Nguyễn Công Phương

Engineering Electromagnetics

Contents

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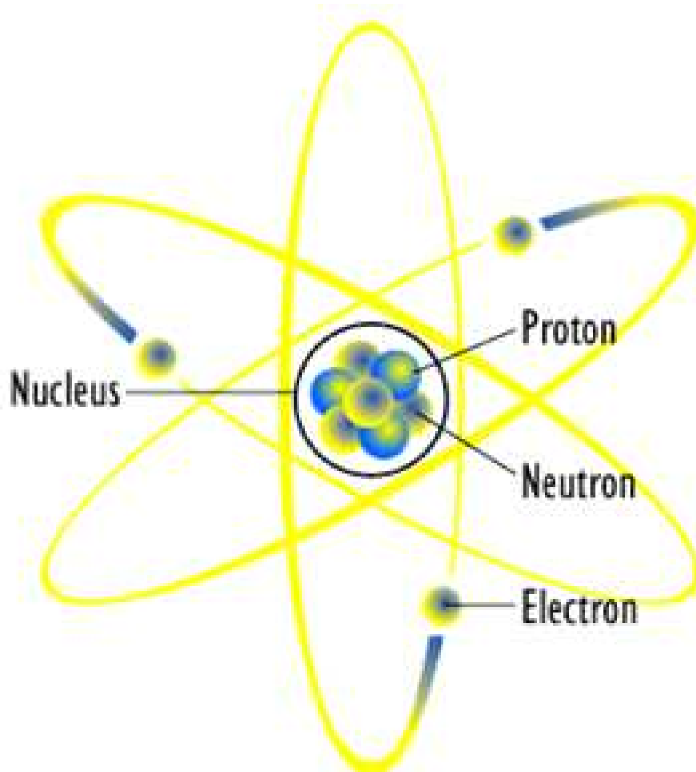
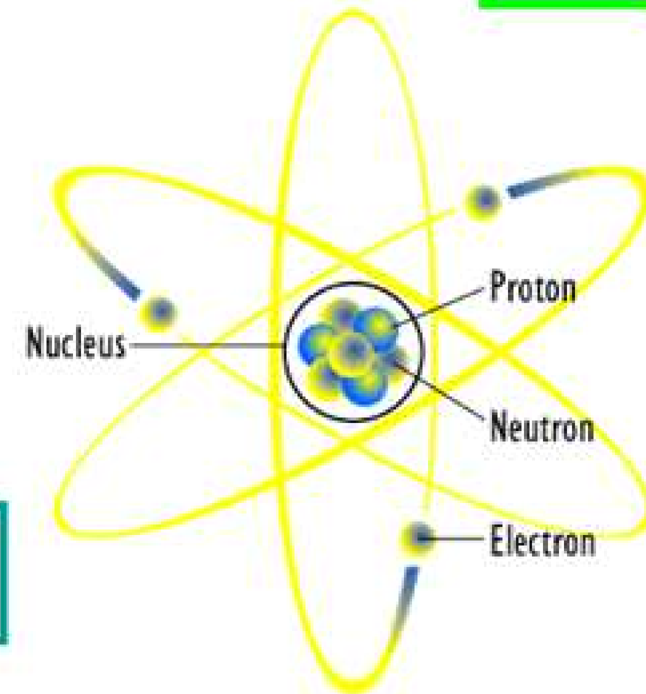


https://kimrendfeld.wordpress.com/2012/11/

Introduction (2)



Electromagnetics

Electrostatics	Magnetostatics	Electromagnetic Waves
$\frac{\partial q}{\partial t} = 0$	$\frac{\partial I}{\partial t} = 0$	$\frac{\partial I}{\partial t} \neq 0$

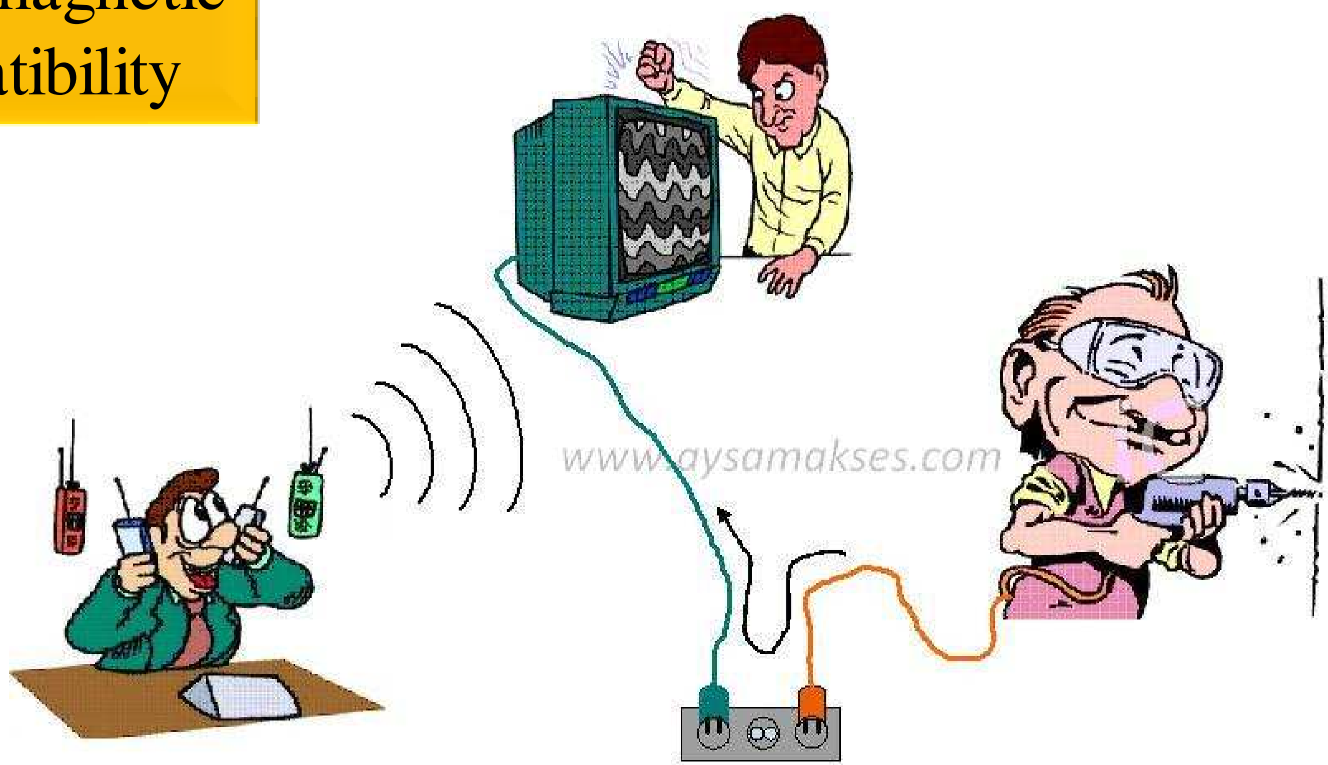
Introduction (3)

ELECTROMAGNETICS

- Electromagnetic compatibility
- Biomedical engineering
- Laser engineering
- Antenna
- Electric machines
- Wireless communication
- Remote sensing
- Military defense

Introduction (4)

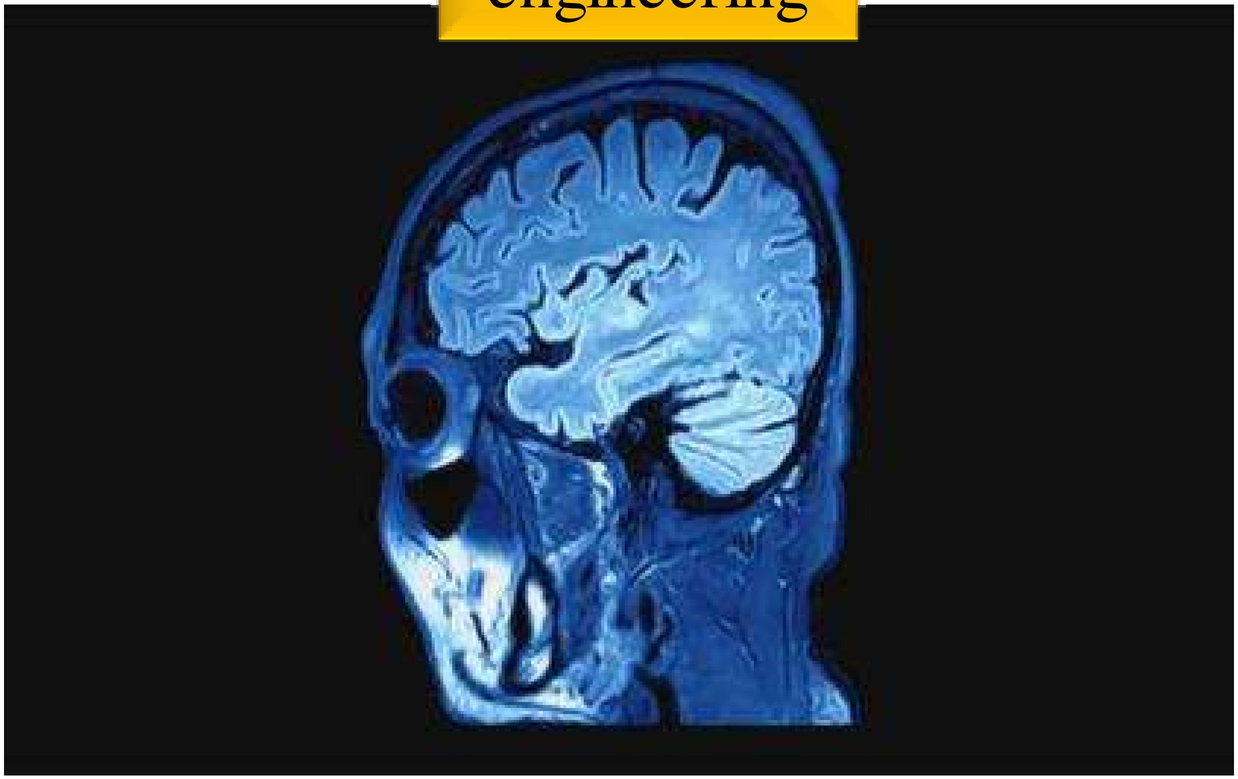
Electromagnetic compatibility



http://www.aysamakses.com/en/bilgi-bankasi/elektromanyetik-uyumluluk-emc/

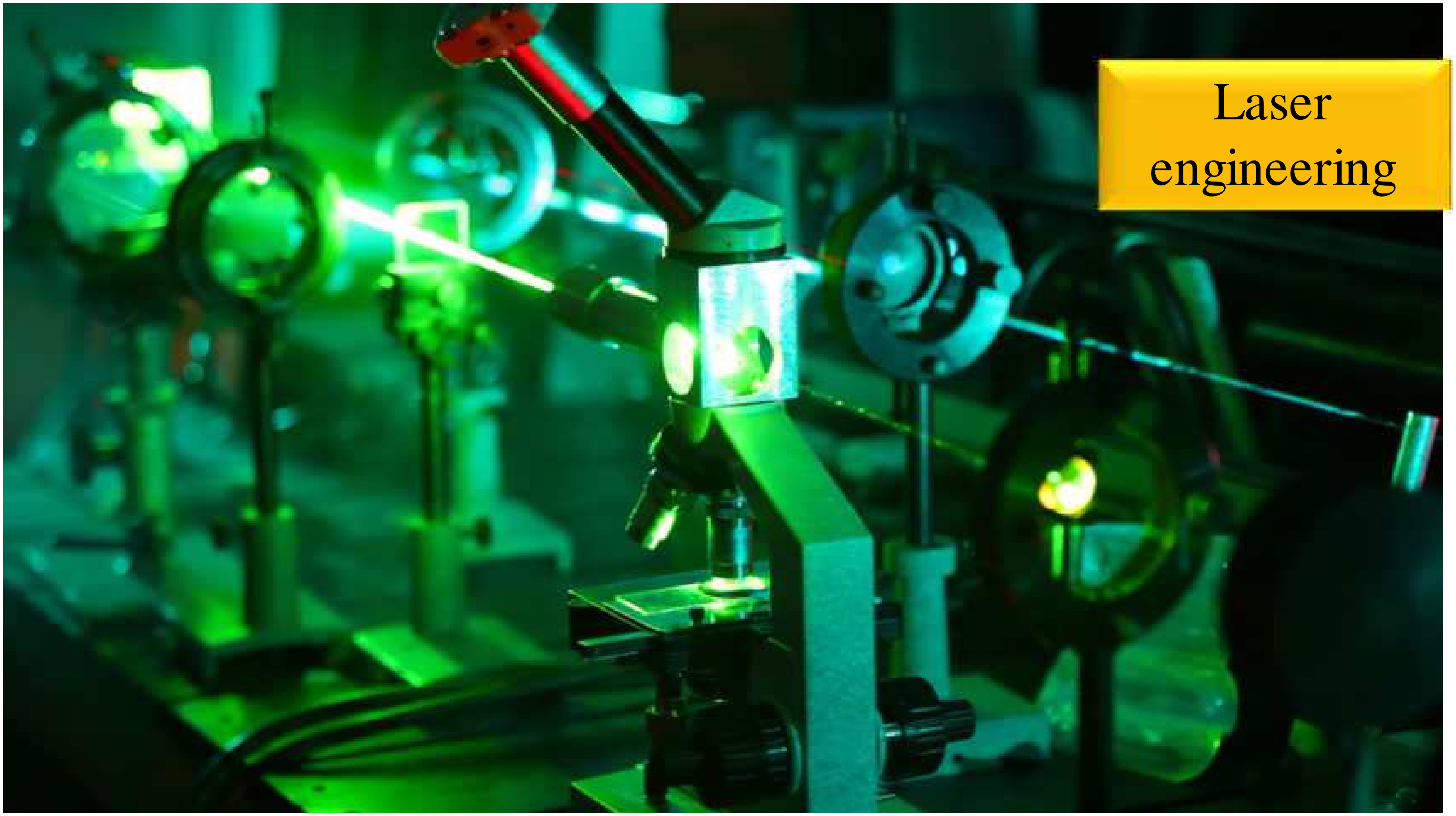
Introduction (5)

