

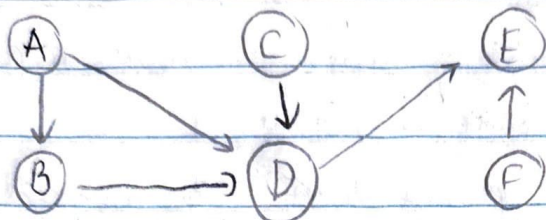
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Assignment 3

COMP 5600

P1: $P(A, B, C, D, E, F) = P(A) P(C) P(B|A, C) P(E|A) P(D|B, C, E) P(F|D)$

P2:



P3:

Part A

Provided probability: $P(H), P(I), P(+e|I), P(U|I, H), P(+e|I, U)$

Elimination order: T, H, I

$$f_1(I, +e, U) = \sum_h P(+e|I) \cdot P(+e|T, U)$$

$$f_2(U, I) = \sum_h P(H) \cdot P(U|I, H)$$

$$f_3(U, +e) = \sum_h P(I) \cdot f_2(U, I) \cdot f_1(I, +e, U)$$

$$\Rightarrow f(U, +e) = \sum_i (P(I) \cdot (\sum_h P(H) \cdot P(U|I, H)) \cdot \sum_t P(+e|I) \cdot P(+e|T, U))$$

$T \quad U \quad +e \quad Val$

+	+	+	0.9	0.9	0.4	T	U	Val
+	+	-	0.1	0.1	0.4	+	+	0.9
+	+	+	0.5	0.5	0.4	+	-	0.5
+	-	+	0.5	0.5	0.4	-	+	0.7
-	+	+	0.7	0.7	0.4	-	-	0.3
-	+	-	0.3	0.3	0.4	-	+	0.3
-	-	+	0.3	0.3	0.4	+	+	0.3
-	-	-	0.7	0.7	0.4	+	-	0.3

Multiply with $P(T|I)$

T	U	val	I	T	val
+	+	.9	+	+	.8
+	-	.5	+	-	.2
-	+	.7	-	+	.5
-	-	.3	-	-	.5

I	T	U	val		T	U	val
+	+	+	0.72		+	+	0.86
+	+	-	0.4	Sum out T	+	-	0.46
+	-	+	0.14		-	+	0.8
+	-	-	0.06		-	-	0.4
-	+	+	0.45		This is $f(T, U)$ with restricted I		
-	+	-	0.25				
-	-	+	0.35				
-	-	-	0.15				

Multiply $P(H)$ with $P(U|I, H)$

H	val	I	H	U	val	I	H	U	val
+	.6	+	+	+	.9	+	+	+	0.54
-	.4	+	+	-	.1	+	+	-	0.06
		+	-	+	.3	+	-	+	0.18
		+	-	-	.7	+	-	-	0.42
		-	+	+	.5	-	+	+	0.2
		-	+	-	.5	-	+	-	0.2
		-	-	+	.1	-	-	+	0.04
		-	-	-	.9	-	-	-	0.36

↓
Sum out I

Sum out I to get $f(U, I)$ then multiply with $f(Ie, U, I)$

I	U	val	I	U	val	I	U	val
+	+	0.72	+	+	0.86	+	+	0.6192
+	-	0.48	+	-	0.46	+	-	0.2704
-	+	0.24	-	+	0.8	-	+	0.192
-	-	0.56	-	-	0.4	-	-	0.224

Finally, multiply with $P(I)$ and sum out I to get $f(U, Ie)$

I	val	I	U	val	I	U	val
+	0.7	+	+	0.6192	+	+	0.43344
-	0.3	+	-	0.2208	+	-	0.15456
		-	+	0.192	-	+	0.0576
		-	-	0.224	-	-	0.0672

\rightarrow	U	val	This is $f(u, e)$
	+	0.49104	
	-	0.22176	

Find $P(+u | +e)$ and $P(-u | +e)$

$$P(+u | +e) = \frac{f(+u, +e)}{f(+u, +e) + f(-u, +e)} = \frac{0.49104}{0.49104 + 0.22176} \approx 0.68$$

$$P(-u | +e) = \frac{f(-u, +e)}{f(+u, +e) + f(-u, +e)} = \frac{0.22176}{0.49104 + 0.22176} = 0.31$$

So, the answer is ≈ 0.68

Part 2

- a) False. They are both dependent on I.
- b) False. Knowing E might couple T and U as it is a child node of both, forming a V-structure.
- c) True, knowing I and H is enough to decouple T and U.
- d) True, E, H, and U form a cascade structure that U decouples E and H.
- e) True, U still decouples E and H in the cascade structure and there is no path from I and T that changed this.
- f) True. The path from I to H is blocked because there exists an E node such that E is the descendant of both I and H and forming a V-structure, therefore they are d-separated.
- g) True, T isn't a child or a descendant of H, so it doesn't couple I and H.
- h) False, T and H are not independent (child of I).
- i) False, E is a child node of T and a descendant of H, therefore it might open up path between T and H.
- j) True, knowing U blocked the path between T and H, and knowing E doesn't provide anything new that U doesn't already.