# 12: Support Vector Machines (SVMs)

holehouse.org/mlclass/12_Support_Vector_Machines.html

**Support Vector Machine (SVM) - Optimization objective**

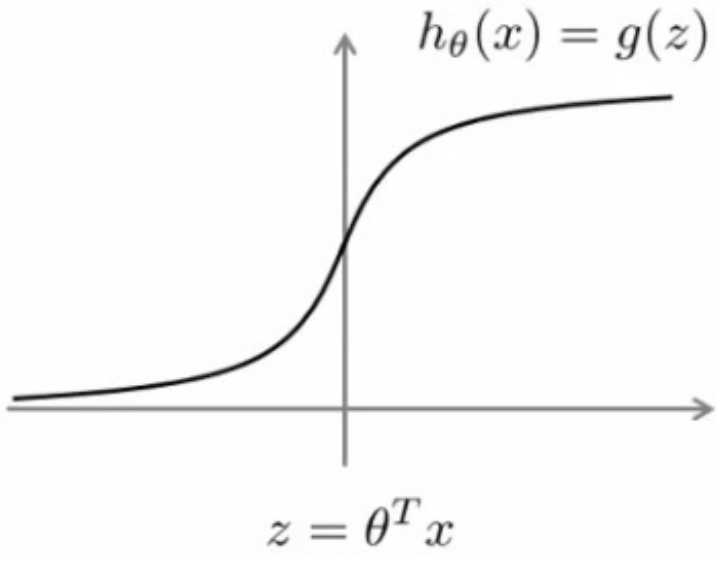
- So far, we've seen a range of different algorithms
  - With supervised learning algorithms - performance is pretty similar
    - What matters more often is;
      - The amount of training data
      - Skill of applying algorithms
- One final supervised learning algorithm that is widely used - **support vector machine (SVM)**
-
  - Compared to both logistic regression and neural networks, a SVM sometimes gives a cleaner way of learning non-linear functions
  - Later in the course we'll do a survey of different supervised learning algorithms

**An alternative view of logistic regression**

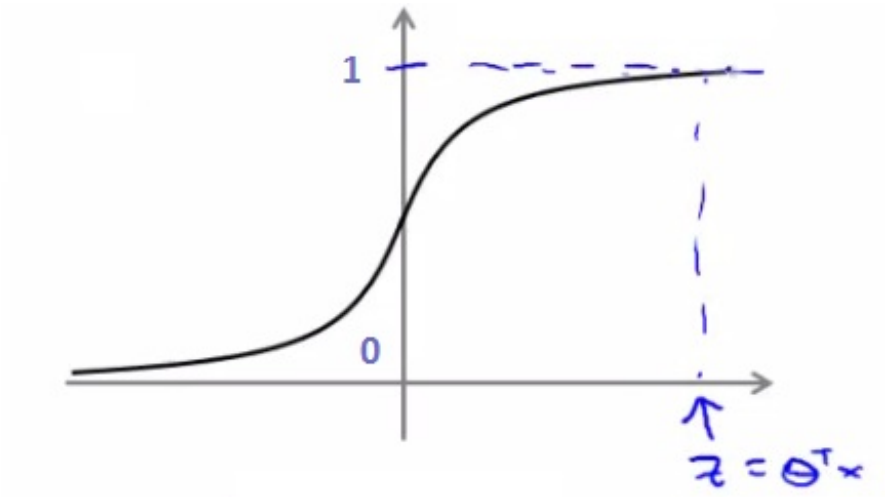
- Start with logistic regression, see how we can modify it to get the SVM
  - As before, the logistic regression hypothesis is as follows
  - And the sigmoid activation function looks like this
  - In order to explain the math, we use z as defined above

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

- What do we want logistic regression to do?
  - We have an example where y = 1
    - Then we hope $h_\theta(x)$ is close to 1




    - With $h_\theta(x)$ close to 1, ($\theta^T x$) must be **much larger** than 0




- Similarly, when y = 0
  - Then we hope $h_\theta(x)$ is close to 0
  - With $h_\theta(x)$ close to 0, $(\theta^T x)$ must be **much less** than 0
- This is our classic view of logistic regression

  Let's consider another way of thinking about the problem

- Alternative view of logistic regression

  If you look at cost function, each example contributes a term like the one below to the overall cost function

$$-(y \log h_\theta(x) + (1-y) \log(1 - h_\theta(x)))$$

  For the overall cost function, we sum over all the training examples using the above function, and have a 1/m term

- If you then plug in the hypothesis definition ($h_\theta(x)$), you get an expanded cost function equation;

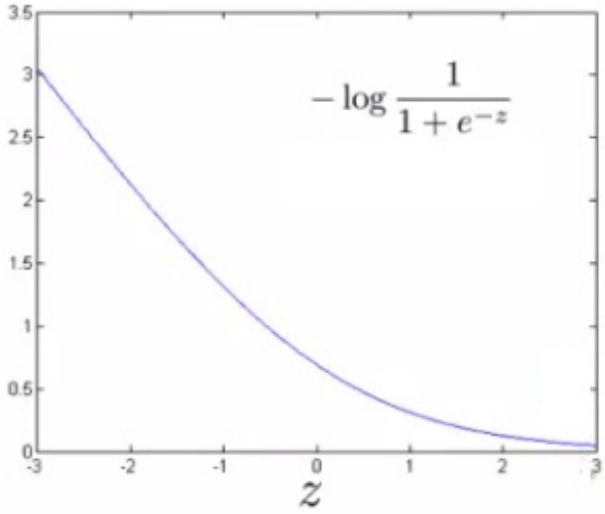
$$= -y \log \frac{1}{1+e^{-\theta^T x}} - (1-y) \log(1 - \frac{1}{1+e^{-\theta^T x}})$$

- So each training example contributes that term to the cost function for logistic regression

- If y = 1 then only the first term in the objective matters

  If we plot the functions vs. z we get the following graph

This plot shows the cost contribution of an example when y = 1 given z



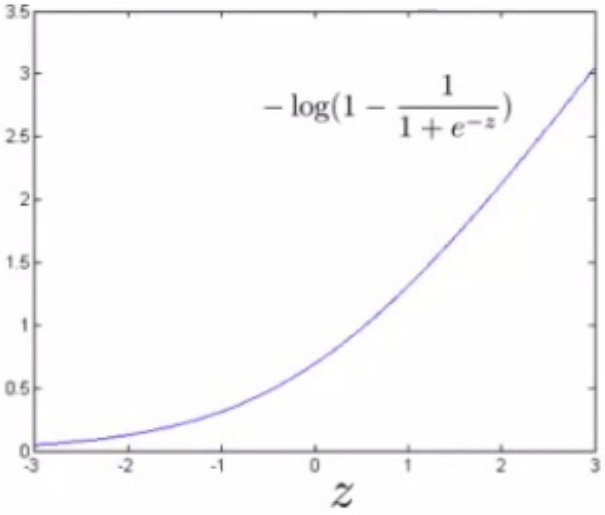

- So if z is big, the cost is low - this is good!
- But if z is 0 or negative the cost contribution is high
- This is why, when logistic regression sees a positive example, it tries to set $\theta^T x$ to be a very large term

