

# MOS Device: Basics

## - MOSFET & Modeling -

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Appendix

October 11, 2010      Rev. 0.00

# Appendix

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# 1. List of symbols

(After Textbook of Size)

Symbol	Description	Unit
$a$	Lattice constant	m
$c$	Speed of light in vacuum	m/s
$C$	Capacitance	F
$E$	Energy	eV
$E_c$	Bottom of conduction band	eV
$E_F$	Fermi energy level	eV
$E_g$	Energy bandgap	eV
$E_v$	Top of valence band	eV
$F(E)$	Fermi-Dirack distribution function	
$I$	Current	A
$J$	Current density	A/m <sup>2</sup>
$k$	Boltzmann constant	J/K
$kT$	Thermal energy	eV
$L$	Length (channel length)	m
$m_0$	Electron rest mass	kg
$m^*$	Effective mass	kg
$n$	Density of free electron	m <sup>-3</sup>
$n_i$	Intrinsic density	m <sup>-3</sup>
$N$	Doping concentration	m <sup>-3</sup>
$N_A$	Acceptor impurity density	m <sup>-3</sup>
$N_C$	Effective density states in conduction band	m <sup>-3</sup>

# Continued...

Symbol	Description	Unit
ND	Donor impurity concentration	m-3
NV	Effective density of states in valence band	m-3
p	Density of free hole	m-3
q	Magnitude of electronic charge	C
R	Resistance	$\Omega$
t	time	s
T	Absolute temperature	K
v	Carrier velocity	m/s
vth	Thermal velocity	m/s
V	Voltage	V
Vbi	Built-in voltage	V
VFB	Flat band voltage	V
Vth	Threshold voltage	V
$\epsilon_0$	Permittivity in vacuum	F/m
$\epsilon_s$ or $\epsilon_{si}$	Semiconductor permittivity	F/m
$\epsilon_i$ or $\epsilon_{ox}$	Insulator permittivity	F/m
$\mu_n$	Electron mobility	m <sup>2</sup> /V.s
$\mu_p$	Hole mobility	m <sup>2</sup> /V.s
$\rho$	Resistivity	$\Omega$ -m
$\phi_m$	Metal work function	V
$\Omega$	Ohm	$\Omega$

## 2. Physical constant

Quantity	Symbol	Value
Elementary charge	q	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass	$m_0$	$0.911 \times 10^{-30} \text{ kg}$
Electron volt	eV	$1.60 \times 10^{-19} \text{ J}$
Permittivity in vacuum	$\epsilon_0$	$8.85 \times 10^{-12} \text{ F/m}$
Speed of light	c	$3.00 \times 10^8 \text{ m/s}$
Thermal voltage @300K	kT/q	0.0259 V

### 3. Properties of Ge, Si, GaAs and SiO<sub>2</sub>

	Ge	Si	GaAs	SiO <sub>2</sub>
Atomic or molecular weight	72.60	28.09	144.63	60.08
Atoms or molecules/cm <sup>3</sup>	$4.42 \times 10^{22}$	$5.00 \times 10^{22}$	$2.21 \times 10^{22}$	$2.3 \times 10^{22}$
Crystal structure	Diamond, 8 atoms/unit cell	Diamond, 8 atoms/unit cell	Zinc-blende, 8 atoms/unit cell	Random network of SiO <sub>4</sub> , tetrahedra, 50% covalent, 50% ionic bonding
Lattice constant (Å)	5.66	5.43	5.65	...
Density, $\rho$ (g/cm <sup>3</sup> )	5.32	2.33	5.32	2.27
Energy gap (eV)	0.67	1.11	1.40	~8
Effective density of states conduction band $N_c$ (cm <sup>-3</sup> ) valence band $N_v$ (cm <sup>-3</sup> )	$1.04 \times 10^{19}$ $6.0 \times 10^{18}$	$2.8 \times 10^{19}$ $1.04 \times 10^{19}$	$4.7 \times 10^{17}$ $7.0 \times 10^{18}$	...
Intrinsic carrier concentration $n_i$ (cm <sup>-3</sup> )	$2.4 \times 10^{13}$	$1.45 \times 10^{10}$	$\sim 9 \times 10^6$	...

