Fundamentals of Electric Circuits

DC Circuits

Chapter 4. Circuit Theorems

4.1. Introduction

+ To reduce the number of equations:

→ The first way: Circuit variable transformations (node voltage and mesh current methods)

→ The second way: Circuit topology transformations (Thevenin’s theorem, Norton’s theorem,...)

+ In this chapter:

- Concept of circuit linearity
- Circuit theorems
- Concept of superposition, source transformation, maximum power transfer

4.2. Linearity property

+ ***Linearity*** → *property of an element describing a linear relationship between cause (input, excitation) and effect (output, response)*

+ Linearity property combines:

Homogeneity (scaling) *property*: $v = iR \rightarrow kiR = kv$

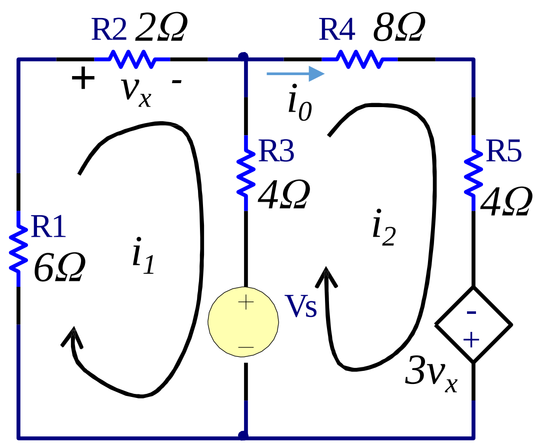
Additivity property: $\begin{matrix} v_1 = i_1 R \\ v_2 = i_2 R \end{matrix} \rightarrow v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$

+ A ***linear circuit*** : its output is linearly related (or directly proportional) to its input

+ A linear circuit consists of only linear elements, linear dependent sources, and independent sources

4.2. Linearity property

+ Example 1: find i_0 when $v_s = 12V$ and $v_s = 24V$



Apply the mesh current method:

$$v_x = R_2 i_1 = 2i_1$$

$$\begin{cases} (R_1 + R_2 + R_3)i_1 - R_3 i_2 + v_s = 0 \\ -R_3 i_1 + (R_3 + R_4 + R_5)i_2 - v_s - 3v_x = 0 \end{cases} \rightarrow \begin{cases} 12i_1 - 4i_2 + v_s = 0 \\ -10i_1 + 16i_2 - v_s = 0 \end{cases} \rightarrow \begin{cases} i_1 = -6i_2 \\ i_2 = \dfrac{v_s}{76} \end{cases}$$

When $v_s = 12V$: $$i_0 = i_2 = \frac{12}{76} = 0.158A$$

When $v_s = 24V$: $$i_0 = i_2 = \frac{24}{76} = 0.316A = 2 \times 0.158A$$

Prove the linearity of this circuit

4.3. Superposition

+ Linear circuits → satisfy superposition principle

> *The* ***superposition*** principle states that the voltage across (or current through) an element ***in a linear circuit*** is the algebraic sum of the voltages across (or currents though) that element due to each independent source acting alone

+ So: if circuit has two or more independent sources → several possible ways to determine the value of a specific variable (voltage, current)

- **Direct approach:** let all sources acting simultaneously → applying nodal or mesh analysis to analyze the given circuit
- **Superposition approach:** determine the contribution of each independent source to the variable, and then add them up (applying NA or MA for each sub-problem)

+ Superposition: applicable in any linear system

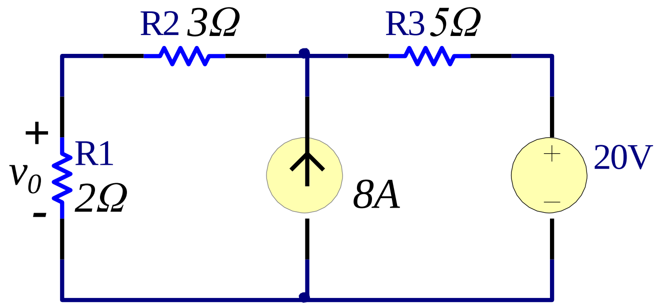
4.3. Superposition

+ To apply superposition principle

- Turn off all ***independent sources*** except one source (*dependent sources are left intact*):
  → *Replace* ***voltage source*** by ***short circuit***
  → *Replace* ***current source*** by ***open circuit***
- Find the output (voltage or/and current) due to that active source (using nodal or mesh analysis)
- Repeat two above steps for each of the other independent sources
- Find the total contribution by adding algebraically all the contributions due to the independent sources

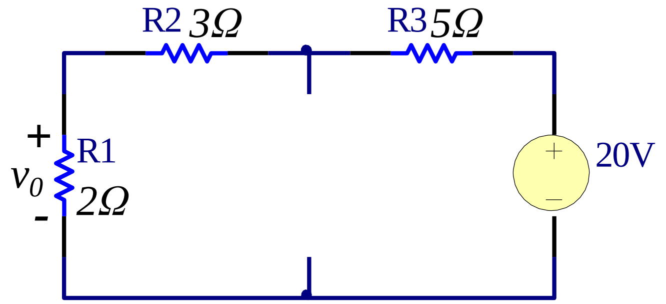
4.3. Superposition

+ Example 2: Using the superposition theorem, find v_0



Let: $$v_0 = v_{01} + v_{02}$$

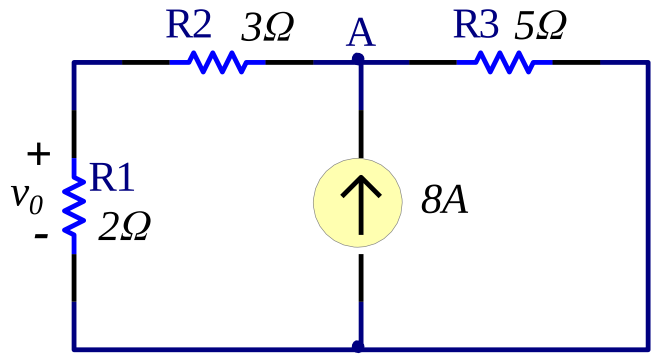
To obtain v_{01}, set the current source to zero



Apply KVL to the obtained loop, we have

$$(3+5+2)i = 20 \rightarrow i = 2A \rightarrow v_{01} = 2.2 = 4V$$

To obtain v_{02}, set the voltage source to zero



Apply KCL to node A, we have

$$\frac{v_A}{R_1+R_2} + \frac{v_A}{R_3} = 8 \rightarrow \frac{2}{5}v_A = 8 \rightarrow v_A = 20V \qquad v_{02} = R_1 i = R_1\frac{v_A}{R_1+R_2} = 8V$$

Or applying current division: $i_{R_1} = \dfrac{8}{2+3+5}5 = 4A \rightarrow v_{02} = R_1 i_{R_1} = 2.4 = 8V$

Finally, calculate v_0:

$$v_0 = v_{01} + v_{02} = 8+4 = 12V$$

4.3. Superposition

+ Example 3: Using the superposition theorem, find i_0

There is a dependent source, so we left intact

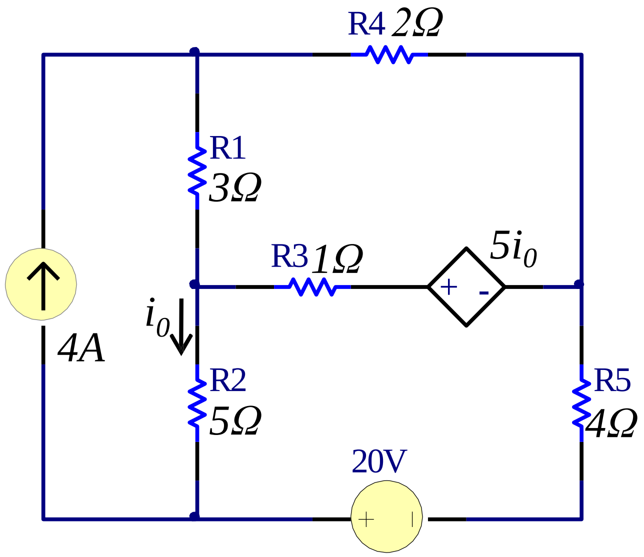
Let: $i_0 = i_{01} + i_{02}$

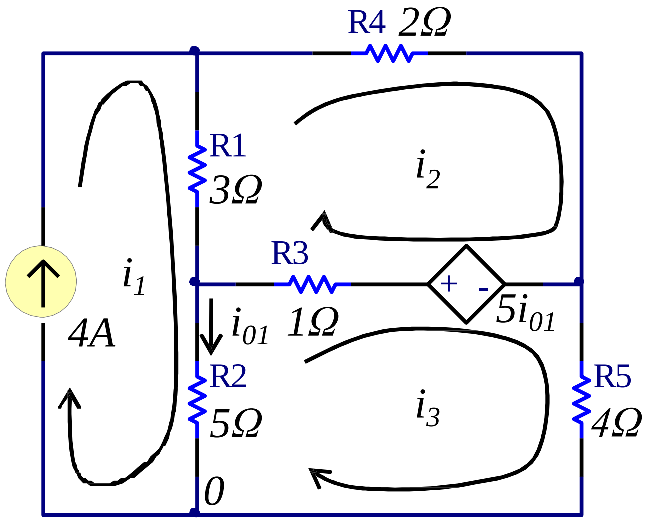
To obtain i_{01}, set the voltage source 20V to zero