- If y = 0 then only the second term matters
-

We can again plot it and get a similar graph

Same deal, if z is small then the cost is low

But if s is large then the cost is massive



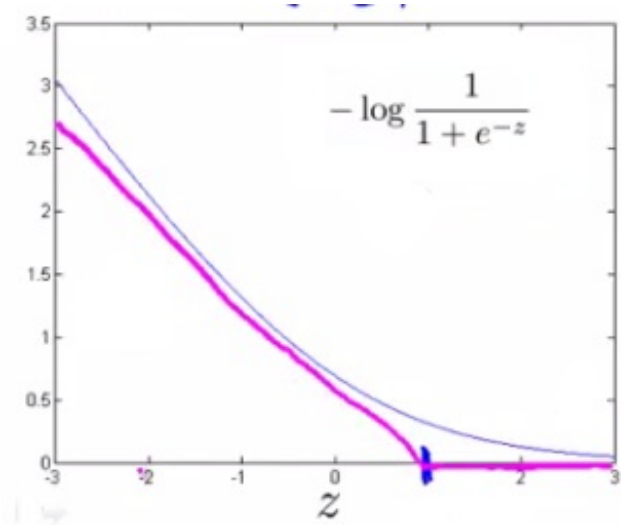

**SVM cost functions from logistic regression cost functions**

To build a SVM we must redefine our cost functions

When y = 1

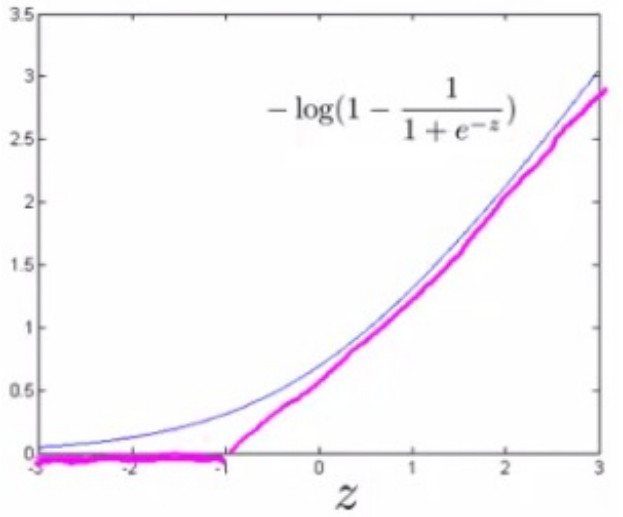
- Take the y = 1 function and create a new cost function
- Instead of a curved line create two straight lines (magenta) which acts as an approximation to the logistic regression y = 1 function

- Take point (1) on the z axis
  - Flat from 1 onwards
  - Grows when we reach 1 or a lower number
- This means we have two straight lines
  - Flat when cost is 0
  - Straight growing line after 1




- So this is the new y=1 cost function
  - Gives the SVM a computational advantage and an easier optimization problem
  - We call this function **$\text{cost}_1(z)$**

- Similarly
  When y = 0
  - Do the equivalent with the y=0 function plot
  - We call this function **$\text{cost}_0(z)$**
- So here we define the two cost function terms for our SVM graphically
  How do we implement this?




**The complete SVM cost function**

- As a comparison/reminder we have logistic regression below

$$\min_\theta \frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\left(-\log h_\theta(x^{(i)})\right) + (1-y^{(i)})\left((-\log(1-h_\theta(x^{(i)}))\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

  If this looks unfamiliar its because we previously had the - sign outside the expression
- For the SVM we take our two logistic regression y=1 and y=0 terms described previously and replace with
  - $\text{cost}_1(\theta^T x)$
  - $\text{cost}_0(\theta^T x)$
- So we get

$$\min_\theta \frac{1}{m} \sum_{i=1}^{m} \left[ y^{(i)} cost_1(\theta^T x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T x^{(i)}) \right] + \frac{\lambda}{2m} \sum_{i=1}^{n} \theta_j^2$$

**SVM notation is slightly different**

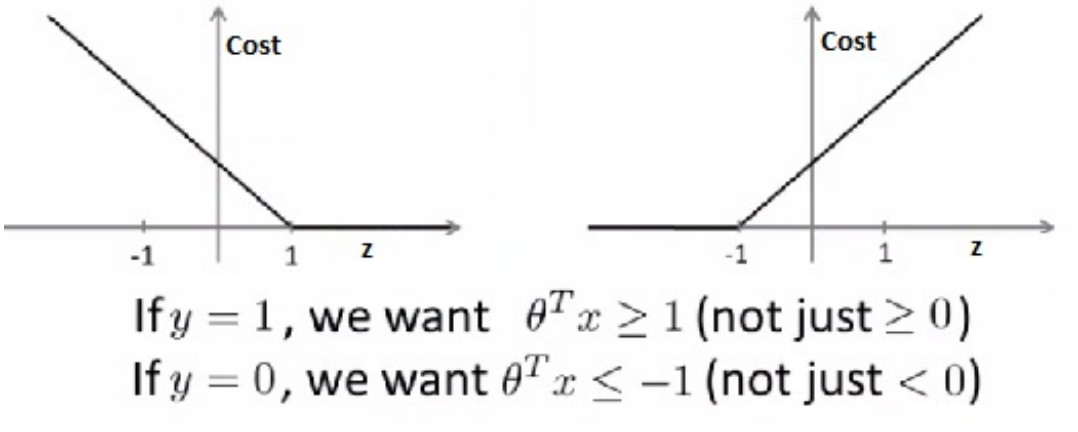
- In convention with SVM notation we rename a few things here
- 1) Get rid of the 1/m terms
  - This is just a slightly different convention
  - By removing 1/m we should get the same optimal values for
    - 1/m is a constant, so should get same optimization
    - e.g. say you have a minimization problem which minimizes to u = 5
      - If your cost function * by a constant, you still generates the minimal value
      - That minimal value is different, but that's irrelevant
- 2) For logistic regression we had two terms;
  - Training data set term (i.e. that we sum over m) = **A**
  - Regularization term (i.e. that we sum over n) = **B**
    - So we could describe it as A + λB
    - Need some way to deal with the trade-off between regularization and data set terms
    - Set different values for λ to parametrize this trade-off
  - Instead of parameterization this as A + λB
    - For SVMs the convention is to use a different parameter called C
    - So do CA + B
    - If C were equal to 1/ λ then the two functions (CA + B and A + λB) would give the same value
- So, our overall equation is

$$\min_\theta C \sum_{i=1}^{m} \left[ y^{(i)} cost_1(\theta^T x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T x^{(i)}) \right] + \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$

- Unlike logistic, $h_\theta(x)$ doesn't give us a probability, but instead we get a direct prediction of 1 or 0
-
  - So if $\theta^T x$ is equal to or greater than 0 --> $h_\theta(x) = 1$
  - Else --> $h_\theta(x) = 0$

## Large margin intuition

- Sometimes people refer to SVM as **large margin classifiers**
  - We'll consider what that means and what an SVM hypothesis looks like
  - The SVM cost function is as above, and we've drawn out the cost terms below




- Left is $cost_1$ and right is $cost_0$
- What does it take to make terms small
  - If y =1

    $cost_1(z) = 0$ only when z >= 1
  - If y = 0

    $cost_0(z) = 0$ only when z <= -1
- Interesting property of SVM
  - If you have a positive example, you only really *need* z to be greater or equal to 0

    If this is the case then you predict 1
  - SVM wants a bit more than that - doesn't want to *just* get it right, but have the value be quite a bit bigger than zero

    Throws in an extra safety margin factor

- Logistic regression does something similar
- What are the consequences of this?
  - Consider a case where we set C to be huge
    - C = 100,000
    - So considering we're minimizing CA + B
      - If C is huge we're going to pick an A value so that A is equal to zero
      - What is the optimization problem here - how do we make A = 0?
    - Making A = 0
      - If y = 1

        Then to make our "A" term 0 need to find a value of θ so ($\theta^T x$) is greater than or equal to 1
      - Similarly, if y = 0

        Then we want to make "A" = 0 then we need to find a value of θ so ($\theta^T x$) is equal to or less than -1
    - So - if we think of our optimization problem a way to ensure that this first "A" term is equal to 0, we re-factor our optimization problem into just minimizing the "B" (regularization) term, because

