Nguyễn Công Phương

Engineering Electromagnetics

The Uniform Plane Wave

Contents

The Uniform Plane Wave

1. Wave Propagation in Free Space
2. Wave Propagation in Dielectrics
3. The Poynting Vector
4. Skin Effect
5. Wave Polarization

Wave Propagation in Free Space (1)

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla . \mathbf{E} = 0$$

$$\nabla . \mathbf{H} = 0$$

Wave Propagation in Free Space (2)

$$\mathbf{E} = E_x \mathbf{a}_x$$

$$\left.\begin{aligned} E_x &= E(x,y,z)\cos(\omega t + \varphi) \\ e^{j\omega t} &= \cos\omega t + j\sin\omega t \end{aligned}\right\}$$

$$\to E_x = \mathrm{Re}\left[E(x,y,z)e^{j(\omega t+\varphi)}\right] = \mathrm{Re}\left[E(x,y,z)e^{j\varphi}e^{j\omega t}\right]$$

$$E_{xs} = E(x,y,z)e^{j\varphi}$$

$$\mathbf{E}_s = E_{xs}\mathbf{a}_x$$

$$E_x = \mathrm{Re}\left[E_{xs}e^{j\omega t}\right]$$

Wave Propagation in Free Space (3)

Ex. 1

Find the time – varying function of the vector field:

$$\mathbf{E}_s = 100\underline{/30^o}\,\mathbf{a}_x + 20\underline{/-50^o}\,\mathbf{a}_y + 40\underline{/210^o}\,\mathbf{a}_z \text{ V/m}$$

If f = 1 MHz

$$\mathbf{E}_s = 100e^{j30^o}\mathbf{a}_x + 20e^{-j50^o}\mathbf{a}_y + 40e^{j210^o}\mathbf{a}_z \text{ V/m}$$

$$\rightarrow \mathbf{E}_s(t) = \left(100e^{j30^o}\mathbf{a}_x + 20e^{-j50^o}\mathbf{a}_y + 40e^{j210^o}\mathbf{a}_z\right)e^{j2\pi10^6 t}$$

$$= 100e^{j(2\pi10^6 t+30^o)}\mathbf{a}_x + 20e^{j(2\pi10^6 t-50^o)}\mathbf{a}_y + 40e^{j(2\pi10^6 t+210^o)}\mathbf{a}_z$$

$$\rightarrow \mathbf{E}(t) = \mathrm{Re}[\mathbf{E}_s(t)] = 100\cos(2\pi10^6 t + 30^o)\mathbf{a}_x +$$

$$+ 20\cos(2\pi10^6 t - 50^o)\mathbf{a}_y + 40\cos(2\pi10^6 t + 210^o)\mathbf{a}_z$$

Wave Propagation in Free Space (3)

$$E_x = E(x,y,z)\cos(\omega t + \varphi)$$

$$\rightarrow \frac{\partial E_x}{\partial t} = \frac{\partial}{\partial t}\left[E(x,y,z)\cos(\omega t+\varphi)\right] = \underline{-\omega E(x,y,z)\sin(\omega t+\varphi)}$$

$$\begin{aligned}
\mathrm{Re}\left[j\omega E_{xs}e^{j\omega t}\right] &= \mathrm{Re}\left\{j\omega\left[E(x,y,z)e^{j\omega t}\right]e^{j\varphi}\right\} \\
&= \mathrm{Re}\left[j\omega E(x,y,z)e^{j(\omega t+\varphi)}\right] \\
&= \mathrm{Re}\left\{\omega E(x,y,z)\,j\left[\cos(\omega t+\varphi)+j\sin(\omega t+\varphi)\right]\right\} \\
&= \mathrm{Re}\left\{\omega E(x,y,z)\left[j\cos(\omega t+\varphi)-\sin(\omega t+\varphi)\right]\right\} \\
&= \underline{-\omega E(x,y,z)\sin(\omega t+\varphi)}
\end{aligned}$$

$$\rightarrow \frac{\partial E_x}{\partial t} = \mathrm{Re}\left[j\omega E_{xs}e^{j\omega t}\right]$$

Wave Propagation in Free Space (4)

$$E_x = E(x, y, z)\cos(\omega t + \varphi)$$

$$\frac{\partial E_x}{\partial t} = \mathrm{Re}\left[j\omega E_{xs} e^{j\omega t} \right] \leftrightarrow j\omega E_{xs}$$

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla . \mathbf{E} = 0$$

$$\nabla . \mathbf{H} = 0$$

$$\Rightarrow$$

$$\nabla \times \mathbf{H}_s = j\omega\varepsilon_0 \mathbf{E}_s$$

$$\nabla \times \mathbf{E}_s = -j\omega\mu_0 \mathbf{H}_s$$

$$\nabla . \mathbf{E}_s = 0$$

$$\nabla . \mathbf{H}_s = 0$$

Wave Propagation in Free Space (5)

$$\left.\begin{array}{r}\nabla\times\mathbf{E}_s = -j\omega\mu_0\mathbf{H}_s \to \nabla\times\nabla\times\mathbf{E}_s = \nabla\times(-j\omega\mu_0\mathbf{H}_s) = -j\omega\mu_0\nabla\times\mathbf{H}_s \\ \nabla\times\mathbf{H}_s = j\omega\varepsilon_0\mathbf{E}_s\end{array}\right\}$$

$$\left.\begin{array}{r}\to \nabla\times\nabla\times\mathbf{E}_s = \omega^2\mu_0\varepsilon_0\mathbf{E}_s \\ \nabla\times\nabla\times\mathbf{E}_s = \nabla(\nabla.\mathbf{E}_s) - \nabla^2\mathbf{E}_s \\ \nabla.\mathbf{E}_s = 0 \to \nabla(\nabla.\mathbf{E}_s) = 0\end{array}\right\}$$

$$\to \boxed{\nabla^2\mathbf{E}_s = -k_0^2\mathbf{E}_s}$$

$$\downarrow \quad k_0 = \omega\sqrt{\mu_0\varepsilon_0} \text{ (wavenumber)}$$

$$\nabla^2 E_{xs} = -k_0^2 E_{xs}$$

$$\left.\begin{array}{l}\to \dfrac{\partial^2 E_{xs}}{\partial x^2} + \dfrac{\partial^2 E_{xs}}{\partial y^2} + \dfrac{\partial^2 E_{xs}}{\partial z^2} = -k_0^2 E_{xs} \\ \text{Suppose } E_{xs} \text{ does not vary with } x \text{ or } y\end{array}\right\} \to \frac{d^2 E_{xs}}{dz^2} = -k_0^2 E_{xs}$$

Wave Propagation in Free Space (6)

$$\frac{d^2E_{xs}}{dz^2} = -k_0^2 E_{xs}$$

$$\rightarrow E_{xs} = E_{x0}e^{-jk_0z} \quad \rightarrow \left.\begin{aligned} E_x(z,t) &= E_{x0}\cos(\omega t - k_0 z) \\ E'_x(z,t) &= E'_{x0}\cos(\omega t + k_0 z) \end{aligned}\right\}$$

$$\left.\begin{aligned} k_0 &= \omega\sqrt{\mu_0\varepsilon_0} \\ \frac{1}{\sqrt{\mu_0\varepsilon_0}} &= 2.998\times10^8 \approx 3\times10^8 \text{ m/s} \end{aligned}\right\} \rightarrow k_0 = \frac{\omega}{c}$$

$$\rightarrow \begin{cases} E_x(z,t) = E_{x0}\cos[\omega(t - z/c)] \\ E'_x(z,t) = E'_{x0}\cos[\omega(t + z/c)] \end{cases}$$

Wave Propagation in Free Space (7)

$$\begin{cases} \boxed{E_x(z,t) = E_{x0}\cos[\omega(t - z/c)]} \\ E'_x(z,t) = E'_{x0}\cos[\omega(t + z/c)] \end{cases}$$

$$\left.\begin{aligned} \nabla \times \mathbf{E}_s = -j\omega\mu_0 \mathbf{H}_s \rightarrow \frac{dE_{xs}}{dz} &= -j\omega\mu_0 H_{ys} \\ E_{xs} &= E_{x0}e^{-jk_0 z} \end{aligned}\right\}$$

$$\rightarrow H_{ys} = -\frac{1}{j\omega\mu_0}(-jk_0)E_{x0}e^{-jk_0 z} = E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}e^{-jk_0 z}$$

$$\left.\begin{aligned} &\rightarrow \boxed{H_y(z,t) = E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t - k_0 z)} \\ &E_x(z,t) = E_{x0}\cos[\omega(t - z/c)] \end{aligned}\right\} \rightarrow \frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Wave Propagation in Free Space (8)

Ex. 2

Given $\mathbf{H} = H_m\cos(\omega t + \beta z)\mathbf{a}_x$ in free space, find $\mathbf{E}$?