(After Textbook of Grove)

## Continued .....

Lattice (intrinsic) mobilities (cm <sup>2</sup> /v sec) electrons holes	3900 1900	1350 480	8600 250	Insulator; $\rho > 10^{16} \Omega\text{-cm}$ at 300°K
Dielectric constant	16.3	11.7	12	3.9
Breakdown field (v/ $\mu$ )	~8	~30	~35	~600
Melting point (°C)	937	1415	1238	~1700
Vapor pressure (Torr)	10 <sup>-7</sup> at 880°C 10 <sup>-9</sup> at 750°C	10 <sup>-5</sup> at 1250°C 10 <sup>-7</sup> at 1050°C	1 at 1050°C 100 at 1220°C	10 <sup>-3</sup> at 1450°C 10 <sup>-1</sup> at 1700°C
Specific heat, C <sub>p</sub> (Joule/g°C)	0.31	0.7	0.35	1.0
Thermal conductivity, K <sub>th</sub> (watt/cm°C)	0.6	1.5	0.81	0.014
Thermal diffusivity $\kappa = \frac{K_{th}}{\rho C_p} \left( \frac{\text{cm}^2}{\text{sec}} \right)$	0.36	0.9	0.44	0.006
Linear coefficient of thermal expansion $\frac{\Delta L}{L \Delta T} \left( \frac{1}{^\circ\text{C}} \right)$	$5.8 \times 10^{-6}$	$2.5 \times 10^{-6}$	$5.9 \times 10^{-6}$	$0.5 \times 10^{-6}$

## 4. Key Physical Parameters of Silicon

Molecular Number ( /cm <sup>3</sup> )	5.00X10 <sup>22</sup>
Crystal Structure	Diamond
Lattice Constant a (nm)	0.543
Material Density (g/cm <sup>3</sup> )	2.33
Energy Gap (eV)	1.11
Density of State: Conduction Band N <sub>c</sub> (cm <sup>-3</sup> ) Valence Band N <sub>v</sub> (cm <sup>-3</sup> )	2.8X10 <sup>19</sup> 1.04X10 <sup>19</sup>
Intrinsic Carrier Density n <sub>i</sub> (cm <sup>-3</sup> )	1.45X10 <sup>10</sup>
Intrinsic Carrier Mobility: Elec. (cm <sup>2</sup> /V.sec) Hole (cm <sup>2</sup> /V.sec)	1350 480
Permittivity ε (si)	11.7
Thermal Conductance k <sub>th</sub> (W/cm.°C)	1.5
Thermal Expansion Constant ( /°C)	2.5X10 <sup>-6</sup>
Melting Temperature (°C)	1415



## 5. Equations of Semiconductor Physics

Charge neutrality	$\rho = q(P - n + N_D - N_A) = 0$
Equilibrium condition	$Pn = n_i^2$
Fermi-Dirac distribution function	$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$
Carrier concentrations in non-degenerate semiconductors: In the extrinsic case, $ N_D - N_A  \geq n_i$ :	$n = N_c e^{-(E_c - E_F)/kT} = n_i e^{(E_F - E_i)/kT}$ $P = N_v e^{-(E_F - E_v)/kT} = n_i e^{(E_i - E_F)/kT}$ $n_n \equiv N_D - N_A \quad P_p \equiv N_A - N_D$ $P_n \equiv \frac{n_i^2}{N_D - N_A} \quad n_p \equiv \frac{n_i^2}{N_A - N_D}$

(After Textbook of Grove)

