Nguyễn Công Phương

Electric Circuit Theory

Three-phase Circuits

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Three-phase Circuits

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2. Three-phase Source
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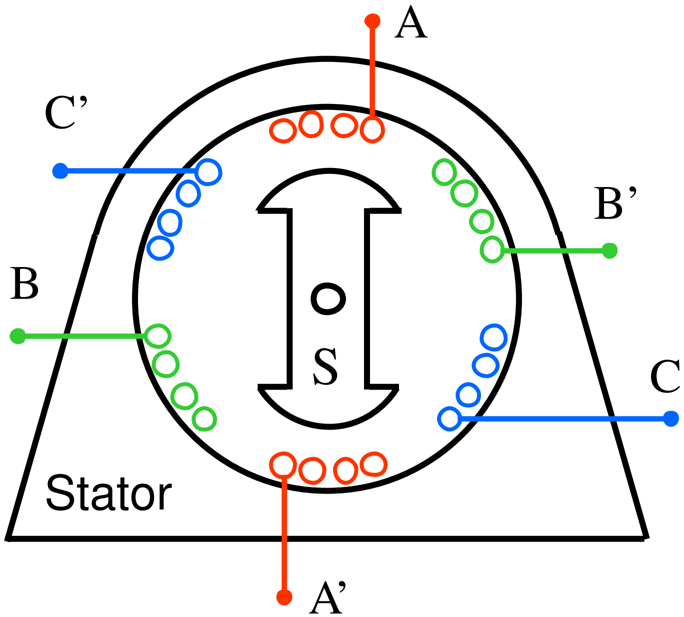
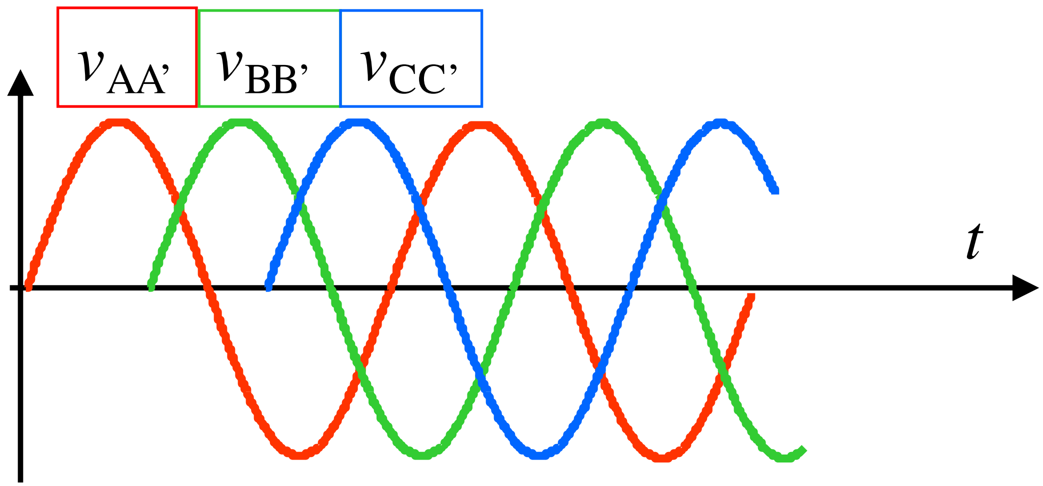
5. Power in Three-phase Circuits

Introduction (1)

- Polyphase:
  - Phase: branch, circuit or winding
  - Poly: many
- Three-phase: three phases
- Advantages:
  - Machine: less space & less cost
  - Transmission & distribution: less conducting material
  - Power delivered to a three-phase load is always constant
  - Single phase source from three-phase source

Three-phase Source (1)





$$v_{AA'} = V_m \sin \omega t$$

$$v_{BB'} = V_m \sin(\omega t - 120^o)$$

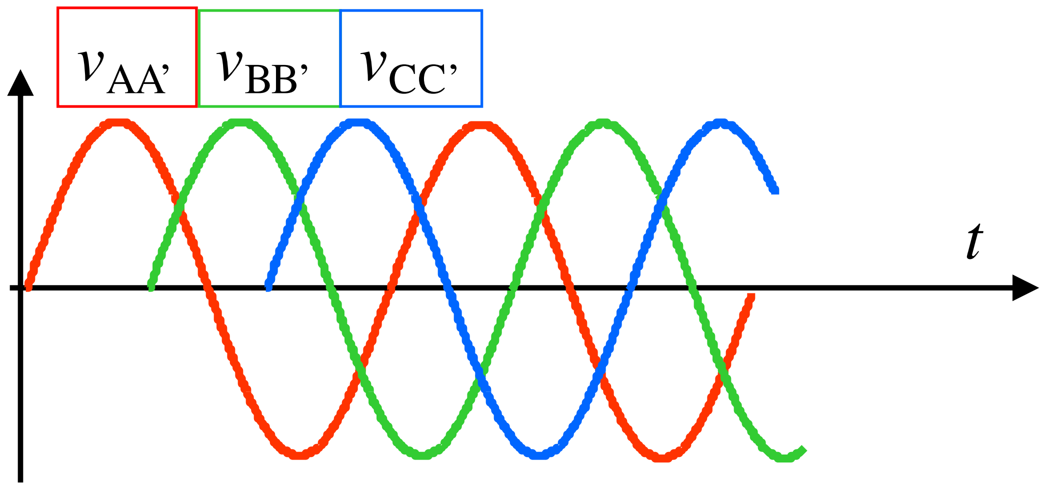
$$v_{CC'} = V_m \sin(\omega t + 120^o)$$

Three-phase Source (2)

$$v_{AA'} = V_m \sin \omega t$$

$$v_{BB'} = V_m \sin(\omega t - 120^o)$$

$$v_{CC'} = V_m \sin(\omega t + 120^o)$$



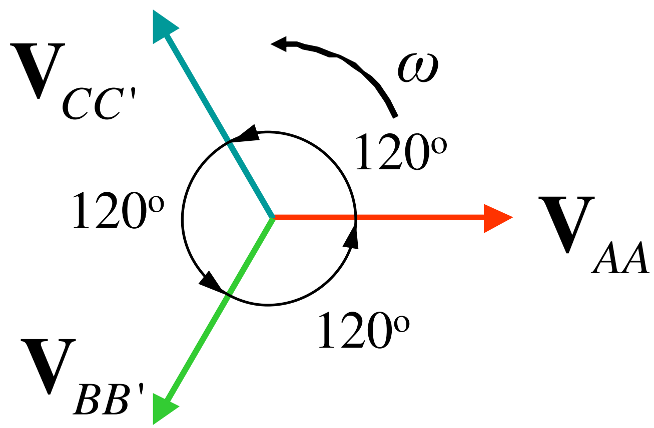
$$v_{AA'} + v_{BB'} + v_{CC'} =$$

$$= V_m(\sin \omega t + \sin \omega t \cos 120^o - \cos \omega t \sin 120^o + \sin \omega t \cos 120^o + \cos \omega t \sin 120^o)$$

$$= V_m(\sin \omega t + 2 \sin \omega t \cos 120^o)$$

$$= V_m\left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$$

$$\boxed{v_{AA'} + v_{BB'} + v_{CC'} = 0}$$

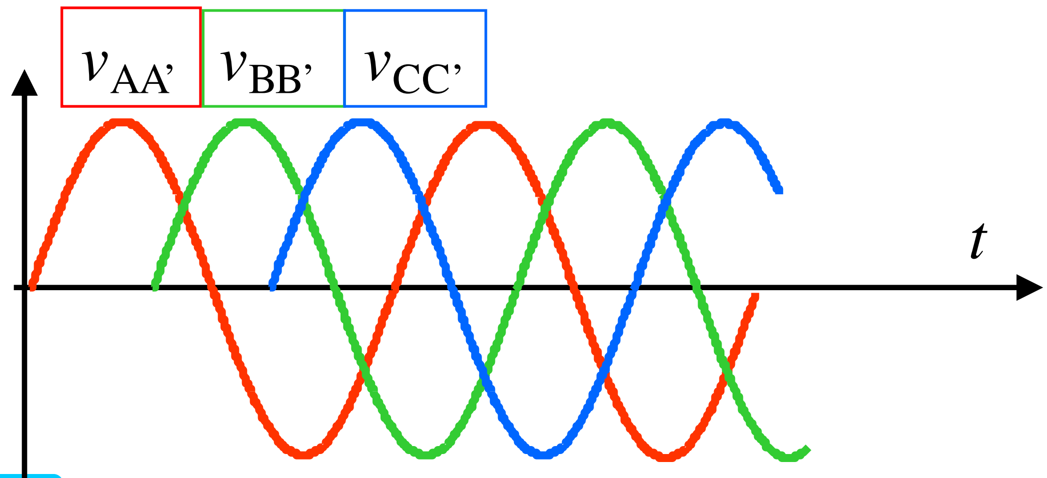


Three-phase Source (3)

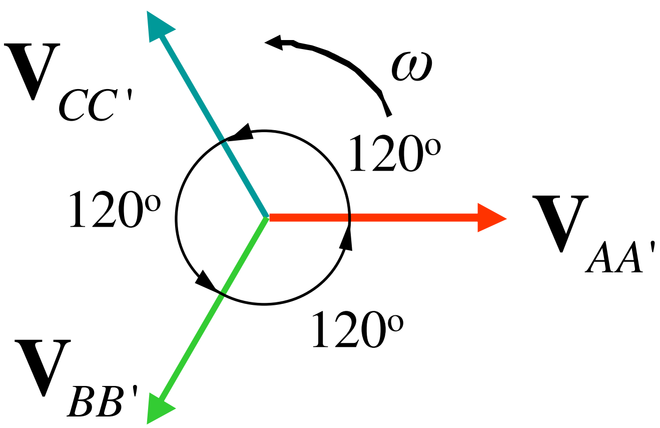
$$v_{AA'} = V_m \sin \omega t$$

$$v_{BB'} = V_m \sin(\omega t - 120^\circ)$$

$$v_{CC'} = V_m \sin(\omega t + 120^\circ)$$



$$\boxed{v_{AA'} + v_{BB'} + v_{CC'} = 0}$$



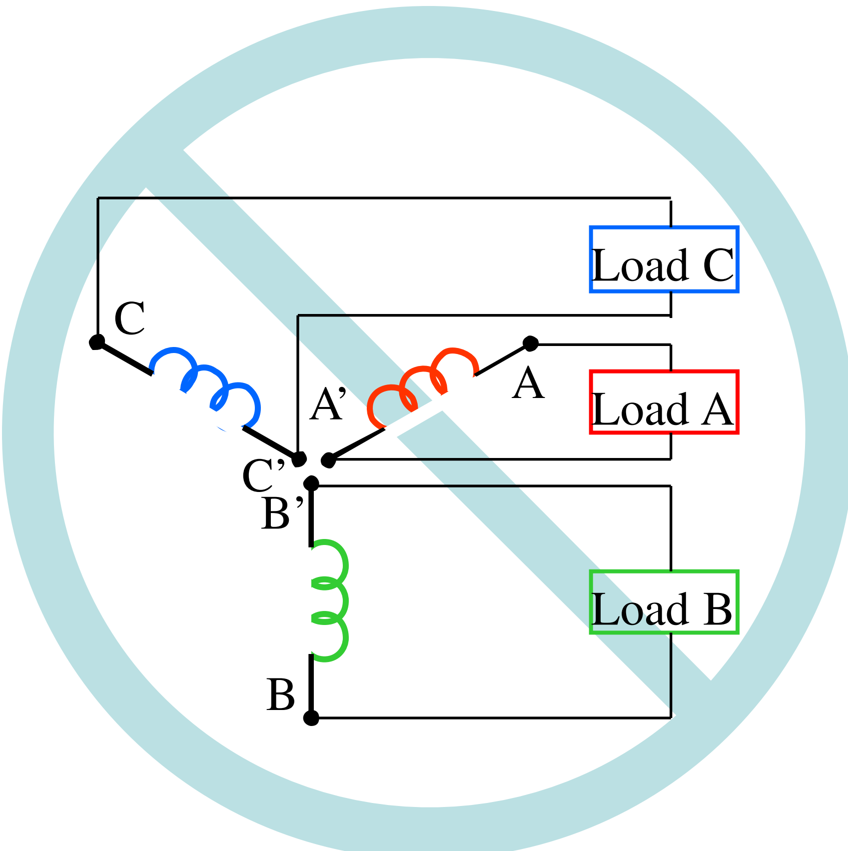
Symmetrical (balanced) Three-phase Source:

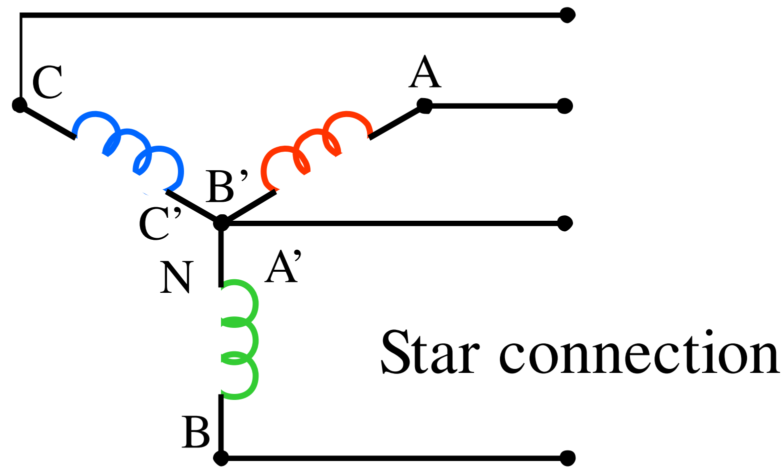
- Same magnitude

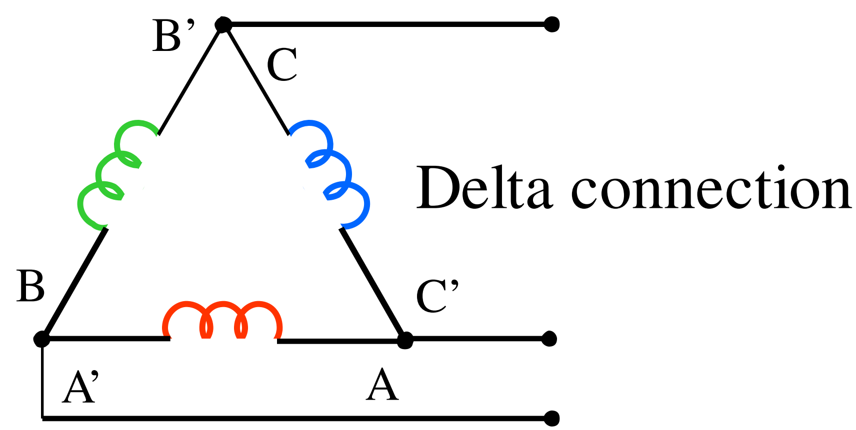
- Same frequency

- Displaced from each other by 120º

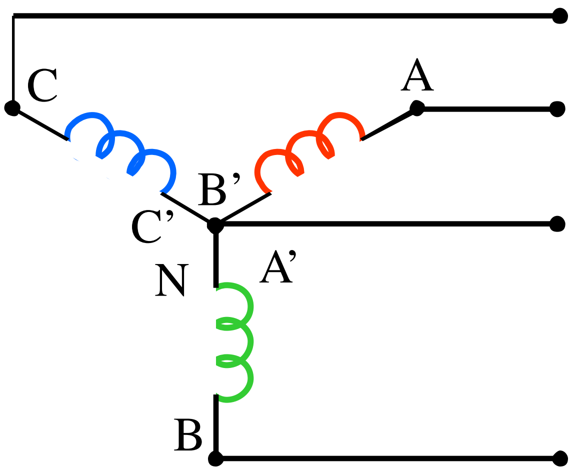
Three-phase Source (4)

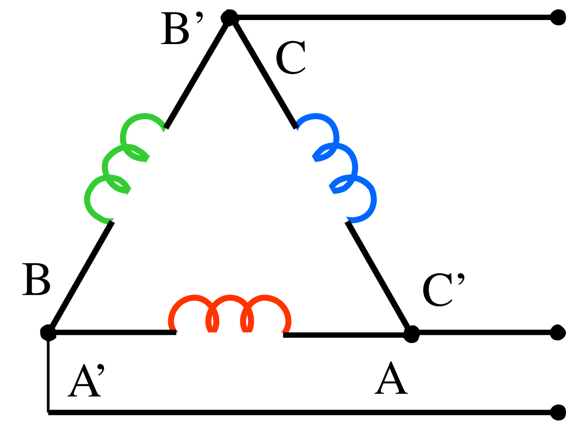


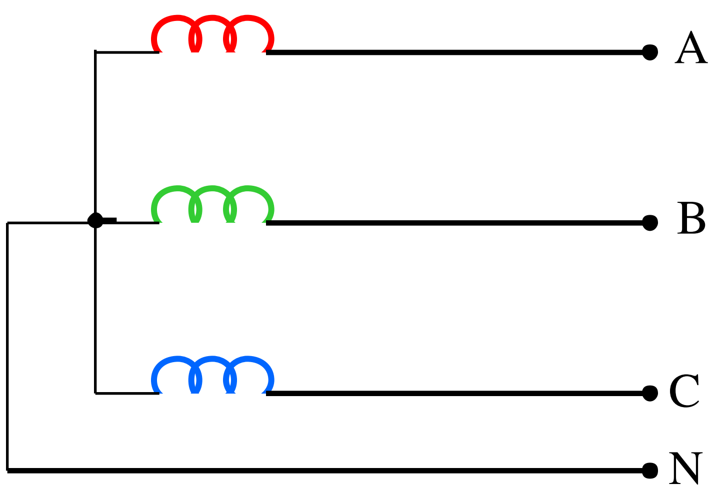


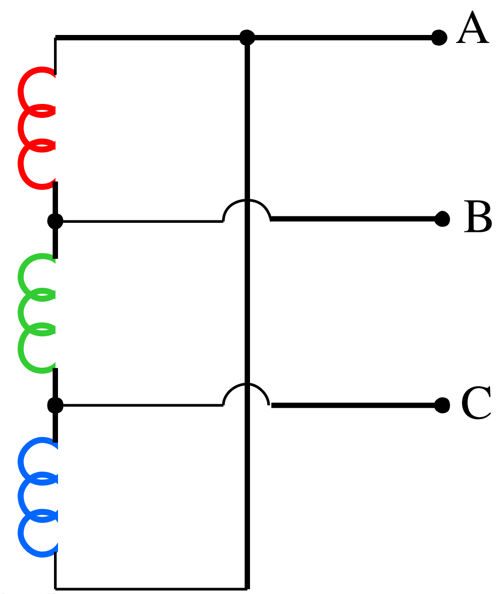


Three-phase Source (5)

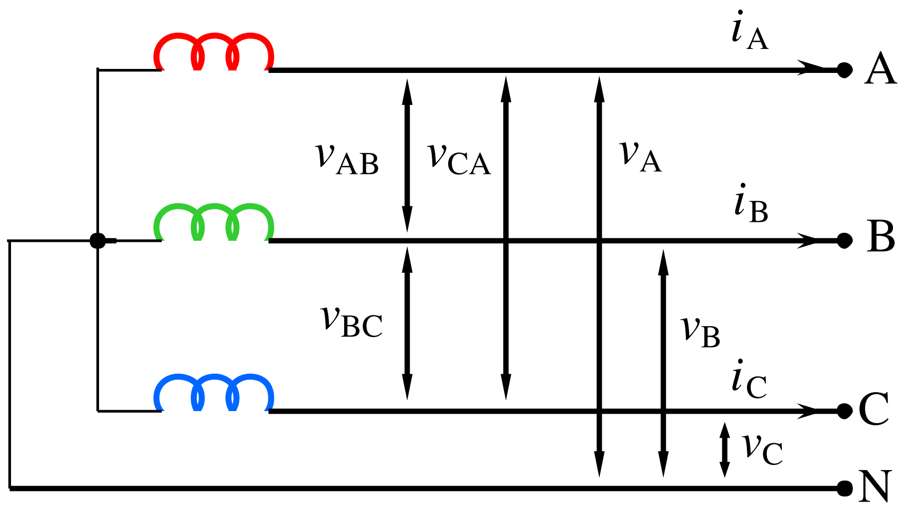


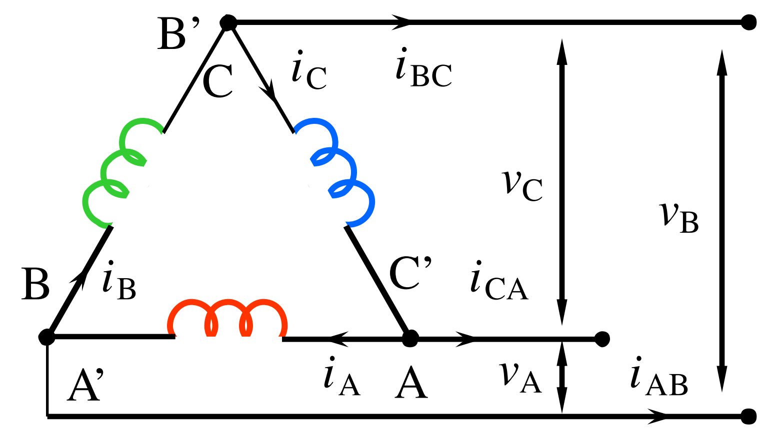






Three-phase Source (6)





v_{AB}, v_{BC}, v_{CA}: line voltages

v_A, v_B, v_C: phase voltages

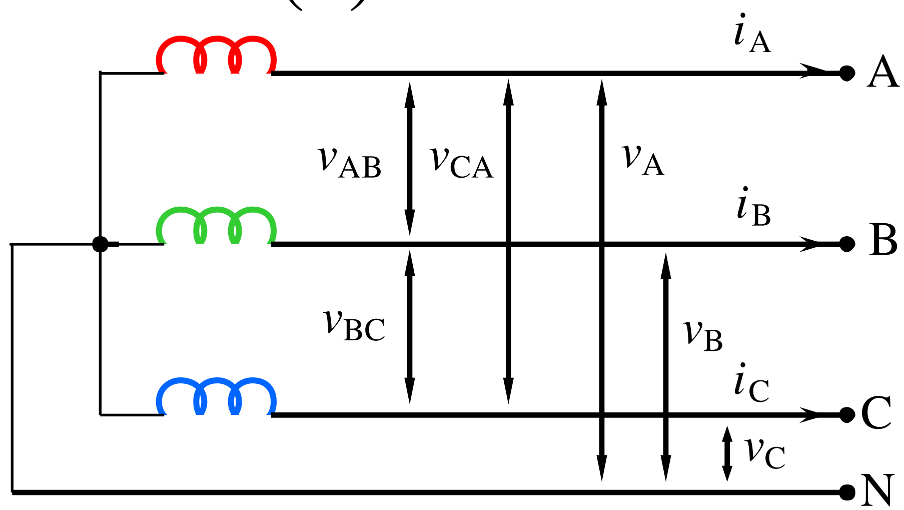
i_A, i_B, i_C: line currents/phase currents

v_A, v_B, v_C: line voltages/phase voltages

i_{AB}, i_{BC}, i_{CA}: line currents

i_A, i_B, i_C: phase currents

Three-phase Source (7)



$$\mathbf{V}_A = V\angle 0^\circ$$

$$\mathbf{V}_B = V\angle -120^\circ$$

$$\mathbf{V}_C = V\angle 120^\circ$$

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

$$= V\angle 0^\circ - V\angle -120^\circ = V\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V\angle 30^\circ$$

$$\mathbf{V}_{BC} = \sqrt{3}V\angle -90^\circ$$

$$\mathbf{V}_{CA} = \sqrt{3}V\angle -210^\circ$$

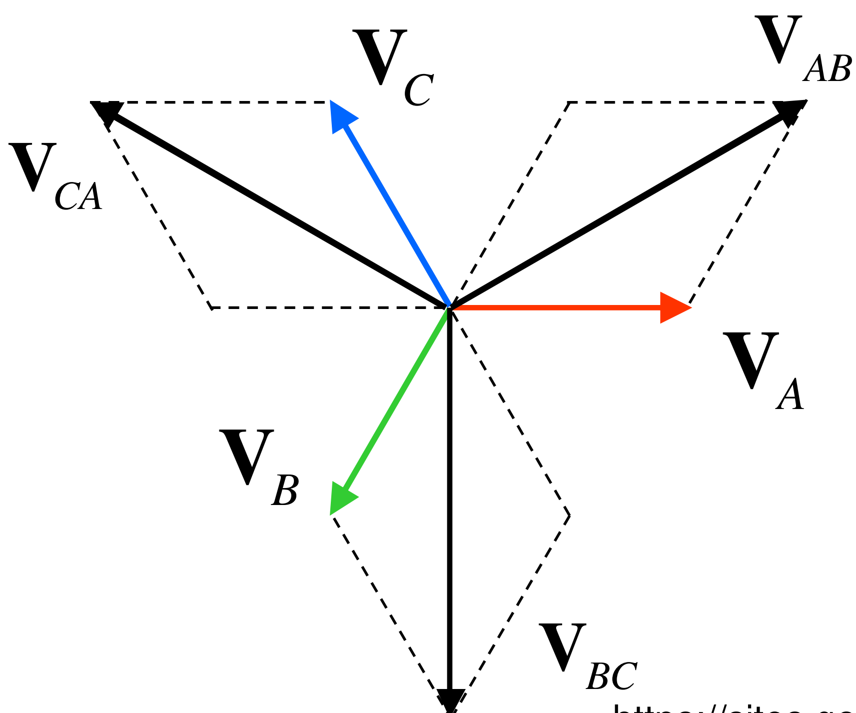
$$\boxed{\mathbf{V}_{line} = \mathbf{V}_{phase}\sqrt{3}\angle 30^\circ}$$