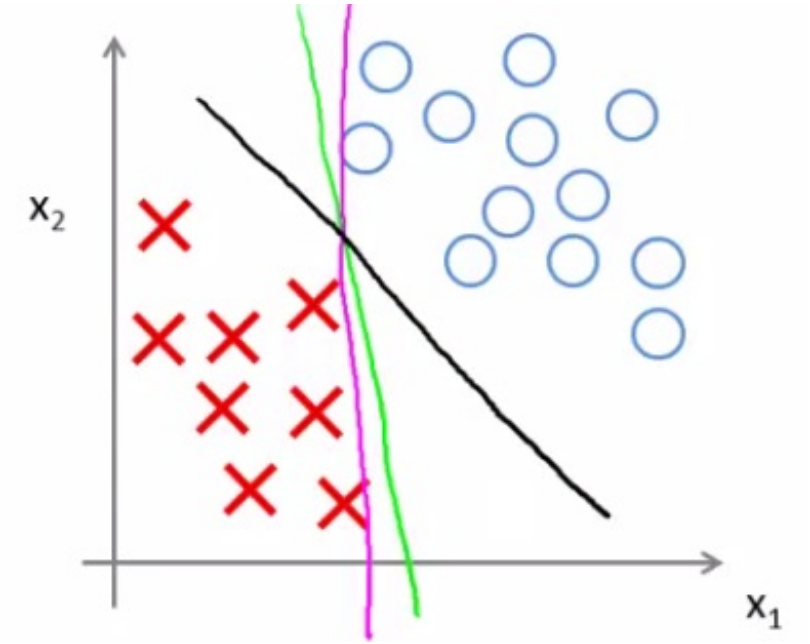
      When A = 0 --> A*C = 0
    - So we're minimizing B, under the constraints shown below

$$\min \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$

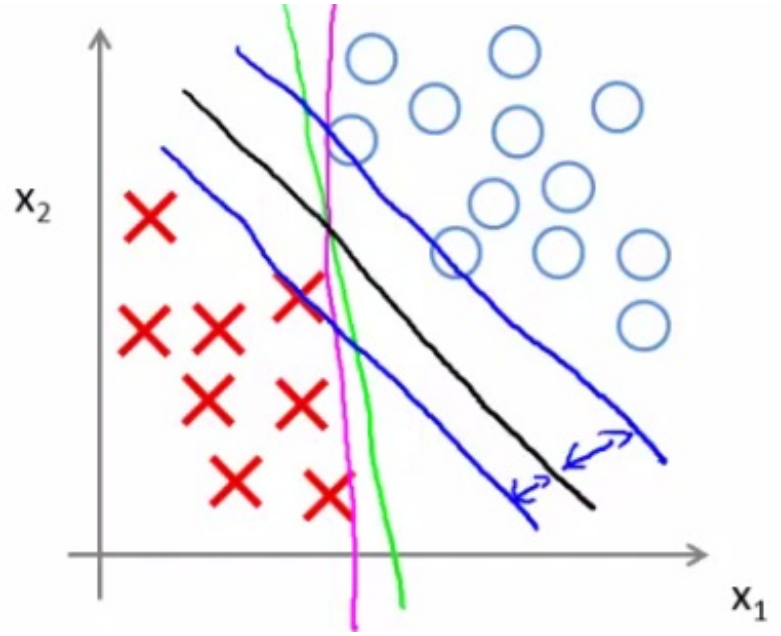
$$\text{s.t.} \quad \theta^T x^{(i)} \geq 1 \quad \text{if} \quad y^{(i)} = 1$$

$$\theta^T x^{(i)} \leq -1 \quad \text{if} \quad y^{(i)} = 0$$

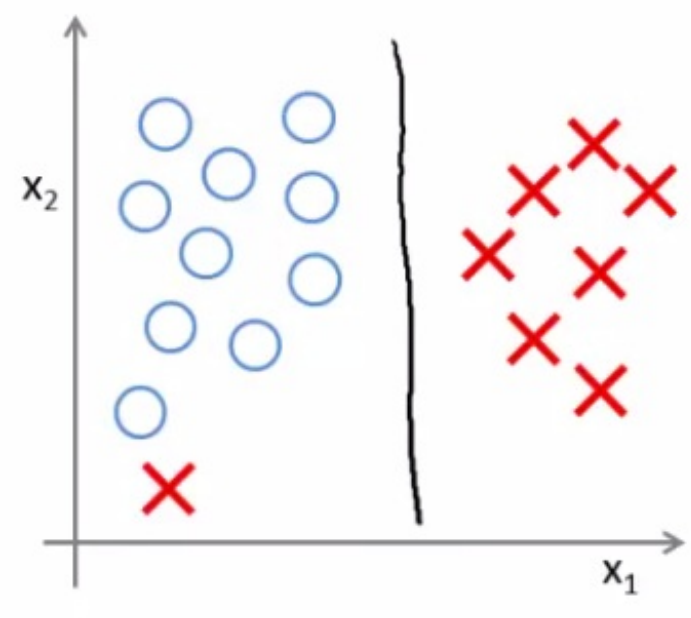
- Turns out when you solve this problem you get interesting decision boundaries




- The green and magenta lines are functional decision boundaries which could be chosen by logistic regression
    - But they probably don't generalize too well
- The black line, by contrast is the the chosen by the SVM because of this safety net imposed by the optimization graph
    - More robust separator
- Mathematically, that black line has a larger minimum distance (margin) from any of the training examples

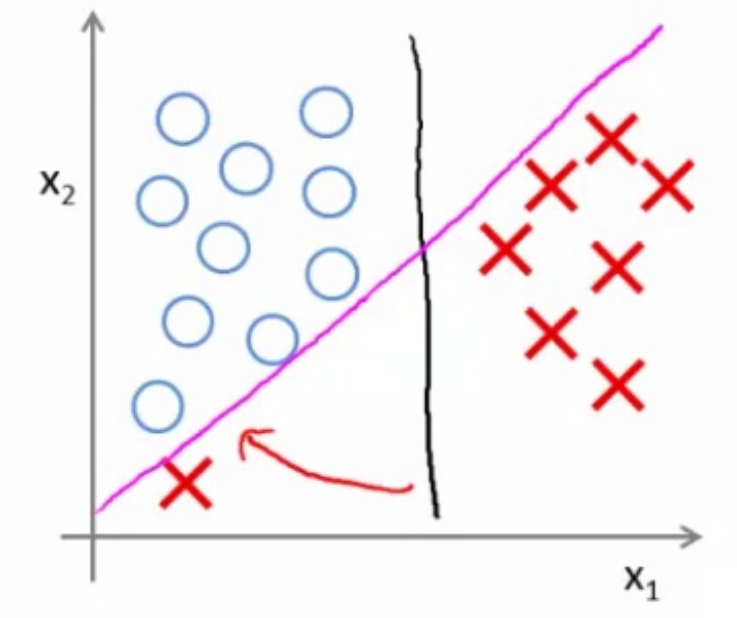



    - By separating with the largest margin you incorporate robustness into your decision making process
- We looked at this at when C is very large
-
    - SVM is more sophisticated than the large margin might look
        - If you were just using large margin then SVM would be very sensitive to outliers
        - You would risk making a ridiculous hugely impact your classification boundary
            - A single example might not represent a good reason to change an algorithm




        - If C is very large then we *do* use this quite naive maximize the margin approach

        - So we'd change the black to the magenta
    - But if C is reasonably small, or a not too large, then you stick with the black decision boundary



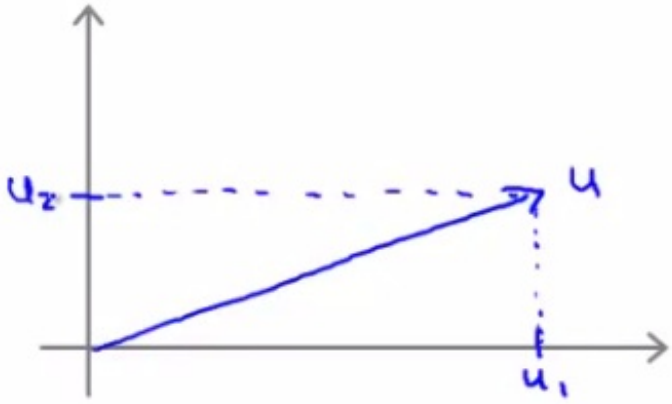

- What about non-linearly separable data?
    - Then SVM still does the right thing if you use a normal size C
    - So the idea of SVM being a large margin classifier is only really relevant when you have no outliers and you can easily linearly separable data
- Means we ignore a few outliers

