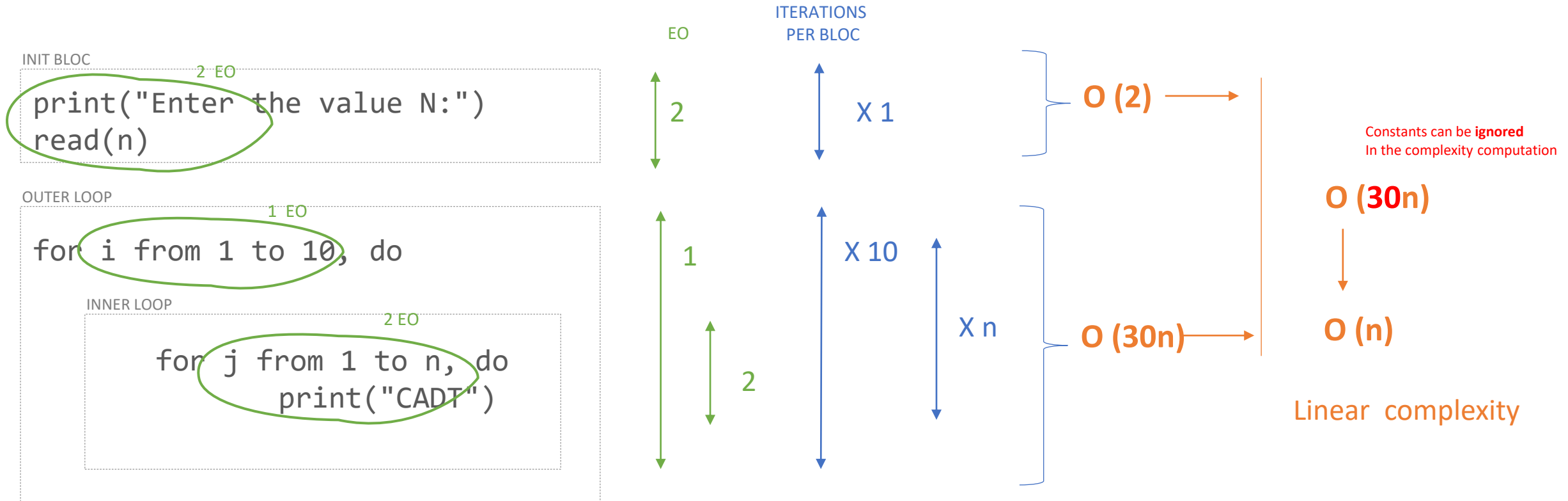
```
print("Enter the value N:")  
read(n)
```

```
for i from 1 to 10, do
```

```
    for j from 1 to n, do  
        print("CADT")
```

With nested loops, complexity are multiplied

In this case, the first loop is a constant loop, which will not impact the final complexity



NOW YOU KNOW

$O(n)$

Linear time complexity

*Algorithm execution time **change linearly** when inputs grow !*



Activity 4

Analyze the complexity of this algorithm following the 4 steps

```
print("Enter the value N:")  
read(n)
```

```
for i from 1 to n, do
```

```
    for j from 1 to i, do  
        print("CADT")
```