Biomedical engineering



https://biomedical.njit.edu/mri/

Introduction (6)



Laser engineering

https://www.shutterstock.com/video/clip-3748037-stock-footage-masked-ninjas-strike-various-dramatic-poses-at-the-bottom-of-the-screen-plenty-of-space-for.html

Introduction (7)

Antenna

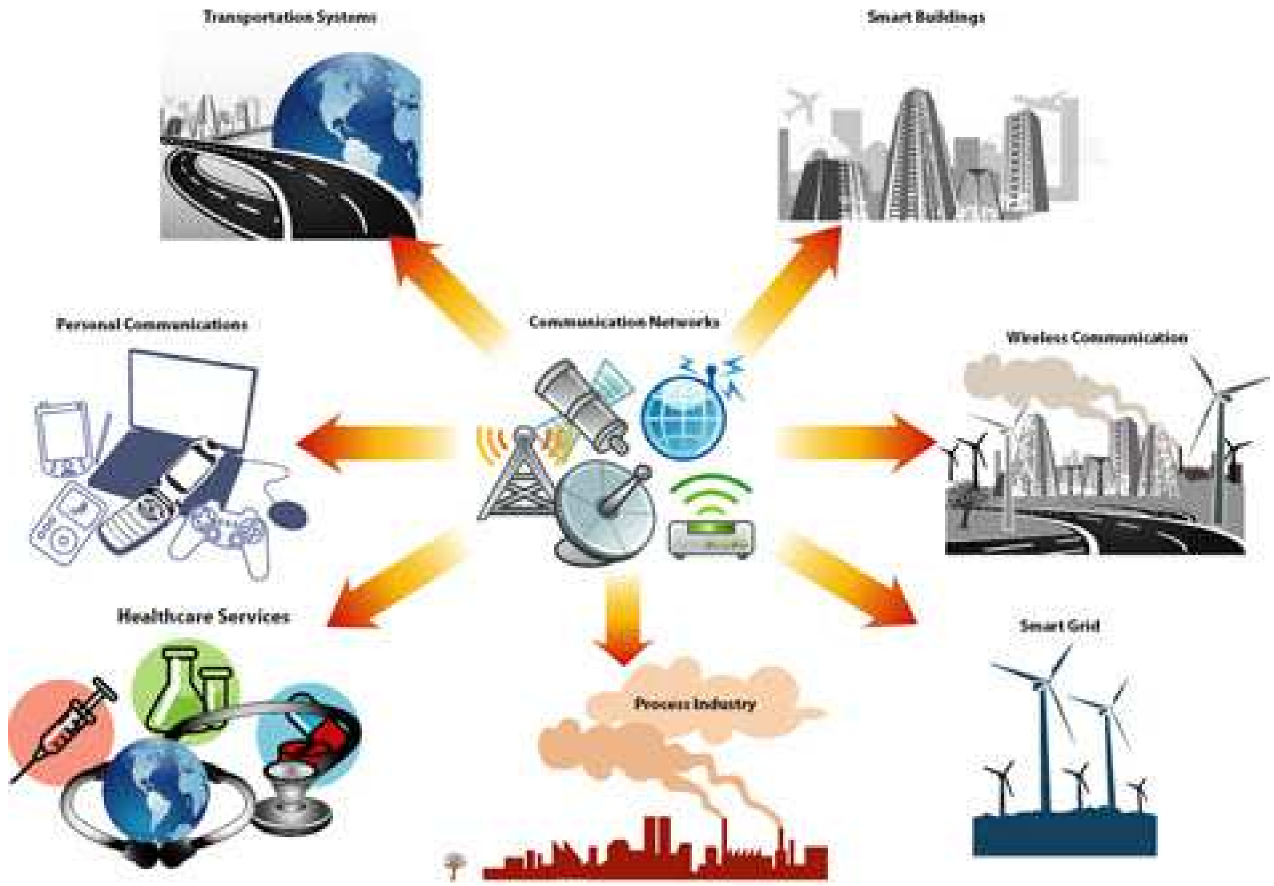
http://www.intertronicsolutions.com/my-product/12m-antenna/

Introduction (8)

Electric machines

http://gibbonsgroup.blogspot.com/2014/05/3-problems-youll-face-if-your-electric.html

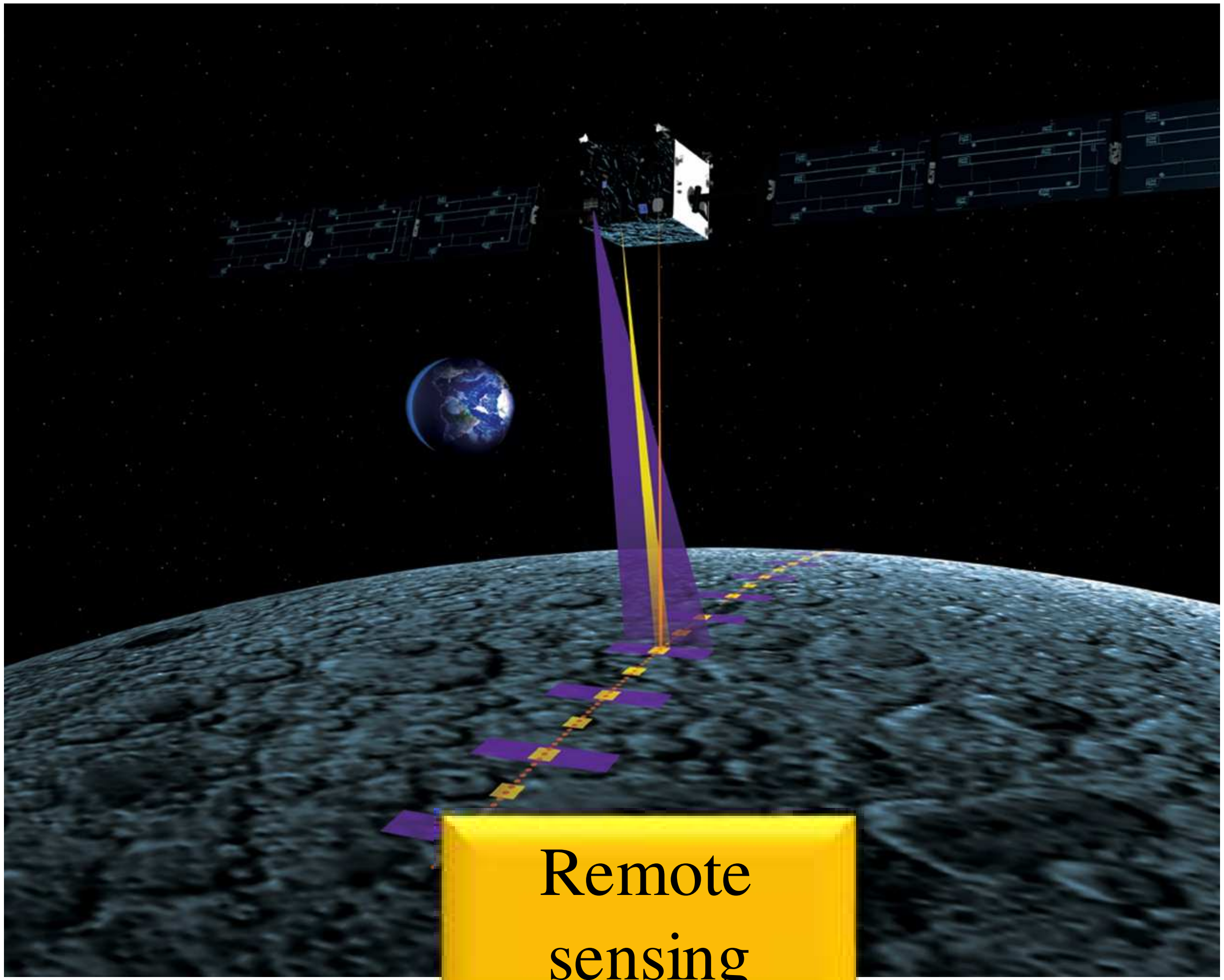
Introduction (9)



https://www.efxkits.us/project-kits-on-wireless-communication-for-electronics-professionals/

Wireless communication

Introduction (10)



Remote sensing

http://m.esa.int/spaceinimages/Images/2003/07/Remote-sensing_instruments_on_SMART-1_scan_the_Moon_s_surface

Introduction (11)

Military defense

russianplanes.net

Introduction (12)

- Electromagnetic compatibility
- Biomedical engineering
- Laser engineering
- Antenna
- **ELECTROMAGNETICS**
- Electric machines
- Wireless communication
- Remote sensing
- Military defense

Introduction (13)

1. W. H. Hayt, J. A. Buck. *Engineering Electromagnetics*. McGraw-Hill, 2007
2. N. Ida. *Engineering Electromagnetics*. Springer, 2015
3. E. J. Rothwell, M. J. Cloud. *Electromagnetics*. CRC Press, 2001
4. M. N. O. Sadiku. *Numerical Techniques in Electromagnetics*. CRC Pres, 2009
5. Nguyễn Bình Thành, Nguyễn Trần Quân, Lê Văn Bảng. *Cơ sở lý thuyết trường điện từ*. NXB Đại học & trung học chuyên nghiệp, 1970
6. Nguyễn Công Phương, Trần Hoài Linh. *Phương pháp số trong trường điện từ – minh họa bằng Python, tập 1*. NXB Khoa học & Kỹ thuật, 2021
7. https://sites.google.com/site/ncpdhbkhn/

Contents

Vector Analysis

1. Scalars & Vectors
2. The Rectangular Coordinate System
3. The Dot Product & The Cross Product
4. The Circular Cylindrical Coordinate System
5. The Spherical Coordinate System