Apply mesh current method:

$$\begin{cases} i_1 = 4A & \leftarrow \text{Loop 1} \\ -R_1 i_1 + (R_1 + R_2 + R_3) i_2 - R_3 i_3 - 5 i_{01} = 0 & \leftarrow \text{Loop 2} \\ -R_2 i_1 - R_3 i_2 + (R_2 + R_3 + R_5) i_3 + 5 i_{01} = 0 & \leftarrow \text{Loop 3} \\ i_3 = i_1 - i_{01} & \leftarrow \text{KCL at node 0} \end{cases}$$

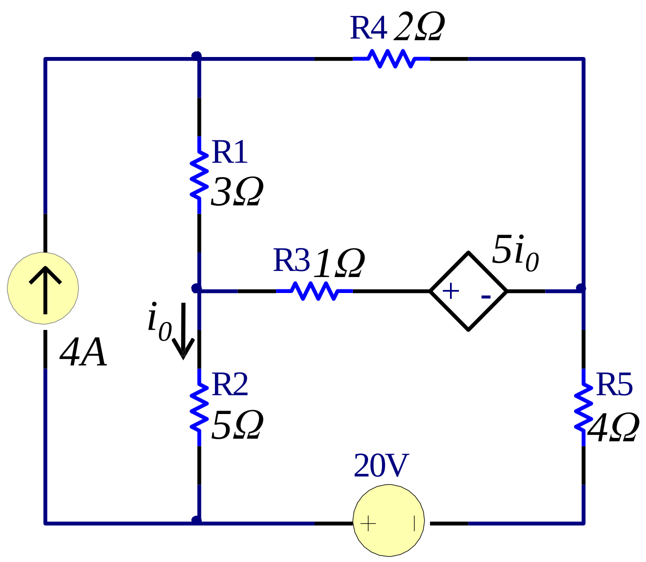
Solve this equation system: $i_{01} = 3.06A$





4.3. Superposition

+ Example 3: Using the superposition theorem, find i_0



There is a dependent source, so we left intact

Let: $i_0 = i_{01} + i_{02}$

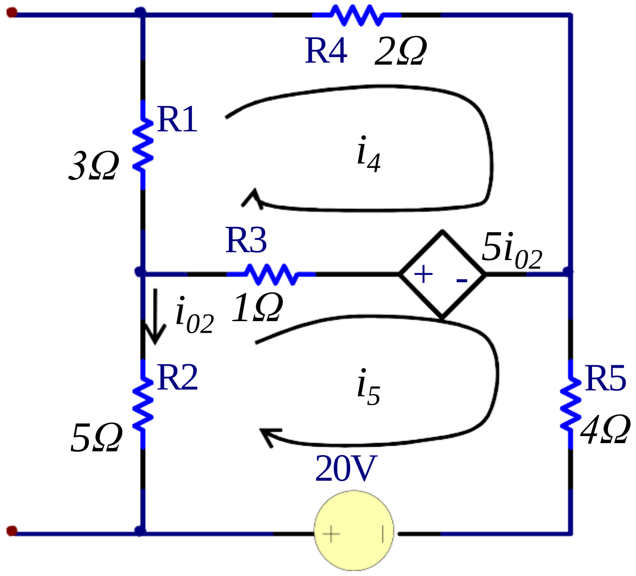
To obtain i_{02}, set the current source 4A to zero

Apply mesh current method:

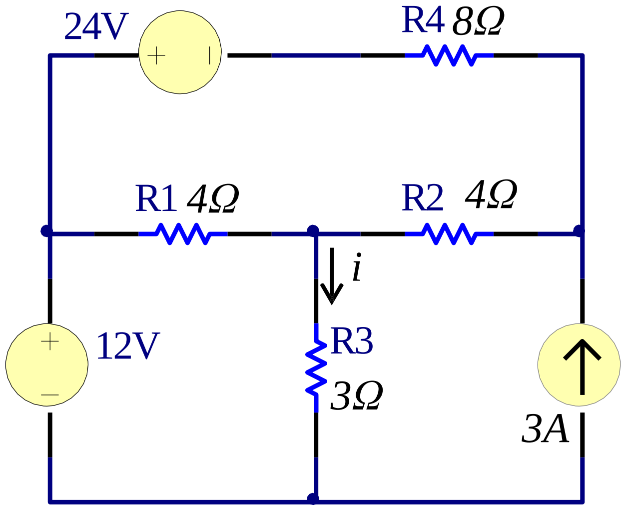
$$\begin{cases} (R_1 + R_3 + R_4)i_4 - R_3 i_5 = 5i_{02} \\ -R_3 i_4 + (R_2 + R_3 + R_5)i_5 = 20 - 5i_{02} \end{cases} \rightarrow \begin{cases} 6i_4 - i_5 = -5i_5 \\ -i_4 + 10i_5 = 20 + 5i_5 \end{cases}$$

Solve this equation system: $i_{02} = -i_5 = -3.53A$

Finally, calculate i_0: $i_0 = i_{01} + i_{02} = 3.06 - 3.53 = -0.47A$



4.3. Superposition



+ Example 4: Using the superposition theorem, find *i*

Let: $i = i_1 + i_2 + i_3$

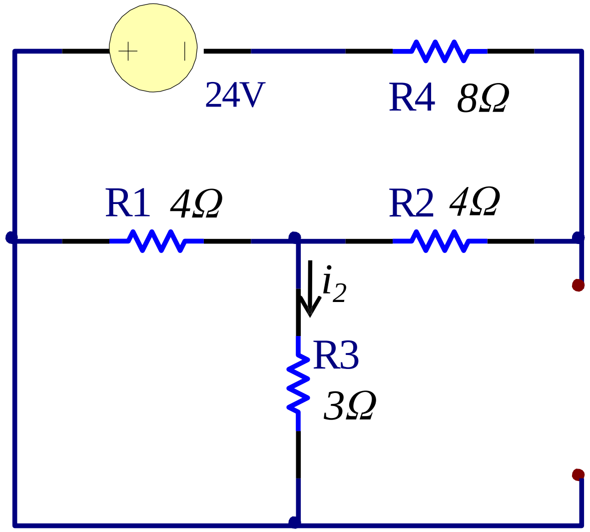
Turn off 3A source, 24V source to calculate i_1 :

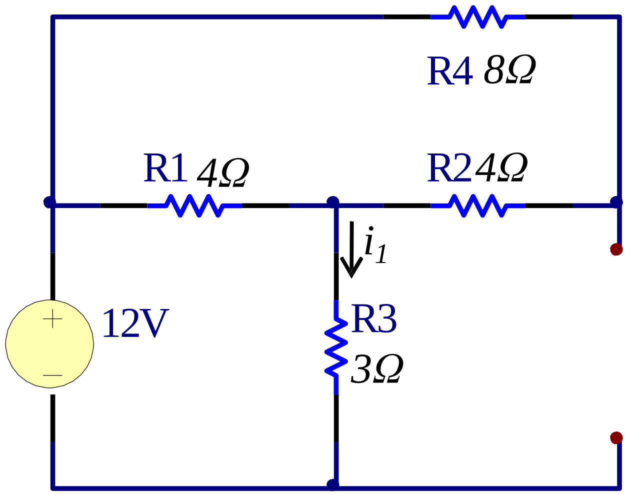
$$i_1 = \frac{12}{[R_1 // (R_2 + R_4)] + R_3} = \frac{12}{6} = 2A$$