**Large margin classification mathematics (optional)**

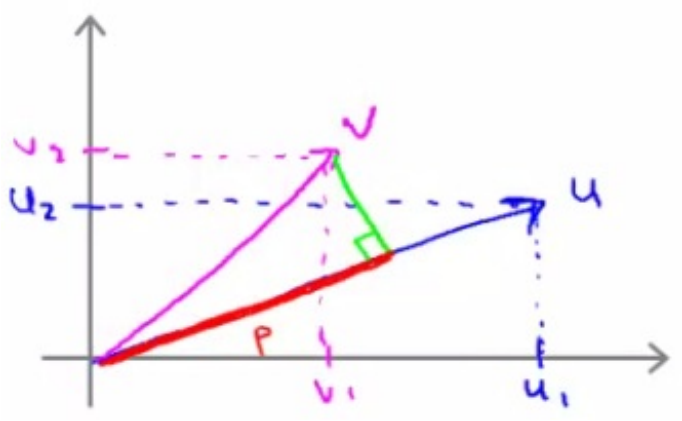
**Vector inner products**

- Have two (2D) vectors u and v - what is the inner product ( $u^T v$)?
    - Plot $u$ on graph
      i.e $u_1$ vs. $u_2$
    - One property which is good to have is the **norm** of a vector
        - Written as ||u||
          This is the euclidean length of vector u
        - So ||u|| = SQRT($u_1^2 + u_2^2$) = real number
            - i.e. length of the arrow above
            - Can show via Pythagoras
    - For the inner product, take $v$ and orthogonally project down onto u
        - First we can plot v on the same axis in the same way ( $v_1$ vs $v_1$)
        - Measure the length/magnitude of the projection

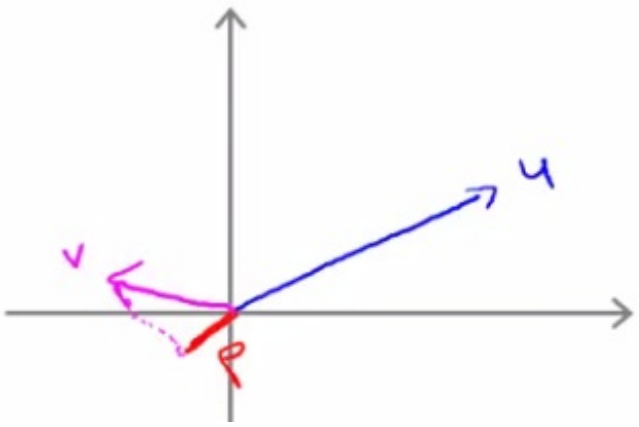
$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



        - So here, the green line is the projection
            - p = length along u to the intersection
            - p is




            the magnitude of the projection of vector *v* onto vector *u*
    - Possible to show that
        - $u^T v$ = p * ||u||
            So this is one way to compute the inner product
        - $u^T v = u_1 v_1 + u_2 v_2$
        - So therefore
            - p * ||u|| = $u_1 v_1 + u_2 v_2$
            - This is an important rule in linear algebra
        - We can reverse this too
            So we could do
            - $v^T u = v_1 u_1 + v_2 u_2$
            - Which would obviously give you the same number
    - p can be negative if the angle between them is 90 degrees or more
        So here p is negative




- Use the vector inner product theory to try and understand SVMs a little better

**SVM decision boundary**

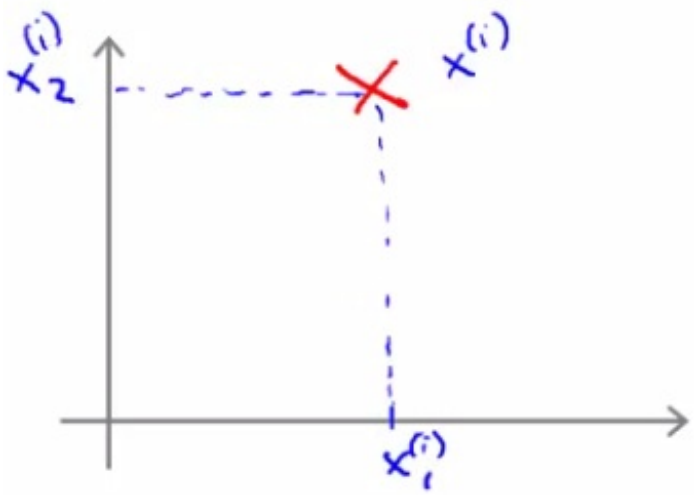
- For the following explanation - two simplification
    - Set $\theta_0 = 0$ (i.e. ignore intercept terms)
    - Set n = 2 - ($x_1$, $x_2$)
        i.e. each example has only 2 features
- Given we only have two parameters we can simplify our function to

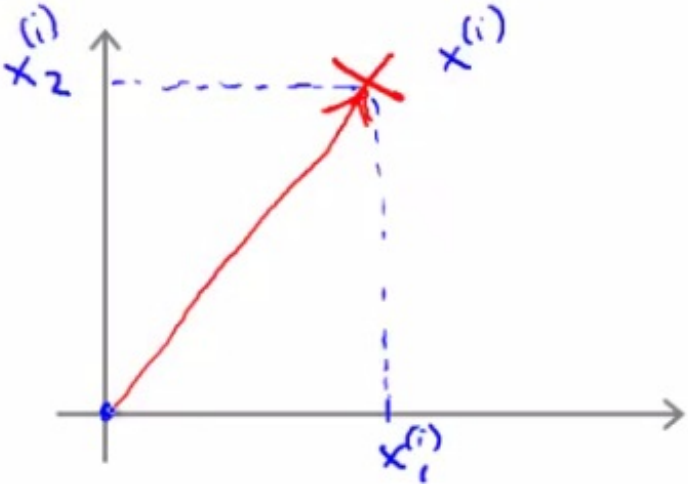
$$\min_\theta \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$$

$$\text{s.t.} \quad \theta^T x^{(i)} \geq 1 \qquad \text{if } y^{(i)} = 1$$

$$\theta^T x^{(i)} \leq -1 \qquad \text{if } y^{(i)} = 0$$

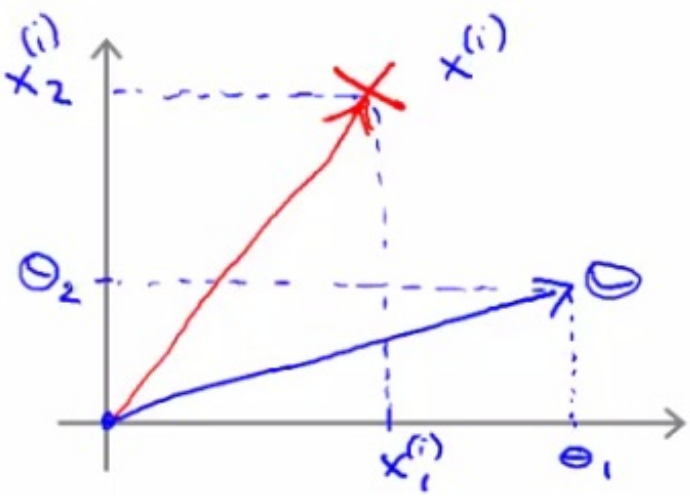
- And, can be re-written as
  Should give same thing
- We may notice that
  The term in red is the norm of θ
  - If we take θ as a 2x1 vector
  - If we assume $\theta_0 = 0$ its still true
- So, finally, this means our optimization function can be re-defined as
- So the SVM is minimizing the squared norm
- Given this, what are the ($\theta^T x$) parameters doing?
  - Given θ and given example x what is this equal to
    We can look at this in a comparable manner to how we just looked at u and v
  - Say we have a single positive training example (red cross below)
  - Although we haven't been thinking about examples as vectors it can be described as such
  - Now, say we have our parameter vector θ and we plot that on the same axis
  - The next question is what is the inner product of these two vectors
    p, is in fact $p^i$, because it's the length of p for example i
    - Given our previous discussion we know
      $(\theta^T x^i)$
      $= p^i * ||\theta||$





