Nguyễn Công Phương

Electric Circuit Theory

Frequency Response

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Frequency Response

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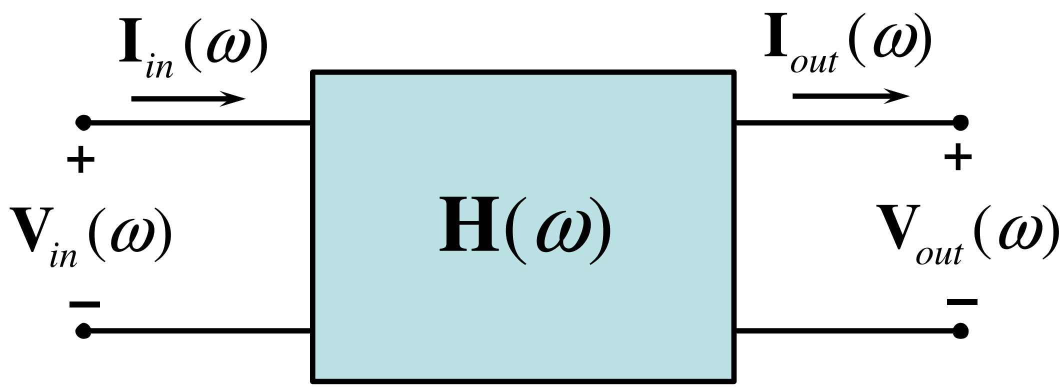
Transfer Function (1)

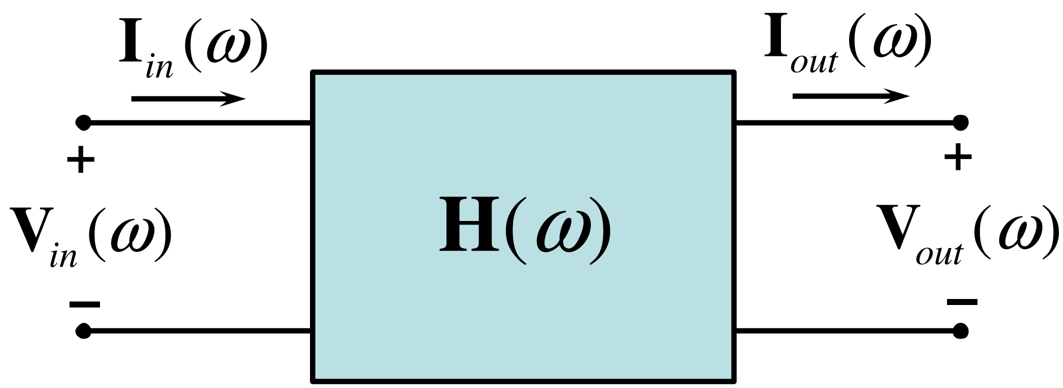


$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{Out}(\omega)}{\mathbf{In}(\omega)}}$$

$$\mathbf{H}_{voltage}(\omega) = \frac{\mathbf{V}_{out}(\omega)}{\mathbf{V}_{in}(\omega)} \qquad \mathbf{H}_{current}(\omega) = \frac{\mathbf{I}_{out}(\omega)}{\mathbf{I}_{in}(\omega)}$$

$$\mathbf{H}_{impedance}(\omega) = \frac{\mathbf{V}_{out}(\omega)}{\mathbf{I}_{in}(\omega)} \qquad \mathbf{H}_{admittance}(\omega) = \frac{\mathbf{I}_{out}(\omega)}{\mathbf{V}_{in}(\omega)}$$

Transfer Function (2)



$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{Out}(\omega)}{\mathbf{In}(\omega)}}$$

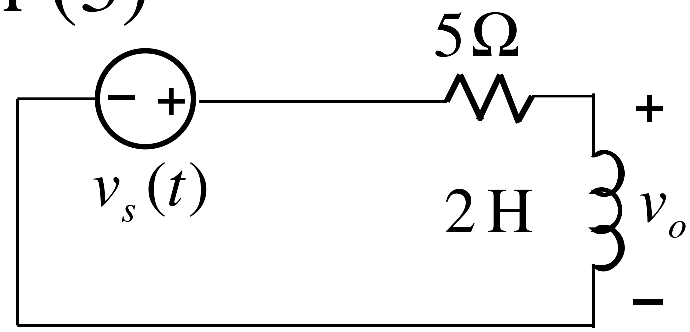
$$\mathbf{Out}(\omega) = 0 \rightarrow z_1, z_2, \ldots \ \text{(zeros)}$$

$$\mathbf{In}(\omega) = 0 \rightarrow p_1, p_2, \ldots \ \text{(poles)}$$

Transfer Function (3)

Ex. 1

$v_s = 100\sin\omega t$ (V). Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and sketch its frequency response.

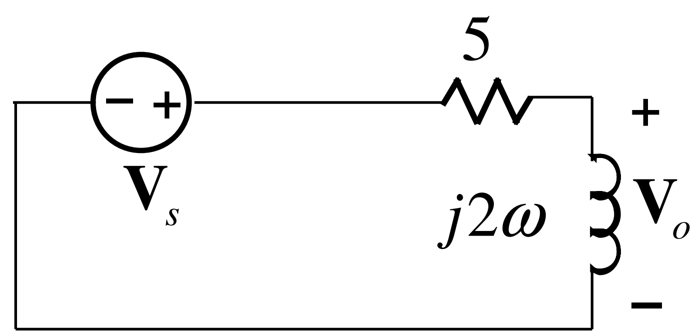


$$\mathbf{V}_o = j2\omega\frac{\mathbf{V}_s}{5 + j2\omega}$$

$$\rightarrow \mathbf{H}_v(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j2\omega}{5 + j2\omega}$$

$$= \frac{j2\omega(5 - j2\omega)}{(5 + j2\omega)(5 - j2\omega)} = \frac{4\omega^2}{25 + 4\omega^2} + j\frac{10\omega}{25 + 4\omega^2} = H_v \underline{/\phi_v}$$

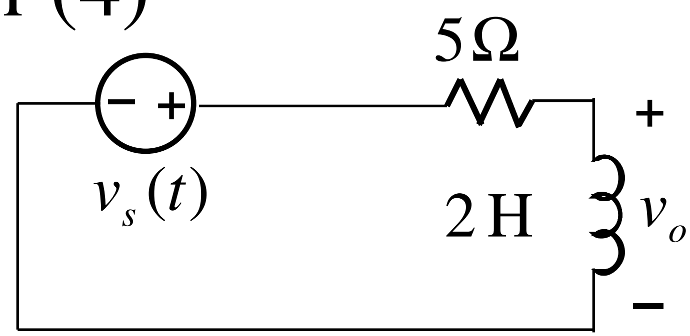
$$\boxed{H_v = \frac{\sqrt{16\omega^4 + 100\omega^2}}{4\omega^2 + 25};\ \phi_v = \tan^{-1}\frac{5}{2\omega}}$$



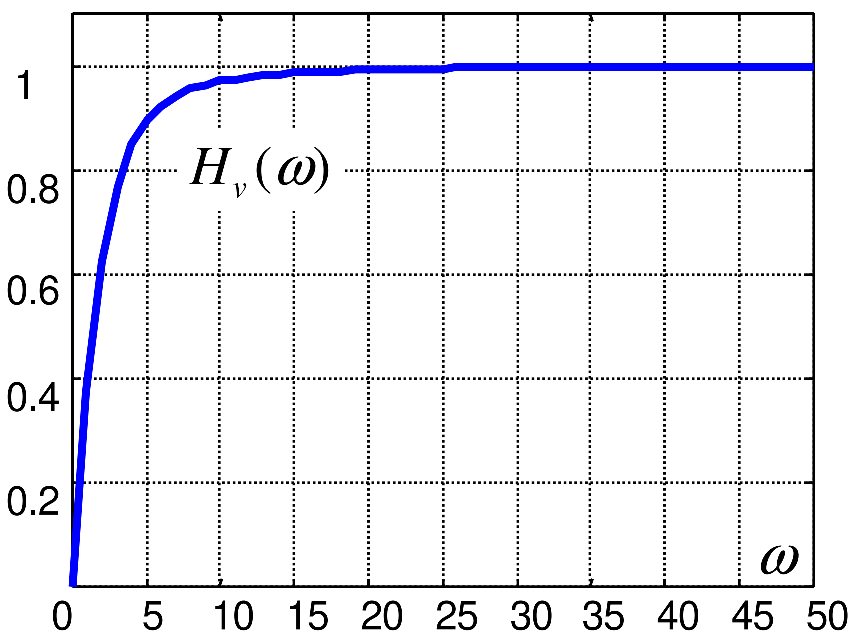
Transfer Function (4)

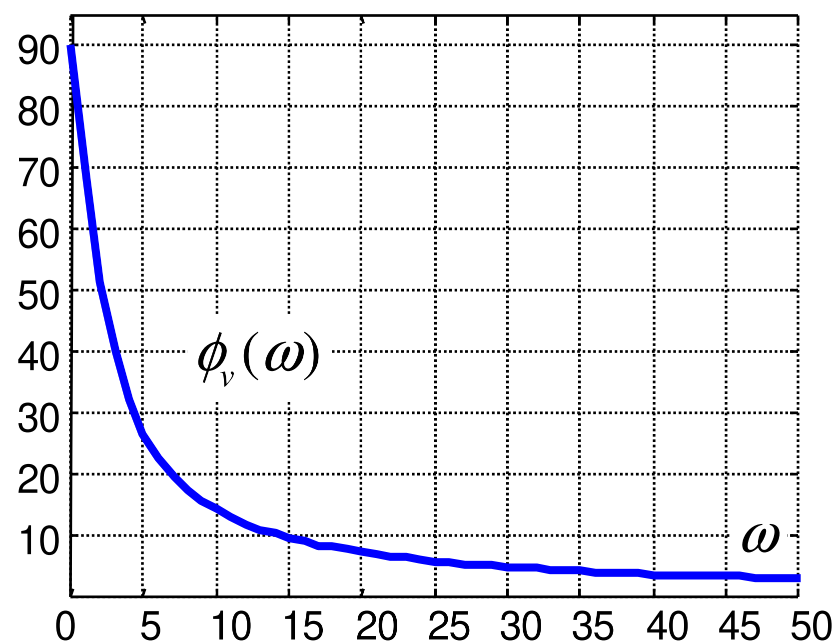
Ex. 1

$v_s = 100\sin\omega t$ (V). Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and sketch its frequency response.



$$H_v = \frac{\sqrt{16\omega^4 + 100\omega^2}}{4\omega^2 + 25};\ \phi_v = \tan^{-1}\frac{5}{2\omega}$$

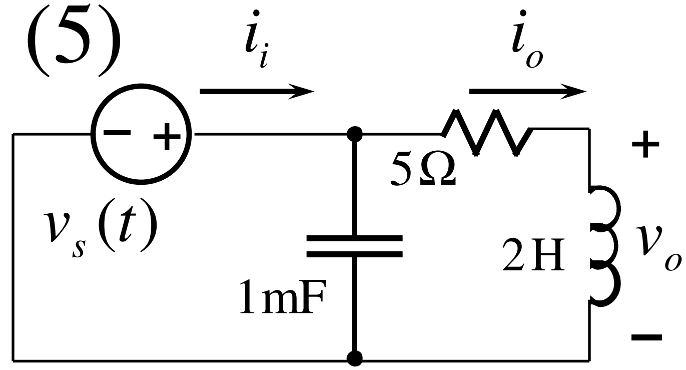




Transfer Function (5)

Ex. 2

$v_s = 100\sin\omega t$ (V). Find the transfer functions $\mathbf{V}_o/\mathbf{V}_s$, $\mathbf{I}_o/\mathbf{I}_i$, $\mathbf{V}_o/\mathbf{I}_i$, & $\mathbf{I}_o/\mathbf{V}_s$.



Transfer Function (6)

Ex. 3

Given a transfer function $\mathbf{H}(j\omega) = \dfrac{j2\omega}{20 + j2\omega}$ & an input of $5\sin 10t$, find the output?