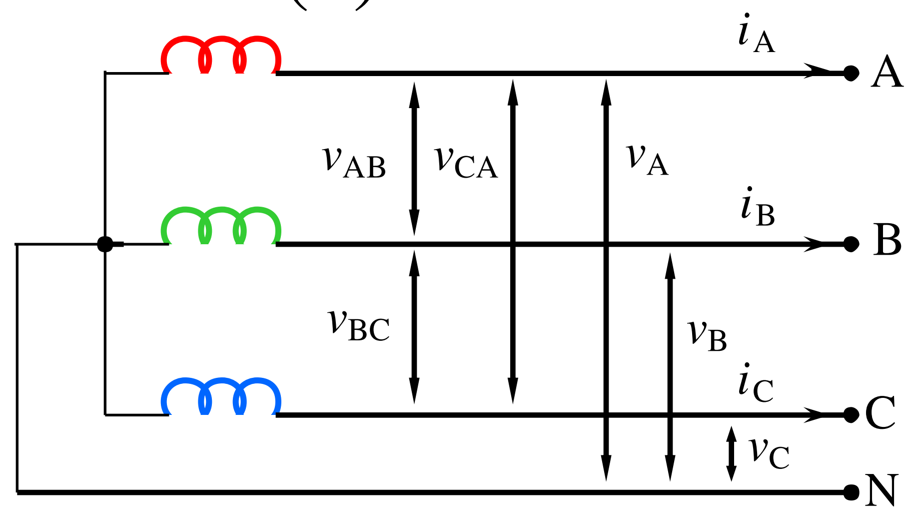
Three-phase Source (8)

$$\mathbf{V}_A = V\angle 0^\circ$$

$$\mathbf{V}_{BN} = V\angle -120^\circ$$

$$\mathbf{V}_{CN} = V\angle 120^\circ$$





$$\mathbf{V}_{AB} = \sqrt{3}V\angle 30^\circ$$

$$\mathbf{V}_{BC} = \sqrt{3}V\angle -90^\circ$$

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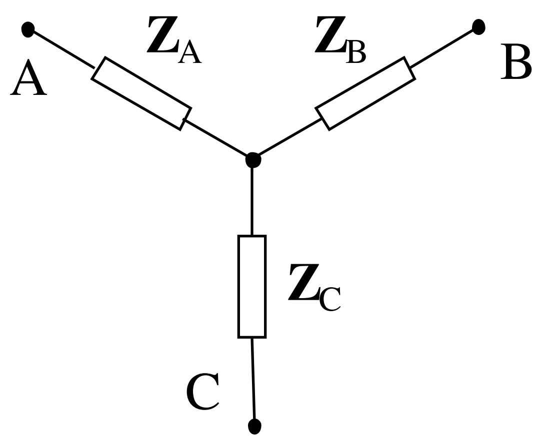
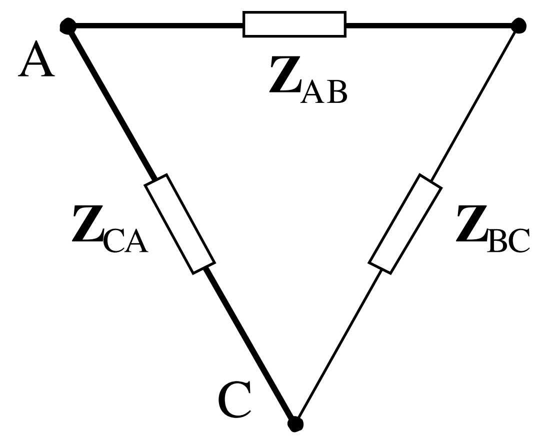
Three-phase Load





$$\mathbf{Z}_A = \frac{\mathbf{Z}_{CA}\mathbf{Z}_{AB}}{\mathbf{Z}_{AB} + \mathbf{Z}_{BC} + \mathbf{Z}_{CA}} \qquad \mathbf{Z}_{AB} = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_C\mathbf{Z}_A}{\mathbf{Z}_C}$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_{AB}\mathbf{Z}_{BC}}{\mathbf{Z}_{AB} + \mathbf{Z}_{BC} + \mathbf{Z}_{CA}} \qquad \mathbf{Z}_{BC} = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_C\mathbf{Z}_A}{\mathbf{Z}_A}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_{BC}\mathbf{Z}_{CA}}{\mathbf{Z}_{AB} + \mathbf{Z}_{BC} + \mathbf{Z}_{CA}} \qquad \mathbf{Z}_{CA} = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_C\mathbf{Z}_A}{\mathbf{Z}_B}$$

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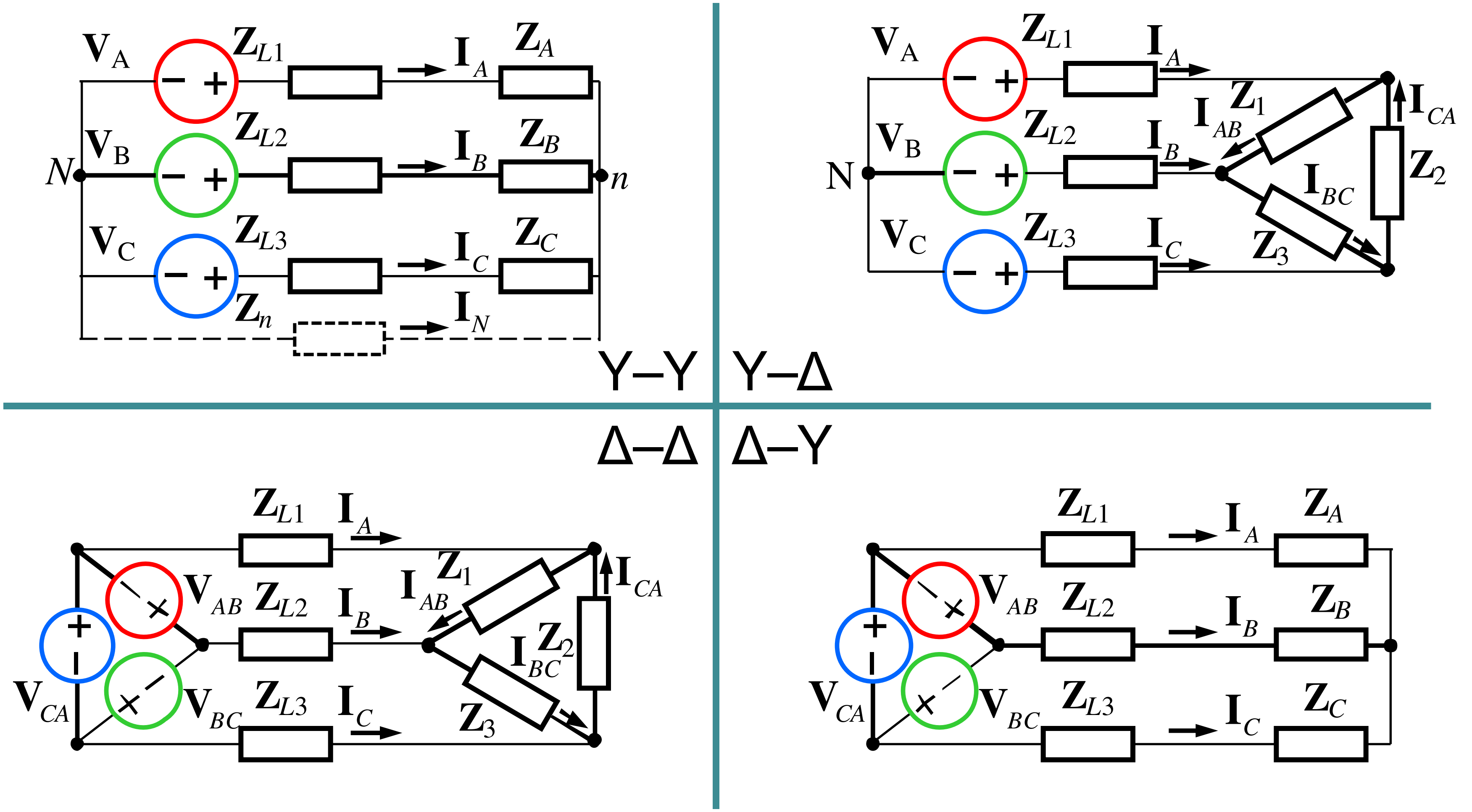
Three-phase Circuit Analysis (1)



Three-phase Circuit Analysis (2)

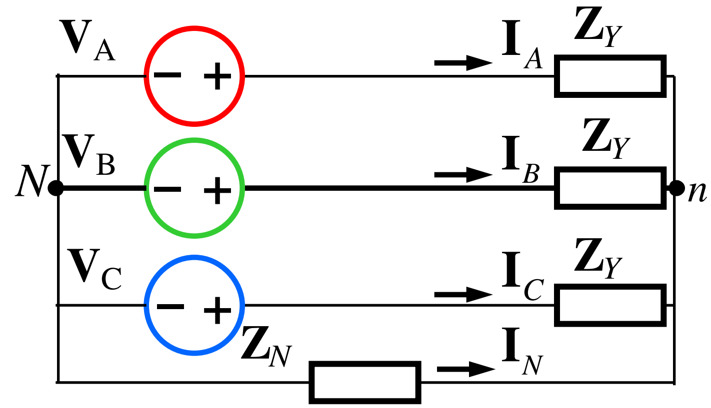
- Y–Y, Y–Δ, Δ–Δ, Δ–Y
- 2 kinds of three-phase circuit: balanced & unbalanced
- Balanced three-phase circuit:
  - Balanced three-phase source <u>and</u> balanced three-phase load
  - ***Balanced three-phase source***: same magnitude, same frequency, displaced from each other by 120^o
  - ***Balanced three-phase load***: three identical loads
- Unbalanced three-phase circuit:
  - Unbalanced three-phase source <u>and/or</u> unbalanced three-phase load
- To solve a balanced one:
  - Exploit the symmetry of a balanced three-phase circuit, <u>or</u>
  - Treat it like a normal three-source circuit
- To solve an unbalanced one:
  - Treat it like a normal three-source circuit

Three-phase Circuit Analysis (3), Y–Y

Suppose $\mathbf{V}_N = 0$

$$\left.\begin{aligned}&\left(\frac{1}{\mathbf{Z}_Y}+\frac{1}{\mathbf{Z}_Y}+\frac{1}{\mathbf{Z}_Y}+\frac{1}{\mathbf{Z}_N}\right)\mathbf{V}_n = \frac{\mathbf{V}_A}{\mathbf{Z}_Y}+\frac{\mathbf{V}_B}{\mathbf{Z}_Y}+\frac{\mathbf{V}_C}{\mathbf{Z}_Y}\\ &\mathbf{V}_A+\mathbf{V}_B+\mathbf{V}_C = 0\end{aligned}\right\}$$

$$\to \mathbf{V}_n = 0 \to \boxed{\mathbf{V}_{Nn} = 0}$$



$$\to \mathbf{I}_A = \frac{\mathbf{V}_A - \mathbf{V}_n}{\mathbf{Z}_Y} = \frac{V\angle 0^\circ - 0}{\mathbf{Z}_Y} = \frac{V\angle 0^\circ}{\mathbf{Z}_Y}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_B}{\mathbf{Z}_Y} = \frac{\mathbf{V}_A \times 1\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_A \times 1\angle -120^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_Y} = \frac{\mathbf{V}_A \times 1\angle 120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_A \times 1\angle 120^\circ$$

$$\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$$

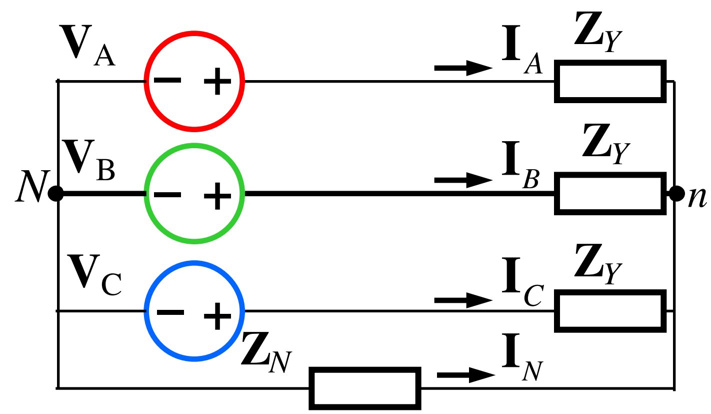
$$\mathbf{I}_N = 0$$

Three-phase Circuit Analysis (4), Y–Y

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{\mathbf{Z}_Y} = \frac{V\angle 0^\circ}{\mathbf{Z}_Y}$$

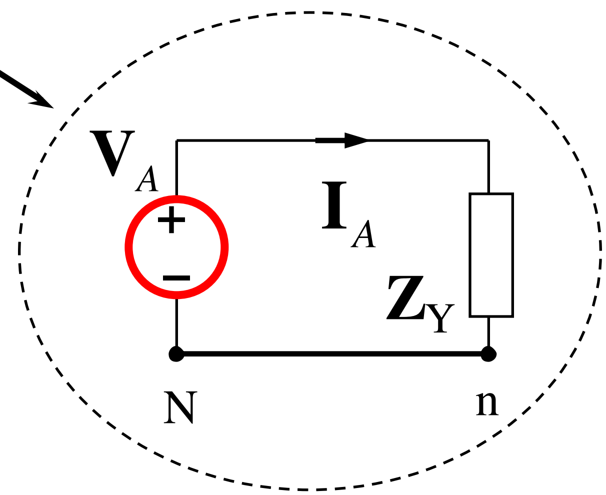
$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ$$

$$\mathbf{I}_C = \mathbf{I}_A \times 1\angle 120^\circ$$



For a balanced Y–Y system:

1. Draw a single-phase equivalent circuit (A-phase)
2. Find the current in the A-phase
3. Write down the currents in the two other phases



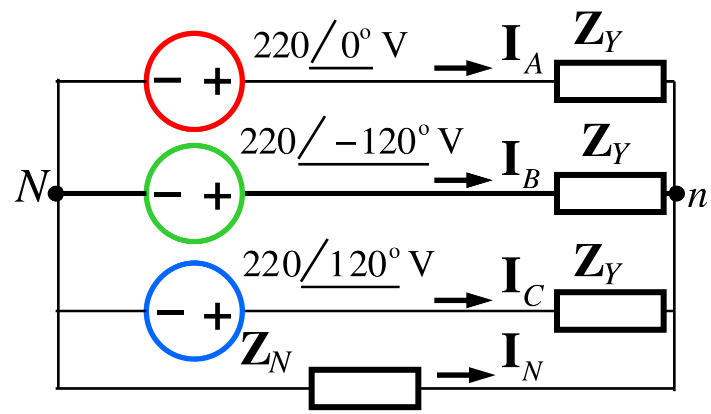
Three-phase Circuit Analysis (5), Y–Y

Ex. 1

$\mathbf{Z}_Y = 30 + j40\ \Omega$; find phase currents?