Turn off 3A source, 12V source to calculate i_2 :

$$i_{R4} = \frac{24}{(R_1 // R_3) + R_2 + R_4} = \frac{24}{42} = 1.75A$$

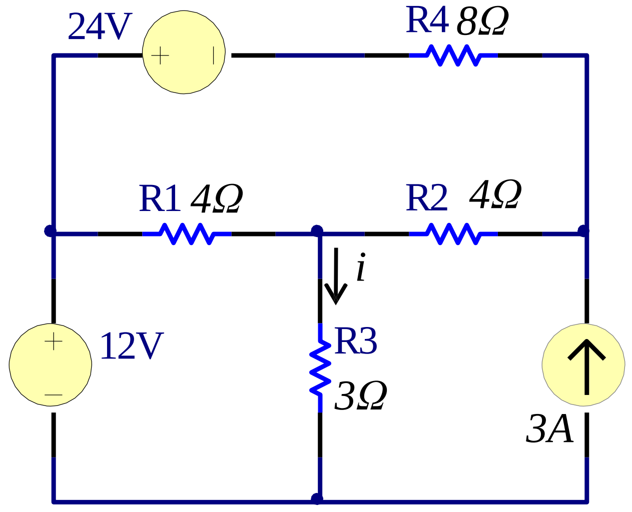
$$i_2 = i_{R4} . \frac{-R_1}{R_1 + R_3} = -1A$$





4.3. Superposition

+ Example 4: Using the superposition theorem, find *i*

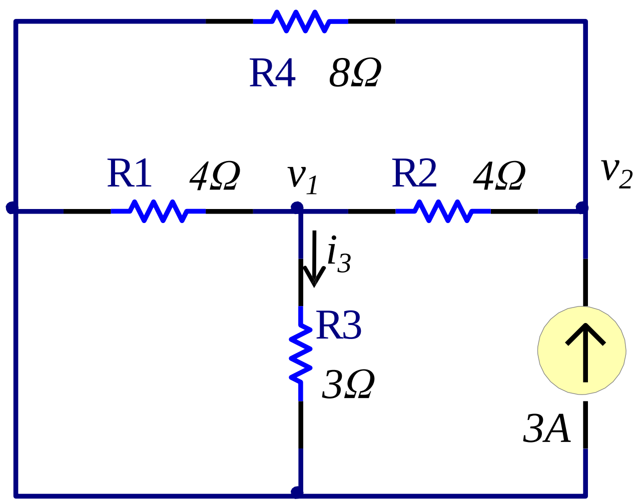


Let: $$i = i_1 + i_2 + i_3$$

Turn off 12V source, 24V source to calculate i_3 :

Apply node voltage method, we have

$$\begin{cases} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_1 - \frac{1}{R_2}v_2 = 0 \\ -\frac{1}{R_2}v_1 + \left(\frac{1}{R_2} + \frac{1}{R_4}\right)v_2 = 3 \end{cases} \rightarrow \begin{cases} 3.33v_1 - v_2 = 0 \\ -2v_1 + 3v_2 = 24 \end{cases}$$



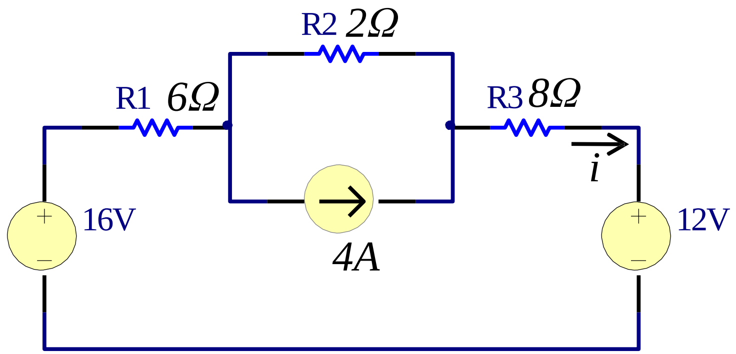
Solving this set of equations gives:

$$v_1 = 3V \rightarrow i_3 = 1A$$

Thus: $$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2A$$

4.3. Superposition

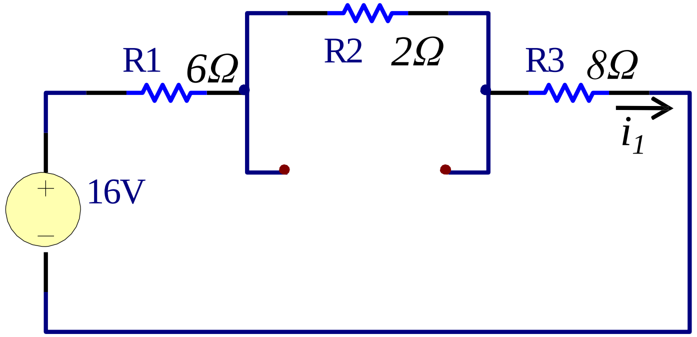
+ Example 5: Using the superposition theorem, find i



Let: $i = i_1 + i_2 + i_3$

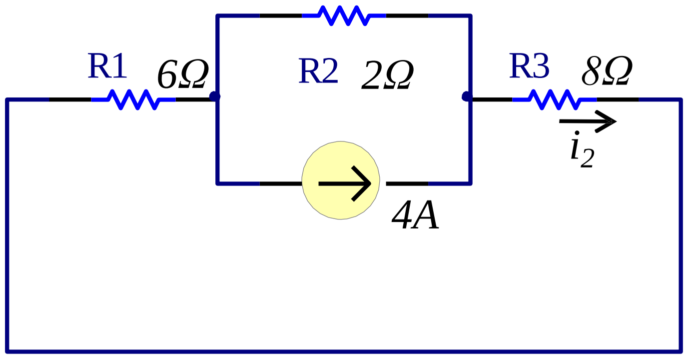
Turn off 4A source, 12V source to calculate i_1 :

$$i_1 = \frac{16}{R_1 + R_2 + R_3} = 1A$$



Turn off 16V source, 12V source to calculate i_2 :

$$i_2 = \frac{4}{R_1 + R_2 + R_3} R_2 = 0.5A$$



Turn off 16V source, 4A source to calculate i_3 :

$$i_3 = \frac{-12}{R_1 + R_2 + R_3} = -0.75A$$

Thus: $i = 1 + 0.5 - 0.75 = 0.75A$

4.4. Source transformation

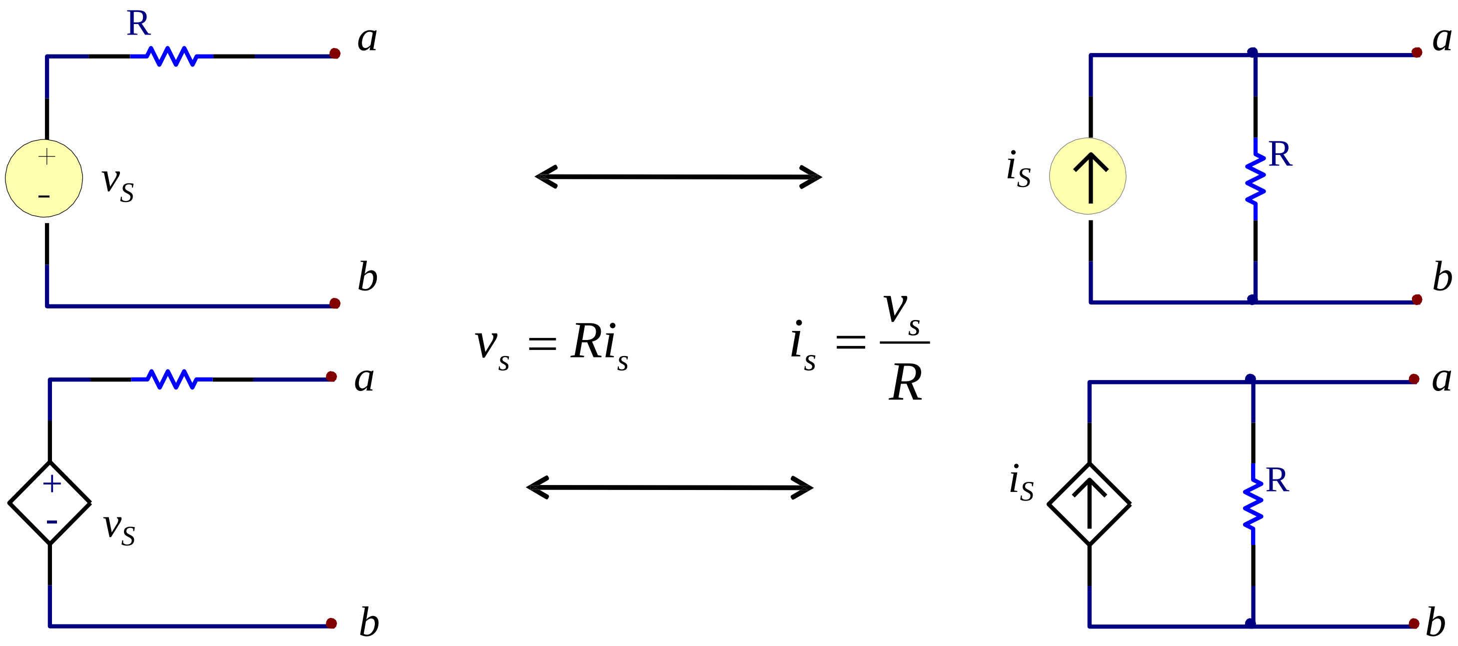
+ ***Source transformation*** → process of replacing a voltage source v_s in series with a resistor *R* by a current source i_s in parallel with a resistor R, or vice versa