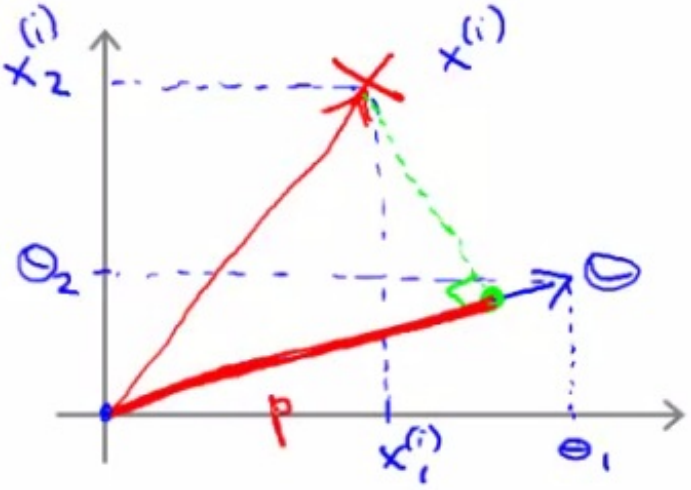
      $= \theta_1 x^i_1 + \theta_2 x^i_2$
    - So these are both equally valid ways of computing $\theta^T x^i$
- What does this mean?
  - The constraints we defined earlier

        - $(\theta^T x) >= 1$ if y = 1
        - $(\theta^T x) <= -1$ if y = 0
    - Can be replaced/substituted with the constraints
        - $p^i * ||\theta|| >= 1$ if y = 1
        - $p^i * ||\theta|| <= -1$ if y = 0




- Writing that into our optimization objective

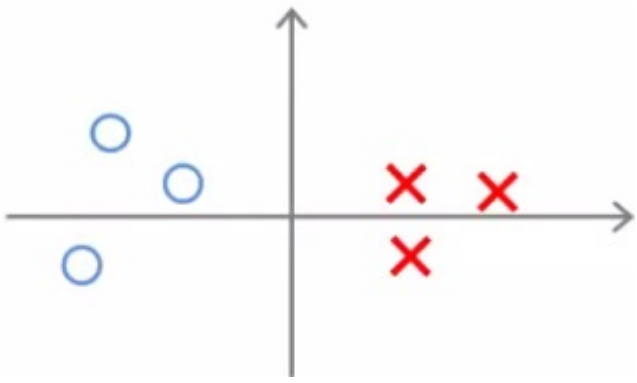
- So, given we've redefined these functions let us now consider the training example below
    - Given this data, what boundary will the SVM choose? Note that we're still assuming $\theta_0 = 0$, which means the boundary has to pass through the origin (0,0)




Green line - small margins

$$\min_{\theta} \frac{1}{2} \sum_{j=1}^{n} \theta_j^2 = \frac{1}{2} \|\theta\|^2$$
$$\text{s.t.} \quad p^{(i)} \cdot \|\theta\| \geq 1 \quad \text{if } y^{(i)} = 1$$
$$p^{(i)} \cdot \|\theta\| \leq -1 \quad \text{if } y^{(i)} = 1$$

SVM would not chose this line

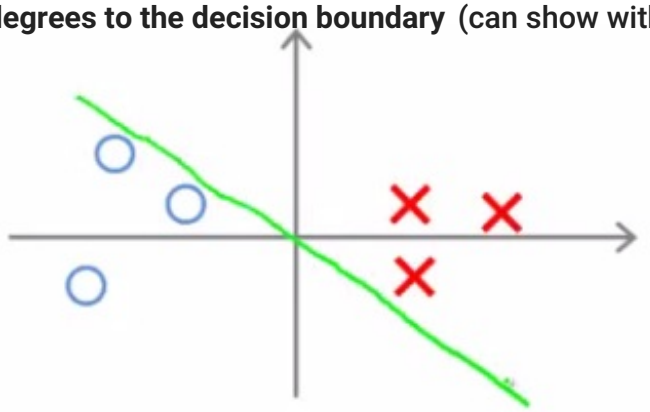


- Decision boundary comes very close to examples
- Lets discuss *why* the SVM would **not** chose this decision boundary

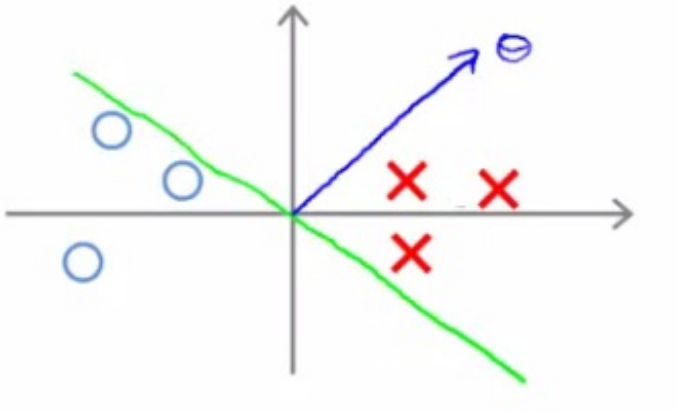
    - Looking at this line

We can show that θ is at 90 degrees to the decision boundary

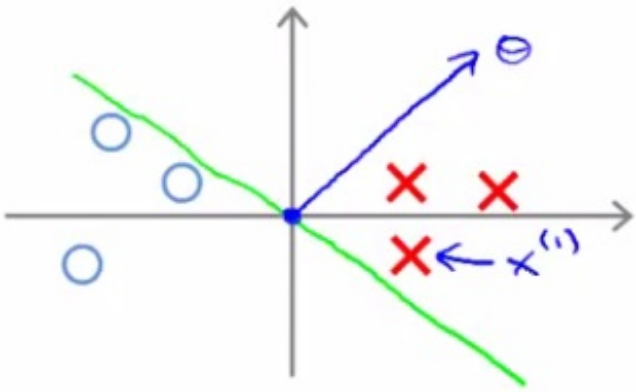
**θ is always at 90 degrees to the decision boundary** (can show with linear algebra, although we're not going to!)



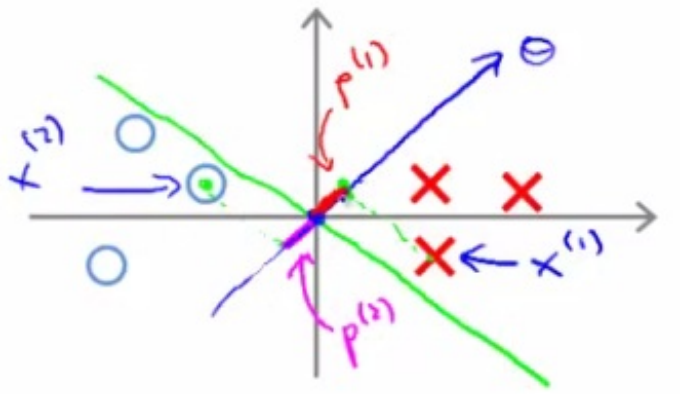
- So now lets look at what this implies for the optimization objective
  - Look at first example ($x^1$)
  - Project a line from $x^1$ on to to the θ vector (so it hits at 90 degrees)
    The distance between the intersection and the origin is ($p^1$)



  - Similarly, look at second example ($x^2$)
  - 
    - Project a line from $x^2$ into to the θ vector
    - This is the magenta line, which will be **negative** ($p^2$)



  - If we overview these two lines below we see a graphical representation of what's going on;
  - We find that both these p values are going to be pretty small
  - If we look back at our optimization objective
    - We know we need $p^1 * ||\theta||$ to be bigger than or equal to 1 for positive examples
      If p is small
      Means that ||θ|| must be pretty large



    - Similarly, for negative examples we need $p^2 * ||\theta||$ to be smaller than or equal to -1

We saw in this example $p^2$ is a small negative number

So $||\theta||$ must be a large number

- Why is this a problem?

