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Midterm Exam

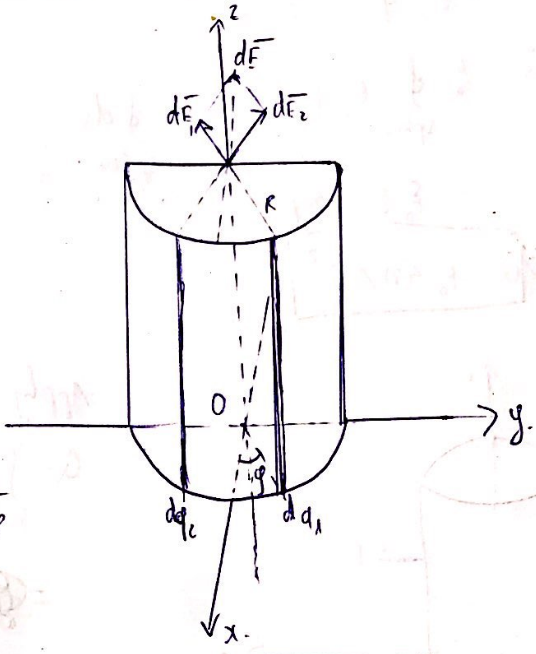
Q1:

a) + We divide the semicylinder into many vertical small infinite line charges: dq

$$dq = \rho_S \cdot ds = \rho_L \cdot dz$$

$$\Leftrightarrow \rho_S R d\varphi dz = \rho_L dz$$

$$\Rightarrow \boxed{\rho_S R d\varphi = \rho_L}$$



+ We have the EFI of a infinite line charge:

$$d\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 R}\bar{a}_\rho = \frac{\rho_S d\varphi R}{2\pi\varepsilon_0 R}\bar{a}_\rho = \frac{\rho_S d\varphi}{2\pi\varepsilon_0}\bar{a}_\rho$$

$$+ \; d\bar{E} = d\bar{E}_1 + d\bar{E}_2$$

$$\Rightarrow dE = 2dE_1 \cos\varphi = \frac{2\cos\varphi \rho_S . d\varphi}{2\pi\varepsilon_0} = \frac{\rho_S \cos\varphi d\varphi}{\pi\varepsilon_0}$$

$$\Rightarrow E = \int_0^{\pi/2} \frac{\rho_S \cos\varphi d\varphi}{\pi\varepsilon_0} = \frac{\rho_S \sin\varphi}{\pi\varepsilon_0}\Big|_0^{\pi/2} = \frac{\rho_S}{\pi\varepsilon_0}$$

From the sketch we can use that $\bar{E}$ is parallell with a axis in opposite direction

$$\Rightarrow \boxed{\bar{E} = \frac{\rho_S}{\pi\varepsilon_0}(-\bar{a}_x)}$$

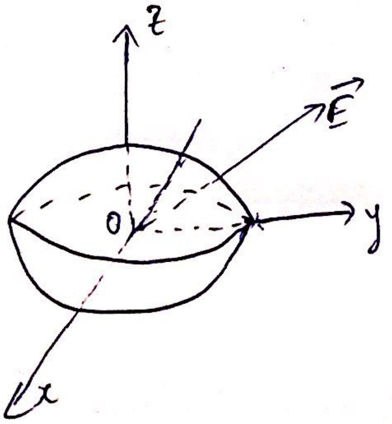
b, We have: $dq = \rho_S ds = \rho_V dv$

$$\Leftrightarrow \rho_S \rho d\varphi dz = \rho_V \rho d\rho d\varphi dz$$

$$\Rightarrow \rho_S = \rho_V d\rho$$

$$dE = \frac{\rho_V d\rho}{\pi\varepsilon_0} \Rightarrow \boxed{E = \int_0^R \frac{\rho_V d\rho}{\pi\varepsilon_0} = \frac{R\rho_V}{\pi\varepsilon_0}} \Rightarrow \boxed{\bar{E} = (-\bar{a}_x)\frac{R\rho_V}{\pi\varepsilon_0}}$$

Q2



+ For the sphere of radius R, the EFI of the surface will be: $\vec{E} = Ar\bar{a}_r = AR\bar{a}_r \quad (r = R)$

+ Apply gauss law: $Q = \oint_{sphere} \bar{D}_S \, d\bar{s} \Rightarrow \frac{Q}{\varepsilon_0} = \oint_{sphere} \bar{E}_S \, d\bar{s}$

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+ Since sphere has $\overline{E_s}$ is everywhere normal to the closed surface