+ ***Source transformation*** → to simplify circuits that bases on the *concept of equivalence*

Note: similar to series-parallel combination and wye-delta transformation→ circuit structure equivalent transformation

+ ***Equivalent circuit*** → v - i characteristics are identical with the original circuit

4.4. Source transformation



$$v_s = Ri_s \qquad i_s = \frac{v_s}{R}$$

Note:

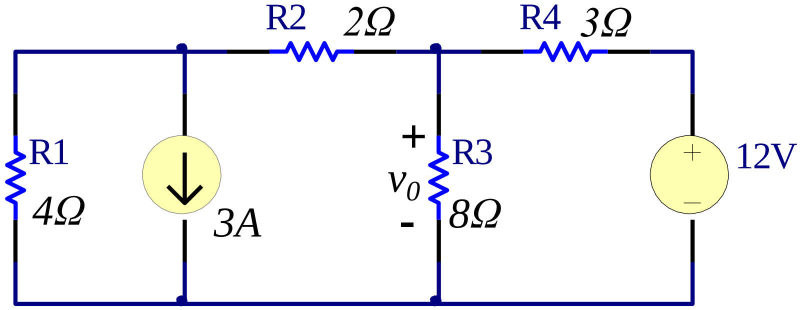
→ arrow of the current source is directed toward the positive terminal of the voltage source

→ source transformation is not possible when $R = 0$ (*ideal voltage source*) or $R = \infty$ (*ideal current source*)

→ One port network transformation

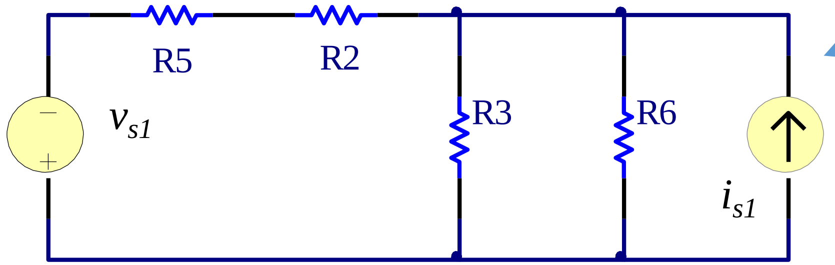
4.4. Source transformation

+ Example 6: Use source transformation to find v_0

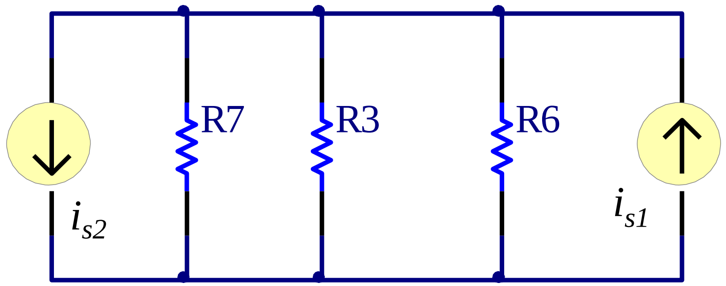


$$\begin{cases} v_{s1} = i_s R_1 = 3.4 = 12V \\ R_5 = R_1 = 4\Omega \end{cases}$$

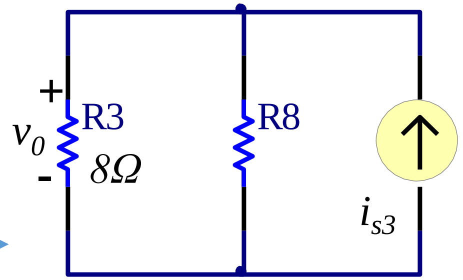
$$\begin{cases} i_{s1} = \dfrac{v_s}{R_4} = \dfrac{12}{3} = 4A \\ R_6 = R_4 = 3\Omega \end{cases}$$



$$\begin{cases} i_{s2} = \dfrac{v_{s1}}{R_5 + R_2} = \dfrac{12}{6} = 2A \\ R_7 = R_5 + R_2 = 6\Omega \end{cases}$$



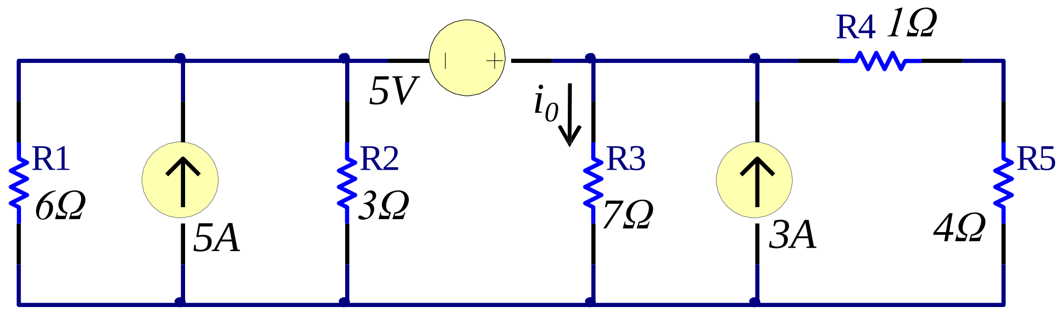
$$\begin{cases} i_{s3} = i_{s1} - i_{s2} = 2A \\ R_8 = R_6 // R_7 = 2\Omega \end{cases}$$



$$\boxed{v_0 = i_{s3} \dfrac{R_3 R_8}{R_3 + R_8} = 2 \dfrac{8.2}{8+2} = 3.2V}$$

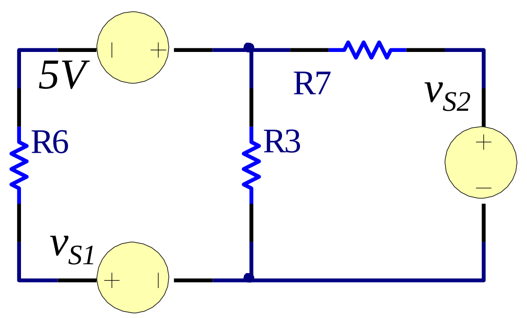
4.4. Source transformation

+ Example 7: Use source transformation to find i_0

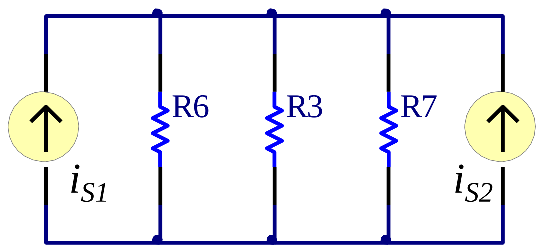


$$\begin{cases} R_6 = R_1 // R_2 = 2\Omega \\ v_{s1} = 5R_6 = 10V \end{cases}$$

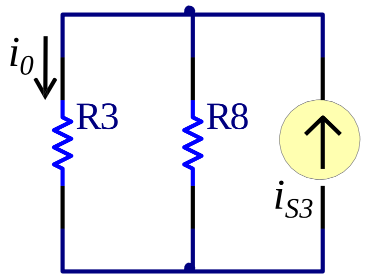
$$\begin{cases} R_7 = R_4 + R_5 = 5\Omega \\ v_{s2} = 3R_7 = 15V \end{cases}$$



$$\begin{cases} i_{s1} = \dfrac{5 + v_{s1}}{R_6} = 7.5A \\ i_{s2} = v_{s2} / R_7 = 3A \end{cases}$$



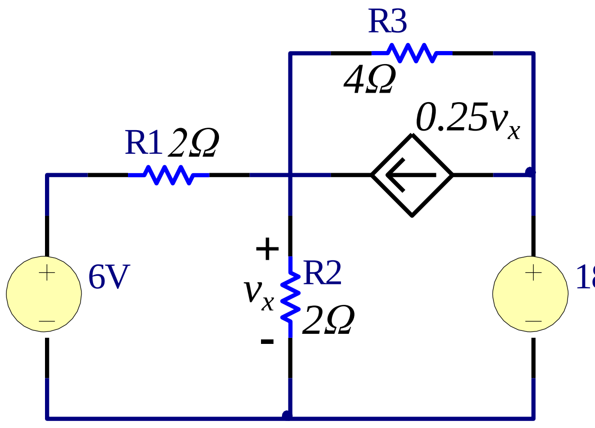
$$\begin{cases} i_{s3} = i_{s1} + i_{s2} = 10.5A \\ R_8 = R_6 // R_7 = 1.43\Omega \end{cases}$$