Method 1

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\rightarrow \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\rightarrow \frac{\partial}{\partial z} H_m \cos(\omega t + \beta z)\mathbf{a}_y = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\rightarrow -\beta H_m \sin(\omega t + \beta z)\mathbf{a}_y = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\rightarrow \mathbf{E} = -\int \frac{\beta}{\varepsilon_0} H_m \sin(\omega t + \beta z)\mathbf{a}_y = \boxed{\frac{\beta}{\varepsilon_0 \omega} H_m \cos(\omega t + \beta z)\mathbf{a}_y}$$

Wave Propagation in Free Space (9)

Ex. 2

Given $\mathbf{H} = H_m\cos(\omega t + \beta z)\mathbf{a}_x$ in free space, find $\mathbf{E}$?

Method 2

$$\boxed{\mathbf{E} = \frac{\beta}{\varepsilon_0\omega} H_m \cos(\omega t + \beta z)\mathbf{a}_y}$$

$$\mathbf{H}_s = H_m e^{j\beta z}\mathbf{a}_x = H_{xs}\mathbf{a}_x$$

$$\nabla\times\mathbf{H}_s = j\omega\varepsilon_0\mathbf{E}_s$$

$$\rightarrow \left(\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y}\right)\mathbf{a}_z = j\omega\varepsilon_0\mathbf{E}_s$$

$$\rightarrow j\beta H_m e^{j\beta z}\mathbf{a}_y = j\omega\varepsilon_0\mathbf{E}_s$$

$$\rightarrow \mathbf{E}_s = \frac{\beta H_m}{\varepsilon_0\omega} e^{j\beta z}\mathbf{a}_y$$

$$\rightarrow \mathbf{E} = \mathrm{Re}\left[\mathbf{E}_s e^{j\omega t}\right] = \mathrm{Re}\left[\frac{\beta H_m}{\varepsilon_0\omega} e^{j(\omega t + \beta z)}\mathbf{a}_y\right] = \boxed{\frac{\beta H_m}{\varepsilon_0\omega}\cos(\omega t + \beta z)\mathbf{a}_y}$$

Wave Propagation in Free Space (10)

Ex. 3

Given a uniform plane wave whose **E** = 100cos(ωt + 6z)$\mathbf{a}_x$ V/m in free space, Find the wave frequency, the wavelength, & **H**?

$$E_x(z,t) = E_{x0}\cos(\omega t - k_0 z),\ \ k_0 = \frac{\omega}{c} \qquad \frac{E_x}{H_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\omega = k_0 c = 6\times 3\times 10^8 = 1.8\times 10^9 \text{ rad/s}$$

$$\lambda = \frac{c}{f} = \frac{c}{\omega/2\pi} = \frac{2\pi}{k_0} = \frac{2\pi}{6} = 1.047 \text{ m}$$

$$\mathbf{E}_s = 100e^{j6z}\mathbf{a}_x = E_{xs}\mathbf{a}_x$$

$$\nabla\times\mathbf{E}_s = -j\omega\mu_0\mathbf{H}_s \rightarrow \frac{\partial E_{xs}}{\partial z}\mathbf{a}_y = j600e^{j6z}\mathbf{a}_y = -j\omega\mu_0\mathbf{H}_s$$

$$\rightarrow \mathbf{H}_s = \frac{j600e^{j6z}}{-j\omega\mu_0}\mathbf{a}_y = -\frac{600e^{j6z}}{(1.8\times 10^9)(4\pi\times 10^{-7})}\mathbf{a}_y = -0.2653e^{j6z}\mathbf{a}_y \text{ A/m}$$

$$\mathbf{H} = \text{Re}\left[\mathbf{H}_s e^{j\omega t}\right] = \text{Re}\left[-0.2653e^{j(1.8\times 10^8 t + 6z)}\mathbf{a}_y\right] = -0.2653\cos(1.8\times 10^8 t + 6z)\mathbf{a}_y \text{ A/m}$$

The Uniform Plane Wave

1. Wave Propagation in Free Space
2. **Wave Propagation in Dielectrics**
3. The Poynting Vector
4. Skin Effect
5. Wave Polarization

Wave Propagation in Dielectrics (1)

$$\nabla^2 \mathbf{E}_s = -k_0^2 \mathbf{E}_s \rightarrow \nabla^2 \mathbf{E}_s = -k^2 \mathbf{E}_s$$

$$k = \omega\sqrt{\mu\varepsilon} = k_0\sqrt{\mu_r \varepsilon_r}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}, \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \approx 120\pi\ \Omega$$

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs}$$

$$jk = \alpha + j\beta$$

$$E_{xs} = E_{x0} e^{-jkz} = E_{x0} e^{-\alpha z} e^{-j\beta z}$$

$$\rightarrow E_x = \mathrm{Re}\left[E_{xs} e^{j\omega t} \right] = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)$$

Wave Propagation in Dielectrics (2)

$$\left.\begin{aligned} \varepsilon &= \varepsilon' - j\varepsilon'' = \varepsilon_0(\varepsilon'_r - j\varepsilon''_r) \\ k &= \omega\sqrt{\mu\varepsilon} = k_0\sqrt{\mu_r\varepsilon_r} \end{aligned}\right\}$$

$$\rightarrow k = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}}$$

$$\mu = \mu' - j\mu'' = \mu_0(\mu'_r - j\mu''_r)$$

$$\alpha = \mathrm{Re}[jk] = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2}$$

$$\beta = \mathrm{Im}[jk] = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2}$$

Wave Propagation in Dielectrics (3)

$$E_x = E_{x0}e^{-\alpha z}\cos(\omega t - \beta z) \to v_p = \frac{\omega}{\beta}$$

$$\beta\lambda = 2\pi \to \lambda = \frac{2\pi}{\beta}$$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \to H_{ys} = \frac{E_{x0}}{\eta}e^{-\alpha z}e^{-j\beta z}$$

$$\left.\begin{array}{l} \alpha = \mathrm{Re}[jk] = \omega\sqrt{\dfrac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2} \\ \beta = \mathrm{Im}[jk] = \omega\sqrt{\dfrac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2} \\ \varepsilon'' = 0 \end{array}\right\} \to \left\{\begin{array}{l} \alpha = 0 \\ \beta = \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu\varepsilon} \\ v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{\sqrt{\mu_r\varepsilon_r}} \\ \lambda = \dfrac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \dfrac{1}{f\sqrt{\mu\varepsilon}} = \dfrac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}} \end{array}\right.$$

Wave Propagation in Dielectrics (4)

$$\left.\begin{aligned} &\alpha = 0 \\ &\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \end{aligned}\right\} \rightarrow \begin{cases} E_x = E_{x0}\cos(\omega t - \beta z) \\ H_y = \dfrac{E_{x0}}{\eta}\cos(\omega t - \beta z) \end{cases}$$

Wave Propagation in Dielectrics (5)

Ex. 1

Find the attenuation of a 2.5 GHz wave propagating in fresh water, given $\varepsilon'_r = 78$, $\varepsilon''_r = 7$, $\mu_r = 1$.

$$\left.\begin{aligned} \alpha &= \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right)^{1/2} \\ \omega\sqrt{\mu\varepsilon'} &= k_0\sqrt{\mu_r\varepsilon'_r} \\ k_0 &= \omega / c \end{aligned}\right\}$$

$$\rightarrow \alpha = \frac{2\pi\times 2.5\times 10^9}{3\times 10^8}\sqrt{\frac{78}{2}}\left(\sqrt{1+\left(\frac{7}{78}\right)^2}-1\right)^{1/2} = 21 \text{ Np/m} \quad \rightarrow \frac{1}{\alpha} \approx 4.8 \text{ cm}$$

Wave Propagation in Dielectrics (6)

