

FORECASTING FINANCIAL INSTRUMENTS PRICES WITH VECM AND ARIMA MODELS

Home Taken Project 1

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TSA_Report

[1] "D:/UW/2. Summer 22-23/1. Time Series Analysis/Project/TSA"

1. Data Preparation

Load necessary packages to memory and some pre-defined functions::

Loading required package: zoo

Attaching package: 'zoo'

The following objects are masked from 'package:base':

as.Date, as.Date.numeric

Warning: package 'tidyverse' was built under R version 4.2.3

— Attaching core tidyverse packages — tidyverse 2.0.0 —

✓ dplyr	1.1.0	✓ readr	2.1.4
✓ forcats	1.0.0	✓ stringr	1.5.0
✓ ggplot2	3.4.1	✓ tibble	3.1.8
✓ lubridate	1.9.2	✓ tidyr	1.3.0
✓ purrr	1.0.1		

— Conflicts — tidyverse_conflicts() —

✗ dplyr::filter() masks stats::filter()
 ✗ dplyr::first() masks xts::first()
 ✗ dplyr::lag() masks stats::lag()
 ✗ dplyr::last() masks xts::last()

ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors

Loading required package: TTR

Registered S3 method overwritten by 'quantmod':

method	from
as.zoo.data.frame	zoo

Warning: package 'forecast' was built under R version 4.2.3

Warning: package 'vars' was built under R version 4.2.3

Loading required package: MASS

Attaching package: 'MASS'

The following object is masked from 'package:dplyr':

select

Loading required package: strucchange

Warning: package 'strucchange' was built under R version 4.2.3

Loading required package: sandwich

Attaching package: 'strucchange'

The following object is masked from 'package:stringr':

boundary

Loading required package: urca

Attaching package: 'kableExtra'

The following object is masked from 'package:dplyr':

group_rows

Load Dataset:

```
Data <- read.csv("TSA_2023_project_data_1.csv",header = TRUE, dec = ".")
```

2. Summary Data

```
str(Data)
```

'data.frame': 970 obs. of 11 variables:

\$ X : chr "11-04-18" "12-04-18" "13-04-18" "14-04-18" ...

\$ x1 : num 105 107 108 108 106 ...

\$ x2 : num 110 110 109 108 108 ...

\$ x3 : num 127 128 129 129 127 ...

\$ x4 : num 117 115 117 117 117 ...

\$ x5 : num 114 116 115 115 116 ...

\$ x6 : num 147 147 146 147 148 ...

\$ x7 : num 155 154 153 155 153 ...

```
$ x8 : num 189 191 188 189 190 ...
$ x9 : num 90.3 89.9 90.4 91 89.5 ...
$ x10: num 167 166 167 166 165 ...
```

Notice that the first column is the Date column, which is under “Character” type We need to transform it into “Date” type.

```
Data$X <- as.Date(Data$X,
                  format = "%d-%m-%y")
```

Until now the class is “Data.Frame” object

```
class(Data)
```

```
[1] "data.frame"
```

Create xts objects

```
Data <- xts(Data[, -1], Data$X)
```

After creating xts objects, the class is “xts”, “zoo”

```
class(Data)
```

```
[1] "xts" "zoo"
```

```
str(Data)
```

An 'xts' object on 2018-04-11/2020-12-05 containing:

```
Data: num [1:970, 1:10] 105 107 108 108 106 ...
- attr(*, "dimnames")=List of 2
..$ : NULL
..$ : chr [1:10] "x1" "x2" "x3" "x4" ...
Indexed by objects of class: [Date] TZ: UTC
xts Attributes:
NULL
```

Checking for missing data

```
summary(Data)
```

Index	x1	x2	x3
Min. :2018-04-11	Min. : 88.29	Min. : 85.64	Min. :124.2
1st Qu.:2018-12-09	1st Qu.: 95.46	1st Qu.: 92.81	1st Qu.:135.5
Median :2019-08-08	Median : 99.20	Median : 97.35	Median :141.3
Mean :2019-08-08	Mean :100.06	Mean : 98.43	Mean :139.9
3rd Qu.:2020-04-06	3rd Qu.:105.11	3rd Qu.:104.99	3rd Qu.:144.4
Max. :2020-12-05	Max. :111.07	Max. :109.92	Max. :150.4

x4		x5		x6		x7	
Min.	: 85.19	Min.	:104.8	Min.	:142.5	Min.	:139.5
1st Qu.	:108.19	1st Qu.	:111.1	1st Qu.	:152.1	1st Qu.	:145.8
Median	:115.68	Median	:113.0	Median	:155.5	Median	:149.3
Mean	:114.87	Mean	:112.7	Mean	:154.8	Mean	:149.5
3rd Qu.	:122.69	3rd Qu.	:114.6	3rd Qu.	:157.9	3rd Qu.	:153.9
Max.	:141.62	Max.	:119.2	Max.	:165.0	Max.	:158.0

x8		x9		x10	
Min.	:179.5	Min.	: 10.70	Min.	:151.5
1st Qu.	:189.2	1st Qu.	: 29.15	1st Qu.	:156.5
Median	:193.9	Median	: 40.96	Median	:158.8
Mean	:194.8	Mean	: 45.37	Mean	:159.4
3rd Qu.	:199.1	3rd Qu.	: 60.91	3rd Qu.	:162.2
Max.	:219.3	Max.	:101.21	Max.	:170.3

```
head(Data,6)
```

	x1	x2	x3	x4	x5	x6	x7
2018-04-11	105.3882	109.6174	127.0583	116.7108	114.2815	147.1509	155.0660
2018-04-12	107.1097	109.9211	128.0893	114.7180	115.5800	146.9616	154.3278
2018-04-13	108.2144	109.4070	129.4782	116.6820	114.9658	146.3560	152.8969
2018-04-14	108.0136	108.3335	128.7952	116.8109	114.5075	147.4023	154.6096
2018-04-15	106.4532	108.4117	127.0486	117.2085	116.3365	147.7198	153.1151
2018-04-16	107.6660	108.6879	127.1959	119.2785	113.8498	146.4102	153.6959

	x8	x9	x10
2018-04-11	189.3256	90.25521	167.4961
2018-04-12	190.7042	89.94702	166.1565
2018-04-13	188.0949	90.42754	166.6610
2018-04-14	188.9110	91.02123	166.1164
2018-04-15	190.1822	89.50997	164.7597
2018-04-16	188.1136	86.33956	165.1651

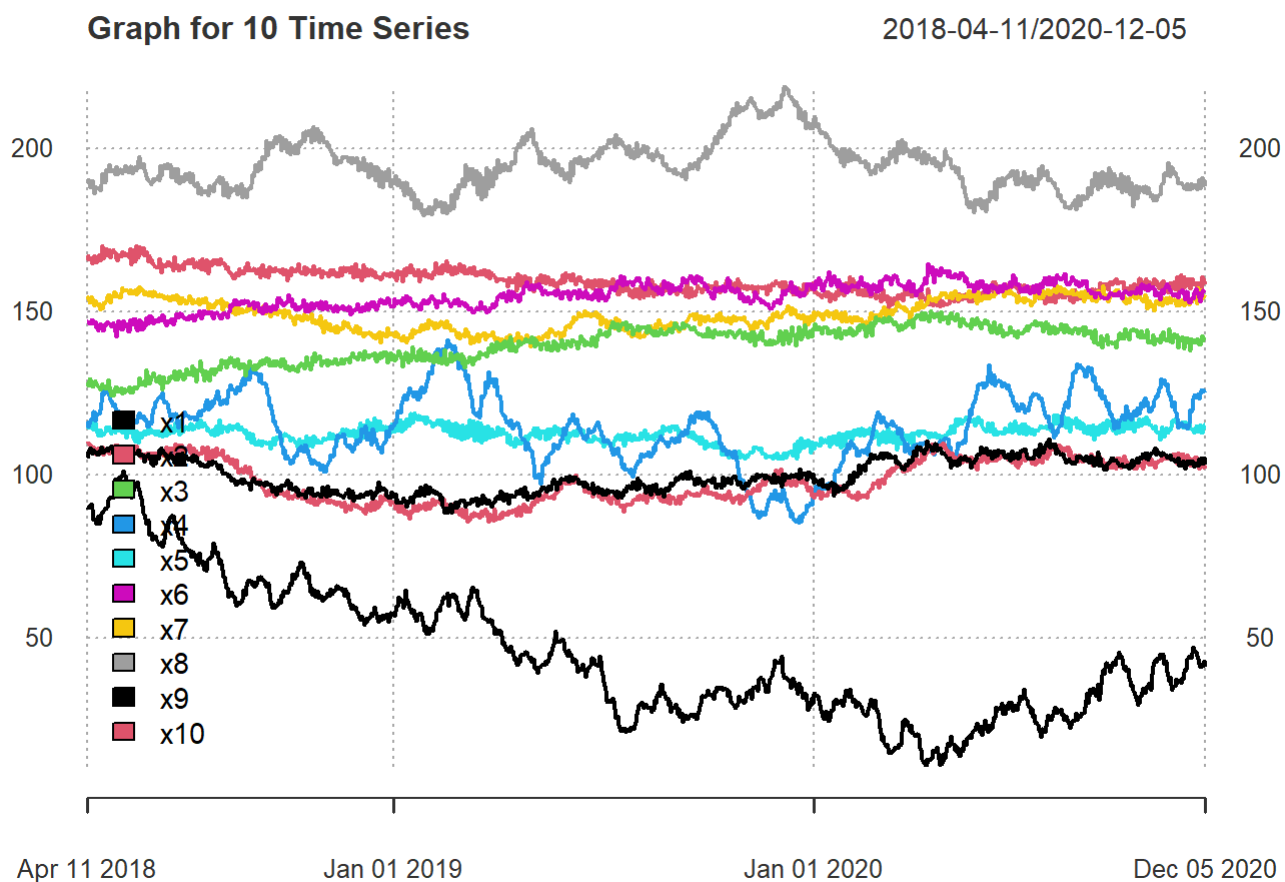
There is no missing data for the table.

3. Checking for Cointegration

Visualization

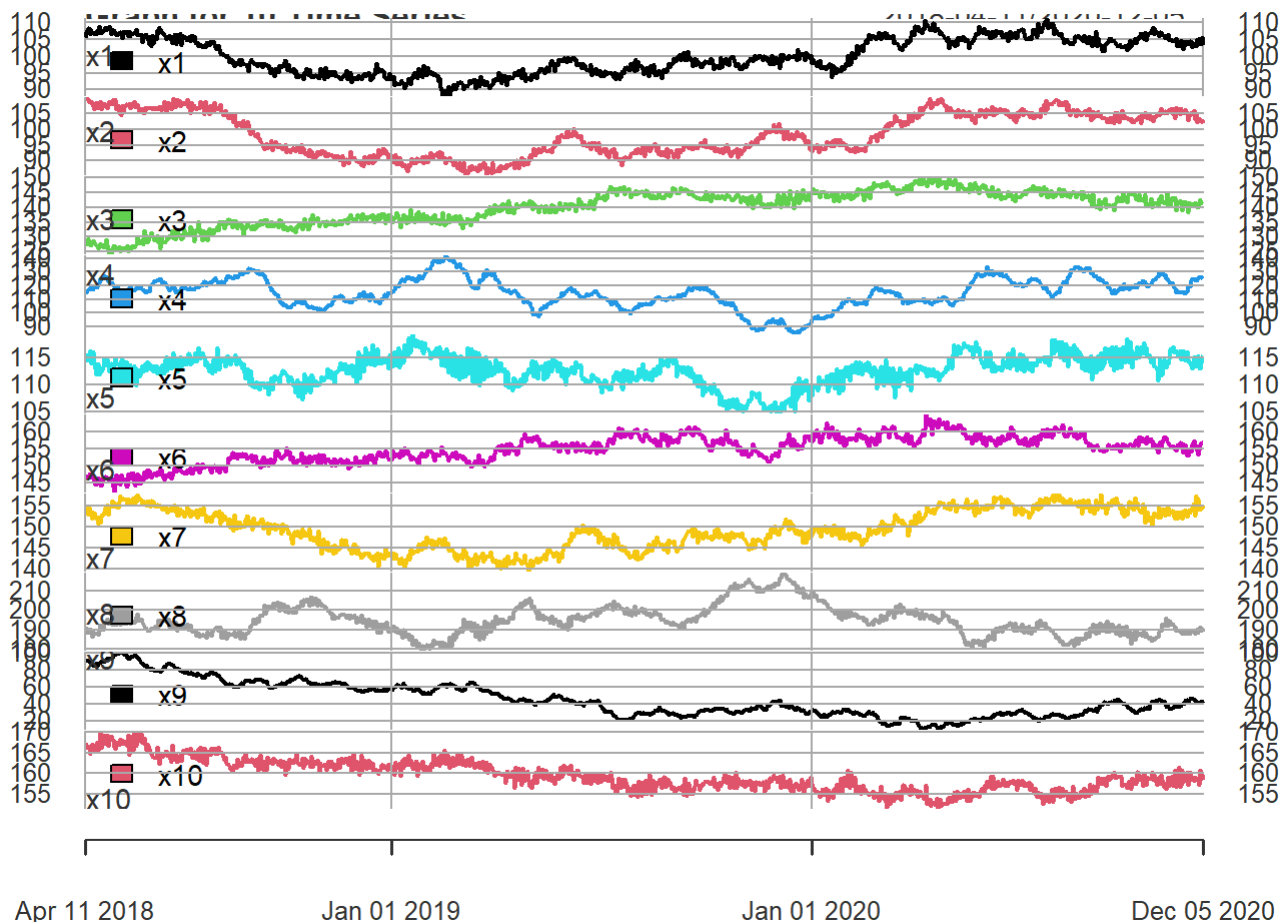
Plot 10 Time Series together

```
plot(Data,
      main = "Graph for 10 Time Series",
      major.ticks = "years",
      grid.ticks.on = "years",
      grid.ticks.lty = 3,
      legend.loc = "bottomleft",
      type="l")
```



Plot 10 Time Series separately

```
plot(Data,  
      main = "Graph for 10 Time Series",  
      major.ticks = "years",  
      grid.ticks.on = "years",  
      legend.loc = "bottomleft",  
      multi.panel = TRUE,  
      yaxis.same = FALSE,  
      type="l")
```



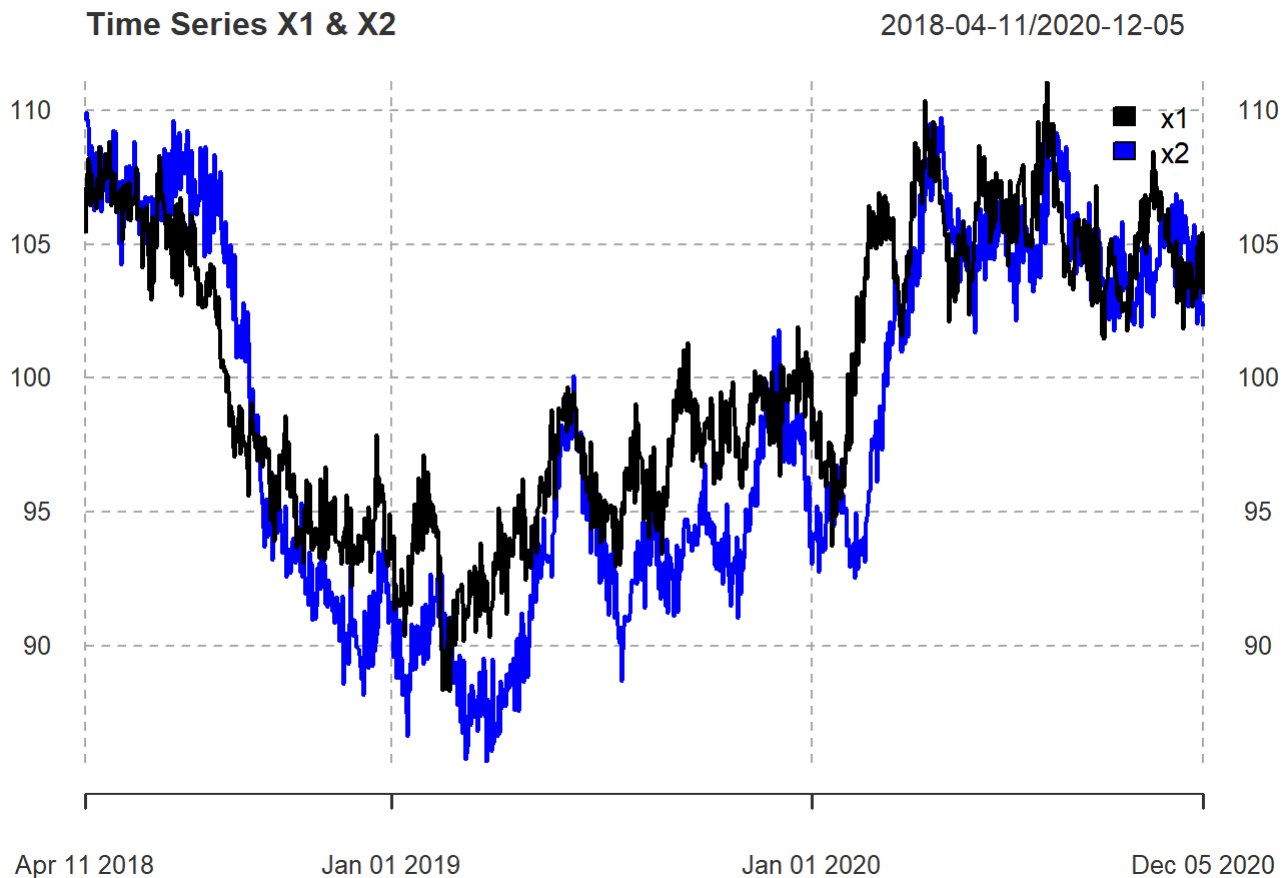
So our group decide to choose Time Series 1 and 2 for analysis.

Create first difference

```
Data$dx1 <- diff.xts(Data$x1)
Data$dx2 <- diff.xts(Data$x2)
```

Plot both variables on the graph:

```
plot(Data[, 1:2],
     col = c("black", "blue"),
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 2,
     main = "Time Series X1 & X2",
     legend.loc = "topright")
```



Test cointegration

We perform the tests of integration order.

Testing the order of the time series 1 and its difference:

```
testdf(variable = Data$x1,
        max.augmentations = 3)
```

Loading required package: fUnitRoots

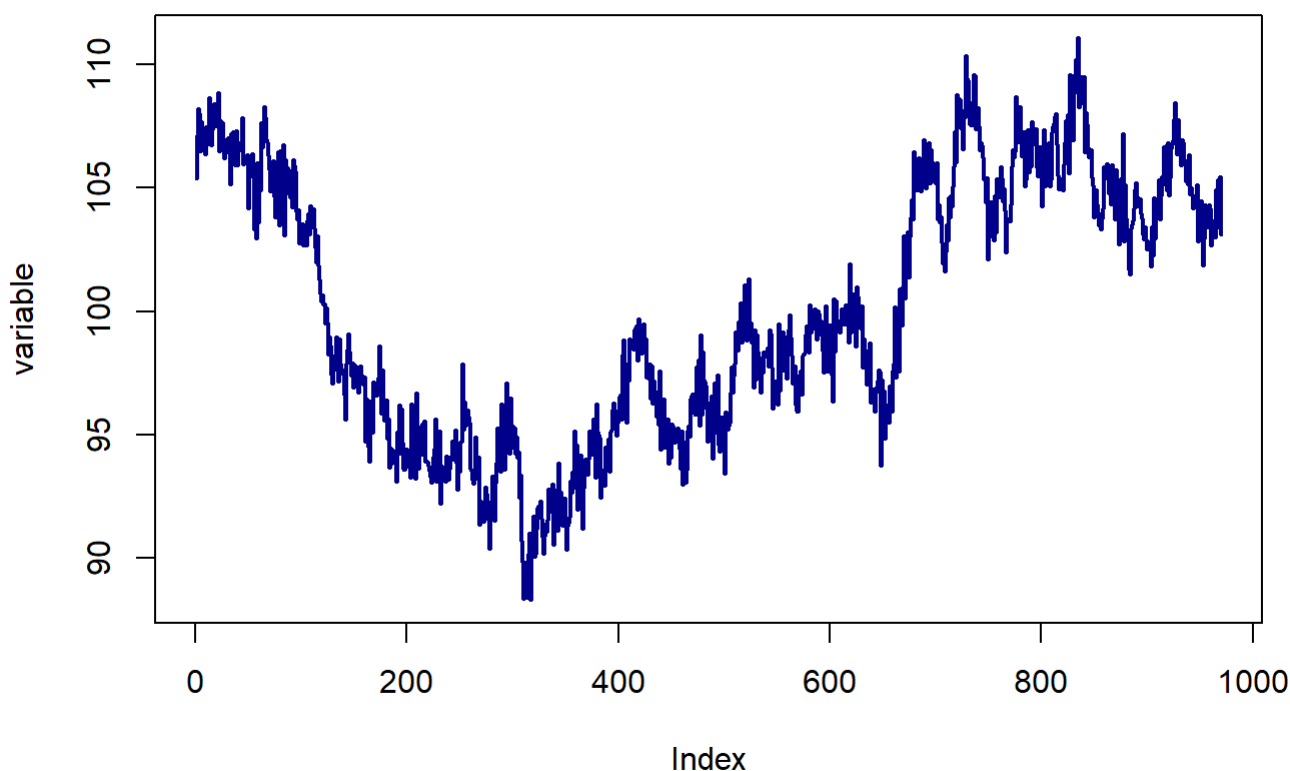
Attaching package: 'fUnitRoots'

The following objects are masked from 'package:urca':

```
punitroot, qunitroot, unitrootTable
```

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Plot of the examined variable



	augmentations	adf	p_adf	bgodfrey	p_bg
1	0	-3.803259	0.0100000	264.5832974	1.719886e-59
2	1	-2.157827	0.2540868	2.0201728	1.552215e-01
3	2	-2.081606	0.2825813	0.3613083	5.477805e-01
4	3	-1.745467	0.4082435	0.9416019	3.318662e-01

Interpret result: The p-bg is very small -> there are auto-correlation in residuals, even when we add the augmentations, we still cant get rid of auto-correlation. Hence, we test for its 1st difference.

```
testdf(variable = Data$dx1,
        max.augmentations = 3)
```

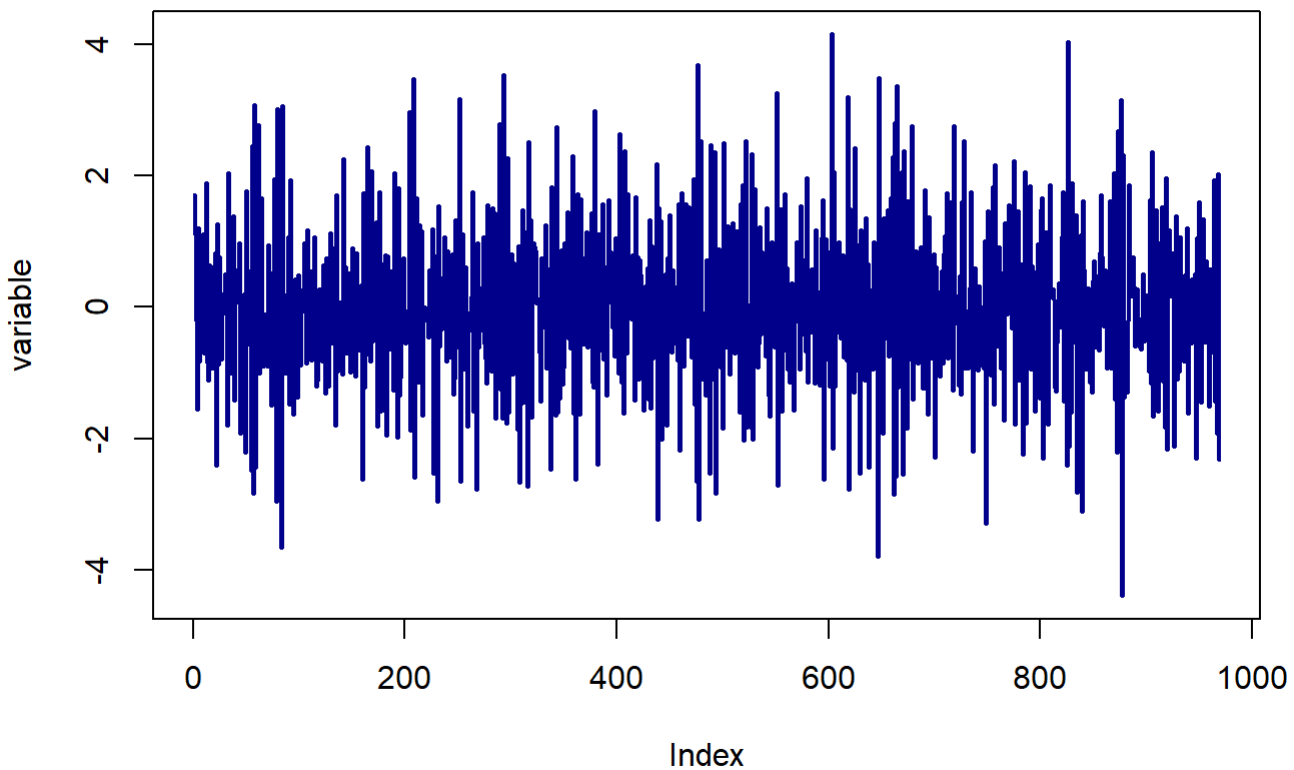
Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Plot of the examined variable



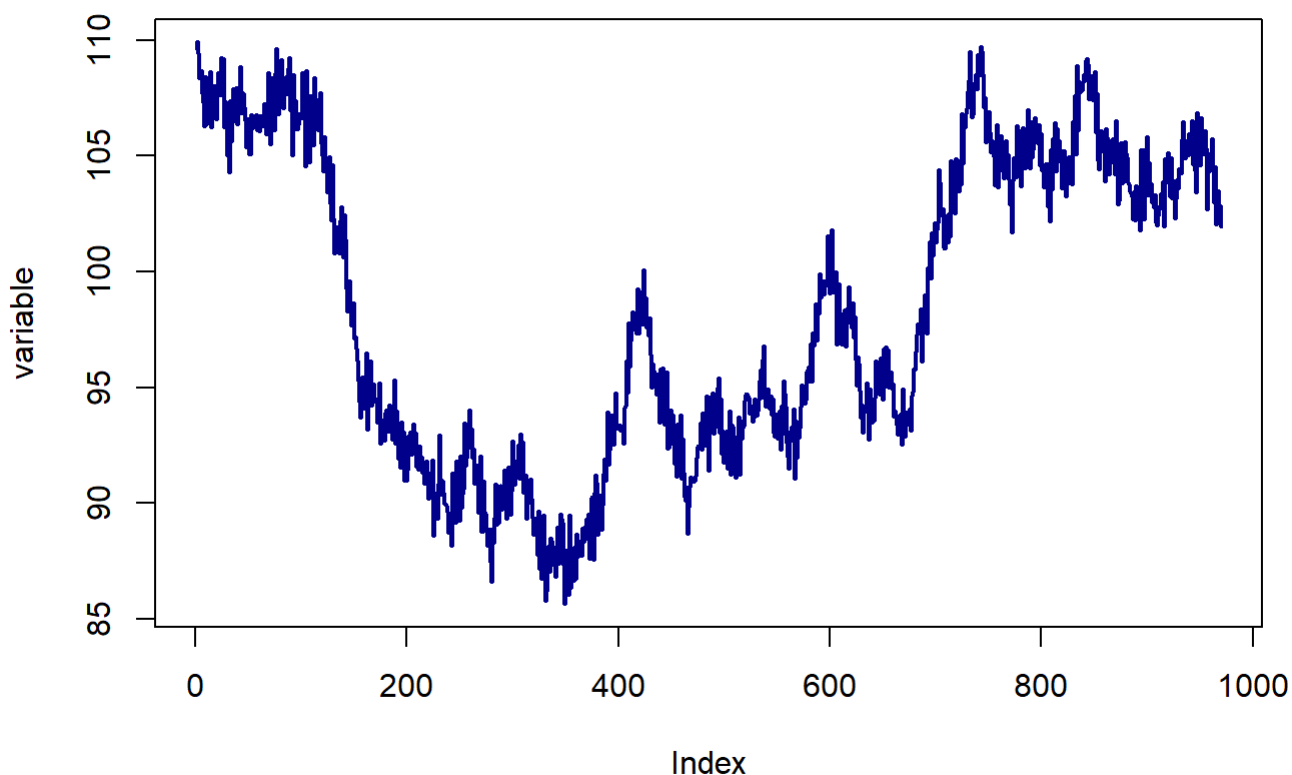
	augmentations	adf	p_adf	bgodfrey	p_bg
1	0	-57.26435	0.01	2.2435617	0.1341716
2	1	-29.78461	0.01	0.3975488	0.5283579
3	2	-27.06769	0.01	0.9268905	0.3356722
4	3	-17.77244	0.01	0.1955419	0.6583436

The p-bg is greater than 5% -> there are no auto-correlations in residual. Next, we test the stationarity of its 1st lag, by checking p-adf. p-adf is smaller than 5%. We can reject the null in the case of the first differences about non-stationary. Its 1st lag is stationary. We can conclude that the Time Series X1 is integrated of order 1.

