Fundamentals of Electric Circuits

DC Circuits

Chapter 2. Basic Laws

Basic Laws

2.1. Introduction

+ Understand some fundamental laws:

→ First important steps to determine the values of current, voltage, and power in an electric circuit

+ In this chapter:

→ Ohm's law, Kirchhoff's laws

→ Some common techniques applied in circuit design and analysis

Combining resistors in series and parallel

Voltage division

Current division

Wye – Delta transformations

2.2. Ohm's law

+ Resisting the flow of electric charge: behavior of materials, in general

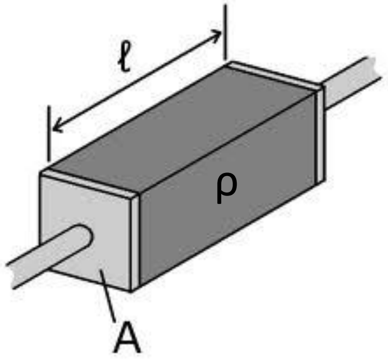
+ The ability to resist electric current → *Resistance* (R)

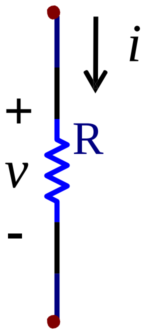
$$R = \rho \frac{l}{A}$$

ρ: **resistivity** of the material [Ωm]

l : **length** of material [m]

A: **cross sectional** area [m²]





+ *Ohm's law:* *The voltage v across a resistor is directly proportional to the current i flowing through the resistor*

$$v = Ri$$

+ The resistance R of an element: → its ability to resist the flow of electric current, measured in Ohms [Ω]

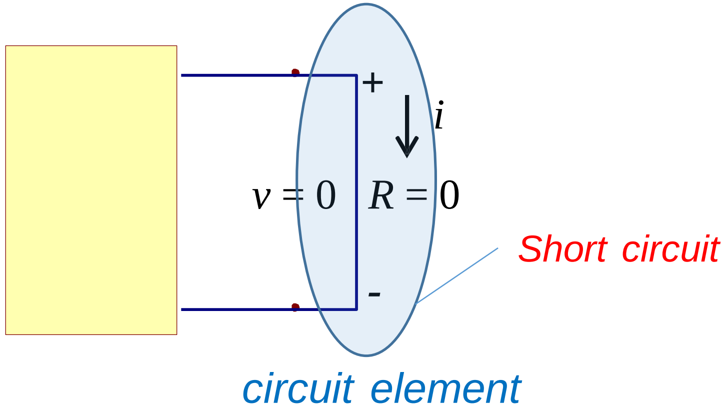
$$R = \frac{v}{i} \rightarrow 1\Omega = 1\frac{V}{A}$$

2.2. Ohm's law

+ A circuit element with resistance approaching zero:

→ *Short circuit* (current could be anything)

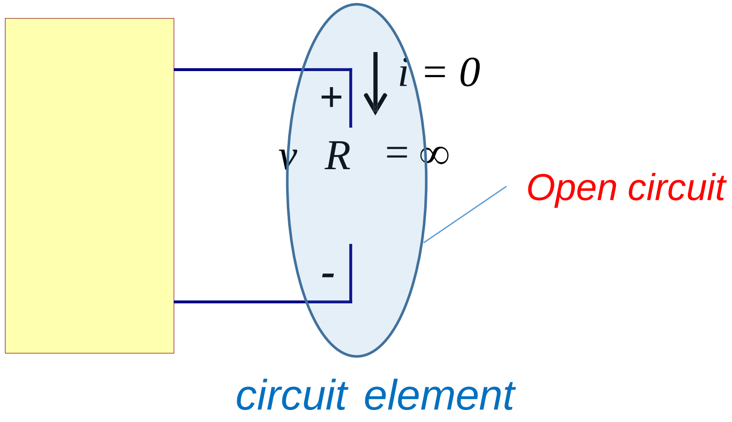
$$R = 0 \rightarrow v = iR = 0$$



+ A circuit element with resistance approaching infinity:

→ *Open circuit* (voltage could be anything)

$$i = \lim_{R\rightarrow\infty} \frac{v}{R} = 0$$



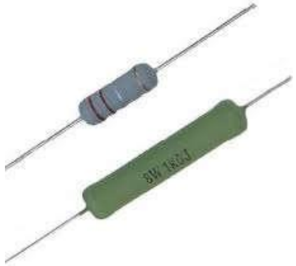
2.2. Ohm's law

+ Resister classification

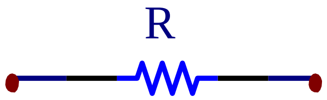
→ Fixed resistors

fixed value

Wire wound (small resistance)



Composition (large resistance)



Symbol for fixed resistor

→ Variable resistors

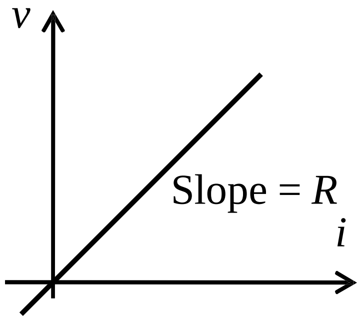
variable value

Two kinds of VR

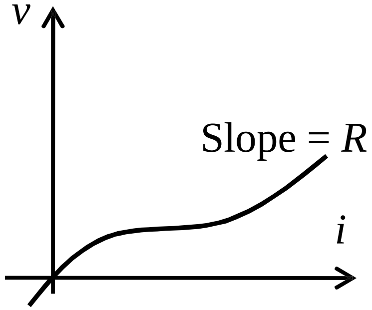
Symbol for variable resistor

→ Linear and nonlinear resistors

Linear resistor: *linear relationship between v and i → R is constant*



Nonlinear resistor. nonlinear relationship between v and i → R is not constant (not be considered)



2.2. Ohm's law

+ The **ability** of an element **to conduct electric current**: ***Conductance*** (measured in Siemens [S])

$$G = \frac{1}{R} = \frac{i}{v} \qquad 1S = 1\frac{A}{V}$$

+ **Power *dissipated by a resistor*** *(conductance)*

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = vi = v^2 G = \frac{i^2}{G}$$

→ a resistor always absorbs power from the circuit

2.3. Nodes, branches, and loops

+ **Nodes, branches, loops**: basic concepts of network (**circuit**) topology

a network → an interconnection of elements or devices

a circuit → a network providing one or more closed paths

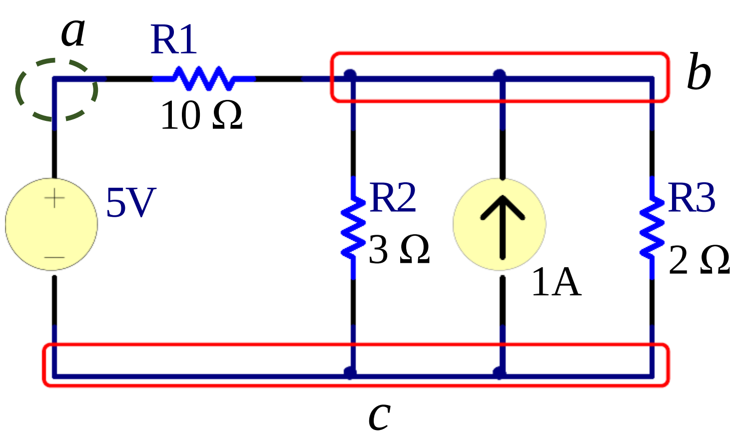
+ **Nodes, branches, loops**

Giving information of

→ ***geometric configuration of the network/circuit***

→ ***properties relating to the placement of elements*** in the network

2.3. Nodes, branches, and loops



+ A *branch* *(b):* represents a single element (any two terminal element) in a network

Ex: Given circuit has five branches: the 5-V voltage source, the 1-A current source, and three resistors

+ A *node* *(n):* → the point of connection between two or more branches

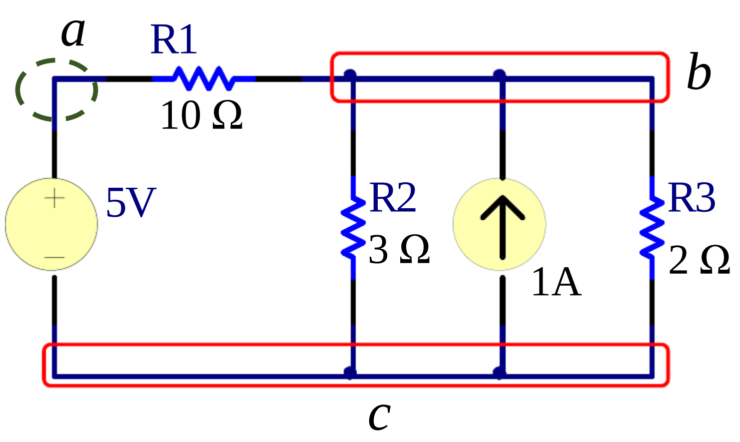
Ex: The given circuit has three nodes: *a, b,* and *c*