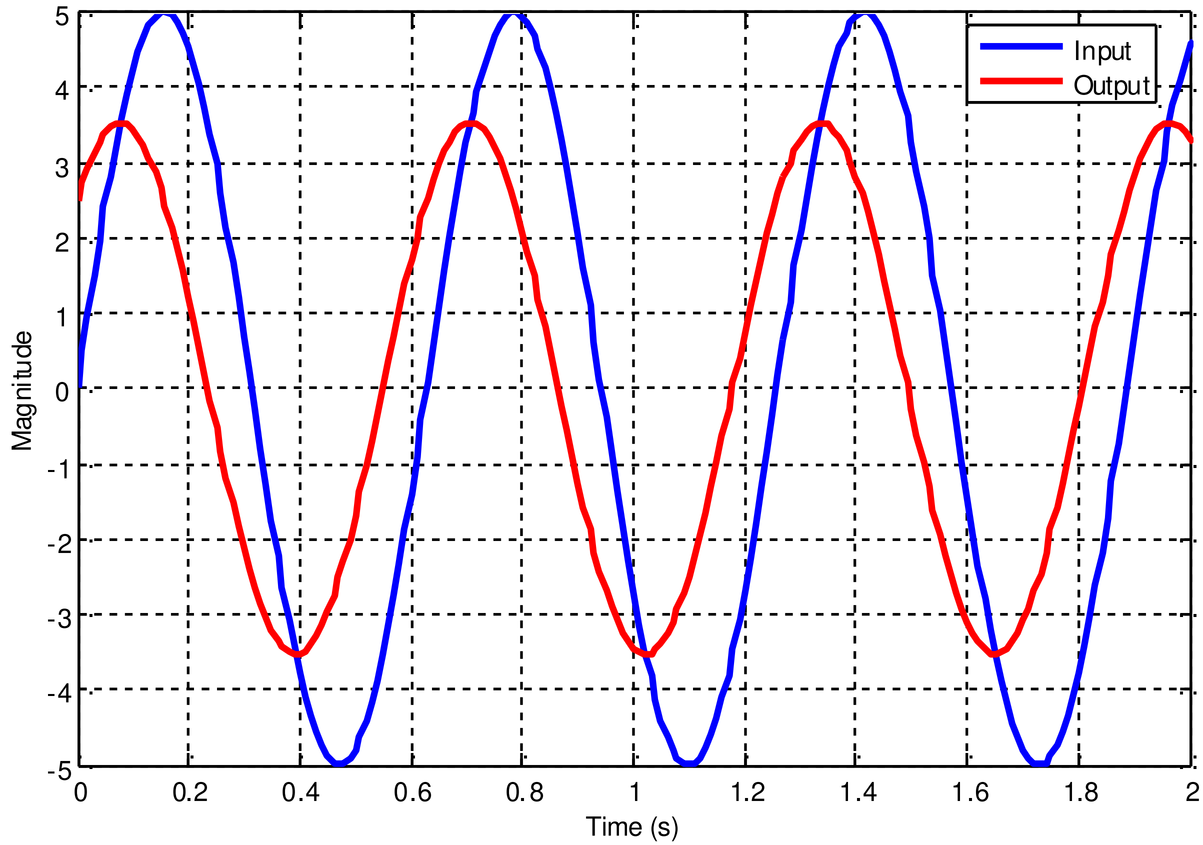
$$\frac{\mathbf{Out}(j\omega)}{\mathbf{In}(j\omega)} = \mathbf{H}(j\omega)$$

$$\rightarrow \mathbf{Out}(j\omega) = \mathbf{H}(j\omega)\mathbf{In}(j\omega)$$

$$= \frac{j2\times 10}{20 + j2\times 10}\times 5$$

$$= 3.53\underline{/45.0^\circ}$$

$$\rightarrow output = 3.53\sin(10t + 45.0^\circ)$$



Frequency Response

1. Transfer Function
2. **The Decibel Scale**
3. Bode Plots
4. Series Resonance
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The Decibel Scale

$$G = \log_{10} \frac{P_2}{P_1}$$

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1}$$

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1}$$

$$G_{dB} = 20 \log_{10} \frac{I_2}{I_1}$$

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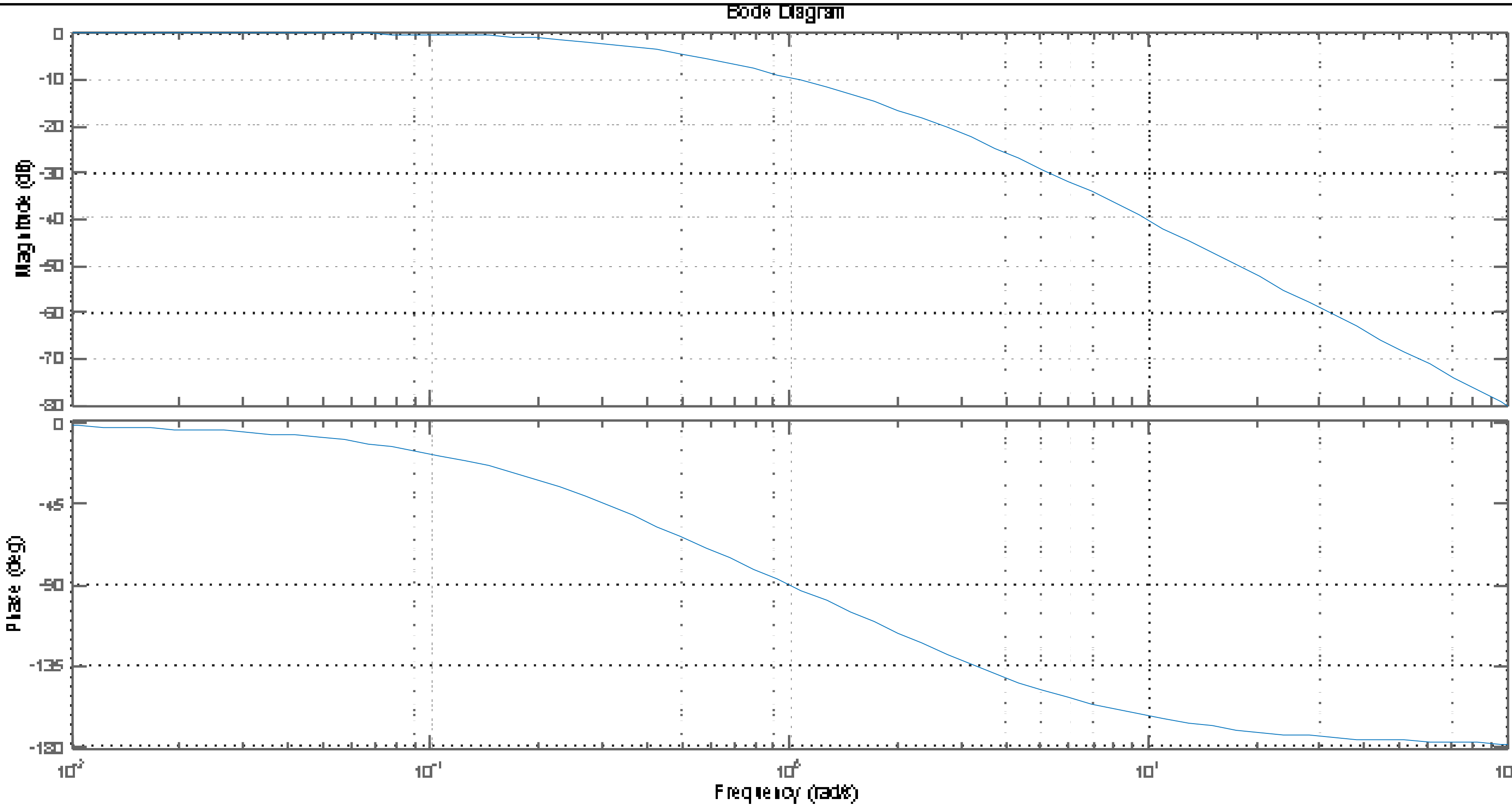
Bode Plots (1)

Semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency



Bode Plots (2)

Semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency

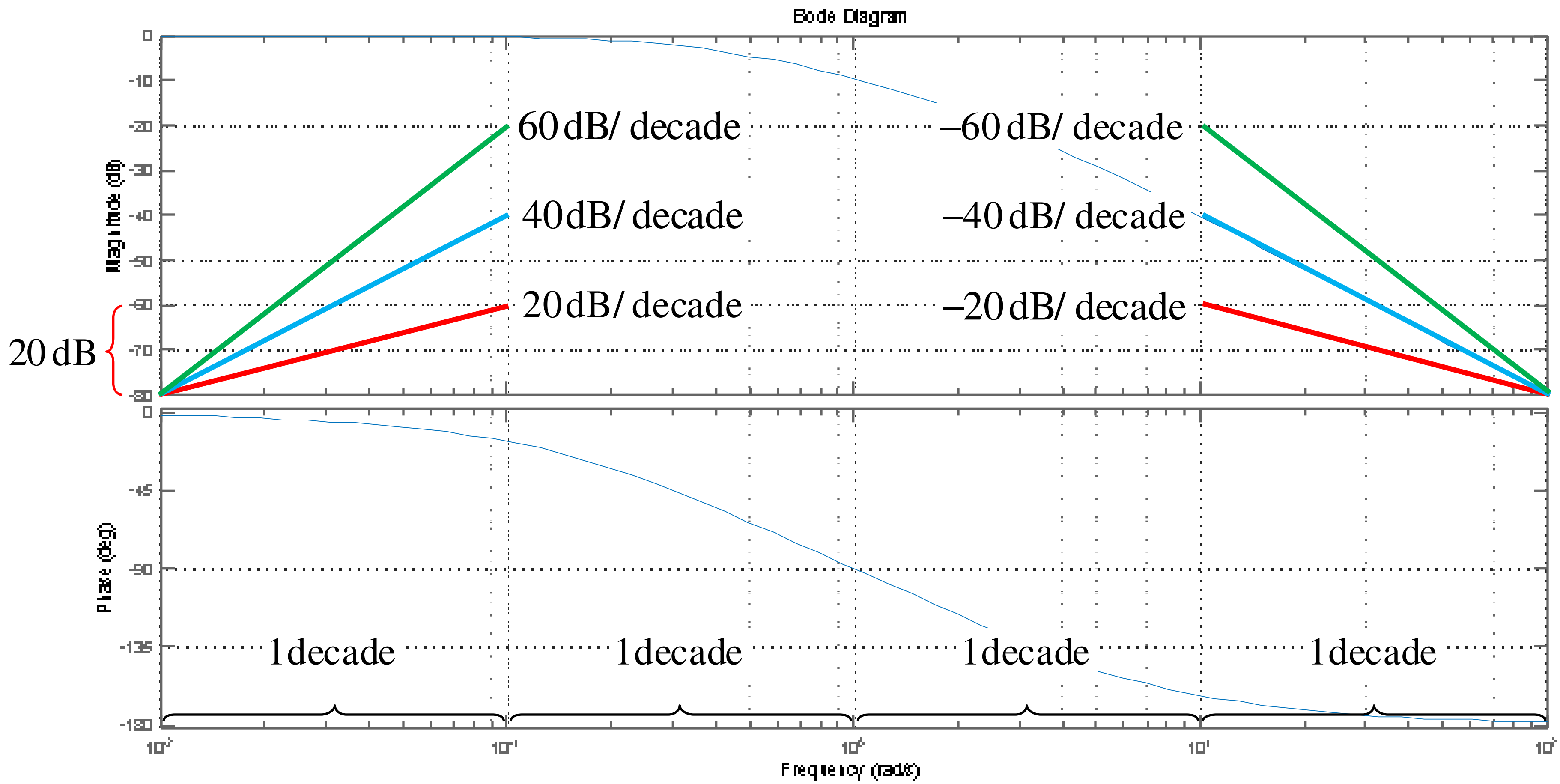
$$\mathbf{H} = H\underline{/\phi} \rightarrow \begin{cases} 20\log_{10} H \\ \phi \end{cases}$$

$$\mathbf{H} = \mathbf{H}_1\mathbf{H}_2\mathbf{H}_3\ldots = \left(H_1\underline{/\phi_1}\right)\left(H_2\underline{/\phi_2}\right)\left(H_3\underline{/\phi_3}\right)\ldots$$

$$= \left(H_1H_2H_3\ldots\right)\underline{/\phi_1+\phi_2+\phi_3+\ldots}$$

$$\rightarrow \begin{cases} 20\log_{10} H = 20\log_{10} H_1 + 20\log_{10} H_2 + 20\log_{10} H_3 + \ldots \\ \phi = \phi_1 + \phi_2 + \phi_3 + \ldots \end{cases}$$

Bode Plots (3)


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
60 dB/ decade
40 dB/ decade
20 dB/ decade
–60 dB/ decade
–40 dB/ decade
–20 dB/ decade
20 dB
1 decade
1 decade
1 decade
1 decade

Bode Plots (4)

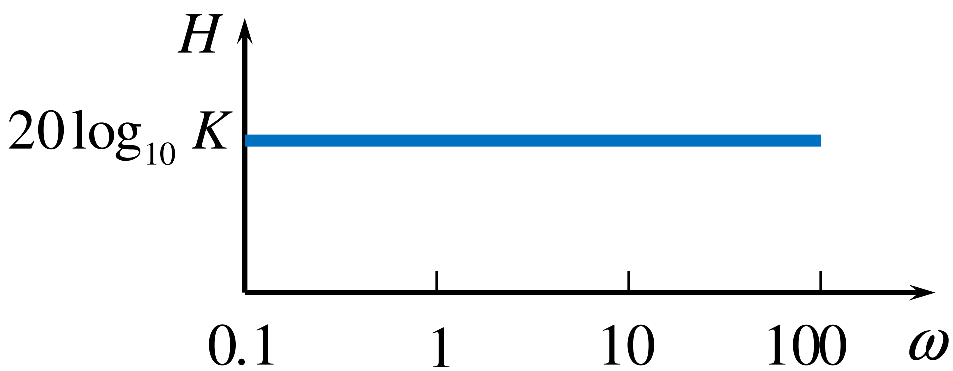
$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+\frac{j2\zeta_1\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\ldots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+\frac{j2\zeta_2\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\ldots}$$