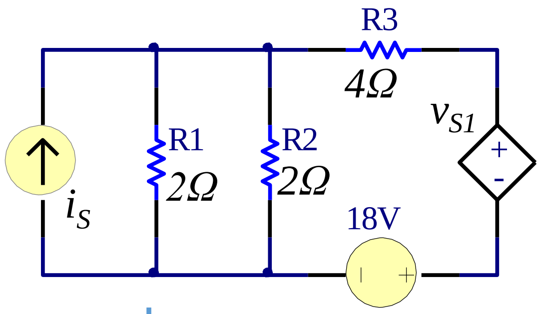
$$\boxed{i_0 = i_{s3} \frac{R_8}{R_3 + R_8} = 1{,}78A}$$

4.4. Source transformation

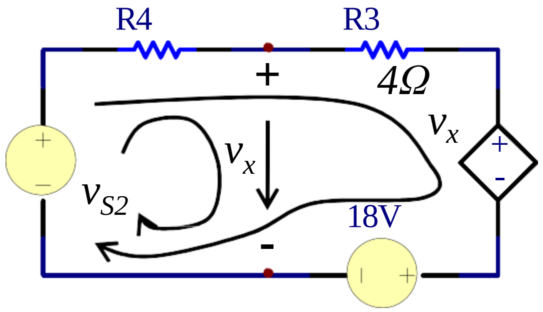
+ Example 8: Use source transformation to find v_x



$$\begin{cases} i_s = \dfrac{6}{R_1} = 3A \\ v_{s1} = 0.25 v_x R_3 = v_x \end{cases}$$



$$\begin{cases} R_4 = R_1 // R_2 = 1\Omega \\ v_{s2} = i_{s2} R_4 = 3V \end{cases}$$

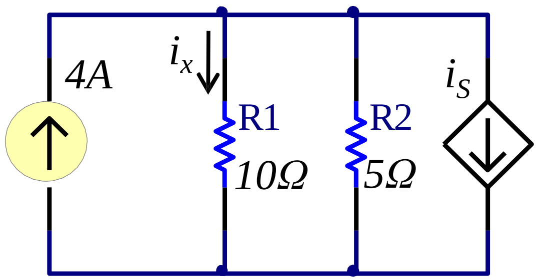


Finally, apply KVL to calculate v_x

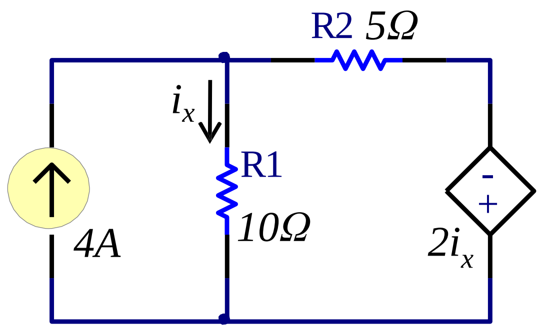
$$\begin{cases} (R_3 + R_4) i + v_x + 18 - v_{s2} = 0 \\ R_4 i + v_x = v_{s2} \end{cases} \rightarrow \begin{cases} 5i + v_x = -15 \\ i + v_x = 3 \end{cases} \rightarrow \boxed{\begin{cases} i = -4.5A \\ v_x = 7.5V \end{cases}}$$

4.4. Source transformation

+ Example 9: Use source transformation to find i_x



$$i_s = \frac{2i_x}{R_2} = 0.4i_x$$



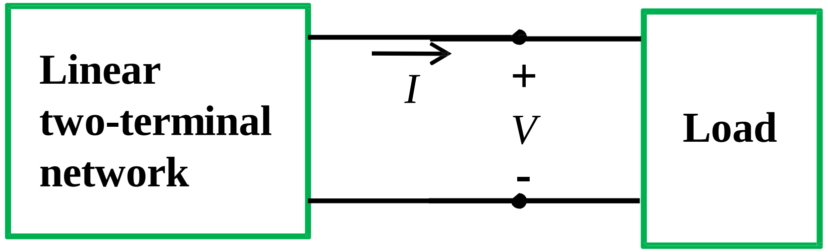
Apply KCL to calculate i_x

$$i_x = (4 - 0.4i_x)\frac{R_2}{R_1 + R_2} \rightarrow 3.4i_x = 4 \rightarrow \boxed{i_x = 1.18A}$$

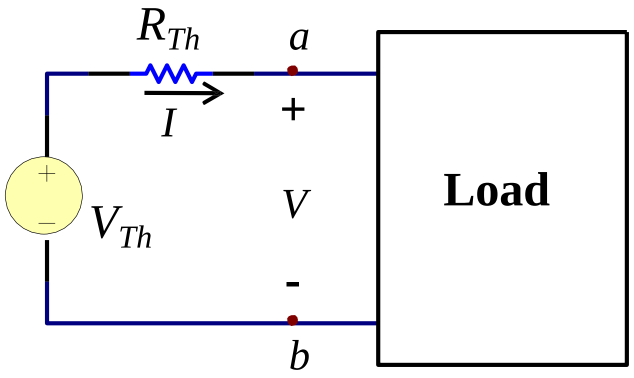
4.5. Thevenin's theorem

+ ***Thevenin's theorem:*** A linear **two terminal network** can be replaced by an **equivalent network** consisting of a voltage source $\boldsymbol{V_{Th}}$ in series with a resister $\boldsymbol{R_{Th}}$ where

- V_{Th} is the *open-circuit voltage* at the terminals
- R_{Th} is the *input or equivalent resistance* at the terminals when all independent sources are turned off



equivalent transformation

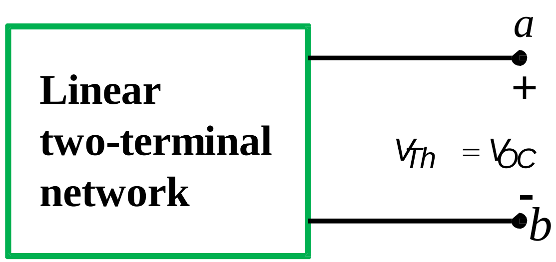


Need to be analyzed all over again ← When variable element (load) is changed

Do not need to be analyzed all over again ← When variable element (load) is changed

4.5. Thevenin's theorem

+ *Find V_{Th}:* V_{Th} is the open-circuit voltage across the terminals

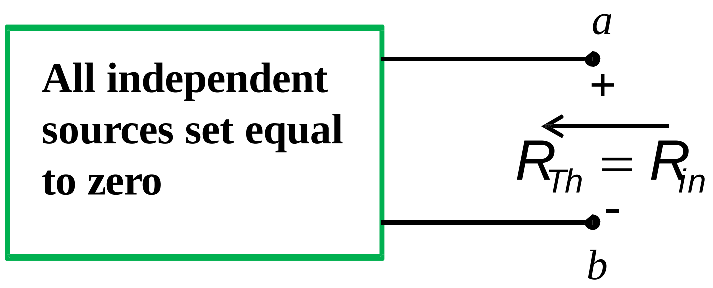


+ *Find R_{Th}:*

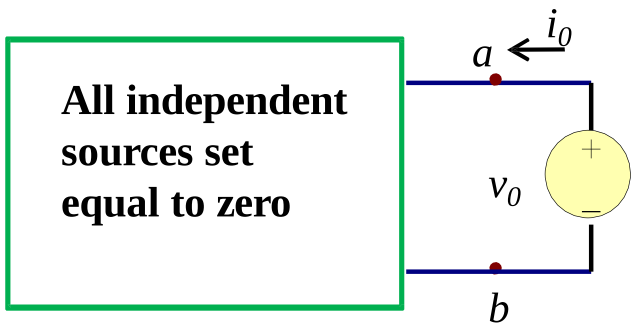
- Network has no dependent sources:

  Turn off all independent sources in network

  R_{Th} → input resistance of the network

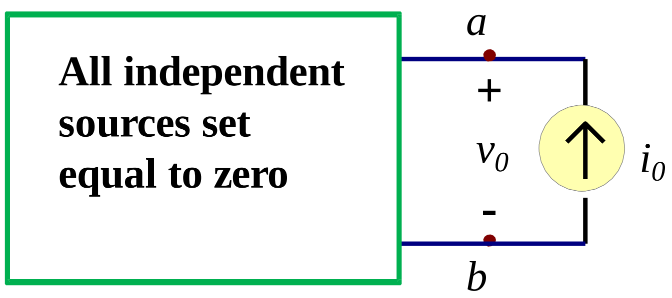


- Network has dependent sources



$$R_{Th} = \frac{v_0}{i_0}$$

then

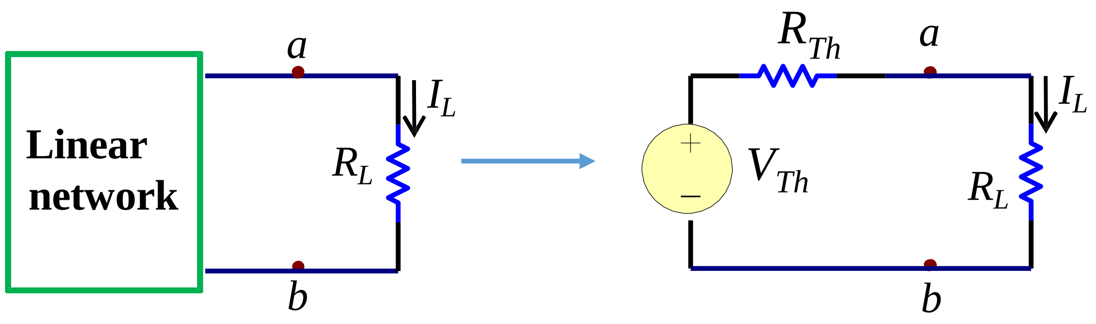


Connect i_0 *or* v_0 *to* terminals, calculate v_0 or i_0 at the port of network

4.5. Thevenin's theorem

+ Advantages of Thevenin's theorem in circuit analysis

- Simplify a circuit: Replace a large circuit by a single independent voltage source and a single resistor
- Easily to determine the current and voltage on the load

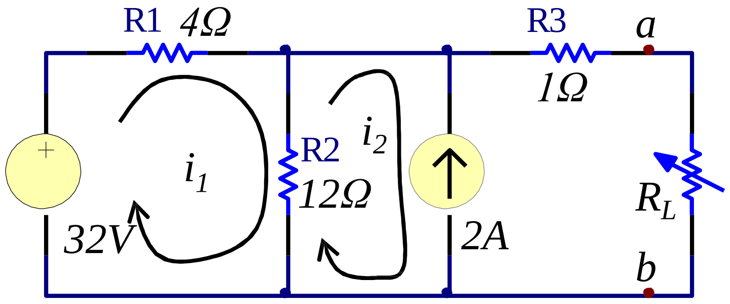


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

4.5. Thevenin's theorem

+ Example 10: Find the Thevenin equivalent network of the circuit. Find the current through $R_L = 6, 16, 36\Omega$



Calculating R_{th} $$R_{th} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 4\Omega$$

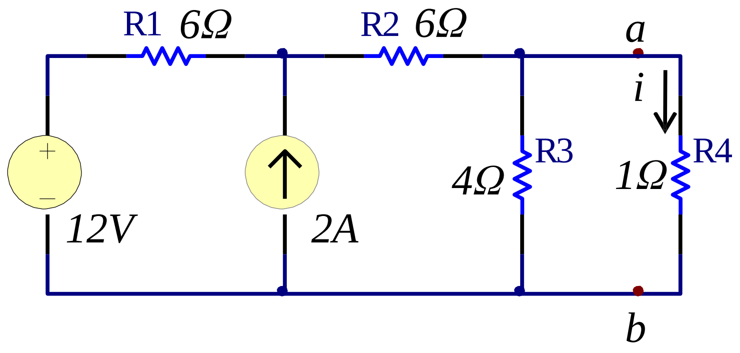
Calculating V_{th} $$\frac{32 - V_{th}}{R_1} - \frac{V_{th}}{R_2} + 2 = 0 \rightarrow V_{th} = 30V$$

Current through R_L $$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{30}{4 + R_L}$$

$$R_L = 6\Omega \rightarrow I_L = \frac{30}{4+6} = 3A \qquad R_L = 16\Omega \rightarrow I_L = \frac{30}{4+16} = 1.5A \qquad R_L = 36\Omega \rightarrow I_L = \frac{30}{4+36} = 0.75A$$

4.5. Thevenin's theorem

+ Example 11: Find *i* using Thevenin theorem



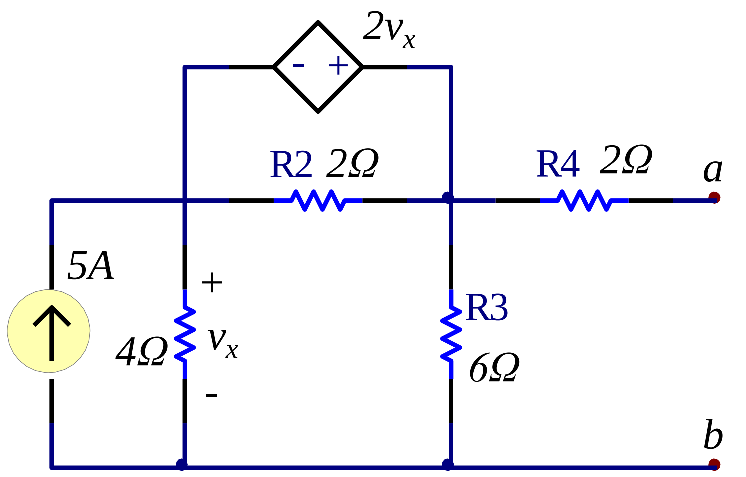
Calculating R_{th} :
$$R_{th} = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} = 3\Omega$$

Calculating V_{th} :
$$\frac{12-V}{R_1} - \frac{V}{R_2+R_3} + 2 = 0 \rightarrow V = 15V \rightarrow V_{th} = \frac{V}{R_2+R_3}R_3 = 6V$$

Current through R_4:
$$i = \frac{V_{th}}{R_{th}+R_4} = \frac{6}{3+1} = 1.5A$$

4.5. Thevenin's theorem

+ Example 12: Find the Thevenin equivalent resistor of the given circuit



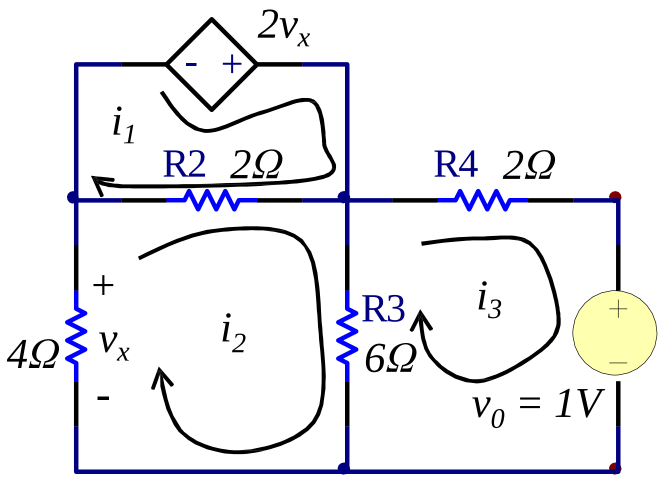
Find R_{Th},:

→ set the independent source equal to zero

→ leave the dependent source intact

→ connect to the terminal a voltage source v_0 = 1V, and find i_0 through the terminal

→ apply mesh current method

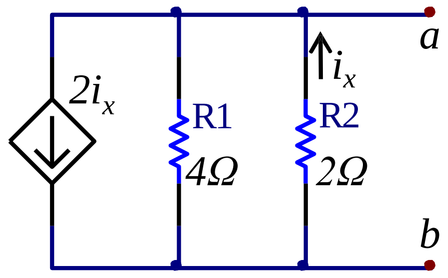


$$\begin{cases} -2v_x + R_2 i_1 - R_2 i_2 = 0 \\ (R_2 + R_3) i_2 - v_x - R_2 i_1 - R_3 i_3 = 0 \\ (R_3 + R_4) i_3 - R_3 i_2 + v_0 = 0 \end{cases} \xrightarrow{v_x = -4i_2} i_0 = -i_3 = \frac{1}{6} A$$

$$R_{th} = \frac{v_0}{i_0} = 6\Omega$$

4.5. Thevenin's theorem

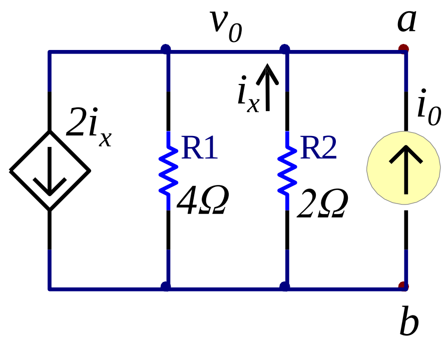
+ Example 13: Find the Thevenin equivalent resistor of the given circuit