First let's compute the number of iterations of the 2 loops

INIT BLOC

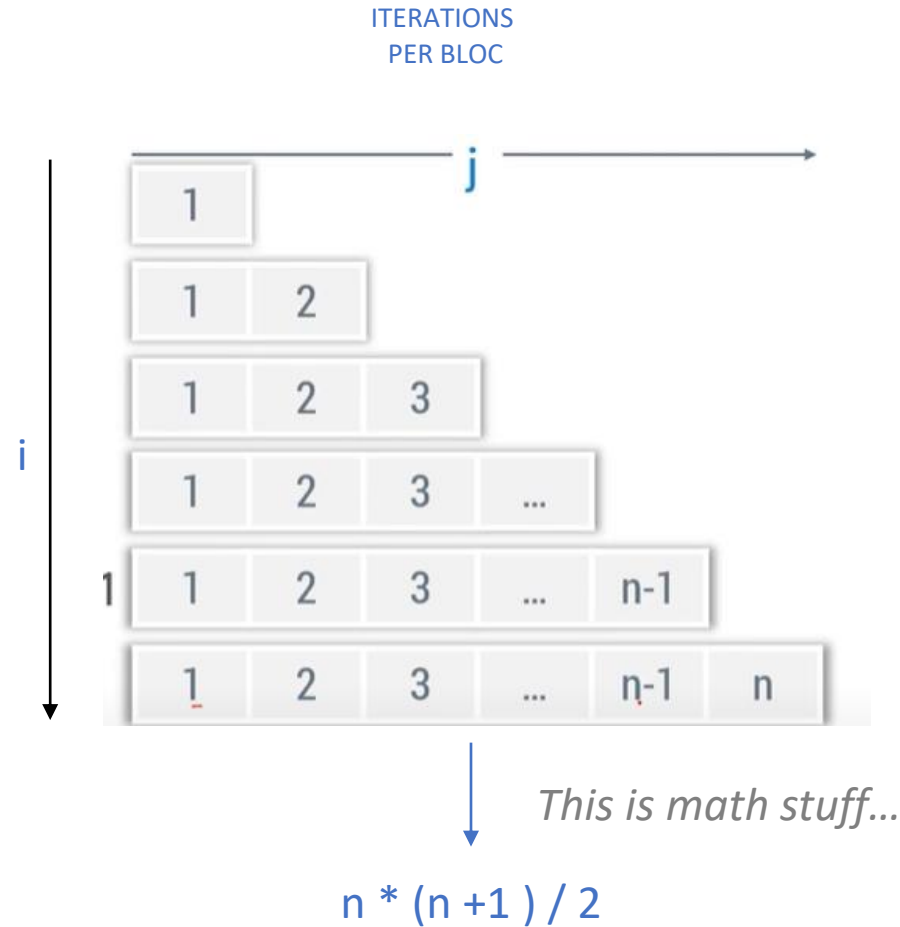
```
print("Enter the value N:")  
read(n)
```

OUTER LOOP

```
for i from 1 to n, do
```

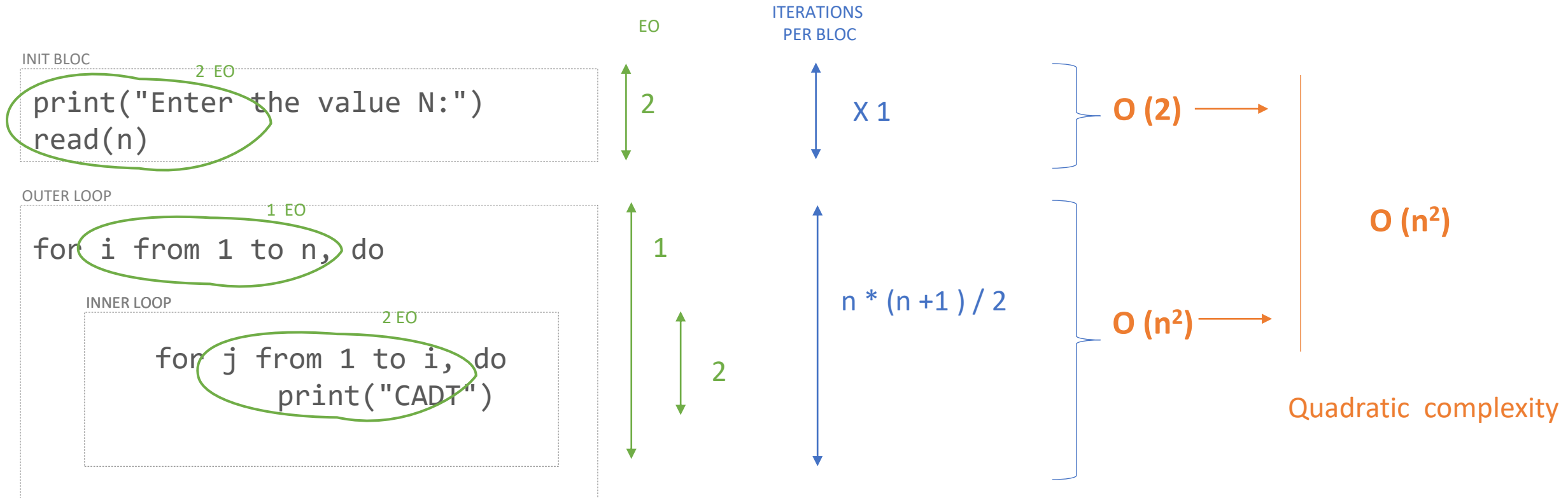
INNER LOOP

```
  for j from 1 to i, do  
    print("CADT")
```



With nested loops, complexity are multiplied

In this case, each nested loop depends on n ... We got a N^2 - quadratic complexity...





