Linear Algebra

1. Scalars

Scalars are single numbers used in basic arithmetic operation.

example 1: calculate body mass index (BMI)

problem: compute BMI given weight = 70kg, height = 175

formula: $BMI = \frac{weight}{height^2}$

- Identify scalars: weight = 70, height = 1.75

- $BMI = \frac{weight}{height^2} = \frac{70}{1.75^2} = 22.86$

2. Vectors

Vectors are arrays of scalars

example 1 : calculate total calories burined.

problem : durations = [0.5, 1.0, 0.75] hours,

calories per hour = [600, 400, 500]

Compute total calories : sum (duration × calories)

- total calories = $0.5 \times 600 + 1.0 \times 400 + 0.75 \times 500$

$= 1075$.

3. Matrices

Matrices are 2D arrays

example : sales data analysis

problem : sales = [[100, 120, 110], [80, 90, 85], [150, 130, 140]]

Compute total sales per product (sum rows)

- Sum each row :

+ product A : 100 + 120 + 110 = 330

+ product B : 80 + 90 + 85 = 255

+ product C : 150 + 130 + 140 = 420

- Total sales = [330, 255, 420].

4. Tensors

Tensors are higher-order arrays

example : batch of grayscale images

problem : Tensor = 2 images of 3×3, first image all 1s, second all 2s
find pixel (1,1) of first image

- tensor structure : shape [2,3,3],
  first image [[1,1,1], [1,1,1], [1,1,1]]
  second image [[2,2,2], ...]

- result: pixel (1,1) = 1.

5. Basic properties of tensor arithmetic

example : image contrast adjustment

problem : image = [[100, 150], [200, 250]],

contrast factor = 1.2

- multiply each element

+ 100 × 1.2 = 120

+ 150 × 1.2 = 180

+ 200 × 1.2 = 240

+ 250 × 1.2 = 300

- result : adjusted image = [[120, 180], [240, 300]] .

6. Reduction

Reduction operations aggregate elements

example : Average exam score

problem : scores = [[85, 90, 88], [78, 82, 80]]

- calculate average score

+ student 1 : $\frac{85 + 90 + 88}{3} = 87.67$

+ student 2 : $\frac{78 + 82 + 80}{3} = 80$

- result : Averages = [87.67, 80]

7. non - reduction sum

non - reduction sums keep axes for broadcasting.

example : normalized sales data

problem : sales = [[100, 120, 110], [80, 90, 75]]

normalize each row to sum to 1.

- Sum each row

product A : 100 + 120 + 110 = 330

product B : 80 + 90 + 75 = 255

2. normalize

product A :

[100 : 330, 120 : 330, 110 : 330]

= [0.3030, 0. 3636, 0. 3333]

product B :

[80 : 255, 90 : 255, 85 : 255]

= [0.3137, 0.3529, 0. 3333]

result : normalize = [[0.3030, 0.3636, 0.3333], [0.3137, 0. 3529, 0.3333]]

8. dot products

dot products is the sum of elementwise products

example : weighted exam score

problem : scores = [85, 90, 88], weights = [0.4, 0.3, 0.3]

- dot product = $85 \times 0.4 + 90 \times 0.3 + 88 \times 0.3$

= 87, 4

9. Matrix - vector products

Matrix - vector products transform vectors

example : neural network layer

problem. weight = [[0.5, 0.3], [0.2, 0.4]]

- dot product. row 1 : $0.5 \times 1 + 0.3 \times 2 = 1.1$

row 2 : $0.2 \times 1 + 0.4 \times 2 = 1.0$

- result : output = [1.1, 1.0].

10. matrix - matrix multiplication

matrix multiplication computes dot products of rows and columns

example : neural network layer composition.

problem : Layer 1 = [[0.5, 0.3], [0.2, 0.4]]

Layer 2 = [[0.1, 0.6], [0.7, 0.2]]

compute composition.

- result matrix : [2, 2]

+ C [0, 0] : 0.5 × 0.1 + 0.3 × 0.7 = 0.26

+ C [0, 1] : 0.5 × 0.6 + 0.3 × 0.2 = 0.36

+ C [1, 0] : 0.2 × 0.1 + 0.4 × 0.7 = 0.30

+ C [1, 1] : 0.2 × 0.6 + 0.4 × 0.2 = 0.20

- result : composition = [[0.26, 0.36], [0.30, 0.20]]

11. norms

norms measure magnitude

example 1: error in predictions

problem: predicted = [2.5, 0, 2.1]

actual = [3, -0.5, 2]

compute l_1 and l_2 norm of error

- error : $[2.5 - 3, 0 - (-0.5), 2.1 - 2]$

$= [-0.5, 0.5, 0.1]$.

- l_1 norm : $|-0.5| + |0.5| + |0.1|$

$= 1.1$

- l_2 norm : $\sqrt{(-0.5)^2 + 0.5^2 + 0.1^2}$ $= 0.7141$

- result : $l_1 = 1.1$, $l_2 = 0.7141$.

example2 : matrix regularization

problem : weight $= [[0.5, 0.3], [0.2, 0.4]]$

compute Frobenius norm

$$\text{Frobenius norm} = \sqrt{0.5^2 + 0.3^2 + 0.2^2 + 0.4^2}$$

$$= 0.78$$