Testing the order of the time series 2 and its difference:

```
testdf(variable = Data$x2,
        max.augmentations = 3)
```

Plot of the examined variable



	augmentations	adf	p_adf	bgodfrey	p_bg
1	0	-3.303984	0.01657912	264.339658	1.943586e-59
2	1	-2.084780	0.28139458	7.143669	7.522910e-03
3	2	-1.876867	0.35912092	2.241790	1.343254e-01
4	3	-1.578488	0.47066701	2.237972	1.346575e-01

Interpret result: Similar to X1, the p-bg here is smaller than 5% -> there are auto-correlation in residuals, even when we add the augmentations, we still cant get rid of auto-correlation. Hence, we test for its 1st difference.

```
testadf(variable = Data$dx2,
        max.augmentations = 3)
```

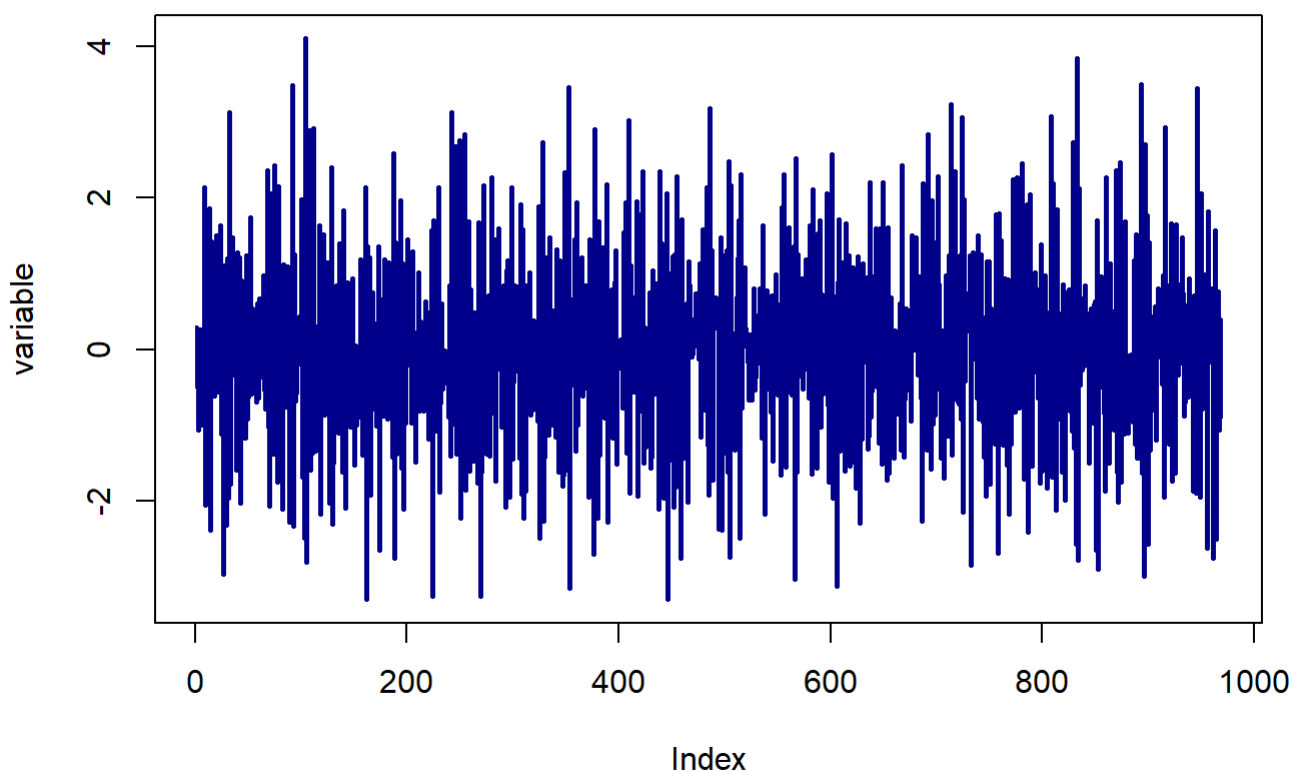
Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Plot of the examined variable



	augmentations	adf	p_adf	bgodfrey	p_bg
1	0	-56.64415	0.01	7.4813480	0.006234138
2	1	-32.10792	0.01	2.3416031	0.125959872
3	2	-30.35197	0.01	2.2137194	0.136788523
4	3	-18.39947	0.01	0.3831258	0.535935076

By adding 1 augmentation, there will be no auto correlation in Residuals. p-adf is greater than 5%, we can reject the null hypothesis in the case the non-stationary of first differences. Its 1st lag is stationary. we can conclude that the Time Series X2 is integrated of order 1.

Conclusion: As a result, both variables has the same order 1: $\sim I(1)$ so in the next step we can check whether they are cointegrated or not.

Estimate Cointegrated Vector [🔗](#)

Granger Causality Test (H_0 : X does not granger cause Y H_1 : X granger causes Y)

```
grangertest(x1 ~ x2,
            data = Data,
            order = 3) # lag assumed
```

Granger causality test

Model 1: $x_1 \sim \text{Lags}(x_1, 1:3) + \text{Lags}(x_2, 1:3)$

Model 2: $x_1 \sim \text{Lags}(x_1, 1:3)$

	Res.Df	Df	F	Pr(>F)
1	960			
2	963	-3	0.2821	0.8383

p-value is big, we can't reject H_0 that X_2 does not granger cause X_1 .

```
grangertest(x2 ~ x1,
            data = Data,
            order = 3) # lag assumed
```

Granger causality test

Model 1: $x_2 \sim \text{Lags}(x_2, 1:3) + \text{Lags}(x_1, 1:3)$

Model 2: $x_2 \sim \text{Lags}(x_2, 1:3)$

	Res.Df	Df	F	Pr(>F)
1	960			
2	963	-3	28.459	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

p-value is big, we reject H_0 , and conclude that X_1 does granger cause X_2 .

Conclusion: At 5% significance level (or 95% confidence level) we have so called one-directional feedback, X_1 does granger cause X_2 .

```
model.coint <- lm(x2 ~ x1, data = Data)
```

Examine the model summary:

```
summary(model.coint)
```

Call:

```
lm(formula = x2 ~ x1, data = Data)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-9.9825	-1.7073	0.0595	1.7705	9.6099

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-16.12716	1.74749	-9.229	<2e-16 ***
x1	1.14496	0.01744	65.648	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.867 on 968 degrees of freedom

Multiple R-squared: 0.8166, Adjusted R-squared: 0.8164

F-statistic: 4310 on 1 and 968 DF, p-value: < 2.2e-16

Both the intercept and Coefficient for x1 are statistically significant.

The model is significantly explained by x1. We further test the stationary of the residuals:

```
testdf(variable = residuals(model.coint), max.augmentations = 3)
```

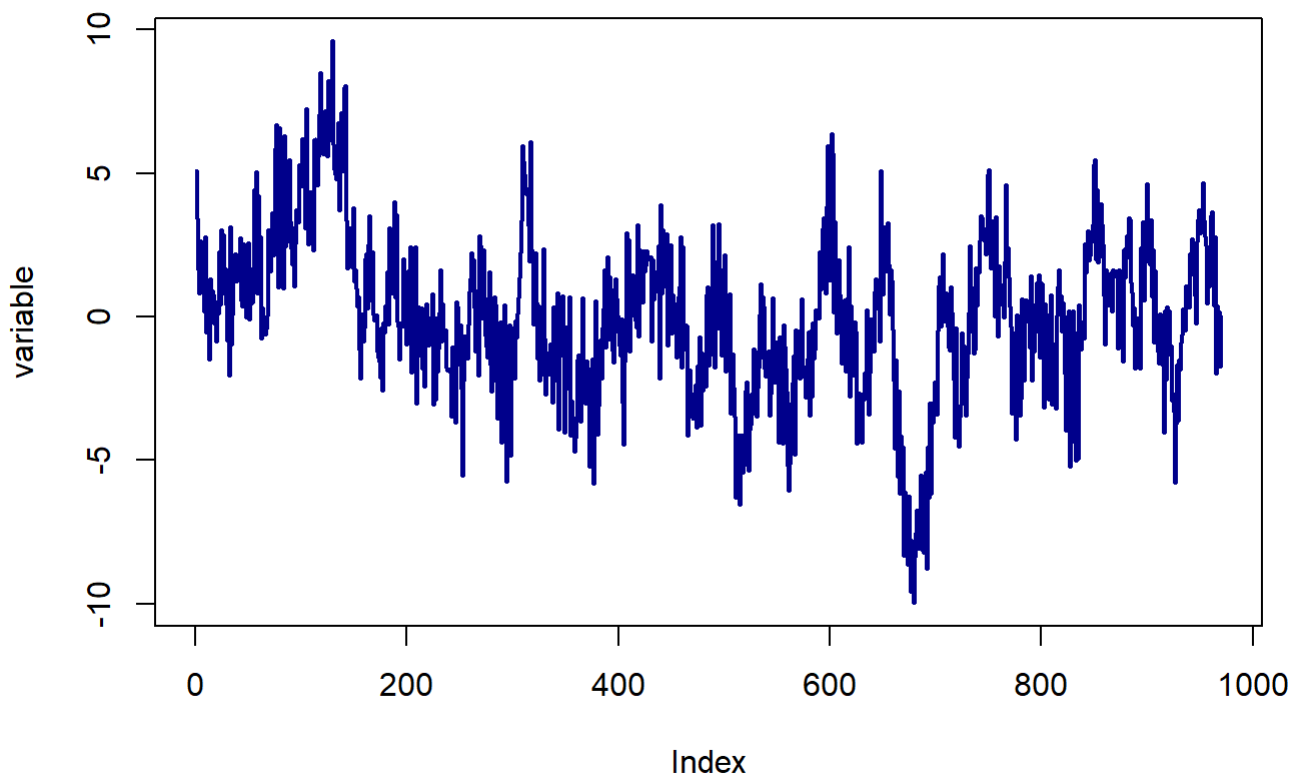
Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Warning in adfTest(variable, lags = augmentations, type = "c"): p-value smaller than printed p-value

Plot of the examined variable



	augmentations	adf	p_adf	bgodfrey	p_bg
1	0	-11.717909	0.01	119.670587	7.468827e-28
2	1	-6.658428	0.01	3.737762	5.319564e-02

```
3          2 -5.656884  0.01  1.945556 1.630666e-01
4          3 -3.962972  0.01  2.230209 1.353355e-01
```

The ADF test with no augmentations can be used. The result is that non-stationarity of residuals is **STRONGLY REJECTED**, so residuals are stationary, which means that x_1 and x_2 are cointegrated.

The cointegrating vector is $[1, 16.127, -1.145]$

which defines the cointegrating relationship as: $1 * x_2 + 16.127 - 1.145 * x_1$.

Create first lags of residuals and adding them to the dataset

```
Data$lresid <- lag.xts(residuals(model.coint))
```

Estimating ECM Model As the intercept is insignificant, we can remove it.

```
model.ecm <- lm(dx2 ~ dx1 + lresid -1, data = Data)
summary(model.ecm)
```

Call:

```
lm(formula = dx2 ~ dx1 + lresid - 1, data = Data)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-3.2494 -0.9248 -0.0335  0.8423  4.5604
```

Coefficients:

```
      Estimate Std. Error t value Pr(>|t|)
dx1      0.02133     0.03276   0.651   0.515
lresid -0.14351     0.01448  -9.911 <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 1.263 on 967 degrees of freedom

(1 observation deleted due to missingness)

Multiple R-squared: 0.09399, Adjusted R-squared: 0.09212

F-statistic: 50.16 on 2 and 967 DF, p-value: < 2.2e-16

The parameter **0.021** describes a **short term** relationship between x_1 and x_2 .

The parameter **1.145** describes a **long term** relationship between x_1 and x_2 .

The value of **-0.144** is the estimate of the adjustment coefficient. Its sign is **negative** and this value means that 0.139 of the unexpected error (increase in gap) will be corrected in the next period, so any unexpected deviation should be corrected finally on average within about **6.9 periods**.

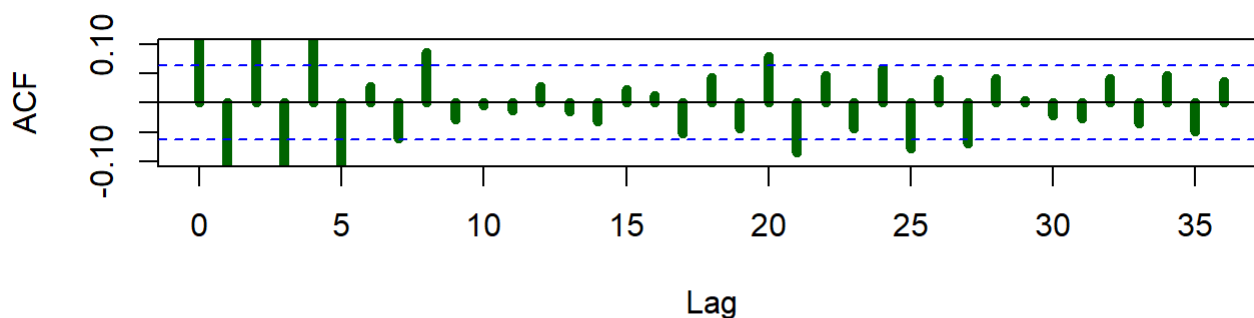
4. Applying Box-Jenkins Procedure

4.1 Applying for Time Series X1

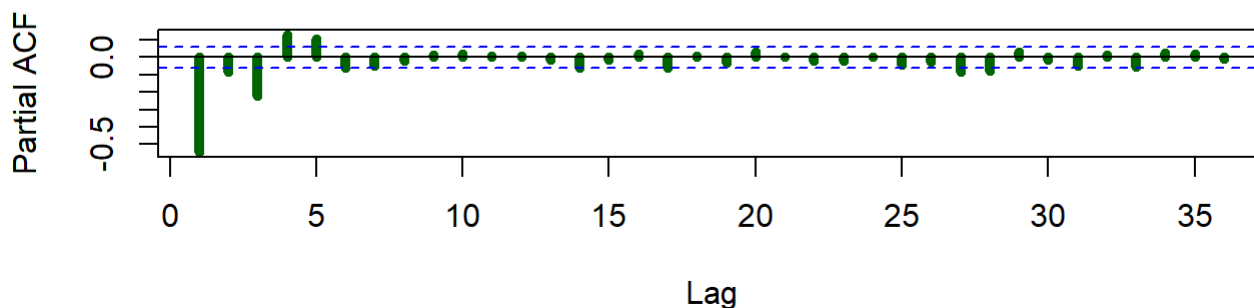
Step 1: Model Parameters

```
par(mfrow = c(2, 1))
acf(Data$dx1,
    lag.max = 36, # max lag for ACF
    ylim = c(-0.1, 0.1), # limits for the y axis - we give c(min, max)
    lwd = 5,
    col = "dark green",
    na.action = na.pass) # do not stop if there are missing values in the data
pacf(Data$dx1,
    lag.max = 36,
    lwd = 5, col = "dark green",
    na.action = na.pass)
```

Series Data\$dx1



Series Data\$dx1



```
par(mfrow = c(1, 1)) # restore the original single panel
```

The PACF shown is suggestive of an AR(5) model or AR(7). So an initial candidate model is an ARIMA(5,1,0). We also have some variations of this model: ARIMA(5,1,1), ARIMA(4,1,0), ARIMA(3,1,0), ARIMA(7,1,0).

Step 2: Model Estimation

```
arima510 <- Arima(Data$x1,
                  order = c(5, 1, 0)
)
```

```
arima511 <- Arima(Data$x1,
                  order = c(5, 1, 1)
)
```

```
arima410 <- Arima(Data$x1,
                  order = c(4, 1, 0)
)
```

```
arima310 <- Arima(Data$x1,
                  order = c(3, 1, 0)
)
```

```
arima313 <- Arima(Data$x1,
                  order = c(3, 1, 3)
)
```

```
arima710 <- Arima(Data$x1,
                  order = c(7, 1, 0),
)
```

The above model is not included constant. Let's include constant into the model:

```
arima510_2 <- Arima(Data$x1,
                   order = c(5, 1, 0),
                   include.constant = TRUE) # including a constant
```

```
coeftest(arima510_2)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
ar1	-0.5962591	0.0320094	-18.6276	< 2.2e-16	***
ar2	-0.1745463	0.0367977	-4.7434	2.102e-06	***
ar3	-0.1253417	0.0370070	-3.3870	0.0007067	***
ar4	0.1966395	0.0367945	5.3443	9.078e-08	***
ar5	0.1079200	0.0320471	3.3675	0.0007584	***
drift	-0.0023145	0.0204875	-0.1130	0.9100535	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Adding the constant did not change the result much for ALL MODELS, so we keep the model without constant.

Summary All The Models:

```
coeftest(arima510)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
ar1	-0.596241	0.032009	-18.6272	< 2.2e-16	***
ar2	-0.174508	0.036796	-4.7426	2.110e-06	***
ar3	-0.125306	0.037006	-3.3861	0.0007089	***
ar4	0.196671	0.036794	5.3453	9.028e-08	***
ar5	0.107940	0.032047	3.3682	0.0007566	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All the terms/coeffs are significant.

```
coeftest(arima511)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
ar1	-0.852356	0.205187	-4.1540	3.267e-05	***
ar2	-0.322847	0.126394	-2.5543	0.0106402	*
ar3	-0.173252	0.058333	-2.9700	0.0029776	**
ar4	0.161202	0.053380	3.0199	0.0025288	**
ar5	0.145414	0.038198	3.8068	0.0001408	***
ma1	0.259179	0.206188	1.2570	0.2087515	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All the terms/coeffs are significant, except MA1.

```
coeftest(arima410)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
ar1	-0.581875	0.031911	-18.2342	< 2.2e-16	***
ar2	-0.189843	0.036733	-5.1681	2.364e-07	***
ar3	-0.145704	0.036726	-3.9673	7.270e-05	***

```
ar4 0.134156 0.031959 4.1978 2.695e-05 ***
```

```
---
```

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All the terms/coeffs are significant.

```
coeftest(arima310)
```

z test of coefficients:

```
      Estimate Std. Error  z value  Pr(>|z|)
ar1 -0.611857   0.031386 -19.4947 < 2.2e-16 ***
ar2 -0.219115   0.036403  -6.0192 1.753e-09 ***
ar3 -0.227520   0.031410  -7.2436 4.369e-13 ***
```

```
---
```

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All the terms/coeffs are significant.

```
coeftest(arima313)
```

z test of coefficients:

```
      Estimate Std. Error  z value  Pr(>|z|)
ar1 -0.709499   0.110391  -6.4272 1.300e-10 ***
ar2 -0.657175   0.122726  -5.3548 8.564e-08 ***
ar3 -0.530463   0.044971 -11.7957 < 2.2e-16 ***
ma1  0.112950   0.117834   0.9586  0.33778
ma2  0.417434   0.090148   4.6306 3.647e-06 ***
ma3  0.131636   0.074994   1.7553  0.07921 .
```

```
---
```

```
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All the terms/coeffs are significant, except MA1, MA3.

```
coeftest(arima710)
```

z test of coefficients:

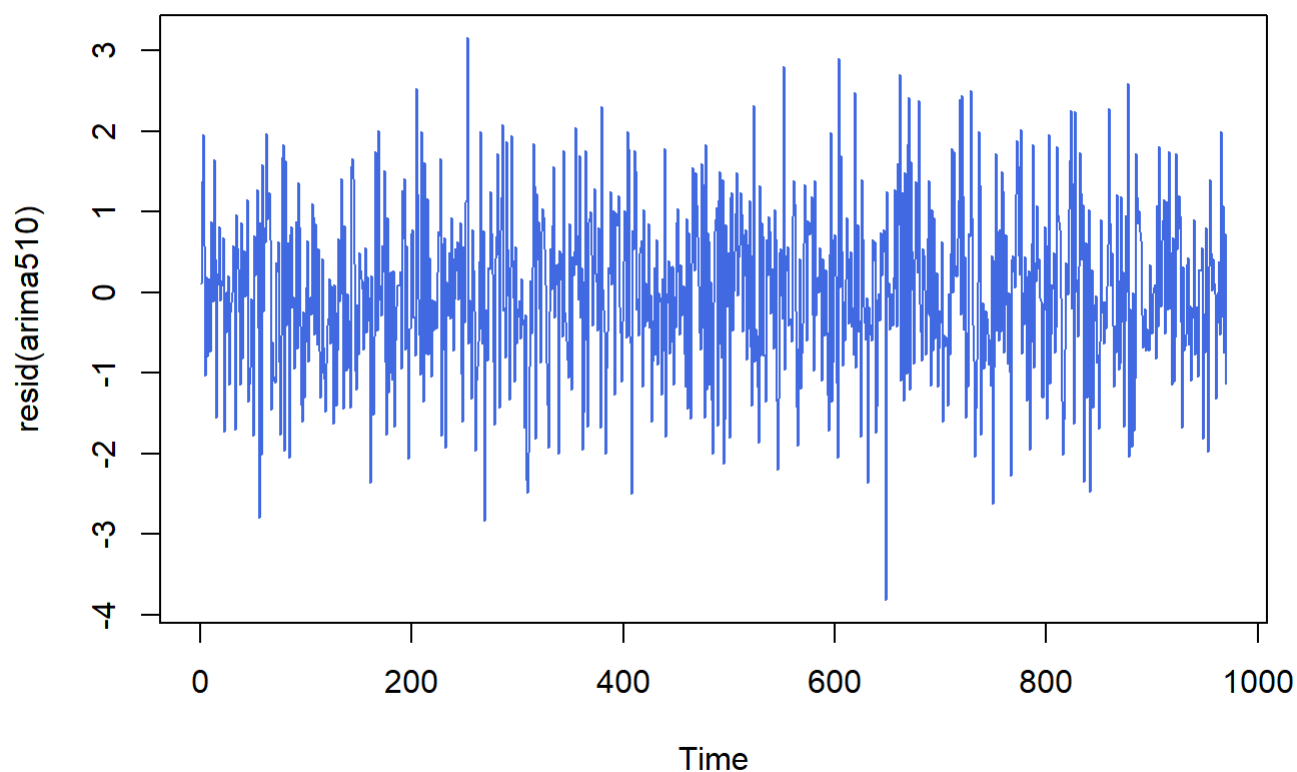
```
      Estimate Std. Error  z value  Pr(>|z|)
ar1 -0.592921   0.032162 -18.4354 < 2.2e-16 ***
ar2 -0.159731   0.037322  -4.2798 1.871e-05 ***
ar3 -0.122414   0.037605  -3.2553 0.001133 **
ar4  0.180564   0.037351   4.8342 1.337e-06 ***
ar5  0.067051   0.037594   1.7836 0.074497 .
ar6 -0.084751   0.037322  -2.2708 0.023160 *
ar7 -0.050815   0.032199  -1.5782 0.114529
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

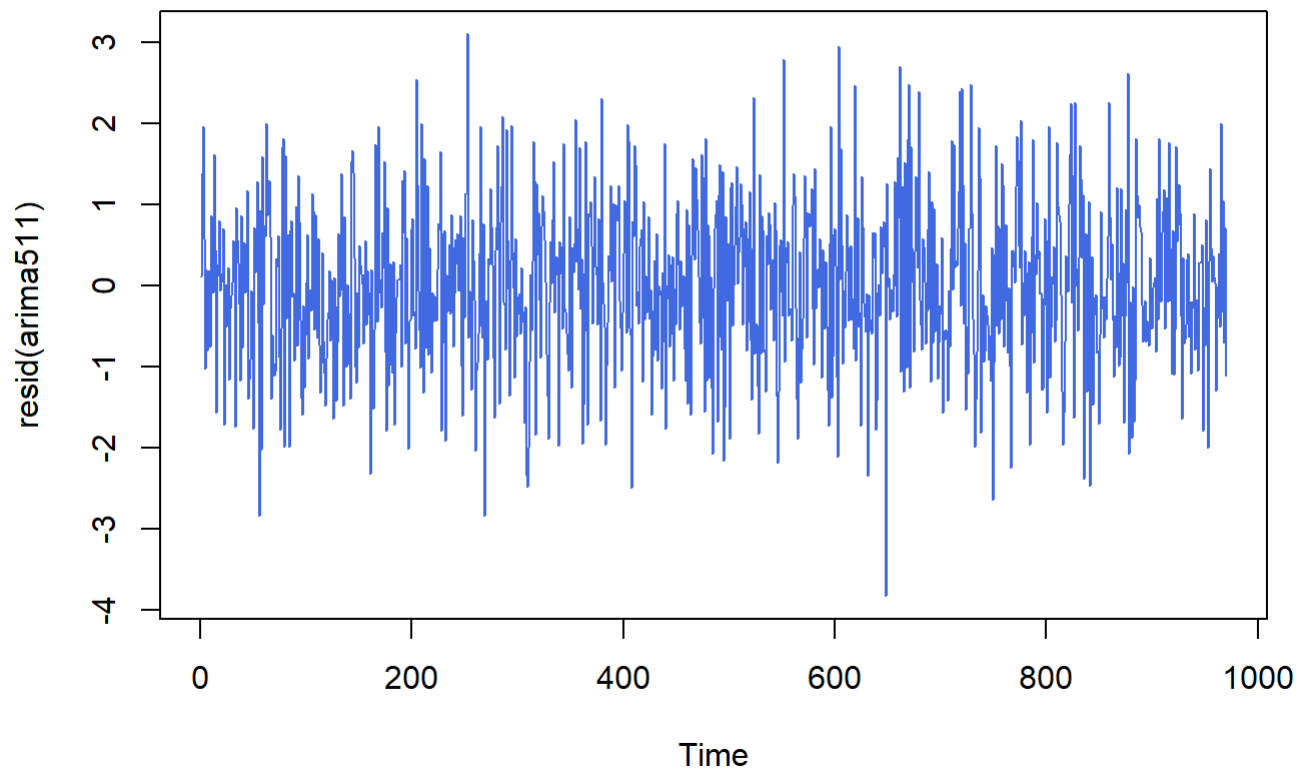
All the terms/coeffs are significant, except AR5, AR7.