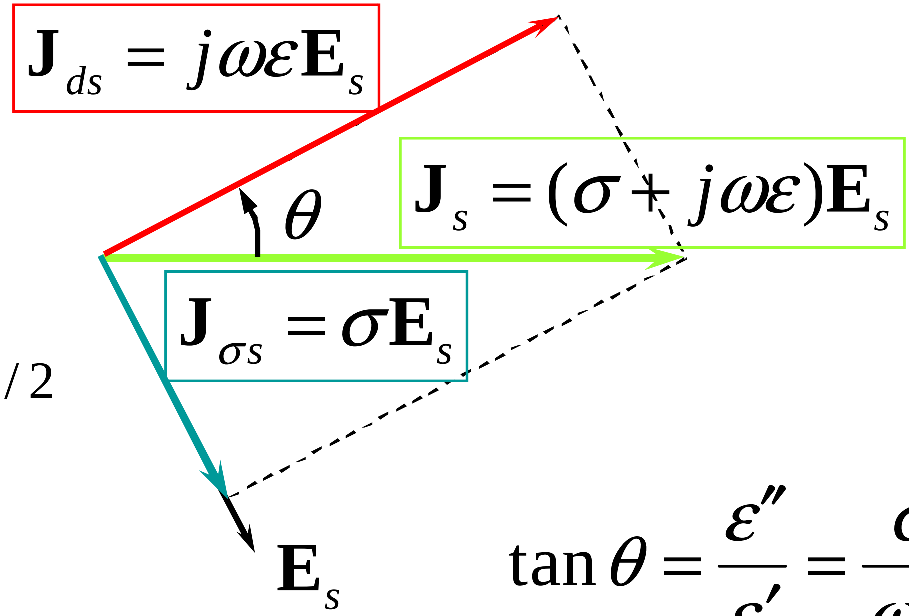
$$\mathbf{J} = \sigma \mathbf{E}$$

$$\left.\begin{aligned} \nabla \times \mathbf{H}_s &= j\omega\varepsilon \mathbf{E}_s \\ \varepsilon &= \varepsilon' - j\varepsilon'' \end{aligned}\right\}$$

$$\left.\begin{aligned} \rightarrow \nabla \times \mathbf{H}_s &= j\omega(\varepsilon' - j\varepsilon'')\mathbf{E}_s = \omega\varepsilon''\mathbf{E}_s + j\omega\varepsilon'\mathbf{E}_s \\ \nabla \times \mathbf{H}_s &= \mathbf{J}_s + j\omega\varepsilon\mathbf{E}_s \\ \rightarrow \nabla \times \mathbf{H}_s &= (\sigma + j\omega\varepsilon')\mathbf{E}_s = \mathbf{J}_{\sigma s} + \mathbf{J}_{ds} \end{aligned}\right\}$$

$$\rightarrow \begin{cases} \mathbf{J}_{\sigma s} = \sigma \mathbf{E}_s, \quad \mathbf{J}_{ds} = j\omega\varepsilon'\mathbf{E}_s \\ \varepsilon'' = \dfrac{\sigma}{\omega} \end{cases}$$

Wave Propagation in Dielectrics (7)



$$\alpha = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right)^{1/2}$$

$$\tan\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'}$$

$$\varepsilon'' = \frac{\sigma}{\omega} \rightarrow \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'}$$

$$\mathbf{J}_{\sigma s} = \sigma\mathbf{E}_s, \quad \mathbf{J}_{ds} = j\omega\varepsilon'\mathbf{E}_s \rightarrow \frac{J_{\sigma s}}{J_{ds}} = \frac{\varepsilon''}{j\varepsilon'} = \frac{\sigma}{j\omega\varepsilon'}$$

Wave Propagation in Dielectrics (8)

$$\text{Good dielectrics : } \frac{\varepsilon''}{\varepsilon'} \ll 1$$

$$\text{Conductor: } \varepsilon'' = \frac{\sigma}{\omega}$$

$$\left.\begin{array}{l} \rightarrow jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\dfrac{\varepsilon''}{\varepsilon'}} = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\dfrac{\sigma}{\omega\varepsilon'}} \\ (1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \ldots \end{array}\right\}$$

$$\rightarrow jk = j\omega\sqrt{\mu\varepsilon'}\left[1 - j\frac{\sigma}{2\omega\varepsilon'} + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2 + \ldots\right] = \alpha + j\beta$$

$$\rightarrow \begin{cases} \alpha = \mathrm{Re}[jk] \approx j\omega\sqrt{\mu\varepsilon'}\left(-j\dfrac{\sigma}{2\omega\varepsilon'}\right) = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon'}} \\ \beta = \mathrm{Im}[jk] \approx \omega\sqrt{\mu\varepsilon'}\left[1 + \dfrac{1}{8}\left(\dfrac{\sigma}{\omega\varepsilon'}\right)^2\right] \approx \omega\sqrt{\mu\varepsilon'} \end{cases}$$

Wave Propagation in Dielectrics (9)

$$\begin{cases} \alpha \approx j\omega\sqrt{\mu\varepsilon'}\left(-j\dfrac{\sigma}{2\omega\varepsilon'}\right) = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon'}} \\ \beta \approx \omega\sqrt{\mu\varepsilon'}\left[1+\dfrac{1}{8}\left(\dfrac{\sigma}{\omega\varepsilon'}\right)^2\right] \approx \omega\sqrt{\mu\varepsilon'} \end{cases}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu}{\varepsilon'}}\frac{1}{\sqrt{1 - j(\varepsilon''/\varepsilon')}}$$

$$\rightarrow \eta \approx \sqrt{\frac{\mu}{\varepsilon'}}\left[1 - \frac{3}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2 + j\frac{\sigma}{2\omega\varepsilon'}\right] \approx \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right)$$

Ex. 2 Wave Propagation in Dielectrics (10)

E = 377cos(10^9t – 5*y*)$\mathbf{a}_z$ V/m represents a uniform plane wave propagating in the *y* direction in a dielectric medium ($\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$), find the relative permittivity, the speed of propagation, the intrinsic impedance, the wavelength, & the magnetic field intensity?

$$\left.\begin{aligned} \nabla^2\mathbf{E}_s &= -k^2\mathbf{E}_s \\ \mathbf{E}_s &= 377e^{-j5y}\mathbf{a}_z \\ \nabla^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \end{aligned}\right\} \rightarrow (-j5)^2 377e^{-j5y}\mathbf{a}_z = -k^2 377e^{-j5y}\mathbf{a}_z$$

$$\left.\begin{aligned} &\rightarrow k = 5 \text{ rad/ m} \\ &k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_r\varepsilon_0} \end{aligned}\right\}$$

$$\rightarrow \varepsilon_r = \frac{k^2}{\omega^2\mu_0\varepsilon_0} = \frac{5^2}{(10^9)^2(4\pi\times10^{-7})(8.854\times10^{-12})} = 2.2469$$

$$v_P = \frac{\omega}{\beta} = \frac{10^9}{5} = 2\times10^8 \text{ m/ s}$$

Ex. 2 Wave Propagation in Dielectrics (11)

E = 377cos(10^9t – 5y)$\mathbf{a}_z$ V/m represents a uniform plane wave propagating in the y direction in a dielectric medium ($\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$), find the relative permittivity, the speed of propagation, the intrinsic impedance, the wavelength, & the magnetic field intensity?

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}} = \sqrt{\frac{4\pi\times10^{-7}}{2.2469\times8.854\times10^{-12}}} = 251.33\ \Omega$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5} = 1.257\ \text{m}$$

$$\left.\begin{matrix} \begin{cases} E_x = E_{x0}\cos(\omega t - \beta z) \\ H_y = \dfrac{E_{x0}}{\eta}\cos(\omega t - \beta z) \end{cases} \\ E_z = 377\cos(10^9 t - 5y) \end{matrix}\right\}$$

$$\rightarrow H_x = \frac{377}{251.33}\cos(10^9 t - 5y) \rightarrow \mathbf{H} = 1.5\cos(10^9 t - 5y)\mathbf{a}_x\ \text{A/m}$$

Ex. 3

Wave Propagation in Dielectrics (12)

A 2-GHz wave propagates in a medium whose $\mu_r = 1.6$, $\varepsilon_r = 25$, $\sigma = 2.5$ S/m. The EFI in this region is $\mathbf{E} = 0.1e^{-\alpha z}\cos(2\pi ft - \beta z)\mathbf{a}_x$ V/m. Find the propagation constant, the attenuation constant, the phase constant, the intrinsic impedance, the phase speed, the wavelength, and the MFI?

$$\left.\begin{aligned} \varepsilon &= \varepsilon' - j\varepsilon'' \\ \varepsilon'' &= \frac{\sigma}{\omega} \end{aligned}\right\} \to \varepsilon = \varepsilon_r\varepsilon_0 - j\frac{\sigma}{2\pi f} = 25\times 8.854\times 10^{-12} - j\frac{2.5}{2\pi\times 2\times 10^9}$$

$$= 2.2135\times 10^{-10} - j1.9894\times 10^{-10} \ \text{F/m}$$

$$jk = j\omega\sqrt{\mu\varepsilon} = j\omega\sqrt{\mu_r\mu_0}\sqrt{\varepsilon' - j\varepsilon''}$$

$$= j2\pi\times 2\times 10^9\sqrt{1.6\times 4\pi\times 10^{-7}}\sqrt{2.2135\times 10^{-10} - j1.9894\times 10^{-10}}$$

$$= j1.7819\times 10^7\sqrt{2.9761\times 10^{-10}\angle -41.9^\circ}$$

$$= 1.7819\times 10^7\angle 90^\circ \times 1.7251\times 10^{-5}\angle -20.1^\circ$$

$$= 307.40\angle 69.0^\circ = 110.03 + j287.04 \ \ 1/\text{m}$$

Wave Propagation in Dielectrics (13)

