Suppose the random variables and have the covariance matrix:

Calculate (by hand, show detail work) the eigenvalue-eigenvector pairs and the principal components and .

***Solution:***

Firstly, to find the eigenvalues we use:

To make Eq. (1) solvable, the characteristic equation is introduced as:

Insert the covariance matrix C into Eq. (2) yields:

With the corresponding eigenvectors are calculated as:

With the ascending order of the eigenvalues, the eigenvectors are the principal components of the data set.

By using these components, we can create the feature vectors to transform data into a new sub-space.