Since the circuit has no independent sources → $V_{Th} = 0$

To find R_{th}, connect a current source i_0 to terminals

Apply KCL, we have:



$$\begin{cases} i_0 + i_x = 2i_x + \dfrac{v_0}{R_1} \\ i_x = -\dfrac{v_0}{R_2} \end{cases} \rightarrow i_0 = \frac{v_0}{R_1} - \frac{v_0}{R_2} = -\frac{v_0}{4} \qquad R_{th} = \frac{v_0}{i_0} = -4\Omega$$

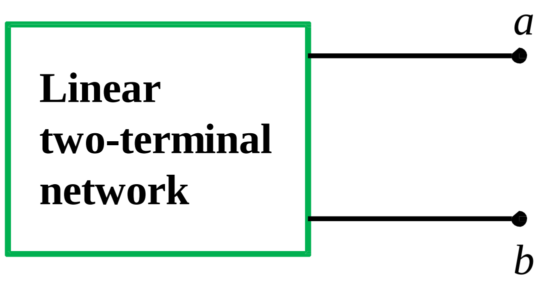
+ Note:

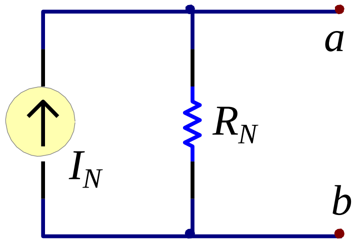
- The negative value of R_{Th} → the circuit is supplying power by the dependent source
- This example shows how a dependent source and resistors could be used to simulated negative resistance

4.6. Norton's theorem

+ *Norton's theorem:* A linear two terminal network can be replaced by an equivalent network consisting of a current source I_N in parallel with a resistor R_N, where:

- I_N is the short circuit current through the terminals
- R_N is the input or equivalent resistance at the terminals when all independent sources are turned off



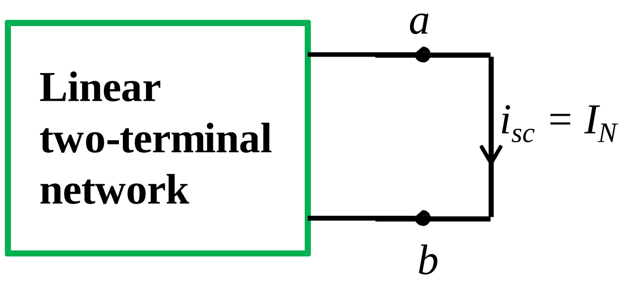


$$\boxed{\begin{aligned} R_N &= R_{Th} \\ I_N &= i_{sc} \end{aligned}}$$

+ Relationship between Norton's and Thevenin's theorems:

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

(Source transformation)



4.6. Norton's theorem

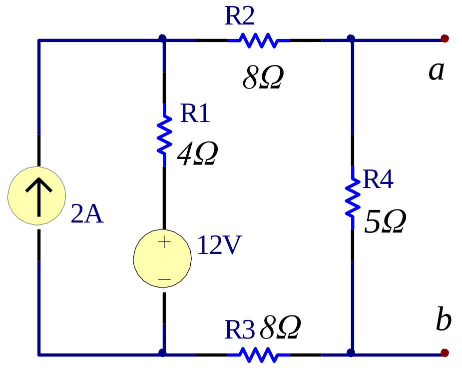
+ To determine the Thevenin or Norton equivalent circuit, we need to find

- open-circuit voltage v_{oc} across terminals *a* and *b*
- short-circuit current i_{sc} at terminals *a* and *b*
- input or equivalent resistance R_{in} at terminals *a* and *b* when all independent sources are turned off

$$V_{Th} = v_{oc} \quad ; \quad I_N = i_{sc} \quad ; \quad R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

4.6. Norton's theorem

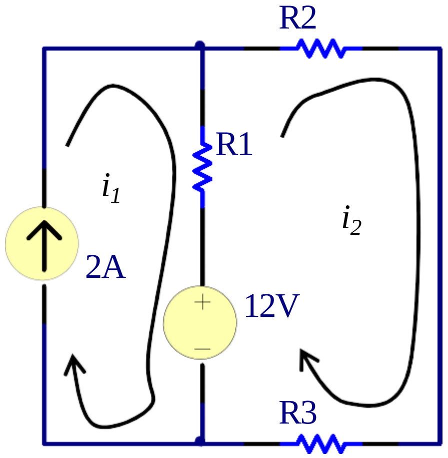
+ Example 14: Find the Norton equivalent circuit for the given circuit



Find R_N in the same way R_{Th} $$R_N = R_4 // (R_1 + R_2 + R_3) = \frac{5(4+8+8)}{5+4+8+8} = 4\Omega$$

Find I_N by shortening circuit terminals *a* and *b*

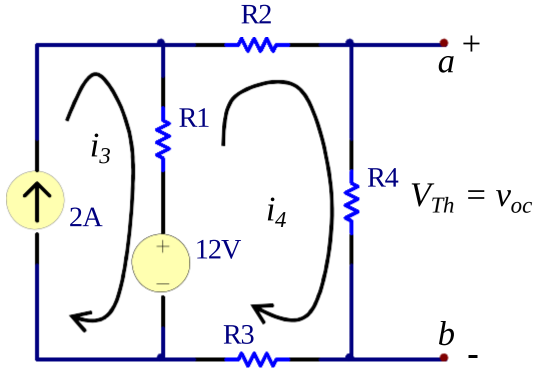
$$\begin{cases} i_1 = 2 \\ (R_1 + R_2 + R_3) i_2 - R_1 i_1 = 12 \end{cases} \rightarrow \begin{cases} i_1 = 2 \\ -4i_1 + 20i_2 = 12 \end{cases} \rightarrow i_2 = i_{sc} = I_N = 1A$$



Another way, we can find I_N by the source transform equation

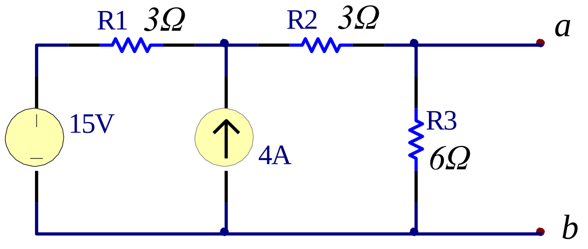
$$\begin{cases} i_3 = 2 \\ (R_1 + R_2 + R_3 + R_4) i_4 - R_1 i_3 = 12 \end{cases} \rightarrow \begin{cases} i_3 = 2 \\ -4i_3 + 25i_4 = 12 \end{cases}$$

$$V_{Th} = v_{os} = R_4 i_4 = 5 \times 0.8 = 4V \qquad I_N = \frac{V_{Th}}{R_{Th}} = 1A$$



4.6. Norton's theorem

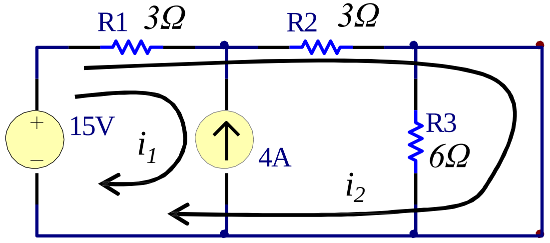
+ Example 15: Find the Norton equivalent circuit for the given circuit



Find R_N in the same way R_{Th}

$$R_N = R_{Th} = R_3 // (R_1 + R_2) = \frac{6(3+3)}{6+3+3} = 3\Omega$$

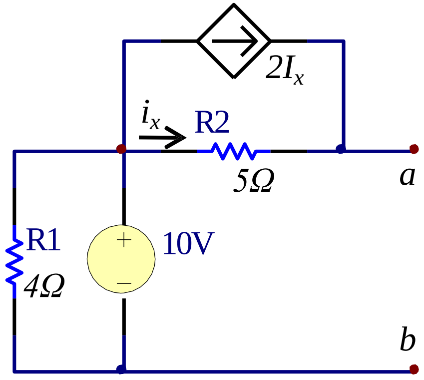
Find I_N by shortening circuit terminals *a* and *b*



$$\begin{cases} i_1 = -4A \\ (R_1 + R_2)i_2 + R_1 i_1 = 15 \end{cases} \rightarrow \begin{cases} i_1 = -4 \\ 3i_1 + 6i_2 = 15 \end{cases} \rightarrow i_2 = i_{sc} = I_N = 4.5A$$