Ex. 3

A 2-GHz wave propagates in a medium whose $\mu_r = 1.6$, $\varepsilon_r = 25$, $\sigma = 2.5$ S/m. The EFI in this region is $\mathbf{E} = 0.1e^{-\alpha z}\cos(2\pi ft - \beta z)\mathbf{a}_x$ V/m. Find the propagation constant, the attenuation constant, the phase constant, the intrinsic impedance, the phase speed, the wavelength, and the MFI?

$$jk = 307.40\angle 69.0^o = 110.03 + j287.04 \quad 1/\text{m}$$

$$\alpha = \text{Re}[jk] = 110.03 \text{ Np/m}$$

$$\beta = \text{Im}[jk] = 287.04 \text{ rad/m}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r\mu_0}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{1.6\times 4\pi\times 10^{-7}}{2.2135\times 10^{-10} - j1.9894\times 10^{-10}}}$$

$$= \sqrt{\frac{1.6\times 4\pi\times 10^{-7}}{2.9761\times 10^{-10}\angle -41.9^o}} = \sqrt{6.7558\times 10^3\angle 41.9^o} = 82.19\angle 21.0^o\ \Omega$$

Wave Propagation in Dielectrics (14)

Ex. 3

A 2-GHz wave propagates in a medium whose $\mu_r = 1.6$, $\varepsilon_r = 25$, $\sigma = 2.5$ S/m. The EFI in this region is $\mathbf{E} = 0.1e^{-\alpha z}\cos(2\pi ft - \beta z)\mathbf{a}_x$ V/m. Find the propagation constant, the attenuation constant, the phase constant, the intrinsic impedance, the phase speed, the wavelength, and the MFI?

$$jk = 307.40\underline{/\ 69.0^{o}} = 110.03 + j287.04 \quad 1/\mathrm{m}$$

$$\alpha = \mathrm{Re}[jk] = 110.03 \text{ Np/m}$$

$$\beta = \mathrm{Im}[jk] = 287.04 \text{ rad/m}$$

$$v_P = \frac{\omega}{\beta} = \frac{2\pi\times 2\times 10^{9}}{287.04} = 4.38\times 10^{7} \text{ m/s}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{287.04} = 0.0218 \text{ m}$$

$$\frac{1}{\alpha} = \frac{1}{110.03} = 0.0091 \text{ m}$$

Ex. 3 Wave Propagation in Dielectrics (15)

A 2-GHz wave propagates in a medium whose $\mu_r = 1.6$, $\varepsilon_r = 25$, $\sigma = 2.5$ S/m. The EFI in this region is $\mathbf{E} = 0.1e^{-\alpha z}\cos(2\pi ft - \beta z)\mathbf{a}_x$ V/m. Find the propagation constant, the attenuation constant, the phase constant, the intrinsic impedance, the phase speed, the wavelength, and the MFI?

$$\alpha = 110.03 \text{ Np/m}; \ \beta = \text{Im}[jk] = 287.04 \text{ rad/m}; \ \eta = 82.19\underline{/21.0^\circ} \ \Omega$$

$$\left.\begin{aligned} &\frac{E_x}{H_y} = \eta \rightarrow \frac{E_{xs}}{H_{ys}} = \eta \\ &E_{xs} = 0.1e^{-\alpha z}e^{-j\beta z} = 0.1e^{-110.03z}e^{-j287.04z} \end{aligned}\right\}$$

$$\rightarrow H_{ys} = \frac{E_{xs}}{\eta} = \frac{0.1e^{-110.03z}e^{-j287.04z}}{82.19e^{j21.0^\circ}} = 0.0012e^{-110.03z}e^{-j287.04z}e^{-j21.0^\circ}$$

$$\rightarrow \mathbf{H} = 1.2e^{-110.03z}\cos(4\pi\times10^9 t - 287.04z - 21.0^\circ)\mathbf{a}_y \ \text{ mA/m}$$

The Uniform Plane Wave

1. Wave Propagation in Free Space
2. Wave Propagation in Dielectrics
3. **The Poynting Vector**
4. Skin Effect
5. Wave Polarization

The Poynting Vector (1)

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\left.\begin{aligned} &\rightarrow \mathbf{E}.\nabla \times \mathbf{H} = \mathbf{E}.\mathbf{J} + \mathbf{E}.\frac{\partial \mathbf{D}}{\partial t} \\ &\nabla.(\mathbf{E} \times \mathbf{H}) = -\mathbf{E}.\nabla \times \mathbf{H} + \mathbf{H}.\nabla \times \mathbf{E} \end{aligned}\right\}$$

$$\left.\begin{aligned} &\rightarrow \mathbf{H}.\nabla \times \mathbf{E} - \nabla.(\mathbf{E} \times \mathbf{H}) = \mathbf{J}.\mathbf{E} + \mathbf{E}.\frac{\partial \mathbf{D}}{\partial t} \\ &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \end{aligned}\right\}$$

$$\rightarrow -\mathbf{H}.\frac{\partial \mathbf{B}}{\partial t} - \nabla.(\mathbf{E} \times \mathbf{H}) = \mathbf{E}.\mathbf{J} + \mathbf{E}.\frac{\partial \mathbf{D}}{\partial t}$$

$$\rightarrow -\nabla.(\mathbf{E} \times \mathbf{H}) = \mathbf{J}.\mathbf{E} + \varepsilon\mathbf{E}.\frac{\partial \mathbf{E}}{\partial t} + \mu\mathbf{H}.\frac{\partial \mathbf{H}}{\partial t}$$

The Poynting Vector (2)

$$\left.\begin{aligned}
-\nabla.(\mathbf{E}\times\mathbf{H}) &= \mathbf{J.E} + \varepsilon\mathbf{E}.\frac{\partial \mathbf{E}}{\partial t} + \mu\mathbf{H}.\frac{\partial \mathbf{H}}{\partial t} \\
\varepsilon\mathbf{E}.\frac{\partial \mathbf{E}}{\partial t} &= \frac{\partial}{\partial t}\left(\frac{\mathbf{D.E}}{2}\right) \\
\mu\mathbf{H}.\frac{\partial \mathbf{H}}{\partial t} &= \frac{\partial}{\partial t}\left(\frac{\mathbf{B.H}}{2}\right)
\end{aligned}\right\}$$

$$\rightarrow \nabla.(\mathbf{E}\times\mathbf{H}) = \mathbf{J.E} + \frac{\partial}{\partial t}\left(\frac{\mathbf{D.E}}{2}\right) + \frac{\partial}{\partial t}\left(\frac{\mathbf{B.H}}{2}\right)$$

$$\left.\begin{aligned}
&\rightarrow -\int_V \nabla.(\mathbf{E}\times\mathbf{H})dv = \int_V \mathbf{J.E}dv + \int_V \frac{\partial}{\partial t}\left(\frac{\mathbf{D.E}}{2}\right)dv + \int_V \frac{\partial}{\partial t}\left(\frac{\mathbf{B.H}}{2}\right)dv \\
&\oint_S \mathbf{D}.d\mathbf{S} = \int_V \nabla.\mathbf{D}dv
\end{aligned}\right\}$$

$$\rightarrow \boxed{-\oint_S (\mathbf{E}\times\mathbf{H}).d\mathbf{S} = \int_V \mathbf{J.E}dv + \frac{d}{dt}\int_V \frac{1}{2}\mathbf{D.E}dv + \frac{d}{dt}\int_V \frac{1}{2}\mathbf{B.H}dv}$$

The Poynting Vector (3)

$$\oint_S (\mathbf{E}\times\mathbf{H}).d\mathbf{S} = \int_V \mathbf{J.E}dv + \frac{\partial}{\partial t}\int_V \left(\frac{\varepsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv$$

$$\boxed{\mathbf{S} = \mathbf{E}\times\mathbf{H} \quad \mathrm{W/m^2}}$$

$$E_x\mathbf{a}_x \times H_y\mathbf{a}_y = S_z\mathbf{a}_z$$

$$\left.\begin{aligned} E_x &= E_{x0}\cos(\omega t - \beta z) \\ H_y &= \frac{E_{x0}}{\eta}\cos(\omega t - \beta z) \end{aligned}\right\} \rightarrow S_z = \frac{E_{x0}^2}{\eta}\cos^2(\omega t - \beta z)$$

$$S_{z,av} = \frac{1}{T}\int_0^T \frac{E_{x0}^2}{\eta}\cos^2(\omega t - \beta z)dt \quad = \frac{1}{2T}\frac{E_{x0}^2}{\eta}\int_0^T [1 + \cos(2\omega t - \beta z)]dt$$

$$= \frac{1}{2T}\frac{E_{x0}^2}{\eta}\left[1 + \frac{1}{2\omega}\sin(2\omega t - 2\beta z)\right]\Bigg|_0^T = \frac{1}{2}\frac{E_{x0}^2}{\eta} \quad \mathrm{W/\,m^2}$$