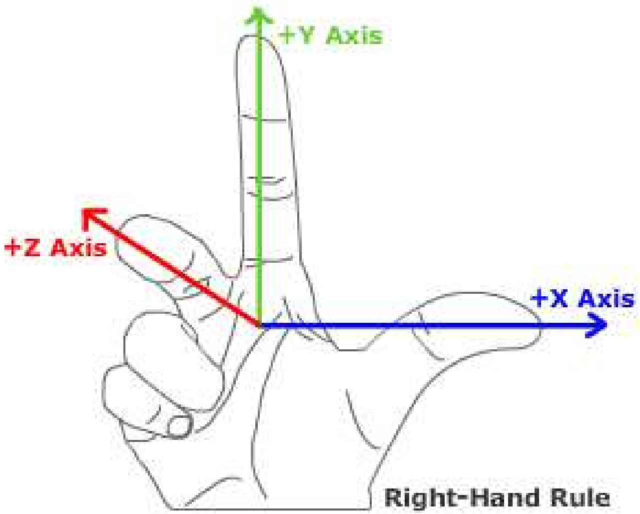
Scalars & Vectors

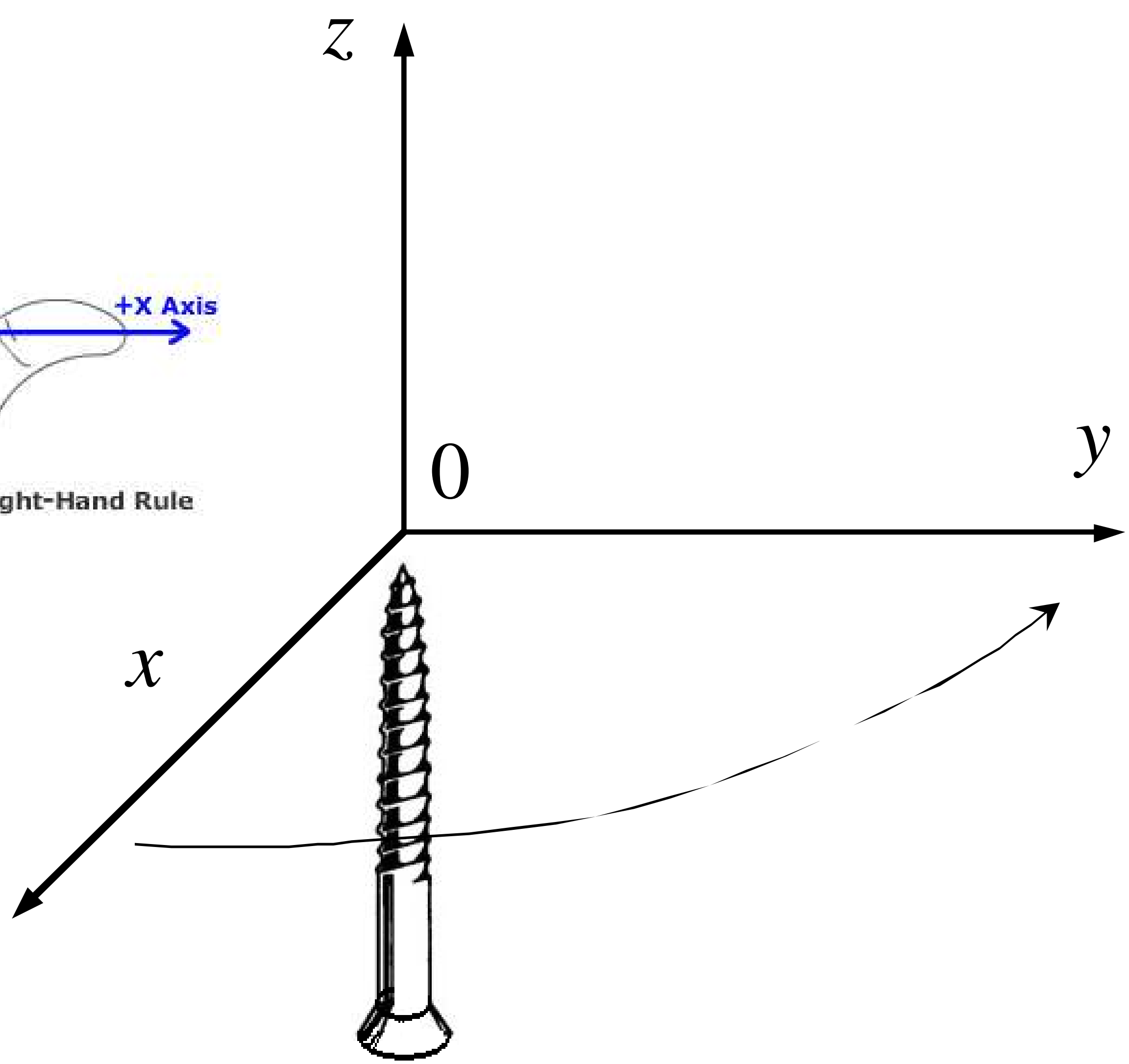
- Scalar: refers to a quantity whose value may be represented by a single (positive/negative) real number
- Ex.: distance, time, temperature, mass, …
- Scalars are in italic type, e.g., t, m, E,…
- Vector: refers to a quantity whose value may be represented by a magnitude and a direction in space (2D, 3D, nD)
- Ex.: force, velocity, acceleration, …
- Vectors are in bold type, e.g. **A**
- **A** may be written as $\bar{A}$
- Write $\mathbf{E} = 5\mathbf{a}_x$ or $\bar{E} = 5\bar{a}_x$: correct
- Write $E = 5a_x$: INCORRECT

Vector Analysis

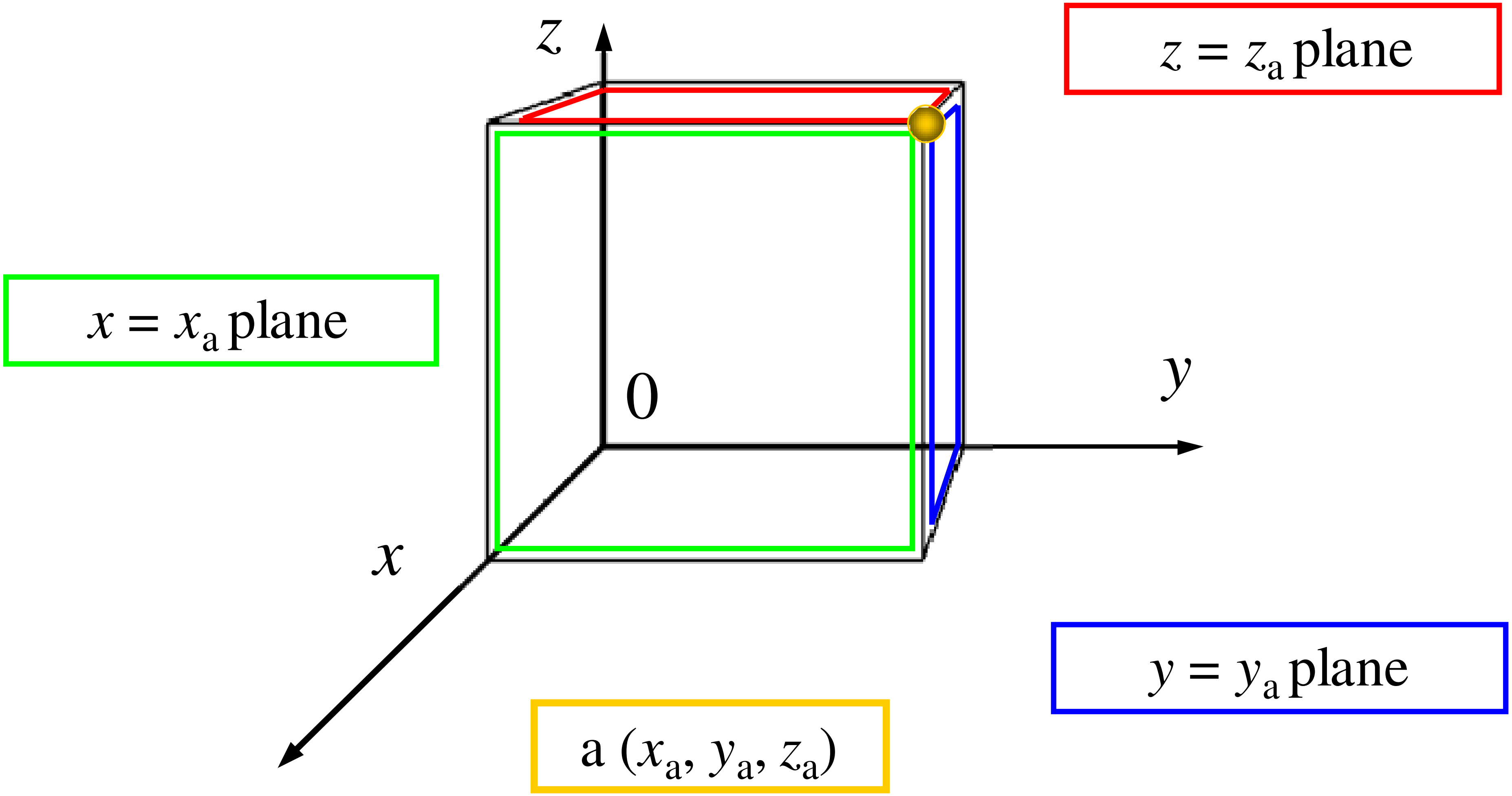
1. Scalars & Vectors
2. **The Rectangular Coordinate System**
3. The Dot Product & The Cross Product
4. The Circular Cylindrical Coordinate System
5. The Spherical Coordinate System

The Rectangular Coordinate System (1)

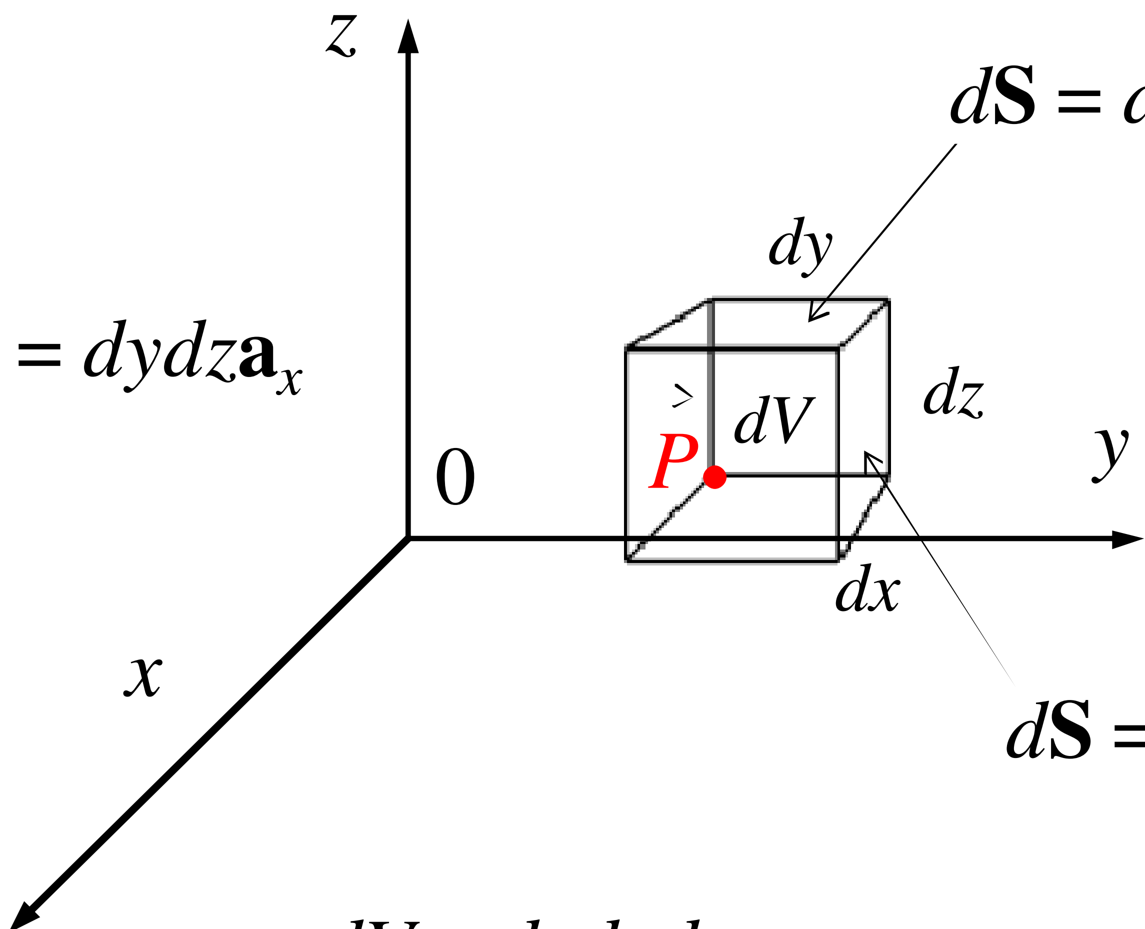




The Rectangular Coordinate System (2)



The Rectangular Coordinate System (3)



$$d\mathbf{S} = dxdy\mathbf{a}_z$$

$$d\mathbf{S} = dydz\mathbf{a}_x$$

$$d\mathbf{S} = dxdz\mathbf{a}_y$$

$$dV = dxdydz$$

The Rectangular Coordinate System (4)

$$|\mathbf{a}_x| = |\mathbf{a}_z| = |\mathbf{a}_z| = 1$$

$$a_x = a_y = a_z = 1$$

$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$

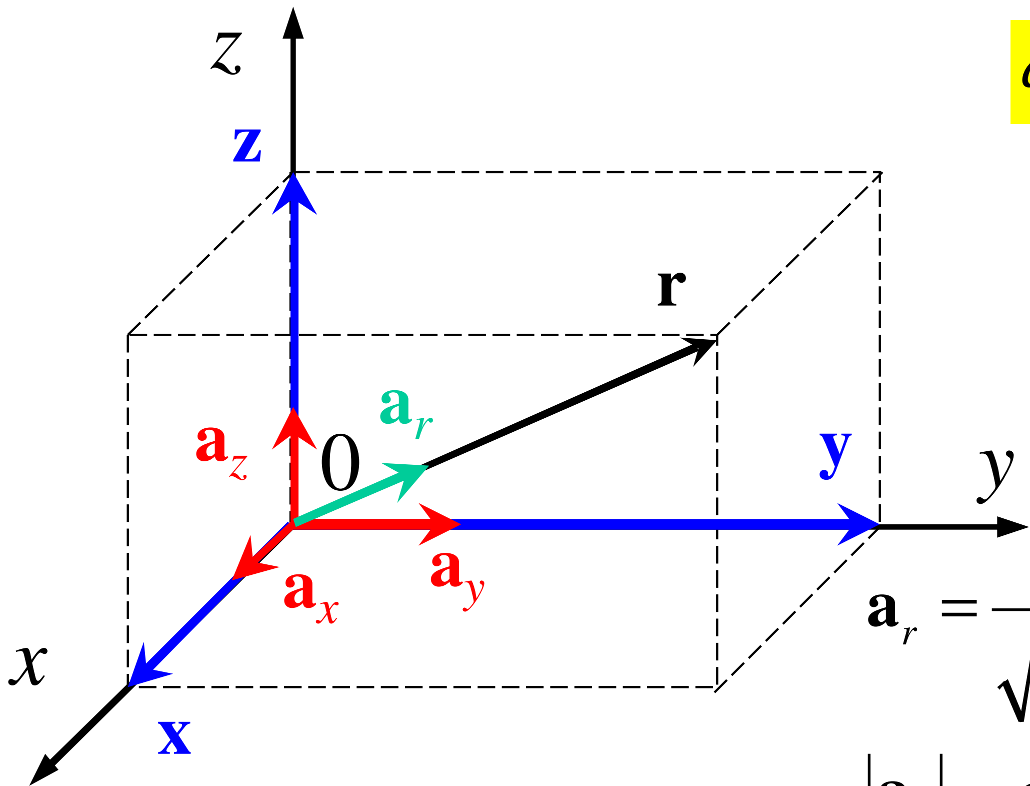
$\bar{a}_x, \bar{a}_y, \bar{a}_y$

$\vec{a}_x, \vec{a}_y, \vec{a}_z$

$\hat{x}, \hat{y}, \hat{z}$

$\bar{i}, \bar{j}, \bar{k}$

…



$$\mathbf{a}_r = \frac{\mathbf{r}}{\sqrt{r_x^2 + r_y^2 + r_z^2}} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

$$|\mathbf{a}_r| = a_r = 1$$

$$\left.\begin{array}{r} \mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z} \\ \mathbf{x} = x\mathbf{a}_x;\ \mathbf{y} = y\mathbf{a}_y;\ \mathbf{z} = z\mathbf{a}_z \end{array}\right\} \rightarrow \mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z = r_x\mathbf{a}_x + r_y\mathbf{a}_y + r_z\mathbf{a}_z$$

$$|\mathbf{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

Ex. 1

The Rectangular Coordinate System (5)