For a balanced Y–Y system:

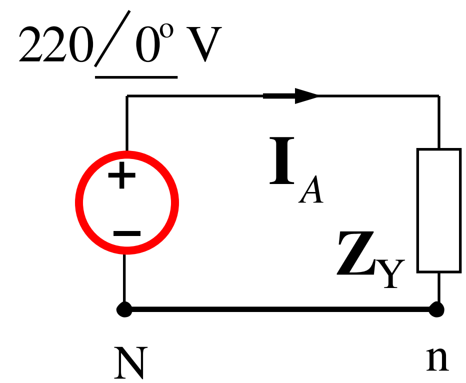
1. ✓ Draw a single-phase equivalent circuit (A-phase)
2. ✓ Find the current in the A-phase
3. ✓ Write down the currents in the two other phases



$$\mathbf{I}_A = \frac{220\angle 0^\circ}{\mathbf{Z}_Y} = \frac{220\angle 0^\circ}{30 + j40} = \boxed{4.4\angle -53.1^\circ \text{ A}}$$

$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ = 4.4\angle -53.1^0 - 120^\circ = \boxed{4.4\angle -173.1^\circ \text{ A}}$$

$$\mathbf{I}_C = \mathbf{I}_A \times 1\angle 120^\circ = 4.4\angle -53.1^\circ + 120^\circ = \boxed{4.4\angle 66.9^\circ \text{ A}}$$



Three-phase Circuit Analysis

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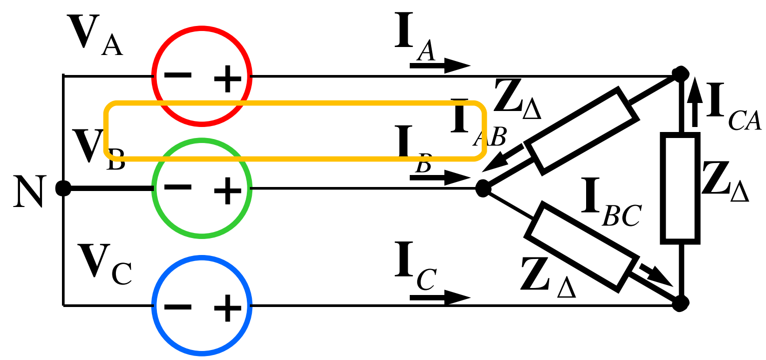
- To solve an unbalanced one:
  - Treat it like a normal three-source circuit

Three-phase Circuit Analysis (6), Y–Δ

$$\mathbf{V}_A = V\angle 0^\circ$$

$$\mathbf{V}_B = V\angle -120^\circ$$

$$\mathbf{V}_C = V\angle 120^\circ$$



$$\boxed{Z_\Delta \mathbf{I}_{AB} = \mathbf{V}_A - \mathbf{V}_B}$$

$$\rightarrow \boxed{\mathbf{I}_{AB}} = \frac{\mathbf{V}_A - \mathbf{V}_B}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}$$

Line currents:
- Same magnitude
- Same frequency
- Displaced from each other by 120°

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} \times 1\angle -120^\circ = \boxed{\mathbf{I}_{AB} \times 1\angle -120^\circ}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} \times 1\angle 120^\circ = \boxed{\mathbf{I}_{AB} \times 1\angle 120^\circ}$$

Three-phase Circuit Analysis (7), Y–Δ

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \mathbf{I}_{AB} \times 1\angle 0^\circ$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB} \times 1\angle -120^\circ$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB} \times 1\angle 120^\circ$$

KCL for a: $\mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA}$

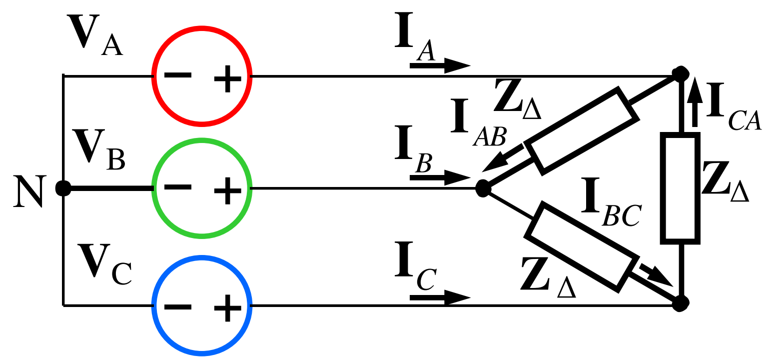
$$\rightarrow \mathbf{I}_A = \mathbf{I}_{AB}(1\angle 0^\circ - 1\angle 120^\circ)$$

$$= \mathbf{I}_{AB}(1 + 0,5 - j0,866)$$

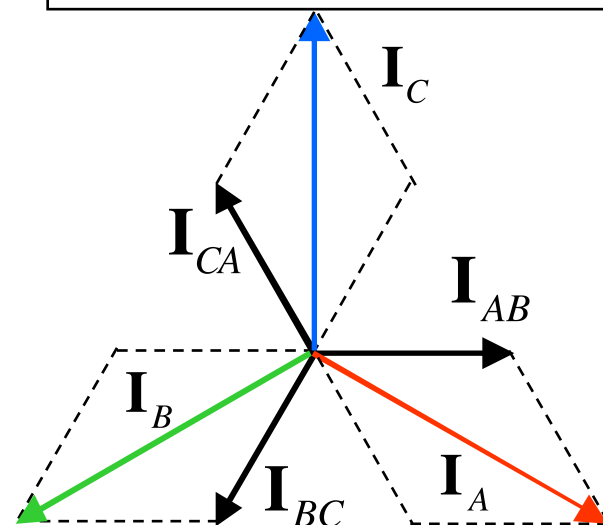
$$= \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\mathbf{I}_B = \mathbf{I}_{AB}\sqrt{3}\angle -150^\circ$$

$$\mathbf{I}_C = \mathbf{I}_{AB}\sqrt{3}\angle 90^\circ$$



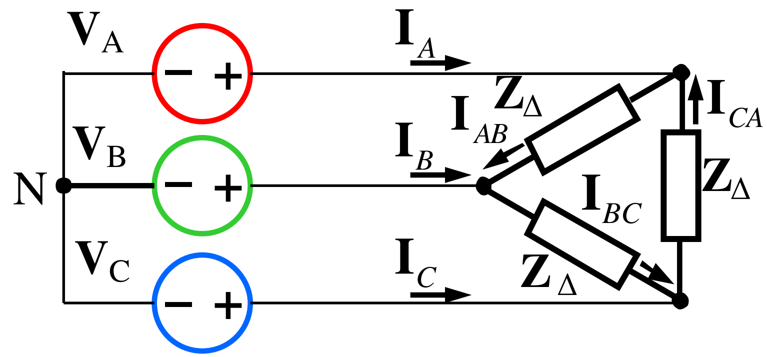
$$\boxed{\mathbf{I}_{phase} = \mathbf{I}_{line}\sqrt{3}\angle -30^\circ}$$



Phase currents:
- Same magnitude
- Same frequency
- Displaced from each other by 120°

Ex. 2 Three-phase Circuit Analysis (8), Y–Δ

$\mathbf{Z}_\Delta = 30 + j40\ \Omega; \mathbf{V}_A = 220\angle 15^\circ$ V . Find currents?



Method 1

$$\boxed{\mathbf{V}_{line} = \mathbf{V}_{phase}\sqrt{3}\angle 30^\circ}$$

$$\mathbf{V}_{AB} = \sqrt{3}\mathbf{V}_A \times 1\angle 30^\circ$$

$$= \sqrt{3} \times 220\angle 15^\circ + 30^\circ = 381\angle 45^\circ \text{ V}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{381\angle 45^\circ}{30 + j40} = 7.62\angle -8.1^\circ \text{ A}$$

$$\boxed{\mathbf{I}_{phase} = \mathbf{I}_{line}\sqrt{3}\angle -30^\circ}$$

$$\mathbf{I}_{BC} = 7.62\angle -8.1^\circ - 120^\circ = 7.62\angle -128.1^\circ \text{ A}$$

$$\mathbf{I}_{CA} = 7.62\angle -8.1^\circ + 120^\circ = 7.62\angle 111.9^\circ \text{ A}$$

$$\mathbf{I}_A = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = 7.62\angle -8.1^\circ \times \sqrt{3}\angle -30^\circ = 13.20\angle -38.1^\circ \text{ A}$$

$$\mathbf{I}_B = \mathbf{I}_A\angle -120^\circ = 13.2\angle -38.1^\circ - 120^\circ = 13.20\angle -158.1^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}_A\angle 120^\circ = 13.2\angle -38.1^\circ + 120^\circ = 13.20\angle 81.9^\circ \text{ A}$$

Ex. 2 Three-phase Circuit Analysis (9), Y–Δ

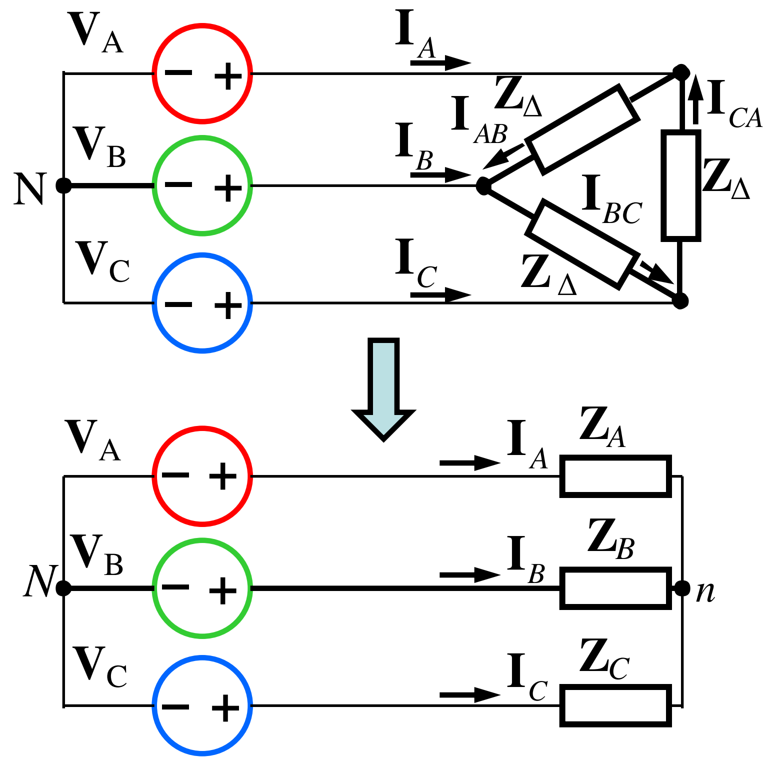
$\mathbf{Z}_\Delta = 30 + j40\ \Omega; \mathbf{V}_A = 220\angle 15^\circ$ V . Find currents?

Method 2

$$\mathbf{Z}_A = \frac{\mathbf{Z}_{CA}\mathbf{Z}_{AB}}{\mathbf{Z}_{AB} + \mathbf{Z}_{BC} + \mathbf{Z}_{CA}}$$

$$= \frac{\mathbf{Z}_\Delta}{3} = \frac{30 + j40}{3} = 10 + j13.33\ \Omega$$

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{\mathbf{Z}_A} = \frac{220\angle 15^\circ}{10 + j13.33} = 13.20\angle -38.1^\circ\ \text{A}$$



Three-phase Circuit Analysis

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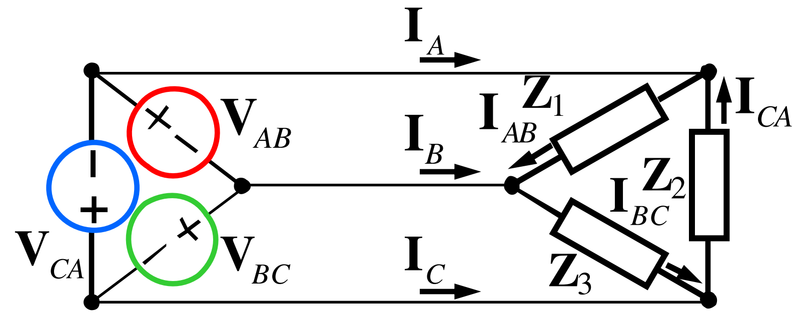
Three-phase Circuit Analysis (10), Δ–Δ

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}$$



$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}} = \mathbf{I}_{AB} \times 1\underline{/-120^{\circ}}$$

$$\left.\begin{aligned} \mathbf{I}_{CA} &= \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}} = \mathbf{I}_{AB} \times 1\underline{/120^{\circ}} \\ \mathbf{I}_{A} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} \end{aligned}\right\} \longrightarrow$$

$$\rightarrow \mathbf{I}_{A} = \mathbf{I}_{AB}(1\underline{/0^{\circ}} - 1\underline{/120^{\circ}}) = \mathbf{I}_{ab}(1 + 0.50 - j0.87) = \mathbf{I}_{ab}\sqrt{3}\underline{/-30^{\circ}}$$

$$\mathbf{I}_{B} = \mathbf{I}_{AB}\sqrt{3}\underline{/-150^{\circ}}$$

$$\mathbf{I}_{C} = \mathbf{I}_{AB}\sqrt{3}\underline{/90^{\circ}}$$

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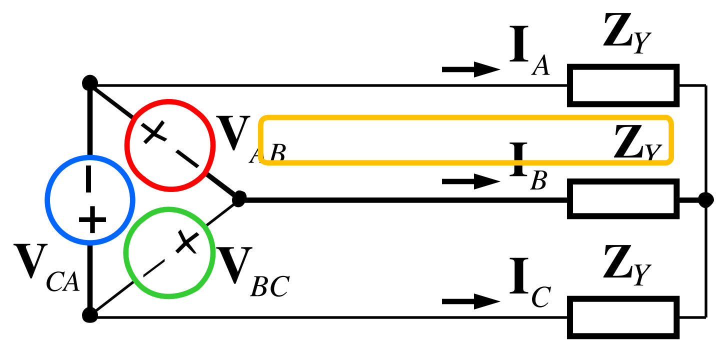
  - Treat it like a normal three-source circuit