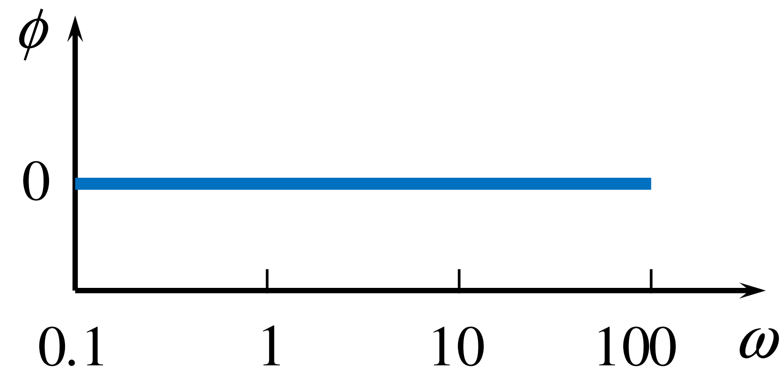
K : gain

$\frac{1}{j\omega}$: pole at the origin	$\frac{1}{1+\frac{j\omega}{p_1}}$: simple pole	$\frac{1}{1+\frac{j2\zeta_2\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2}$: quadratic pole
$j\omega$: zero at the origin	$1+\frac{j\omega}{z_1}$: simple zero	$1+\frac{j2\zeta_1\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2$: quadratic zero

Bode Plots (5)

$$\mathbf{H}(\omega) = K \rightarrow \begin{cases} H_{dB} = 20\log_{10} K \\ \phi = 0 \end{cases}$$

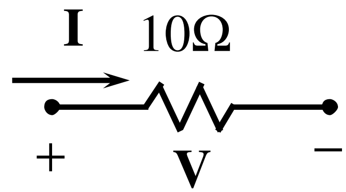


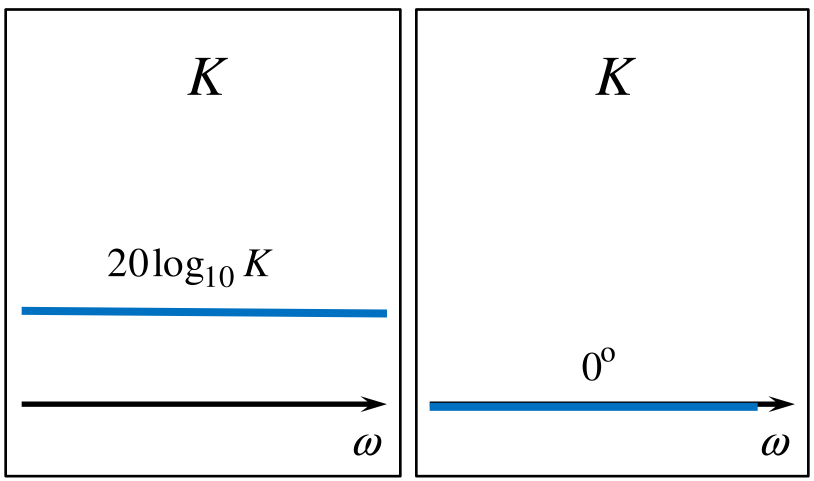


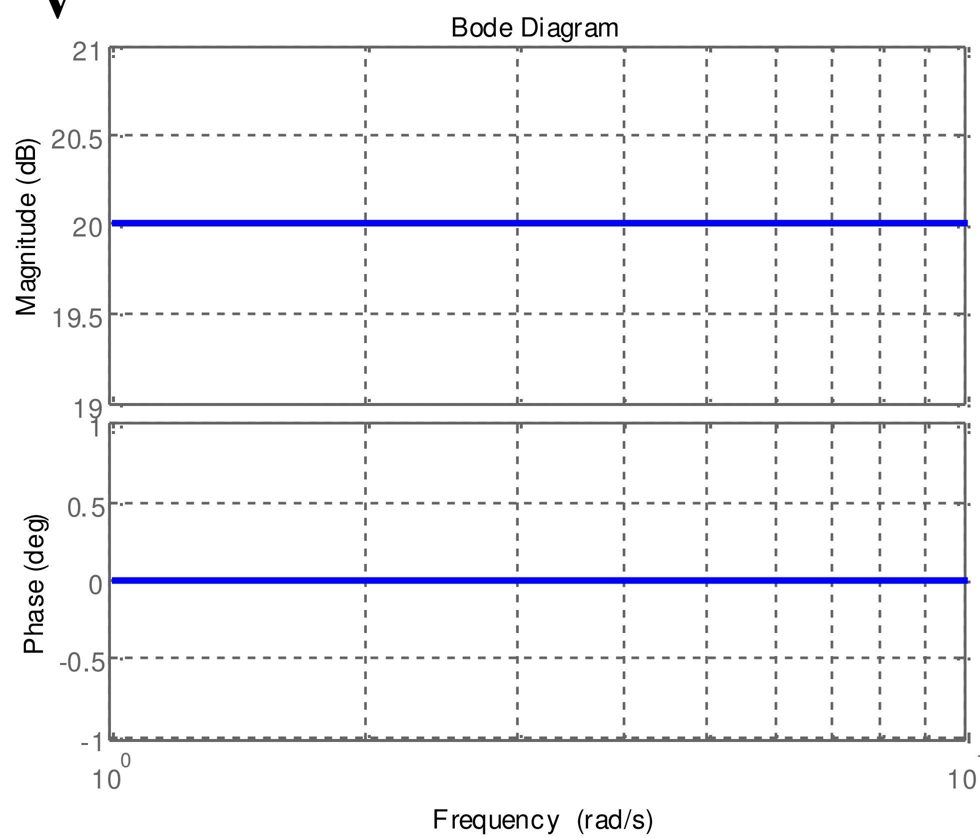
Bode Plots (6)

Ex. 1

$$\mathbf{H}(\omega) = \frac{\mathbf{V}}{\mathbf{I}} = \frac{10\mathbf{I}}{\mathbf{I}} = 10$$

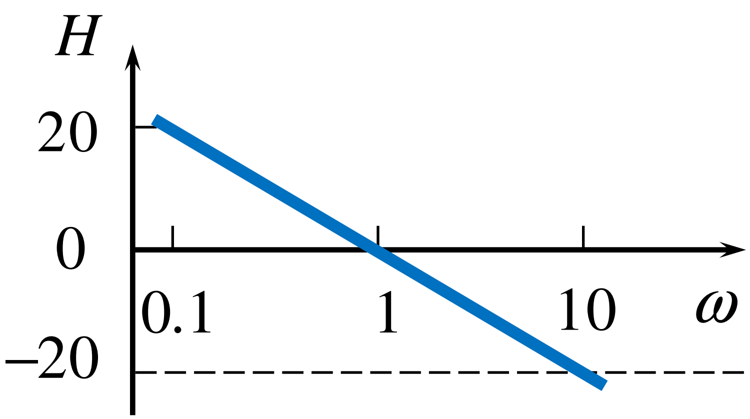


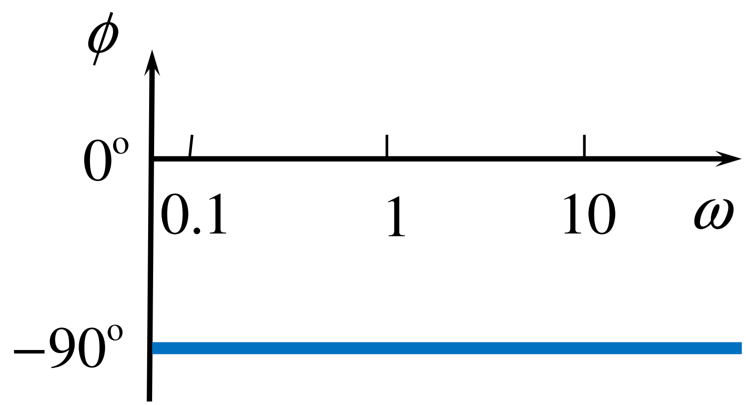




Bode Plots (7)

$$\mathbf{H}(\omega) = \frac{1}{j\omega} \rightarrow \begin{cases} H_{dB} = -20\log_{10}\omega \\ \phi = -90^{\circ} \end{cases}$$

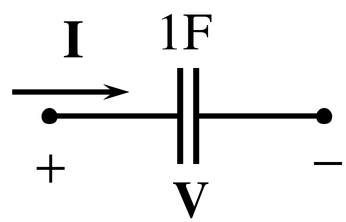


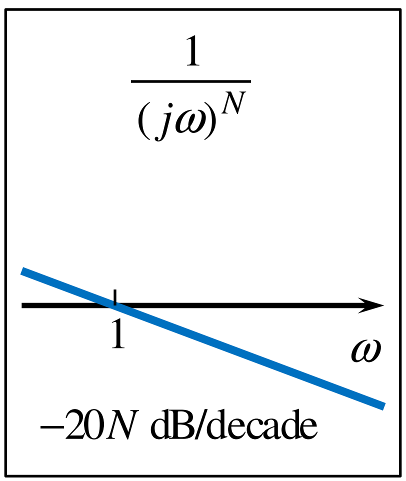


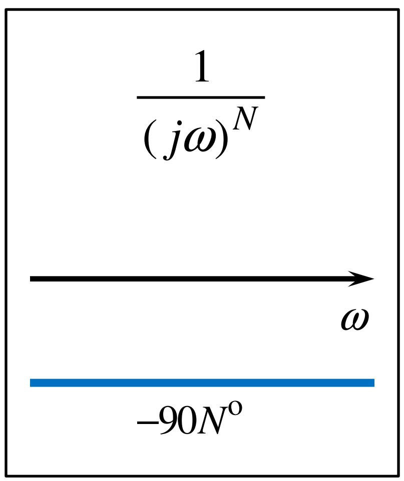
Bode Plots (8)

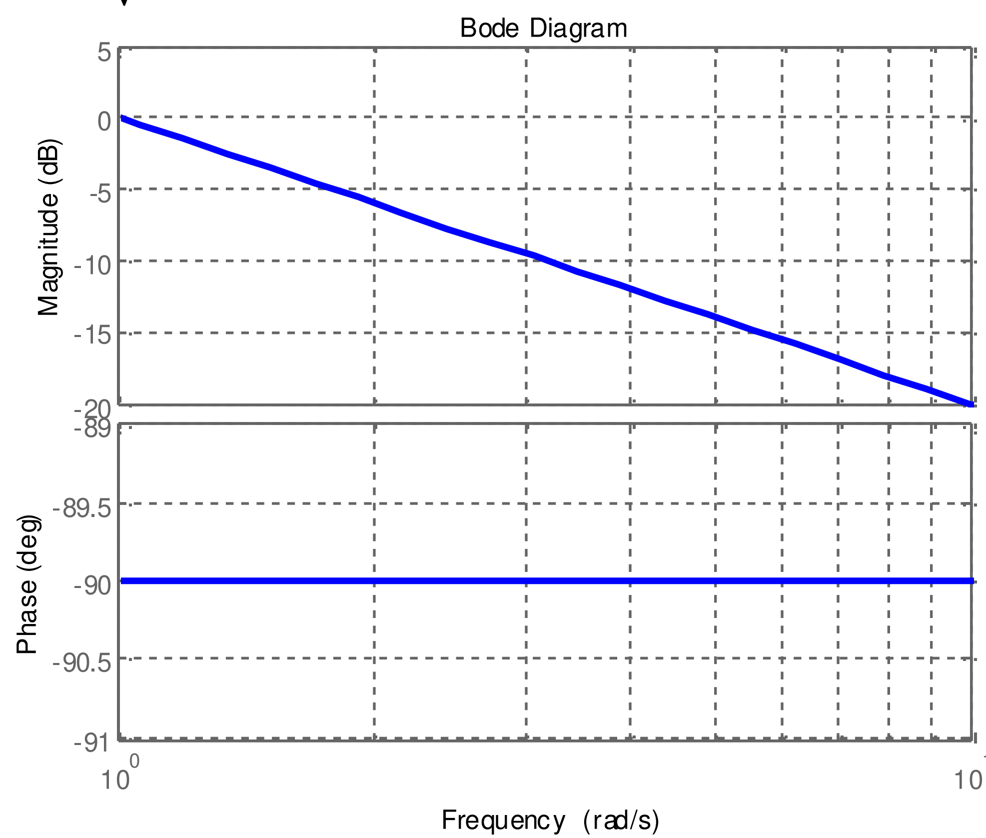
Ex. 2

$$\mathbf{H}(\omega) = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega}$$



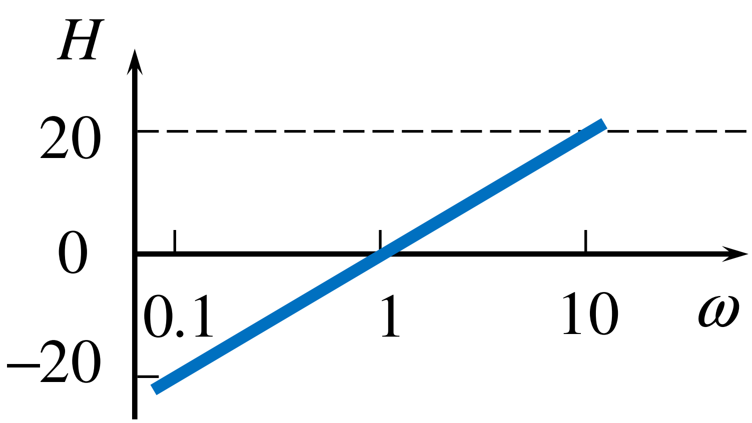


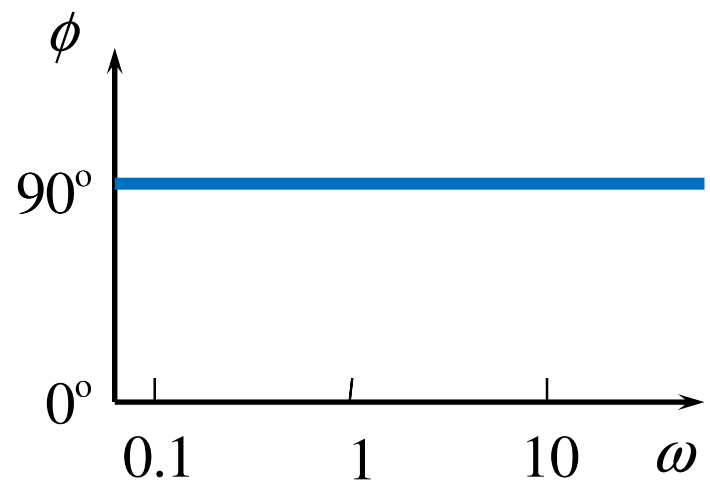




Bode Plots (9)

$$\mathbf{H}(\omega) = j\omega \rightarrow \begin{cases} H_{dB} = 20\log_{10}\omega \\ \phi = 90^{\text{o}} \end{cases}$$