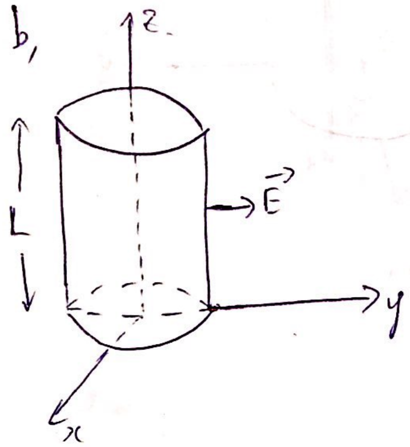
$\Rightarrow \vec{E_s}\, d\bar{s} = E_s ds$

$\Rightarrow Q = \varepsilon_0 \oint_{sphere} \overline{E_s}\, d\bar{s} = \varepsilon_0 E \oint_{sphere} ds$

$= \varepsilon_0 E 4\pi R^2$

$\Rightarrow \boxed{Q = \varepsilon_0 4\pi A R^3}$

b,



Apply gauss's law:

$Q = \int_S \bar{D}\, d\bar{s} = \varepsilon_0 \int_S \bar{E}\, d\bar{s}$

$= \varepsilon_0 \int_{bottom} \bar{E}\, d\bar{s} + \varepsilon_0 \int_{side} \bar{E}\, d\bar{s} + \varepsilon_0 \int_{top} \bar{E}\, d\bar{s}$

+ $Q_{bottom} = \varepsilon_0 \int_{bottom} \bar{E}\, d\bar{s} = \varepsilon_0 \int_{bot} A\rho^2 (\bar{a}_\rho)\, \rho\, d\rho\, d\varphi\, (-\bar{a}_z) = 0$ ①

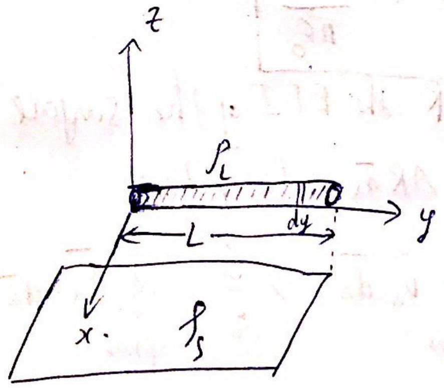
$(\bar{a}_\rho \cdot \bar{a}_z = 0)$

+ $Q_{top} = \varepsilon_0 \int_{top} \bar{E}\, d\bar{s} = \varepsilon_0 \int_{top} A\rho^2 (\bar{a}_\rho)\, \rho\, d\rho\, d\varphi\, (\bar{a}_z) = 0 \quad (\bar{a}_\rho \cdot \bar{a}_z = 0)$ ②

+ $Q_{side} = \varepsilon_0 \int_{side} A\rho^2 (\bar{a}_\rho)\, \rho\, d\varphi\, dz\, (\bar{a}_\rho) = \varepsilon_0 \int_0^{2\pi} \int_0^{L} A\rho^3 dz\, d\varphi = \varepsilon_0 \int_0^{2\pi} A\rho^3 L\, d\varphi = \varepsilon_0 2\pi L A\rho^3$ ③

①②③ $\Rightarrow \boxed{Q = \varepsilon_0 2\pi L A R^3}$

$\boxed{Q_3}$



+ The electric field due to a infinite sheet with surface charge density ρ_s is:

$\bar{E} = \frac{\rho_s}{2\varepsilon_0} \bar{a}_z$

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We divide the line charge into many small pieces of charge dq, length dy

$\Rightarrow dq = \rho_L dy$

We have work needed to rotate the pieces of charge dq is:

$$\begin{cases} dW = -dq \int_a^b \bar{E}\, d\bar{L} \\ d\bar{L} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z \\ \bar{E} = \dfrac{\rho_s}{2\varepsilon_0} \bar{a}_z \end{cases}$$

$$\Rightarrow dW = -dq \int_a^b \frac{\rho_s}{2\varepsilon_0} dz = -dq \int_0^y \frac{\rho_s}{2\varepsilon_0} dz$$

$$= \frac{-\rho_L . dy . y\, \rho_s}{2\varepsilon_0} \quad (dq = dy\, \rho_L)$$

$$\Rightarrow dW = \frac{-\rho_L \rho_s\, y\, dy}{2\varepsilon_0}$$

$$\Rightarrow W = \int_0^L \frac{-\rho_s \rho_L\, y\, dy}{2\varepsilon_0} = \boxed{\frac{-\rho_L \rho_s L^2}{4\varepsilon_0}}$$