The Poynting Vector (4)

$$\left.\begin{array}{l} E_x = E_{x0} e^{-\alpha z}\cos(\omega t - \beta z) \\ \eta = |\eta| \underline{/\theta_\eta} \end{array}\right\} \rightarrow H_y = \frac{E_{x0}}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)$$

$$\rightarrow S_z = E_x H_y = \frac{E_{x0}^2}{|\eta|} e^{-2\alpha z}\cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta_\eta)$$

$$= \frac{E_{x0}^2}{2|\eta|} e^{-2\alpha z}[\cos(2\omega t - 2\beta z - 2\theta_\eta) + \cos\theta_\eta]$$

$$\rightarrow S_{z,\,av} = \frac{1}{2}\frac{E_{x0}^2}{\eta} e^{-2\alpha z}\cos\theta_\eta \quad = \boxed{\frac{1}{2}\text{Re}\left[\mathbf{E}_s \times \hat{\mathbf{H}}_s\right] \quad \text{W/ m}^2}$$

$$\mathbf{E}_s = E_{x0} e^{-j\beta z}\mathbf{a}_x$$

$$\hat{\mathbf{H}}_s = \frac{E_{x0}}{\hat{\eta}} e^{j\beta z}\mathbf{a}_y = \frac{E_{x0}}{|\eta|} e^{j\theta_\eta} e^{j\beta z}\mathbf{a}_y$$

The Poynting Vector (5)

Ex. 1

Given a uniform plane wave whose **E** = 100cos(ωt + 6z)$\mathbf{a}_x$ V/m in free space, Find the average power density in the medium?

$$\omega = k_0 c = 6\times 3\times 10^8 = 1.8\times 10^9 \text{ rad/s}$$

$$\mathbf{E}_s = 100e^{j6z}\mathbf{a}_x = E_{xs}\mathbf{a}_x$$

$$\nabla\times\mathbf{E}_s = -j\omega\mu_0\mathbf{H}_s \rightarrow \frac{\partial E_{xs}}{\partial z}\mathbf{a}_y = j600e^{j6z}\mathbf{a}_y = -j\omega\mu_0\mathbf{H}_s$$

$$\rightarrow \mathbf{H}_s = \frac{j600e^{j6z}}{-j\omega\mu_0}\mathbf{a}_y = -\frac{600e^{j6z}}{(1.8\times 10^9)(4\pi\times 10^{-7})}\mathbf{a}_y = -0.2653e^{j6z}\mathbf{a}_y \text{ A/m}$$

$$\mathbf{H} = \text{Re}\left[\mathbf{H}_s e^{j\omega t}\right] = \text{Re}\left[-0.2653e^{j(1.8\times 10^8 t+6z)}\mathbf{a}_y\right] = -0.2653\cos(1.8\times 10^8 t + 6z)\mathbf{a}_y \text{ A/m}$$

$$\mathbf{S}_{av} = \frac{1}{2}\text{Re}\left[\mathbf{E}_s\times\hat{\mathbf{H}}_s\right] = \frac{1}{2}\text{Re}\left[100e^{j6z}\mathbf{a}_x\times(-0.2653e^{-j6z}\mathbf{a}_y)\right] = -13.265\mathbf{a}_z \text{ W/m}^2$$

The Poynting Vector (6)

Ex. 2

In a source-free dielectric region, there is a field $\mathbf{E} = C\sin\alpha x\cos(\omega t - kz)\mathbf{a}_y$ V/m. Find the magnetic field intensity & the average power density?

$$\mathbf{E}_s = E_{ys}\mathbf{a}_y = C\sin\alpha x e^{-jkz}\mathbf{a}_y$$

$$\nabla\times\mathbf{E}_s = -j\omega\mu\mathbf{H}_s$$

$$\rightarrow \left(\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y}\right)\mathbf{a}_z = -j\omega\mu\mathbf{H}_s$$

$$\rightarrow jkC\sin\alpha x e^{-jkz}\mathbf{a}_x + \alpha C\cos\alpha x e^{-jkz}\mathbf{a}_z = -j\omega\mu\mathbf{H}_s$$

$$\rightarrow \mathbf{H}_s = -\frac{kC}{\omega\mu}\sin\alpha x e^{-jkz}\mathbf{a}_x + j\frac{\alpha C}{\omega\mu}\cos\alpha x e^{-jkz}\mathbf{a}_z$$

The Poynting Vector (7)

Ex. 2

In a source-free dielectric region, there is a field $\mathbf{E} = C\sin\alpha x\cos(\omega t - kz)\mathbf{a}_y$ V/m. Find the magnetic field intensity & the average power density?

$$\mathbf{E}_s = E_{ys}\mathbf{a}_y = C\sin\alpha x e^{-jkz}\mathbf{a}_y$$

$$\mathbf{H}_s = -\frac{kC}{\omega\mu}\sin\alpha x e^{-jkz}\mathbf{a}_x + j\frac{\alpha C}{\omega\mu}\cos\alpha x e^{-jkz}\mathbf{a}_z$$

$$\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \hat{\mathbf{H}}_s\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\left(C\sin\alpha x e^{-jkz}\mathbf{a}_y\right)\times\left(-\frac{kC}{\omega\mu}\sin\alpha x e^{jkz}\mathbf{a}_x - j\frac{\alpha C}{\omega\mu}\cos\alpha x e^{jkz}\mathbf{a}_z\right)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{kC^2}{\omega\mu}\sin^2\alpha x\mathbf{a}_z - j\frac{\alpha C^2}{\omega\mu}\sin\alpha x\cos\alpha x\mathbf{a}_x\right] = \frac{kC^2}{2\omega\mu}\sin^2\alpha x\mathbf{a}_z$$

The Poynting Vector (8)

Ex. 3

A 2-GHz wave propagates in a medium whose ($\mu_r = 1.6$, $\varepsilon_r = 25$, $\sigma = 2.5$ S/m). The EFI in this region is $\mathbf{E} = 0.1e^{-\alpha z}\cos(2\pi ft - \beta z)\mathbf{a}_x$ V/m. Find the average power density?

$$\mathbf{E}_s = 0.1e^{-110.03z}e^{-j287.04z}\mathbf{a}_x \ \text{V/m}$$

$$\mathbf{H}_s = 0.0012e^{-110.03z}e^{-j(287.04z+21.0^\circ)}\mathbf{a}_y \ \text{A/m}$$

$$\mathbf{S}_{av} = \frac{1}{2}\text{Re}\left[\mathbf{E}_s \times \hat{\mathbf{H}}_s\right]$$

$$= \frac{1}{2}\text{Re}\left[\left(0.1e^{-110.03z}e^{-j287.04z}\mathbf{a}_x\right)\times\left(0.0012e^{-110.03z}e^{+j(287.04z+21.0^\circ)}\mathbf{a}_y\right)\right]$$

$$= \frac{1}{2}\text{Re}\left[121.7\times10^{-6}e^{-2\times110.03z}e^{j21.0^\circ}\mathbf{a}_z\right] = \frac{1}{2}\text{Re}\left[121.7\times10^{-6}e^{-220.06z}e^{j21.0^\circ}\mathbf{a}_z\right]$$

$$= \frac{1}{2}121.7\times10^{-6}e^{-220.06z}\cos 21.0^\circ\mathbf{a}_z = 56.79e^{-220.06z}\mathbf{a}_z \ \mu\text{W/m}^2$$

The Uniform Plane Wave

1. Wave Propagation in Free Space
2. Wave Propagation in Dielectrics
3. The Poynting Vector
4. **Skin Effect**
5. Wave Polarization

Skin Effect (1)

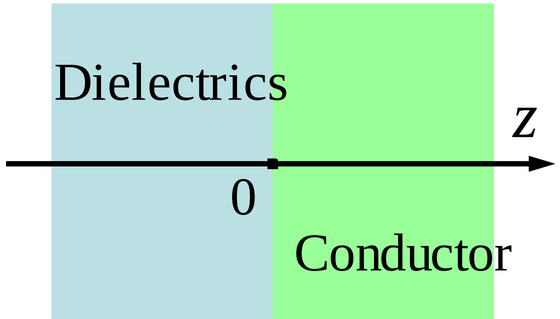
$$\left.\begin{aligned} jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1-j\frac{\sigma}{\omega\varepsilon'}} \approx j\omega\sqrt{\mu\varepsilon'}\sqrt{-j\frac{\sigma}{\omega\varepsilon'}} = j\sqrt{-j\omega\mu\sigma} \\ -j = 1\underline{/-90^{\circ}} \\ \sqrt{1\underline{/-90^{\circ}}} = 1\underline{/-45^{\circ}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \end{aligned}\right\}$$

$$\rightarrow jk = j\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\sqrt{\omega\mu\sigma} = (j1+1)\sqrt{\pi f\mu\sigma} = \alpha + j\beta$$

$$\rightarrow \boxed{\alpha = \beta = \sqrt{\pi f\mu\sigma}}$$

$$\rightarrow E_x = E_{x0}e^{-\alpha z}\cos(\omega t - \beta z) = E_{x0}e^{-z\sqrt{\pi f\mu\sigma}}\cos(\omega t - z\sqrt{\pi f\mu\sigma})$$