Three-phase Circuit Analysis (11), Δ–Y

$$\mathbf{V}_{AB} = V\angle 0^\circ$$
$$\mathbf{V}_{BC} = V\angle -120^\circ$$
$$\mathbf{V}_{CA} = V\angle 120^\circ$$



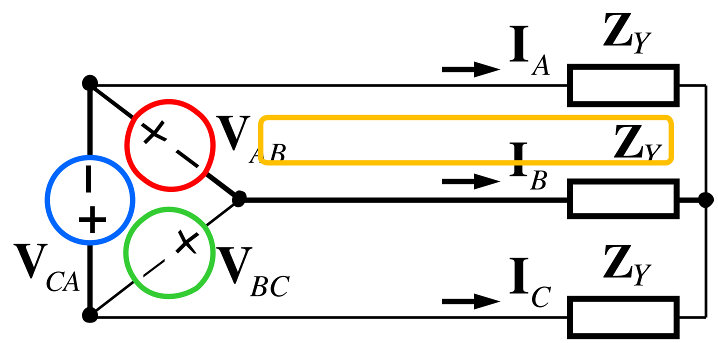
$$\boxed{Z_Y\mathbf{I}_A - Z_Y\mathbf{I}_B = \mathbf{V}_{AB}} \to \mathbf{I}_A - \mathbf{I}_B = \frac{\mathbf{V}_{AB}}{Z_Y} = \frac{V\angle 0^\circ}{Z_Y}$$
$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ \to \mathbf{I}_A - \mathbf{I}_B = \mathbf{I}_A(1 - 1\angle -120^\circ) = \mathbf{I}_A\sqrt{3}\angle 30^\circ$$

$$\to \mathbf{I}_A = \frac{V}{\sqrt{3}Z_Y}\angle -30^\circ \qquad \mathbf{I}_B = \frac{V}{\sqrt{3}Z_Y}\angle -150^\circ \qquad \mathbf{I}_C = \frac{V}{\sqrt{3}Z_Y}\angle 90^\circ$$

$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ \qquad \mathbf{I}_C = \mathbf{I}_A \times 1\angle 120^\circ$$

Ex. 3 Three-phase Circuit Analysis (12), Δ–Y

$\mathbf{Z}_Y = 30 + j40\ \Omega; \mathbf{V}_{AB} = 220$ V. Find currents?



Method 1

$$\boxed{Z_Y \mathbf{I}_A - Z_Y \mathbf{I}_B = \mathbf{V}_{AB}} \rightarrow \mathbf{I}_A - \mathbf{I}_B = \frac{\mathbf{V}_{AB}}{Z_Y}$$

$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ$$

$$\rightarrow \mathbf{I}_A - \mathbf{I}_B = \mathbf{I}_A(1 - 1\angle -120^\circ) = \mathbf{I}_A \sqrt{3}\angle 30^\circ$$

$$\rightarrow \mathbf{I}_A = \frac{\mathbf{V}_{AB}}{\sqrt{3}\angle 30^\circ Z_Y} = \frac{220}{\sqrt{3}\angle 30^\circ (30 + j40)} = 2.54\angle -83.13^\circ \text{ A}$$

$$\mathbf{I}_B = \mathbf{I}_A \times 1\angle -120^\circ = 2.54\angle -203.13^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}_A \times 1\angle 120^\circ = 2.54\angle 36.87^\circ \text{ A}$$

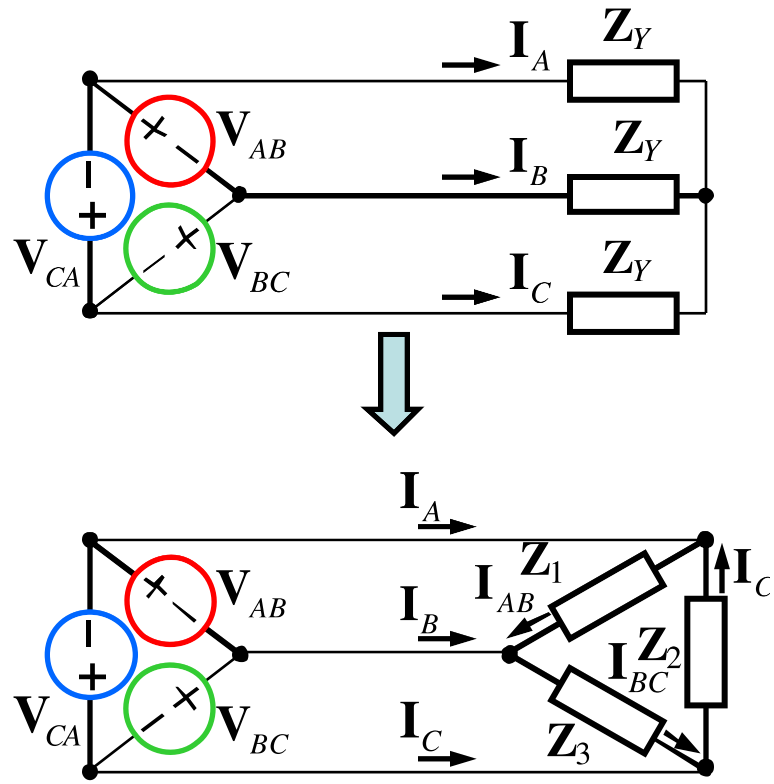
Ex. 3 Three-phase Circuit Analysis (13), Δ–Y

$\boxed{\mathbf{Z}_Y = 30 + j40\ \Omega; \mathbf{V}_{AB} = 220\text{ V. Find currents?}}$

Method 2

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_A\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_C + \mathbf{Z}_C\mathbf{Z}_A}{\mathbf{Z}_C}$$

$$= 3\mathbf{Z}_Y = 3(30 + j40) = 90 + j120\,\Omega$$



$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_1} = \frac{220}{90 + j120} = 0.88 - j1.17\text{ A}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_1} = \frac{220\angle 120^\circ}{90 + j120} = 0.58 + j1.35\text{ A}$$

$$\mathbf{I}_A + \mathbf{I}_{CA} - \mathbf{I}_{AB} = 0 \rightarrow \mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA} = (0.88 - j1.17) - (0.58 + j1.35)$$

$$= \boxed{2.54\angle -83.13^\circ\text{ A}}$$

Three-phase Circuit Analysis

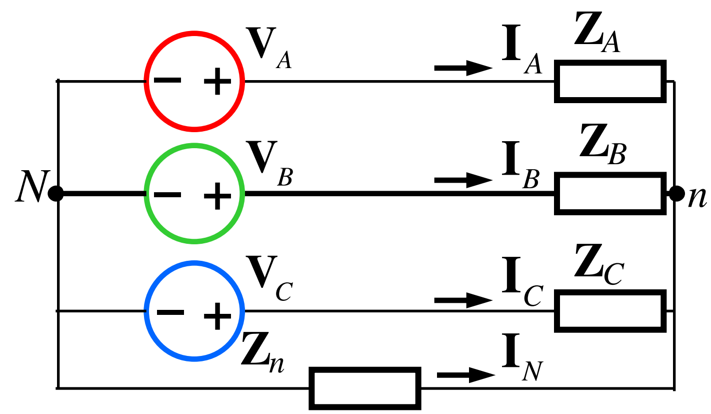
- Y–Y, Y–Δ, Δ–Δ, Δ–Y
- 2 kinds of three-phase circuit: balanced & unbalanced
- Balanced three-phase circuit:
  - Balanced three-phase source and balanced three-phase load
  - ***Balanced three-phase source***: same magnitude, same frequency, displaced from each other by 120^o
  - ***Balanced three-phase load***: three identical loads
- Unbalanced three-phase circuit:
  - Unbalanced three-phase source and/or unbalanced three-phase load
- **To solve a balanced one:**
  - **Exploit the symmetry of a balanced three-phase circuit, or**
  - **Treat it like a normal three-source circuit**
- To solve an unbalanced one:
  - Treat it like a normal three-source circuit

Three-phase Circuit Analysis

- Y–Y, Y–Δ, Δ–Δ, Δ–Y
- 2 kinds of three-phase circuit: balanced & unbalanced
- Balanced three-phase circuit:
  - Balanced three-phase source <u>and</u> balanced three-phase load
  - ***Balanced three-phase source***: same magnitude, same frequency, displaced from each other by 120^o
  - ***Balanced three-phase load***: three identical loads
- **Unbalanced three-phase circuit**:
  - Unbalanced three-phase source <u>and/or</u> unbalanced three-phase load
- To solve a balanced one:
  - Exploit the symmetry of a balanced three-phase circuit, <u>or</u>
  - Treat it like a normal three-source circuit
- **To solve an unbalanced one:**
  - **Treat it like a normal three-source circuit**

Ex. 4 Three-phase Circuit Analysis (14)

$$\boxed{\begin{aligned}&\mathbf{Z}_A = 20\ \Omega;\ \mathbf{Z}_B = j10\ \Omega;\ \mathbf{Z}_C = -j10\ \Omega;\ \mathbf{V}_A = 220\,\text{V};\\ &\mathbf{V}_B = 220\angle -120^{\circ}\ \text{V};\ \mathbf{V}_C = 220\angle 120^{\circ}\ \text{V};\ \mathbf{Z}_n = 1 + j2\ \Omega.\end{aligned}}$$



$$\mathbf{V}_N = 0 \rightarrow \mathbf{V}_n = \frac{\mathbf{V}_A / \mathbf{Z}_A + \mathbf{V}_B / \mathbf{Z}_B + \mathbf{V}_C / \mathbf{Z}_C}{1/\mathbf{Z}_A + 1/\mathbf{Z}_B + 1/\mathbf{Z}_C + 1/\mathbf{Z}_N}$$

$$= 57.46\angle -122^{\circ}\ \text{V}$$

$$\rightarrow \mathbf{I}_{Aa} = \frac{220\angle 0^{\circ} - \mathbf{V}_n}{20} = \frac{220\angle 0^{\circ} - 57.46\angle -122^{\circ}}{20} = \boxed{12.76\angle 11^{\circ}\ \text{A}}$$

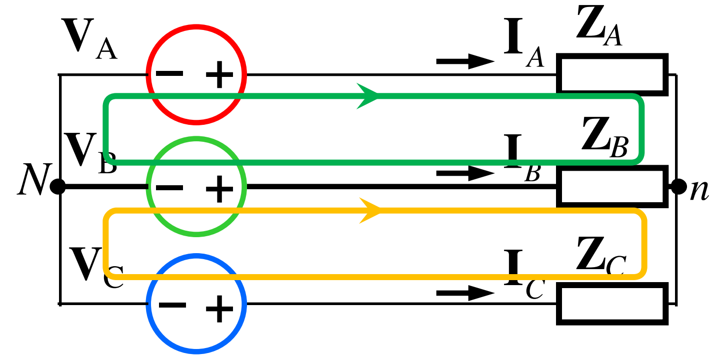
$$\mathbf{I}_{Bb} = \frac{220\angle -120^{\circ} - \mathbf{V}_n}{j10} = \frac{220\angle -120^{\circ} - 57.46\angle -122^{\circ}}{j10} = \boxed{16.26\angle 150.7^{\circ}\ \text{A}}$$

$$\mathbf{I}_{Cc} = \frac{220\angle 120^{\circ} - \mathbf{V}_n}{-j10} = \frac{220\angle 120^{\circ} - 57.46\angle -122^{\circ}}{-j10} = \boxed{25.21\angle -161.6^{\circ}\ \text{A}}$$

$$\mathbf{I}_{nN} = \frac{\mathbf{V}_n}{1 + j2} = \frac{57.46\angle -122^{\circ}}{1 + j2} = \boxed{25,70\angle 174,6^{\circ}\ \text{A}}$$

Ex. 5 Three-phase Circuit Analysis (15)

$\mathbf{Z}_A = 20\ \Omega;\ \mathbf{Z}_B = j10\ \Omega;\ \mathbf{Z}_C = -j5\ \Omega;\ \mathbf{V}_A = 220\,\text{V};$
$\mathbf{V}_B = 220\underline{/-120^\circ}\ \text{V};\ \mathbf{V}_C = 220\underline{/120^\circ}\ \text{V}.$



$$\begin{cases} \boxed{\mathbf{Z}_A\mathbf{I}_g + \mathbf{Z}_B(\mathbf{I}_g - \mathbf{I}_y) = \mathbf{V}_A - \mathbf{V}_B} \\ \boxed{\mathbf{Z}_B(\mathbf{I}_y - \mathbf{I}_g) + \mathbf{Z}_C\mathbf{I}_y = \mathbf{V}_B - \mathbf{V}_C} \end{cases}$$