+ A *loop:* → any closed path in a circuit, formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once

Ex: abca is a loop, containing the R1, R2 and voltage source, or containing the R1, R3 and voltage source

→ Important note: different definition of node, branch, and loop

2.3. Nodes, branches, and loops



+ An *independent* loop: → contains a branch which is not in any other loop

Ex: The given circuit → six loops, three of them are independent

+ A network with ***b*** branches, ***n*** nodes, and ***l*** independent loops, has an equation

$$\boxed{b = l + n - 1}$$

+ Two or more elements are in ***series:*** → they are cascaded or connected sequentially and consequently carry the same current

+ Two or more elements are in ***parallel:*** → they are connected to the same two nodes and sequentially have the same voltage across them

2.4. Kirchhoff's laws

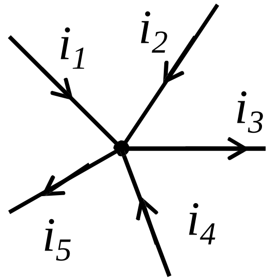
+ *Kirchhoff's current law (KCL):* the algebraic sum of currents entering a node (or a closed boundary) is zero

$$\sum_{n=1}^{N} i_n = 0$$

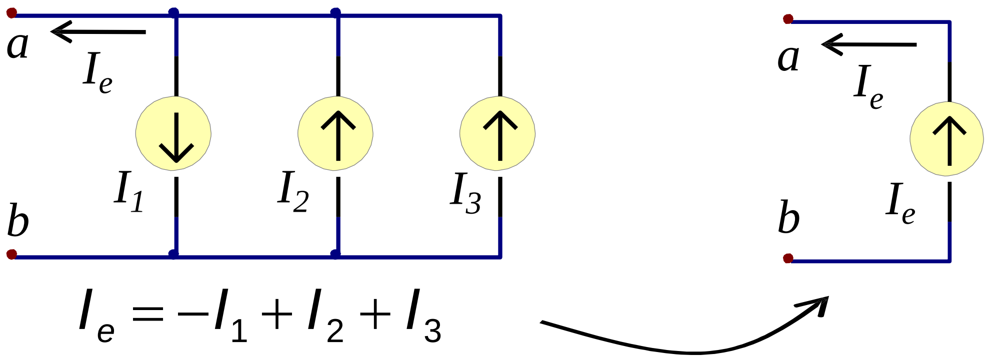
Common Convention:

- Current **entering** a node → regarded as **positive**
- Current **leaving** the node → taken as **negative**

For example:



$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$



$$I_e = -I_1 + I_2 + I_3$$

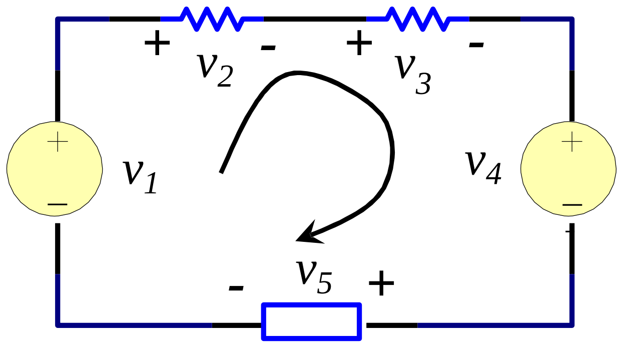
(KCL applied to current sources which connected in parallel)

2.4. Kirchhoff's laws

+ *Kirchhoff's voltage law (KVL):* the algebraic sum of all voltages around a closed path (or loop) is zero

$$\boxed{\sum_{m=1}^{M} v_m = 0}$$

For example:



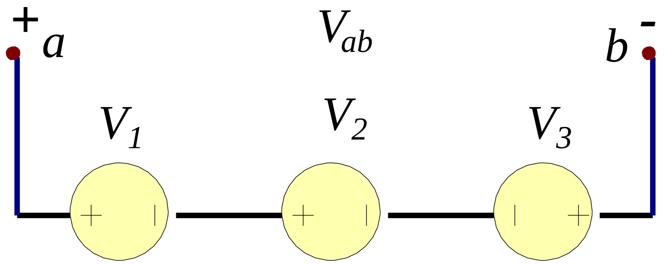
$$-v_1 + v_2 + v_3 + v_4 + v_5 = 0$$

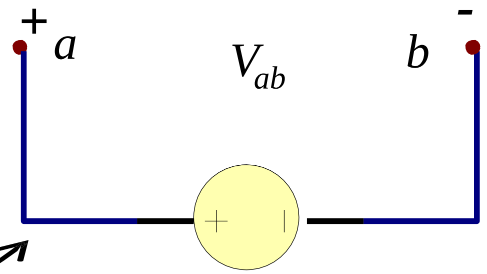
$$\Rightarrow \quad v_2 + v_3 + v_5 = v_1 - v_4$$

(go around the loop either clockwise or counterclockwise)

$$V_{ab} = V_1 + V_2 - V_3$$

(KVL applied to voltage sources which connected in series)

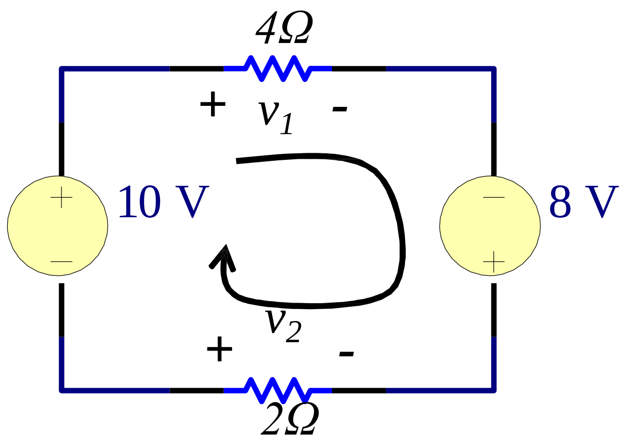




2.4. Kirchhoff's laws

+ Kirchhoff's laws, coupled with Ohm's law → a sufficient and powerful set of tools for analyzing electric circuits

Example 1: find v1 and v2 in the given circuit



From Ohm's law: $v_1 = 4i$; $v_2 = -2i$

Applying KVL around the loop gives: $v_1 - v_2 = 10 + 8 = 18$

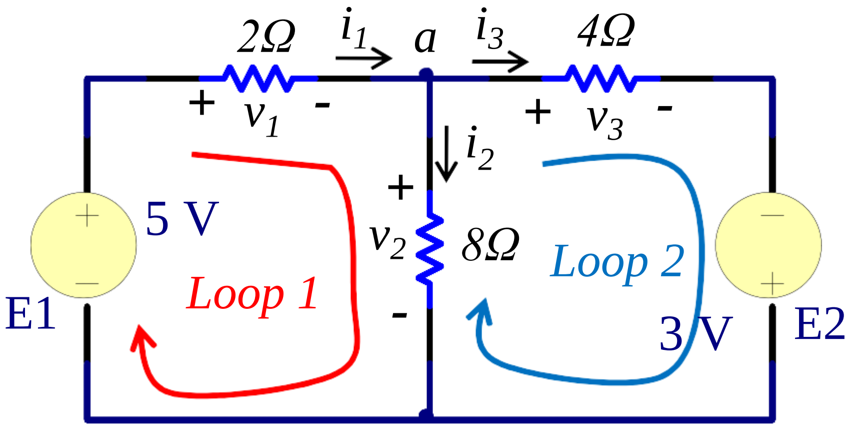
Substituting i in Ohm's law to KVL: $6i = 18 \rightarrow i = 3A$