Step 3: Model Diagnostics

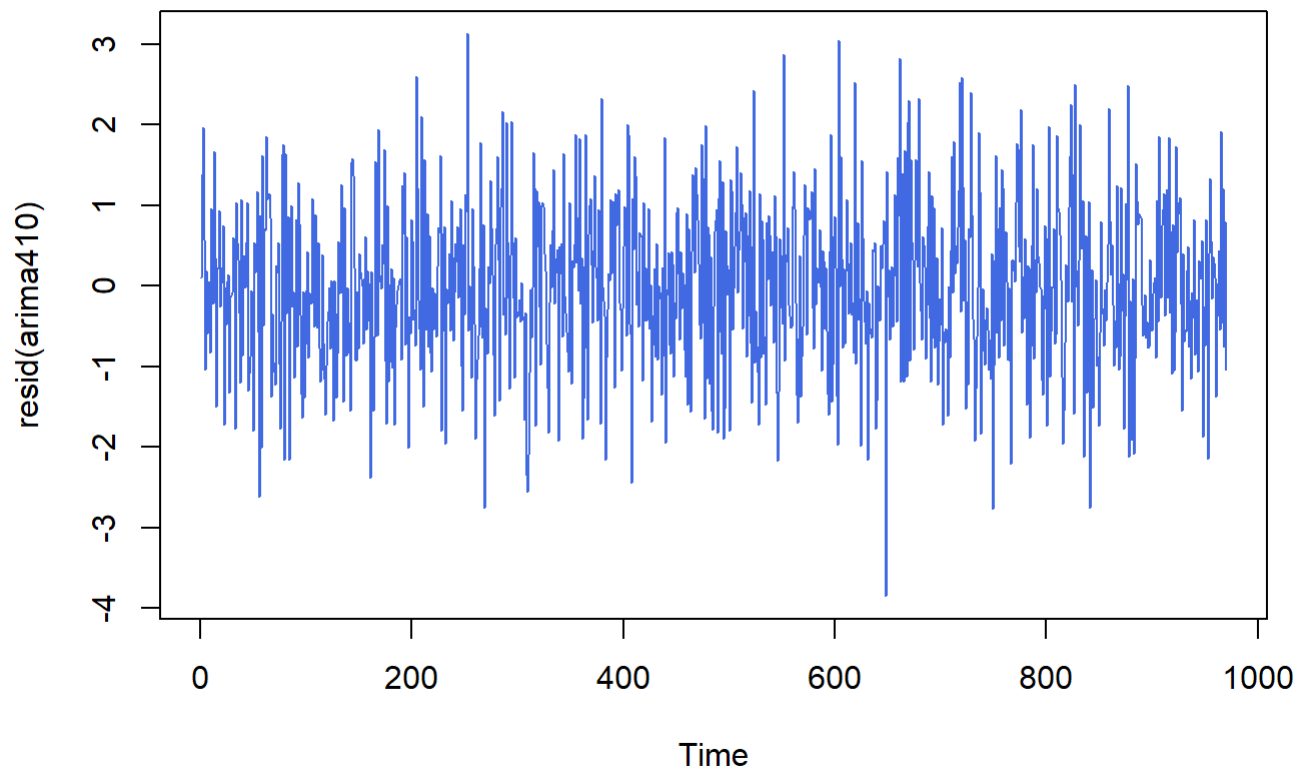
```
plot(resid(arima510),col = "royalblue")
```



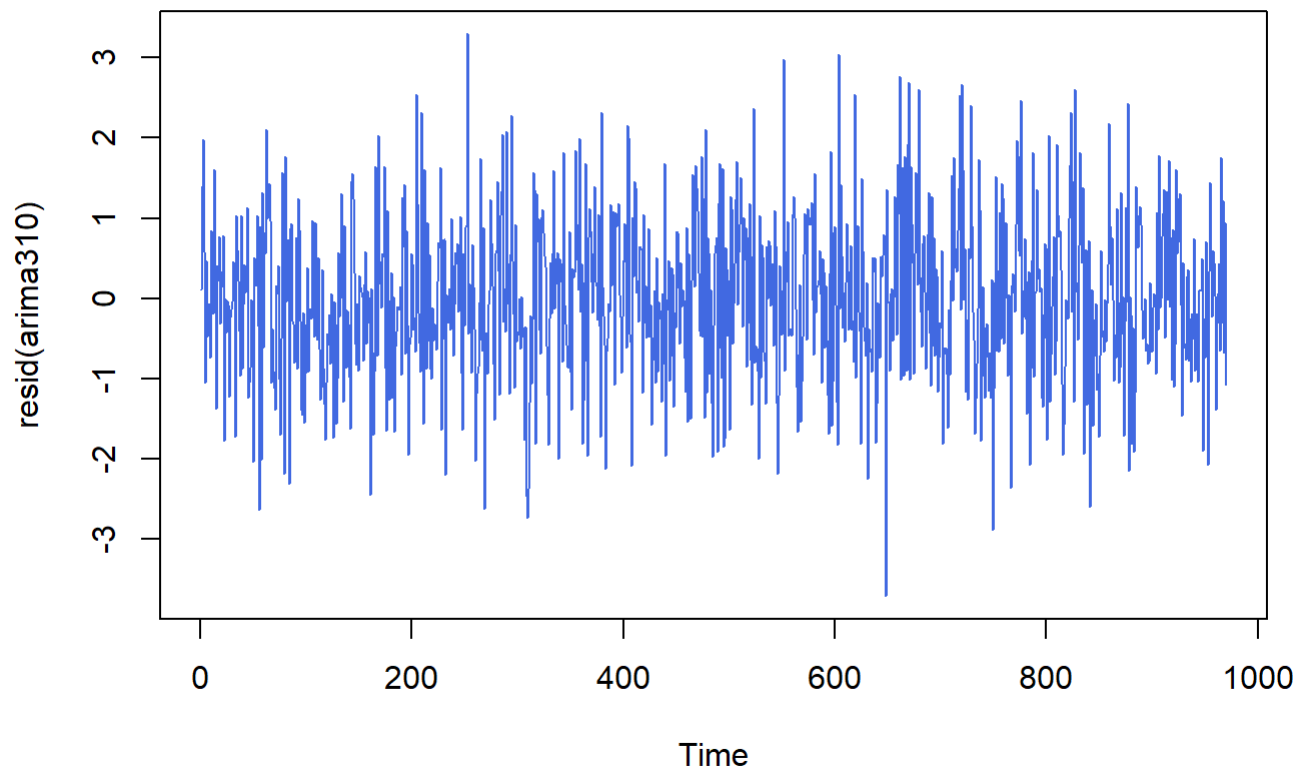
```
plot(resid(arima511),col = "royalblue")
```



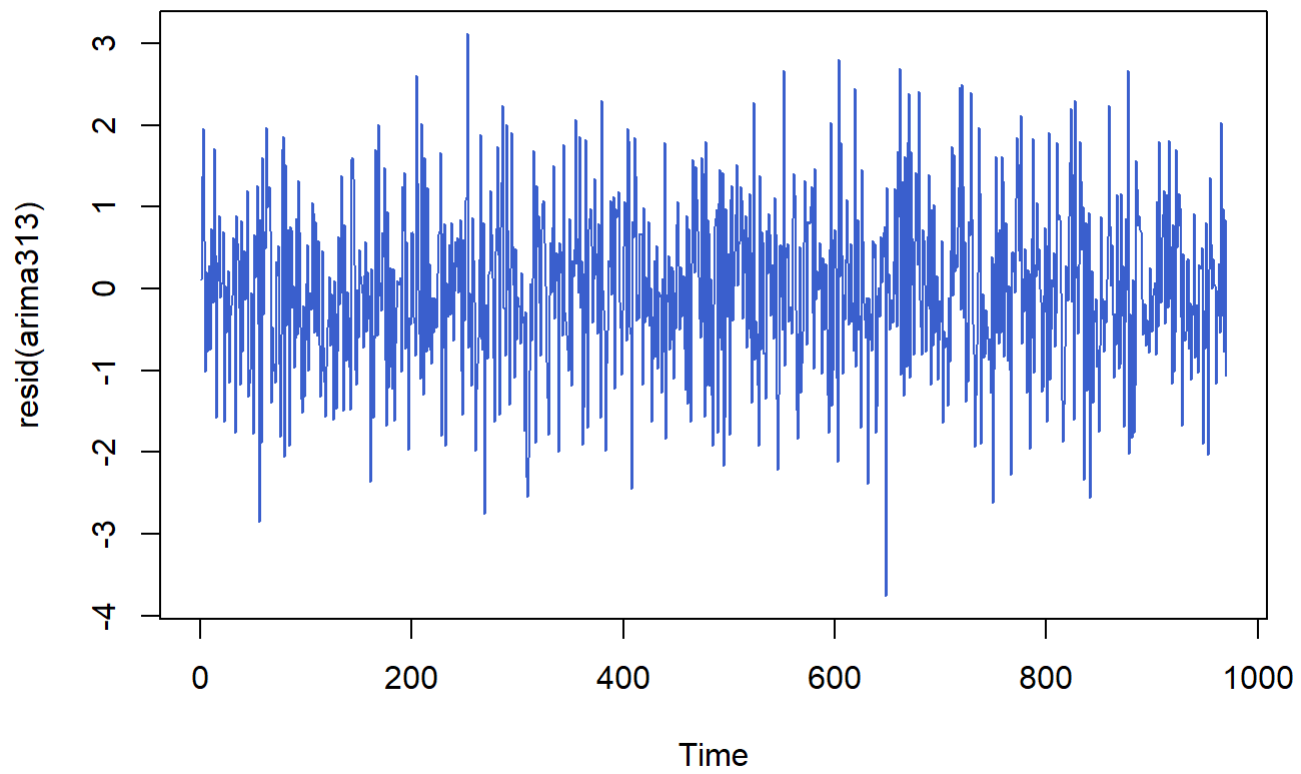
```
plot(resid(arima410),col = "royalblue")
```



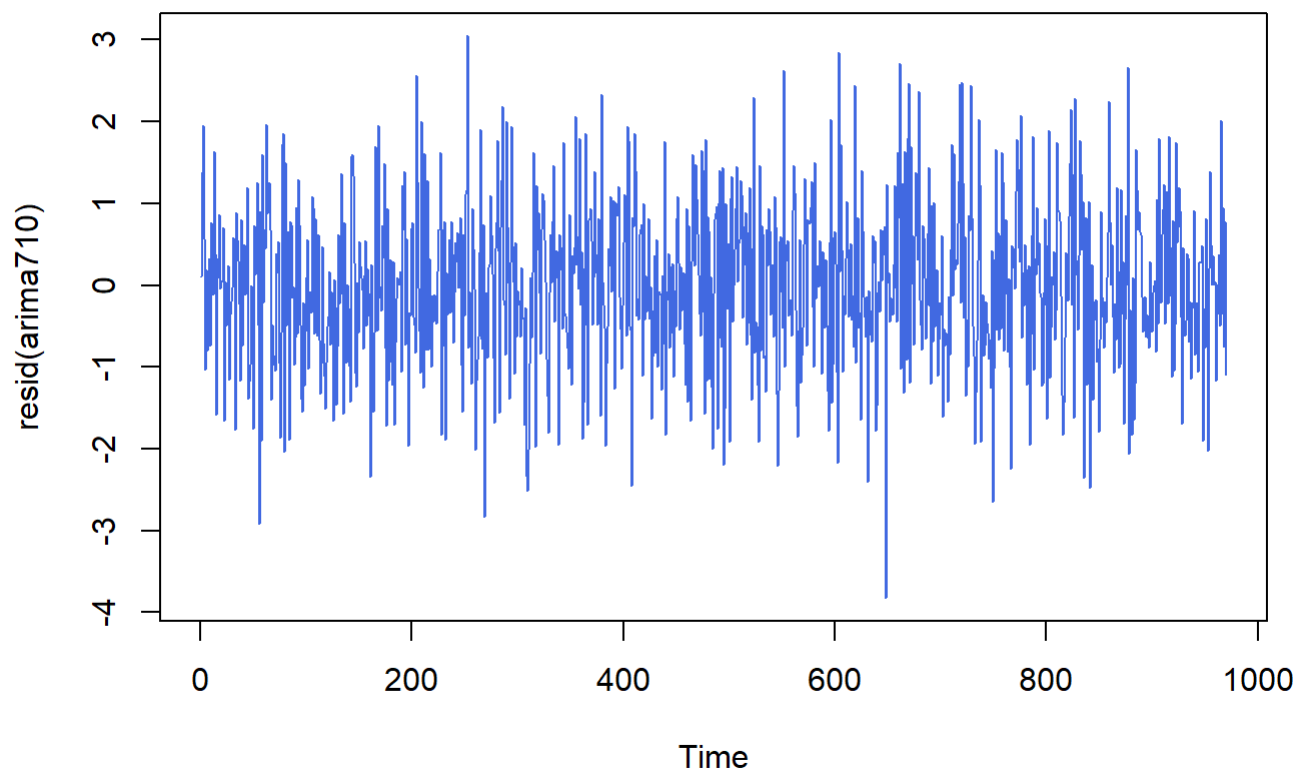
```
plot(resid(arima310),col = "royalblue")
```



```
plot(resid(arima313), col = "royalblue3")
```

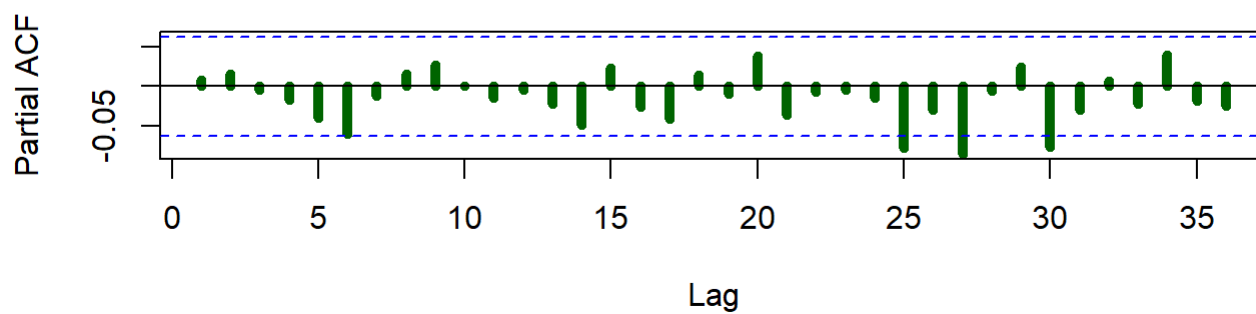


```
plot(resid(arima710),col = "royalblue")
```



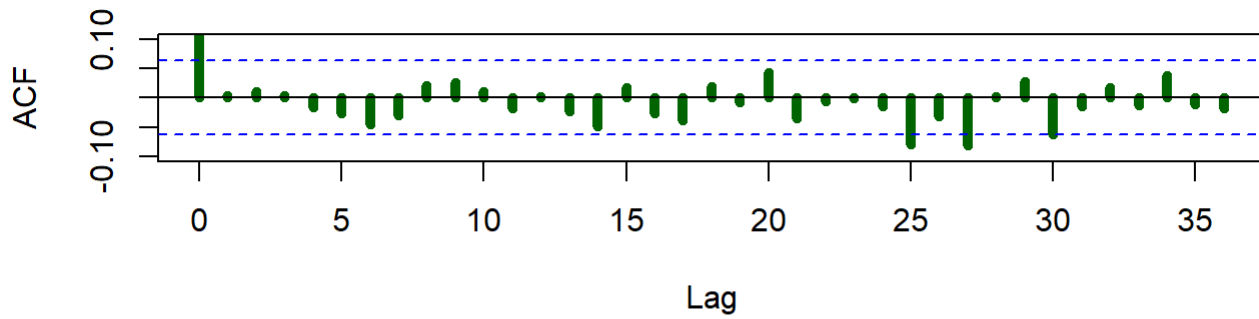
Lets check the ACF and the PACF of the Residual values:

```
plot_ACF_PACF_resids(arima510)
```

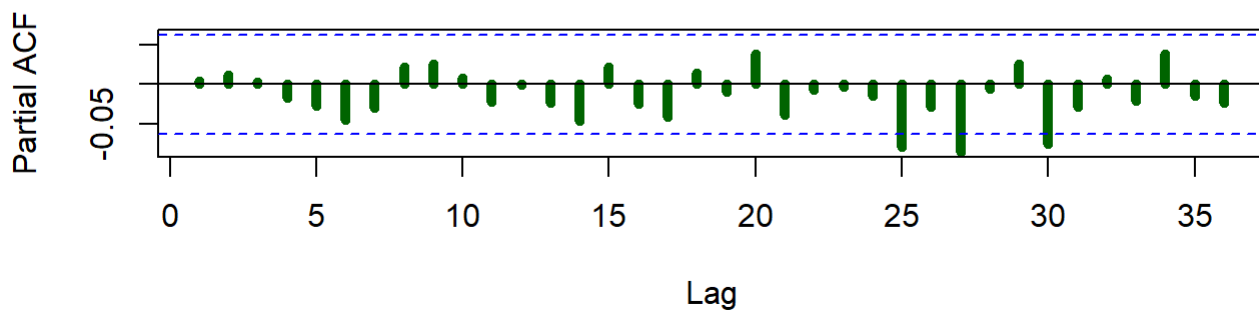

Series resid(ARIMA_model)**Series resid(ARIMA_model)**

```
plot_ACF_PACF_resids(arima511)
```

Series resid(ARIMA_model)

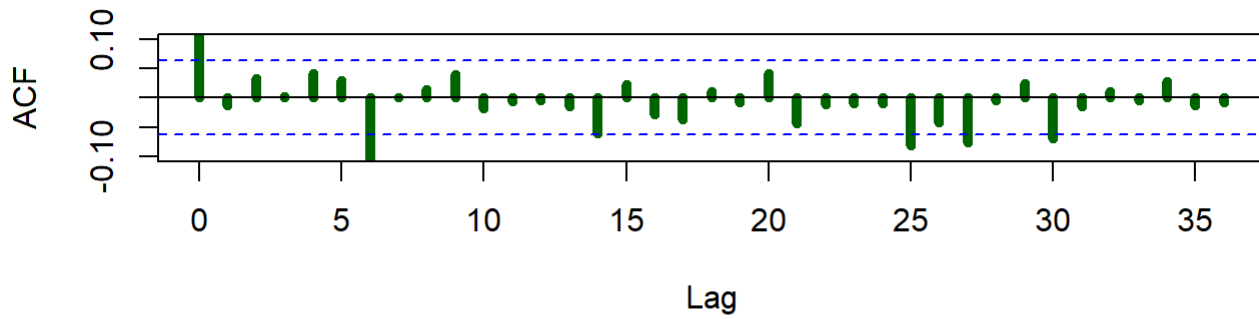


Series resid(ARIMA_model)

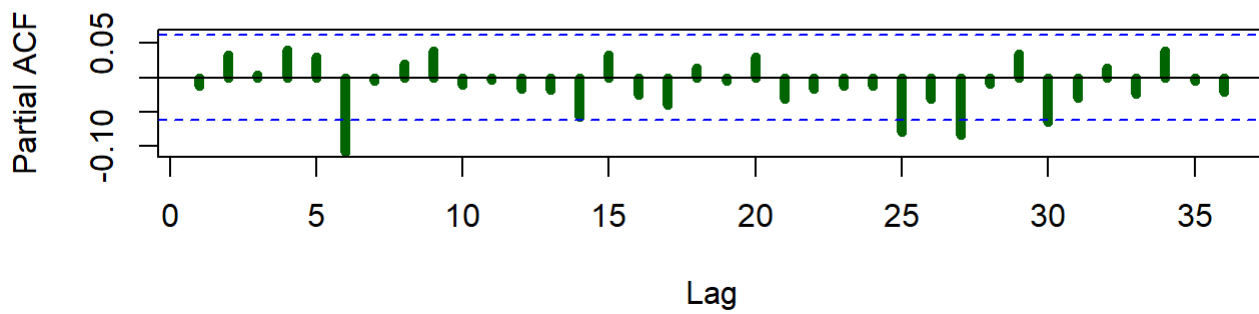


```
plot_ACF_PACF_resids(arima410)
```

Series resid(ARIMA_model)

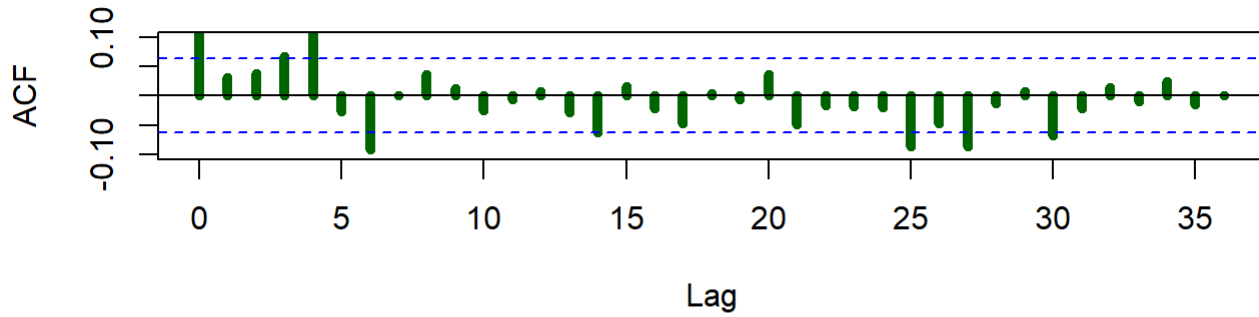


Series resid(ARIMA_model)

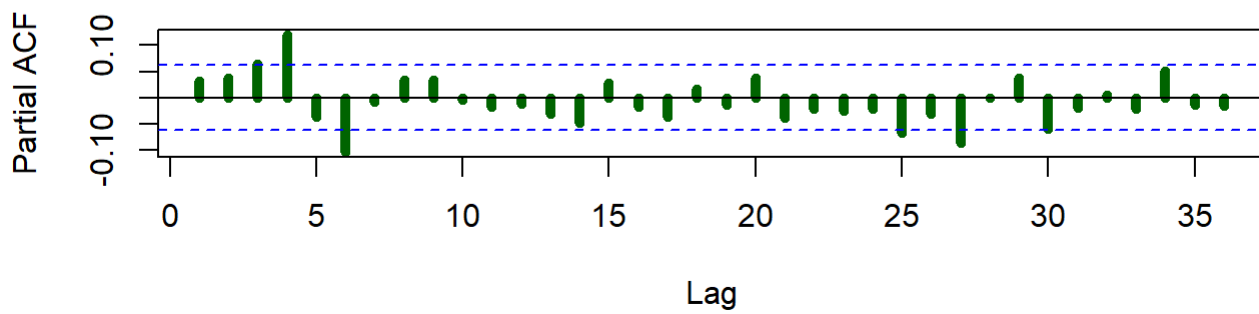


```
plot_ACF_PACF_resids(arima310)
```

Series resid(ARIMA_model)

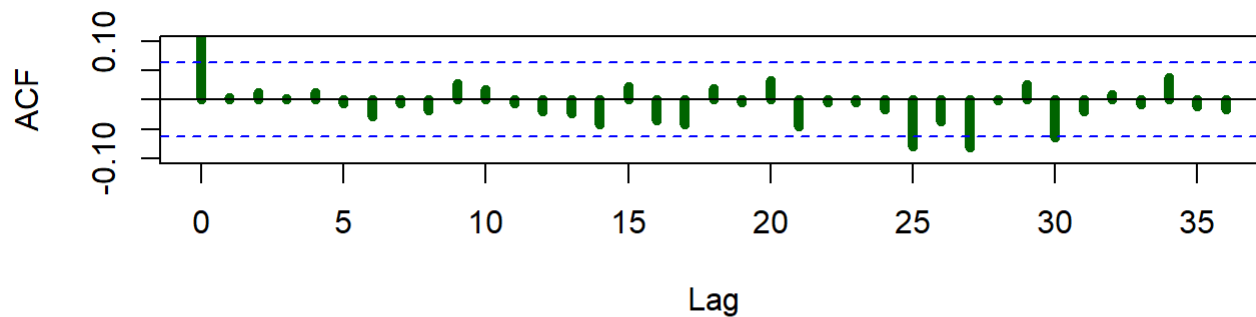


Series resid(ARIMA_model)

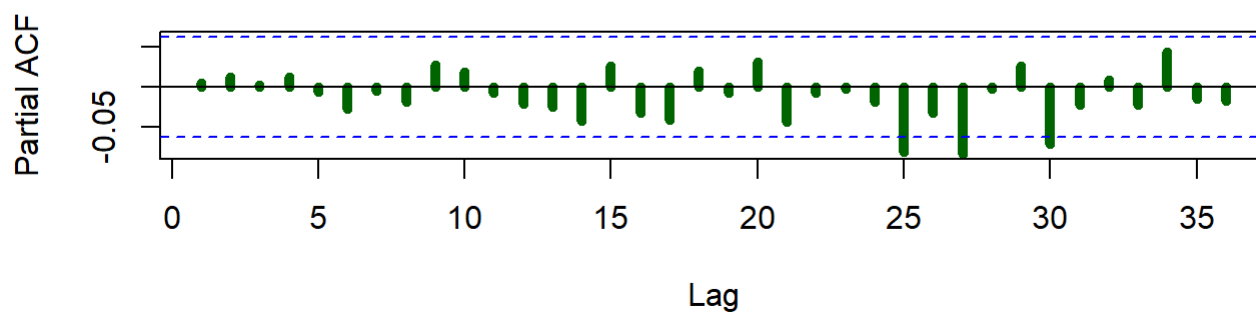


```
plot_ACF_PACF_resids(arima313)
```

Series resid(ARIMA_model)

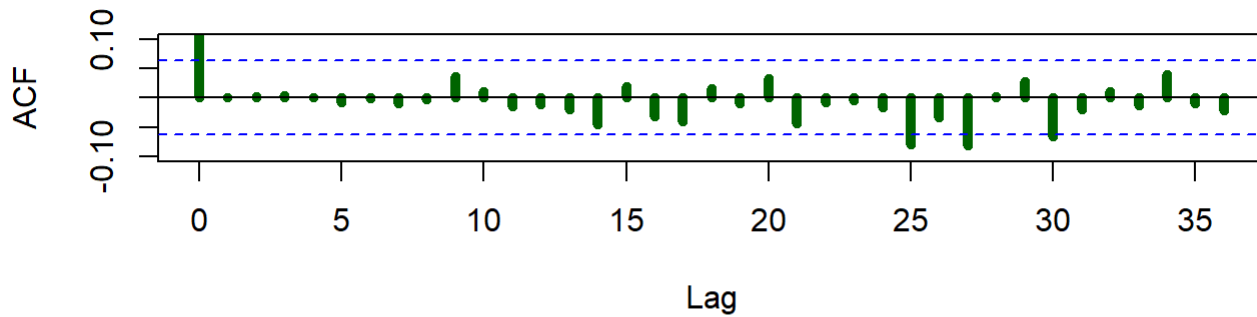


Series resid(ARIMA_model)

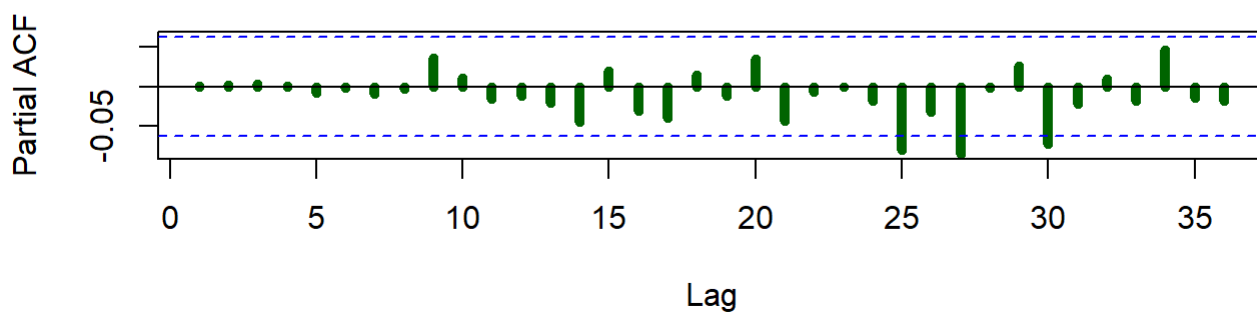


```
plot_ACF_PACF_resids(arima710)
```

Series resid(ARIMA_model)



Series resid(ARIMA_model)



The Ljung-Box test:

```
Box.test(resid(arima510), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

data: resid(arima510)

X-squared = 7.0428, df = 10, p-value = 0.7214

```
Box.test(resid(arima510), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

data: resid(arima510)

X-squared = 10.704, df = 15, p-value = 0.7733

```
Box.test(resid(arima510), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima510)
X-squared = 14.992, df = 20, p-value = 0.7768
```

```
Box.test(resid(arima510), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima510)
X-squared = 22.683, df = 25, p-value = 0.5961
```

```
Box.test(resid(arima511), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima511)
X-squared = 5.2863, df = 10, p-value = 0.8713
```

```
Box.test(resid(arima511), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima511)
X-squared = 8.8698, df = 15, p-value = 0.8842
```

```
Box.test(resid(arima511), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima511)
X-squared = 13.267, df = 20, p-value = 0.8656
```

```
Box.test(resid(arima511), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima511)
X-squared = 21.201, df = 25, p-value = 0.6813
```

```
Box.test(resid(arima410), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima410)
X-squared = 16.975, df = 10, p-value = 0.07492
```

```
Box.test(resid(arima410), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima410)
X-squared = 21.351, df = 15, p-value = 0.126
```

```
Box.test(resid(arima410), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima410)
X-squared = 25.428, df = 20, p-value = 0.1856
```

```
Box.test(resid(arima410), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima410)
X-squared = 34.231, df = 25, p-value = 0.1031
```

```
Box.test(resid(arima310), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima310)
X-squared = 32.859, df = 10, p-value = 0.0002877
```

```
Box.test(resid(arima310), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima310)
X-squared = 37.895, df = 15, p-value = 0.0009351
```

```
Box.test(resid(arima310), type = "Ljung-Box", lag = 20)
```


Box-Ljung test

```
data: resid(arima310)
X-squared = 41.845, df = 20, p-value = 0.002897
```

```
Box.test(resid(arima310), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima310)
X-squared = 52.643, df = 25, p-value = 0.0009933
```

We have very low p-values, greater than 5% -> Reject H_0 about no autocorrelation

Hence, the residual is auto-correlated in model ARIMA(3,1,0).

```
Box.test(resid(arima313), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima313)
X-squared = 2.5635, df = 10, p-value = 0.9899
```

```
Box.test(resid(arima313), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima313)
X-squared = 5.8069, df = 15, p-value = 0.9828
```

```
Box.test(resid(arima313), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima313)
X-squared = 10.277, df = 20, p-value = 0.9629
```

```
Box.test(resid(arima313), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima313)
X-squared = 18.939, df = 25, p-value = 0.8001
```

```
Box.test(resid(arima710), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima710)
X-squared = 1.5401, df = 10, p-value = 0.9988
```

```
Box.test(resid(arima710), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima710)
X-squared = 4.7483, df = 15, p-value = 0.9941
```

```
Box.test(resid(arima710), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima710)
X-squared = 8.7753, df = 20, p-value = 0.9854
```

```
Box.test(resid(arima710), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima710)
X-squared = 17.31, df = 25, p-value = 0.8702
```

We have very large p-values, greater than 5% for all models (except ARIMA(3,1,0)) -> fail to reject H_0 about no autocorrelation

Hence, the residual is white-noise.