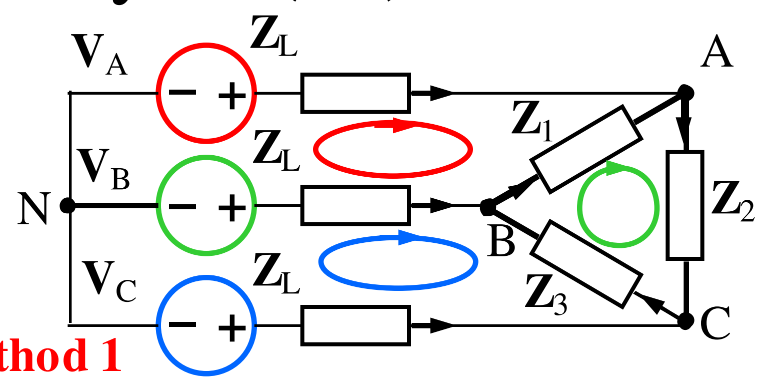
$$\rightarrow \begin{cases} \mathbf{I}_g = 24.63 - j16.26\ \text{A} \\ \mathbf{I}_y = -26.95 - j32.53\ \text{A} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_A = \mathbf{I}_g = 24.63 - j16.26\ \text{A} \\ \mathbf{I}_B = \mathbf{I}_y - \mathbf{I}_g = -51.58 - j16.26\ \text{A} \\ \mathbf{I}_C = -\mathbf{I}_y = 26.95 + j32.53\ \text{A} \end{cases}$$

Three-phase Circuit Analysis (16)

Ex. 6

$$\mathbf{V}_A = 220\angle 0^\circ \text{ V};\ \mathbf{V}_B = 220\angle -120^\circ \text{ V};\ \mathbf{V}_C = 220\angle 120^\circ \text{ V}$$
$$\mathbf{Z}_L = 5\Omega;\ \mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = -j30\Omega.$$



Method 1

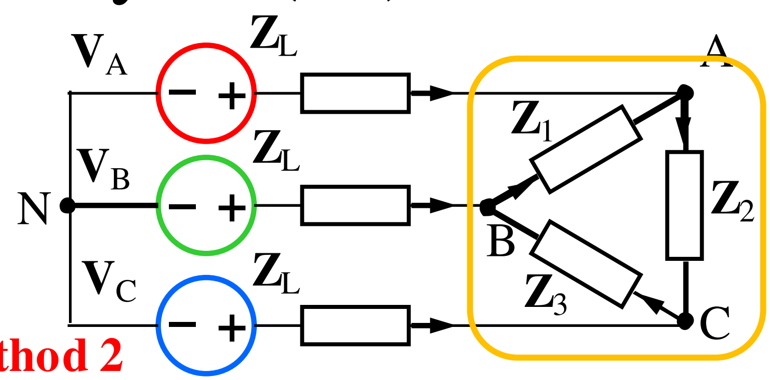
$$\begin{cases} (2\mathbf{Z}_L + \mathbf{Z}_1)\mathbf{I}_{red} - \mathbf{Z}_1\mathbf{I}_{green} - \mathbf{Z}_L\mathbf{I}_{blue} = \mathbf{V}_A - \mathbf{V}_B \\ -\mathbf{Z}_1\mathbf{I}_{red} + (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_{green} - \mathbf{Z}_3\mathbf{I}_{blue} = 0 \\ -\mathbf{Z}_L\mathbf{I}_{red} - \mathbf{Z}_3\mathbf{I}_{green} + (2\mathbf{Z}_L + \mathbf{Z}_3)\mathbf{I}_{blue} = \mathbf{V}_B - \mathbf{V}_C \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_{red} = 13.57 + j0.48 \text{ A} \\ \mathbf{I}_{green} = -6.89 - j12.59 \text{ A} \\ \mathbf{I}_{blue} = 2.06 - j11.02 \text{ A} \end{cases} \rightarrow \begin{cases} \mathbf{I}_A = \mathbf{I}_r = 13,36 + j0.15 \text{ A} \\ \mathbf{I}_B = \mathbf{I}_b - \mathbf{I}_r = -11.39 - j11.21 \text{ A} \\ \mathbf{I}_C = -\mathbf{I}_b = -1.98 + j11.06 \text{ A} \\ \mathbf{I}_{ab} = \mathbf{I}_g - \mathbf{I}_r = -20.38 - j12.94 \text{ A} \\ \mathbf{I}_{bc} = \mathbf{I}_g = -7.02 - j12.79 \text{ A} \\ \mathbf{I}_{ca} = \mathbf{I}_g - \mathbf{I}_b = -8.99 - j1.74 \text{ A} \end{cases}$$

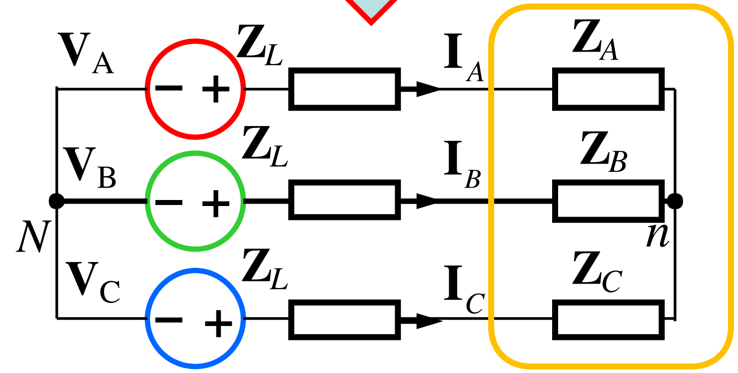
Three-phase Circuit Analysis (17)

Ex. 6

$\mathbf{V}_A = 220\angle 0^\circ$ V; $\mathbf{V}_B = 220\angle -120^\circ$ V; $\mathbf{V}_C = 220\angle 120^\circ$ V
$\mathbf{Z}_L = 5\Omega$; $\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = j20\Omega$; $\mathbf{Z}_3 = -j30\Omega$.



Method 2



$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} = \frac{10(j20)}{10 + j20 - j30} = -10 + j10\,\Omega$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} = \frac{10(-j30)}{10 + j20 - j30} = 15 - j15\,\Omega$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} = \frac{j20(-j30)}{10 + j20 - j30} = 30 + j30\,\Omega$$

$$\mathbf{V}_N = 0 \rightarrow \mathbf{V}_n = \frac{\dfrac{\mathbf{V}_A}{\mathbf{Z}_L + \mathbf{Z}_A} + \dfrac{\mathbf{V}_B}{\mathbf{Z}_L + \mathbf{Z}_B} + \dfrac{\mathbf{V}_C}{\mathbf{Z}_L + \mathbf{Z}_C}}{\dfrac{1}{\mathbf{Z}_L + \mathbf{Z}_A} + \dfrac{1}{\mathbf{Z}_L + \mathbf{Z}_B} + \dfrac{1}{\mathbf{Z}_L + \mathbf{Z}_C}} = 288.3 - j132.9\,\text{V}$$

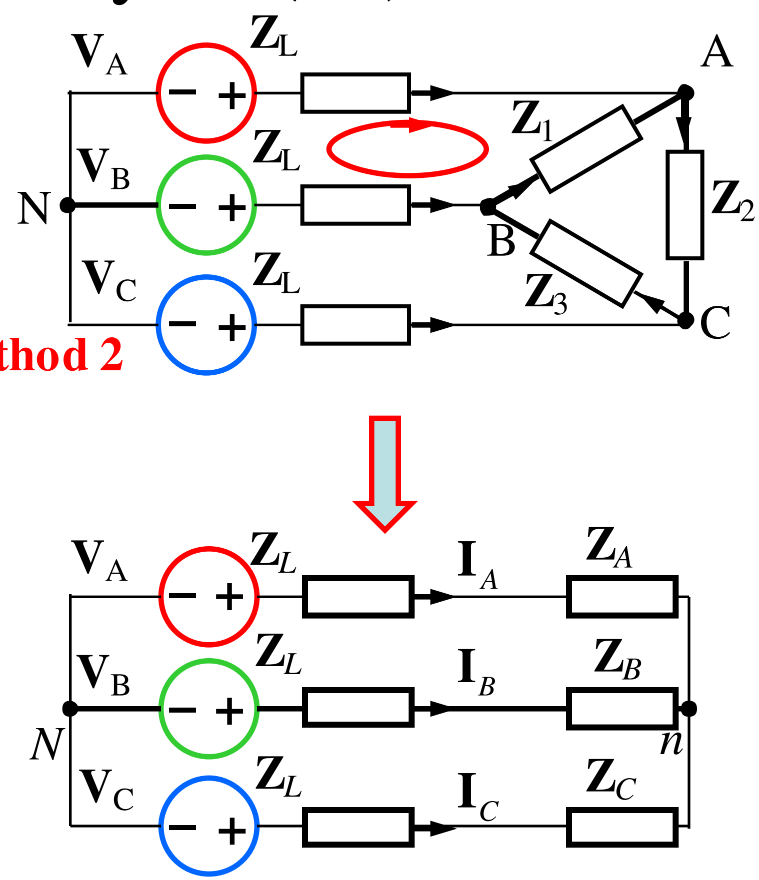
Three-phase Circuit Analysis (18)

Ex. 6

$$\mathbf{V}_A = 220\angle 0^o \text{ V}; \ \mathbf{V}_B = 220\angle -120^o \text{ V}; \ \mathbf{V}_C = 220\angle 120^o \text{ V}$$
$$\mathbf{Z}_L = 5\Omega; \ \mathbf{Z}_1 = 10\Omega; \ \mathbf{Z}_2 = j20\Omega; \ \mathbf{Z}_3 = -j30\Omega.$$

$$\mathbf{V}_n = 288,3 - j132,9 \text{ V}$$

Method 2



$$\mathbf{I}_A = (\mathbf{V}_A - \mathbf{V}_n)/(\mathbf{Z}_L + \mathbf{Z}_A) = 13.36 + j0.15 \text{ A}$$

$$\mathbf{I}_B = (\mathbf{V}_B - \mathbf{V}_n)/(\mathbf{Z}_L + \mathbf{Z}_B) = -11.39 - j11.21 \text{ A}$$

$$\mathbf{I}_C = (\mathbf{V}_C - \mathbf{V}_n)/(\mathbf{Z}_L + \mathbf{Z}_C) = -1.98 + j11.06 \text{ A}$$

$$\mathbf{Z}_L\mathbf{I}_A - \mathbf{Z}_1\mathbf{I}_{ab} - \mathbf{Z}_L\mathbf{I}_B = \mathbf{V}_A - \mathbf{V}_B$$

$$\rightarrow \mathbf{I}_{ab} = \frac{\mathbf{V}_B - \mathbf{V}_A + \mathbf{Z}_L(\mathbf{I}_A - \mathbf{I}_B)}{\mathbf{Z}_1} = -20.38 - j12.94 \text{ A}$$

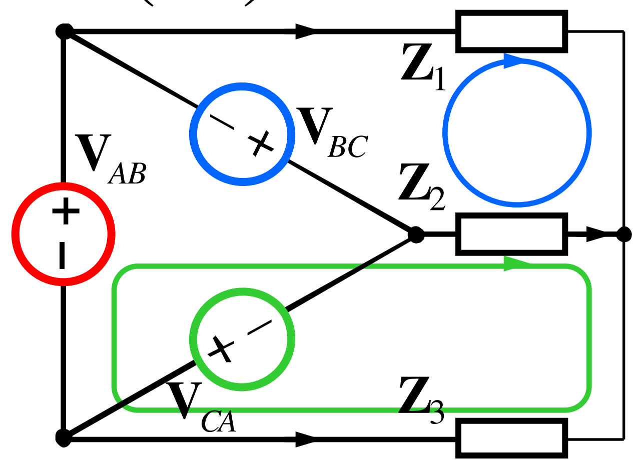
$$\mathbf{I}_{bc} = \mathbf{I}_A + \mathbf{I}_{ab} = 13.36 + j0.15 - 20.38 - j12.94 = -7.02 - j12.79 \text{ A}$$

$$\mathbf{I}_{ca} = \mathbf{I}_C + \mathbf{I}_{bc} = -1.98 + j11.06 - 7.02 - j12.79 = 9.00 - j1.73 \text{ A}$$

Three-phase Circuit Analysis (19)

Ex. 7

$\mathbf{V}_{AB} = 220\angle 0^{o}$ V; $\mathbf{V}_{BC} = 215\angle -120^{o}$ V;
$\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = j20\Omega$; $\mathbf{Z}_3 = -j30\Omega$.