NOW YOU KNOW

$O(n^2)$

Quadratic time complexity

*the execution time increases proportionally to
the **square** of the size of the input data*



Activity 5

Analyze the complexity of this algorithm following the 4 steps

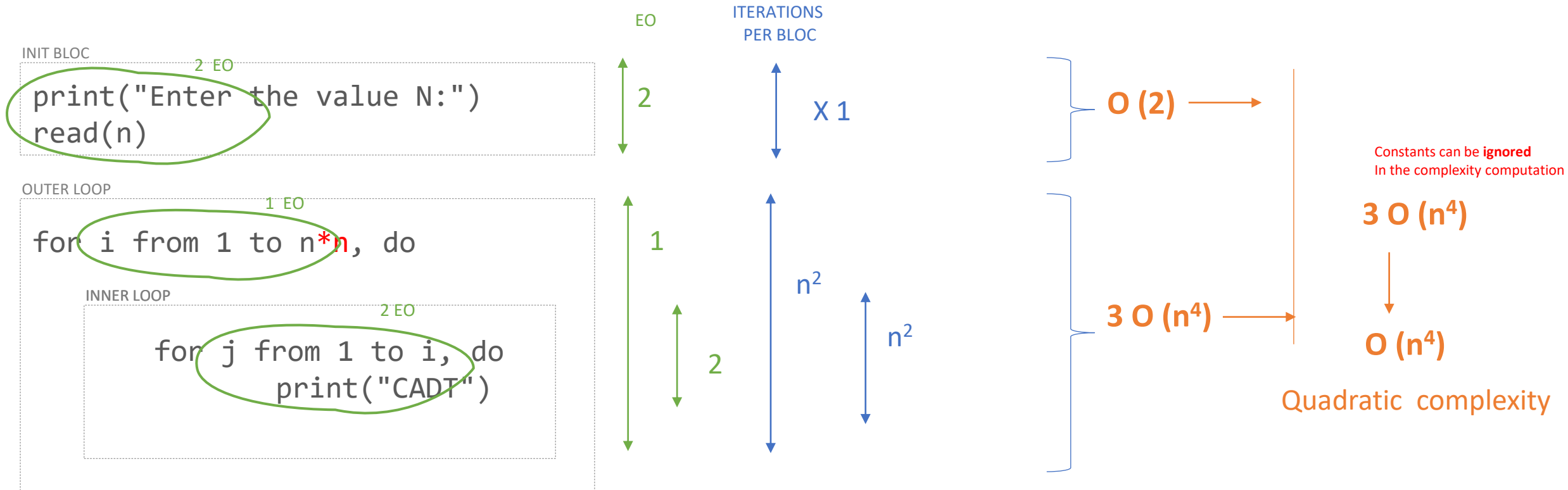
```
print("Enter the value N:")  
read(n)
```

```
for i from 1 to  $n \cdot n$ , do
```

```
    for j from 1 to i, do  
        print("CADD")
```

With nested loops, complexity are multiplied

In this case, each nested loop depends on n ... We got a N^2 - quadratic complexity...





Activity 6

Analyze the complexity of this algorithm following the 4 steps

```
print("enter a number:")  
read(n)
```

```
for i from 1 to n, do
```

```
    t = 1
```