## 6. Equations of PN-Diode

Built-in voltage	$\phi_B \cong 2 \frac{kT}{q} \ln \frac{C_B}{n_i}$
Depletion region width	$W = \sqrt{\frac{2K_S \epsilon_0 [\phi_B \pm  V_J ]}{qC_B}}$ where $\begin{matrix} +: \text{reverse} \\ -: \text{forward} \end{matrix} \}$ bias
Maximum electric field	$\epsilon_{\max} = 2 \frac{\phi_B \pm  V_J }{W}$
Capacitance per unit area	$C = \frac{K_S \epsilon_0}{W}$
Reverse current	$I_R = I_{\text{gen}} + I_{\text{diff}}$ $I_{\text{gen}} = \frac{1}{2} q \frac{n_i}{\tau} W A_J, I_{\text{diff}} = qD \frac{n_i^2}{C_B L} A_J$
Forward current	$I_F = I_{\text{rec}} + I_{\text{diff}}$ $I_{\text{rec}} = -\frac{1}{2} q \frac{n_i}{\tau} W e^{q V_F /2kT} A_J, I_{\text{diff}} = -qD \frac{n_i^2}{C_B L} e^{q V_F /kT} A_J$
Avalanche breakdown voltage	$BV = \frac{K_S \epsilon_0 E_{\text{crit}}^2}{2qC_B}$

(After Textbook of Grove)

## 7. Equations of MOS-Diode

Metal and semiconductor work-function difference ( $\phi_{MS}$ )	$\phi_{MS} = \chi_M - \left[ \chi_{Si} + E_g(Si) - \frac{kT}{q} \ln \left( \frac{N_C}{N_A} \right) \right]$
Flat-band Voltage ( $V_{FB}$ )	$V_{FB} = \phi_{MS} - \frac{Q_{SS}}{C_o}$
Physical Threshold Voltage ( $V_{th}$ )	$V_{th} = V_{FB} + 2\phi_F \frac{Q_B}{C_o}, \quad \phi_F = \frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right), \quad Q_B = \sqrt{2\epsilon_S q N_A (2\phi_F)}$
MOS Capacitance at Accumulation Condition	$C_{Acc} = C_o$
MOS Capacitance at Flat-Band Condition	$C_{FB} = C_o \frac{1}{1 + \frac{\epsilon_S/L_D}{\epsilon_{OX}/T_{OX}}}, \quad L_D = \sqrt{\frac{kT\epsilon_S}{q^2 N_A}}$
MOS Capacitance at Inversion Condition	$C_{Inv} = C_o \frac{1}{1 + \frac{\epsilon_S/W_m}{\epsilon_{OX}/T_{OX}}}, \quad W_m = \sqrt{\frac{4kT\epsilon_S \ln(N_A/n_i)}{q^2 N_A}}$

## 8. Equations of MOSFET (simple)

	Linear region	Saturation region
Current-voltage characteristics	$I_{ds} = \frac{W}{L} \mu_n C_o \left( V_g - V_{th} - \frac{V_d}{2} \right) V_d$	$I_{ds} = \frac{1}{2} \frac{W}{L} \mu_n C_o (V_g - V_{th})^2$
Saturation voltage	$V_{Dsat} = V_g - V_{th}$	—————
Drain conductance	$g_d = \frac{1}{2} \frac{W}{L} \mu_n C_o (V_g - V_{th} - V_d)$	$g_d = 0$
Mutual conductance	$g_m = \frac{W}{L} \mu_n C_o V_d$	$g_m = \frac{W}{L} \mu_n C_o (V_g - V_{th})$
Max. frequency	$f_o = \frac{g_m}{C_G} = \frac{\mu_n V_d}{L^2}$	$f_o = \frac{g_m}{C_G} = \frac{\mu_n (V_g - V_{th})}{L^2}$