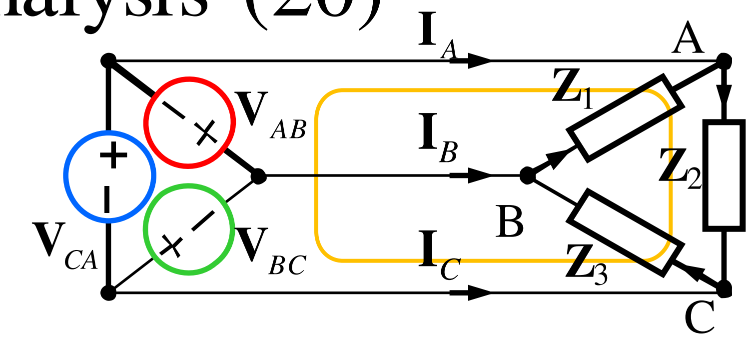


$$\begin{cases} (\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{I}_b - \mathbf{Z}_2\mathbf{I}_g = -\mathbf{V}_{BC} \\ -\mathbf{Z}_2\mathbf{I}_b + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_g = \mathbf{V}_{AB} + \mathbf{V}_{BC} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_g = 1.14 + j4.42 \text{ A} \\ \mathbf{I}_b = 8.74 + j3.42 \text{ A} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = \mathbf{I}_b = 8.74 + j3.42 \text{ A} \\ \mathbf{I}_2 = \mathbf{I}_g - \mathbf{I}_b = -7.60 + j1.01 \text{ A} \\ \mathbf{I}_3 = -\mathbf{I}_g = -1.14 - j4.42 \text{ A} \end{cases}$$

Three-phase Circuit Analysis (20)



Ex. 8

$\mathbf{V}_{AB} = 220\angle 0^\circ$ V; $\mathbf{V}_{BC} = 215\angle -120^\circ$ V;
$\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = j20\Omega$; $\mathbf{Z}_3 = -j30\Omega$.

$$\mathbf{I}_{BA} = \frac{\mathbf{V}_A}{\mathbf{Z}_1} = \frac{220}{10} = 22\text{ A}$$

$$\mathbf{Z}_2\mathbf{I}_{AC} = -\mathbf{V}_{AB} - \mathbf{V}_{BC} \rightarrow \mathbf{I}_{AC} = \frac{-\mathbf{V}_{AB} - \mathbf{V}_{BC}}{\mathbf{Z}_2} = \frac{-220 - 215\angle -120^\circ}{j20} = 9.31 + j5.63\text{ A}$$

$$\mathbf{I}_{CB} = \frac{\mathbf{V}_B}{\mathbf{Z}_3} = \frac{215\angle -120^\circ}{-j30} = 6.21 - j3.58\text{ A}$$

$$\mathbf{I}_A = \mathbf{I}_{AC} - \mathbf{I}_{BA} = 9.31 + j5.63 - 22 = -12.69 + j5.63\text{ A}$$

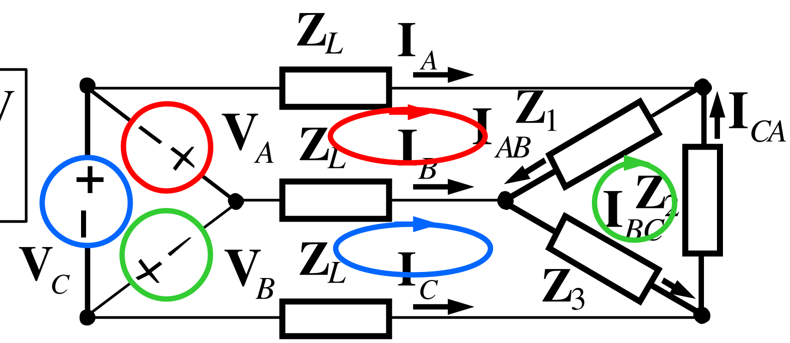
$$\mathbf{I}_B = \mathbf{I}_{BA} - \mathbf{I}_{CB} = 22 - (6.21 - j3.58) = 15.79 + j3.58\text{ A}$$

$$\mathbf{I}_C = \mathbf{I}_{CB} - \mathbf{I}_{AC} = 6.21 - j3.58 - (9.14 + j5.63) = -3.10 - j9.21\text{ A}$$

Three-phase Circuit Analysis (21)

Ex. 9

$\mathbf{V}_A = 220\angle 0^\circ$ V; $\mathbf{V}_B = 220\angle -120^\circ$ V; $\mathbf{V}_C = 220\angle 120^\circ$ V
$\mathbf{Z}_L = 5\Omega$; $\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = j20\Omega$; $\mathbf{Z}_3 = -j30\Omega$.



Method 1

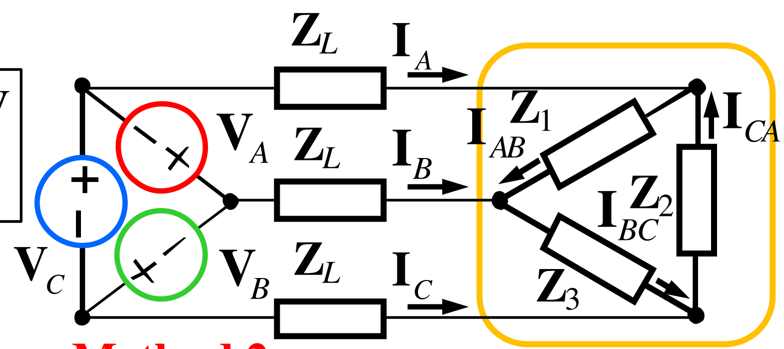
$$\begin{cases} (2\mathbf{Z}_L + \mathbf{Z}_1)\mathbf{I}_{red} - \mathbf{Z}_1\mathbf{I}_{green} - \mathbf{Z}_L\mathbf{I}_{blue} = \mathbf{V}_A \\ -\mathbf{Z}_1\mathbf{I}_{red} + (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_{green} - \mathbf{Z}_3\mathbf{I}_{blue} = 0 \\ -\mathbf{Z}_L\mathbf{I}_{red} - \mathbf{Z}_3\mathbf{I}_{green} + (2\mathbf{Z}_L + \mathbf{Z}_3)\mathbf{I}_{blue} = \mathbf{V}_B \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_{red} = 6.92 - j3.68 \text{ A} \\ \mathbf{I}_{green} = -7.08 - j4.31 \text{ A} \\ \mathbf{I}_{blue} = -2.15 - j6.10 \text{ A} \end{cases} \rightarrow \begin{cases} \mathbf{I}_A = \mathbf{I}_r = 6.92 - j3.68 \text{ A} \\ \mathbf{I}_B = \mathbf{I}_b - \mathbf{I}_r = -9.07 - j2.42 \text{ A} \\ \mathbf{I}_C = -\mathbf{I}_b = 2.15 + j6.10 \text{ A} \\ \mathbf{I}_{ab} = \mathbf{I}_r - \mathbf{I}_g = 14.00 + j0.63 \text{ A} \\ \mathbf{I}_{bc} = \mathbf{I}_b - \mathbf{I}_g = 4.93 - j1.80 \text{ A} \\ \mathbf{I}_{ca} = -\mathbf{I}_g = 7.08 - j4.31 \text{ A} \end{cases}$$

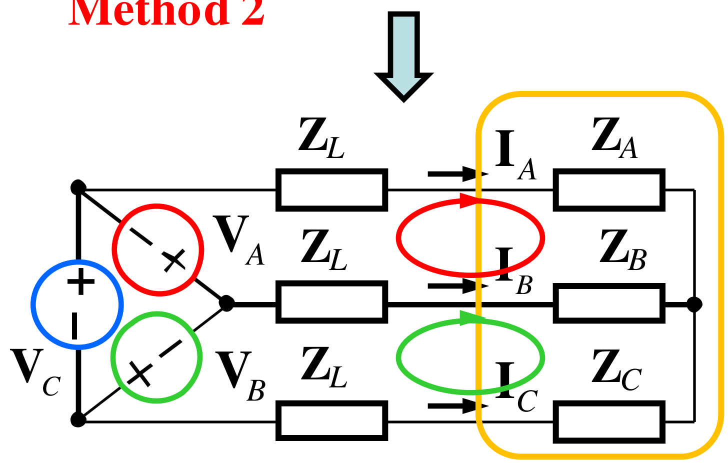
Three-phase Circuit Analysis (22)

Ex. 9

$\mathbf{V}_A = 220\angle 0^{\circ}$ V; $\mathbf{V}_B = 220\angle -120^{\circ}$ V; $\mathbf{V}_C = 220\angle 120^{\circ}$ V
$\mathbf{Z}_L = 5\Omega$; $\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = j20\Omega$; $\mathbf{Z}_3 = -j30\Omega$.



Method 2



$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3} = \frac{10(j20)}{10+j20-j30} = -10+j10\,\Omega$$

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3} = \frac{10(-j30)}{10+j20-j30} = 15-j15\,\Omega$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3} = \frac{j20(-j30)}{10+j20-j30} = 30+j30\,\Omega$$

$$\begin{cases}(2\mathbf{Z}_L+\mathbf{Z}_A+\mathbf{Z}_B)\mathbf{I}_r-(\mathbf{Z}_L+\mathbf{Z}_B)\mathbf{I}_g=\mathbf{V}_A\\ -(\mathbf{Z}_L+\mathbf{Z}_B)\mathbf{I}_r+(2\mathbf{Z}_L+\mathbf{Z}_B+\mathbf{Z}_C)\mathbf{I}_g=\mathbf{V}_B\end{cases}$$

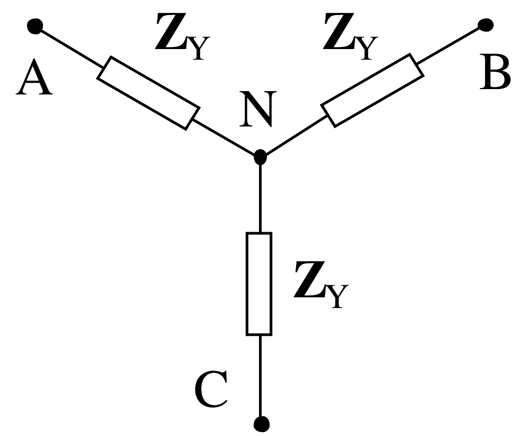
$$\rightarrow\begin{cases}\mathbf{I}_r = 6.92-j3.68 \text{ A}\\ \mathbf{I}_g = -2.15-j6.10 \text{ A}\end{cases}$$

Three-phase Circuits

1. Introduction
2. Three-phase Source
3. Three-phase Load
4. Three-phase Circuit Analysis
5. **Power in Three-phase Circuits**
   a) **Balanced Three-phase Circuits**
   b) **Unbalanced Three-phase Circuits**

Balanced Three-phase Circuits (1)

$$\mathbf{Z}_Y = Z\underline{/\phi}$$

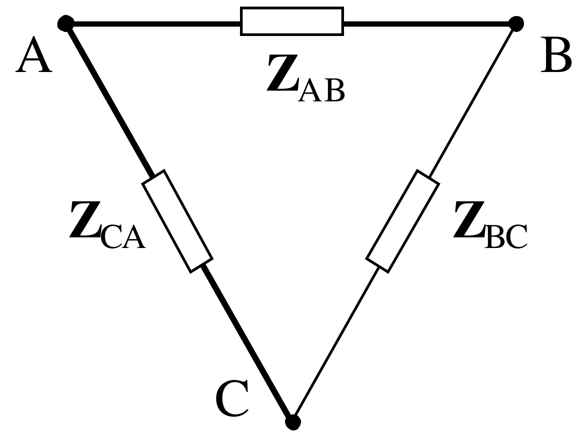


$$v_{AN} = V\sqrt{2}\sin\omega t \qquad i_A = I\sqrt{2}\sin(\omega t - \phi)$$

$$v_{BN} = V\sqrt{2}\sin(\omega t - 120^o) \qquad i_B = I\sqrt{2}\sin(\omega t - \phi - 120^o)$$

$$v_{CN} = V\sqrt{2}\sin(\omega t + 120^o) \qquad i_C = I\sqrt{2}\sin(\omega t - \phi + 120^o)$$

$$\begin{cases} p_{total} = p_a + p_b + p_c = v_{AN}i_A + v_{BN}i_B + v_{CN}i_C \\ \quad = 2VI[\sin\omega t\sin(\omega t - \phi) + \sin(\omega t - 120^o)\sin(\omega t - \phi - 120^o) + \\ \quad + \sin(\omega t + 120^o)\sin(\omega t - \phi + 120^o)] \\ \sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)] \end{cases}$$



$$\rightarrow \boxed{p_{total} = 3VI\cos\phi}$$

Balanced Three-phase Circuits (2)

$$\boxed{p_{total} = 3VI\cos\phi}$$

$$P_p = VI\cos\phi$$

$$S_p = VI$$

$$Q_p = VI\sin\phi$$

$$\mathbb{S}_p = P_p + jQ_p = \mathbf{V}\hat{\mathbf{I}}$$

Balanced Three-phase Circuits (3)

Ex. 1

A three-phase balanced Y–Y system has a phase voltage of 220 V. The total real power absorbed by the load is 2400 W, the power factor angle of the load is 20°. Find the line current?