Skin Effect (2)



$$E_x = E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$$

$$E_x\big|_{z=0} = E_{x0} \cos \omega t$$

$$J_x = \sigma E_x = \sigma E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}}$$

$$\delta_{Cu} = \frac{0.066}{\sqrt{f}}$$

$$\delta_{Cu;\,50\,\mathrm{Hz}} = 9.3\text{ mm}$$

$$\delta_{Cu;\,10{,}000\,\mathrm{MHz}} = 6.61\times 10^{-4}\text{ mm}$$

Skin Effect (3)

$$\left.\begin{array}{l} \alpha = \beta = \dfrac{1}{\delta} = \sqrt{\pi f \mu \sigma} \\ \\ \beta = \dfrac{2\pi}{\lambda} \end{array}\right\}$$

$$\rightarrow \lambda = 2\pi\delta$$

$$v_p = \frac{\omega}{\beta}$$

$$\rightarrow v_p = \omega\delta$$

Skin Effect (4)

Ex.

Consider an 1 MHz wave propagating in seawater, $\sigma = 4$ S/m, $\varepsilon'_r = 81$.

$$\frac{\sigma}{\omega\varepsilon'} = \frac{4}{(2\pi\times10^6)(81)(8.85\times10^{-12})} = 8.9\times10^2 \gg 1$$

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\sqrt{(\pi\times10^6)(4\pi\times10^{-7})(4)}} = 0.25 \text{ m}$$

$$\lambda = 2\pi\delta = 1.6 \text{ m}$$

$$v_p = \omega\delta = (2\pi\times10^6)(0.25) = 1.6\times10^6 \text{ m/s}$$

Skin Effect (5)

$$\left.\begin{array}{r}\eta=\sqrt{\dfrac{\mu}{\varepsilon}}=\sqrt{\dfrac{\mu}{\varepsilon'-j\varepsilon''}}=\sqrt{\dfrac{j\omega\mu}{\sigma+j\omega\varepsilon'}}\\ \sigma\gg\omega\varepsilon'\end{array}\right\}\to\eta=\sqrt{\dfrac{j\omega\mu}{\sigma}}$$

$$\sqrt{j}=1\underline{/\,45^{\mathrm{o}}} \qquad \delta=\frac{1}{\sqrt{\pi f\mu\sigma}}$$

$$\to\boxed{\eta=\frac{\sqrt{2}\underline{/\,45^{\mathrm{o}}}}{\sigma\delta}=\frac{1}{\sigma\delta}+j\frac{1}{\sigma\delta}}$$

$$E_x=E_{x0}e^{-z\sqrt{\pi f\mu\sigma}}\cos(\omega t-z\sqrt{\pi f\mu\sigma})=E_{x0}e^{-z/\delta}\cos(\omega t-z/\delta)$$

$$\frac{E_x}{H_y}=\eta$$

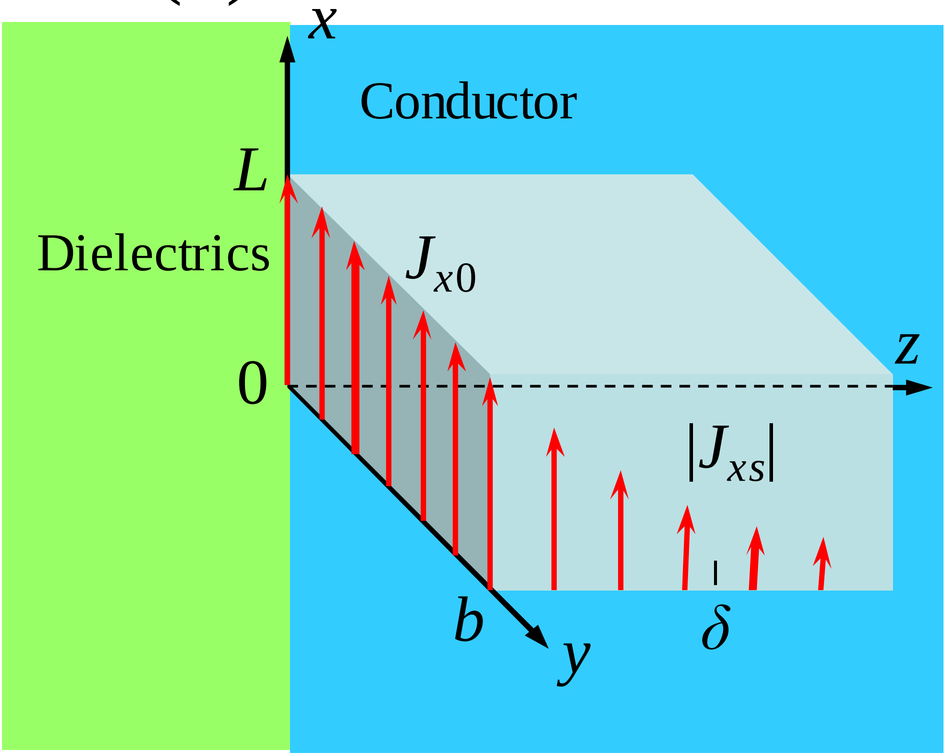
$$\to H_y=\frac{\sigma\delta E_{x0}}{\sqrt{2}}e^{-z/\delta}\cos\left(\omega t-\frac{z}{\delta}-\frac{\pi}{4}\right)$$

Skin Effect (6)

$$\left.\begin{aligned} E_x &= E_{x0}e^{-z/\delta}\cos(\omega t - z/\delta) \\ H_y &= \frac{\sigma\delta E_{x0}}{\sqrt{2}}e^{-z/\delta}\cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right) \\ S_{av} &= \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \hat{\mathbf{H}}_s\right] \end{aligned}\right\}$$

$$\to S_{av} = \frac{1}{2}\frac{\sigma\delta E_{x0}^2}{\sqrt{2}}e^{-2z/\delta}\cos\left(\frac{\pi}{4}\right)$$

$$= \boxed{\frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}}$$



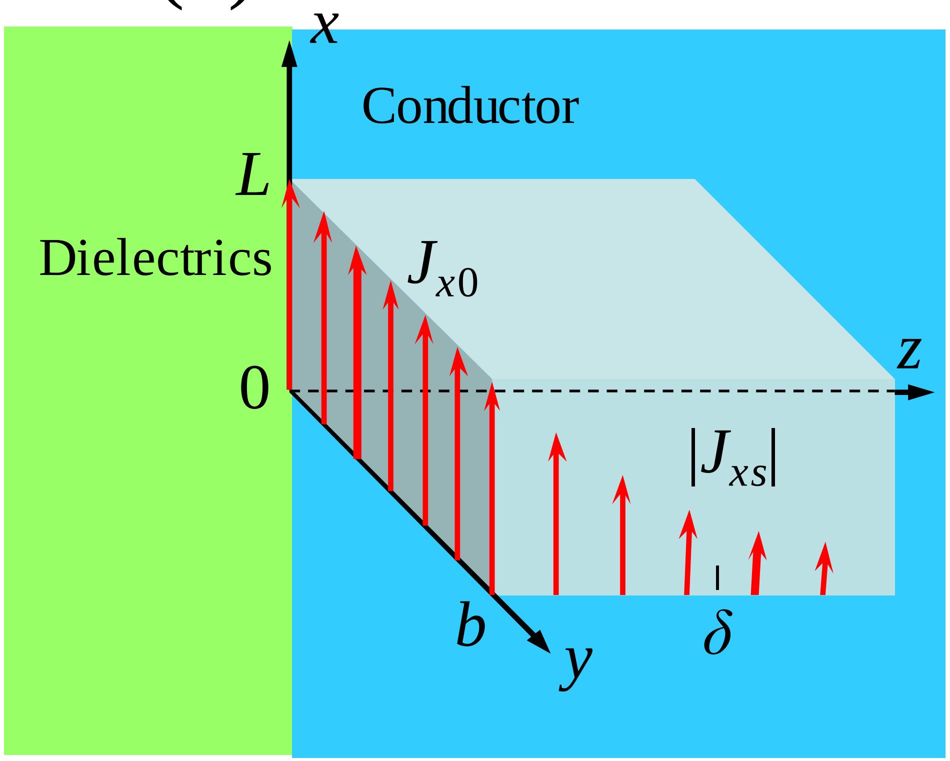
$$S_{L,av} = \int_S S_{z,av}dS = \int_0^b\int_0^L \frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}\Big|_{z=0} dxdy = \frac{1}{4}\sigma\delta bLE_{x0}^2$$