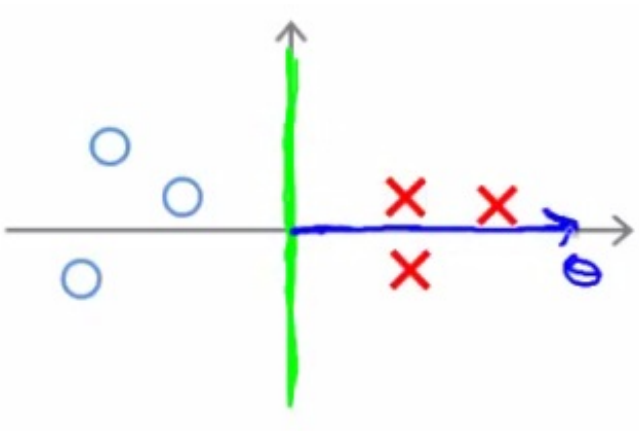
The optimization objective is trying to find a set of parameters where the norm of theta is small

So this doesn't seem like a good direction for the parameter vector (because as p values get smaller $||\theta||$ must get larger to compensate)

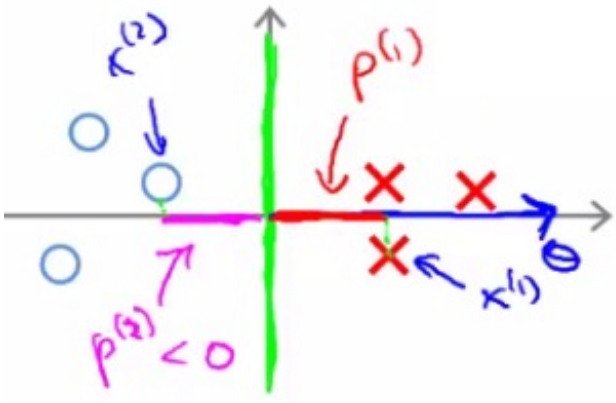
So we should make p values larger which allows $||\theta||$ to become smaller

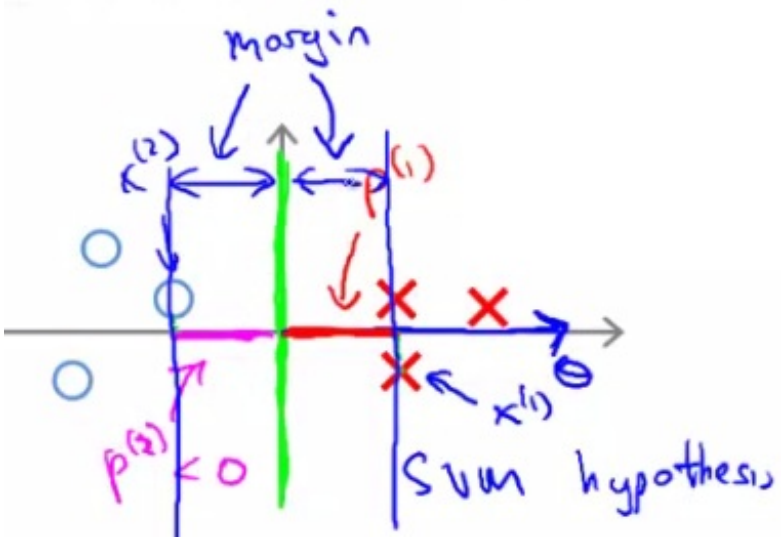
- So lets chose a different boundary

Now if you look at the projection of the examples to θ we find that $p^1$ becomes large and $||\theta||$ can become small



- So with some values drawn in
- This means that by choosing this second decision boundary we can make $||\theta||$ smaller
  - Which is why the SVM choses this hypothesis as better
  - This is how we generate the large margin effect



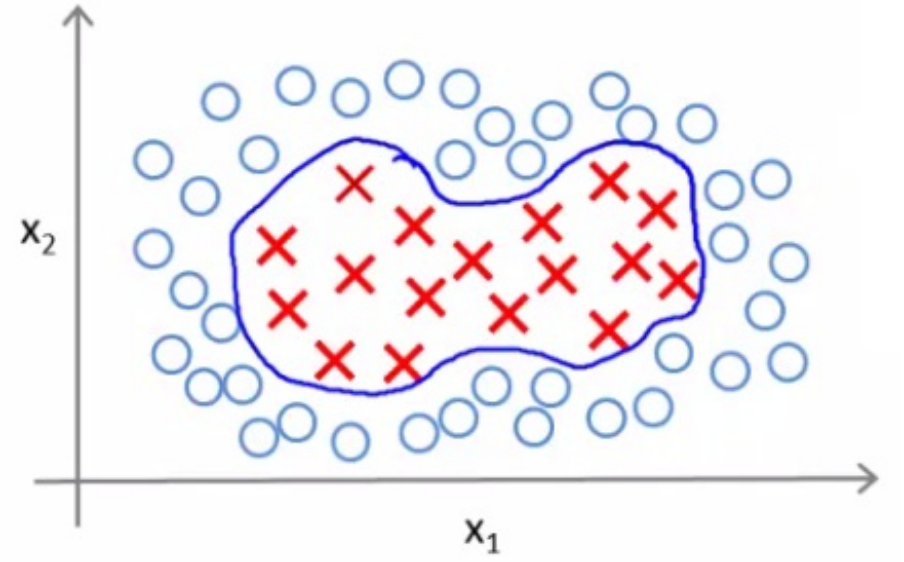

- The magnitude of this margin is a function of the p values

So by maximizing these p values we minimize $||\theta||$

- Finally, we did this derivation assuming $\theta_0 = 0$,
  - If this is the case we're entertaining only decision boundaries which pass through (0,0)
  - If you allow $\theta_0$ to be other values then this simply means you can have decision boundaries which cross through the x and y values at points other than (0,0)
  - Can show with basically same logic that this works, and even when $\theta_0$ is non-zero when you have optimization objective described above (when C is very large) that the SVM is looking for a large margin separator between the classes

**Kernels - 1: Adapting SVM to non-linear classifiers**

- What are kernels and how do we use them
  - We have a training set
  - We want to find a non-linear boundary



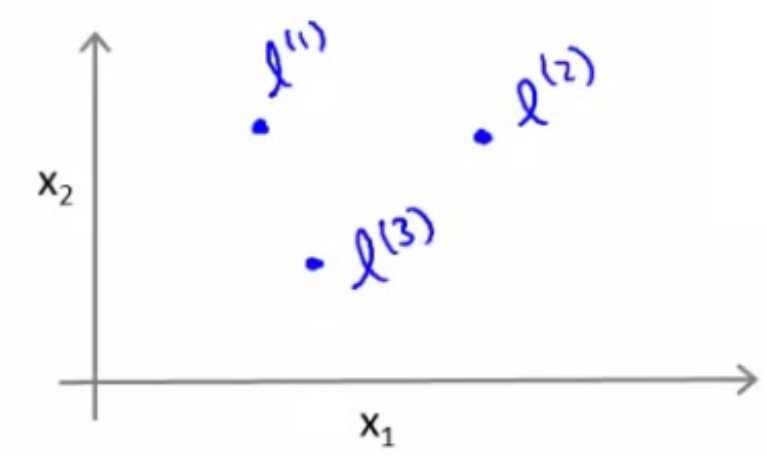

  - Come up with a complex set of polynomial features to fit the data
    - Have $h_\theta(x)$ which
      - Returns 1 if the combined weighted sum of vectors (weighted by the parameter vector) is less than or equal to 0
      - Else return 0
    - Another way of writing this (new notation) is
      - That a hypothesis computes a decision boundary by taking the sum of the parameter vector multiplied by a **new feature vector f**, which simply contains the various high order x terms
      - e.g.
        - $h_\theta(x) = \theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3$
        - Where
          - $f_1 = x_1$
          - $f_2 = x_1 x_2$
          - $f_3 = ...$
          - i.e. not specific values, but each of the terms from

your complex polynomial function

- Is there a better choice of feature f than the high order polynomials?