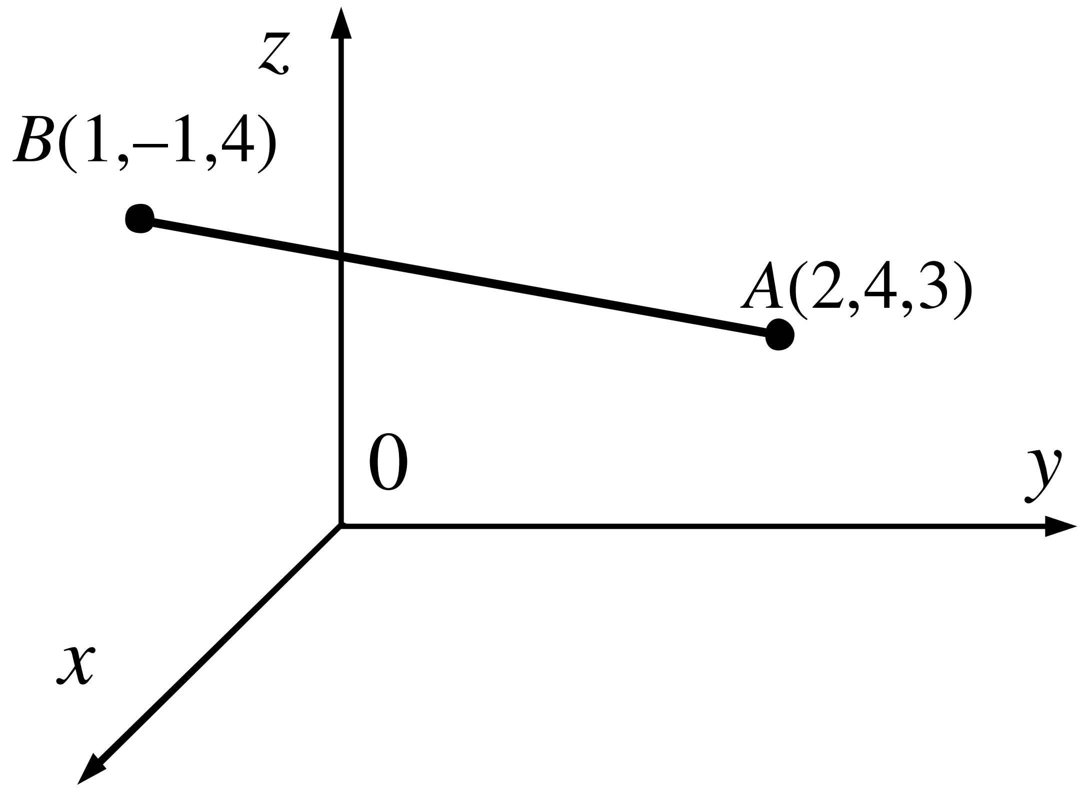
Given a vector $\mathbf{V} = 5\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z$, find:

a) Its components?

b) Its magnitude?

c) Its unit vector ?

Ex. 2 The Rectangular Coordinate System (6)



$$\mathbf{R}_1 = \overline{BA} = (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z$$

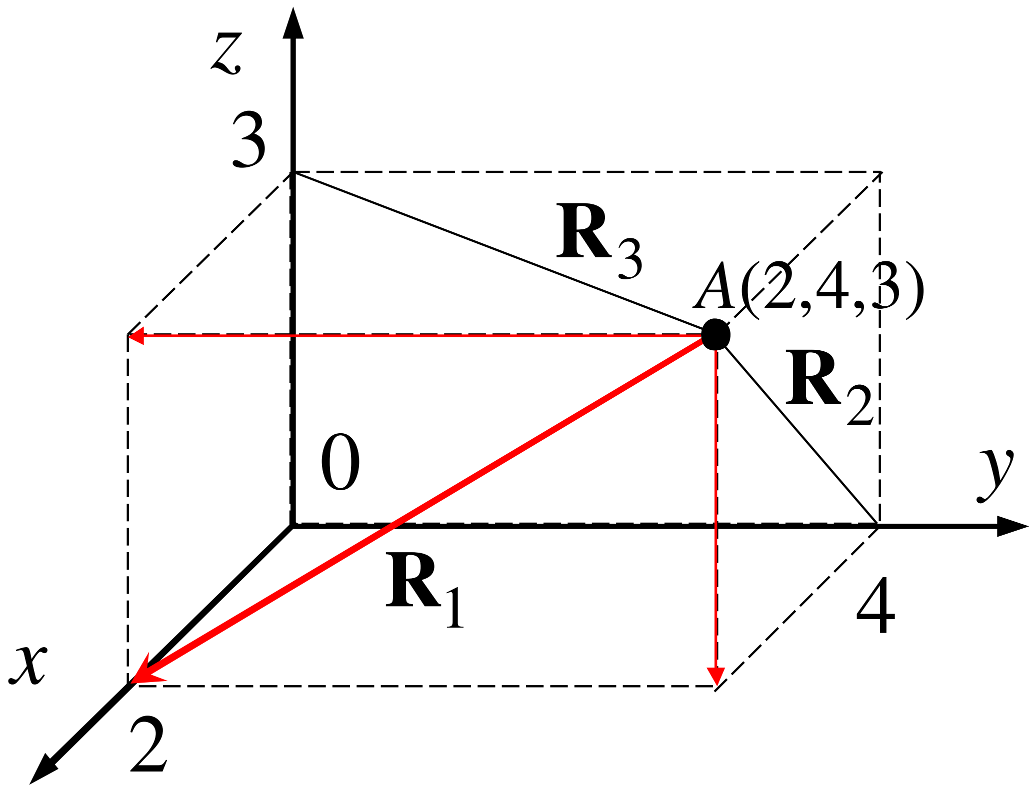
$$= (2-1)\mathbf{a}_x + [4-(-1)]\mathbf{a}_y + (3-4)\mathbf{a}_z = \boxed{\mathbf{a}_x + 5\mathbf{a}_y - \mathbf{a}_z}$$

$$\mathbf{R}_2 = \overline{AB} = (B_x - A_x)\mathbf{a}_x + (B_y - A_y)\mathbf{a}_y + (B_z - A_z)\mathbf{a}_z$$

$$= (1-2)\mathbf{a}_x + (-1-4)\mathbf{a}_y + (4-3)\mathbf{a}_z = \boxed{-\mathbf{a}_x - 5\mathbf{a}_y + \mathbf{a}_z} = -\mathbf{R}_1$$

The Rectangular Coordinate System (7)

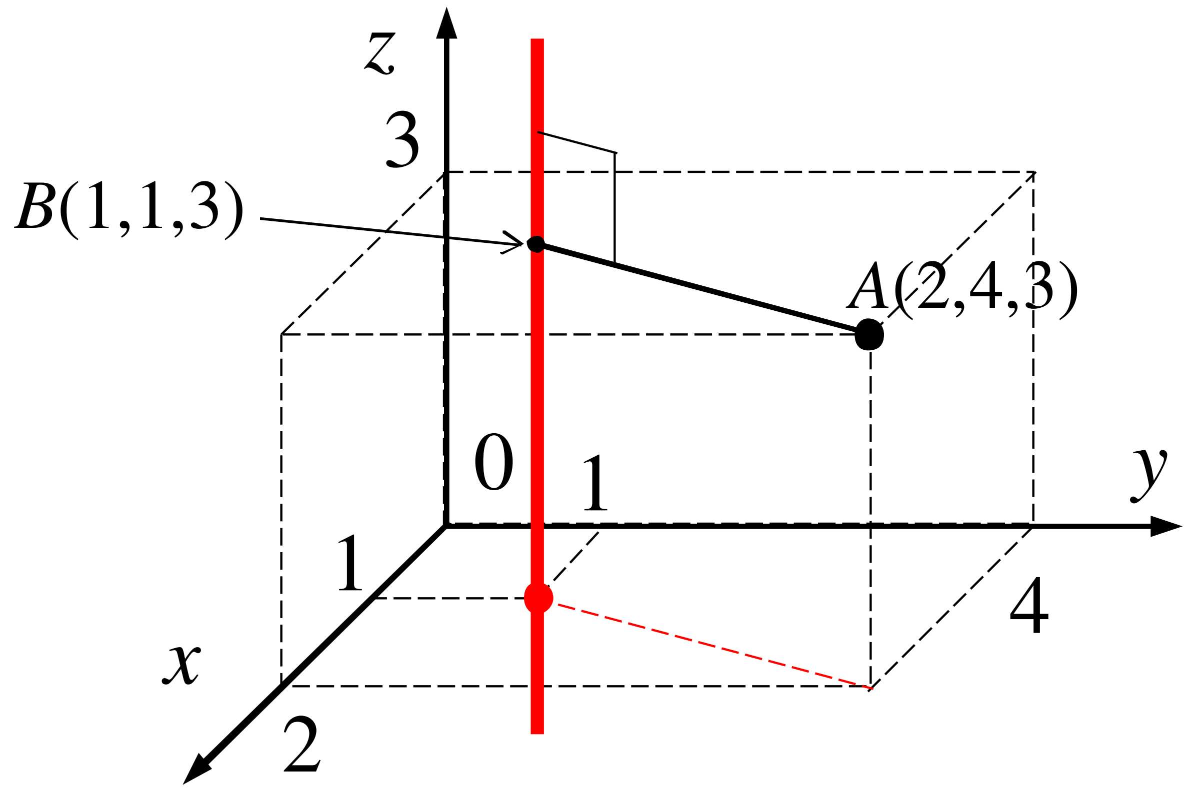
Ex. 3



$$\mathbf{R}_1 = (2-2)\mathbf{a}_x + (0-4)\mathbf{a}_y + (0-3)\mathbf{a}_z = -4\mathbf{a}_y - 3\mathbf{a}_z$$

Ex. 4

The Rectangular Coordinate System (8)



$$\mathbf{R}_1 = \overline{BA} = (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z$$
$$= (2-1)\mathbf{a}_x + (4-1)\mathbf{a}_y + (3-3)\mathbf{a}_z = \boxed{\mathbf{a}_x + 3\mathbf{a}_y}$$
$$\mathbf{R}_2 = \overline{AB} = (B_x - A_x)\mathbf{a}_x + (B_y - A_y)\mathbf{a}_y + (B_z - A_z)\mathbf{a}_z$$
$$= (1-2)\mathbf{a}_x + (1-4)\mathbf{a}_y + (3-3)\mathbf{a}_z = \boxed{-\mathbf{a}_x - 3\mathbf{a}_y} = -\mathbf{R}_1$$