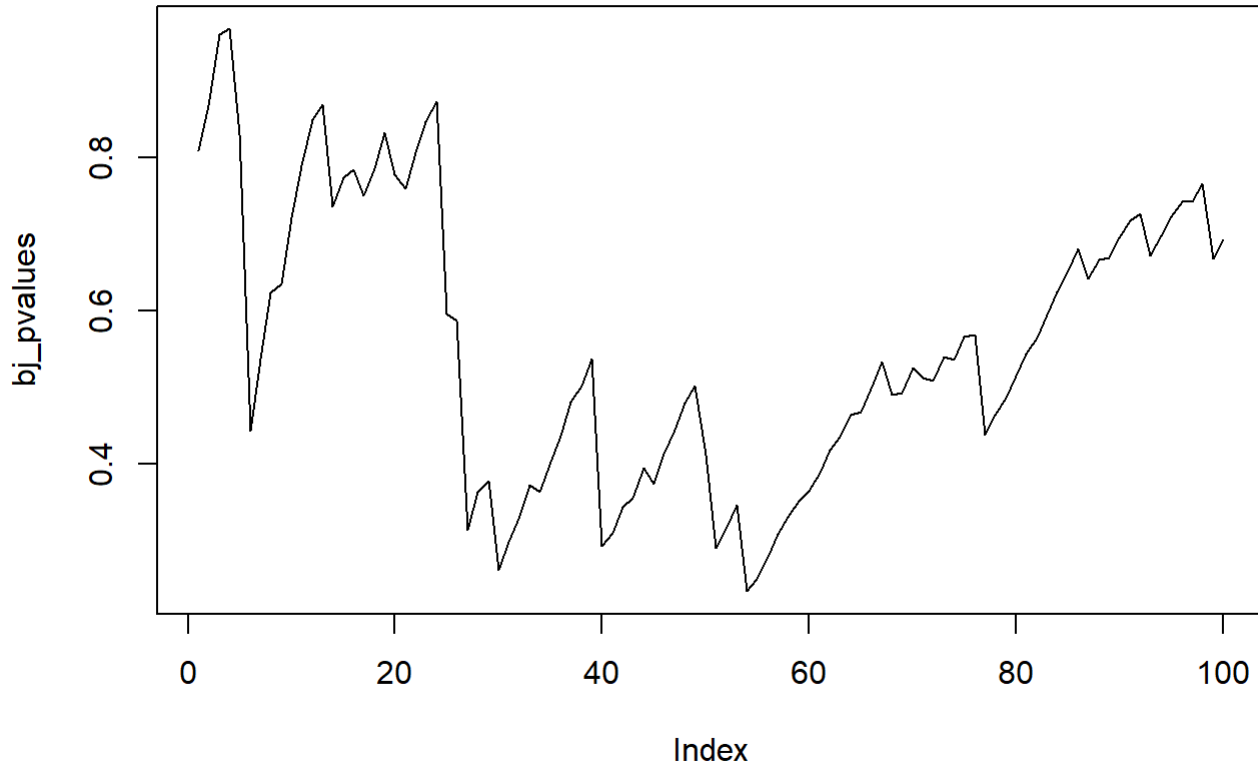
Plot graph for Ljung-Box test:

For model ARIMA(5,1,0)

```
bj_pvalues = c()
for(i in c(1:100)){
  bj = Box.test(resid(arima510), type = "Ljung-Box", lag = i)
  bj_pvalues = append(bj_pvalues,bj$p.value)
}
```

```
plot(bj_pvalues, type='l')

abline(h = 0.05, col='red')
```



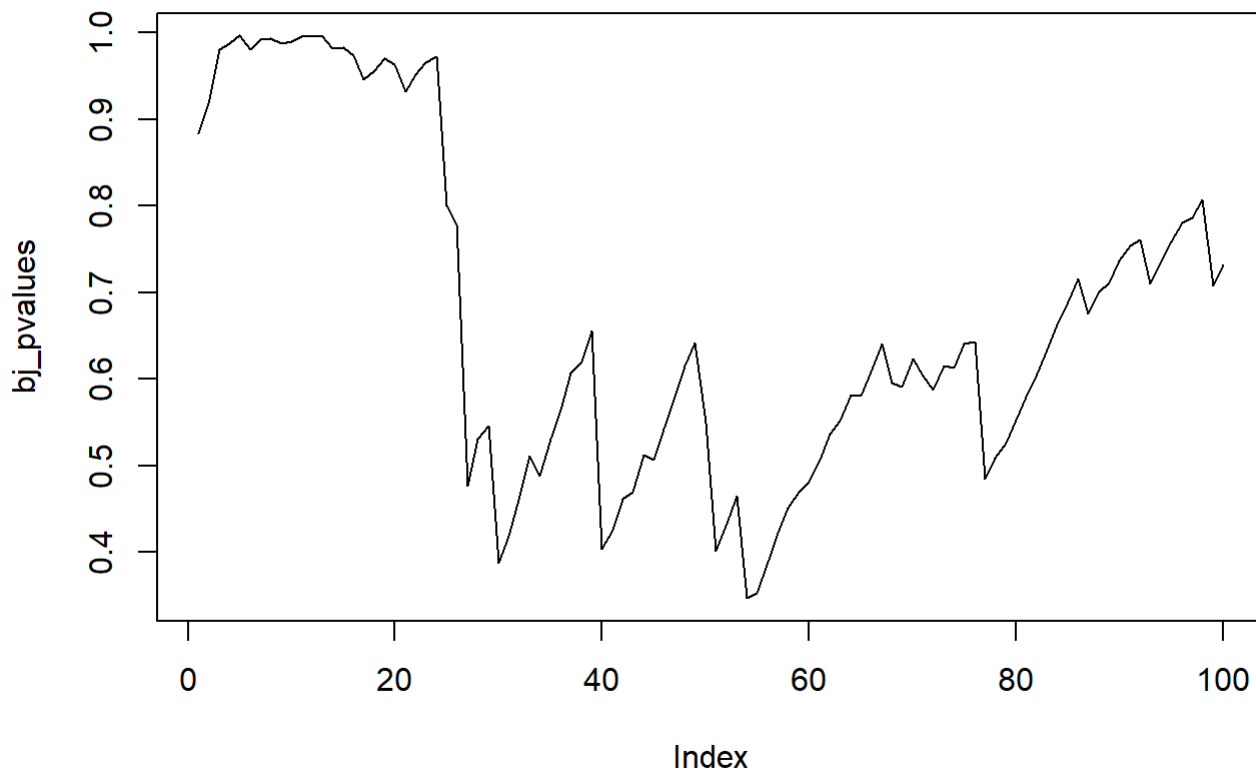
ARIMA(3,1,3)

```
bj_pvalues = c()

for(i in c(1:100)){
  bj = Box.test(resid(arima313), type = "Ljung-Box", lag = i)
  bj_pvalues = append(bj_pvalues,bj$p.value)
}

plot(bj_pvalues, type='l')

abline(h = 0.05, col='red')
```



Step 4. Evaluate Model

```
AIC(arima510, arima511,  
    arima410,  
    arima310, arima313,  
    arima710)
```

	df	AIC
arima510	6	2791.488
arima511	7	2791.961
arima410	5	2800.763
arima310	4	2816.218
arima313	7	2789.785
arima710	8	2790.102

Model ARIMA(3,1,3) returns the lowest AIC test.

```
BIC(arima510, arima511,  
    arima410,  
    arima310, arima313,  
    arima710)
```

	df	BIC
arima510	6	2820.745
arima511	7	2826.095
arima410	5	2825.144
arima310	4	2835.723
arima313	7	2823.919
arima710	8	2829.112

Model ARIMA(5,1,0) returns the lowest BIC test; Model ARIMA(3,1,3) comes second.

However, we should prefer AIC over BIC.

Conclusion: From the perspective of sensibility, the ARIMA(3,1,3) seems to be the most attractive one: all terms are significant, residuals are white noise and we observe low values of information criteria (IC).

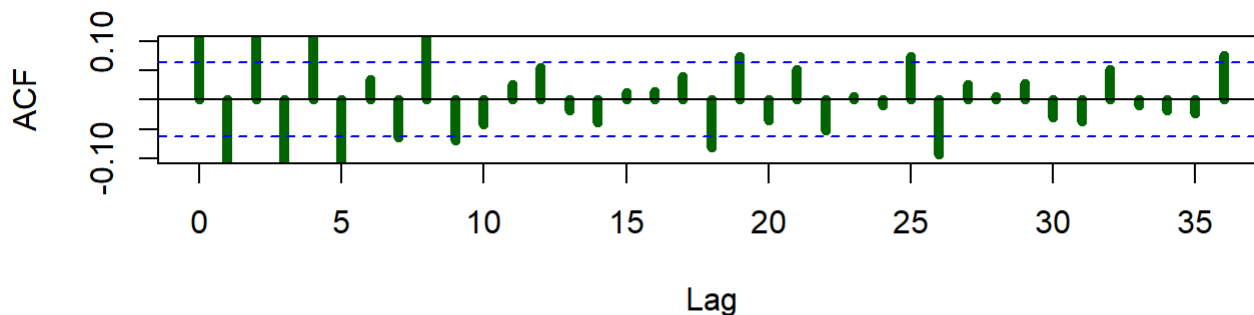
Hence, we use ARIMA(3,1,3) for forecasting Time Series X1.

4.1 Applying for Time Series X2

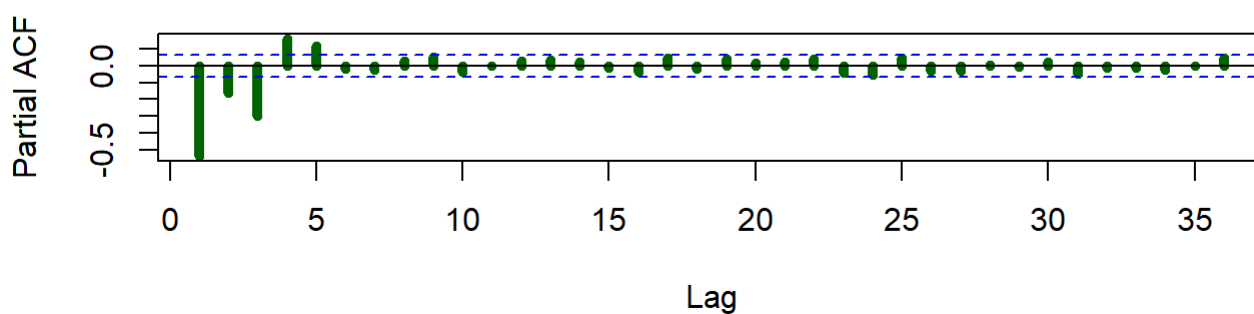
Step 1: Model Parameters

```
par(mfrow = c(2, 1))
acf(Data$dx2,
     lag.max = 36, # max lag for ACF
     ylim = c(-0.1, 0.1), # limits for the y axis - we give c(min, max)
     lwd = 5,
     col = "dark green",
     na.action = na.pass) # do not stop if there are missing values in the data
pacf(Data$dx2,
     lag.max = 36,
     lwd = 5, col = "dark green",
     na.action = na.pass)
```

Series Data\$dx2



Series Data\$dx2



```
par(mfrow = c(1, 1)) # restore the original single panel
```

The PACF shown is suggestive of an AR(5) model. So an initial candidate model is an ARIMA(5,1,0). We also have some variations of this model: ARIMA(5,1,1), ARIMA(4,1,0), ARIMA(3,1,0).

Step 2: Model Estimation

```
arima510_x2 <- Arima(Data$x2,
  order = c(5, 1, 0)
)
```

```
arima511_x2 <- Arima(Data$x2,
  order = c(5, 1, 1)
)
```

```
arima410_x2 <- Arima(Data$x2,
  order = c(4, 1, 0)
)
```

```
arima310_x2 <- Arima(Data$x2,
  order = c(3, 1, 0)
)
```

)

```

arima313_x2 <- Arima(Data$x2,
                      order = c(3, 1, 3)
)

```

The above model is not included constant. Let's include constant into the model:

```

arima510_2_x2 <- Arima(Data$x2,
                       order = c(5, 1, 0),
                       include.constant = TRUE)

```

Adding the constant did not change the result much for ALL MODELS, so we keep the model without constant.

Summary Results:

```
coeftest(arima510_x2)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)
ar1	-0.644866	0.031897	-20.2169	< 2.2e-16 ***
ar2	-0.269632	0.037238	-7.2408	4.462e-13 ***
ar3	-0.152155	0.037949	-4.0095	6.084e-05 ***
ar4	0.239044	0.037248	6.4176	1.385e-10 ***
ar5	0.121498	0.031871	3.8122	0.0001377 ***

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All parameters are significant.

```
coeftest(arima511_x2)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)
ar1	-0.750998	0.210052	-3.5753	0.0003498 ***
ar2	-0.335858	0.136327	-2.4636	0.0137545 *
ar3	-0.183105	0.073713	-2.4840	0.0129914 *
ar4	0.219352	0.056620	3.8741	0.0001070 ***
ar5	0.139151	0.044528	3.1251	0.0017777 **
ma1	0.107671	0.211118	0.5100	0.6100480

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All parameters are significant, except MA1.

```
coeftest(arima410_x2)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)
ar1	-0.624896	0.031703	-19.7112	< 2.2e-16 ***
ar2	-0.292513	0.037034	-7.8985	2.824e-15 ***
ar3	-0.187860	0.037057	-5.0695	3.990e-07 ***
ar4	0.162862	0.031675	5.1417	2.723e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All parameters are significant.

```
coeftest(arima310_x2)
```

z test of coefficients:

	Estimate	Std. Error	z value	Pr(> z)
ar1	-0.673600	0.030668	-21.9643	< 2.2e-16 ***
ar2	-0.349818	0.035809	-9.7691	< 2.2e-16 ***
ar3	-0.297916	0.030659	-9.7170	< 2.2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

All parameters are significant.

```
coeftest(arima313_x2)
```

z test of coefficients:

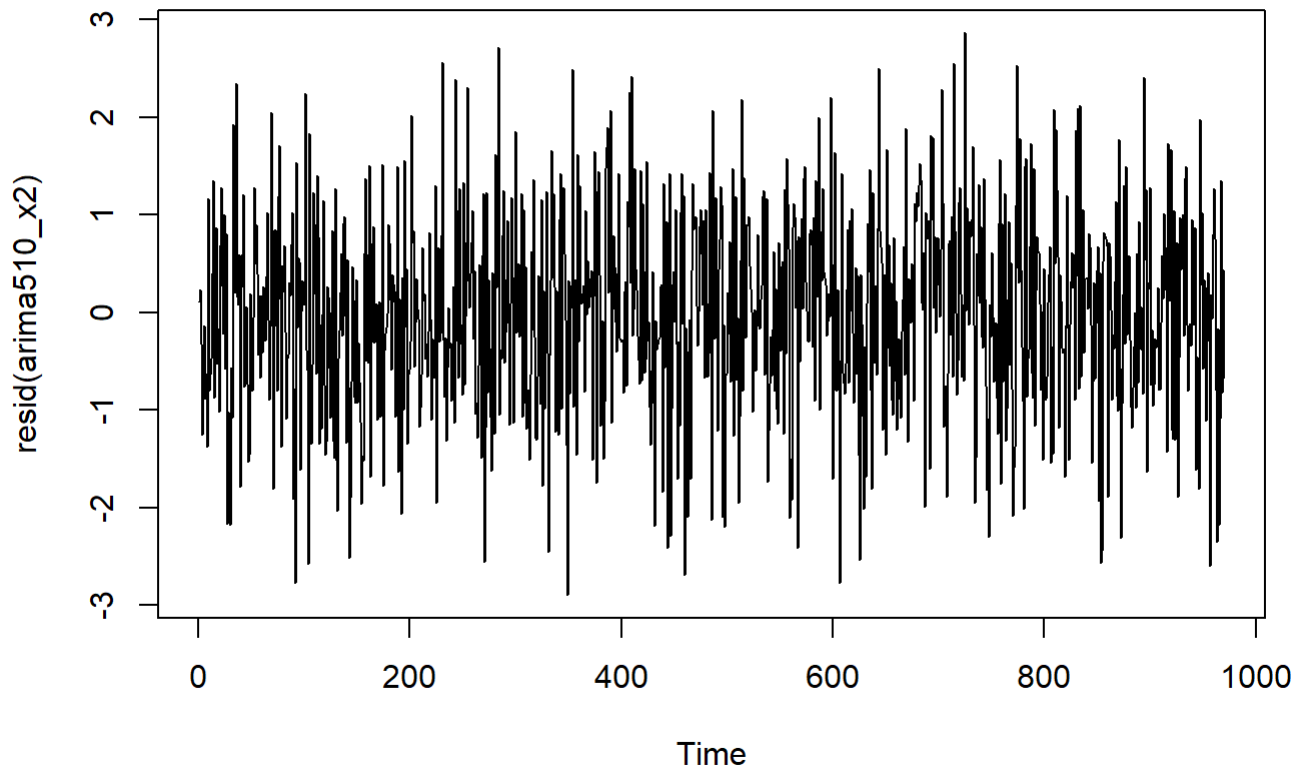
	Estimate	Std. Error	z value	Pr(> z)
ar1	-0.792133	0.063771	-12.4214	< 2.2e-16 ***
ar2	-0.768797	0.068468	-11.2285	< 2.2e-16 ***
ar3	-0.566525	0.040594	-13.9560	< 2.2e-16 ***
ma1	0.151590	0.072020	2.1048	0.03531 *
ma2	0.417740	0.079320	5.2665	1.391e-07 ***
ma3	0.105005	0.054887	1.9131	0.05573 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

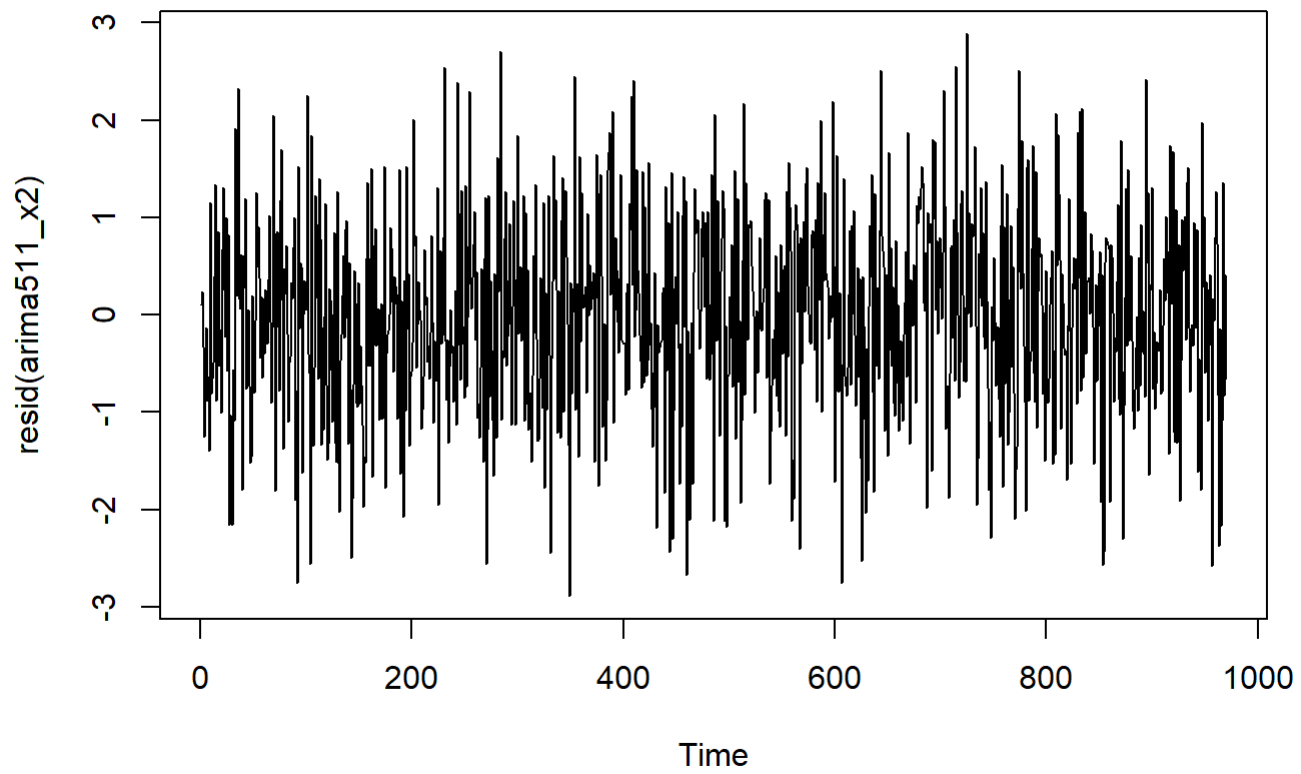
All parameters are significant, except MA3.

Step 3: Model Diagnostics

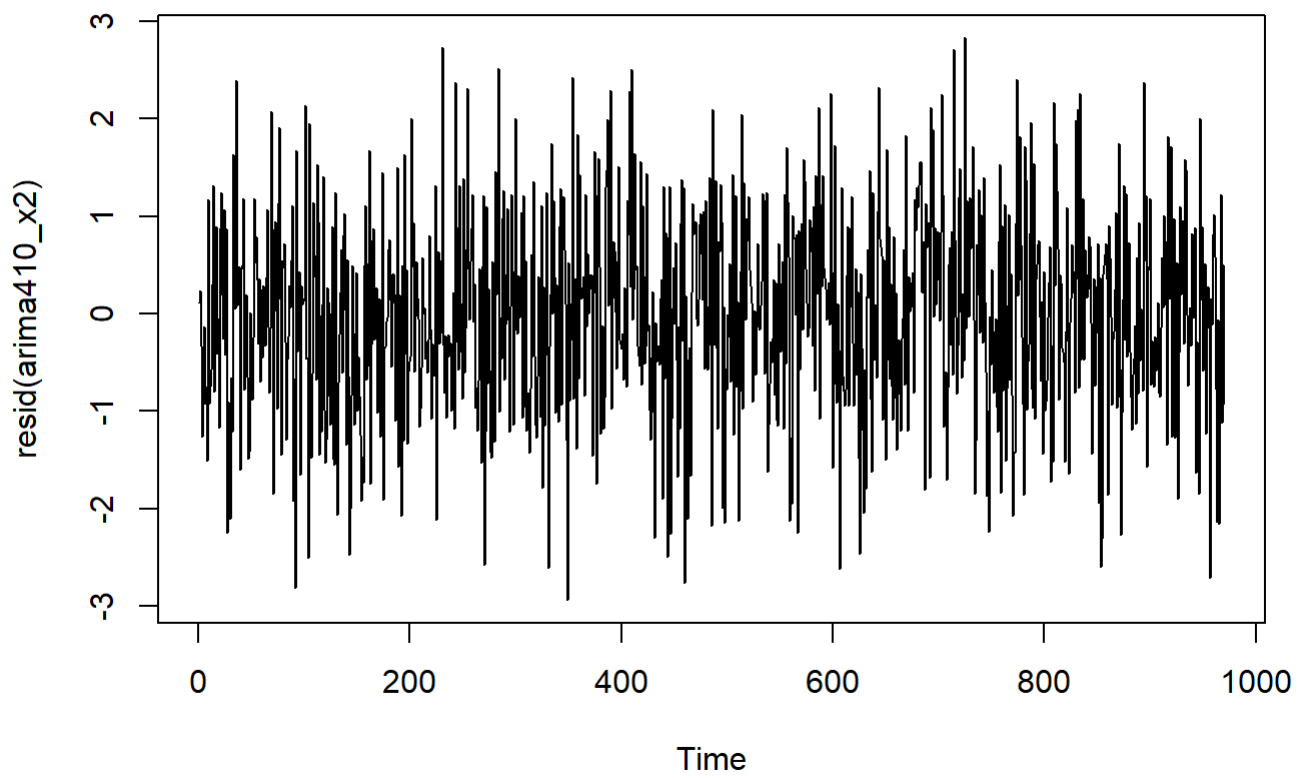

```
plot(resid(arima510_x2))
```



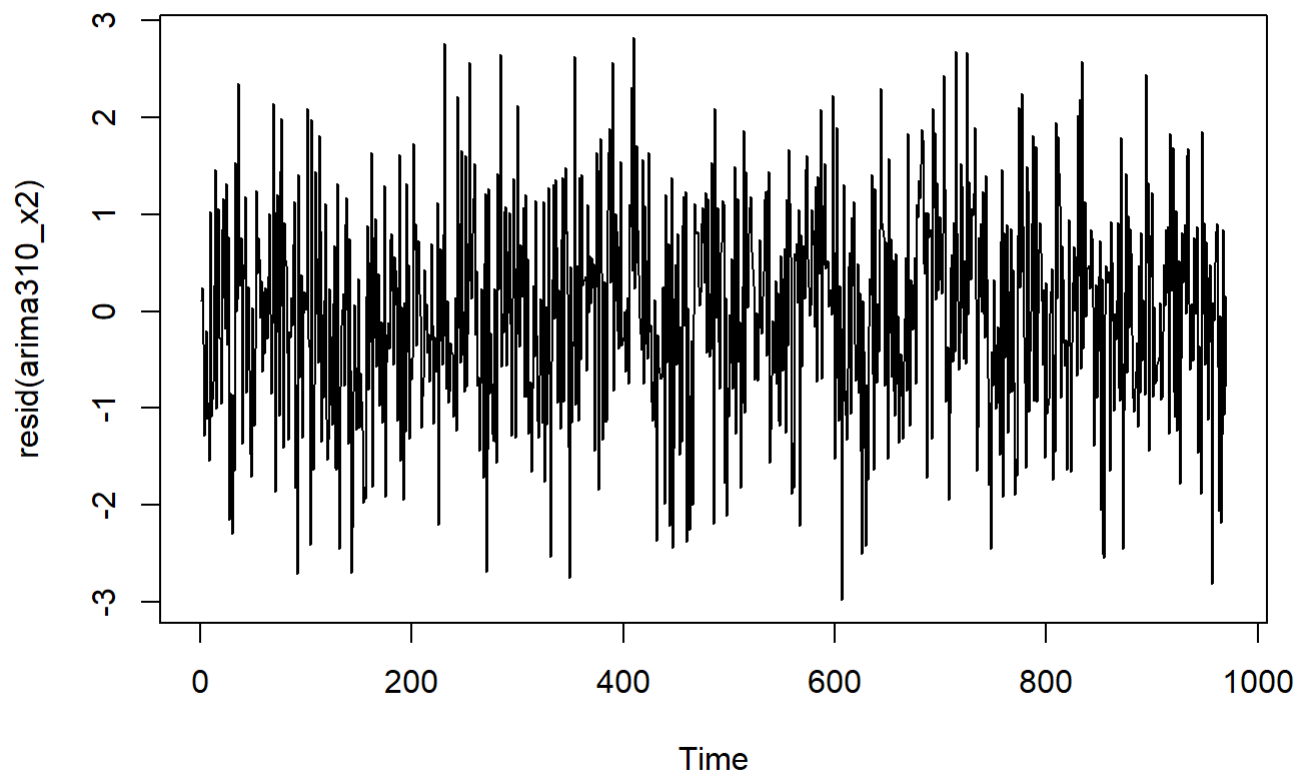
```
plot(resid(arima511_x2))
```



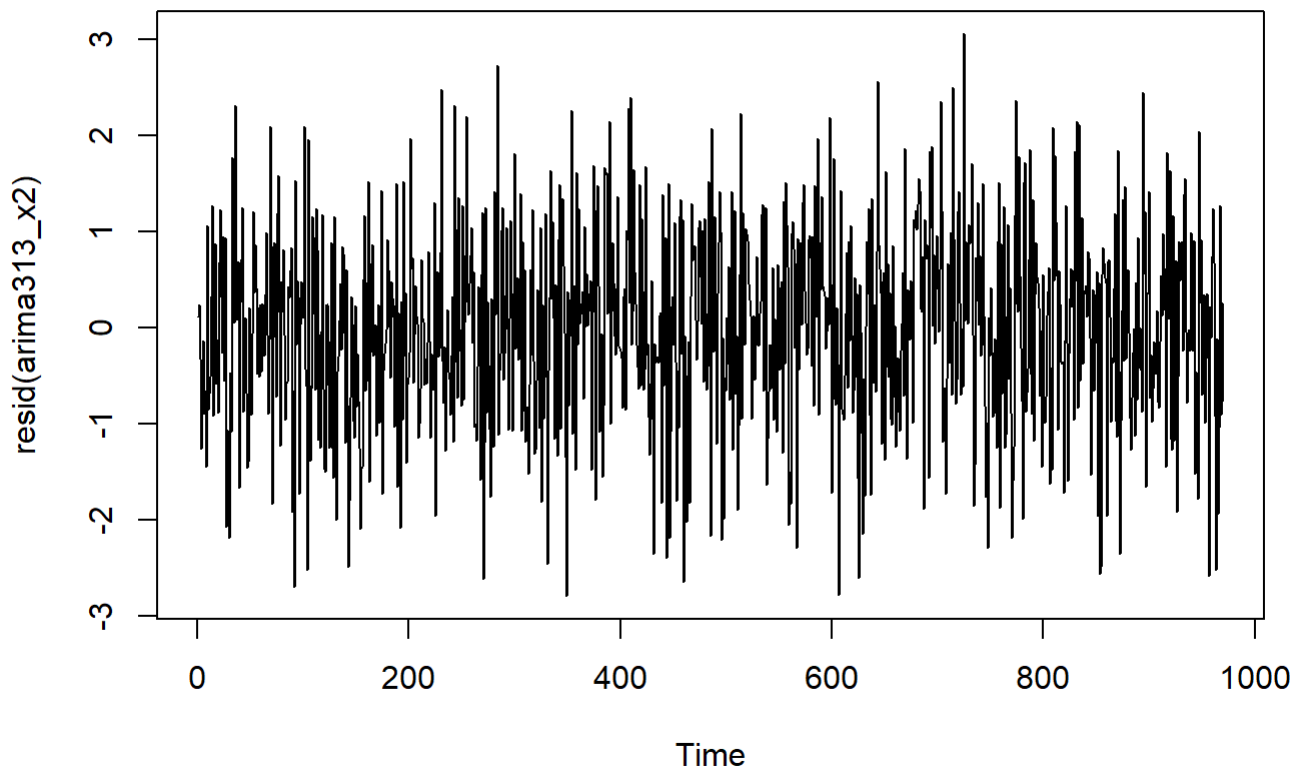
```
plot(resid(arima410_x2))
```



```
plot(resid(arima310_x2))
```



```
plot(resid(arima313_x2))
```



The Ljung-Box test:

```
Box.test(resid(arima510_x2), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima510_x2)
X-squared = 2.6884, df = 10, p-value = 0.9878
```

```
Box.test(resid(arima510_x2), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima510_x2)
X-squared = 8.8657, df = 15, p-value = 0.8844
```

```
Box.test(resid(arima510_x2), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima510_x2)
X-squared = 16.452, df = 20, p-value = 0.6882
```

```
Box.test(resid(arima510_x2), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima510_x2)
X-squared = 19.863, df = 25, p-value = 0.754
-> Very large p-values: The residual is white-noise
```

```
Box.test(resid(arima511_x2), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima511_x2)
X-squared = 2.7223, df = 10, p-value = 0.9872
```

```
Box.test(resid(arima511_x2), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima511_x2)
X-squared = 8.7444, df = 15, p-value = 0.8905
```

```
Box.test(resid(arima511_x2), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima511_x2)
X-squared = 16.459, df = 20, p-value = 0.6878
```

```
Box.test(resid(arima511_x2), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima511_x2)
X-squared = 19.927, df = 25, p-value = 0.7506
```

-> Very large p-values: The residual is white-noise

```
Box.test(resid(arima410_x2), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima410_x2)
X-squared = 15.929, df = 10, p-value = 0.1017
```

```
Box.test(resid(arima410_x2), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima410_x2)
X-squared = 23.17, df = 15, p-value = 0.08059
```

```
Box.test(resid(arima410_x2), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima410_x2)
X-squared = 32.02, df = 20, p-value = 0.04308
```