Skin Effect (7)

$$\left.\begin{aligned} S_{L,av} &= \frac{1}{4}\sigma\delta b L E_{x0}^2 \\ J_{x0} &= \sigma E_{x0} \end{aligned}\right\}$$

$$\rightarrow S_{L,av} = \frac{1}{4\sigma}\delta b L J_{x0}^2$$

$$\left.\begin{aligned} I &= \int_0^\infty \int_0^b J_x dydz \\ J_x &= J_{x0} e^{-z/\delta}\cos(\omega t - z/\delta) \\ \rightarrow J_{xs} &= J_{x0} e^{-z/\delta} e^{-jz/\delta} \\ &= J_{x0} e^{-(1+j)z/\delta} \end{aligned}\right\}$$



$$\rightarrow I_s = \int_0^\infty \int_0^b J_{x0} e^{-(1+j)z/\delta} dydz = J_{x0} b e^{-(1+j)z/\delta} \frac{-\delta}{1+j}\Bigg|_0^\infty = \frac{J_{x0} b\delta}{1+j}$$

$$\rightarrow I = \frac{J_{x0} b\delta}{\sqrt{2}}\cos\left(\omega t - \frac{\pi}{4}\right)$$

Skin Effect (8)

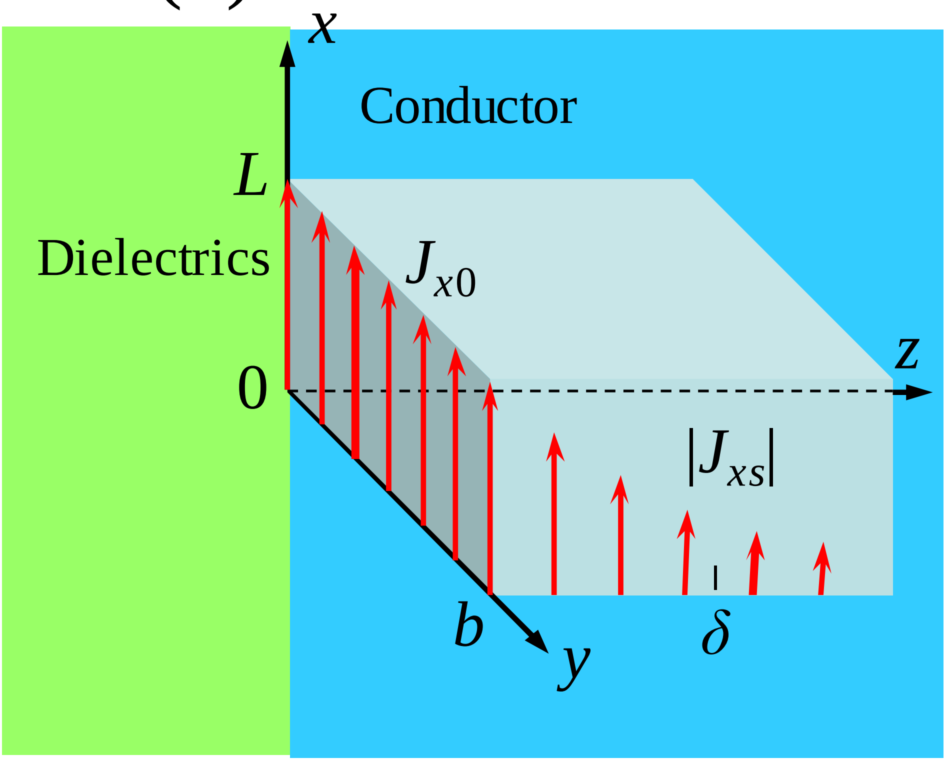
$$I = \frac{J_{x0} b\delta}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$\rightarrow J' = \frac{I}{b\delta} = \frac{J_{x0}}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$\rightarrow S_L = \frac{1}{\sigma}(J')^2 bL\delta$$

$$= \frac{J_{x0}^2}{2\sigma} bL\delta \cos^2\left(\omega t - \frac{\pi}{4}\right)$$

$$\rightarrow S_{L,av} = \frac{1}{4\sigma} J_{x0}^2 bL\delta$$



(if the current is distributed uniformly throughout $0 < z < \delta$)

$$S_{L,av} = \frac{1}{4\sigma} J_{x0}^2 bL\delta$$

(if the total current is distributed throuthout $0 < z < \infty$)

Skin Effect (9)

$$R = \frac{L}{\sigma S} = \frac{L}{\sigma 2\pi a \delta}$$

$$R_{Cu,\,1\,\mathrm{MHz},\,a=1\,\mathrm{mm},\,l=1\,\mathrm{km}} = \frac{10^3}{(5.8\times10^7)(2\pi)(10^{-3})(0.066\times10^{-3})} = 41.5\Omega$$

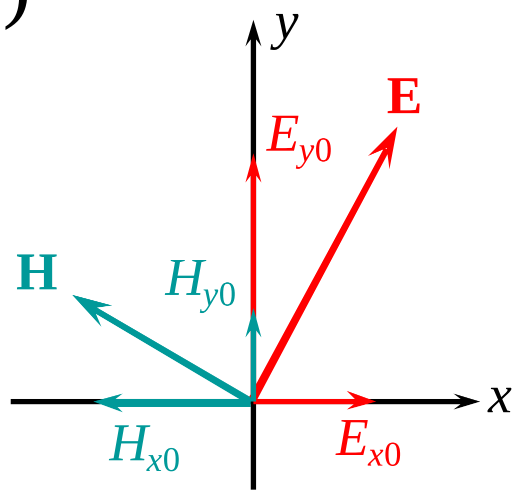
The Uniform Plane Wave

1. Wave Propagation in Free Space
2. Wave Propagation in Dielectrics
3. The Poynting Vector
4. Skin Effect
5. **Wave Polarization**

Wave Polarization (1)

- In the previous sections, **E** & **H** are supposed to lie in fix directions
- However, the directions of **E** & **H** within the plane perpendicular to $\mathbf{a}_z$ may change as functions of time and position
- λ, v_p, **S**, …
- The instantaneous orientation of field vectors
- *Wave polarization*: its electric field vector orientation as a function of time, at a fixed point in space
- **H** can be found from **E**

Wave Polarization (2)



$$\mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y)e^{-\alpha z}e^{-j\beta z}$$

$$\mathbf{H}_s = (H_{x0}\mathbf{a}_x + H_{y0}\mathbf{a}_y)e^{-\alpha z}e^{-j\beta z}$$

$$= \left[-\frac{E_{y0}}{\eta}\mathbf{a}_x + \frac{E_{x0}}{\eta}\mathbf{a}_y\right]e^{-\alpha z}e^{-j\beta z}$$

$$S_{z,av} = \frac{1}{2}\mathrm{Re}[\mathbf{E}_s \times \hat{\mathbf{H}}_s]$$

$$= \frac{1}{2}\mathrm{Re}\left[E_{x0}\hat{H}_{y0}(\mathbf{a}_x \times \mathbf{a}_y) + E_{y0}\hat{H}_{x0}(\mathbf{a}_y \times \mathbf{a}_x)\right]e^{-2\alpha z}$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{E_{x0}\hat{E}_{x0}}{\hat{\eta}} + \frac{E_{y0}\hat{E}_{y0}}{\hat{\eta}}\right]e^{-2\alpha z}\mathbf{a}_z$$