## 9. Equations of MOSFET (First-order)

	n-channel $V_D > 0, V_G > V_T, I_D$ flows from source to drain.	n-channel $V_D > 0, V_G > V_T, I_D$ flows from drain to source.
Current-voltage characteristics	$I_D = \frac{Z}{L} \mu_n C_o \left\{ \left[ V_G - V_{FB} - 2\phi_{Fp} - \frac{V_D}{2} \right] V_D - \frac{2}{3} \frac{\sqrt{2K_s \epsilon_o q N_A}}{C_o} \left[ (V_D + 2\phi_{Fp})^{\frac{3}{2}} - (2\phi_{Fp})^{\frac{3}{2}} \right] \right\}$	$I_D = \frac{Z}{L} \mu_n C_o \left\{ \left[ V_G - V_{FB} - 2\phi_{Fn} - \frac{V_D}{2} \right] V_D - \frac{2}{3} \frac{\sqrt{2K_s \epsilon_o q N_D}}{C_o} \left[  V_D + 2\phi_{Fn} ^{\frac{3}{2}} -  2\phi_{Fn} ^{\frac{3}{2}} \right] \right\}$
Saturation voltage	$V_{Dsat} = V_G - V_{FB} - 2\phi_{Fp} + \frac{K_s \epsilon_o q N_A}{C_o^2} \left[ 1 - \sqrt{1 + \frac{2C_o^2 (V_G - V_{FB})}{K_s \epsilon_o q N_A}} \right]$	$V_{Dsat} = V_G - V_{FB} - 2\phi_{Fn} + \frac{K_s \epsilon_o q N_D}{C_o^2} \left[ 1 - \sqrt{1 - \frac{2C_o^2 (V_G - V_{FB})}{K_s \epsilon_o q N_D}} \right]$
Turn-on voltage	$V_T = V_{FB} + 2\phi_{Fp} + \frac{\sqrt{2K_s \epsilon_o q N_A (2\phi_{Fp})}}{C_o}$	$V_T = V_{FB} + 2\phi_{Fn} + \frac{\sqrt{2K_s \epsilon_o q N_D  2\phi_{Fn} }}{C_o}$
Conductance (linear area)	$g = \frac{Z}{L} \mu_n C_o (V_G - V_T)$	$g = \frac{Z}{L} \mu_p C_o (V_G - V_T)$
Mutual conductance	$g_m = \frac{Z}{L} \mu_n C_o V_D \quad \text{When } V_D \leq V_{Dsat}$	$g_m = \frac{Z}{L} \mu_p C_o V_D \quad \text{When }  V_D  \leq  V_{Dsat} $
Max. frequency	$f_o = \frac{g_m}{C_G} \leq \frac{\mu_n V_D}{L^2} \quad \text{When } V_D \leq V_{Dsat}$	$f_o = \frac{g_m}{C_G} \leq \frac{\mu_p  V_D }{L^2} \quad \text{When }  V_D  \leq  V_{Dsat} $
Series resistance effect	<p>Linear area <math display="block">g(\text{obs}) = \frac{g}{1 + (R_s + R_d)g}</math></p> <p>Saturated area <math display="block">g_{msat}(\text{obs}) = \frac{g_{msat}}{1 + R_s g_{msat}}</math></p>	

(After Textbook of Grove)

## 10. High-K Materials

Material	$\kappa$	$E_g$ [eV]	$\Delta E_c$ [eV]	Stable on Si	Crystal structure
SiO <sub>2</sub>	3.9	9	3.1	Yes	Amorphous
Si <sub>3</sub> N <sub>4</sub>	7.8	5.3	2.4	Yes	Amorphous
Y <sub>2</sub> O <sub>3</sub>	15	6	2.3	Yes	Cubic
La <sub>2</sub> O <sub>3</sub>	30	6	2.3	Yes	Hexagonal, cubic
Pr <sub>2</sub> O <sub>3</sub>	31			Yes	
Dy <sub>2</sub> O <sub>3</sub>	14	4.8		Yes	
Ta <sub>2</sub> O <sub>5</sub>	26	4.4	0.3	No	Orthorhombic
TiO <sub>2</sub>	80	3.05	0.05	No	Tetragonal
HfO <sub>2</sub>	25	~6	1.5	Yes	Monoclinic, tetragonal, cubic
ZrO <sub>2</sub>	25	5.8	1.4	Yes	Monoclinic, tetragonal, cubic
HfSi <sub>x</sub> O <sub>y</sub>	~10	~6	1.5	Yes	Amorphous
ZrSi <sub>x</sub> O <sub>y</sub>	~10	~6.5	1.5	Yes	Amorphous
Al <sub>2</sub> O <sub>3</sub>	9	8.8	2.8	Yes	Amorphous



FIN !

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