  As we saw with computer imaging, high order polynomials become computationally expensive

- New features
-
  - Define three features in this example (ignore $x_0$)
  - Have a graph of $x_1$ vs. $x_2$ (don't plot the values, just define the space)
  - Pick three points in that space




  - These points $l^1$, $l^2$, and $l^3$, were chosen manually and are called **landmarks**
    - Given x, define f1 as the similarity between (x, $l^1$)
      - = exp(- (|| x - $l^1$ ||$^2$ ) / 2$\sigma^2$)
        =
      - || x - $l^1$ || is the euclidean distance between the point x and the landmark $l^1$ squared

        Disussed more later

        $$\exp\left(-\frac{\|x-l^{(1)}\|^2}{2\sigma^2}\right)$$
      - If we remember our statistics, we know that
        - σ is the **standard deviation**
        - $\sigma^2$ is commonly called the **variance**
    - Remember, that as discussed
  - So, f2 is defined as

    f2 = similarity(x, $l^1$)
    = exp(- (|| x - $l^2$ ||$^2$) / 2$\sigma^2$)

    $$\|x-l^{(1)}\|^2 = \sum_{j=1}^{n}(x_j - l_j^{(1)})^2$$
  - And similarly

    f3 = similarity(x, $l^2$) = exp(- (|| x - $l^1$ ||$^2$) / 2$\sigma^2$)
  - This similarity function is called a **kernel**

    This function is a **Gaussian Kernel**
  - So, instead of writing similarity between x and l we might write
  -

    f1 = k(x, $l^1$)

**Diving deeper into the kernel**

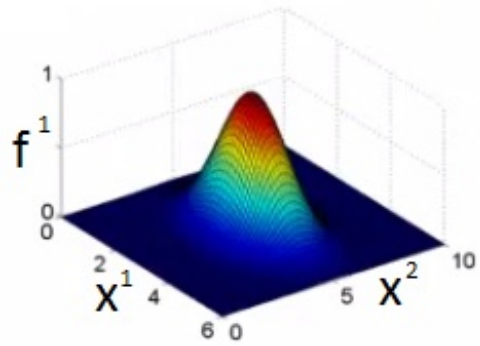
- So lets see what these kernels do and why the functions defined make sense
  Say x is close to a landmark
  - Then the squared distance will be ~0
    So
    Which is basically $e^{-0}$

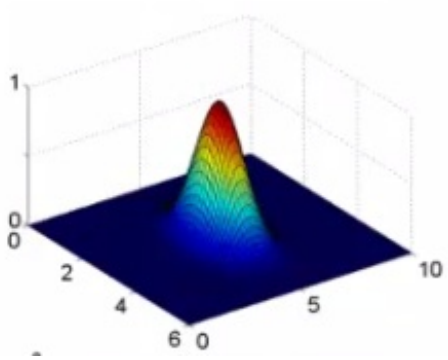
$$f_1 \approx \exp\left(-\frac{0^2}{2\sigma^2}\right)$$

    Which is close to 1
  - Say x is far from a landmark
    Then the squared distance is big
    Gives $e^{-\text{large number}}$
    Which is close to zero
  - Each landmark defines a new features
- If we plot f1 vs the kernel function we get a plot like this
  - Notice that when x = [3,5] then f1 = 1
  - As x moves away from [3,5] then the feature takes on values close to zero
  - So this measures how close x is to this landmark

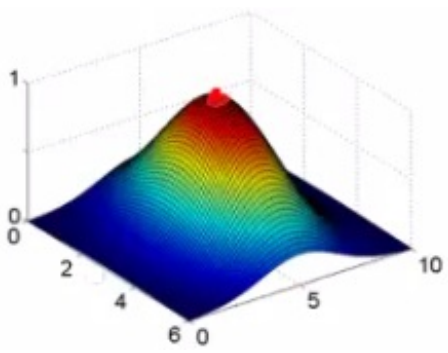
**What does σ do?**

- $\sigma^2$ is a parameter of the Gaussian kernel
  Defines the steepness of the rise around the landmark
- Above example $\sigma^2 = 1$
- Below $\sigma^2 = 0.5$
  We see here that as you move away from 3,5 the feature f1 falls to zero much more rapidly
- The inverse can be seen if $\sigma^2 = 3$



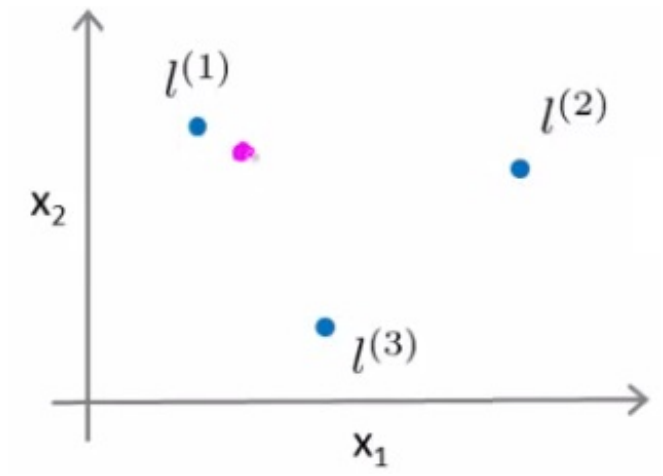

- Given this definition, what kinds of hypotheses can we learn?
  - With training examples x we predict "1" when
  - $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 >= 0$
    - For our example, lets say we've already run an algorithm and got the
      - $\theta_0 = -0.5$
      - $\theta_1 = 1$
      - $\theta_2 = 1$
      - $\theta_3 = 0$
    - Given our placement of three examples, what happens if we evaluate an example at the



magenta dot below?

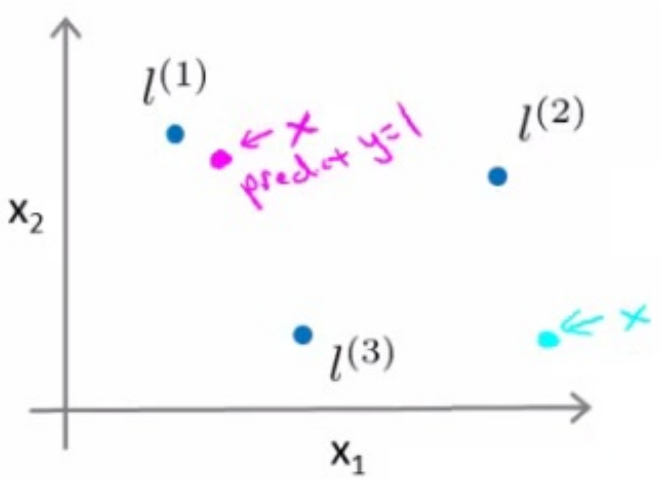
- Looking at our formula, we know f1 will be close to 1, but f2 and f3 will be close to 0

  So if we look at the formula we have




  - $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 >= 0$
  - $-0.5 + 1 + 0 + 0 = 0.5$