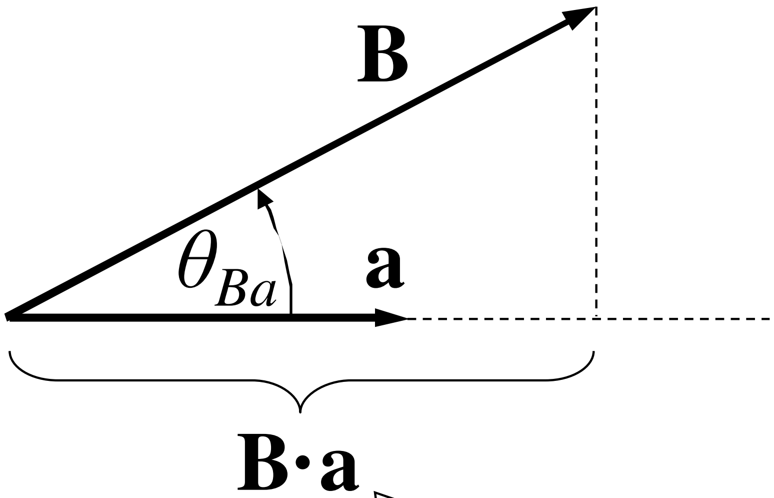
Vector Analysis

1. Scalars & Vectors
2. The Rectangular Coordinate System
3. **The Dot Product & The Cross Product**
4. The Circular Cylindrical Coordinate System
5. The Spherical Coordinate System

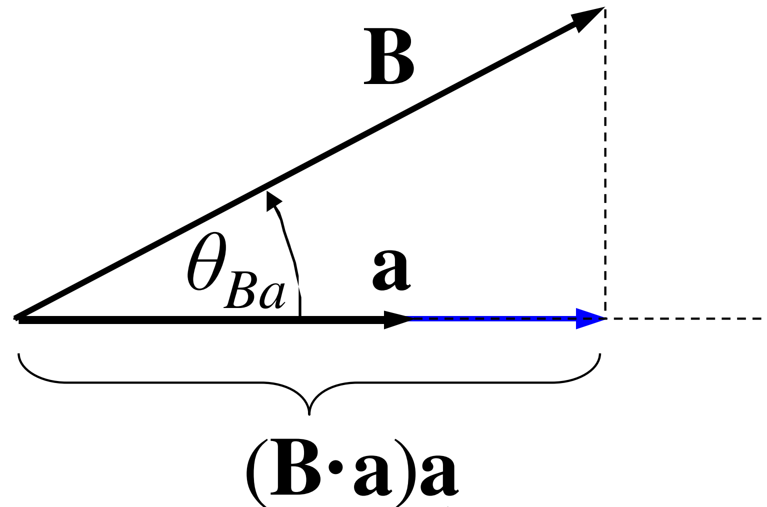
The Dot Product (1)

- $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$
  - $|\mathbf{A}|$: magnitude of **A**
  - $|\mathbf{B}|$: magnitude of **B**
  - θ_{AB}: smaller angle between **A** & **B**
- $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
- $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

The Dot Product (2)



The scalar component of **B** in the direction of the unit vector **a**



The vector component of **B** in the direction of the unit vector **a**

Ex.: $B_x = \mathbf{B}\cdot\mathbf{a}_x$

Ex.: $B_x\mathbf{a}_x = (\mathbf{B}\cdot\mathbf{a}_x)\mathbf{a}_x$

Ex.

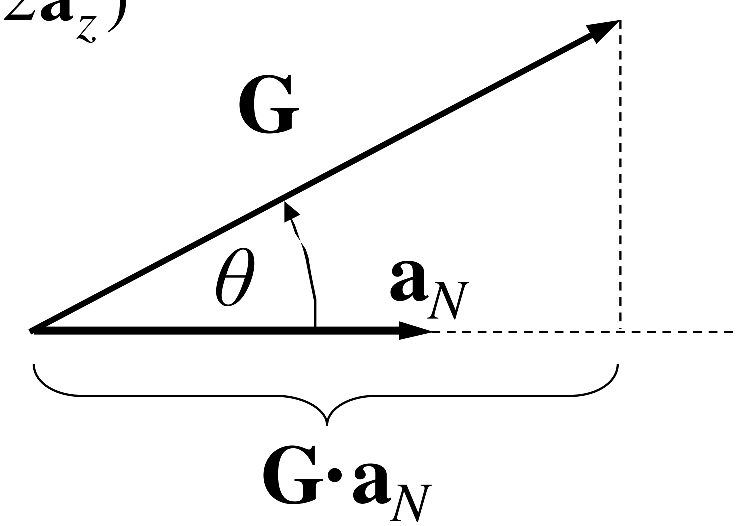
The Dot Product (3)

Consider the vector field $\mathbf{G} = z\mathbf{a}_x - 2x\mathbf{a}_y + 3y\mathbf{a}_z$ and the point $Q(4, 3, 2)$. Find:
a) $\mathbf{G}$ at Q ?
b) The scalar component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N = ⅓(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$?
c) The vector component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N$?
d) The angle between $\mathbf{G}(\mathbf{r}_Q)$ & $\mathbf{a}_N$?

a) $\mathbf{G}(\mathbf{r}_Q) = 2\mathbf{a}_x - 2\times4\mathbf{a}_y + 3\times3\mathbf{a}_z = 2\mathbf{a}_x - 8\mathbf{a}_y + 9\mathbf{a}_z$

b) $\mathbf{G}\cdot\mathbf{a}_N = (2\mathbf{a}_x - 8\mathbf{a}_y + 9\mathbf{a}_z)\cdot\frac{1}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$

$$= \frac{1}{3}(2\times1 - 8\times2 - 9\times2) = -10.67$$



The Dot Product (4)

Ex.

Consider the vector field $\mathbf{G} = z\mathbf{a}_x - 2x\mathbf{a}_y + 3y\mathbf{a}_z$ and the point $Q(4, 3, 2)$. Find:

a) $\mathbf{G}$ at Q ?

b) The scalar component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N = \frac{1}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$?

c) The vector component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N$?

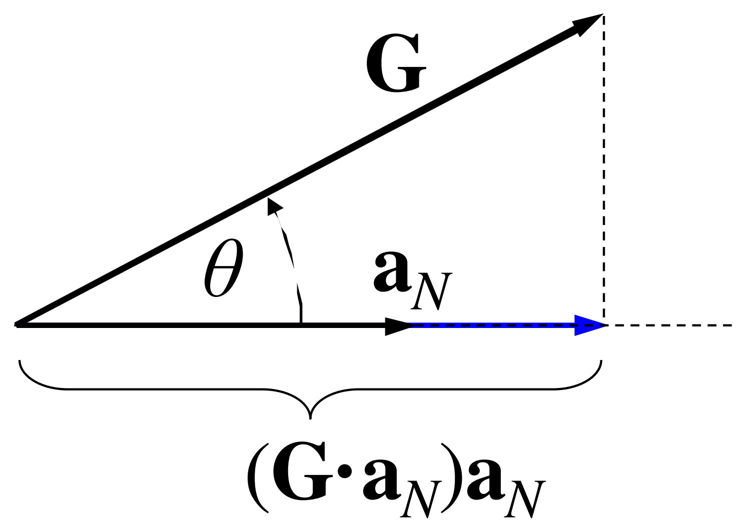
d) The angle between $\mathbf{G}(\mathbf{r}_Q)$ & $\mathbf{a}_N$?

a) $\mathbf{G}(\mathbf{r}_Q) = 2\mathbf{a}_x - 2\times4\mathbf{a}_y + 3\times3\mathbf{a}_z = 2\mathbf{a}_x - 8\mathbf{a}_y + 9\mathbf{a}_z$

b) $\mathbf{G} \cdot \mathbf{a}_N = -10.67$

c) $(\mathbf{G} \cdot \mathbf{a}_N)\mathbf{a}_N = (-10.67)\frac{1}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$

$$= -3.55\mathbf{a}_x - 7.11\mathbf{a}_y + 7.11\mathbf{a}_z$$



The Dot Product (5)

Ex.

Consider the vector field $\mathbf{G} = z\mathbf{a}_x - 2x\mathbf{a}_y + 3y\mathbf{a}_z$ and the point $Q(4, 3, 2)$. Find:
a) $\mathbf{G}$ at Q ?
b) The scalar component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N = \frac{1}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$?
c) The vector component of $\mathbf{G}$ at Q in the direction of $\mathbf{a}_N$?
d) The angle between $\mathbf{G}(\mathbf{r}_Q)$ & $\mathbf{a}_N$?

a) $\mathbf{G}(\mathbf{r}_Q) = 2\mathbf{a}_x - 2\times 4\mathbf{a}_y + 3\times 3\mathbf{a}_z = 2\mathbf{a}_x - 8\mathbf{a}_y + 9\mathbf{a}_z$

b) $\mathbf{G} \bullet \mathbf{a}_N = -10.67$

c) $(\mathbf{G} \bullet \mathbf{a}_N)\mathbf{a}_N = -3.55\mathbf{a}_x - 7.11\mathbf{a}_y + 7.11\mathbf{a}_z$

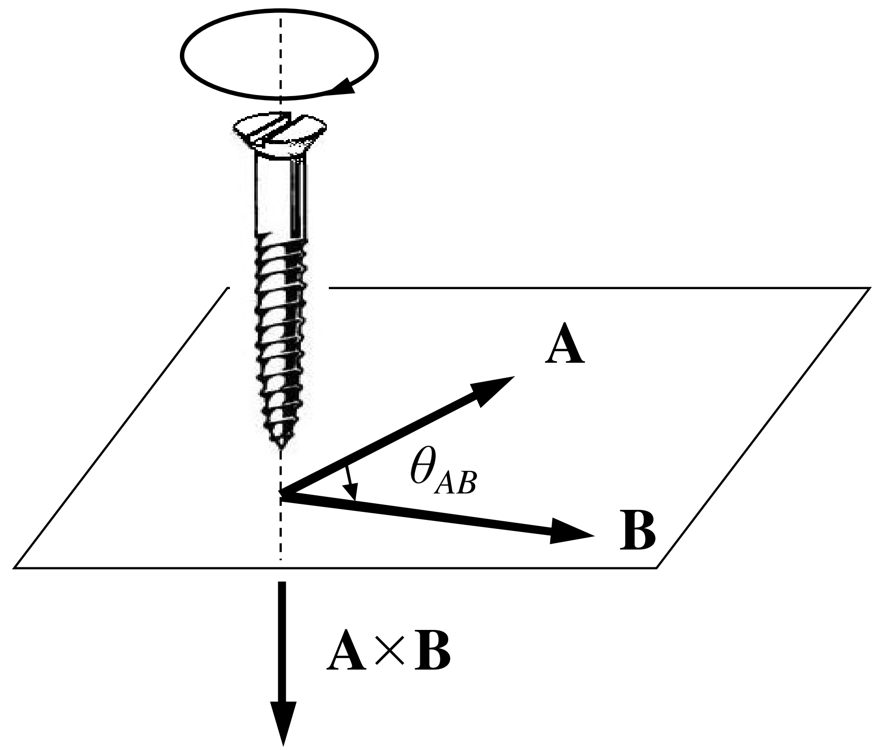
d) $\mathbf{G} \bullet \mathbf{a}_N = G a_N \cos\theta = -10.67$

$$\rightarrow \sqrt{2^2 + 8^2 + 9^2} \times 1 \times \cos\theta = -10.67$$

$$\rightarrow \theta = 151^{\circ}$$

The Cross Product (1)

- $\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}||\mathbf{B}| \sin\theta_{AB}$
  - $\mathbf{a}_N$: normal (unit) vector
- $\mathbf{B} \times \mathbf{A} = -(\mathbf{A} \times \mathbf{B})$



$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$: unit vectors of x, y, z axes

The Cross Product (2)

Ex. 1

Given $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -4\mathbf{a}_x + 5\mathbf{a}_y - 6\mathbf{a}_z$. Find their cross product ?

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 1 & -2 & 3 \\ -4 & 5 & -6 \end{vmatrix}$$

$$= \mathbf{a}_x \begin{vmatrix} -2 & 3 \\ 5 & -6 \end{vmatrix} - \mathbf{a}_y \begin{vmatrix} 1 & 3 \\ -4 & -6 \end{vmatrix} + \mathbf{a}_z \begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix}$$

$$= -3\mathbf{a}_x - 6\mathbf{a}_y - 3\mathbf{a}_z$$

The Cross Product (3)

Ex. 2

Given $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -4\mathbf{a}_x + 5\mathbf{a}_y - 6\mathbf{a}_z$. Find the angle between **A** & **B**?

Method 1:

$$\left.\begin{aligned}
&\mathbf{A}\times\mathbf{B} = \mathbf{a}_N |\mathbf{A}||\mathbf{B}|\sin\theta \quad \rightarrow |\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin\theta \rightarrow \sin\theta = \frac{|\mathbf{A}\times\mathbf{B}|}{|\mathbf{A}||\mathbf{B}|} \\
&\mathbf{A}\times\mathbf{B} = -3\mathbf{a}_x - 6\mathbf{a}_y - 3\mathbf{a}_z \rightarrow |\mathbf{A}\times\mathbf{B}| = \sqrt{3^2+6^2+3^2} = 7.35 \\
&|\mathbf{A}| = \sqrt{1^2+2^2+3^2} = 3.74 \\
&|\mathbf{B}| = \sqrt{4^2+5^2+6^2} = 8.75
\end{aligned}\right\}$$

$$\rightarrow \sin\theta = \frac{7.35}{3.74\times 8.75} = 0.22 \rightarrow \theta = \mathrm{asin}(0.22) = \boxed{12.9^{\circ}}$$

The Cross Product (4)

Ex. 2

Given $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -4\mathbf{a}_x + 5\mathbf{a}_y - 6\mathbf{a}_z$. Find the angle between **A** & **B**?