4.6. Norton's theorem

+ Example 16: Using Norton theorem find R_N and I_N at terminals *a-b*

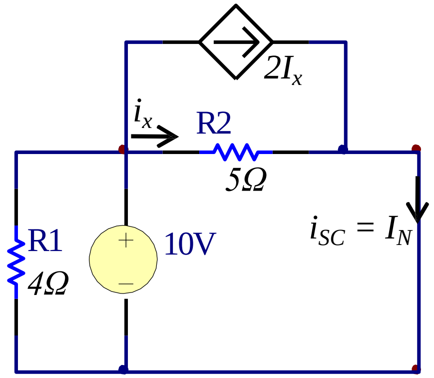


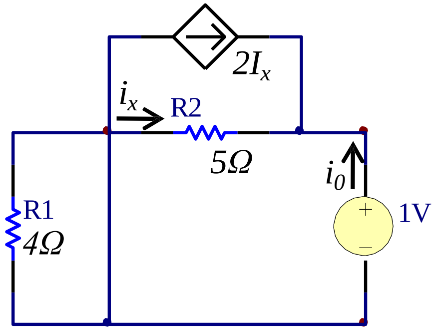
Find R_N: set the independent voltage source equal to zero and connect a voltage source of $v_0 = 1V$ to *a-b*

$$\begin{cases} i_0 + i_x + 2i_x = 0 \\ R_2 i_x = -v_0 = -1 \end{cases} \rightarrow \begin{cases} i_x = -\dfrac{v_0}{5} = -0.2A \\ i_0 = -3i_x = 0.6A \end{cases} \rightarrow R_N = \frac{v_0}{i_0} = \frac{1}{0.6} = 1.67\Omega$$

Find I_N by shortening circuit terminals *a* and *b*

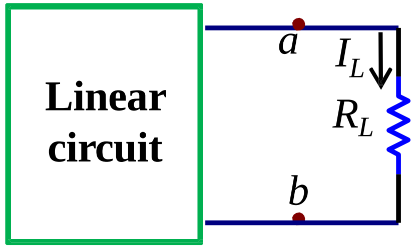
$$\begin{cases} R_2 i_x = 10 \\ I_N = i_{sc} = 3i_x \end{cases} \rightarrow \begin{cases} i_x = 2A \\ I_N = 6A \end{cases}$$





4.7. Maximum power transfer

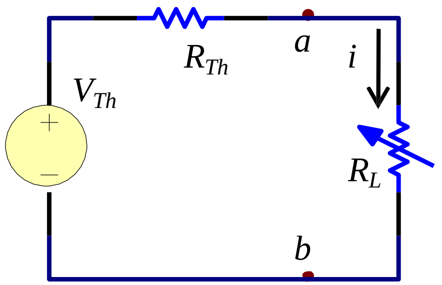
+ In many practical situations, a circuit is designed to provide power to a load
    - Electric utilities: Minimizing power losses in the process of distribution
    - Communications: Maximize the power delivered to a load



+ Problem: Delivering p_{max} to a load when given a system with known internal losses

→ Assuming that the load resistance R_L can be adjusted

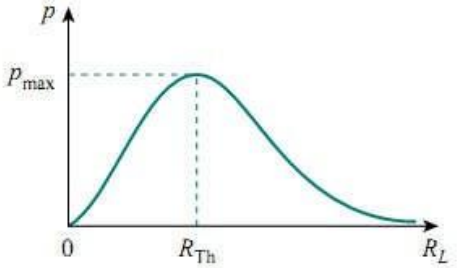
→ Replacing entire circuit by Thevenin equivalent circuit



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \rightarrow \boxed{p = p_{\max} \Leftrightarrow R_L = R_{Th}}$$

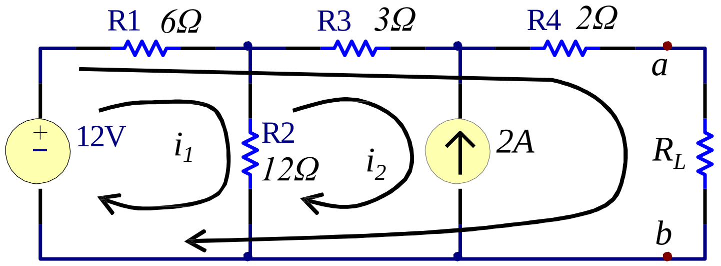
+ Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)

$$\boxed{p_{\max} = \frac{V_{Th}^2}{4R_{Th}}}$$



4.7. Maximum power transfer

+ Example 17: Find the value of R_L for maximum power transfer then find the maximum power



Find R_{Th}

$$R_{Th} = R_4 + R_3 + \frac{R_1R_2}{R_1+R_2} = 9\Omega$$

Find V_{Th}

$$\begin{cases} R_1i_1 + R_2(i_1 - i_2) = 12 \\ i_2 = -2 \end{cases} \rightarrow \begin{cases} i_1 = -\frac{2}{3}A \\ i_2 = -2A \end{cases}$$

Apply KVL for the outer loop (open circuit) to get V_{Th}

$$R_1i_1 + R_3i_2 + V_{Th} = 12 \rightarrow V_{Th} = 22V$$

Another way to calculate V_{Th}

$$V_{Th} = R_2(i_1 - i_2) + R_3I_s \rightarrow V_{Th} = 22V$$

For maximum power transfer:

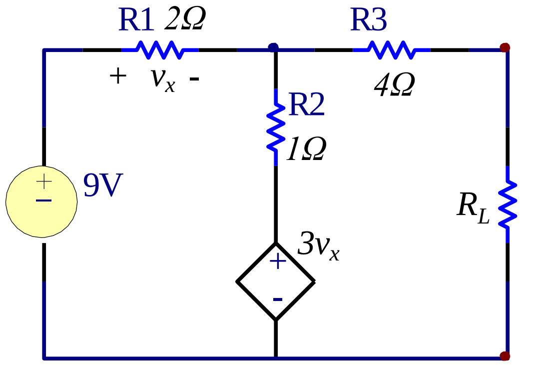
$$R_L = R_{Th} = 9\Omega$$

Maximum power:

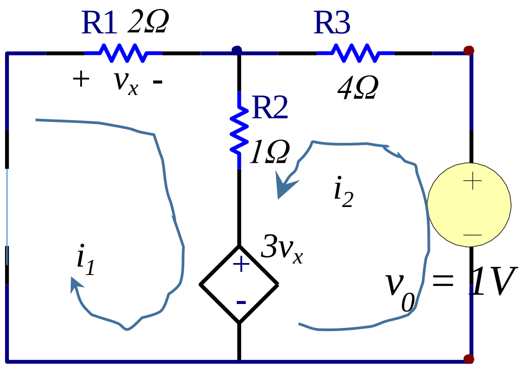
$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{22^2}{4\times 9} = 13.44W$$

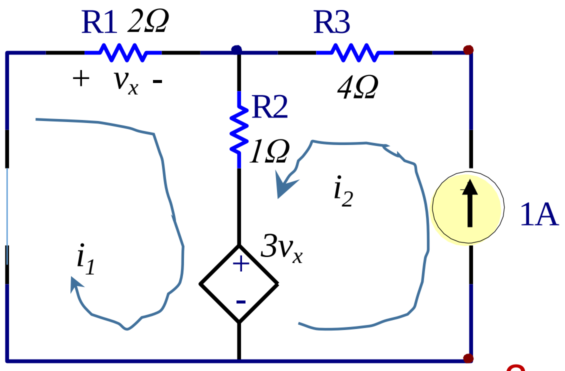
4.7. Maximum power transfer

+ Example 18: Find the value of R_L for maximum power transfer then find the maximum power



Find $\boldsymbol{R_{Th}}$:





1st approach:

$$\begin{cases}(R_1+R_2)i_1+R_2i_2+3R_1i_1=0\\ R_2i_1+(R_2+R_3)i_2+3R_1i_1=1\end{cases}\rightarrow\begin{cases}9i_1+i_2=0\\ 7i_1+5i_2=1\end{cases}\rightarrow\begin{cases}i_1=\dfrac{-1}{38}\\ i_2=\dfrac{9}{38}\end{cases}$$

$$i_0=i_2=\frac{9}{38}A\rightarrow\boxed{R_{Th}=\frac{v_0}{i_0}=\frac{38}{9}\approx 4.22\Omega}$$

2nd approach:

$$\begin{cases}i_2=1A\\ R_1i_1+R_2(i_1+i_2)+3R_1i_1=0\end{cases}\rightarrow\begin{cases}i_1=-\dfrac{1}{9}A\\ i_2=1A\end{cases}$$

$$-v_0+R_3i_2-v_x=0\rightarrow v_0=4.1+2\frac{1}{9}=\frac{38}{9}V\rightarrow\boxed{R_{Th}=\frac{v_0}{i_0}\approx 4.22\Omega}$$

Find $\boldsymbol{V_{Th}}$: $(R_1+R_2)i+3R_1i=9\rightarrow i=1A\rightarrow\boxed{V_{Th}=R_2i+3R_1i=7V}$

$$\boxed{R_L=4.22\Omega\quad P_{\max}=\frac{7^2}{4\times 4.22}=2.9W}$$