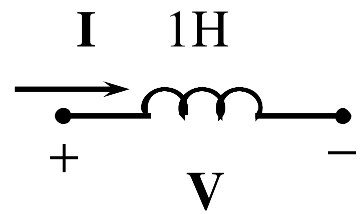


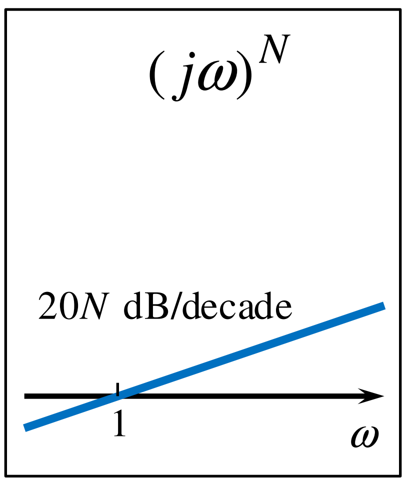


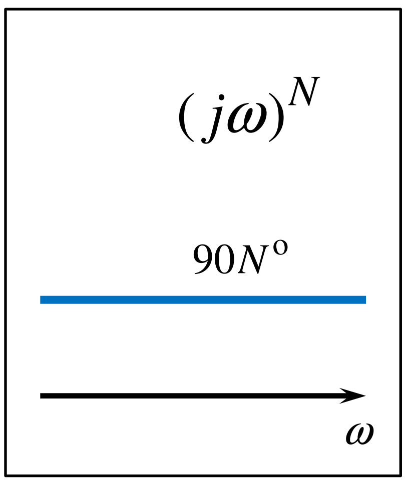
Bode Plots (10)

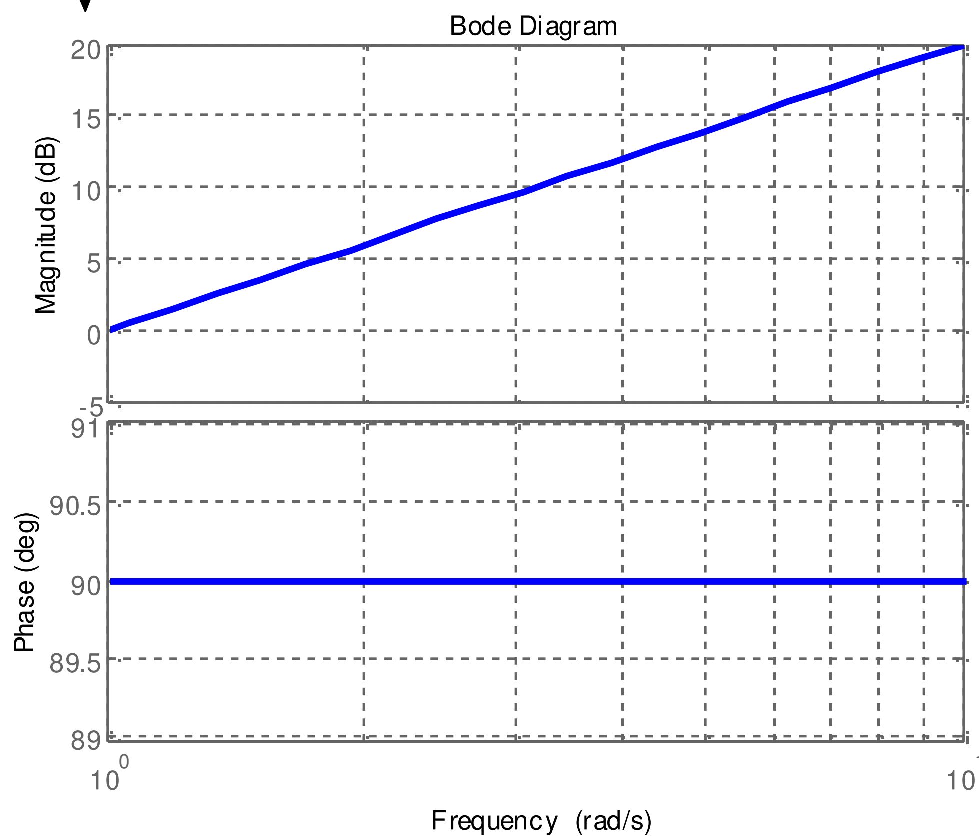
Ex. 3

$$\mathbf{H}(\omega) = \frac{\mathbf{V}}{\mathbf{I}} = j\omega$$



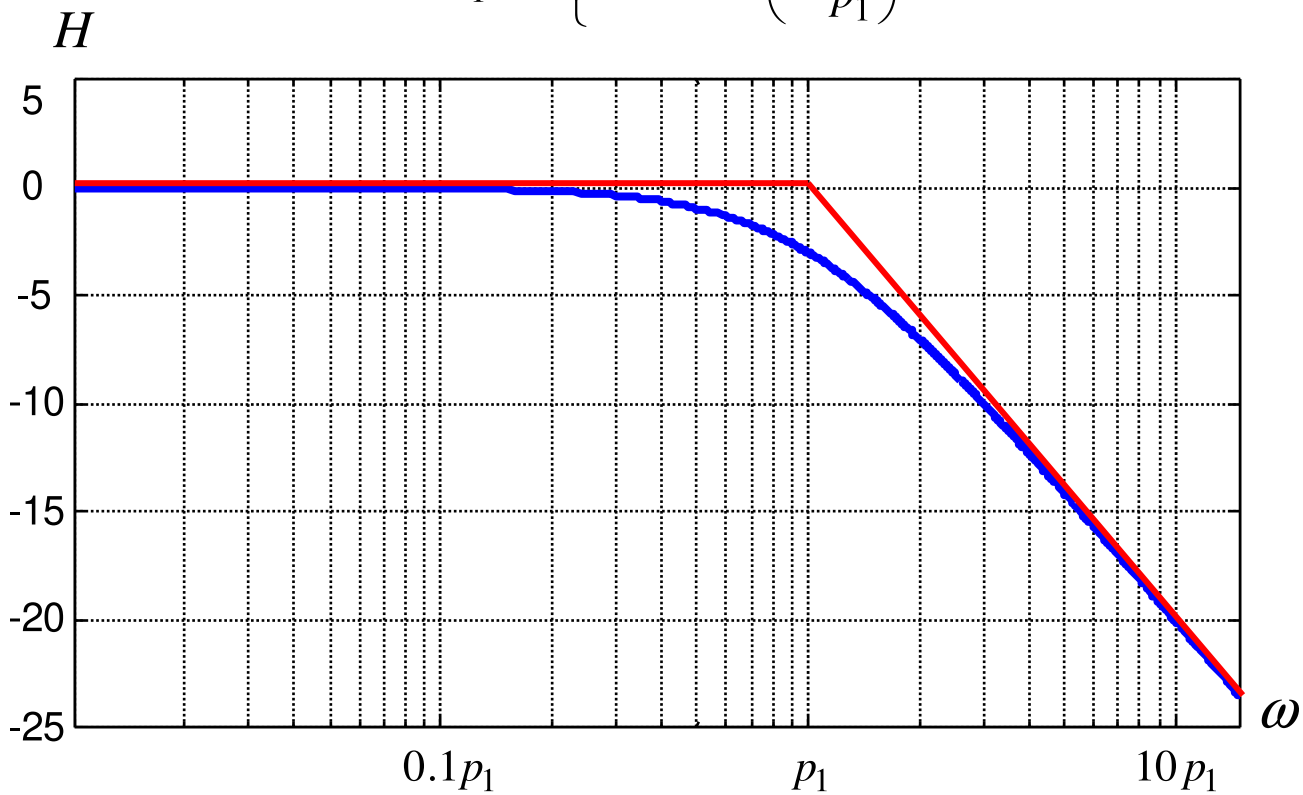






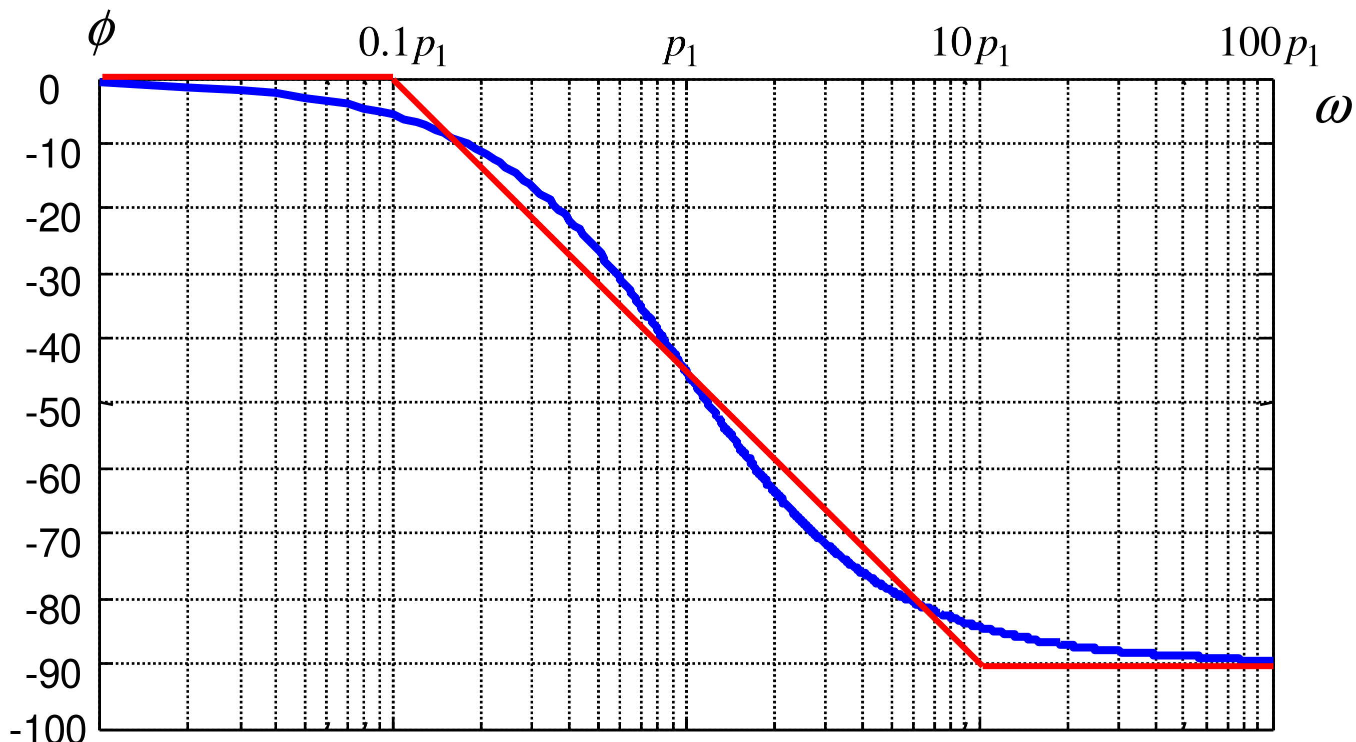
Bode Plots (11)

$$\mathbf{H}(\omega) = \frac{1}{1+\dfrac{j\omega}{p_1}} \rightarrow \begin{cases} H_{dB} = -20\log_{10}\left|1+\dfrac{j\omega}{p_1}\right| \\ \phi = \tan^{-1}\left(-\dfrac{\omega}{p_1}\right) \end{cases}$$



Bode Plots (12)

$$\mathbf{H}(\omega) = \frac{1}{1+\frac{j\omega}{p_1}} \rightarrow \begin{cases} H_{dB} = -20\log_{10}\left|1+\frac{j\omega}{p_1}\right| \\ \phi = \tan^{-1}\left(-\frac{\omega}{p_1}\right) \end{cases}$$



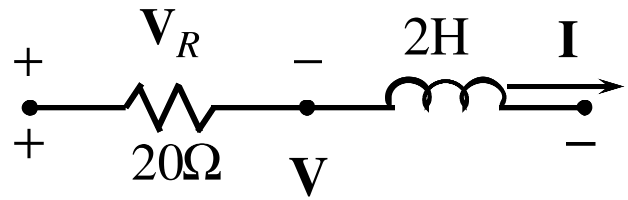
Bode Plots (13)