Method 2:

$$\left.\begin{aligned}
&\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta \rightarrow \cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|} \\
&\mathbf{A}\cdot\mathbf{B} = 1(-4) - 2(5) + 3(-6) = -32 \\
&|\mathbf{A}| = \sqrt{1^2 + 2^2 + 3^2} = 3.74 \\
&|\mathbf{B}| = \sqrt{4^2 + 5^2 + 6^2} = 8.75
\end{aligned}\right\}$$

$$\rightarrow \cos\theta = \frac{-32}{3.74\times 8.75} = -0.97 \rightarrow \theta = \mathrm{a}\cos(-0.97) = \boxed{12.9^{\circ}}$$

Ex. 3

The Cross Product (5)

Given $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{B} = -4\mathbf{a}_x + 5\mathbf{a}_y - 6\mathbf{a}_z$, and $\mathbf{C} = \mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$. Find:

a) $\mathbf{A} \pm \mathbf{B}$, $\mathbf{B} \pm \mathbf{C}$, $\mathbf{C} \pm \mathbf{A}$

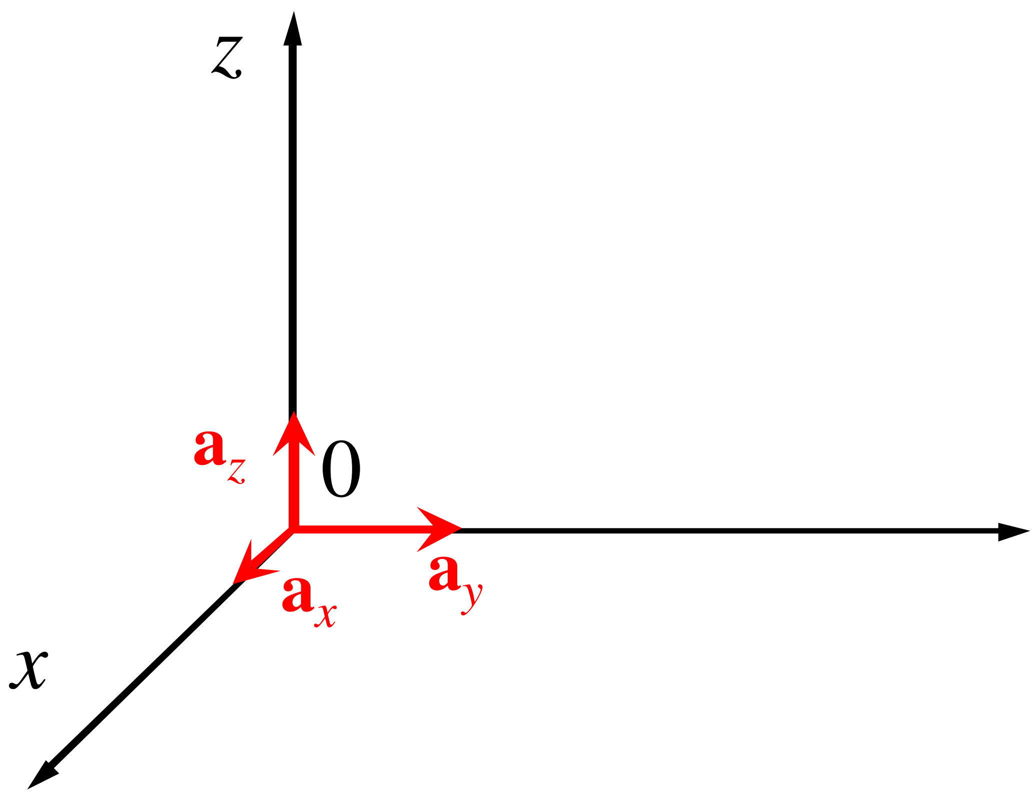
b) $\mathbf{A.B}$, $\mathbf{B.C}$, $\mathbf{C.A}$

c) $\mathbf{A}\times\mathbf{B}$, $\mathbf{B}\times\mathbf{C}$, $\mathbf{C}\times\mathbf{A}$

d) $(\mathbf{A}\times\mathbf{B}).\mathbf{C}$, $\mathbf{A}.(\mathbf{B}\times\mathbf{C})$

e) What is the angle between $\mathbf{B}$ and $\mathbf{C}\times\mathbf{A}$?

The Rectangular Coordinate System (6)



$$\mathbf{a}_x . \mathbf{a}_y = 0$$

$$\mathbf{a}_x . \mathbf{a}_x = 1$$

$$\mathbf{a}_x \times \mathbf{a}_x = 0$$

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

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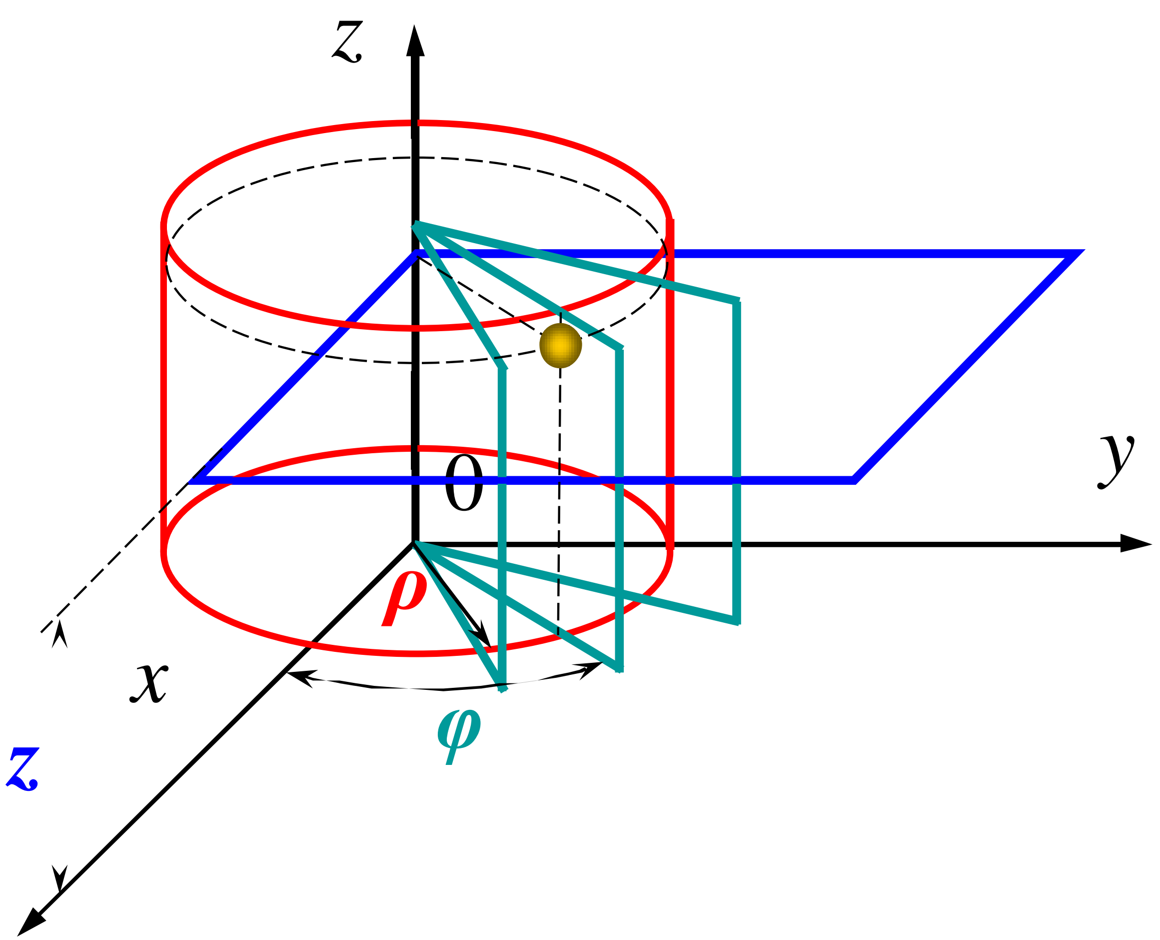
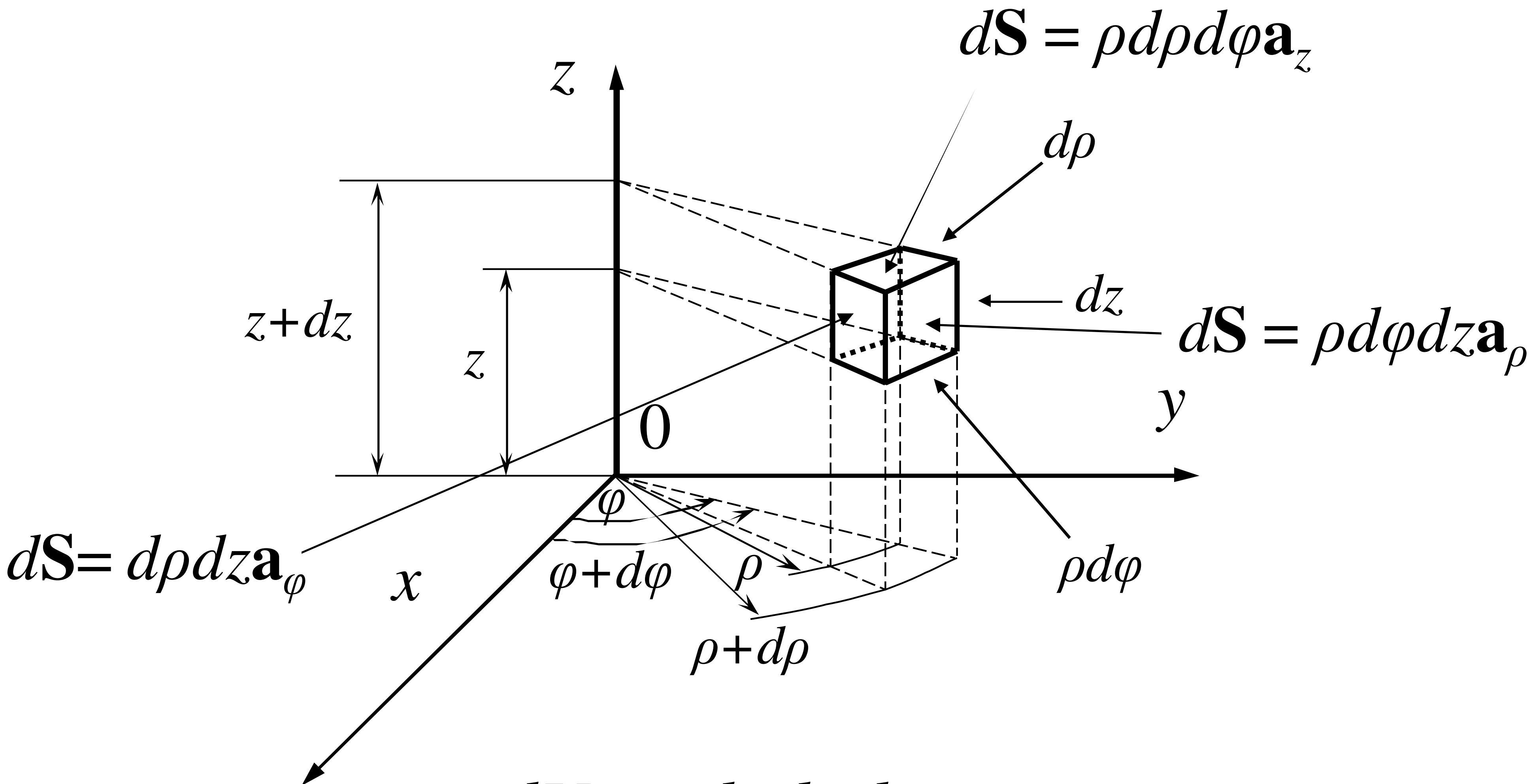
The Circular Cylindrical Coordinate System (1)