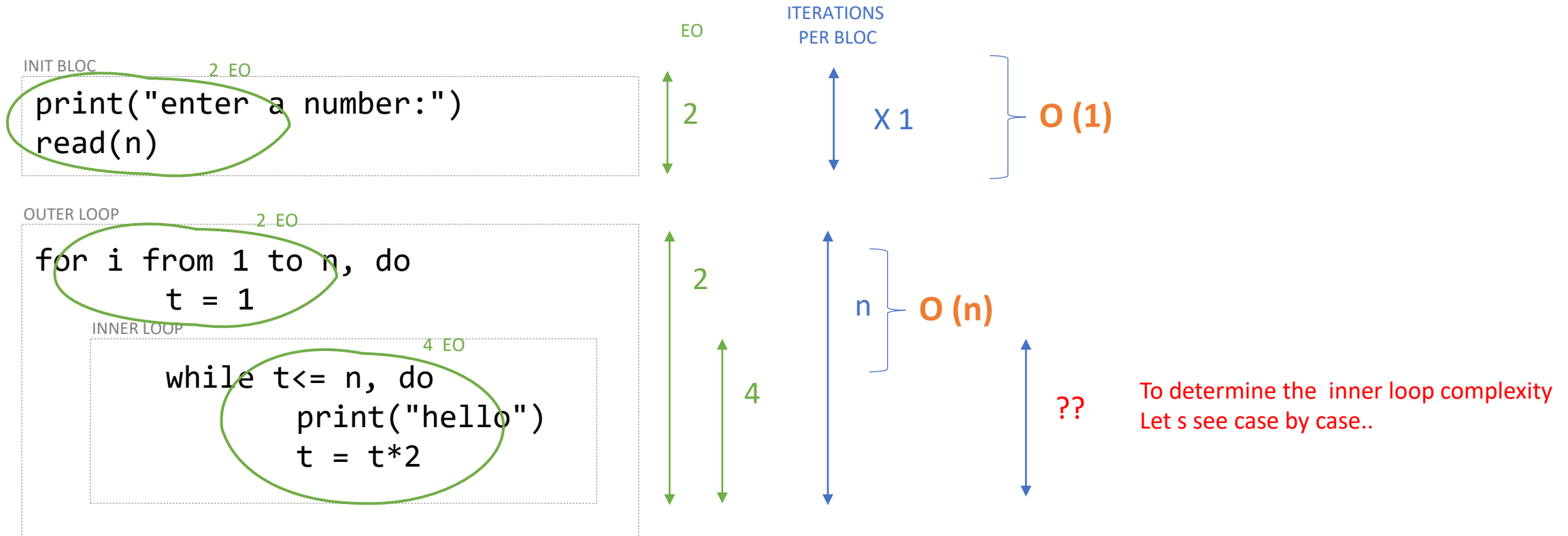
INNER LOOP

```
    while t<= n, do
```

```
        print("hello")
```

```
        t = t*2
```

We need to **identity** the **inner loop** iterations



We need to **identity** the **inner loop** iterations

```
print("enter a number:")  
read(n)
```

```
for i from 1 to n, do  
    t = 1
```

INNER LOOP

```
while t <= n, do  
    print("hello")  
    t = t*2
```

N	1	10	300
INNER LOOP ITERATIONS	1	4	9

If $n = 10$

- Iteration 1 : $t = 1$
 - Iteration 2 : $t = 2$
 - Iteration 3 : $t = 4$
 - Iteration 4 : $t = 8$
- $t=16 \Rightarrow$ we stop

If $n = 300$

- Iteration 1 : $t = 1$
 - Iteration 2 : $t = 2$
 - ...
 - Iteration 9 : $t = 256$
- $t=512 \Rightarrow$ we stop

NOW YOU KNOW

Logarithm complexity : $O(\log N)$

The **number of steps** required to complete the bloc grows **logarithmically**

INNER LOOP

```
while t<= n, do
    print("hello")
    t = t*2
```

Log2 as we multiply by 2

Bloc	N=1	N=10	N=300
INNER LOOP	1	4	9
$1 + \text{Log}_2(n)$	1	4.3	9.2

CHECK THE LOG2 CALCULATOR

x

300

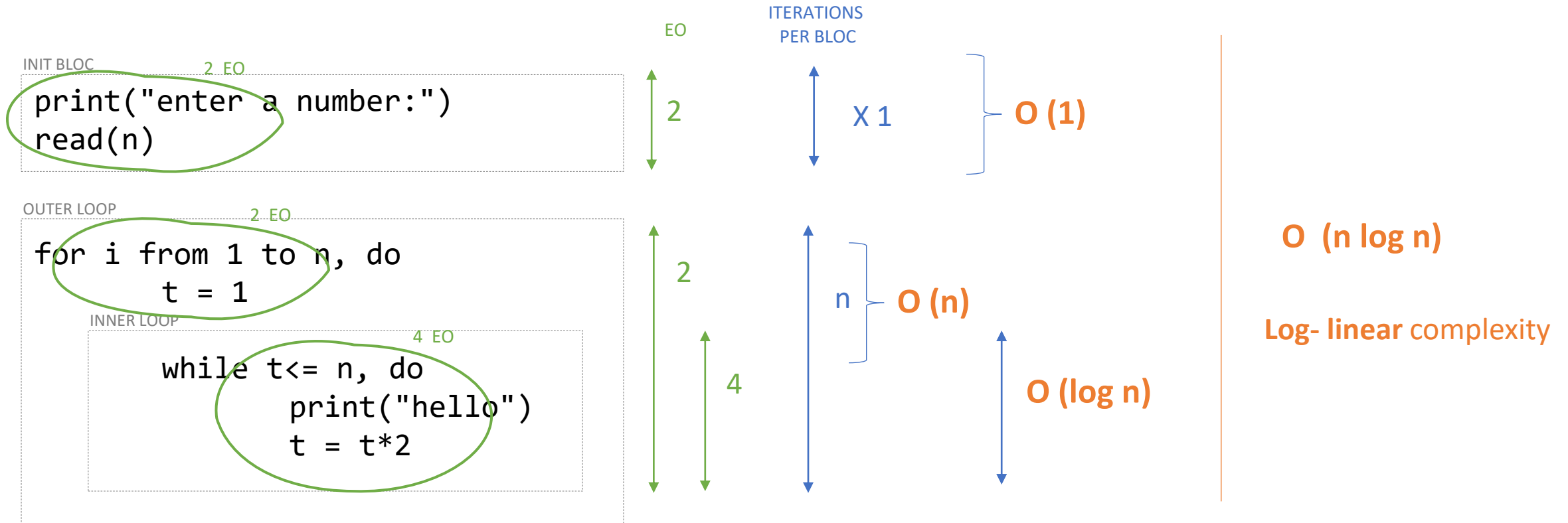
...