Ex. 4

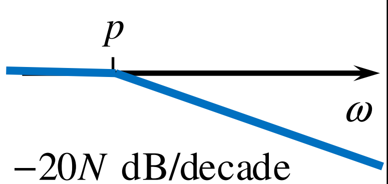
$$\mathbf{H}(\omega)=\frac{\mathbf{V}_R}{\mathbf{V}}$$

$$=\frac{R\mathbf{I}}{(R+j\omega L)\mathbf{I}}$$

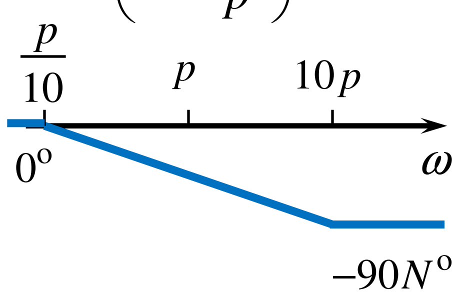
$$=\frac{20}{20+j\omega 2}=\frac{1}{1+\dfrac{j\omega}{10}}$$

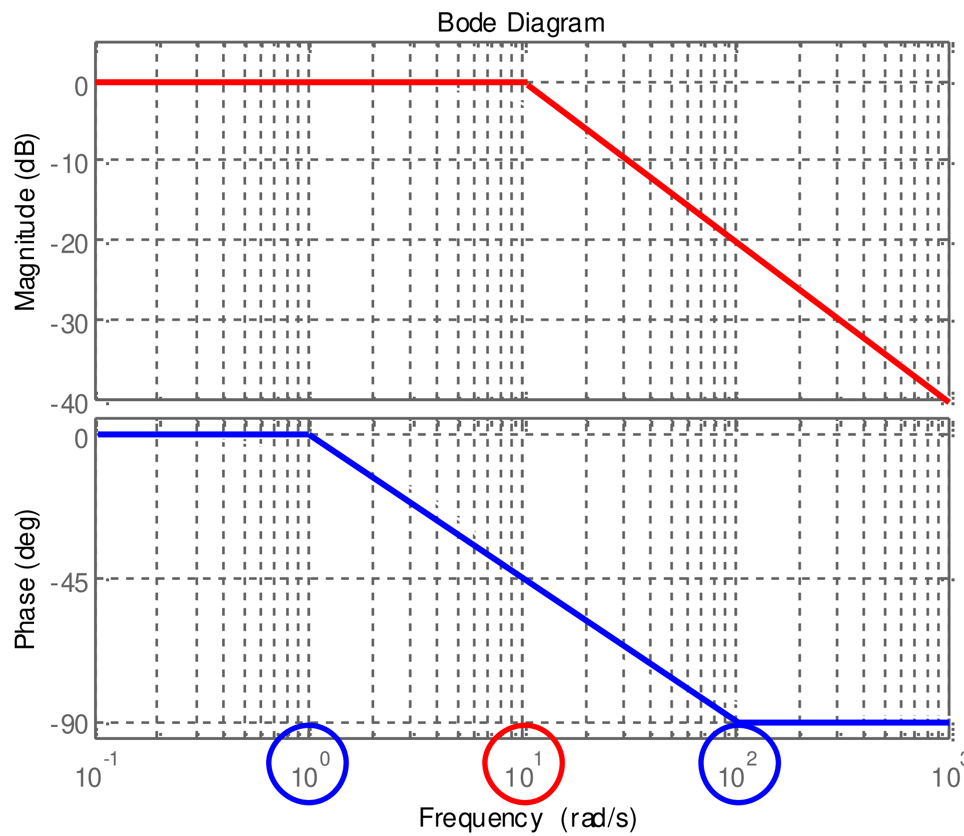


$$\frac{1}{\left(1+\dfrac{j\omega}{p}\right)^N}$$



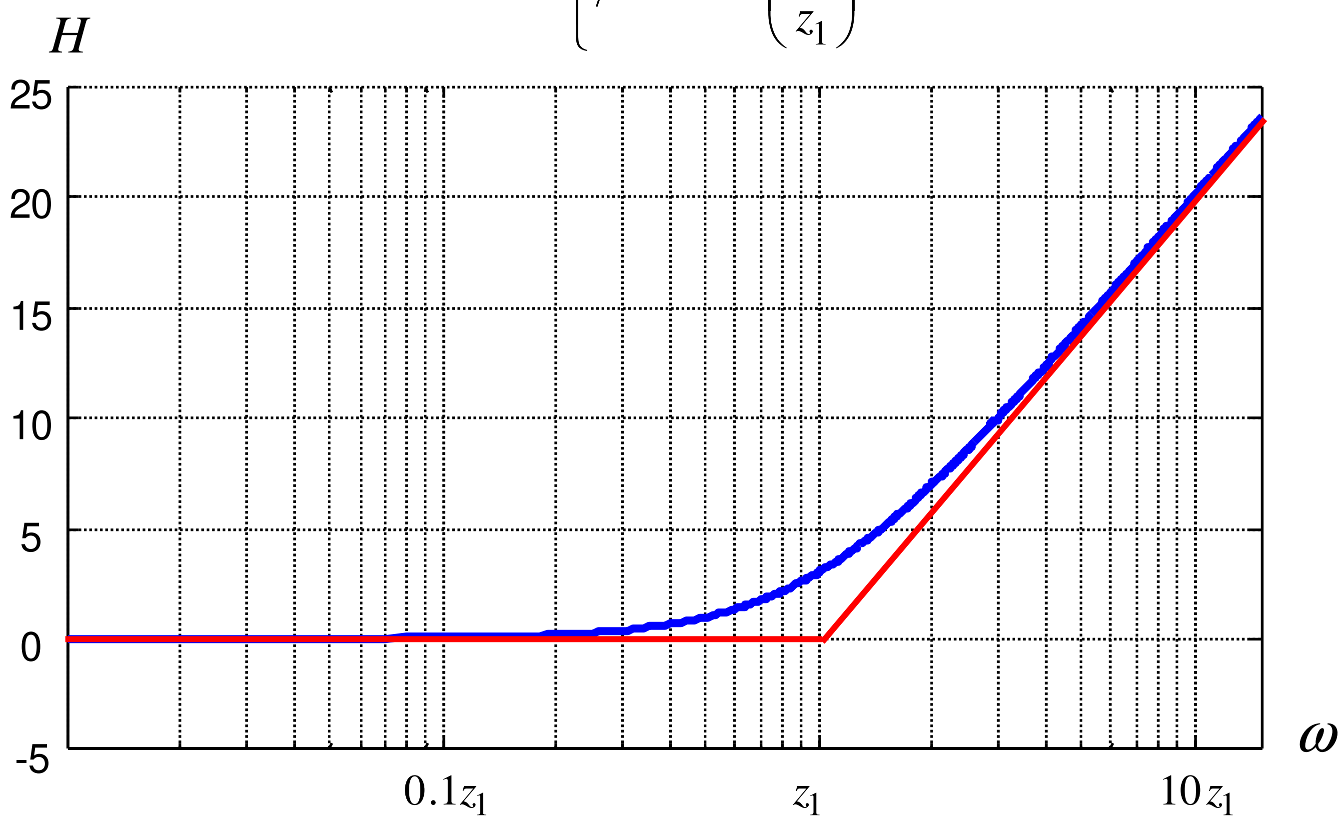
$$\frac{1}{\left(1+\dfrac{j\omega}{p}\right)^N}$$





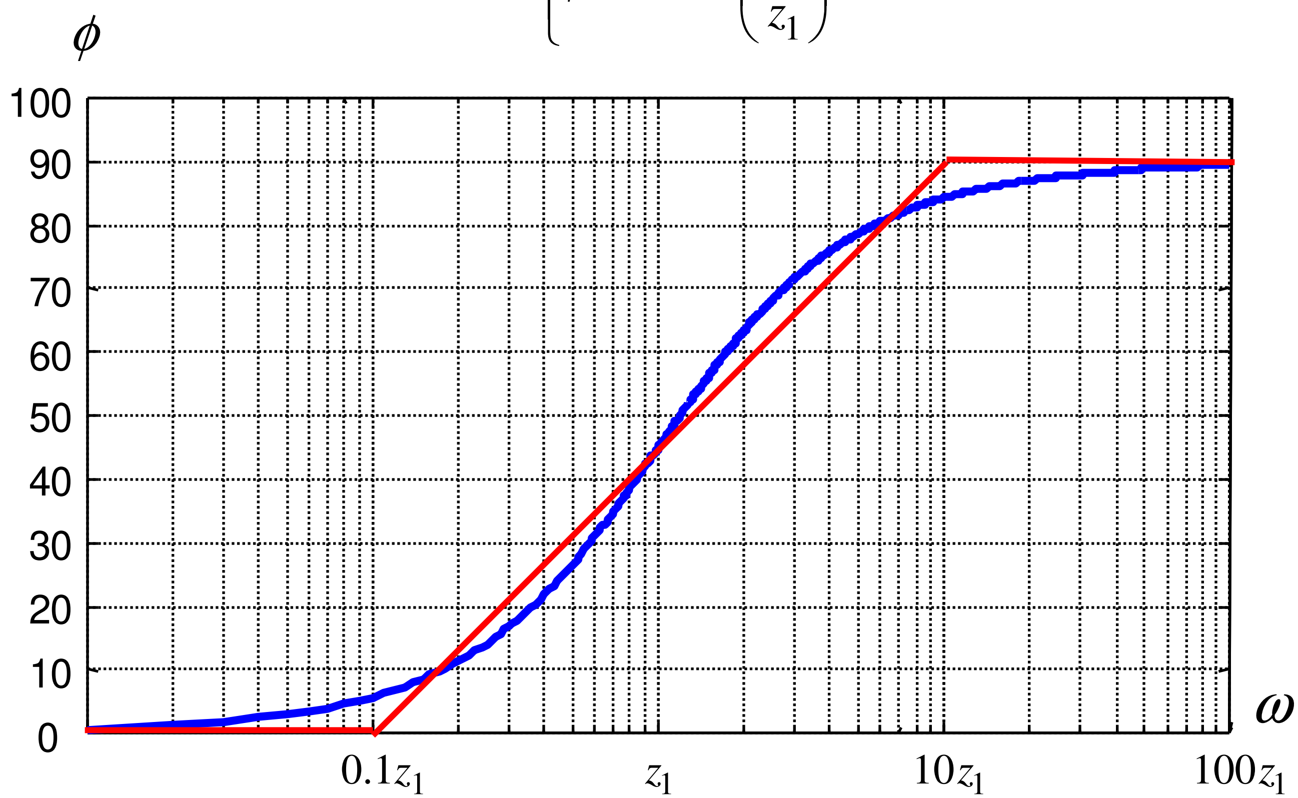
Bode Plots (14)

$$\mathbf{H}(\omega) = 1 + \frac{j\omega}{z_1} \rightarrow \begin{cases} H_{dB} = 20\log_{10}\left|1 + \dfrac{j\omega}{z_1}\right| \\ \phi = \tan^{-1}\left(\dfrac{\omega}{z_1}\right) \end{cases}$$



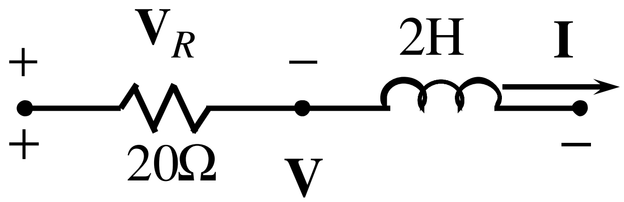
Bode Plots (15)

$$\mathbf{H}(\omega) = 1 + \frac{j\omega}{z_1} \rightarrow \begin{cases} H_{dB} = 20\log_{10}\left|1 + \dfrac{j\omega}{z_1}\right| \\ \phi = \tan^{-1}\left(\dfrac{\omega}{z_1}\right) \end{cases}$$



Bode Plots (16)