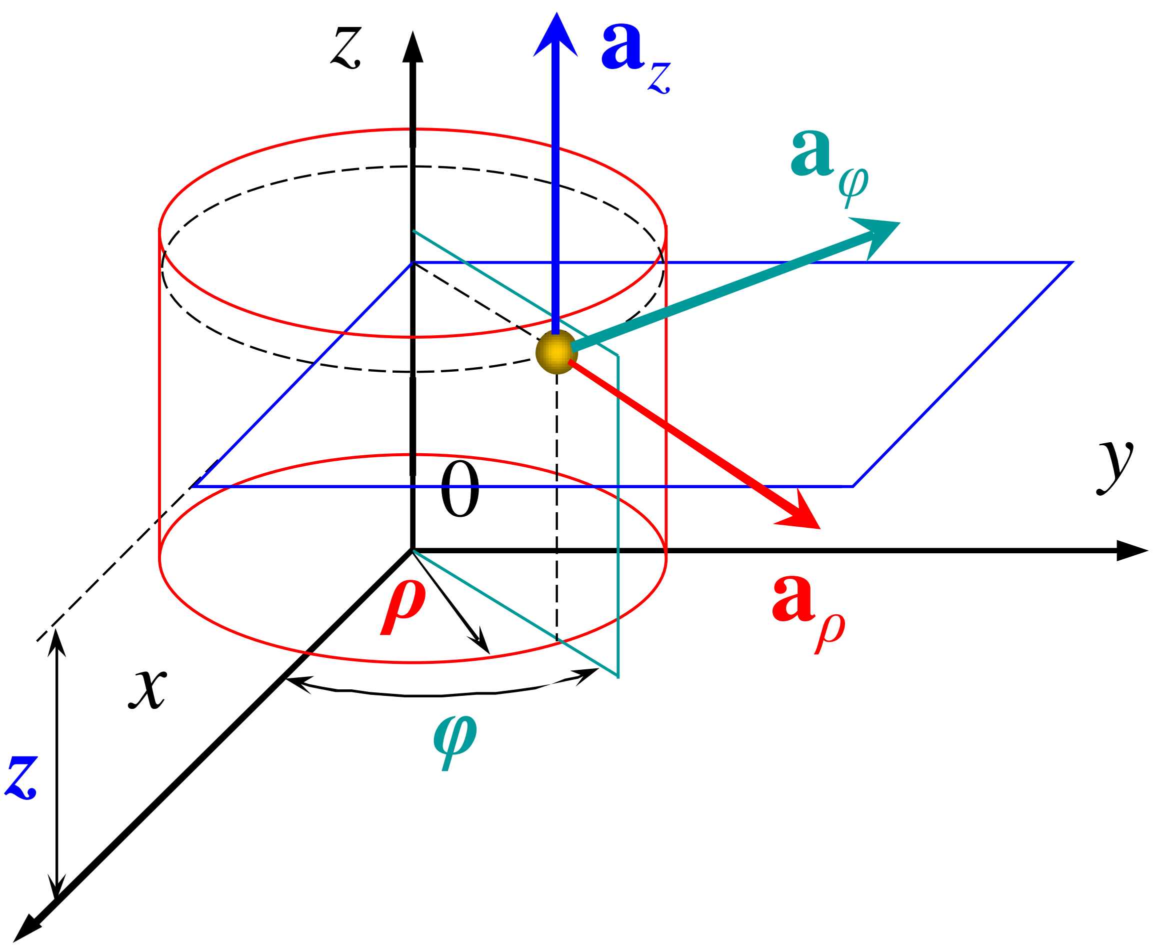
$$\rho,\ \varphi,\ z$$

The Circular Cylindrical Coordinate System (2)



$$dV = \rho d\rho d\varphi dz$$

The Circular Cylindrical Coordinate System (3)



$$\mathbf{a}_\rho.\mathbf{a}_\varphi = 0$$

$$\mathbf{a}_\rho.\mathbf{a}_\rho = 1$$

$$\mathbf{a}_\rho \times \mathbf{a}_\rho = 0$$

$$\mathbf{a}_\rho \times \mathbf{a}_\varphi = \mathbf{a}_z$$

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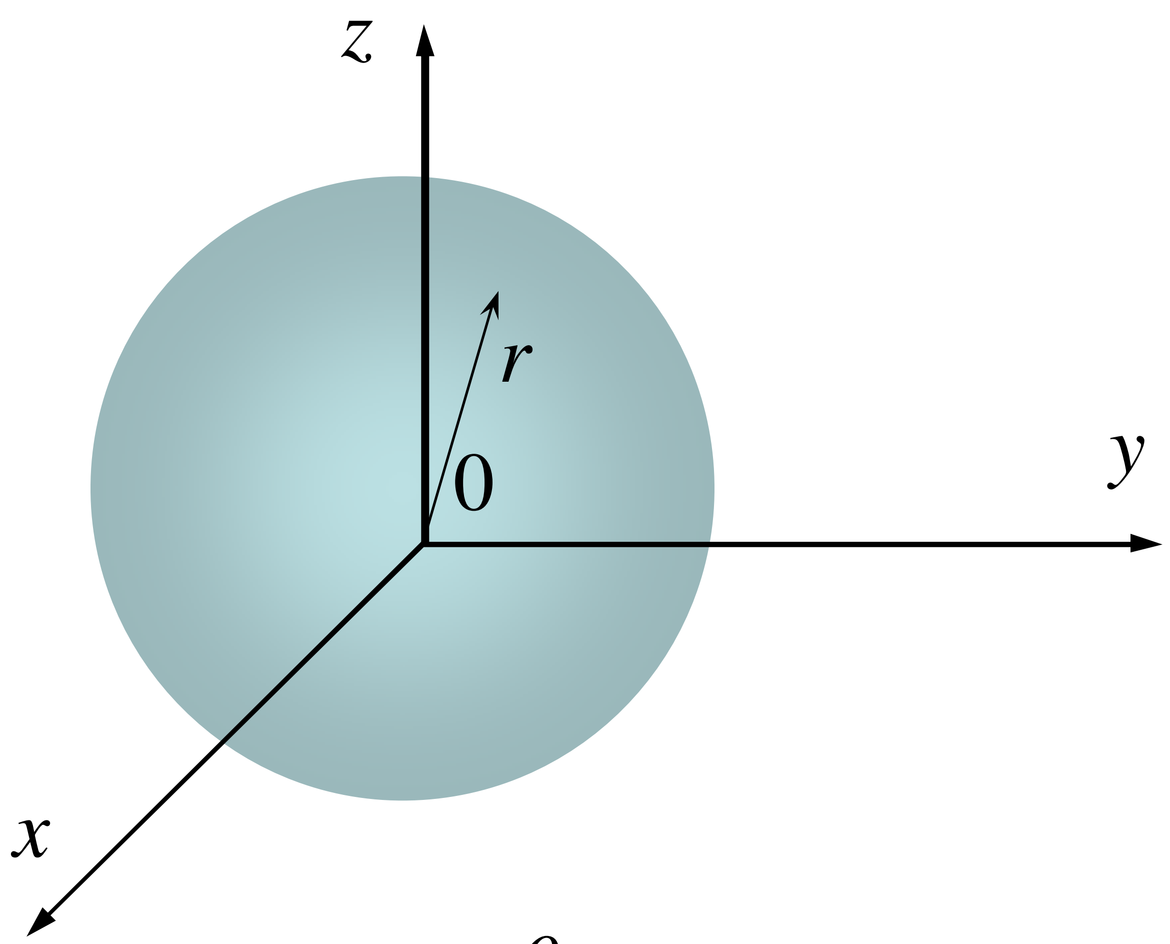
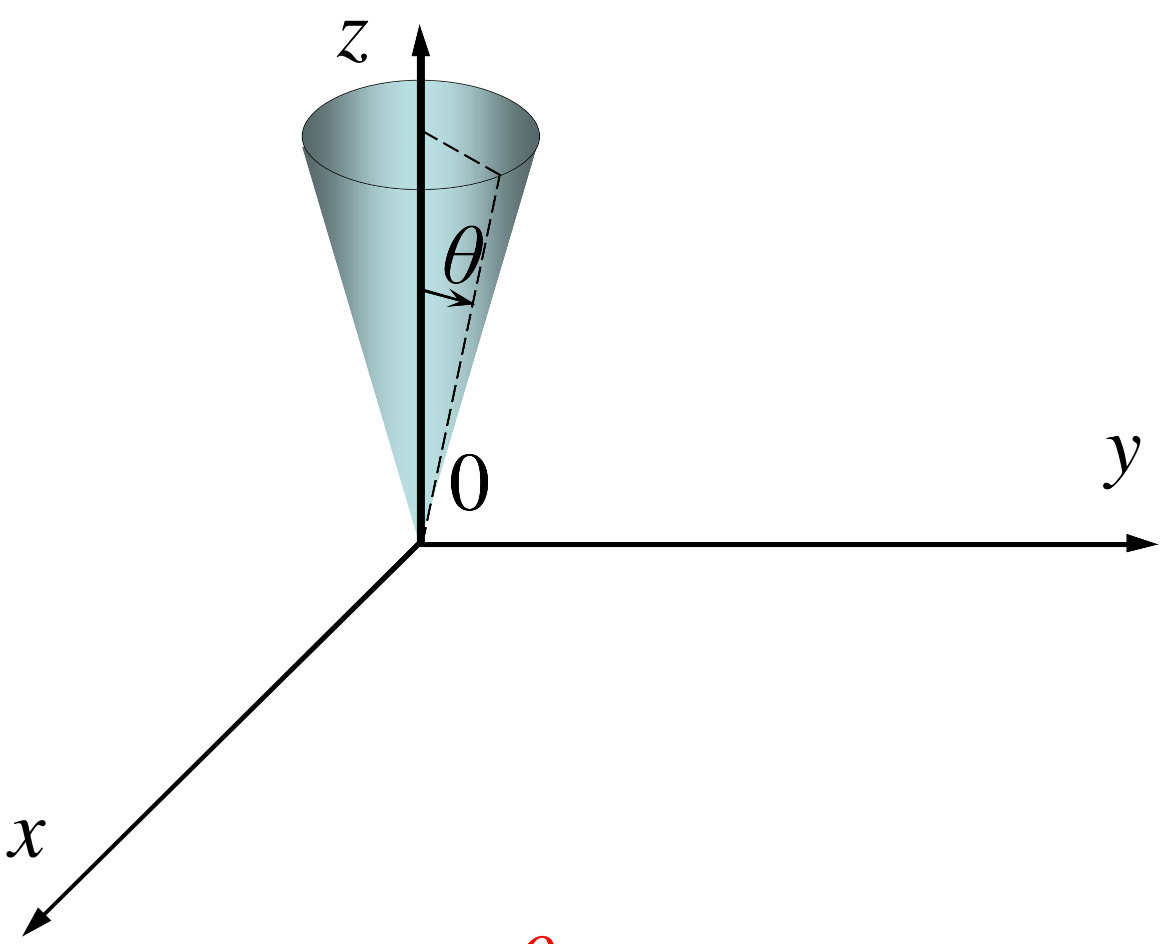
2. The Rectangular Coordinate System
3. The Dot Product & The Cross Product
4. The Circular Cylindrical Coordinate System
5. **The Spherical Coordinate System**

The Spherical Coordinate System (1)



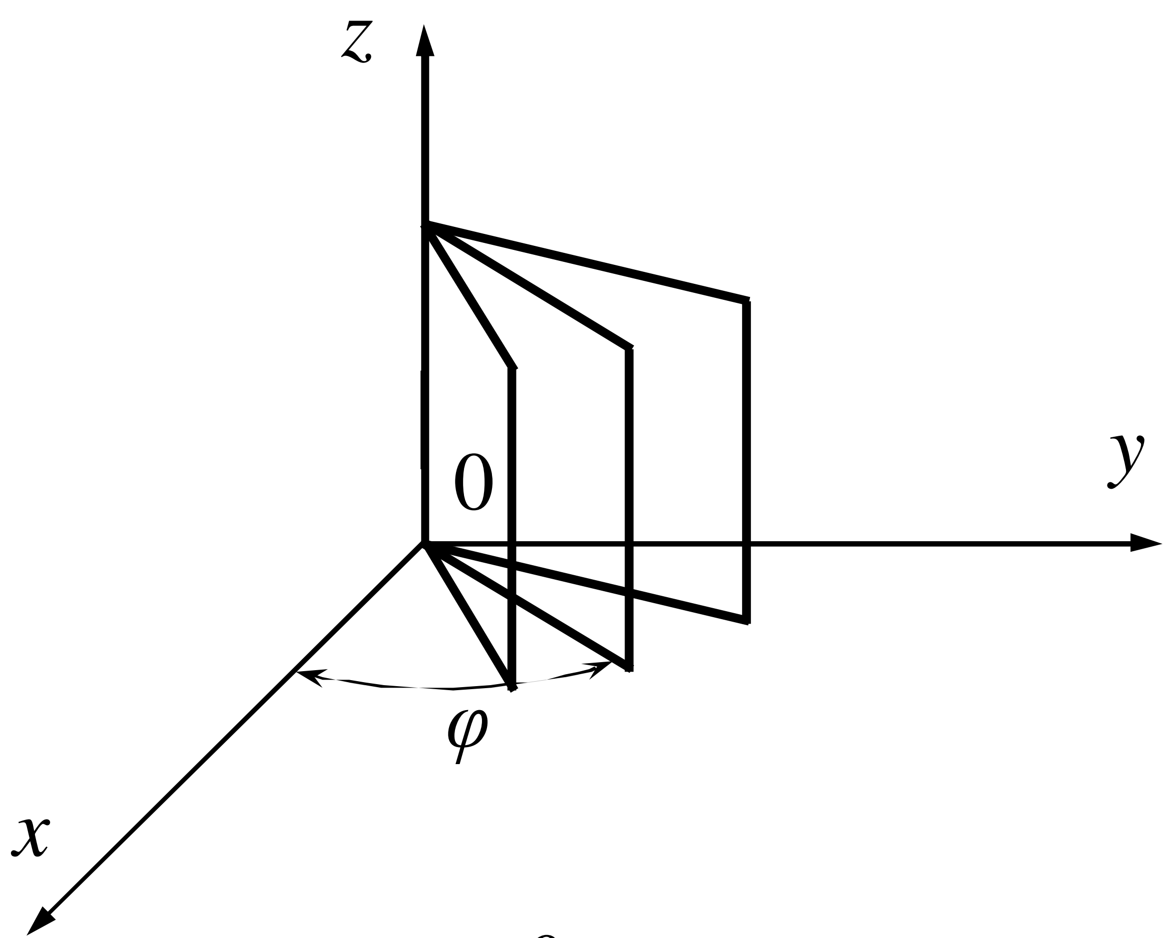
r, θ, φ

The Spherical Coordinate System (1)