```
Box.test(resid(arima410_x2), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima410_x2)
X-squared = 35.549, df = 25, p-value = 0.07872
-> Not so large p-values: The residual is not really a white-noise for lag20.
```

```
Box.test(resid(arima310_x2), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima310_x2)
X-squared = 46.307, df = 10, p-value = 1.262e-06
```

```
Box.test(resid(arima310_x2), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima310_x2)
X-squared = 54.692, df = 15, p-value = 2.012e-06
```

```
Box.test(resid(arima310_x2), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima310_x2)
X-squared = 63.082, df = 20, p-value = 2.35e-06
```

```
Box.test(resid(arima310_x2), type = "Ljung-Box", lag = 25)
```

Box-Ljung test

```
data: resid(arima310_x2)
X-squared = 66.416, df = 25, p-value = 1.292e-05
-> Very small p-values: The residual is auto-correlated
```

```
Box.test(resid(arima313_x2), type = "Ljung-Box", lag = 10)
```

Box-Ljung test

```
data: resid(arima313_x2)
X-squared = 2.7105, df = 10, p-value = 0.9874
```

```
Box.test(resid(arima313_x2), type = "Ljung-Box", lag = 15)
```

Box-Ljung test

```
data: resid(arima313_x2)
X-squared = 7.035, df = 15, p-value = 0.9567
```

```
Box.test(resid(arima313_x2), type = "Ljung-Box", lag = 20)
```

Box-Ljung test

```
data: resid(arima313_x2)
X-squared = 13.67, df = 20, p-value = 0.8468
```

```
Box.test(resid(arima313_x2), type = "Ljung-Box", lag = 25)
```

Box-Ljung test


```
data: resid(arima313_x2)
```

```
X-squared = 16.922, df = 25, p-value = 0.8846
```

-> Very large p-values: The residual is white-noise

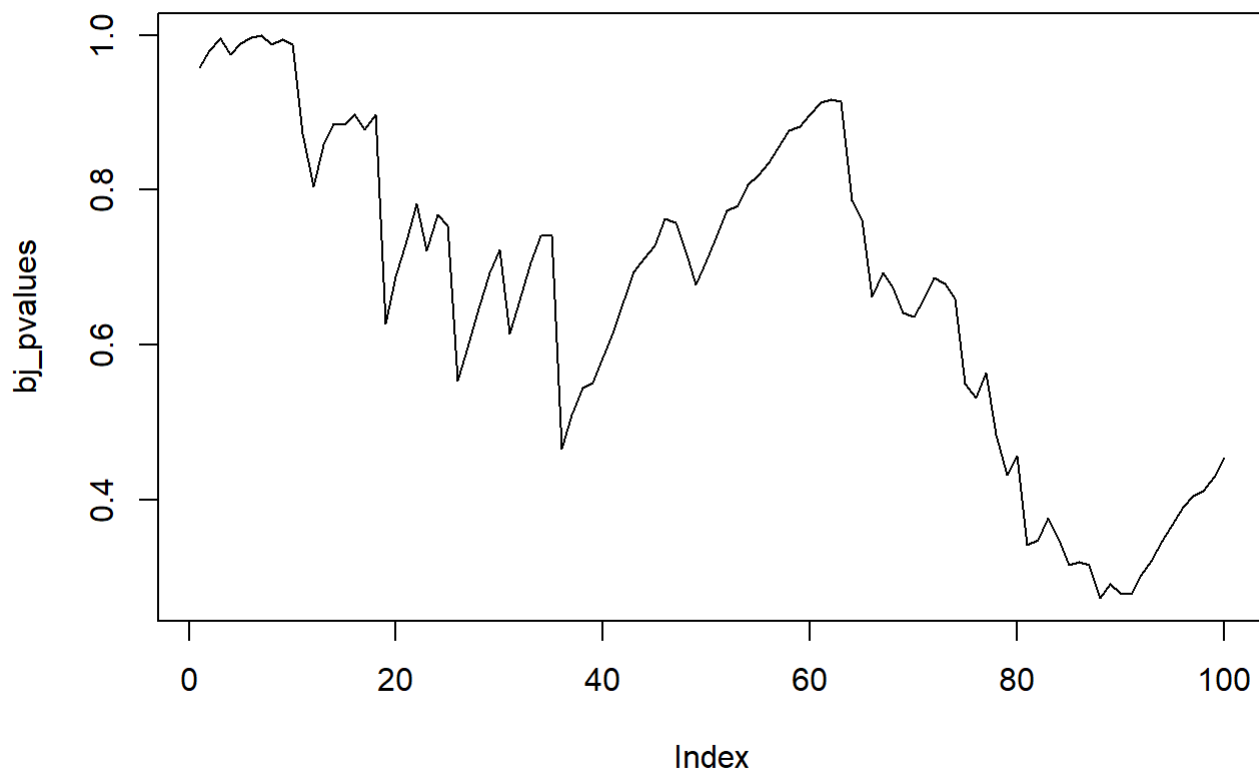
Plot graph for Ljung-Box test:

```
bj_pvalues = c()

for(i in c(1:100)){
  bj = Box.test(resid(arima510_x2), type = "Ljung-Box", lag = i)
  bj_pvalues = append(bj_pvalues,bj$p.value)
}

plot(bj_pvalues, type='l')

abline(h=0.05, col='red')
```



Step 4. Evaluate Model

```
AIC(arima510_x2,arima511_x2,
    arima410_x2,
    arima310_x2, arima313_x2)
```

	df	AIC
arima510_x2	6	2822.100
arima511_x2	7	2823.846
arima410_x2	5	2834.519
arima310_x2	4	2858.586
arima313_x2	7	2821.547

-> Suggestion model: arima313_x2 - ARIMA(3,1,3)

```
BIC(arima510_x2, arima511_x2,
    arima410_x2,
    arima310_x2, arima313_x2)
```

	df	BIC
arima510_x2	6	2851.357
arima511_x2	7	2857.980
arima410_x2	5	2858.900
arima310_x2	4	2878.091
arima313_x2	7	2855.681

-> Suggestion model: arima510_x2 - ARIMA(5,1,0)

Conclusion: From the perspective of sensibility, the ARIMA(5,1,0) seems to be the most attractive one:

all terms are significant, residuals are white noise and we observe low values of information criteria (IC).

-> Suggestion model: arima510_x2 - ARIMA(5,1,0)

4.3 ARIMA Forecast for X1 & X2

Import Data Forecast:

```
out_of_sample <- read.csv("Out_of_sample.csv", header = TRUE, dec = ".")
```

```
class(out_of_sample)
```

```
[1] "data.frame"
```

Change Date

```
out_of_sample$X <- as.Date(out_of_sample$X,
    format = "%d-%m-%y")
```

Create xts objects

```
out_of_sample <- xts(out_of_sample[, -1], out_of_sample$X)
```

Assign forecast X1 to object oos_x1

```
oos_x1 <- out_of_sample$x1
oos_x1
```

```

              x1
2020-12-06 104.6044
2020-12-07 103.6145
2020-12-08 102.8175
2020-12-09 101.3033
2020-12-10 102.4395
2020-12-11 103.7279
2020-12-12 102.8575
2020-12-13 103.0086
2020-12-14 102.8093
2020-12-15 103.5626
2020-12-16 102.5536
2020-12-17 104.9300
2020-12-18 104.3301
2020-12-19 104.1219
2020-12-20 103.8526
2020-12-21 105.4009
2020-12-22 105.4246
2020-12-23 103.2777
2020-12-24 102.4349
2020-12-25 103.7180
2020-12-26 103.1486
2020-12-27 101.4948
2020-12-28 101.8467
2020-12-29 102.5797
2020-12-30 103.6637
2020-12-31 101.0638
2021-01-01 100.5104
2021-01-02 102.3766
2021-01-03 101.5015
2021-01-04 102.1252
```

Forecast for X1

```
tail(Data, 12)
```

```

              x1      x2      x3      x4      x5      x6      x7
2020-11-24 104.3013 104.3355 140.7577 119.5638 114.1140 154.3604 153.2460
2020-11-25 104.1657 105.1016 141.4567 123.3867 113.5237 154.0791 153.7337
2020-11-26 102.6537 104.9208 140.7758 124.4254 112.9245 156.0804 154.2043
2020-11-27 103.2396 105.7320 140.4508 124.7773 114.9400 157.1855 155.4884
2020-11-28 103.6736 102.9653 141.7815 123.5558 115.1752 155.7699 152.7099
2020-11-29 102.9761 104.5524 140.6848 124.9638 113.3780 155.9102 157.3308
2020-11-30 104.9255 102.0271 140.3751 123.5249 114.2484 154.4416 154.2609
2020-12-01 103.4802 102.7281 139.8656 124.8164 115.4927 153.0521 156.7857
```

```

2020-12-02 105.3159 103.5042 141.6949 125.9501 113.2420 154.6303 154.1157
2020-12-03 103.3953 102.4242 142.7535 126.2372 112.9752 156.1399 154.1009
2020-12-04 105.4361 102.8317 141.2816 125.7234 115.3071 156.6897 154.6461
2020-12-05 103.1150 101.9375 141.2156 125.2633 115.3152 156.7915 155.2692
      x8      x9      x10      dx1      dx2      lresid
2020-11-24 187.7767 45.31686 160.3198 0.2285039 -0.1819140 1.4851947
2020-11-25 188.6371 47.13610 158.2888 -0.1355977 0.7660739 1.0416523
2020-11-26 189.0998 46.20568 159.0346 -1.5119562 -0.1807765 1.9629805
2020-11-27 188.8698 45.66828 158.1373 0.5859083 0.8111557 3.5133371
2020-11-28 187.4831 44.49097 158.9628 0.4339784 -2.7666582 3.6536498
2020-11-29 189.1835 43.55099 158.6368 -0.6975379 1.5870744 0.3901026
2020-11-30 190.5172 42.14053 158.4194 1.9494719 -2.5252541 2.7758317
2020-12-01 187.9916 41.04858 158.8728 -1.4453358 0.7009970 -1.9814946
2020-12-02 189.1746 41.14571 157.2235 1.8357427 0.7760933 0.3743577
2020-12-03 191.1886 41.65493 160.8470 -1.9206885 -1.0800434 -0.9514055
2020-12-04 190.2398 42.99143 159.4651 2.0407973 0.4074824 0.1676674
2020-12-05 188.6125 41.27213 158.3798 -2.3210339 -0.8941768 -1.7614866

```

```

forecasts_x1 <- forecast(arima313, # model for prediction
                        h = 30) # how many periods outside the sample

```

```
forecasts_x1
```

```

      Point Forecast      Lo 80      Hi 80      Lo 95      Hi 95
971      104.5678 103.26545 105.8702 102.57601 106.5596
972      103.6466 102.24226 105.0510 101.49883 105.7944
973      104.4368 102.83785 106.0357 101.99142 106.8822
974      103.7109 102.00794 105.4139 101.10645 106.3153
975      104.1953 102.23661 106.1540 101.19974 107.1909
976      103.9095 101.83371 105.9853 100.73484 107.0842
977      104.1790 101.97856 106.3794 100.81371 107.5443
978      103.9187 101.61508 106.2222 100.39564 107.4417
979      104.0779 101.62708 106.5287 100.32971 107.8260
980      103.9930 101.43417 106.5519 100.07959 107.9065
981      104.0867 101.42542 106.7480 100.01662 108.1568
982      103.9915 101.23499 106.7481 99.77576 108.2073
983      104.0425 101.17663 106.9084 99.65953 108.4255
984      104.0192 101.05724 106.9812 99.48927 108.5491
985      104.0527 101.00037 107.1051 99.38456 108.7209
986      104.0172 100.87791 107.1565 99.21605 108.8184
987      104.0327 100.80217 107.2633 99.09200 108.9735
988      104.0273 100.71073 107.3438 98.95505 109.0995
989      104.0398 100.64126 107.4383 98.84219 109.2374
990      104.0263 100.54797 107.5046 98.70667 109.3459
991      104.0305 100.47156 107.5895 98.58755 109.4735
992      104.0298 100.39268 107.6668 98.46733 109.5922
993      104.0347 100.32216 107.7472 98.35688 109.7125
994      104.0294 100.24312 107.8157 98.23877 109.8201
995      104.0303 100.17050 107.8902 98.12723 109.9334
996      104.0305 100.09875 107.9623 98.01739 110.0437

```

```

997      104.0326 100.03065 108.0345  97.91216 110.1530
998      104.0305 99.95974 108.1013  97.80481 110.2562
999      104.0305 99.89157 108.1695  97.70053 110.3605
1000     104.0308 99.82477 108.2368  97.59824 110.4633

```

Extract the forecast points

```
forecasts_x1$mean
```

Time Series:

Start = 971

End = 1000

Frequency = 1

```

[1] 104.5678 103.6466 104.4368 103.7109 104.1953 103.9095 104.1790 103.9187
[9] 104.0779 103.9930 104.0867 103.9915 104.0425 104.0192 104.0527 104.0172
[17] 104.0327 104.0273 104.0398 104.0263 104.0305 104.0298 104.0347 104.0294
[25] 104.0303 104.0305 104.0326 104.0305 104.0305 104.0308

```

```
class(forecasts_x1$mean)
```

```
[1] "ts"
```

It is a ts object, not xts! However, the xts objects are more convenient and modern. In terms of plotting the real data and forecast data to compare.

```
forecasts_x1$lower
```

Time Series:

Start = 971

End = 1000

Frequency = 1

```

      80%      95%
971 103.26545 102.57601
972 102.24226 101.49883
973 102.83785 101.99142
974 102.00794 101.10645
975 102.23661 101.19974
976 101.83371 100.73484
977 101.97856 100.81371
978 101.61508 100.39564
979 101.62708 100.32971
980 101.43417 100.07959
981 101.42542 100.01662
982 101.23499  99.77576
983 101.17663  99.65953
984 101.05724  99.48927
985 101.00037  99.38456
986 100.87791  99.21605
987 100.80217  99.09200
988 100.71073  98.95505

```

```

989 100.64126 98.84219
990 100.54797 98.70667
991 100.47156 98.58755
992 100.39268 98.46733
993 100.32216 98.35688
994 100.24312 98.23877
995 100.17050 98.12723
996 100.09875 98.01739
997 100.03065 97.91216
998 99.95974 97.80481
999 99.89157 97.70053
1000 99.82477 97.59824

```

```
forecasts_x1$upper
```

Time Series:

Start = 971

End = 1000

Frequency = 1

```

      80%      95%
971 105.8702 106.5596
972 105.0510 105.7944
973 106.0357 106.8822
974 105.4139 106.3153
975 106.1540 107.1909
976 105.9853 107.0842
977 106.3794 107.5443
978 106.2222 107.4417
979 106.5287 107.8260
980 106.5519 107.9065
981 106.7480 108.1568
982 106.7481 108.2073
983 106.9084 108.4255
984 106.9812 108.5491
985 107.1051 108.7209
986 107.1565 108.8184
987 107.2633 108.9735
988 107.3438 109.0995
989 107.4383 109.2374
990 107.5046 109.3459
991 107.5895 109.4735
992 107.6668 109.5922
993 107.7472 109.7125
994 107.8157 109.8201
995 107.8902 109.9334
996 107.9623 110.0437
997 108.0345 110.1530
998 108.1013 110.2562
999 108.1695 110.3605
1000 108.2368 110.4633

```

Create xts object: (We use the second column (95% confidence interval))

```
forecasts_x1_data <- data.frame(f_mean = as.numeric(forecasts_x1$mean),
                                f_lower = as.numeric(forecasts_x1$lower[, 2]),
                                f_upper = as.numeric(forecasts_x1$upper[, 2]))
```

```
head(forecasts_x1_data, 30)
```

	f_mean	f_lower	f_upper
1	104.5678	102.57601	106.5596
2	103.6466	101.49883	105.7944
3	104.4368	101.99142	106.8822
4	103.7109	101.10645	106.3153
5	104.1953	101.19974	107.1909
6	103.9095	100.73484	107.0842
7	104.1790	100.81371	107.5443
8	103.9187	100.39564	107.4417
9	104.0779	100.32971	107.8260
10	103.9930	100.07959	107.9065
11	104.0867	100.01662	108.1568
12	103.9915	99.77576	108.2073
13	104.0425	99.65953	108.4255
14	104.0192	99.48927	108.5491
15	104.0527	99.38456	108.7209
16	104.0172	99.21605	108.8184
17	104.0327	99.09200	108.9735
18	104.0273	98.95505	109.0995
19	104.0398	98.84219	109.2374
20	104.0263	98.70667	109.3459
21	104.0305	98.58755	109.4735
22	104.0298	98.46733	109.5922
23	104.0347	98.35688	109.7125
24	104.0294	98.23877	109.8201
25	104.0303	98.12723	109.9334
26	104.0305	98.01739	110.0437
27	104.0326	97.91216	110.1530
28	104.0305	97.80481	110.2562
29	104.0305	97.70053	110.3605
30	104.0308	97.59824	110.4633

Adding real value X1 below the current Dataset:

```
Data_x1 <- rbind(Data[, "x1"], oos_x1)
tail(Data_x1, n = 40)
```

	x1
2020-11-26	102.6537
2020-11-27	103.2396
2020-11-28	103.6736

```

2020-11-29 102.9761
2020-11-30 104.9255
2020-12-01 103.4802
2020-12-02 105.3159
2020-12-03 103.3953
2020-12-04 105.4361
2020-12-05 103.1150
2020-12-06 104.6044
2020-12-07 103.6145
2020-12-08 102.8175
2020-12-09 101.3033
2020-12-10 102.4395
2020-12-11 103.7279
2020-12-12 102.8575
2020-12-13 103.0086
2020-12-14 102.8093
2020-12-15 103.5626
2020-12-16 102.5536
2020-12-17 104.9300
2020-12-18 104.3301
2020-12-19 104.1219
2020-12-20 103.8526
2020-12-21 105.4009
2020-12-22 105.4246
2020-12-23 103.2777
2020-12-24 102.4349
2020-12-25 103.7180
2020-12-26 103.1486
2020-12-27 101.4948
2020-12-28 101.8467
2020-12-29 102.5797
2020-12-30 103.6637
2020-12-31 101.0638
2021-01-01 100.5104
2021-01-02 102.3766
2021-01-03 101.5015
2021-01-04 102.1252

```

Turn Forecast with Index into Forecast with Date

```

forecasts_x1_xts <- xts(forecasts_x1_data,
                        order.by = index(oos_x1))
forecasts_x1_xts

```

```

      f_mean  f_lower f_upper
2020-12-06 104.5678 102.57601 106.5596
2020-12-07 103.6466 101.49883 105.7944
2020-12-08 104.4368 101.99142 106.8822
2020-12-09 103.7109 101.10645 106.3153
2020-12-10 104.1953 101.19974 107.1909
2020-12-11 103.9095 100.73484 107.0842

```



```

2020-12-12 104.1790 100.81371 107.5443
2020-12-13 103.9187 100.39564 107.4417
2020-12-14 104.0779 100.32971 107.8260
2020-12-15 103.9930 100.07959 107.9065
2020-12-16 104.0867 100.01662 108.1568
2020-12-17 103.9915 99.77576 108.2073
2020-12-18 104.0425 99.65953 108.4255
2020-12-19 104.0192 99.48927 108.5491
2020-12-20 104.0527 99.38456 108.7209
2020-12-21 104.0172 99.21605 108.8184
2020-12-22 104.0327 99.09200 108.9735
2020-12-23 104.0273 98.95505 109.0995
2020-12-24 104.0398 98.84219 109.2374
2020-12-25 104.0263 98.70667 109.3459
2020-12-26 104.0305 98.58755 109.4735
2020-12-27 104.0298 98.46733 109.5922
2020-12-28 104.0347 98.35688 109.7125
2020-12-29 104.0294 98.23877 109.8201
2020-12-30 104.0303 98.12723 109.9334
2020-12-31 104.0305 98.01739 110.0437
2021-01-01 104.0326 97.91216 110.1530
2021-01-02 104.0305 97.80481 110.2562
2021-01-03 104.0305 97.70053 110.3605
2021-01-04 104.0308 97.59824 110.4633

```

Merge it together with the original data

```

Data_x1_combined <- merge(Data_x1, forecasts_x1_xts)
head(Data_x1_combined)

```

```

      x1 f_mean f_lower f_upper
2018-04-11 105.3882    NA     NA     NA
2018-04-12 107.1097    NA     NA     NA
2018-04-13 108.2144    NA     NA     NA
2018-04-14 108.0136    NA     NA     NA
2018-04-15 106.4532    NA     NA     NA
2018-04-16 107.6660    NA     NA     NA

```

```
tail(Data_x1_combined,40)
```

```

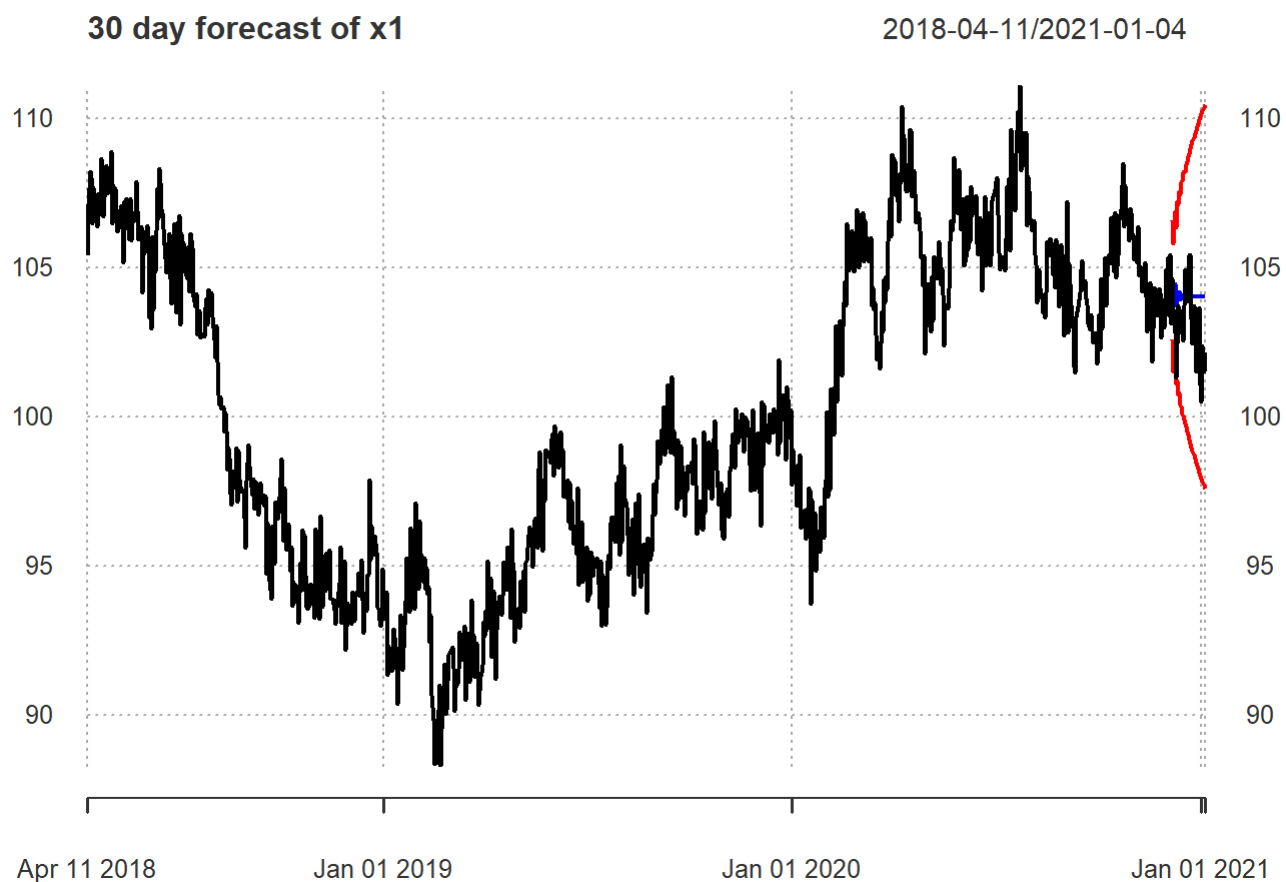
      x1  f_mean  f_lower  f_upper
2020-11-26 102.6537    NA     NA     NA
2020-11-27 103.2396    NA     NA     NA
2020-11-28 103.6736    NA     NA     NA
2020-11-29 102.9761    NA     NA     NA
2020-11-30 104.9255    NA     NA     NA
2020-12-01 103.4802    NA     NA     NA
2020-12-02 105.3159    NA     NA     NA
2020-12-03 103.3953    NA     NA     NA
2020-12-04 105.4361    NA     NA     NA

```

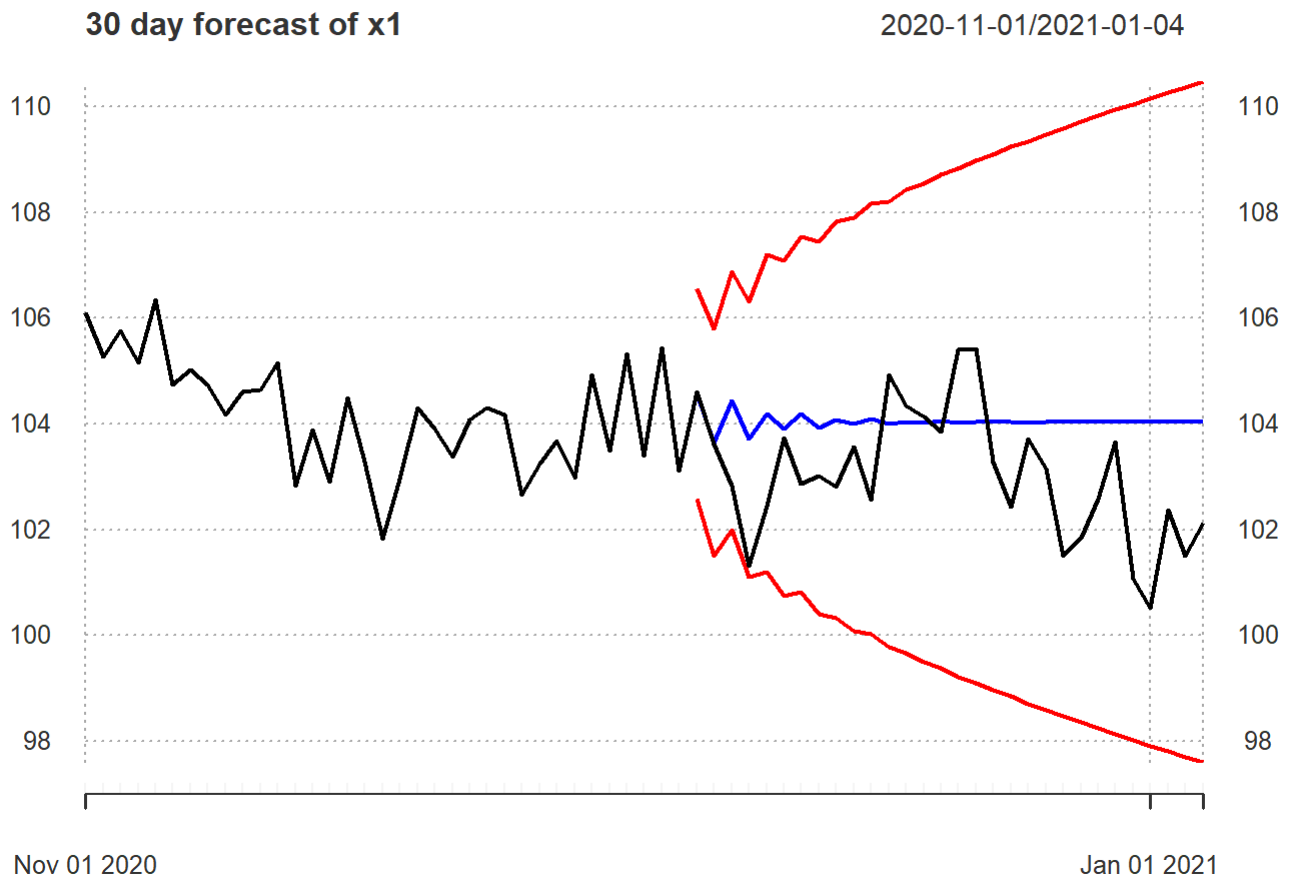
	2020-12-05	103.1150	NA	NA	NA
2020-12-06	104.6044	104.5678	102.5760	106.5596	
2020-12-07	103.6145	103.6466	101.4988	105.7944	
2020-12-08	102.8175	104.4368	101.9914	106.8822	
2020-12-09	101.3033	103.7109	101.1064	106.3153	
2020-12-10	102.4395	104.1953	101.1997	107.1909	
2020-12-11	103.7279	103.9095	100.7348	107.0842	
2020-12-12	102.8575	104.1790	100.8137	107.5443	
2020-12-13	103.0086	103.9187	100.3956	107.4417	
2020-12-14	102.8093	104.0779	100.3297	107.8260	
2020-12-15	103.5626	103.9930	100.0795	107.9065	
2020-12-16	102.5536	104.0867	100.0166	108.1568	
2020-12-17	104.9300	103.9915	99.7757	108.2073	
2020-12-18	104.3301	104.0425	99.6595	108.4255	
2020-12-19	104.1219	104.0192	99.4892	108.5491	
2020-12-20	103.8526	104.0527	99.3845	108.7209	
2020-12-21	105.4009	104.0172	99.2160	108.8184	
2020-12-22	105.4246	104.0327	99.0920	108.9735	
2020-12-23	103.2777	104.0273	98.9550	109.0995	
2020-12-24	102.4349	104.0398	98.8421	109.2374	
2020-12-25	103.7180	104.0263	98.7066	109.3459	
2020-12-26	103.1486	104.0305	98.5875	109.4735	
2020-12-27	101.4948	104.0298	98.4673	109.5922	
2020-12-28	101.8467	104.0347	98.3568	109.7125	
2020-12-29	102.5797	104.0294	98.2387	109.8201	
2020-12-30	103.6637	104.0303	98.1272	109.9334	
2020-12-31	101.0638	104.0305	98.0173	110.0437	
2021-01-01	100.5104	104.0326	97.9121	110.1530	
2021-01-02	102.3766	104.0305	97.8048	110.2562	
2021-01-03	101.5015	104.0305	97.7005	110.3605	
2021-01-04	102.1252	104.0308	97.5982	110.4633	

Plot Chart