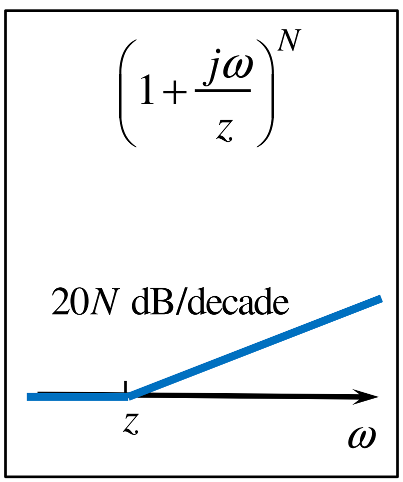
Ex. 5

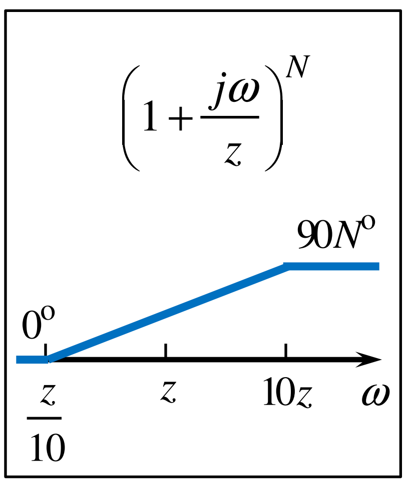


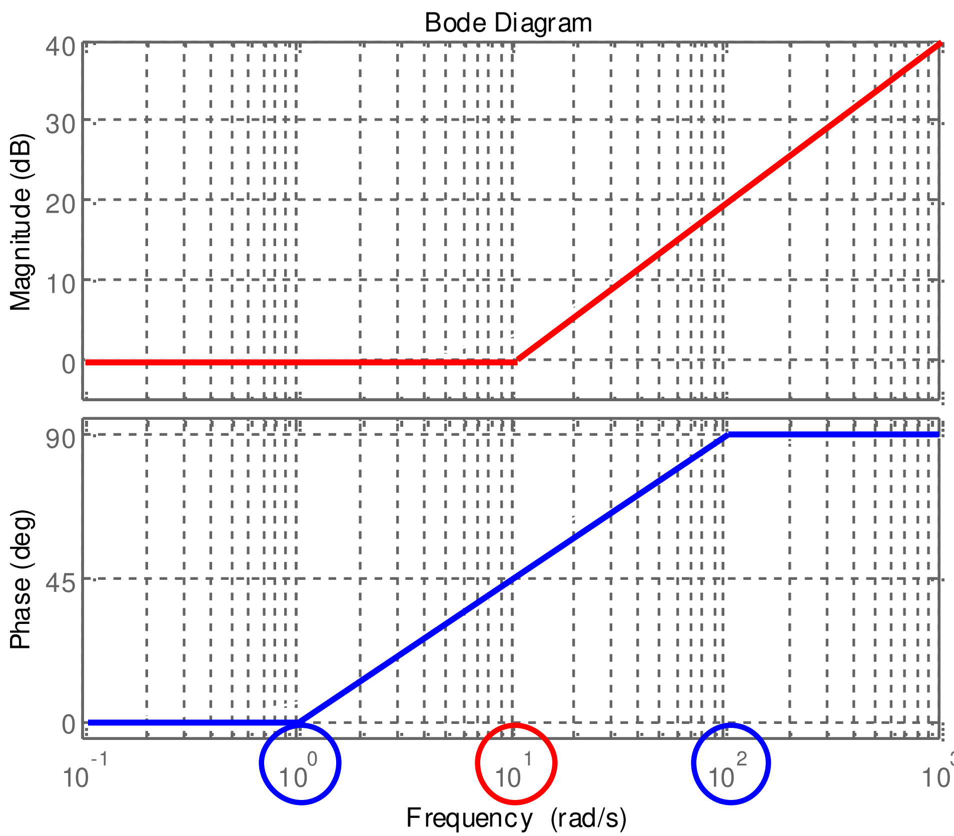
$$\mathbf{H}(\omega) = \frac{\mathbf{V}}{\mathbf{V}_R}$$

$$= \frac{(R + j\omega L)\mathbf{I}}{R\mathbf{I}}$$

$$= \frac{20 + j\omega 2}{20} = 1 + \frac{j\omega}{10}$$

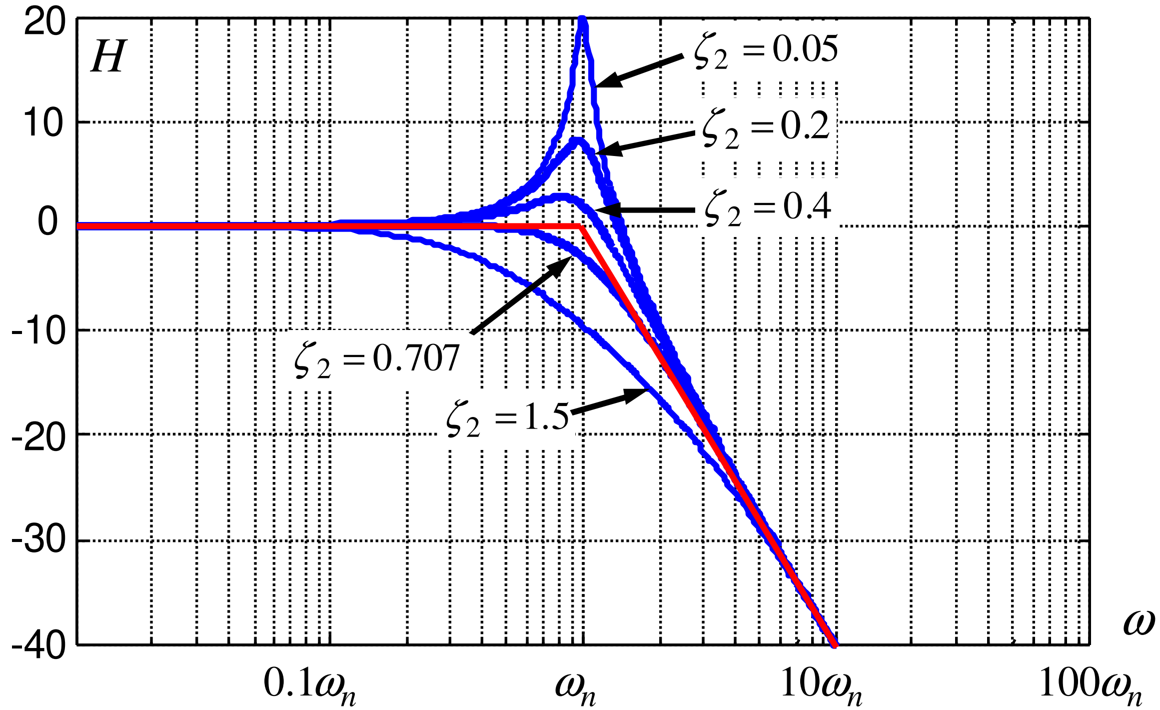






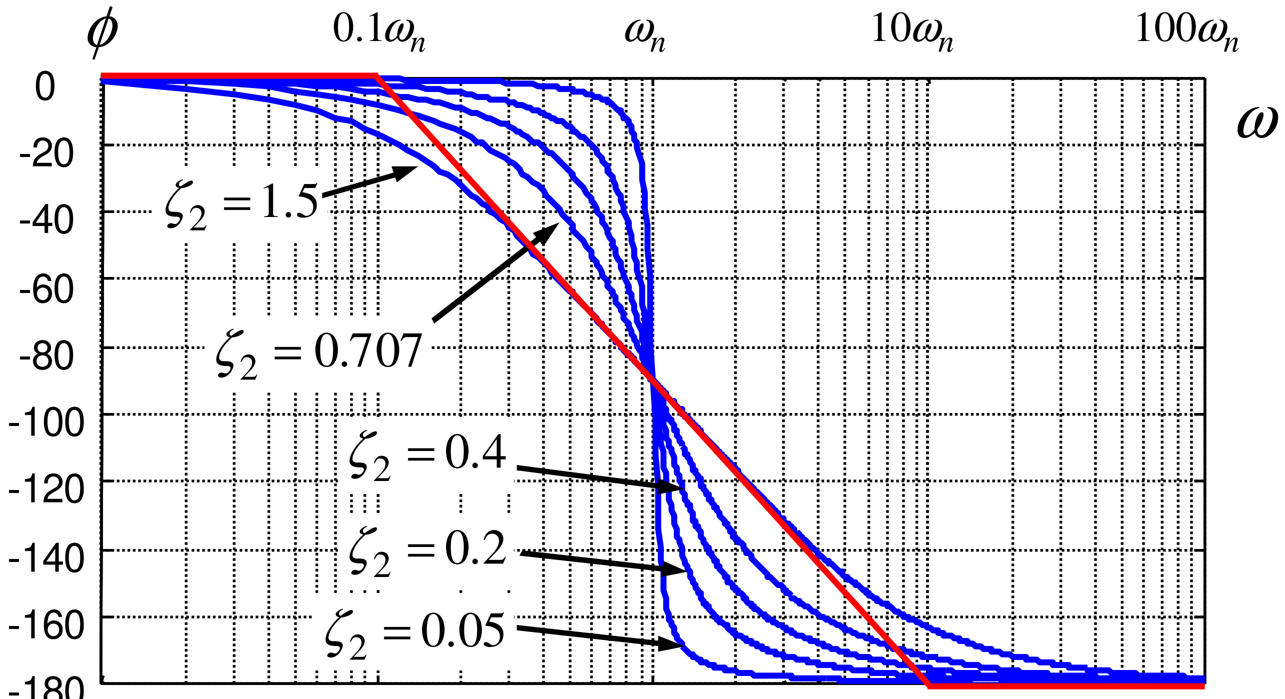
Bode Plots (17)

$$\mathbf{H}(\omega) = \frac{1}{1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2} \to \begin{cases} H_{dB} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \\ \phi = -\tan^{-1}\left(\frac{j2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2}\right) \end{cases}$$



Bode Plots (18)

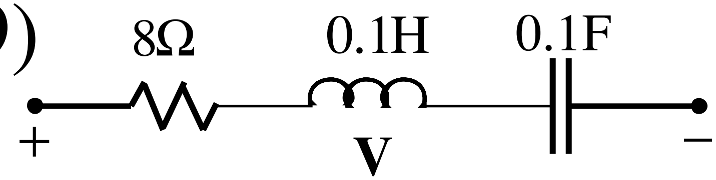
$$\mathbf{H}(\omega)=\frac{1}{1+\dfrac{j2\zeta_2\omega}{\omega_n}+\left(\dfrac{j\omega}{\omega_n}\right)^2}\rightarrow\begin{cases}H_{dB}=-20\log_{10}\left|1+\dfrac{j2\zeta_2\omega}{\omega_n}+\left(\dfrac{j\omega}{\omega_n}\right)^2\right|\\[2ex]\phi=-\tan^{-1}\left(\dfrac{j2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2}\right)\end{cases}$$



Ex. 6

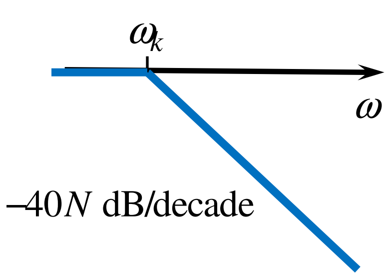
Bode Plots (19)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_C}{\mathbf{V}}$$

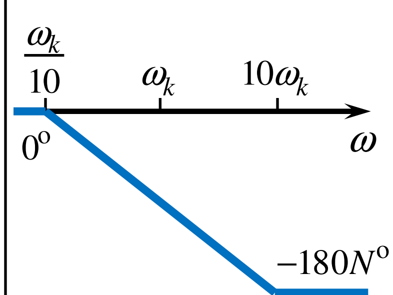


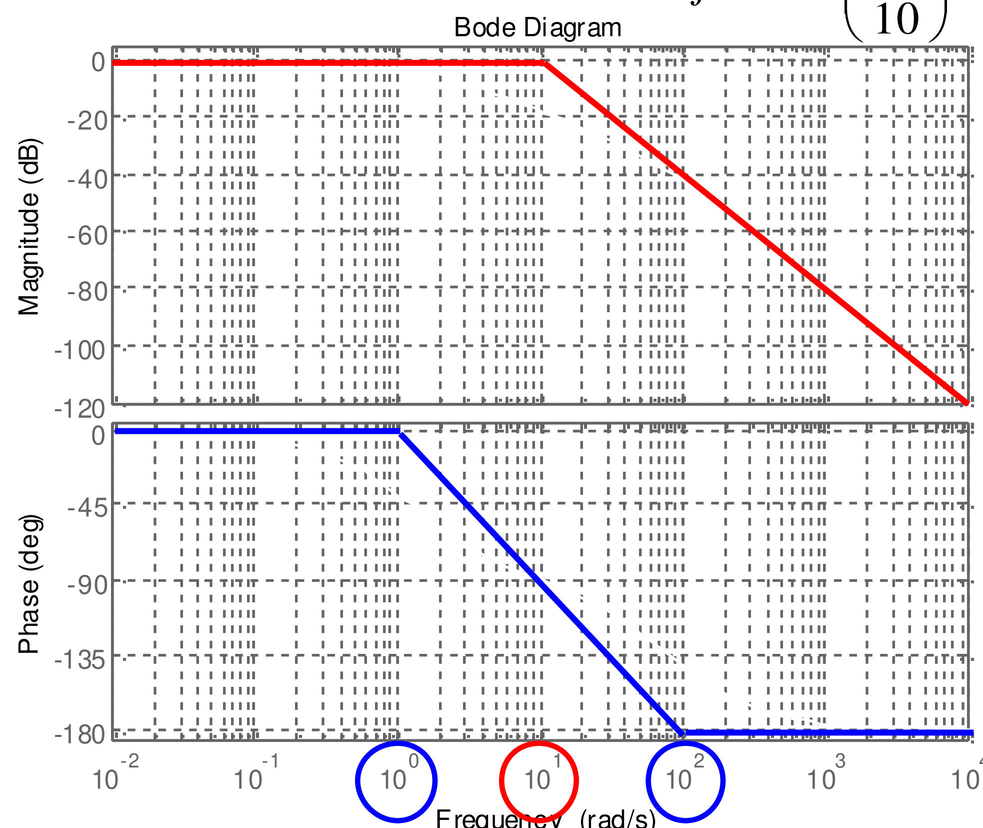
$$= \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 + RCj\omega + LC(j\omega)^2} = \frac{1}{1 + 8\times 0.1 j\omega + 0.1\times 0.1(j\omega)^2} = \frac{1}{1 + j0.8\omega + \left(\dfrac{j\omega}{10}\right)^2}$$

$$\frac{1}{\left[1 + \dfrac{j2\zeta_2\omega}{\omega_k} + \left(\dfrac{j\omega}{\omega_k}\right)^2\right]^N}$$