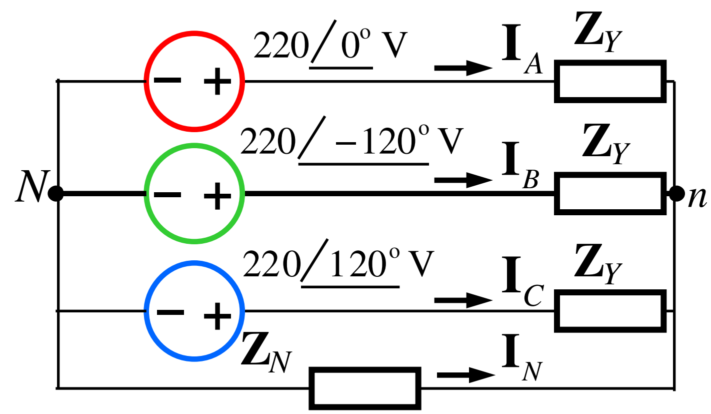
$$P_p = \frac{p_{total}}{3} = \frac{2400}{3} = 800\,\mathrm{W} = V_p I_p \cos 20^{\circ} = 220 I_p \times 0.94$$

$$\rightarrow I_p = \frac{800}{0.94 \times 220} = 3.87\,\mathrm{A}$$

$$\rightarrow I_L = I_p = \boxed{3.87\,\mathrm{A}}$$

Balanced Three-phase Circuits (4)

Ex. 2



$\boxed{\mathbf{Z}_Y = 30 + j40\ \Omega\text{; find phase currents?}}$

$$\begin{cases} \mathbf{I}_A = 4.4\underline{/-53.13^\circ}\ \text{A} \\ \mathbf{I}_B = 4.4\underline{/-173.13^\circ}\ \text{A} \\ \mathbf{I}_C = 4.4\underline{/66.87^\circ}\ \text{A} \end{cases}$$

$$\cos\varphi = \frac{R}{|Z|} = \frac{30}{\sqrt{30^2 + 40^2}} = 0.6$$

$$p_\Sigma = 3UI\cos\varphi = 3 \times 220 \times 4.4 \times 0.6 = 1742.4\ \text{W}$$

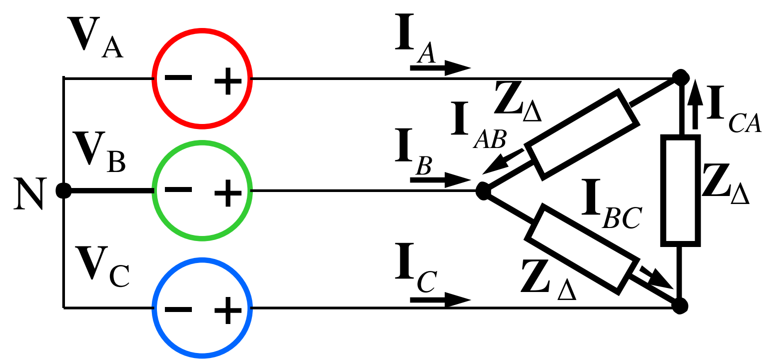
$$\rightarrow P_A = \frac{p_\Sigma}{3} = \frac{1742.4}{3} = \boxed{580.8\ \text{W}}$$

$$P_A = RI_A^2 = 30(4.4)^2 = \boxed{580.8\ \text{W}}$$

Balanced Three-phase Circuits (5)

Ex. 3

$$\boxed{\mathbf{V}_A = 220\angle 15^\circ \text{ V};\ \mathbf{V}_B = 220\angle -105^\circ \text{ V};\ \mathbf{V}_C = 220\angle 135^\circ \text{ V};\ Z_\Delta = 30 + j40\ \Omega.}$$



$$\begin{cases} \mathbf{I}_{AB} = 7.62\angle -8.1^\circ \text{ A} \\ \mathbf{I}_{BC} = 7.62\angle -128.1^\circ \text{ A} \\ \mathbf{I}_{CA} = 7.62\angle 111.9^\circ \text{ A} \end{cases}$$

$$\begin{cases} \mathbf{V}_{AB} = \mathbf{Z}\mathbf{I}_{AB} = 381\angle 45.0^\circ \text{ V} \\ \mathbf{V}_{BC} = \mathbf{Z}\mathbf{I}_{BC} = 381\angle -75.0^\circ \text{ V} \\ \mathbf{V}_{CA} = \mathbf{Z}\mathbf{I}_{CA} = 381\angle 165^\circ \text{ V} \end{cases}$$

$$\cos\varphi = \frac{R}{|Z|} = \frac{30}{\sqrt{30^2 + 40^2}} = 0,6$$

$$p_\Sigma = 3UI\cos\varphi = 3 \times 381 \times 7.62 \times 0.6 = 5225.8 \text{ W}$$

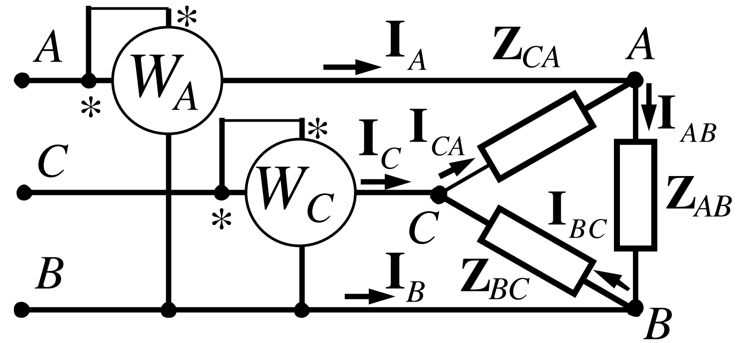
$$\rightarrow P_A = \frac{p_\Sigma}{3} = \frac{5225.8}{3} = \boxed{1741.9 \text{ W}}$$

$$P_A = RI_A^2 = 30(7.62)^2 = \boxed{1741.9 \text{ W}}$$

Three-phase Circuits

1. Introduction
2. Three-phase Source
3. Three-phase Load
4. Three-phase Circuit Analysis
5. **Power in Three-phase Circuits**
   a) Balanced Three-phase Circuits
   b) **Unbalanced Three-phase Circuits**

Unbalanced Three-phase Circuits (1)



$$\left.\begin{aligned} W_A &= \mathrm{Re}\{\mathbf{V}_{AB}\mathbf{I}_A^*\} \\ \mathbf{I}_A &= \mathbf{I}_{AB} + \mathbf{I}_{AC} \end{aligned}\right\}$$

$$\left.\begin{aligned} \rightarrow W_A &= \mathrm{Re}\{\mathbf{V}_{AB}\mathbf{I}_{AB}^*\} + \mathrm{Re}\{\mathbf{V}_{AB}\mathbf{I}_{AC}^*\} \\ \mathrm{Re}\{\mathbf{V}_{AB}\mathbf{I}_{AB}^*\} &= P_{AB} \end{aligned}\right\}$$

$$\rightarrow W_A = P_{AB} + \mathrm{Re}\{\mathbf{V}_{AB}\mathbf{I}_{AC}^*\}$$

$$\left.\begin{aligned} W_C &= \mathrm{Re}\{\mathbf{V}_{CB}\mathbf{I}_C^*\} \\ \mathbf{I}_C &= \mathbf{I}_{CA} + \mathbf{I}_{CB} \end{aligned}\right\} \left.\begin{aligned} \rightarrow W_C &= \mathrm{Re}\{\mathbf{V}_{CB}\mathbf{I}_{CA}^*\} + \mathrm{Re}\{\mathbf{V}_{CB}\mathbf{I}_{CB}^*\} \\ \mathrm{Re}\{\mathbf{V}_{CB}\mathbf{I}_{CB}^*\} &= P_{CB} \end{aligned}\right\} \rightarrow W_C = \mathrm{Re}\{\mathbf{V}_{CB}\mathbf{I}_{CA}^*\} + P_{CB}$$

$$\left.\begin{aligned} &\rightarrow W_A + W_C = P_{AB} + \mathrm{Re}\{(\mathbf{V}_{AB} - \mathbf{V}_{CB})\mathbf{I}_{AC}^*\} + P_{CB} \\ &\mathbf{V}_{AB} - \mathbf{V}_{CB} = \mathbf{V}_{AC} \rightarrow \mathrm{Re}\{(\mathbf{V}_{AB} - \mathbf{V}_{CB})\mathbf{I}_{AC}^*\} = P_{AC} \end{aligned}\right\} \rightarrow \boxed{W_A + W_C = P_{AB} + P_{AC} + P_{CB}}$$

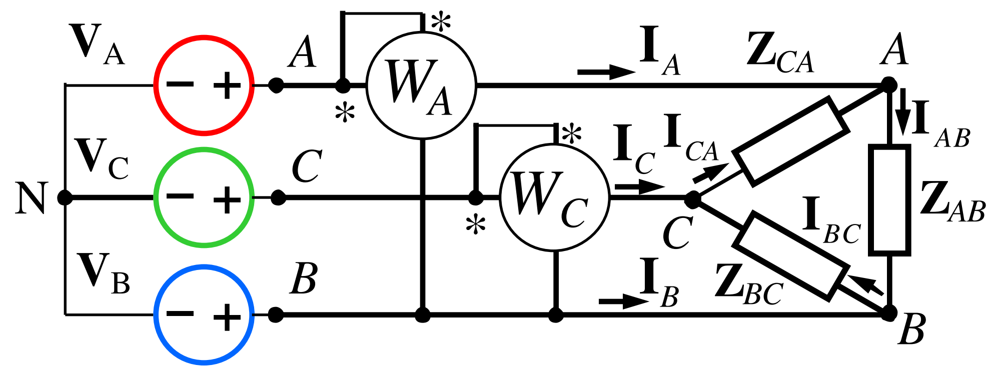
Unbalanced Three-phase Circuits (2)

Ex.

$\mathbf{V}_A = 220\angle 0^\circ$ V; $\mathbf{V}_C = 220\angle 120^\circ$ V

$\mathbf{V}_B = 220\angle -120^\circ$ V; $\mathbf{Z}_{AB} = 50\,\Omega$;

$\mathbf{Z}_{BC} = j75\,\Omega$; $\mathbf{Z}_{CA} = -j100\,\Omega$.



$$\mathbf{Z}_{AB}\mathbf{I}_{AB} = \mathbf{V}_A - \mathbf{V}_B \rightarrow \mathbf{I}_{AB} = \frac{\mathbf{V}_A - \mathbf{V}_B}{\mathbf{Z}_{AB}} = \frac{220 - 220\angle -120^\circ}{50} = 6.60 + j3.81\,\text{A}$$

$$\mathbf{Z}_{BC}\mathbf{I}_{BC} = \mathbf{V}_B - \mathbf{V}_C \rightarrow \mathbf{I}_{BC} = \frac{\mathbf{V}_B - \mathbf{V}_C}{\mathbf{Z}_{BC}} = \frac{220\angle -120^\circ - 220\angle 120^\circ}{j75} = -5.08\,\text{A}$$

$$\mathbf{Z}_{CA}\mathbf{I}_{CA} = \mathbf{V}_C - \mathbf{V}_A \rightarrow \mathbf{I}_{CA} = \frac{\mathbf{V}_C - \mathbf{V}_A}{\mathbf{Z}_{CA}} = \frac{220\angle 120^\circ - 220}{-j100} = -1.91 - j3.30\,\text{A}$$

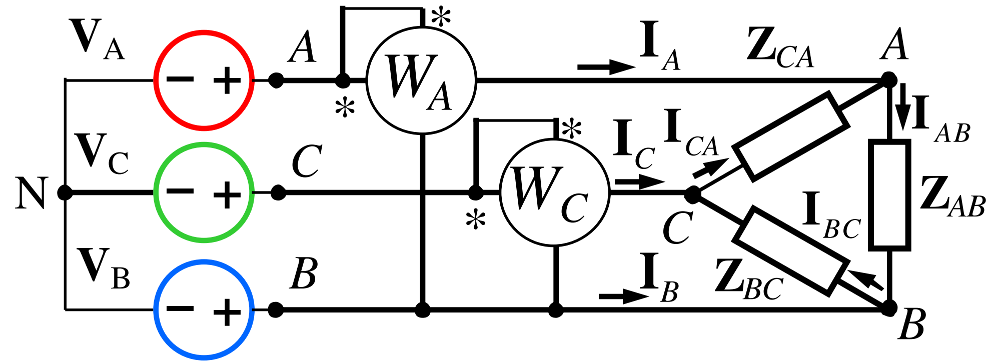
$$\mathbf{I}_A + \mathbf{I}_{CA} - \mathbf{I}_{AB} = 0 \rightarrow \mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 6.60 + j3.81 - (-1.91 - j3.30) = 8.50 + j7.11\,\text{A}$$

$$\mathbf{I}_C + \mathbf{I}_{BC} - \mathbf{I}_{CA} = 0 \rightarrow \mathbf{I}_C = \mathbf{I}_{CA} - \mathbf{I}_{BC} = -1.91 - j3.30 - (-5.08) = 3.18 - j3.30\,\text{A}$$

Unbalanced Three-phase Circuits (3)

Ex.

$$\boxed{\begin{array}{l}\mathbf{V}_A = 220\underline{/0^\circ}\ \text{V};\ \mathbf{V}_C = 220\underline{/120^\circ}\ \text{V}\\ \mathbf{V}_B = 220\underline{/-120^\circ}\ \text{V};\mathbf{Z}_{AB} = 50\,\Omega;\\ \mathbf{Z}_{BC} = j75\,\Omega;\mathbf{Z}_{CA} = -j100\,\Omega.\end{array}}$$



$$\mathbf{I}_A = 8.50 + j7.11\,\text{A};\ \mathbf{I}_C = 3.18 - j3.30\,\text{A}$$

$$\mathbf{I}_{AB} = 6.60 + j3.81\,\text{A};\ \mathbf{I}_{BC} = -5.08\,\text{A};\ \mathbf{I}_{CA} = -1.91 - j3.30\,\text{A}$$

$$W_A = \text{Re}\left\{\mathbf{V}_{AB}\mathbf{I}_A^*\right\} = \text{Re}\left\{(\mathbf{V}_A - \mathbf{V}_B)\mathbf{I}_A^*\right\}$$

$$= \text{Re}\left\{(220 - 220\underline{/-120^\circ})(8.50 - j7.11)\right\} = 4161.5\,\text{W}$$

$$W_C = \text{Re}\left\{\mathbf{V}_{CB}\mathbf{I}_C^*\right\} = \text{Re}\left\{(\mathbf{V}_C - \mathbf{V}_B)\mathbf{I}_C^*\right\}$$

$$= \text{Re}\left\{(220\underline{/120^\circ} - 220\underline{/-120^\circ})(3.18 + j3.30)\right\} = -1257.5\,\text{W}$$

$$P_{total} = W_A + W_C = 4161.5 - 1257.5 = \boxed{2904.0\,\text{W}}$$

$$P_{total} = P_{AB} + P_{BC} + P_{CA} = R_{AB}I_{AB}^2 + 0 + 0 = 50(6.60^2 + 3.81^2) = \boxed{2903.8\,\text{W}}$$