```
plot(Data_x1_combined[, c("x1", "f_mean", "f_lower", "f_upper")],
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 3,
     main = "30 day forecast of x1",
     col = c("black", "blue", "red", "red"))
```



```
plot(Data_x1_combined ["2020-11/", c("x1", "f_mean", "f_lower", "f_upper")],
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 3,
     main = "30 day forecast of x1",
     col = c("black", "blue", "red", "red"))
```



Extract Data for Evaluating the Forecast:

```
Data_x1_Eva <- tail(Data_x1_combined, 30)
Data_x1_Eva
```

	x1	f_mean	f_lower	f_upper
2020-12-06	104.6044	104.5678	102.5760	106.5596
2020-12-07	103.6145	103.6466	101.4988	105.7944
2020-12-08	102.8175	104.4368	101.9914	106.8822
2020-12-09	101.3033	103.7109	101.1064	106.3153
2020-12-10	102.4395	104.1953	101.1997	107.1909
2020-12-11	103.7279	103.9095	100.7348	107.0842
2020-12-12	102.8575	104.1790	100.8137	107.5443
2020-12-13	103.0086	103.9187	100.3956	107.4417
2020-12-14	102.8093	104.0779	100.3297	107.8260
2020-12-15	103.5626	103.9930	100.0795	107.9065
2020-12-16	102.5536	104.0867	100.0166	108.1568
2020-12-17	104.9300	103.9915	99.7757	108.2073
2020-12-18	104.3301	104.0425	99.6595	108.4255
2020-12-19	104.1219	104.0192	99.4892	108.5491
2020-12-20	103.8526	104.0527	99.3845	108.7209
2020-12-21	105.4009	104.0172	99.2160	108.8184
2020-12-22	105.4246	104.0327	99.0920	108.9735
2020-12-23	103.2777	104.0273	98.9550	109.0995

```

2020-12-24 102.4349 104.0398 98.84219 109.2374
2020-12-25 103.7180 104.0263 98.70667 109.3459
2020-12-26 103.1486 104.0305 98.58755 109.4735
2020-12-27 101.4948 104.0298 98.46733 109.5922
2020-12-28 101.8467 104.0347 98.35688 109.7125
2020-12-29 102.5797 104.0294 98.23877 109.8201
2020-12-30 103.6637 104.0303 98.12723 109.9334
2020-12-31 101.0638 104.0305 98.01739 110.0437
2021-01-01 100.5104 104.0326 97.91216 110.1530
2021-01-02 102.3766 104.0305 97.80481 110.2562
2021-01-03 101.5015 104.0305 97.70053 110.3605
2021-01-04 102.1252 104.0308 97.59824 110.4633

```

```

Data_x1_Eva$mae <- abs(Data_x1_Eva$x1 - Data_x1_Eva$f_mean)
Data_x1_Eva$mse <- (Data_x1_Eva$x1 - Data_x1_Eva$f_mean) ^ 2
Data_x1_Eva$mape <- abs((Data_x1_Eva$x1 - Data_x1_Eva$f_mean)/Data_x1_Eva$x1)
Data_x1_Eva$amape <- abs((Data_x1_Eva$x1 - Data_x1_Eva$f_mean)/(Data_x1_Eva$x1 + Data_x1_Eva$f_m
Data_x1_Eva

```

	x1	f_mean	f_lower	f_upper	mae	mse
2020-12-06	104.6044	104.5678	102.57601	106.5596	0.03660466	0.001339901
2020-12-07	103.6145	103.6466	101.49883	105.7944	0.03214078	0.001033030
2020-12-08	102.8175	104.4368	101.99142	106.8822	1.61928051	2.622069373
2020-12-09	101.3033	103.7109	101.10645	106.3153	2.40761193	5.796595205
2020-12-10	102.4395	104.1953	101.19974	107.1909	1.75578459	3.082779544
2020-12-11	103.7279	103.9095	100.73484	107.0842	0.18163929	0.032992831
2020-12-12	102.8575	104.1790	100.81371	107.5443	1.32152181	1.746419902
2020-12-13	103.0086	103.9187	100.39564	107.4417	0.91001295	0.828123569
2020-12-14	102.8093	104.0779	100.32971	107.8260	1.26855923	1.609242532
2020-12-15	103.5626	103.9930	100.07959	107.9065	0.43042133	0.185262518
2020-12-16	102.5536	104.0867	100.01662	108.1568	1.53311149	2.350430849
2020-12-17	104.9300	103.9915	99.77576	108.2073	0.93851010	0.880801205
2020-12-18	104.3301	104.0425	99.65953	108.4255	0.28757158	0.082697416
2020-12-19	104.1219	104.0192	99.48927	108.5491	0.10270791	0.010548914
2020-12-20	103.8526	104.0527	99.38456	108.7209	0.20015706	0.040062848
2020-12-21	105.4009	104.0172	99.21605	108.8184	1.38369455	1.914610617
2020-12-22	105.4246	104.0327	99.09200	108.9735	1.39185676	1.937265232
2020-12-23	103.2777	104.0273	98.95505	109.0995	0.74960674	0.561910257
2020-12-24	102.4349	104.0398	98.84219	109.2374	1.60483551	2.575497003
2020-12-25	103.7180	104.0263	98.70667	109.3459	0.30829309	0.095044631
2020-12-26	103.1486	104.0305	98.58755	109.4735	0.88191051	0.777766144
2020-12-27	101.4948	104.0298	98.46733	109.5922	2.53492685	6.425854112
2020-12-28	101.8467	104.0347	98.35688	109.7125	2.18801355	4.787403309
2020-12-29	102.5797	104.0294	98.23877	109.8201	1.44970001	2.101630123
2020-12-30	103.6637	104.0303	98.12723	109.9334	0.36666172	0.134440817
2020-12-31	101.0638	104.0305	98.01739	110.0437	2.96676333	8.801684647
2021-01-01	100.5104	104.0326	97.91216	110.1530	3.52221649	12.406008988
2021-01-02	102.3766	104.0305	97.80481	110.2562	1.65387995	2.735318895

	2021-01-03	101.5015	104.0305	97.70053	110.3605	2.52903620	6.396024080
	2021-01-04	102.1252	104.0308	97.59824	110.4633	1.90555415	3.631136608
		mape		amape			
	2020-12-06	0.0003499342	0.0001749977				
	2020-12-07	0.0003101958	0.0001550739				
	2020-12-08	0.0157490725	0.0078130124				
	2020-12-09	0.0237663751	0.0117436357				
	2020-12-10	0.0171397199	0.0084970415				
	2020-12-11	0.0017511137	0.0008747909				
	2020-12-12	0.0128480866	0.0063830384				
	2020-12-13	0.0088343360	0.0043977424				
	2020-12-14	0.0123389531	0.0061316475				
	2020-12-15	0.0041561456	0.0020737633				
	2020-12-16	0.0149493696	0.0074192284				
	2020-12-17	0.0089441500	0.0044921643				
	2020-12-18	0.0027563633	0.0013800836				
	2020-12-19	0.0009864198	0.0004934533				
	2020-12-20	0.0019273194	0.0009627319				
	2020-12-21	0.0131279183	0.0066073294				
	2020-12-22	0.0132023912	0.0066450610				
	2020-12-23	0.0072581686	0.0036159617				
	2020-12-24	0.0156668747	0.0077725515				
	2020-12-25	0.0029724173	0.0014840031				
	2020-12-26	0.0085499008	0.0042567530				
	2020-12-27	0.0249759214	0.0123339350				
	2020-12-28	0.0214834099	0.0106275470				
	2020-12-29	0.0141324215	0.0070166298				
	2020-12-30	0.0035370320	0.0017653939				
	2020-12-31	0.0293553610	0.0144653625				
	2021-01-01	0.0350433171	0.0172199367				
	2021-01-02	0.0161548571	0.0080127065				
	2021-01-03	0.0249162464	0.0123048282				
	2021-01-04	0.0186589942	0.0092432621				

```
ARIMA_x1 <- colMeans(Data_x1_Eva[, c("mae", "mse", "mape", "amape")])
apply(Data_x1_Eva[, c("mae", "mse", "mape", "amape")], 2, FUN = median)
```

	mae	mse	mape	amape
	1.352608183	1.830515260	0.012988002	0.006495184

Forecast for X2

Assign forecast X2 to object oos_x2

```
oos_x2 <- out_of_sample$x2
oos_x2
```

	x2
2020-12-06	101.99487

```

2020-12-07 103.34756
2020-12-08 103.38339
2020-12-09 102.65495
2020-12-10 103.14643
2020-12-11 101.29482
2020-12-12 102.82414
2020-12-13 102.60263
2020-12-14 102.48089
2020-12-15 102.83048
2020-12-16 103.74833
2020-12-17 104.77165
2020-12-18 102.25474
2020-12-19 101.73088
2020-12-20 102.61676
2020-12-21 100.11499
2020-12-22 100.82839
2020-12-23 101.55266
2020-12-24 101.81775
2020-12-25 101.75292
2020-12-26 100.70069
2020-12-27 102.97327
2020-12-28 103.15157
2020-12-29 102.18853
2020-12-30 99.62443
2020-12-31 104.57416
2021-01-01 102.09902
2021-01-02 100.91690
2021-01-03 98.09291
2021-01-04 102.21843

```

```
tail(Data, 12)
```

	x1	x2	x3	x4	x5	x6	x7
2020-11-24	104.3013	104.3355	140.7577	119.5638	114.1140	154.3604	153.2460
2020-11-25	104.1657	105.1016	141.4567	123.3867	113.5237	154.0791	153.7337
2020-11-26	102.6537	104.9208	140.7758	124.4254	112.9245	156.0804	154.2043
2020-11-27	103.2396	105.7320	140.4508	124.7773	114.9400	157.1855	155.4884
2020-11-28	103.6736	102.9653	141.7815	123.5558	115.1752	155.7699	152.7099
2020-11-29	102.9761	104.5524	140.6848	124.9638	113.3780	155.9102	157.3308
2020-11-30	104.9255	102.0271	140.3751	123.5249	114.2484	154.4416	154.2609
2020-12-01	103.4802	102.7281	139.8656	124.8164	115.4927	153.0521	156.7857
2020-12-02	105.3159	103.5042	141.6949	125.9501	113.2420	154.6303	154.1157
2020-12-03	103.3953	102.4242	142.7535	126.2372	112.9752	156.1399	154.1009
2020-12-04	105.4361	102.8317	141.2816	125.7234	115.3071	156.6897	154.6461
2020-12-05	103.1150	101.9375	141.2156	125.2633	115.3152	156.7915	155.2692
	x8	x9	x10	dx1	dx2	lresid	
2020-11-24	187.7767	45.31686	160.3198	0.2285039	-0.1819140	1.4851947	
2020-11-25	188.6371	47.13610	158.2888	-0.1355977	0.7660739	1.0416523	
2020-11-26	189.0998	46.20568	159.0346	-1.5119562	-0.1807765	1.9629805	
2020-11-27	188.8698	45.66828	158.1373	0.5859083	0.8111557	3.5133371	

```

2020-11-28 187.4831 44.49097 158.9628 0.4339784 -2.7666582 3.6536498
2020-11-29 189.1835 43.55099 158.6368 -0.6975379 1.5870744 0.3901026
2020-11-30 190.5172 42.14053 158.4194 1.9494719 -2.5252541 2.7758317
2020-12-01 187.9916 41.04858 158.8728 -1.4453358 0.7009970 -1.9814946
2020-12-02 189.1746 41.14571 157.2235 1.8357427 0.7760933 0.3743577
2020-12-03 191.1886 41.65493 160.8470 -1.9206885 -1.0800434 -0.9514055
2020-12-04 190.2398 42.99143 159.4651 2.0407973 0.4074824 0.1676674
2020-12-05 188.6125 41.27213 158.3798 -2.3210339 -0.8941768 -1.7614866

```

```

forecasts_x2 <- forecast(arima510_x2, # model for prediction
                        h = 30) # how many periods outside the sample
forecasts_x2

```

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
971	102.8393	101.51445	104.1641	100.81312	104.8654
972	102.2730	100.86707	103.6789	100.12284	104.4231
973	102.4972	100.94235	104.0521	100.11924	104.8753
974	102.2039	100.54844	103.8593	99.67212	104.7356
975	102.5257	100.57887	104.4725	99.54830	105.5031
976	102.3373	100.27407	104.4006	99.18185	105.4928
977	102.4015	100.21039	104.5925	99.05050	105.7524
978	102.3190	100.01093	104.6271	98.78909	105.8490
979	102.4248	99.95891	104.8908	98.65353	106.1961
980	102.3631	99.78713	104.9392	98.42346	106.3028
981	102.3794	99.69382	105.0650	98.27216	106.4866
982	102.3575	99.56504	105.1500	98.08679	106.6283
983	102.3919	99.48463	105.2992	97.94560	106.8382
984	102.3713	99.36505	105.3775	97.77365	106.9689
985	102.3750	99.27200	105.4781	97.62935	107.1207
986	102.3697	99.17140	105.5680	97.47832	107.2611
987	102.3808	99.08693	105.6747	97.34325	107.4184
988	102.3738	98.99061	105.7569	97.19967	107.5478
989	102.3745	98.90408	105.8450	97.06694	107.6821
990	102.3734	98.81713	105.9297	96.93455	107.8123
991	102.3770	98.73610	106.0179	96.80872	107.9453
992	102.3745	98.65212	106.0970	96.68159	108.0675
993	102.3747	98.57235	106.1770	96.55953	108.1898
994	102.3745	98.49367	106.2554	96.43927	108.3098
995	102.3757	98.41764	106.3337	96.32238	108.4290
996	102.3748	98.34144	106.4082	96.20631	108.5433
997	102.3748	98.26746	106.4821	96.09317	108.6564
998	102.3748	98.19471	106.5550	95.98188	108.7678
999	102.3752	98.12348	106.6269	95.87274	108.8777
1000	102.3749	98.05288	106.6969	95.76495	108.9848

Extract the forecast points

```
forecasts_x2$mean
```


Time Series:

Start = 971

End = 1000

Frequency = 1

```
[1] 102.8393 102.2730 102.4972 102.2039 102.5257 102.3373 102.4015 102.3190
[9] 102.4248 102.3631 102.3794 102.3575 102.3919 102.3713 102.3750 102.3697
[17] 102.3808 102.3738 102.3745 102.3734 102.3770 102.3745 102.3747 102.3745
[25] 102.3757 102.3748 102.3748 102.3748 102.3752 102.3749
```

```
class(forecasts_x2$mean)
```

```
[1] "ts"
```

It is a ts object, not xts! However, the xts objects are more convenient and modern. In terms of plotting the real data and forecast data to compare.

```
forecasts_x2$lower
```

Time Series:

Start = 971

End = 1000

Frequency = 1

	80%	95%
971	101.51445	100.81312
972	100.86707	100.12284
973	100.94235	100.11924
974	100.54844	99.67212
975	100.57887	99.54830
976	100.27407	99.18185
977	100.21039	99.05050
978	100.01093	98.78909
979	99.95891	98.65353
980	99.78713	98.42346
981	99.69382	98.27216
982	99.56504	98.08679
983	99.48463	97.94560
984	99.36505	97.77365
985	99.27200	97.62935
986	99.17140	97.47832
987	99.08693	97.34325
988	98.99061	97.19967
989	98.90408	97.06694
990	98.81713	96.93455
991	98.73610	96.80872
992	98.65212	96.68159
993	98.57235	96.55953
994	98.49367	96.43927
995	98.41764	96.32238
996	98.34144	96.20631
997	98.26746	96.09317

```

998  98.19471  95.98188
999  98.12348  95.87274
1000 98.05288  95.76495

```

```
forecasts_x2$upper
```

Time Series:

Start = 971

End = 1000

Frequency = 1

```

      80%      95%
971 104.1641 104.8654
972 103.6789 104.4231
973 104.0521 104.8753
974 103.8593 104.7356
975 104.4725 105.5031
976 104.4006 105.4928
977 104.5925 105.7524
978 104.6271 105.8490
979 104.8908 106.1961
980 104.9392 106.3028
981 105.0650 106.4866
982 105.1500 106.6283
983 105.2992 106.8382
984 105.3775 106.9689
985 105.4781 107.1207
986 105.5680 107.2611
987 105.6747 107.4184
988 105.7569 107.5478
989 105.8450 107.6821
990 105.9297 107.8123
991 106.0179 107.9453
992 106.0970 108.0675
993 106.1770 108.1898
994 106.2554 108.3098
995 106.3337 108.4290
996 106.4082 108.5433
997 106.4821 108.6564
998 106.5550 108.7678
999 106.6269 108.8777
1000 106.6969 108.9848

```

Create xts object: (We use the second column (95% confidence interval))

```

forecasts_x2_data <- data.frame(f_mean = as.numeric(forecasts_x2$mean),
                                f_lower = as.numeric(forecasts_x2$lower[, 2]),
                                f_upper = as.numeric(forecasts_x2$upper[, 2]))

```

```
head(forecasts_x2_data, 30)
```

	f_mean	f_lower	f_upper
1	102.8393	100.81312	104.8654
2	102.2730	100.12284	104.4231
3	102.4972	100.11924	104.8753
4	102.2039	99.67212	104.7356
5	102.5257	99.54830	105.5031
6	102.3373	99.18185	105.4928
7	102.4015	99.05050	105.7524
8	102.3190	98.78909	105.8490
9	102.4248	98.65353	106.1961
10	102.3631	98.42346	106.3028
11	102.3794	98.27216	106.4866
12	102.3575	98.08679	106.6283
13	102.3919	97.94560	106.8382
14	102.3713	97.77365	106.9689
15	102.3750	97.62935	107.1207
16	102.3697	97.47832	107.2611
17	102.3808	97.34325	107.4184
18	102.3738	97.19967	107.5478
19	102.3745	97.06694	107.6821
20	102.3734	96.93455	107.8123
21	102.3770	96.80872	107.9453
22	102.3745	96.68159	108.0675
23	102.3747	96.55953	108.1898
24	102.3745	96.43927	108.3098
25	102.3757	96.32238	108.4290
26	102.3748	96.20631	108.5433
27	102.3748	96.09317	108.6564
28	102.3748	95.98188	108.7678
29	102.3752	95.87274	108.8777
30	102.3749	95.76495	108.9848

Adding real value X2 below the current Dataset:

```
Data_x2 <- rbind(Data[, "x2"], oos_x2)
tail(Data_x2, n = 40)
```

	x2
2020-11-26	104.92083
2020-11-27	105.73198
2020-11-28	102.96533
2020-11-29	104.55240
2020-11-30	102.02715
2020-12-01	102.72814
2020-12-02	103.50424
2020-12-03	102.42419
2020-12-04	102.83168
2020-12-05	101.93750
2020-12-06	101.99487
2020-12-07	103.34756
2020-12-08	103.38339

```

2020-12-09 102.65495
2020-12-10 103.14643
2020-12-11 101.29482
2020-12-12 102.82414
2020-12-13 102.60263
2020-12-14 102.48089
2020-12-15 102.83048
2020-12-16 103.74833
2020-12-17 104.77165
2020-12-18 102.25474
2020-12-19 101.73088
2020-12-20 102.61676
2020-12-21 100.11499
2020-12-22 100.82839
2020-12-23 101.55266
2020-12-24 101.81775
2020-12-25 101.75292
2020-12-26 100.70069
2020-12-27 102.97327
2020-12-28 103.15157
2020-12-29 102.18853
2020-12-30 99.62443
2020-12-31 104.57416
2021-01-01 102.09902
2021-01-02 100.91690
2021-01-03 98.09291
2021-01-04 102.21843

```

Turn Forecast with Index into Forecast with Date

```

forecasts_x2_xts <- xts(forecasts_x2_data,
                        order.by = index(oos_x2))
forecasts_x2_xts

```

```

      f_mean  f_lower f_upper
2020-12-06 102.8393 100.81312 104.8654
2020-12-07 102.2730 100.12284 104.4231
2020-12-08 102.4972 100.11924 104.8753
2020-12-09 102.2039 99.67212 104.7356
2020-12-10 102.5257 99.54830 105.5031
2020-12-11 102.3373 99.18185 105.4928
2020-12-12 102.4015 99.05050 105.7524
2020-12-13 102.3190 98.78909 105.8490
2020-12-14 102.4248 98.65353 106.1961
2020-12-15 102.3631 98.42346 106.3028
2020-12-16 102.3794 98.27216 106.4866
2020-12-17 102.3575 98.08679 106.6283
2020-12-18 102.3919 97.94560 106.8382
2020-12-19 102.3713 97.77365 106.9689
2020-12-20 102.3750 97.62935 107.1207
2020-12-21 102.3697 97.47832 107.2611

```

```

2020-12-22 102.3808 97.34325 107.4184
2020-12-23 102.3738 97.19967 107.5478
2020-12-24 102.3745 97.06694 107.6821
2020-12-25 102.3734 96.93455 107.8123
2020-12-26 102.3770 96.80872 107.9453
2020-12-27 102.3745 96.68159 108.0675
2020-12-28 102.3747 96.55953 108.1898
2020-12-29 102.3745 96.43927 108.3098
2020-12-30 102.3757 96.32238 108.4290
2020-12-31 102.3748 96.20631 108.5433
2021-01-01 102.3748 96.09317 108.6564
2021-01-02 102.3748 95.98188 108.7678
2021-01-03 102.3752 95.87274 108.8777
2021-01-04 102.3749 95.76495 108.9848

```

Merge it together with the original data

```

Data_x2_combined <- merge(Data_x2, forecasts_x2_xts)
head(Data_x2_combined)

```

```

      x2 f_mean f_lower f_upper
2018-04-11 109.6174      NA      NA      NA
2018-04-12 109.9211      NA      NA      NA
2018-04-13 109.4070      NA      NA      NA
2018-04-14 108.3335      NA      NA      NA
2018-04-15 108.4117      NA      NA      NA
2018-04-16 108.6879      NA      NA      NA

```

```
tail(Data_x2_combined,40)
```

```

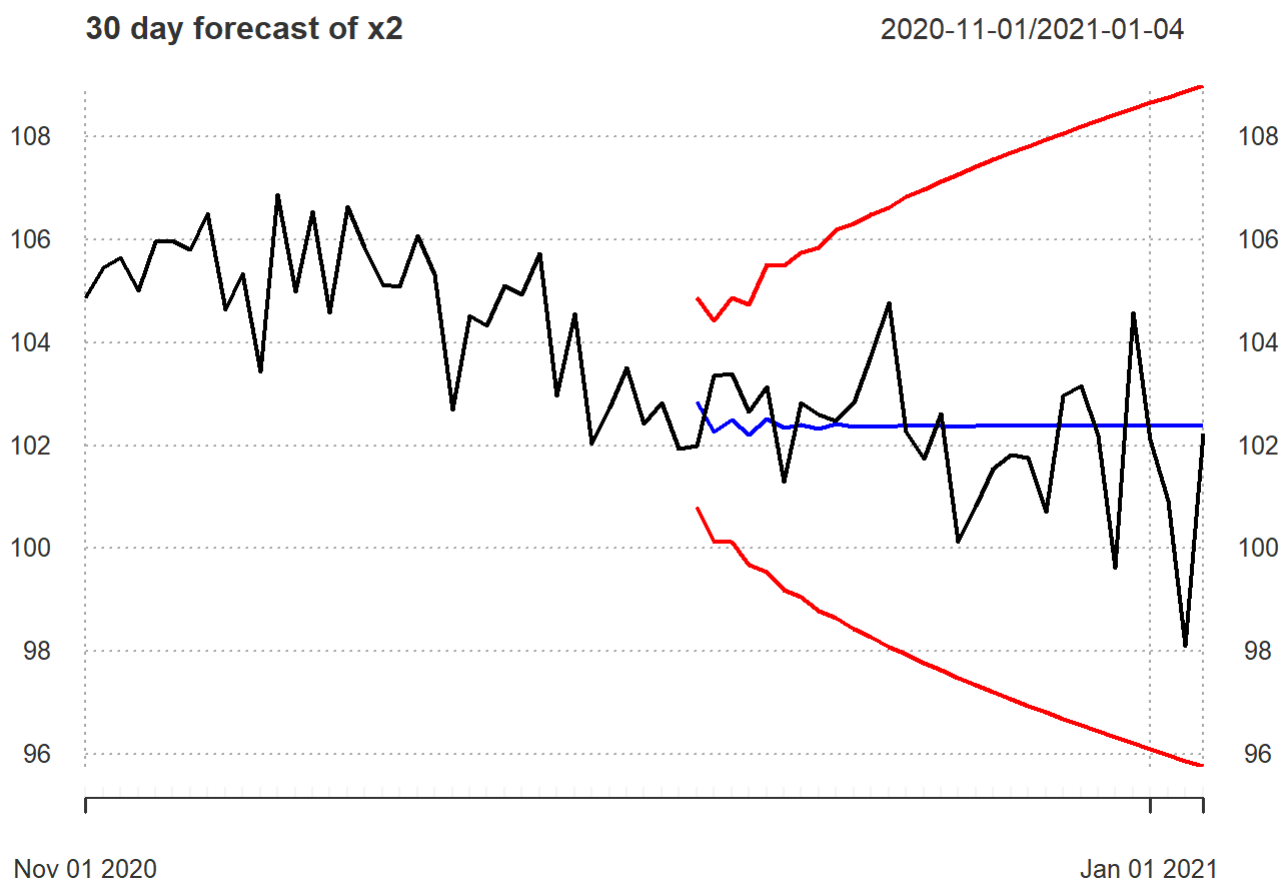
      x2  f_mean  f_lower  f_upper
2020-11-26 104.92083      NA      NA      NA
2020-11-27 105.73198      NA      NA      NA
2020-11-28 102.96533      NA      NA      NA
2020-11-29 104.55240      NA      NA      NA
2020-11-30 102.02715      NA      NA      NA
2020-12-01 102.72814      NA      NA      NA
2020-12-02 103.50424      NA      NA      NA
2020-12-03 102.42419      NA      NA      NA
2020-12-04 102.83168      NA      NA      NA
2020-12-05 101.93750      NA      NA      NA
2020-12-06 101.99487 102.8393 100.81312 104.8654
2020-12-07 103.34756 102.2730 100.12284 104.4231
2020-12-08 103.38339 102.4972 100.11924 104.8753
2020-12-09 102.65495 102.2039 99.67212 104.7356
2020-12-10 103.14643 102.5257 99.54830 105.5031
2020-12-11 101.29482 102.3373 99.18185 105.4928
2020-12-12 102.82414 102.4015 99.05050 105.7524
2020-12-13 102.60263 102.3190 98.78909 105.8490
2020-12-14 102.48089 102.4248 98.65353 106.1961

```

2020-12-15	102.83048	102.3631	98.42346	106.3028
2020-12-16	103.74833	102.3794	98.27216	106.4866
2020-12-17	104.77165	102.3575	98.08679	106.6283
2020-12-18	102.25474	102.3919	97.94560	106.8382
2020-12-19	101.73088	102.3713	97.77365	106.9689
2020-12-20	102.61676	102.3750	97.62935	107.1207
2020-12-21	100.11499	102.3697	97.47832	107.2611
2020-12-22	100.82839	102.3808	97.34325	107.4184
2020-12-23	101.55266	102.3738	97.19967	107.5478
2020-12-24	101.81775	102.3745	97.06694	107.6821
2020-12-25	101.75292	102.3734	96.93455	107.8123
2020-12-26	100.70069	102.3770	96.80872	107.9453
2020-12-27	102.97327	102.3745	96.68159	108.0675
2020-12-28	103.15157	102.3747	96.55953	108.1898
2020-12-29	102.18853	102.3745	96.43927	108.3098
2020-12-30	99.62443	102.3757	96.32238	108.4290
2020-12-31	104.57416	102.3748	96.20631	108.5433
2021-01-01	102.09902	102.3748	96.09317	108.6564
2021-01-02	100.91690	102.3748	95.98188	108.7678
2021-01-03	98.09291	102.3752	95.87274	108.8777
2021-01-04	102.21843	102.3749	95.76495	108.9848

Plot Chart

```
plot(Data_x2_combined ["2020-11/", c("x2", "f_mean", "f_lower", "f_upper")],
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 3,
     main = "30 day forecast of x2",
     col = c("black", "blue", "red", "red"))
```



Extract Data for Evaluating the Forecast:

```
Data_x2_Eva <- tail(Data_x2_combined, 30)
Data_x2_Eva
```

	x2	f_mean	f_lower	f_upper
2020-12-06	101.99487	102.8393	100.81312	104.8654
2020-12-07	103.34756	102.2730	100.12284	104.4231
2020-12-08	103.38339	102.4972	100.11924	104.8753
2020-12-09	102.65495	102.2039	99.67212	104.7356
2020-12-10	103.14643	102.5257	99.54830	105.5031
2020-12-11	101.29482	102.3373	99.18185	105.4928
2020-12-12	102.82414	102.4015	99.05050	105.7524
2020-12-13	102.60263	102.3190	98.78909	105.8490
2020-12-14	102.48089	102.4248	98.65353	106.1961
2020-12-15	102.83048	102.3631	98.42346	106.3028
2020-12-16	103.74833	102.3794	98.27216	106.4866
2020-12-17	104.77165	102.3575	98.08679	106.6283
2020-12-18	102.25474	102.3919	97.94560	106.8382
2020-12-19	101.73088	102.3713	97.77365	106.9689
2020-12-20	102.61676	102.3750	97.62935	107.1207
2020-12-21	100.11499	102.3697	97.47832	107.2611
2020-12-22	100.82839	102.3808	97.34325	107.4184
2020-12-23	101.55266	102.3738	97.19967	107.5478