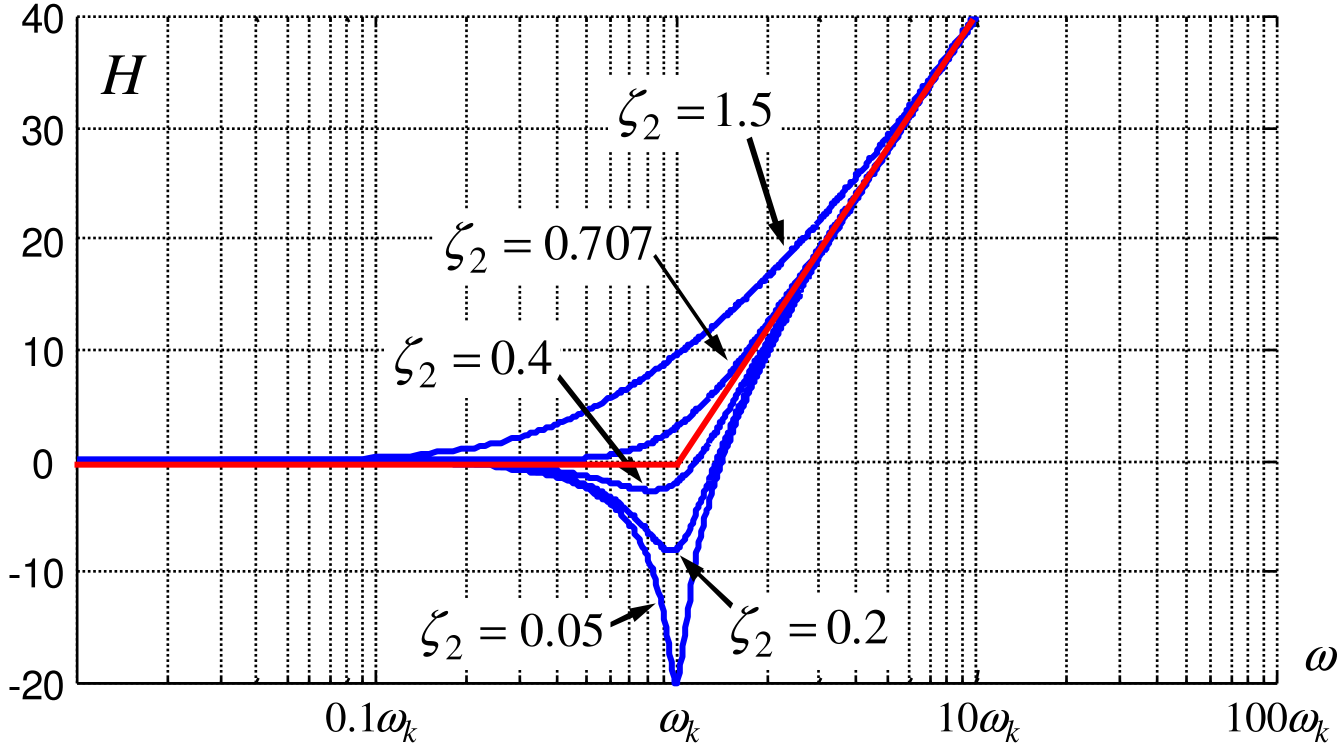
$$\frac{1}{\left[1 + \dfrac{j2\zeta_2\omega}{\omega_k} + \left(\dfrac{j\omega}{\omega_k}\right)^2\right]^N}$$





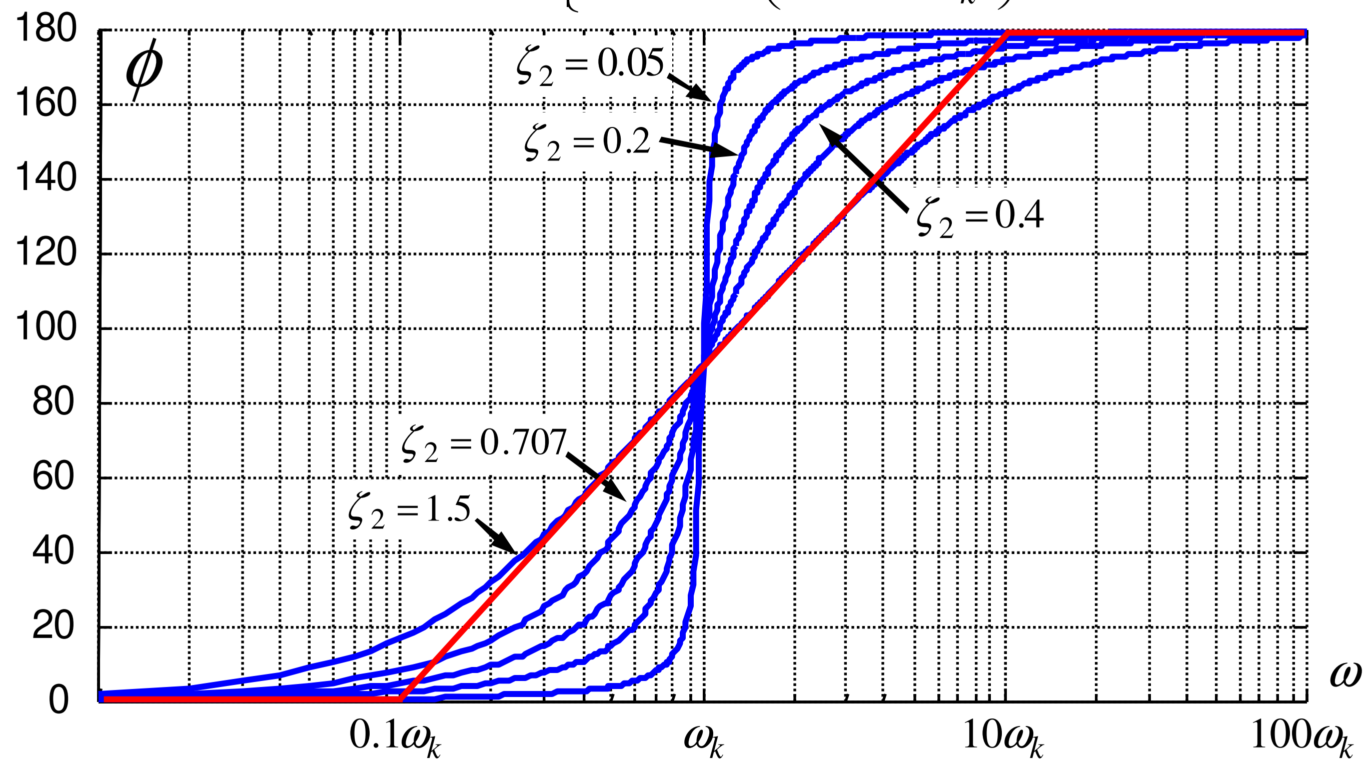
Bode Plots (20)

$$\mathbf{H}(\omega) = 1 + \frac{j2\zeta_2\omega}{\omega_k} + \left(\frac{j\omega}{\omega_k}\right)^2 \rightarrow \begin{cases} H_{dB} = 20\log_{10}\left|1 + \dfrac{j2\zeta_2\omega}{\omega_k} + \left(\dfrac{j\omega}{\omega_k}\right)^2\right| \\ \phi = \tan^{-1}\left(\dfrac{j2\zeta_2\omega/\omega_k}{1-\omega^2/\omega_k^2}\right) \end{cases}$$



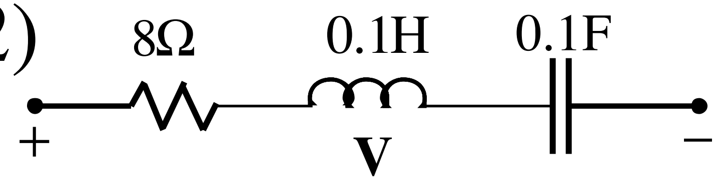
Bode Plots (21)

$$\mathbf{H}(\omega) = 1 + \frac{j2\zeta_2\omega}{\omega_k} + \left(\frac{j\omega}{\omega_k}\right)^2 \rightarrow \begin{cases} H_{dB} = 20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_k} + \left(\frac{j\omega}{\omega_k}\right)^2\right| \\ \phi = \tan^{-1}\left(\frac{j2\zeta_2\omega/\omega_k}{1-\omega^2/\omega_k^2}\right) \end{cases}$$



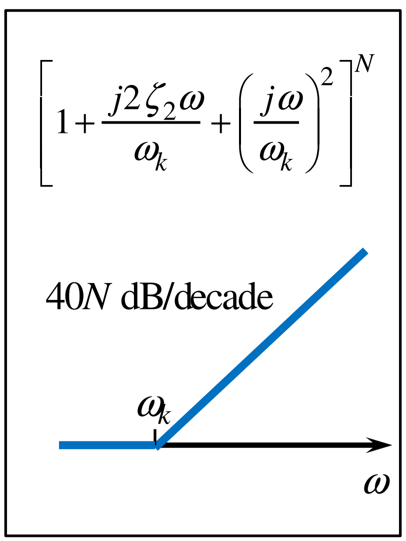
Ex. 7

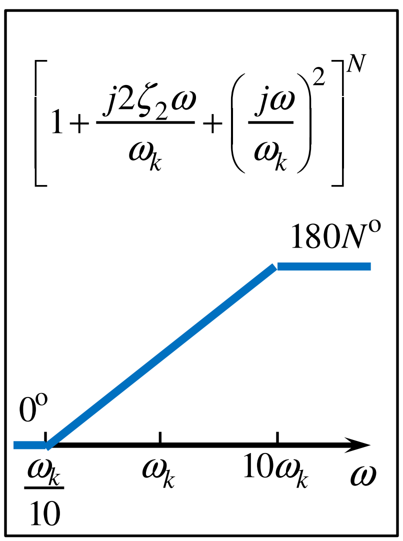
Bode Plots (22)