log₂(x)

8.229

...

We need to **identity** the **inner loop** iterations



NOW YOU KNOW

$O(n \log N)$

Log Linear time complexity

*the execution time increases proportionally
with the input size multiplied by the logarithm of n*

EXERCICES

EXERCICE 1

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")
read(n)

isPrime = true

for i from 2 to n-1, do
    if n mod i == 0 then
        isPrime = false
        break

if isPrime then
    print("Prime number")
else
    print("Not a prime number")
```

EXERCICE 2

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")  
read(n)
```

```
sum = 0
```

```
for i from 1 to n, do  
    sum = sum + i
```

```
print("Sum is: ", sum)
```

EXERCICE 3

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")
read(n)
print("Enter the value x:")
read(x)

left = 1
right = n

while left <= right, do
    mid = (left + right) // 2

    if array[mid] == x then
        print("Element found at: ", mid)
        break
    elif array[mid] < x then
        left = mid + 1
    else
        right = mid - 1
```


EXERCICE 4

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")  
read(n)  
  
for i from 1 to n, do  
    for j from 1 to n - i, do  
        if array[j] > array[j + 1] then  
            temp = array[j]  
            array[j] = array[j + 1]  
            array[j + 1] = temp
```

EXERCICE 5

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")
read(n)
print("Enter the value x:")
read(x)

count = 0

for i from 1 to n, do
    if array[i] == x then
        count = count + 1

print("Occurrence of x: ", count)
```

EXERCICE 6

Evaluate the complexity based on the five types you have learned

```
print("Enter the size of the matrix N:")
read(n)

for i from 1 to n, do
  for j from 1 to n, do
    result[i, j] = 0
    for k from 1 to n, do
      result[i, j] = result[i, j] + matrix1[i, k] * matrix2[k, j]
```

EXERCICE 7

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")
read(n)
print("Enter the value x:")
read(x)

found = false

for i from 1 to n, do
    if array[i] == x then
        found = true
        print("Element found at: ", i)
        break

if not found then
    print("Element not found")
```

EXERCICE 8

Evaluate the complexity based on the five types you have learned

```
print("Enter the value of the radius:")  
read(radius)
```

```
area = 3.14 * radius * radius
```

```
print("Area of the circle is: ", area)
```

EXERCICE 9

Evaluate the complexity based on the five types you have learned

```
print("Enter the value N:")
read(n)

maxValue = array[1]

for i from 2 to n, do
    if array[i] > maxValue then
        maxValue = array[i]

print("Maximum value is: ", maxValue)
```



You should know...



Understand the concept of **time complexity**



Understand **5 Big O Notations**

✓ $O(1)$

Constant time complexity

✓ $O(\log n)$

Logarithmic time complexity

✓ $O(n)$

Linear time complexity

✓ $O(n \log n)$

Log-linear time complexity

✓ $O(n^2)$

Quadratic time complexity



Calculate the time complexity in different use cases

FOR NEXT TIME !

1 – REVIEW THE THEORY

2 – FINALIZE THE EXERICES

3-2-1 Challenge

- ✓ List three things you **learned** today.
- ✓ List two **questions** you still have.
- ✓ List one aspect of the lesson or topic you **enjoyed**.