So we have:

$$\boxed{\begin{aligned} v_1 &= 4i = 12V \\ v_2 &= -2i = -6V \end{aligned}}$$

2.4. Kirchhoff's laws

Example 2: find the currents and voltages in the given circuit



From Ohm's law $v_1 = 2i_1$ $v_2 = 8i_2$ $v_3 = 4i_3$

At node *a*, applying KCL gives $i_1 - i_2 - i_3 = 0$

Applying KVL to loop 1 and loop 2

$$\begin{cases} v_1 + v_2 = 5 \\ -v_2 + v_3 = 3 \end{cases} \rightarrow \begin{cases} 2i_1 + 8i_2 = 5 \\ -8i_2 + 4i_3 = 3 \end{cases}$$

So we have

$$\begin{cases} i_1 - i_2 - i_3 = 0 \\ 2i_1 + 8i_2 = 5 \\ -8i_2 + 4i_3 = 3 \end{cases} \rightarrow \begin{cases} i_1 = 1.5A \\ i_2 = 0.25A \\ i_3 = 1.25A \end{cases} \rightarrow \begin{cases} v_1 = 3V \\ v_2 = 2V \\ v_3 = 5V \end{cases}$$

2.5. Series resistors and voltage division

+ ***Equivalent resistance*** of any number of resistors connected in series: → the sum of the individual resistances

$$R_{eq} = R_1 + R_2 + ... + R_N = \sum_{n=1}^{N} R_n$$

+ ***Voltage division***: *The **voltage** v is divided among the resistors in **direct proportion** to their **resistances**, the larger the resistance, the larger the voltage drop*

$$v_n = \frac{R_n}{R_1 + R_2 + ... + R_N} v$$

2.6. Parallel resistors and current division

+ ***Equivalent resistance*** of two parallel resistors: → equal to the product of their resistances divided by their sum

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}$$

(*case N parallel resistors*)

+ ***Equivalent conductance*** of resistors connected in parallel: → the sum of their individual conductance

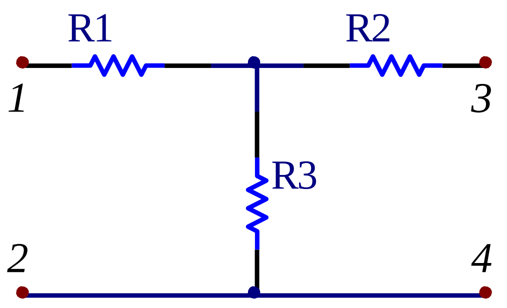
$$G_{eq} = G_1 + G_2 + \dots + G_N = \sum_{n=1}^{N} G_n$$

+ ***Current division***

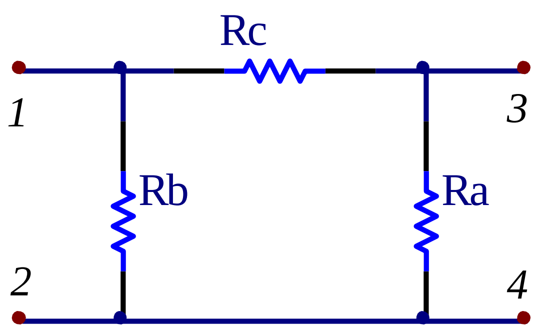
$$i_n = \frac{G_n}{G_1 + G_2 + \dots + G_N} i$$

2.7. Wye-delta transformations

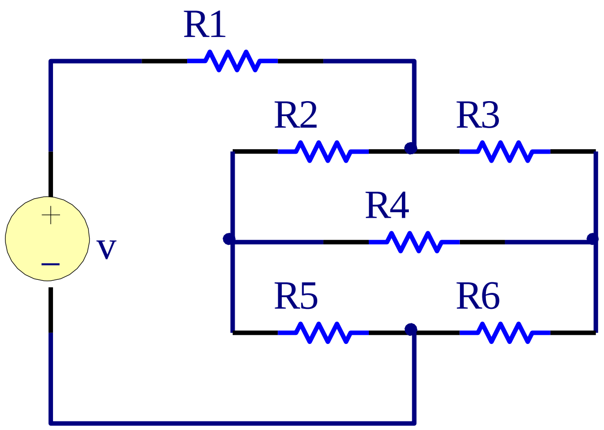
+ Resistors are neither in series nor in parallel