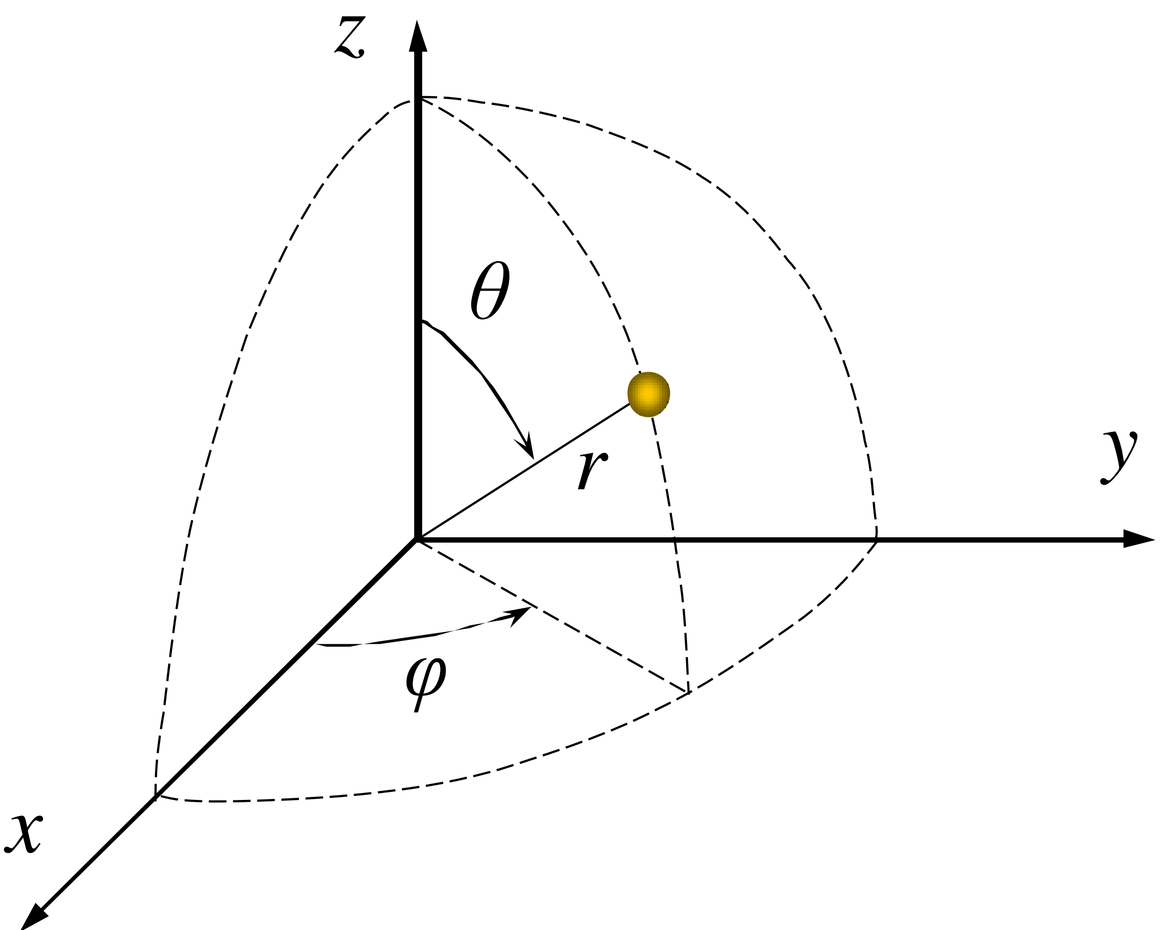
r, θ, φ

The Spherical Coordinate System (1)



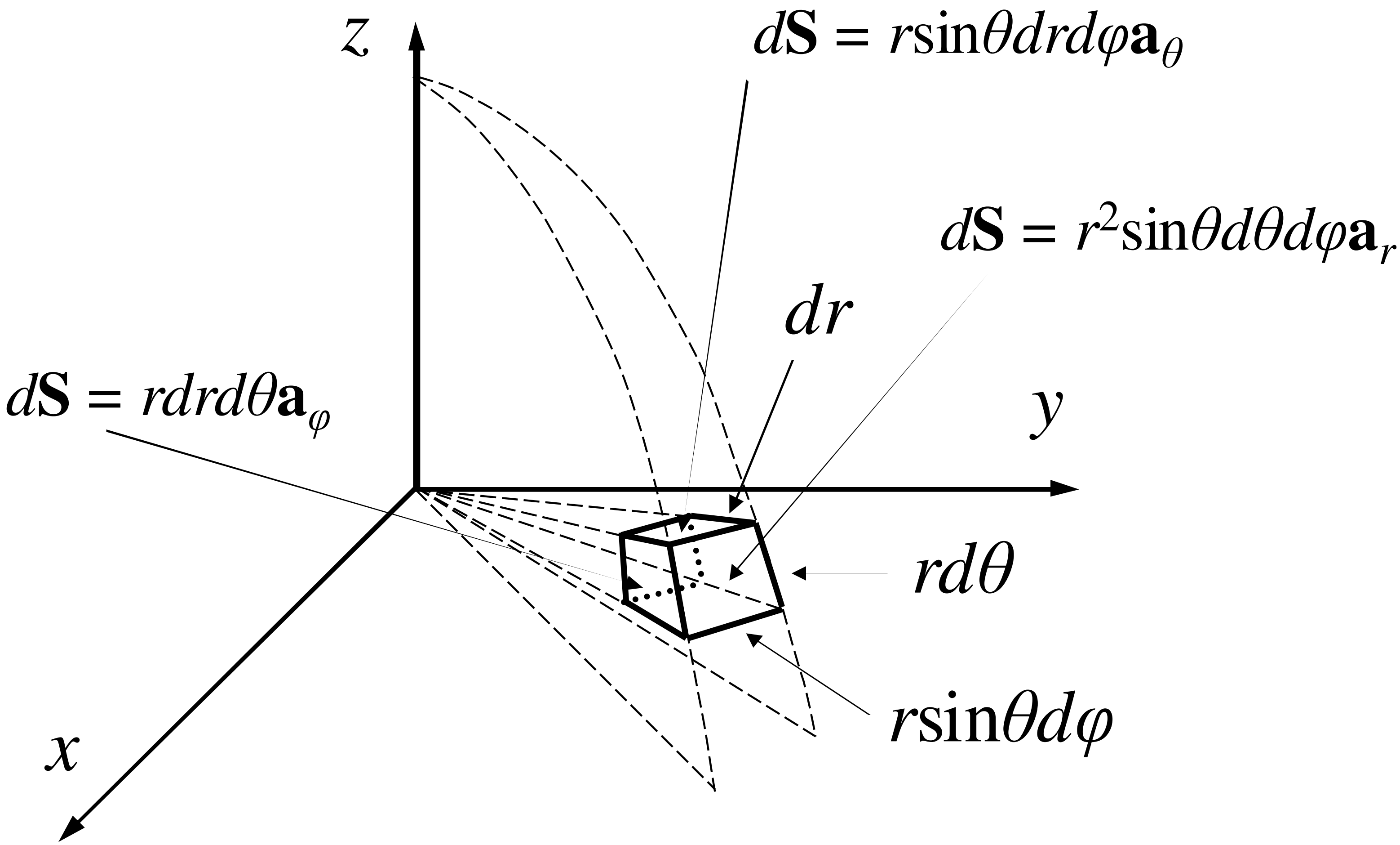
r, θ, φ

The Spherical Coordinate System (2)



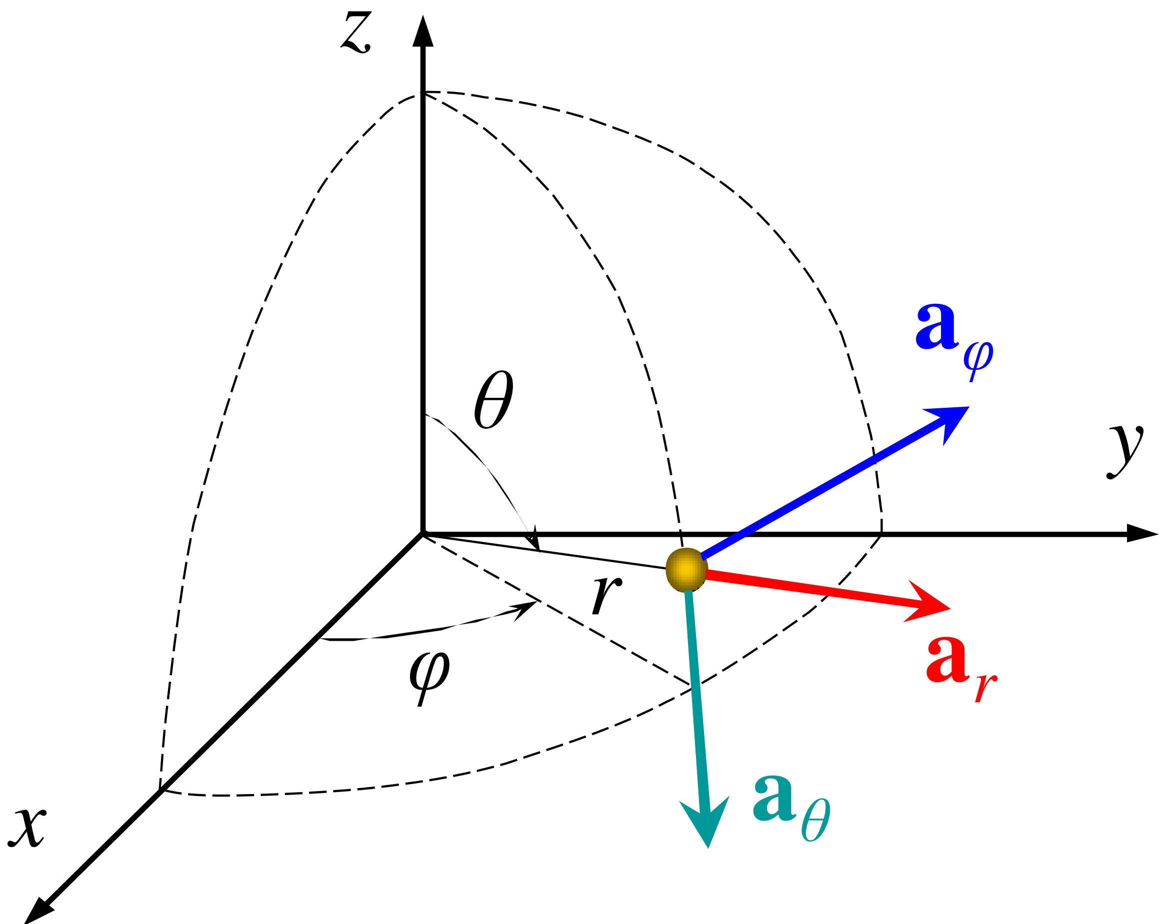
r, θ, φ

The Spherical Coordinate System (3)



$$dV = r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

The Spherical Coordinate System (4)



r, θ, φ

$$\mathbf{a}_r.\mathbf{a}_\theta = 0$$

$$\mathbf{a}_r.\mathbf{a}_r = 1$$

$$\mathbf{a}_r \times \mathbf{a}_r = 0$$

$$\mathbf{a}_r \times \mathbf{a}_\theta = \mathbf{a}_\varphi$$

RECTANGULAR	CYLINDRICAL	SPHERICAL
x	$\rho\cos\varphi$	$r\sin\theta\cos\varphi$
y	$\rho\sin\varphi$	$r\sin\theta\sin\varphi$
z	z	$r\cos\theta$
CYLINDRICAL	**RECTANGULAR**	**SPHERICAL**
ρ	$\sqrt{x^2+y^2}$	$r\sin\theta$
φ	$\mathrm{atan}(y/x)$	φ
z	z	$r\cos\theta$
SPHERICAL	**RECTANGULAR**	**CYLINDRICAL**
r	$\sqrt{x^2+y^2+z^2}$	$\sqrt{\rho^2+z^2}$
θ	$\mathrm{acos}(z/\sqrt{x^2+y^2+z^2})$	$\mathrm{acos}(z/\sqrt{\rho^2+z^2})$
φ	$\mathrm{acot}(x/y)$	φ