```

2020-12-24 101.81775 102.3745 97.06694 107.6821
2020-12-25 101.75292 102.3734 96.93455 107.8123
2020-12-26 100.70069 102.3770 96.80872 107.9453
2020-12-27 102.97327 102.3745 96.68159 108.0675
2020-12-28 103.15157 102.3747 96.55953 108.1898
2020-12-29 102.18853 102.3745 96.43927 108.3098
2020-12-30 99.62443 102.3757 96.32238 108.4290
2020-12-31 104.57416 102.3748 96.20631 108.5433
2021-01-01 102.09902 102.3748 96.09317 108.6564
2021-01-02 100.91690 102.3748 95.98188 108.7678
2021-01-03 98.09291 102.3752 95.87274 108.8777
2021-01-04 102.21843 102.3749 95.76495 108.9848

```

```

Data_x2_Eva$mae <- abs(Data_x2_Eva$x2 - Data_x2_Eva$f_mean)
Data_x2_Eva$mse <- (Data_x2_Eva$x2 - Data_x2_Eva$f_mean) ^ 2
Data_x2_Eva$mape <- abs((Data_x2_Eva$x2 - Data_x2_Eva$f_mean)/Data_x2_Eva$x2)
Data_x2_Eva$amape <- abs((Data_x2_Eva$x2 - Data_x2_Eva$f_mean)/(Data_x2_Eva$x2 + Data_x2_Eva$f_m
Data_x2_Eva

```

	x2	f_mean	f_lower	f_upper	mae	mse
2020-12-06	101.99487	102.8393	100.81312	104.8654	0.84440428	0.713018594
2020-12-07	103.34756	102.2730	100.12284	104.4231	1.07459191	1.154747779
2020-12-08	103.38339	102.4972	100.11924	104.8753	0.88613819	0.785240898
2020-12-09	102.65495	102.2039	99.67212	104.7356	0.45108762	0.203480039
2020-12-10	103.14643	102.5257	99.54830	105.5031	0.62075767	0.385340080
2020-12-11	101.29482	102.3373	99.18185	105.4928	1.04250402	1.086814624
2020-12-12	102.82414	102.4015	99.05050	105.7524	0.42267568	0.178654727
2020-12-13	102.60263	102.3190	98.78909	105.8490	0.28359718	0.080427363
2020-12-14	102.48089	102.4248	98.65353	106.1961	0.05605681	0.003142366
2020-12-15	102.83048	102.3631	98.42346	106.3028	0.46733257	0.218399735
2020-12-16	103.74833	102.3794	98.27216	106.4866	1.36894247	1.874003492
2020-12-17	104.77165	102.3575	98.08679	106.6283	2.41410282	5.827892427
2020-12-18	102.25474	102.3919	97.94560	106.8382	0.13717593	0.018817235
2020-12-19	101.73088	102.3713	97.77365	106.9689	0.64039817	0.410109819
2020-12-20	102.61676	102.3750	97.62935	107.1207	0.24173247	0.058434587
2020-12-21	100.11499	102.3697	97.47832	107.2611	2.25470204	5.083681304
2020-12-22	100.82839	102.3808	97.34325	107.4184	1.55243228	2.410045994
2020-12-23	101.55266	102.3738	97.19967	107.5478	0.82109757	0.674201215
2020-12-24	101.81775	102.3745	97.06694	107.6821	0.55676647	0.309988905
2020-12-25	101.75292	102.3734	96.93455	107.8123	0.62050327	0.385024305
2020-12-26	100.70069	102.3770	96.80872	107.9453	1.67632087	2.810051675
2020-12-27	102.97327	102.3745	96.68159	108.0675	0.59873445	0.358482936
2020-12-28	103.15157	102.3747	96.55953	108.1898	0.77691565	0.603597926
2020-12-29	102.18853	102.3745	96.43927	108.3098	0.18600274	0.034597021
2020-12-30	99.62443	102.3757	96.32238	108.4290	2.75124974	7.569375138
2020-12-31	104.57416	102.3748	96.20631	108.5433	2.19935975	4.837183304
2021-01-01	102.09902	102.3748	96.09317	108.6564	0.27578302	0.076056276
2021-01-02	100.91690	102.3748	95.98188	108.7678	1.45794267	2.125596825

	mae	mse	mape	amape
2021-01-03	98.09291	102.3752	95.87274	108.8777
2021-01-04	102.21843	102.3749	95.76495	108.9848
	0.15646094	0.024480026		
2020-12-06	0.0082788895	0.0041223804		
2020-12-07	0.0103978453	0.0052260927		
2020-12-08	0.0085713790	0.0043041357		
2020-12-09	0.0043942120	0.0022019439		
2020-12-10	0.0060182175	0.0030181908		
2020-12-11	0.0102917805	0.0051195456		
2020-12-12	0.0041106659	0.0020595661		
2020-12-13	0.0027640341	0.0013839297		
2020-12-14	0.0005469977	0.0002735737		
2020-12-15	0.0045446891	0.0022775199		
2020-12-16	0.0131948382	0.0066412341		
2020-12-17	0.0230415664	0.0116550586		
2020-12-18	0.0013415117	0.0006703062		
2020-12-19	0.0062950225	0.0031376355		
2020-12-20	0.0023556821	0.0011792300		
2020-12-21	0.0225211227	0.0111351731		
2020-12-22	0.0153967770	0.0076395761		
2020-12-23	0.0080854363	0.0040264404		
2020-12-24	0.0054682654	0.0027266776		
2020-12-25	0.0060981374	0.0030398001		
2020-12-26	0.0166465680	0.0082545788		
2020-12-27	0.0058144646	0.0029157089		
2020-12-28	0.0075317870	0.0037801291		
2020-12-29	0.0018201921	0.0009092685		
2020-12-30	0.0276162157	0.0136200409		
2020-12-31	0.0210315797	0.0106275469		
2021-01-01	0.0027011329	0.0013487449		
2021-01-02	0.0144469619	0.0071716765		
2021-01-03	0.0436555433	0.0213614978		
2021-01-04	0.0015306529	0.0007647412		

```
ARIMA_x2 <- colMeans(Data_x2_Eva[, c("mae", "mse", "mape", "amape")])
apply(Data_x2_Eva[, c("mae", "mse", "mape", "amape")], 2, FUN = median)
```

mae	mse	mape	amape
0.708656911	0.506853873	0.006913405	0.003458882

5. VECM Model and Analysis

5.1. Johansen Cointegration Test

Determine the lag for Johansen test:

```
VARselect(Data[,1:2],
          lag.max = 7)
```

```
$selection
```

```
AIC(n)  HQ(n)  SC(n)  FPE(n)
      6      6      5      6
```

```
$criteria
```

	1	2	3	4	5	6	7
AIC(n)	0.8952490	0.2695796	0.2436797	0.07889688	0.0553301	0.03714266	0.04116647
HQ(n)	0.9068026	0.2888355	0.2706379	0.11355751	0.0976931	0.08720801	0.09893419
SC(n)	0.9255921	0.3201513	0.3144800	0.16992592	0.1665878	0.16862904	0.19288153
FPE(n)	2.4479455	1.3094141	1.2759362	1.08209391	1.0568915	1.03784447	1.04203081

Three out of four tests suggests $K = 6$.

We will choose the $K=6$ lag structure:

```
johan.test.trace <-
ca.jo(Data[,1:2], # data
      ecdet = "const", # "none" for no intercept in cointegrating equation,
                        # "const" for constant term in cointegrating equation and
                        # "trend" for trend variable in cointegrating equation
      type = "trace", # type of the test: trace or eigen
      K = 6           # lag order of the series (levels) in the VAR
)

summary(johan.test.trace)
```

```
#####
# Johansen-Procedure #
#####
```

Test type: trace statistic , without linear trend and constant in cointegration

Eigenvalues (lambda):

```
[1] 5.327670e-02 3.750917e-03 3.797504e-19
```

Values of teststatistic and critical values of test:

	test	10pct	5pct	1pct
$r \leq 1$		3.62	7.52	9.24 12.97
$r = 0$		56.40	17.85	19.96 24.60

Eigenvectors, normalised to first column:

(These are the cointegration relations)

	x1.l6	x2.l6	constant
--	-------	-------	----------

```

x1.l6      1.0000000    1.000000    1.000000
x2.l6      -0.7769762    1.079965   -2.614802
constant -23.6658920  -204.788105  253.716917

```

Weights W:

(This is the loading matrix)

```

           x1.l6      x2.l6      constant
x1.d -0.008001286 -0.0050723664 -2.360290e-18
x2.d  0.127375901 -0.0002914929  9.264392e-17

```

```

cbind(summary(johan.test.trace)@teststat,
summary(johan.test.trace)@cval)

```

```

           10pct  5pct  1pct
r <= 1 |  3.622683  7.52  9.24 12.97
r = 0  | 56.400153 17.85 19.96 24.60

```

Test Statistic < Critical Value: CANNOT reject the null

Test Statistic > Critical Value: reject the null

First we start with $r=0$, (no cointegrating vector): t-test statistic > Critical value: we reject the null hypothesis about NO cointegrating vector.

Next, we test the hypothesis of $r=1$: Test Statistic < Critical Value: we CANNOT reject the null about 1 cointegrating vector.

The model has ONLY 1 cointegrating vector.

```
summary(johan.test.trace)@V
```

```

           x1.l6      x2.l6      constant
x1.l6      1.0000000    1.000000    1.000000
x2.l6      -0.7769762    1.079965   -2.614802
constant -23.6658920  -204.788105  253.716917

```

Weights W:

```
summary(johan.test.trace)@W
```

```

           x1.l6      x2.l6      constant
x1.d -0.008001286 -0.0050723664 -2.360290e-18
x2.d  0.127375901 -0.0002914929  9.264392e-17

```

Check for another type of test: for Eigen:

```

johan.test.eigen <-
  ca.jo(Data[,1:2], # data

```

```

ecdet = "const", # "none" for no intercept in cointegrating equation,
# "const" for constant term in cointegrating equation and
# "trend" for trend variable in cointegrating equation
type = "eigen", # type of the test: trace or eigen
K = 6           # lag order of the series (levels) in the VAR
)
summary(johan.test.eigen)

```

```

#####
# Johansen-Procedure #
#####

```

Test type: maximal eigenvalue statistic (lambda max) , without linear trend and constant in cointegration

Eigenvalues (lambda):

```
[1] 5.327670e-02 3.750917e-03 3.797504e-19
```

Values of teststatistic and critical values of test:

	test	10pct	5pct	1pct
r <= 1		3.62	7.52	9.24 12.97
r = 0		52.78	13.75	15.67 20.20

Eigenvectors, normalised to first column:
(These are the cointegration relations)

	x1.l6	x2.l6	constant
x1.l6	1.0000000	1.000000	1.000000
x2.l6	-0.7769762	1.079965	-2.614802
constant	-23.6658920	-204.788105	253.716917

Weights W:

(This is the loading matrix)

	x1.l6	x2.l6	constant
x1.d	-0.008001286	-0.0050723664	-2.360290e-18
x2.d	0.127375901	-0.0002914929	9.264392e-17

The conclusion stays the same: There is only one cointegrating vector.

5.2. VECM Model

Define the specification of the VECM model, with cointegrating vector from either trace or eigen test from Johansen test.

```

Data.vec6 <- cajorls(johan.test.eigen, # defined specification
                     r = 1) # number of cointegrating vectors

```

```
summary(Data.vec6)
```

```
      Length Class  Mode
rlm    12      mlm    list
beta    3      -none- numeric
```

Summary results:

```
summary(Data.vec6$rlm)
```

Response x1.d :

Call:

```
lm(formula = x1.d ~ ect1 + x1.dl1 + x2.dl1 + x1.dl2 + x2.dl2 +
    x1.dl3 + x2.dl3 + x1.dl4 + x2.dl4 + x1.dl5 + x2.dl5 - 1,
    data = data.mat)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-3.6229 -0.6404 -0.0185  0.6448  3.1156
```

Coefficients:

```
      Estimate Std. Error t value Pr(>|t|)
ect1   -0.008001   0.017896  -0.447  0.65490
x1.dl1  -0.597241   0.032149 -18.577 < 2e-16 ***
x2.dl1   0.022922   0.033115   0.692  0.48899
x1.dl2  -0.169316   0.036961  -4.581 5.24e-06 ***
x2.dl2  -0.011240   0.040525  -0.277  0.78156
x1.dl3  -0.123468   0.037689  -3.276  0.00109 **
x2.dl3  -0.015389   0.042094  -0.366  0.71475
x1.dl4   0.192674   0.038085   5.059 5.05e-07 ***
x2.dl4  -0.072815   0.042015  -1.733  0.08341 .
x1.dl5   0.102480   0.034159   3.000  0.00277 **
x2.dl5  -0.060323   0.035927  -1.679  0.09347 .
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.016 on 953 degrees of freedom

Multiple R-squared: 0.3641, Adjusted R-squared: 0.3567

F-statistic: 49.6 on 11 and 953 DF, p-value: < 2.2e-16

Response x2.d :

Call:

```
lm(formula = x2.d ~ ect1 + x1.dl1 + x2.dl1 + x1.dl2 + x2.dl2 +
    x1.dl3 + x2.dl3 + x1.dl4 + x2.dl4 + x1.dl5 + x2.dl5 - 1,
    data = data.mat)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2.77300	-0.64814	0.01029	0.67165	2.88981

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
ect1	0.127376	0.017424	7.311	5.63e-13 ***
x1.dl1	0.035245	0.031300	1.126	0.2604
x2.dl1	-0.729593	0.032242	-22.629	< 2e-16 ***
x1.dl2	0.198563	0.035986	5.518	4.42e-08 ***
x2.dl2	-0.400538	0.039456	-10.152	< 2e-16 ***
x1.dl3	0.212915	0.036694	5.802	8.89e-09 ***
x2.dl3	-0.303041	0.040983	-7.394	3.11e-13 ***
x1.dl4	0.150872	0.037080	4.069	5.12e-05 ***
x2.dl4	0.079746	0.040906	1.949	0.0515 .
x1.dl5	0.158115	0.033257	4.754	2.30e-06 ***
x2.dl5	-0.005628	0.034979	-0.161	0.8722

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9889 on 953 degrees of freedom

Multiple R-squared: 0.4524, Adjusted R-squared: 0.446

F-statistic: 71.56 on 11 and 953 DF, p-value: < 2.2e-16

extract the cointegrating vector:

```
Data.vec6$beta
```

	ect1
x1.l6	1.0000000
x2.l6	-0.7769762
constant	-23.6658920

extract the adjustment coefficients (check for sign to determine whether ECM works or not):

```
johan.test.eigen@W
```

	x1.l6	x2.l6	constant
x1.d	-0.008001286	-0.0050723664	-2.360290e-18
x2.d	0.127375901	-0.0002914929	9.264392e-17

Conclusion about whether Error Correction Mechanism work: The adjustment Coeff has different sign here -> ECM works.

Reparametrizing the VEC model into VAR:

```
Data.vec6.asVAR <- vec2var(johan.test.eigen, r = 1)
```

Check result:

```
Data.vec6.asVAR
```

Coefficient matrix of lagged endogenous variables:

A1:

	x1.l1	x2.l1
x1	0.40275879	0.02292209
x2	0.03524544	0.27040741

A2:

	x1.l2	x2.l2
x1	0.4279249	-0.03416221
x2	0.1633176	0.32905429

A3:

	x1.l3	x2.l3
x1	0.04584829	-0.004149253
x2	0.01435199	0.097497511

A4:

	x1.l4	x2.l4
x1	0.31614231	-0.05742534
x2	-0.06204285	0.38278641

A5:

	x1.l5	x2.l5
x1	-0.090194324	0.01249198
x2	0.007242869	-0.08537378

A6:

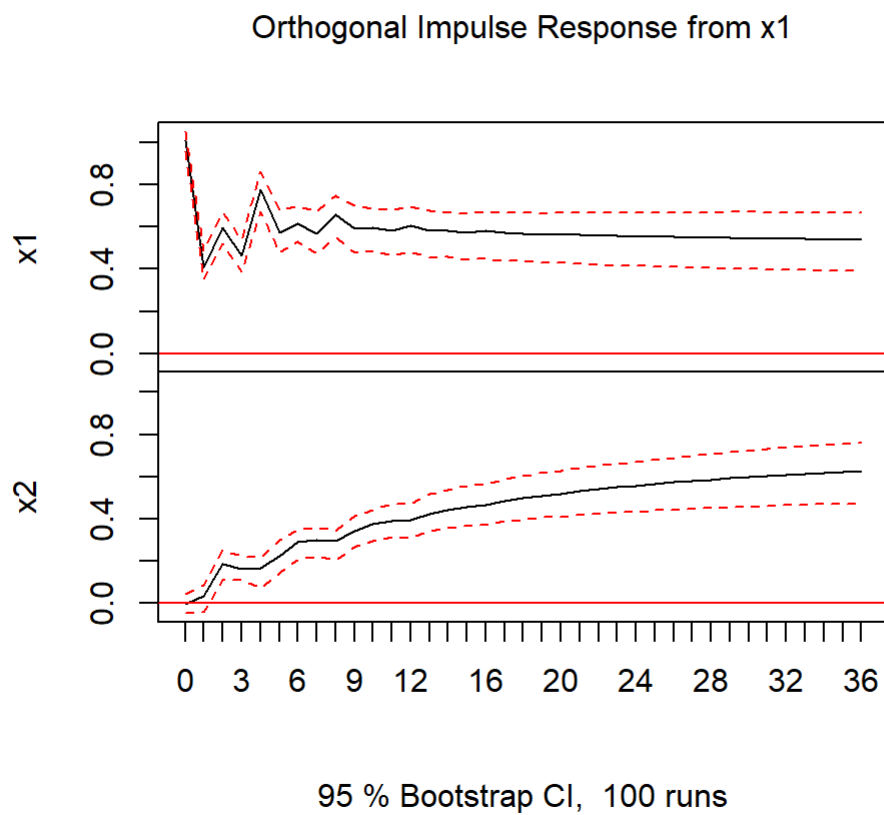
	x1.l6	x2.l6
x1	-0.11048130	0.06653954
x2	-0.03073914	-0.09333988

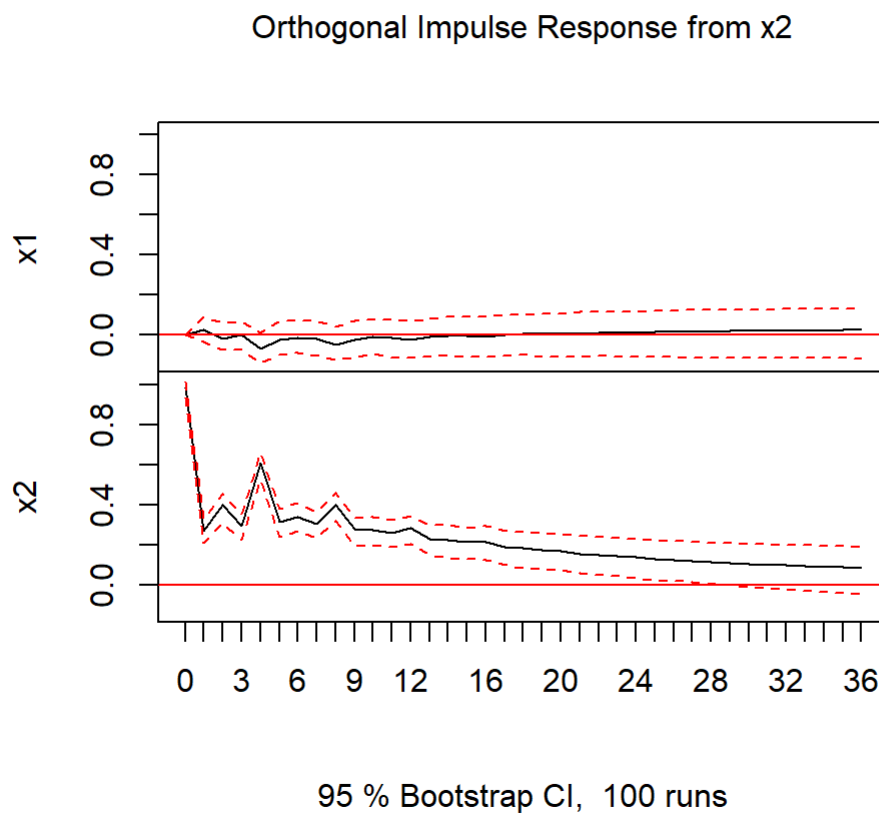
Coefficient matrix of deterministic regressor(s).

	constant
x1	0.1893576
x2	-3.0144643

Calculate and plot Impulse Response Functions:

```
plot(irf(Data.vec6.asVAR, n.ahead = 36), ask = FALSE)
```



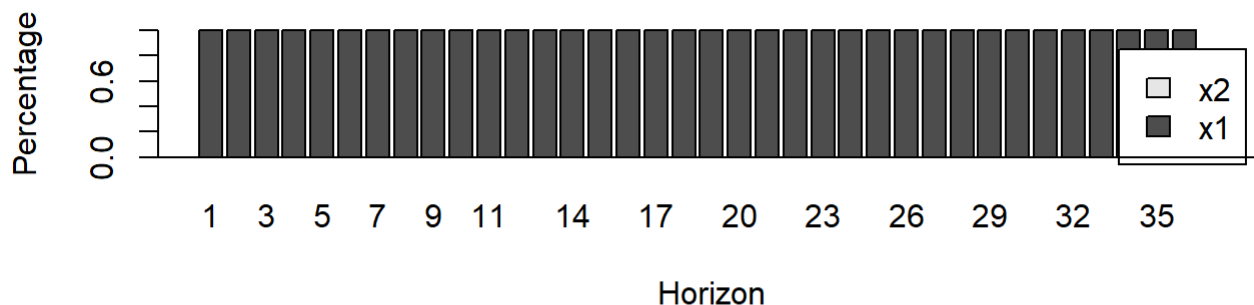


The residuals seem to be stable: first increases then decreases.

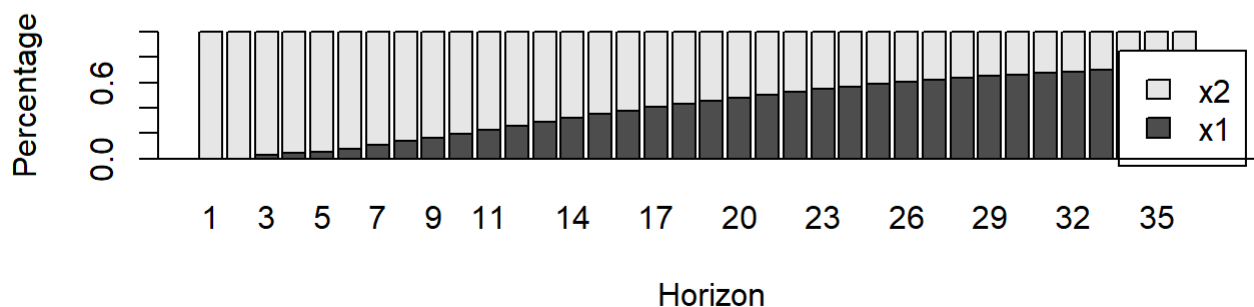
Perform variance decomposition:

```
plot(fevd(Data.vec6.asVAR, n.ahead = 36), ask = FALSE)
```

FEVD for x1



FEVD for x2



Check if model residuals are autocorrelated or not: Residuals can be extracted only from the VAR reparametrized model.

```
head(residuals(Data.vec6.asVAR))
```

	resids of x1	resids of x2
[1,]	-0.8484499	-0.7442952
[2,]	-0.4114669	-0.6779789
[3,]	0.1044630	-1.4051325
[4,]	-0.7057218	1.1074373
[5,]	0.7418924	-0.7982018
[6,]	-0.1534021	-0.4547940

```
serial.test(Data.vec6.asVAR)
```

Portmanteau Test (asymptotic)

data: Residuals of VAR object Data.vec6.asVAR
Chi-squared = 37.666, df = 42, p-value = 0.6616

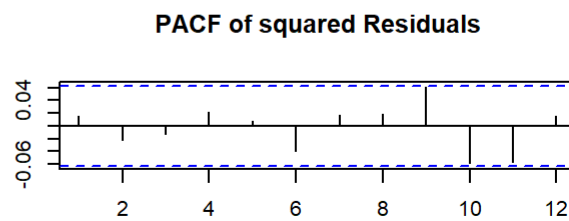
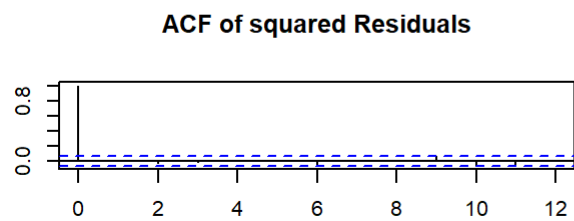
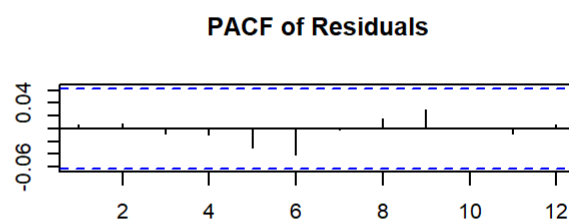
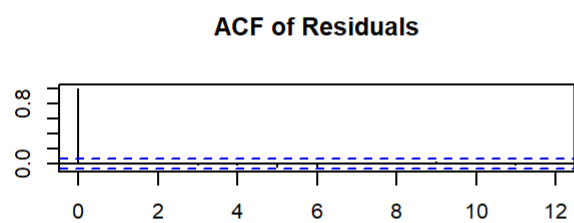
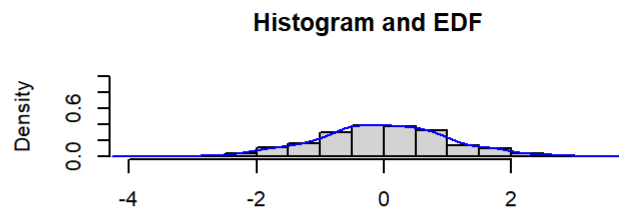
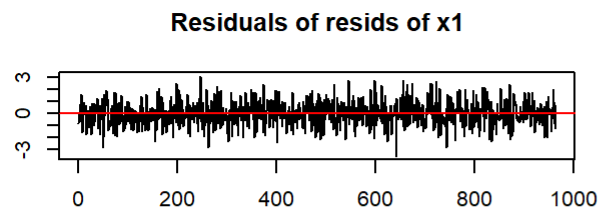
p-value = 0.6616 > p-critical = 5%

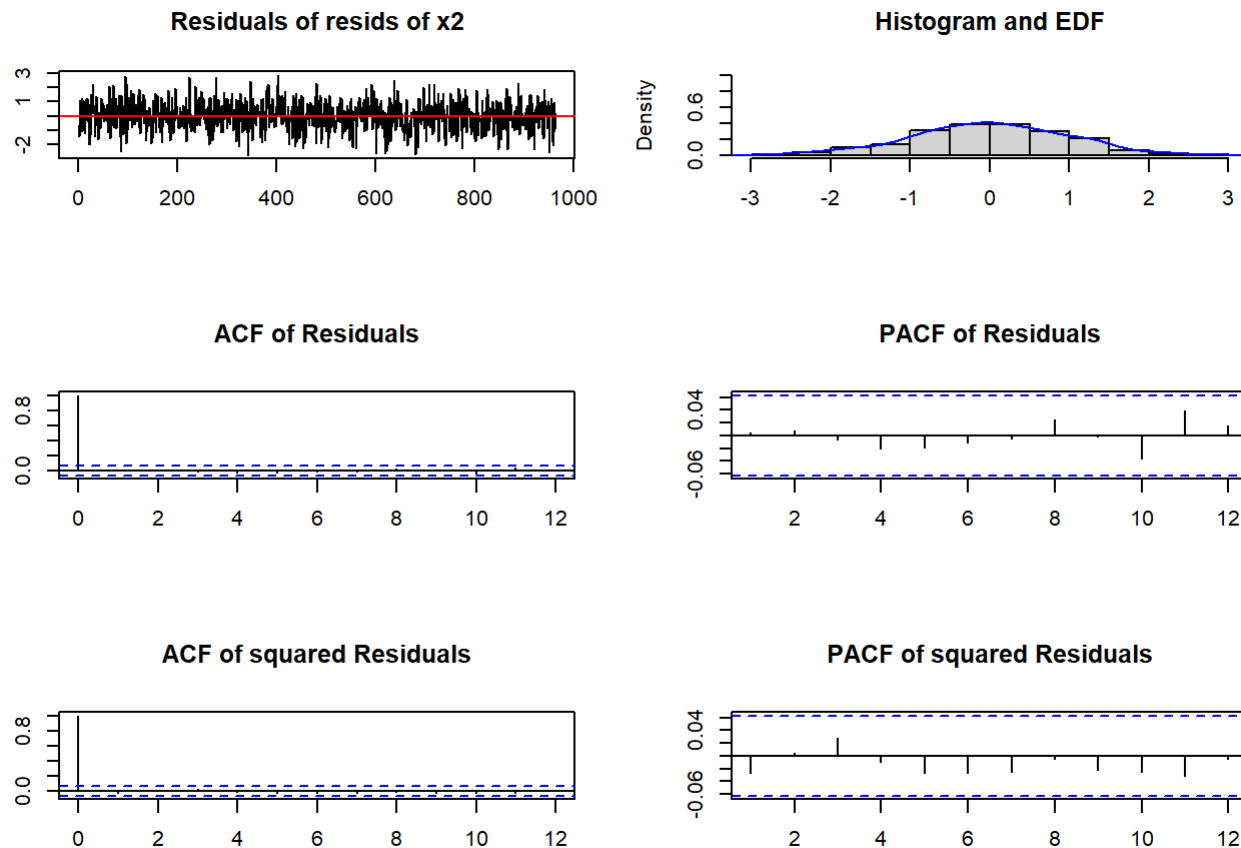
The null about no-autocorrelation is fail to Reject.

=> There is no auto-correlation in Residuals.

Plot ACF and PACF for the model:

```
plot(serial.test(Data.vec6.asVAR))
```





Checking the Normality for x1 and x2 by creating Histogram:

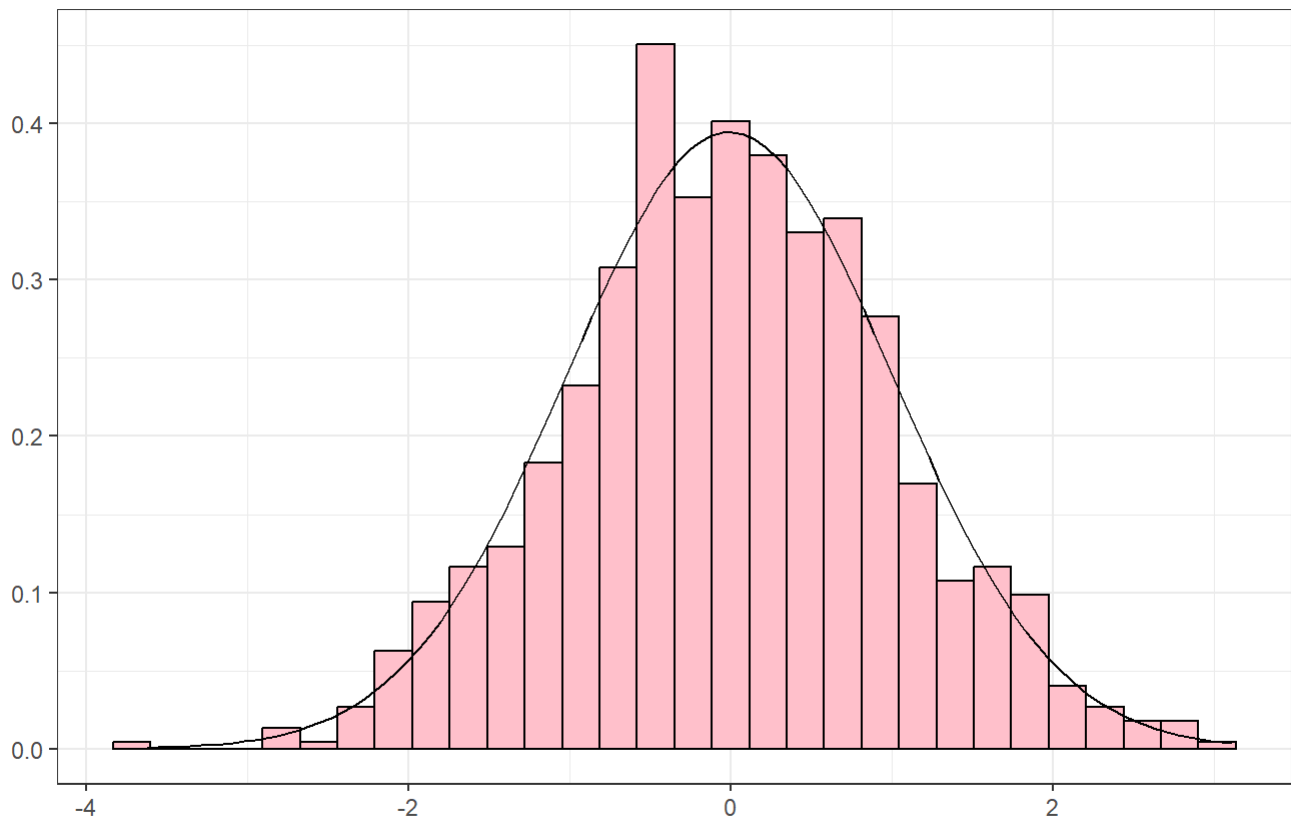
```
Data.vec6.asVAR %>%
  residuals() %>%
  as_tibble() %>%
  ggplot(aes(`resids of x1`)) +
  geom_histogram(aes(y = ..density..),
                 colour = "black",
                 fill = "pink") +
  stat_function(fun = dnorm,
               args = list(mean = mean(residuals(Data.vec6.asVAR)[, 1]),
                           sd = sd(residuals(Data.vec6.asVAR)[, 1]))) +
  theme_bw() +
  labs(
    title = "Density of x1 residuals",
    y = "", x = "",
    caption = "source: own calculations"
  )
```

Warning: The dot-dot notation (`..density..`) was deprecated in ggplot2 3.4.0.

! Please use `after_stat(density)` instead.

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

Density of x1 residuals

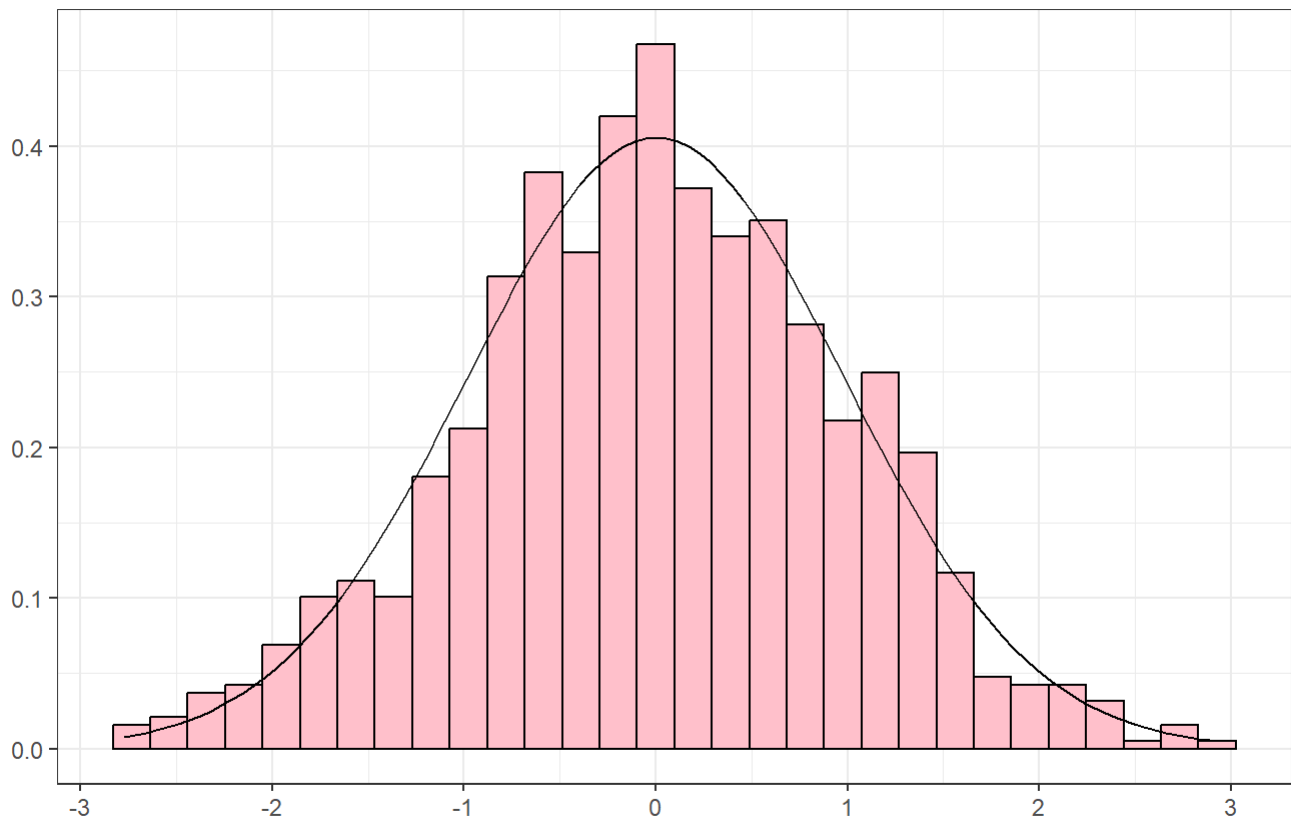


source: own calculations

```
Data.vec6.asVAR %>%
  residuals() %>%
  as_tibble() %>%
  ggplot(aes(`resids of x2`)) +
  geom_histogram(aes(y = ..density..),
    colour = "black",
    fill = "pink") +
  stat_function(fun = dnorm,
    args = list(mean = mean(residuals(Data.vec6.asVAR)[, 2]),
      sd = sd(residuals(Data.vec6.asVAR)[, 2]))) +
  theme_bw() +
  labs(
    title = "Density of x2 residuals",
    y = "", x = "",
    caption = "source: own calculations"
  )
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

Density of x2 residuals



source: own calculations

We can also check it formally by using the Jarque-Bera (JB) test.

```
normality.test(Data.vec6.asVAR)
```

```
$JB
```

```
JB-Test (multivariate)
```

```
data: Residuals of VAR object Data.vec6.asVAR
```

```
Chi-squared = 0.8198, df = 4, p-value = 0.9358
```

```
$Skewness
```

```
Skewness only (multivariate)
```

```
data: Residuals of VAR object Data.vec6.asVAR
```

```
Chi-squared = 0.52028, df = 2, p-value = 0.7709
```

```
$Kurtosis
```

```
Kurtosis only (multivariate)
```

data: Residuals of VAR object Data.vec6.asVAR
 Chi-squared = 0.29952, df = 2, p-value = 0.8609
 p-value > 0.05 => Fail to reject Ho about the normality

Conclusion: the residual has a normal distribution.

5.3 VECM Forecasting

```
Data.vec6.fore <-
  predict(
    vec2var(
      johan.test.eigen,
      r = 1),      # no of cointegrating vectors
    n.ahead = 30, # forecast horizon
    ci = 0.95)    # confidence level for intervals

summary(Data.vec6.fore)
```

	Length	Class	Mode
fcst	2	-none-	list
endog	1940	-none-	numeric
model	12	vec2var	list
exo.fcst	0	-none-	NULL

VEC forecasts for x1

```
Data.vec6.fore$fcst$x1
```

	fcst	lower	upper	CI
[1,]	104.4753	102.49586	106.4547	1.979405
[2,]	103.6826	101.54823	105.8169	2.134341
[3,]	104.4214	101.98719	106.8555	2.434163
[4,]	103.7435	101.14509	106.3419	2.598400
[5,]	104.1078	101.09226	107.1234	3.015553
[6,]	103.8848	100.66734	107.1023	3.217457
[7,]	104.1109	100.67626	107.5455	3.434638
[8,]	103.9060	100.29479	107.5172	3.611197
[9,]	103.9979	100.16297	107.8328	3.834915
[10,]	103.9318	99.92567	107.9380	4.006172
[11,]	104.0007	99.82811	108.1733	4.172580
[12,]	103.9341	99.60896	108.2593	4.325175
[13,]	103.9541	99.46903	108.4392	4.485065
[14,]	103.9335	99.30547	108.5615	4.628038
[15,]	103.9530	99.18763	108.7184	4.765388
[16,]	103.9295	99.03302	108.8259	4.896441
[17,]	103.9313	98.90472	108.9580	5.026615
[18,]	103.9240	98.77437	109.0736	5.149603
[19,]	103.9282	98.65988	109.1966	5.268371

```
[20,] 103.9187 98.53532 109.3021 5.383397
[21,] 103.9168 98.42060 109.4130 5.496178
[22,] 103.9133 98.30828 109.5184 5.605040
[23,] 103.9132 98.20229 109.6240 5.710869
[24,] 103.9086 98.09446 109.7226 5.814089
[25,] 103.9063 97.99114 109.8215 5.915171
[26,] 103.9041 97.89045 109.9178 6.013697
[27,] 103.9029 97.79293 110.0129 6.109991
[28,] 103.9003 97.69596 110.1046 6.204314
[29,] 103.8984 97.60157 110.1953 6.296853
[30,] 103.8968 97.50931 110.2844 6.387536
```

VEC forecasts for x2

```
Data.vec6.fore$fcst$x2
```

```
      fcst      lower      upper      CI
[1,] 103.0279 101.10074 104.9551 1.927173
[2,] 102.3982 100.40064 104.3958 1.997568
[3,] 102.6393 100.46457 104.8139 2.174685
[4,] 102.4612 100.18883 104.7336 2.272381
[5,] 102.8081 100.22055 105.3957 2.587584
[6,] 102.6593 99.96367 105.3549 2.695634
[7,] 102.7086 99.87473 105.5425 2.833893
[8,] 102.7254 99.77097 105.6799 2.954443
[9,] 102.8334 99.72424 105.9427 3.109206
[10,] 102.8134 99.58622 106.0405 3.227131
[11,] 102.8272 99.47429 106.1801 3.352919
[12,] 102.8651 99.38871 106.3416 3.476436
[13,] 102.9071 99.30352 106.5108 3.603621
[14,] 102.9157 99.18963 106.6418 3.726104
[15,] 102.9265 99.07559 106.7774 3.850920
[16,] 102.9558 98.97979 106.9319 3.976046
[17,] 102.9779 98.87713 107.0787 4.100780
[18,] 102.9907 98.76475 107.2167 4.225966
[19,] 103.0015 98.64961 107.3534 4.351898
[20,] 103.0215 98.54342 107.4996 4.478083
[21,] 103.0360 98.43237 107.6397 4.603656
[22,] 103.0475 98.31773 107.7772 4.729756
[23,] 103.0573 98.20144 107.9132 4.855899
[24,] 103.0710 98.08911 108.0529 4.981873
[25,] 103.0816 97.97432 108.1888 5.107234
[26,] 103.0908 97.85826 108.3234 5.232548
[27,] 103.0991 97.74162 108.4566 5.357468
[28,] 103.1087 97.62681 108.5905 5.481867
[29,] 103.1167 97.51110 108.7222 5.605549
[30,] 103.1239 97.39514 108.8526 5.728736
```

Lets store it as an xts object. The correct set of dates (index) can be extracted from the out_of_sample xts data object.


```
tail(index(out_of_sample), 30)
```

```
[1] "2020-12-06" "2020-12-07" "2020-12-08" "2020-12-09" "2020-12-10"
[6] "2020-12-11" "2020-12-12" "2020-12-13" "2020-12-14" "2020-12-15"
[11] "2020-12-16" "2020-12-17" "2020-12-18" "2020-12-19" "2020-12-20"
[16] "2020-12-21" "2020-12-22" "2020-12-23" "2020-12-24" "2020-12-25"
[21] "2020-12-26" "2020-12-27" "2020-12-28" "2020-12-29" "2020-12-30"
[26] "2020-12-31" "2021-01-01" "2021-01-02" "2021-01-03" "2021-01-04"
```

```
x1_forecast <- xts(Data.vec6.fore$fcst$x1[, -4],
  # we exclude the last column with CI
  tail(index(out_of_sample), 30))
```

Correction of the variable names:

```
names(x1_forecast) <- c("x1_fore", "x1_lower", "x1_upper")
```

Apply similarly for x2:

```
x2_forecast <- xts(Data.vec6.fore$fcst$x2[, -4],
  # we exclude the last column with CI
  tail(index(out_of_sample), 30))

names(x2_forecast) <- c("x2_fore", "x2_lower", "x2_upper")
```

Merge forecast into original data:

```
Data.fore <- merge(out_of_sample[, 1:2],
  x1_forecast,
  x2_forecast)
```

```
tail(Data.fore, 40)
```

	x1	x2	x1_fore	x1_lower	x1_upper	x2_fore	x2_lower
2020-12-06	104.6044	101.99487	104.4753	102.49586	106.4547	103.0279	101.10074
2020-12-07	103.6145	103.34756	103.6826	101.54823	105.8169	102.3982	100.40064
2020-12-08	102.8175	103.38339	104.4214	101.98719	106.8555	102.6393	100.46457
2020-12-09	101.3033	102.65495	103.7435	101.14509	106.3419	102.4612	100.18883
2020-12-10	102.4395	103.14643	104.1078	101.09226	107.1234	102.8081	100.22055
2020-12-11	103.7279	101.29482	103.8848	100.66734	107.1023	102.6593	99.96367
2020-12-12	102.8575	102.82414	104.1109	100.67626	107.5455	102.7086	99.87473
2020-12-13	103.0086	102.60263	103.9060	100.29479	107.5172	102.7254	99.77097
2020-12-14	102.8093	102.48089	103.9979	100.16297	107.8328	102.8334	99.72424
2020-12-15	103.5626	102.83048	103.9318	99.92567	107.9380	102.8134	99.58622
2020-12-16	102.5536	103.74833	104.0007	99.82811	108.1733	102.8272	99.47429
2020-12-17	104.9300	104.77165	103.9341	99.60896	108.2593	102.8651	99.38871

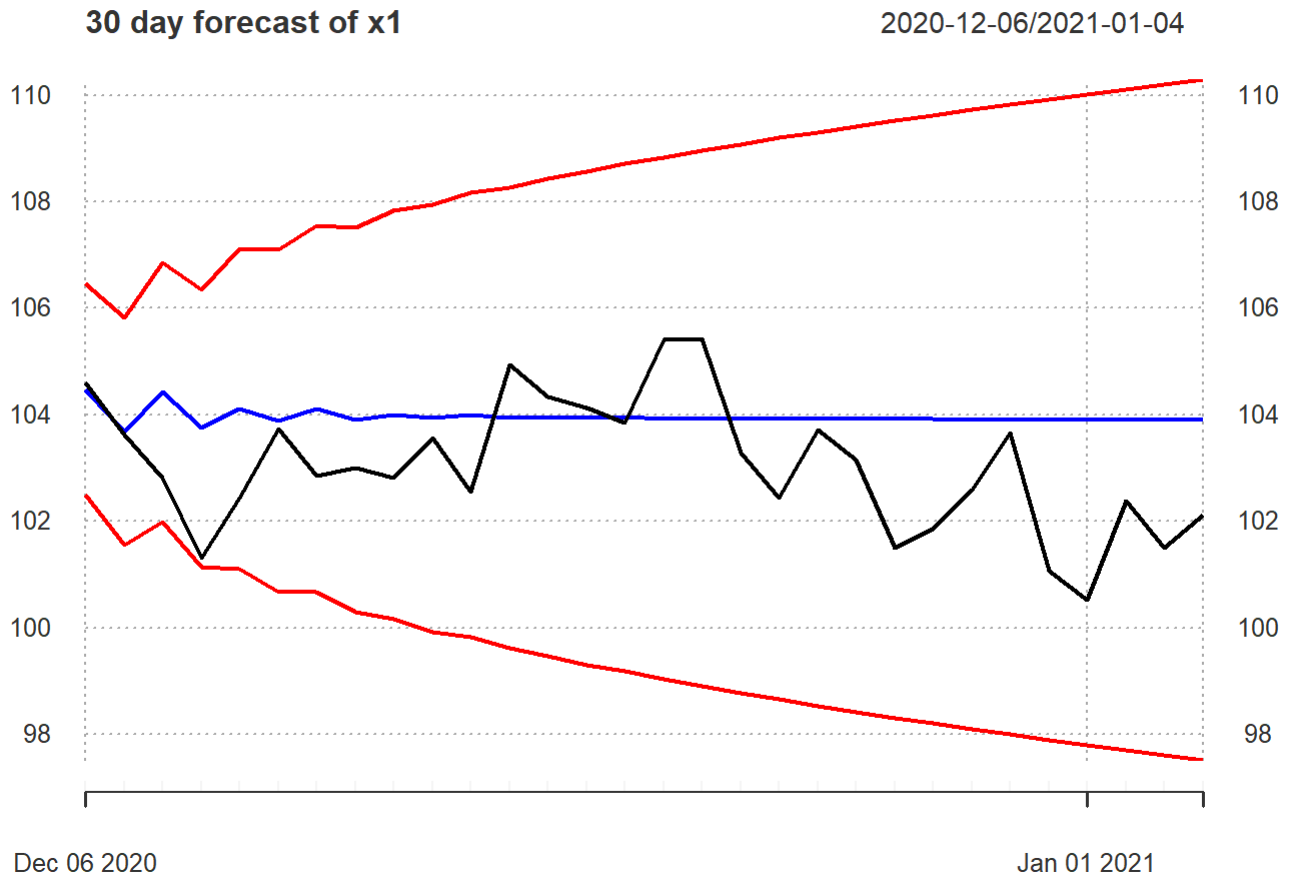
2020-12-18	104.3301	102.25474	103.9541	99.46903	108.4392	102.9071	99.30352
2020-12-19	104.1219	101.73088	103.9335	99.30547	108.5615	102.9157	99.18963
2020-12-20	103.8526	102.61676	103.9530	99.18763	108.7184	102.9265	99.07559
2020-12-21	105.4009	100.11499	103.9295	99.03302	108.8259	102.9558	98.97979
2020-12-22	105.4246	100.82839	103.9313	98.90472	108.9580	102.9779	98.87713
2020-12-23	103.2777	101.55266	103.9240	98.77437	109.0736	102.9907	98.76475
2020-12-24	102.4349	101.81775	103.9282	98.65988	109.1966	103.0015	98.64961
2020-12-25	103.7180	101.75292	103.9187	98.53532	109.3021	103.0215	98.54342
2020-12-26	103.1486	100.70069	103.9168	98.42060	109.4130	103.0360	98.43237
2020-12-27	101.4948	102.97327	103.9133	98.30828	109.5184	103.0475	98.31773
2020-12-28	101.8467	103.15157	103.9132	98.20229	109.6240	103.0573	98.20144
2020-12-29	102.5797	102.18853	103.9086	98.09446	109.7226	103.0710	98.08911
2020-12-30	103.6637	99.62443	103.9063	97.99114	109.8215	103.0816	97.97432
2020-12-31	101.0638	104.57416	103.9041	97.89045	109.9178	103.0908	97.85826
2021-01-01	100.5104	102.09902	103.9029	97.79293	110.0129	103.0991	97.74162
2021-01-02	102.3766	100.91690	103.9003	97.69596	110.1046	103.1087	97.62681
2021-01-03	101.5015	98.09291	103.8984	97.60157	110.1953	103.1167	97.51110
2021-01-04	102.1252	102.21843	103.8968	97.50931	110.2844	103.1239	97.39514

x2_upper

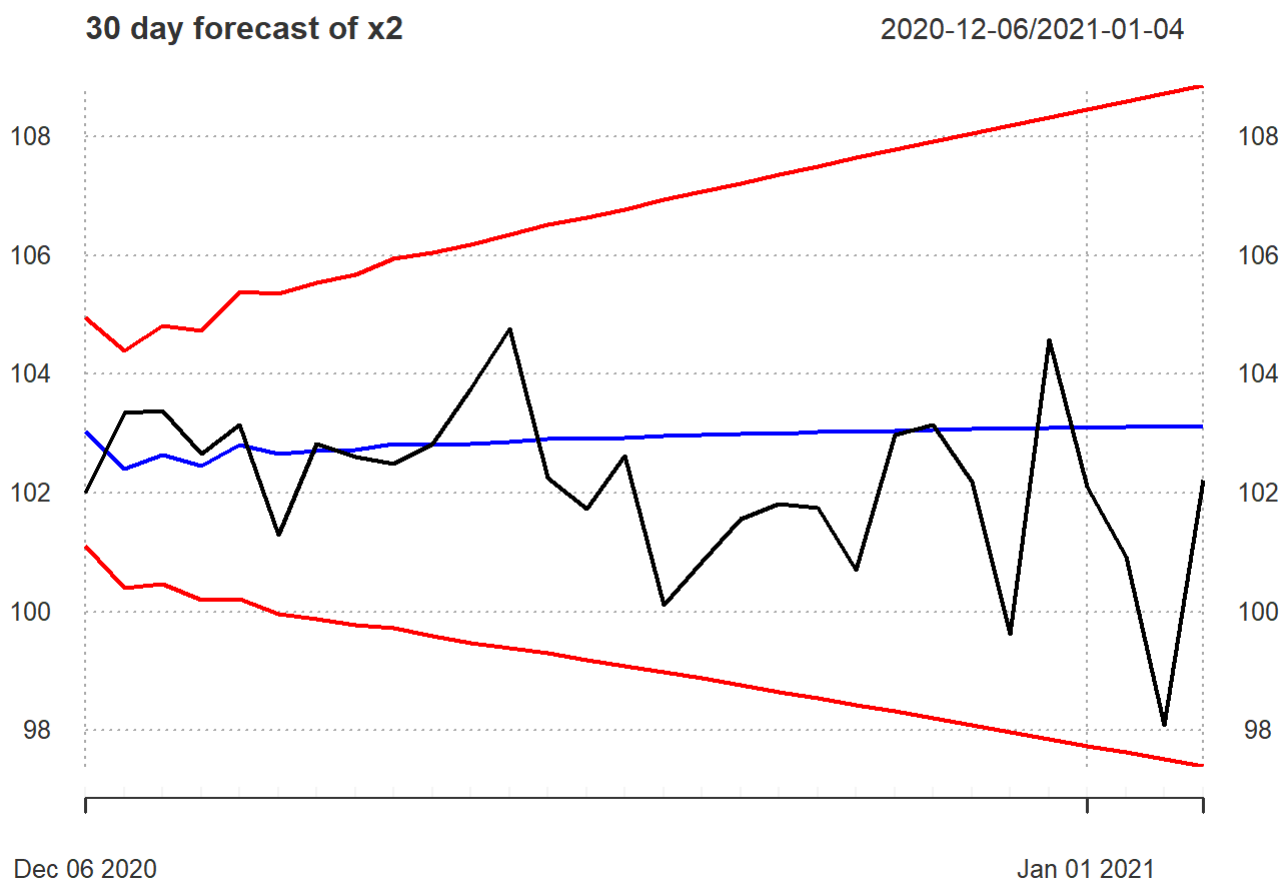
2020-12-06	104.9551
2020-12-07	104.3958
2020-12-08	104.8139
2020-12-09	104.7336
2020-12-10	105.3957
2020-12-11	105.3549
2020-12-12	105.5425
2020-12-13	105.6799
2020-12-14	105.9427
2020-12-15	106.0405
2020-12-16	106.1801
2020-12-17	106.3416
2020-12-18	106.5108
2020-12-19	106.6418
2020-12-20	106.7774
2020-12-21	106.9319
2020-12-22	107.0787
2020-12-23	107.2167
2020-12-24	107.3534
2020-12-25	107.4996
2020-12-26	107.6397
2020-12-27	107.7772
2020-12-28	107.9132
2020-12-29	108.0529
2020-12-30	108.1888
2020-12-31	108.3234
2021-01-01	108.4566
2021-01-02	108.5905
2021-01-03	108.7222
2021-01-04	108.8526

Plot chart for Forecast:

```
plot(Data.fore ["2020-11/", c("x1", "x1_fore", "x1_lower", "x1_upper")],
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 3,
     main = "30 day forecast of x1",
     col = c("black", "blue", "red", "red"))
```



```
plot(Data.fore ["2020-11/", c("x2", "x2_fore", "x2_lower", "x2_upper")],
     major.ticks = "years",
     grid.ticks.on = "years",
     grid.ticks.lty = 3,
     main = "30 day forecast of x2",
     col = c("black", "blue", "red", "red"))
```



5.4. Evaluate Forecast Accuracy

Extract the out-of-sample data to evaluate:

```
Data.fore2 <- Data.fore[, -30]
```

```
Data.fore2$mae.x1 <- abs(Data.fore2$x1 - Data.fore2$x1_fore)
Data.fore2$mse.x1 <- (Data.fore2$x1 - Data.fore2$x1_fore)^2
Data.fore2$mape.x1 <- abs(Data.fore2$x1 - Data.fore2$x1_fore)/Data.fore2$x1
Data.fore2$amape.x1 <- abs(Data.fore2$x1 - Data.fore2$x1_fore)/(Data.fore2$x1 + Data.fore2$x1_fore)
```

```
Data.fore2$mae.x2 <- abs(Data.fore2$x2 - Data.fore2$x2_fore)
Data.fore2$mse.x2 <- (Data.fore2$x2 - Data.fore2$x2_fore)^2
Data.fore2$mape.x2 <- abs(Data.fore2$x2 - Data.fore2$x2_fore)/Data.fore2$x2
Data.fore2$amape.x2 <- abs(Data.fore2$x2 - Data.fore2$x2_fore)/(Data.fore2$x2 + Data.fore2$x2_fore)
```

and calculate its averages

```
VECM_x1 <- colMeans(Data.fore2[, c("mae.x1",
                                   "mse.x1",
                                   "mape.x1",
                                   "amape.x1")], na.rm = TRUE)
```

```
VECM_x2 <- colMeans(Data.fore2[, c("mae.x2",
                                   "mse.x2",
                                   "mape.x2",
                                   "amape.x2")], na.rm = TRUE)
```

6. Comparing Models

Comparing VAR model's forecasts with ARIMAs:

```
result <- rbind(ARIMA_x1, VECM_x1, ARIMA_x2, VECM_x2)

result %>%
  knitr::kable(digits = 4) %>%
  kableExtra::kable_styling(full_width = F,
                             bootstrap_options = c("striped",
                                                     "hover",
                                                     "condensed"))
```

	mae	mse	mape	amape
ARIMA_x1	1.2821	2.4851	0.0125	0.0062
VECM_x1	1.2313	2.2985	0.0120	0.0060
ARIMA_x2	1.0373	1.9546	0.0102	0.0051
VECM_x2	1.2178	2.7043	0.0120	0.0060

Conclusion:

For Timeseries x1: The forecasts from VECM outperforms that of ARIMA.

For Timeseries x2: The forecasts from ARIMA model outperforms that of VECM.