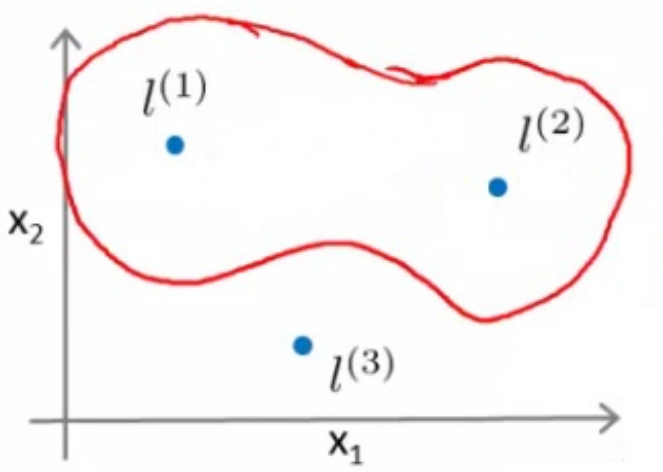
    0.5 is greater than 1
- If we had **another point** far away from all three




  This equates to -0.5

  So we predict 0

  - Considering our parameter, for points near $l^1$ and $l^2$ you predict 1, but for points near $l^3$ you predict 0
  - Which means we create a non-linear decision boundary that goes a lil' something like this;
    - Inside we predict y = 1
    - Outside we predict y = 0




- So this show how we can create a non-linear boundary with landmarks and the kernel function in the support vector machine
-
  - But
  -

        - How do we get/chose the landmarks
        - What other kernels can we use (other than the Gaussian kernel)

**Kernels II**

- Filling in missing detail and practical implications regarding kernels
- Spoke about picking landmarks manually, defining the kernel, and building a hypothesis function
    - Where do we get the landmarks from?
    - For complex problems we probably want lots of them

**Choosing the landmarks**

- Take the training data
- For each example place a landmark at exactly the same location
- So end up with m landmarks
    - One landmark per location per training example
    - Means our features measure how close to a training set example something is
- Given a new example, compute all the f values
    Gives you a feature vector f ($f_0$ to $f_m$)
        $f_0 = 1$ always
- A more detailed look at generating the f vector
    If we had a training example - features we compute would be using ($x^i$, $y^i$)
        - So we just cycle through each landmark, calculating how close to that landmark actually $x^i$ is
            - $f_1^i, = k(x^i, l^1)$
            - $f_2^i, = k(x^i, l^2)$
            - ...
            - $f_m^i, = k(x^i, l^m)$
        - Somewhere in the list we compare x to itself... (i.e. when we're at $f_i^i$)
            So because we're using the Gaussian Kernel this evalues to 1
        Take these m features ($f_1$, $f_2$ ... $f_m$) group them into an [m +1 x 1] dimensional vector called f
            - $f^i$ is the f feature vector for the ith example
            - And add a 0th term = 1
- Given these kernels, how do we use a support vector machine

**SVM hypothesis prediction with kernels**

- Predict y = 1 if ($\theta^T f$) >= 0
    - Because θ = [m+1 x 1]
    - And f = [m +1 x 1]
- So, this is how you make a prediction assuming you already have θ
    How do you get θ?

**SVM training with kernels**

- Use the SVM learning algorithm

$$\min_{\theta} C \sum_{i=1}^{m} y^{(i)} cost_1(\theta^T f^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T f^{(i)}) + \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$$

    - Now, we minimize using f as the feature vector instead of x
    - By solving this minimization problem you get the parameters for your SVM
- In this setup, m = n

  Because number of features is the number of training data examples we have
- One final mathematic detail (not crucial to understand)
    - If we ignore $\theta_0$ then the following is true
    - What many implementations do is

      $\theta^T M \theta$

      $$\sum_{j=1}^{n} \theta_j^2 = \theta^T \theta$$

        - Where the matrix M depends on the kernel you use
        - Gives a slightly different minimization - means we determine a rescaled version of θ
        - Allows more efficient computation, and scale to much bigger training sets
        - If you have a training set with 10 000 values, means you get 10 000 features
            - Solving for all these parameters can become expensive
            - So by adding this in we avoid a for loop and use a matrix multiplication algorithm instead
- You can apply kernels to other algorithms
    - But they tend to be very computationally expensive
    - But the SVM is far more efficient - so more practical
- Lots of good off the shelf software to minimize this function

- **SVM parameters (C)**
    - Bias and variance trade off
    - Must chose C

      C plays a role similar to 1/LAMBDA (where LAMBDA is the regularization parameter)
    - Large C gives a hypothesis of **low bias high variance** --> overfitting
    - Small C gives a hypothesis of **high bias low variance** --> underfitting
- **SVM parameters ($\sigma^2$)**

  Parameter for calculating f values
    - Large $\sigma^2$ - f features vary more smoothly - higher bias, lower variance
    - Small $\sigma^2$ - f features vary abruptly - low bias, high variance

**SVM - implementation and use**

- So far spoken about SVM in a very abstract manner
- What do you need to do this
    - Use SVM software packages (e.g. liblinear, libsvm) to solve parameters θ
    - Need to specify
        - Choice of parameter C

        - Choice of kernel

**Choosing a kernel**

- We've looked at the **Gaussian kernel**
    - Need to define σ ($\sigma^2$)
        Discussed$\sigma^2$
    - When would you chose a Gaussian?
        If n is small and/or m is large
            e.g. 2D training set that's large
    - If you're using a Gaussian kernel then you may need to implement the kernel function
        - e.g. a function
          fi = kernel(x1,x2)
            Returns a real number
        - Some SVM packages will expect you to define kernel
        - Although, some SVM implementations include the Gaussian and a few others
            Gaussian is probably most popular kernel
    - NB - make sure you perform **feature scaling** before using a Gaussian kernel
        If you don't features with a large value will dominate the f value
- Could use no kernel - **linear kernel**
    - Predict y = 1 if ($\theta^T x$) >= 0
        - So no f vector
        - Get a standard linear classifier
    - Why do this?
        If n is large and m is small then
            - Lots of features, few examples
            - Not enough data - risk overfitting in a high dimensional feature-space
- Other choice of kernel
    - Linear and Gaussian are most common
    - Not all similarity functions you develop are valid kernels
        - Must satisfy **Merecer's Theorem**
        - SVM use numerical optimization tricks
            Mean certain optimizations can be made, but they must follow the theorem
    - **Polynomial Kernel**
        - We measure the similarity of x and l by doing one of
            - $(x^T l)^2$
            - $(x^T l)^3$
            - $(x^T l+1)^3$
        - General form is
            $(x^T l+Con)^D$
        - If they're similar then the inner product tends to be large
        - Not used that often
        - Two parameters

        - 
            - Degree of polynomial (D)
            - Number you add to l (Con)
        - Usually performs worse than the Gaussian kernel
        - Used when x and l are both non-negative
    - **String kernel**
        - Used if input is text strings
        - Use for text classification
    - **Chi-squared kernel**
    - **Histogram intersection kernel**

**Multi-class classification for SVM**

- Many packages have built in multi-class classification packages
- Otherwise use one-vs all method
- Not a big issue

**Logistic regression vs. SVM**

- When should you use SVM and when is logistic regression more applicable
- If n (features) is large vs. m (training set)
- 
    - e.g. text classification problem
    - 
        - Feature vector dimension is 10 000
        - Training set is 10 - 1000
        - Then use logistic regression or SVM with a linear kernel
- If n is small and m is intermediate
    - n = 1 - 1000
    - m = 10 - 10 000
    - Gaussian kernel is good
- If n is small and m is large
    - n = 1 - 1000
    - m = 50 000+

        SVM will be slow to run with Gaussian kernel
    - In that case
        - Manually create or add more features
        - Use logistic regression of SVM with a linear kernel
- Logistic regression and SVM with a linear kernel are pretty similar
    - Do similar things
    - Get similar performance
- A lot of SVM's power is using diferent kernels to learn complex non-linear functions
- For all these regimes a well designed NN should work

    But, for some of these problems a NN might be slower - SVM well implemented would be faster
- SVM has a convex optimization problem - so you get a global minimum
- It's not always clear how to chose an algorithm
    - Often more important to get enough data
    - Designing new features

    - Debugging the algorithm
- SVM is widely perceived a very powerful learning algorithm