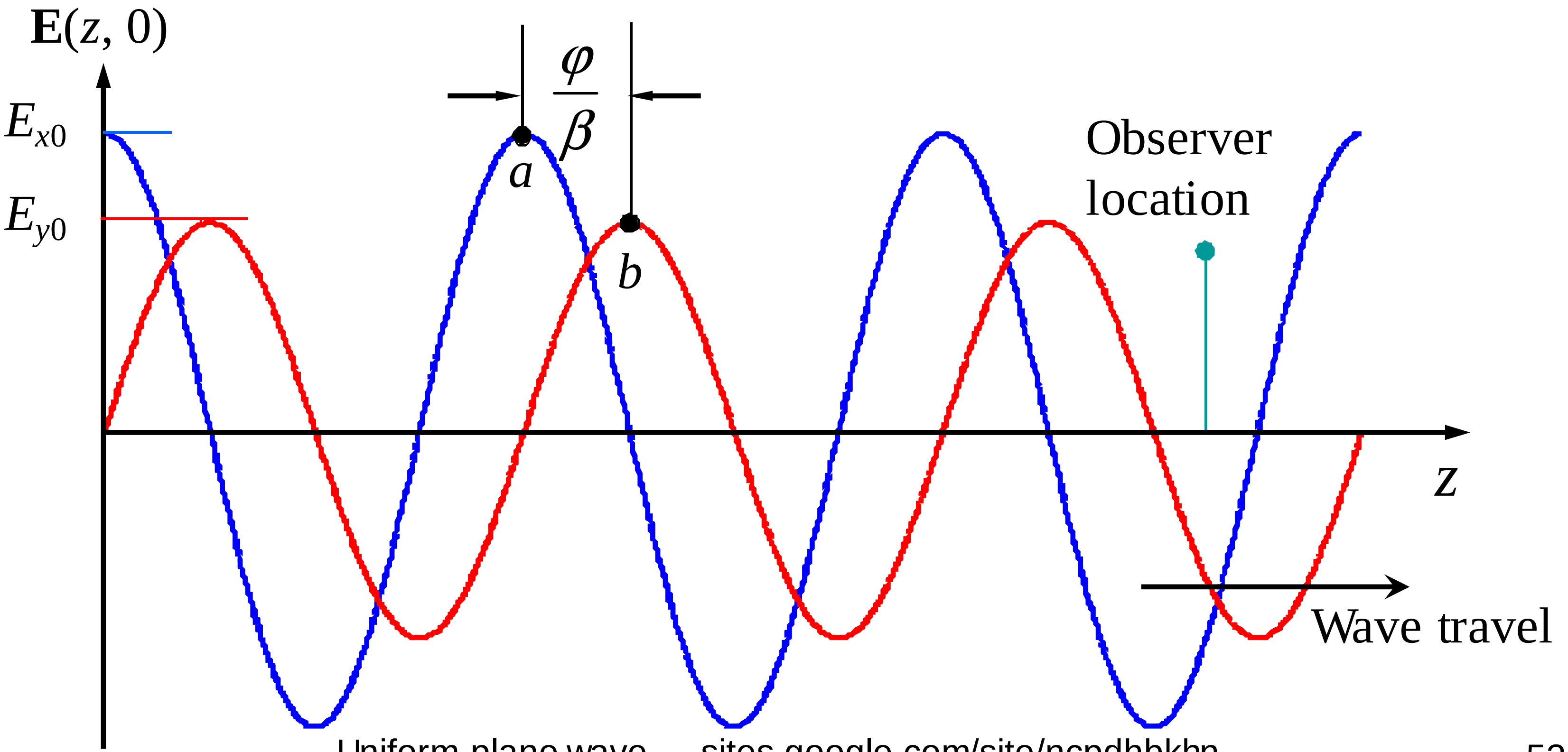
$$= \frac{1}{2}\mathrm{Re}\left[\frac{1}{\hat{\eta}}\right]\left(|E_{x0}|^2 + |E_{y0}|^2\right)e^{-2\alpha z}\mathbf{a}_z \quad \mathrm{W/m^2}$$

Wave Polarization (3)

$$\mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y)e^{-j\beta z}$$

$$\rightarrow \mathbf{E}(z,t) = E_{x0}\cos(\omega t - \beta z)\mathbf{a}_x + E_{y0}\cos(\omega t - \beta z + \varphi)\mathbf{a}_y$$

$$\rightarrow \mathbf{E}(z,0) = E_{x0}\cos(\beta z)\mathbf{a}_x + E_{y0}\cos(\beta z - \varphi)\mathbf{a}_y$$



Wave Polarization (4)

Ex.

If EFI in a region is given by $\mathbf{E}_s = e^{-0.2z}e^{-j0.5z}(3\mathbf{a}_x + j4\mathbf{a}_y)$ V/m, find the polarization of the wave?

$$\mathbf{E} = \text{Re}\left[e^{-0.2z}e^{-j0.5z}(3\mathbf{a}_x + j4\mathbf{a}_y)e^{j\omega t} \right]$$

$$= \text{Re}\left[3e^{-0.2z}e^{j(\omega t-0.5z)}\mathbf{a}_x + j4e^{-0.2z}e^{j(\omega t-0.5z)}\mathbf{a}_y) \right]$$

$$= \text{Re}\left\{ \begin{array}{l} 3e^{-0.2z}[\cos(\omega t - 0.5z) + j\sin(\omega t - 0.5z)]\mathbf{a}_x + \\ + j4e^{-0.2z}[\cos(\omega t - 0.5z) + j\sin(\omega t - 0.5z)]\mathbf{a}_y \end{array} \right\}$$

$$= \text{Re}\left\{ \begin{array}{l} 3e^{-0.2z}[\cos(\omega t - 0.5z) + j\sin(\omega t - 0.5z)]\mathbf{a}_x + \\ +4e^{-0.2z}[j\cos(\omega t - 0.5z) - \sin(\omega t - 0.5z)]\mathbf{a}_y \end{array} \right\}$$

$$= 3e^{-0.2z}\cos(\omega t - 0.5z)\mathbf{a}_x - 4e^{-0.2z}\sin(\omega t - 0.5z)\mathbf{a}_y$$

$$\rightarrow \begin{cases} E_x(z,t) = 3e^{-0.2z}\cos(\omega t - 0.5z) \\ E_y(z,t) = -4e^{-0.2z}\sin(\omega t - 0.5z) \end{cases}$$

Wave Polarization (5)

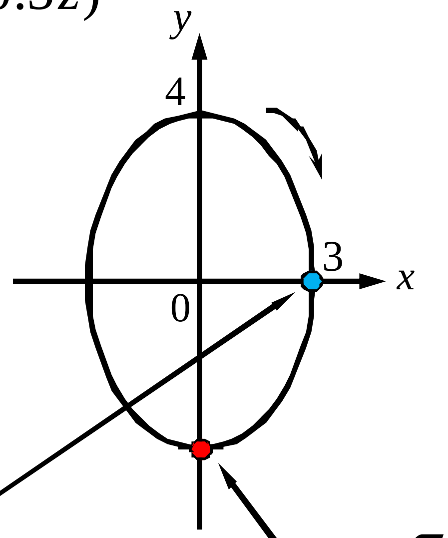
Ex.

If EFI in a region is given by $\mathbf{E}_s = e^{-0.2z}e^{-j0.5z}(3\mathbf{a}_x + j4\mathbf{a}_y)$ V/m, find the polarization of the wave?

$$\begin{cases} E_x(z,t) = 3e^{-0.2z}\cos(\omega t - 0.5z) \\ E_y(z,t) = -4e^{-0.2z}\sin(\omega t - 0.5z) \end{cases}$$

$$\rightarrow \begin{cases} E_x(0,t) = 3\cos\omega t \\ E_y(0,t) = -4\sin\omega t \end{cases}$$

$$\rightarrow \frac{1}{9}E_x^2(0,t) + \frac{1}{16}E_y^2(0,t) = 1$$



$$t = 0 \rightarrow \begin{cases} E_x(0,0) = 3 \\ E_y(0,0) = 0 \end{cases}$$

$$t = \frac{\pi}{2\omega} \rightarrow \begin{cases} E_x(0,\pi/2\omega) = 0 \\ E_y(0,\pi/2\omega) = -4 \end{cases}$$

$$Q \longrightarrow \mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon R^2}\mathbf{a}_R \longrightarrow \mathbf{E} = \frac{Q}{4\pi\varepsilon R^2}\mathbf{a}_R \longrightarrow \mathbf{D} = \varepsilon\mathbf{E}$$

$$W = -Q\int \mathbf{E}.d\mathbf{L} \longrightarrow V = -\int \mathbf{E}.d\mathbf{L} \longrightarrow C = \frac{Q}{V}$$

$$I = \frac{dQ}{dt} \longrightarrow R = \frac{V}{I}$$

$$\boxed{\begin{array}{l} \nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t};\ \nabla\times\mathbf{H} = \mathbf{J} + \dfrac{\partial\mathbf{D}}{\partial t} \\ \nabla.\mathbf{D} = \rho_v;\ \nabla.\mathbf{B} = 0 \end{array}}$$

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\varphi \longrightarrow \mathbf{B} = \mu\mathbf{H} \longrightarrow \Phi = \int \mathbf{B}.d\mathbf{S} \longrightarrow L = \frac{\Phi}{I}$$

$$V_{m,ab} = -\int_b^a \mathbf{H}.d\mathbf{L} \qquad \mathbf{B} = \nabla\times\mathbf{A} \qquad \text{sđđ} = -\frac{d\Phi}{dt} \qquad M_{12} = \frac{N_2\Phi_{12}}{I_1}$$

$$\mathbf{F} = -I\oint \mathbf{B}\times d\mathbf{L} \longrightarrow \mathbf{T} = \mathbf{R}\times\mathbf{F}$$

$$\mathbf{E}(x, y, z, t) \longrightarrow \mathbf{S} = \mathbf{E}\times\mathbf{H}$$