Wye (Y) or T network



Delta (Δ) or Π network

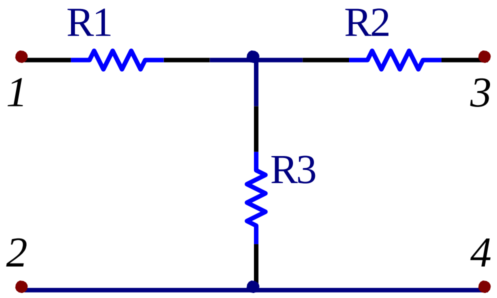


Bridge circuit

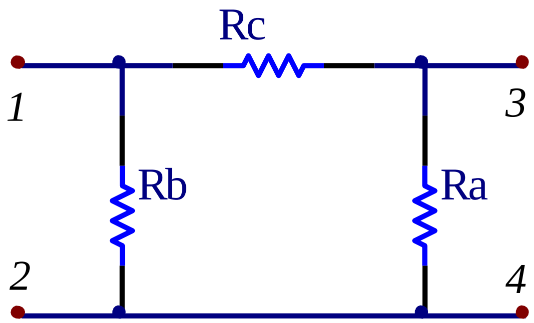
+ ***Delta to Wye conversion***

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad ; \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad ; \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

2.7. Wye-delta transformations



Wye (Y) or T network



Delta (Δ) or Π network

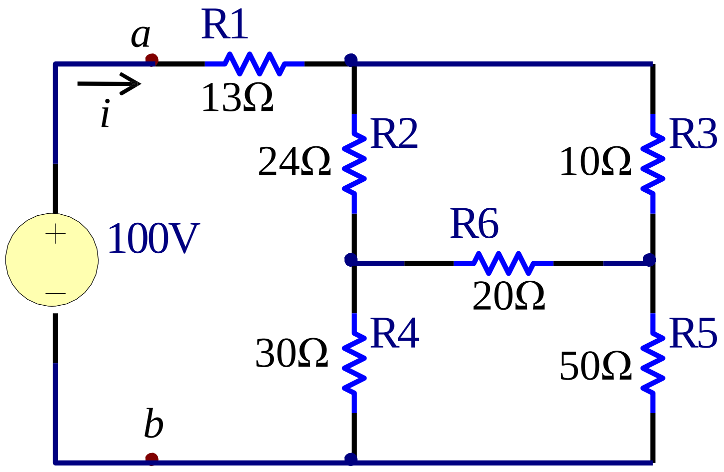
+ ***Wye to Delta conversion***

$$R_a = R_2 + R_3 + \frac{R_2 R_3}{R_1} \quad ; \quad R_b = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_c = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

2.7. Wye-delta transformations

For example: Given a bridge circuit as beside figure, find R_{eq} and i

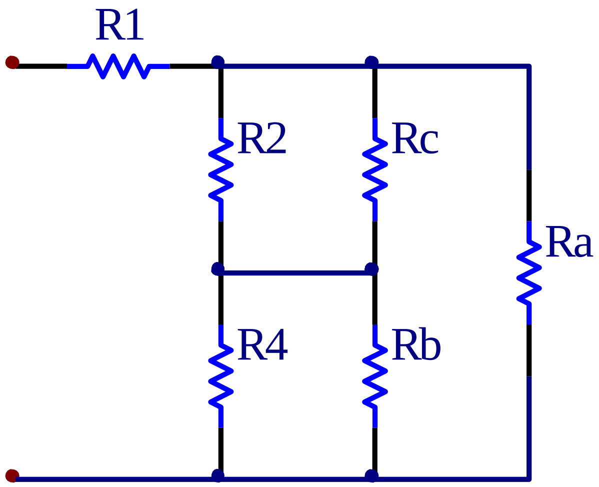


Having two Y networks: (R_2, R_4, R_6) and (R_3, R_5, R_6)
→ transforming just one of them to simplify the circuit

Applying the Y to Δ transformation:

$$R_a = R_3 + R_5 + \frac{R_3 R_5}{R_6} = 85\Omega \qquad R_b = R_5 + R_6 + \frac{R_5 R_6}{R_3} = 170\Omega$$

$$R_c = R_3 + R_6 + \frac{R_3 R_6}{R_5} = 34\Omega$$



The equivalent resistor:

$$\boldsymbol{R_{eq} = R_1 + \{[(R_2 // R_c) + (R_4 // R_b)] // R_a\}} \Rightarrow \boldsymbol{R_{eq} = 40\Omega}$$

The current needed to find: $$i = \frac{u_{ab}}{R_{eq}} = \frac{100}{40} = 2.5A$$