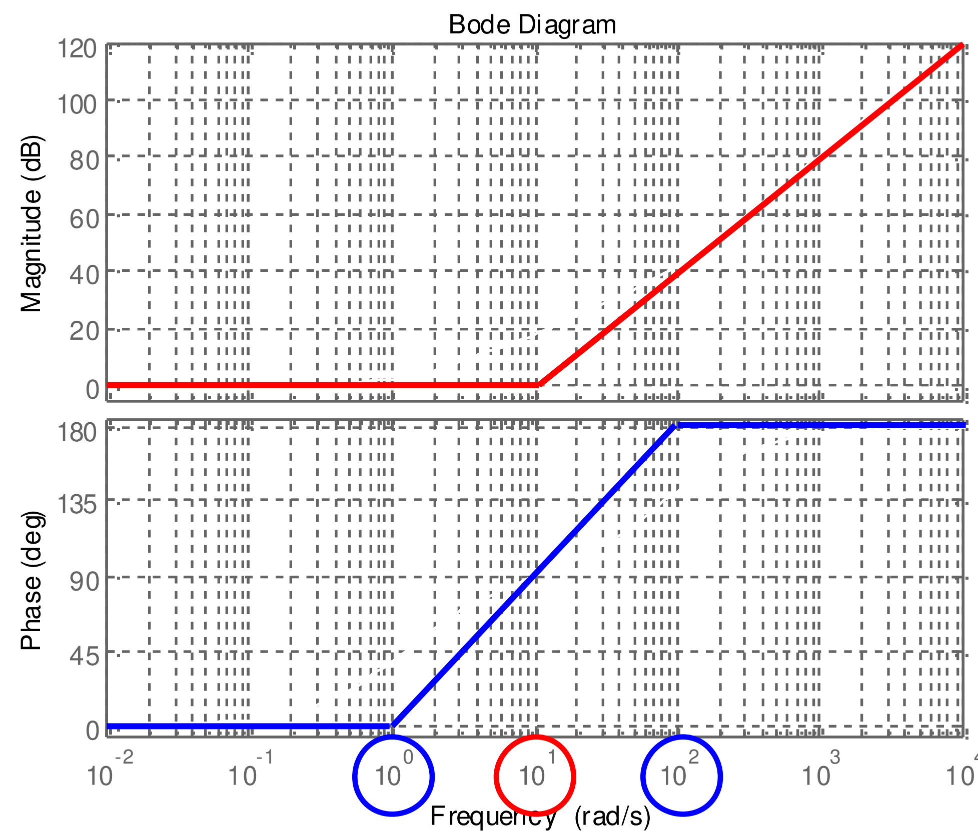


$$\mathbf{H}(\omega)=\frac{\mathbf{V}}{\mathbf{V}_C}$$

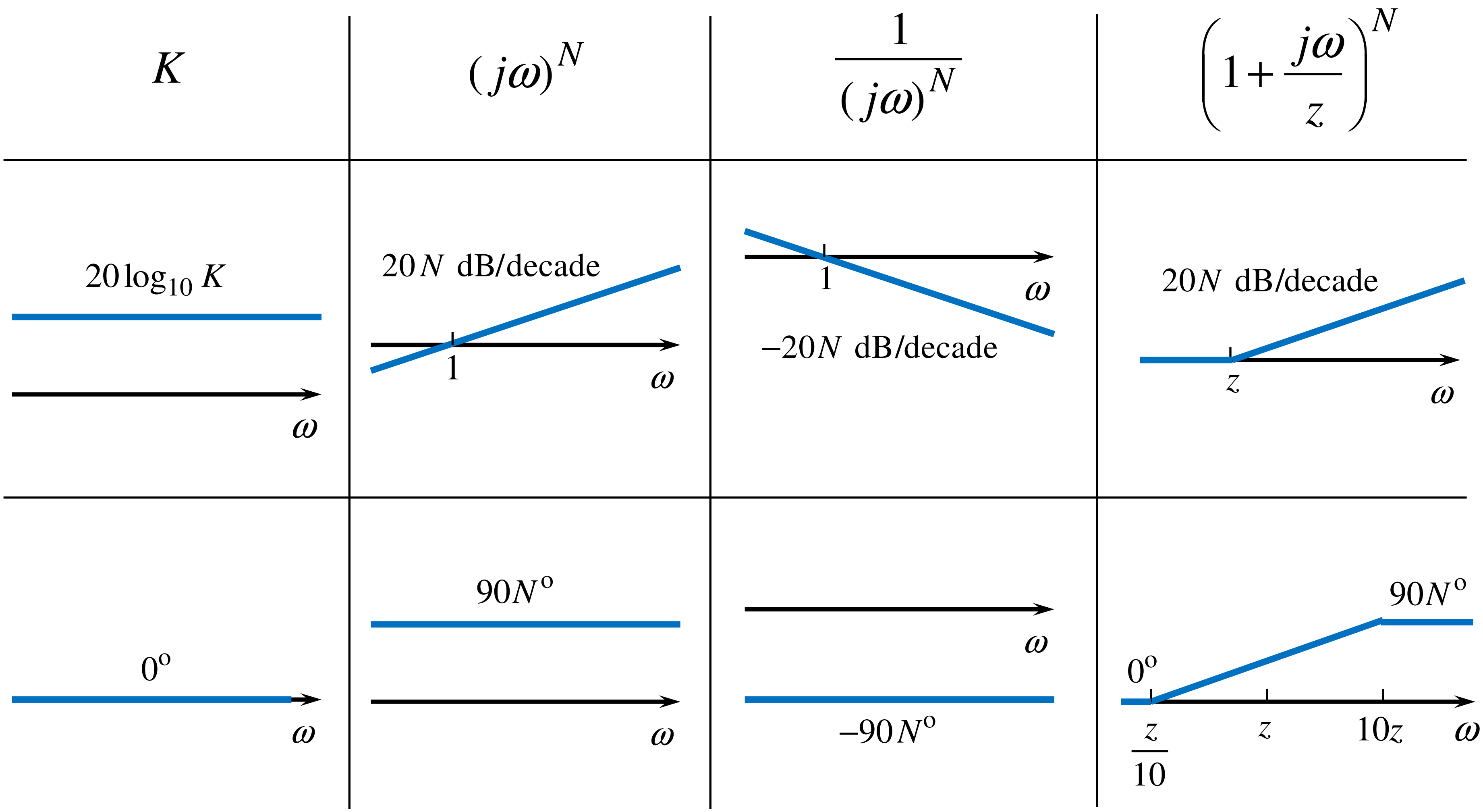
$$=\frac{R+j\omega L+\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C}}=1+RCj\omega+LC(j\omega)^2=1+8\times0.1j\omega+0.1\times0.1(j\omega)^2=1+j0.8\omega+\left(\frac{j\omega}{10}\right)^2$$





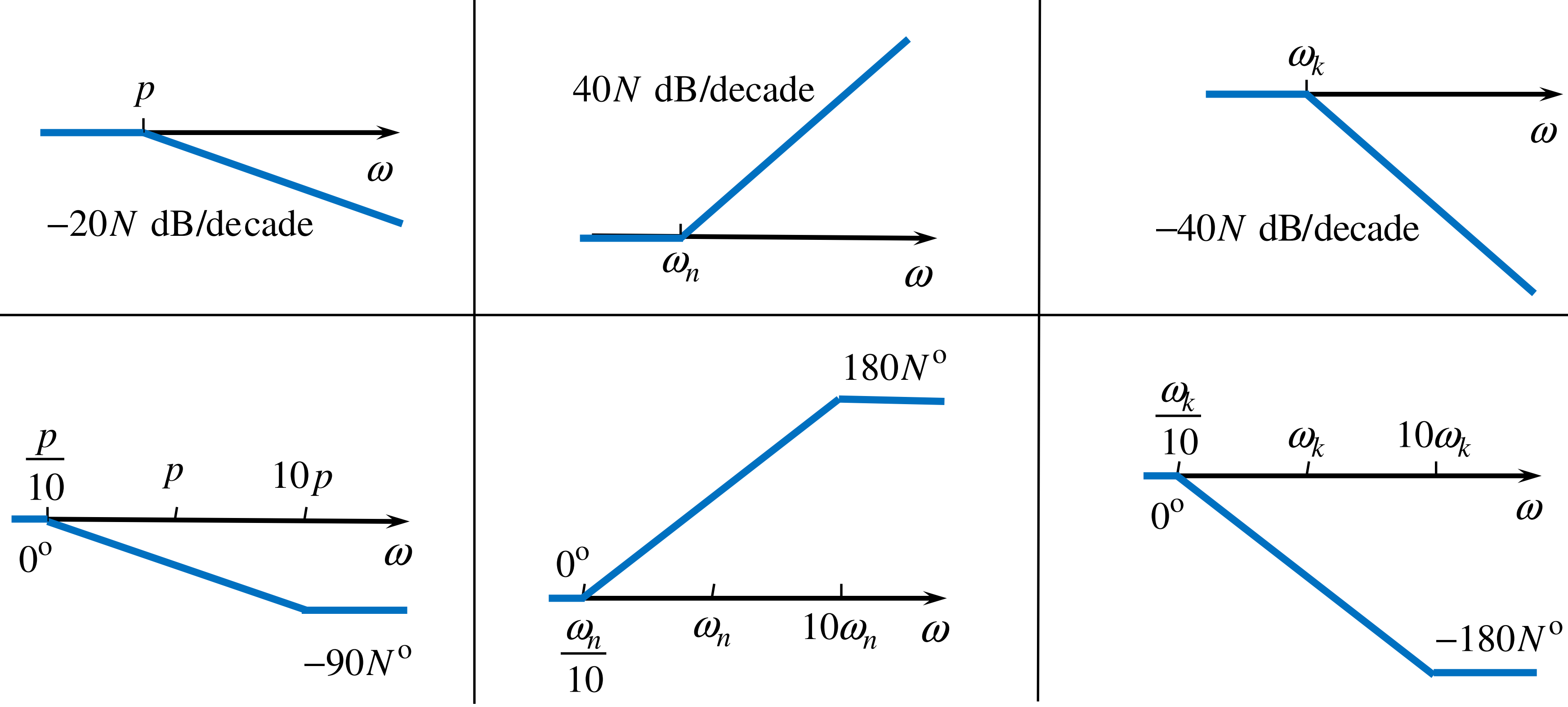


Bode Plots (23)



Bode Plots (24)

$$\frac{1}{\left(1+\dfrac{j\omega}{p}\right)^N} \qquad \left[1+\frac{j2\zeta_1\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]^N \qquad \frac{1}{\left[1+\dfrac{j2\zeta_2\omega}{\omega_k}+\left(\dfrac{j\omega}{\omega_k}\right)^2\right]^N}$$



Bode Plots (25)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)}$ $= \dfrac{10\,j\omega}{(1+j\omega/5)(1+j\omega/10)}$

$$\mathbf{H}(\omega) = \frac{10\,j\omega}{(1+j\omega/5)(1+j\omega/10)} = \frac{|10\,j\omega|}{|1+j\omega/5||1+j\omega/10|}\angle\underline{90^{\text{o}} - \tan^{-1}(\omega/5) - \tan^{-1}(\omega/10)}$$

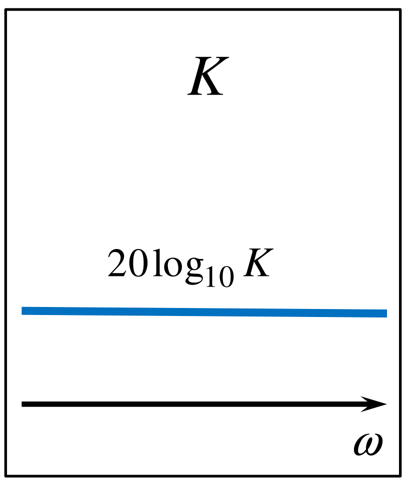
$$\rightarrow \begin{cases} H_{dB} = 20\log_{10}10 + 20\log_{10}|j\omega| + 20\log_{10}\left|\dfrac{1}{1+j\omega/5}\right| + 20\log_{10}\left|\dfrac{1}{1+j\omega/10}\right| \\ \phi = 90^{\text{o}} + \tan^{-1}\left(\dfrac{1}{\omega/5}\right) + \tan^{-1}\left(\dfrac{1}{\omega/10}\right) \end{cases}$$

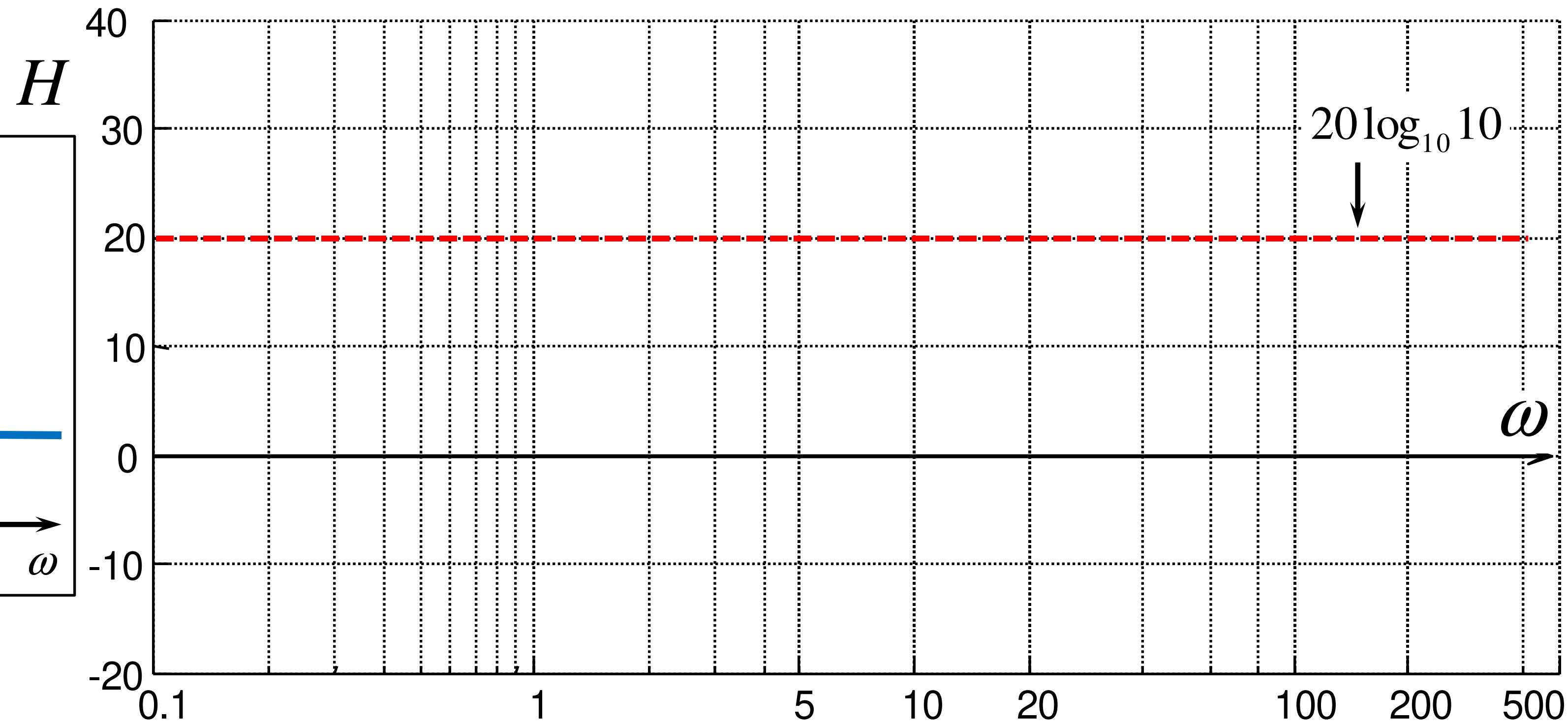
Bode Plots (26)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{\boxed{10}\,j\omega}{(1+j\omega/5)(1+j\omega/10)}$

$$H_{dB} = \boxed{20\log_{10}10} + 20\log_{10}|j\omega| + 20\log_{10}\left|\frac{1}{1+j\omega/5}\right| + 20\log_{10}\left|\frac{1}{1+j\omega/10}\right|$$



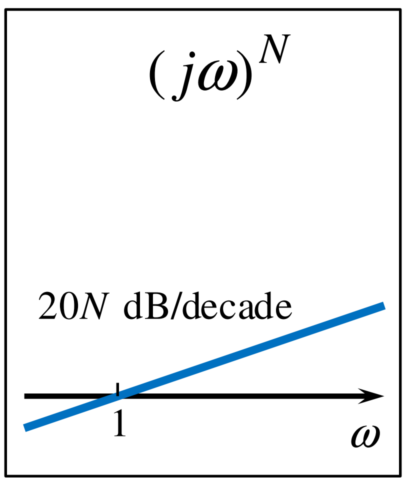


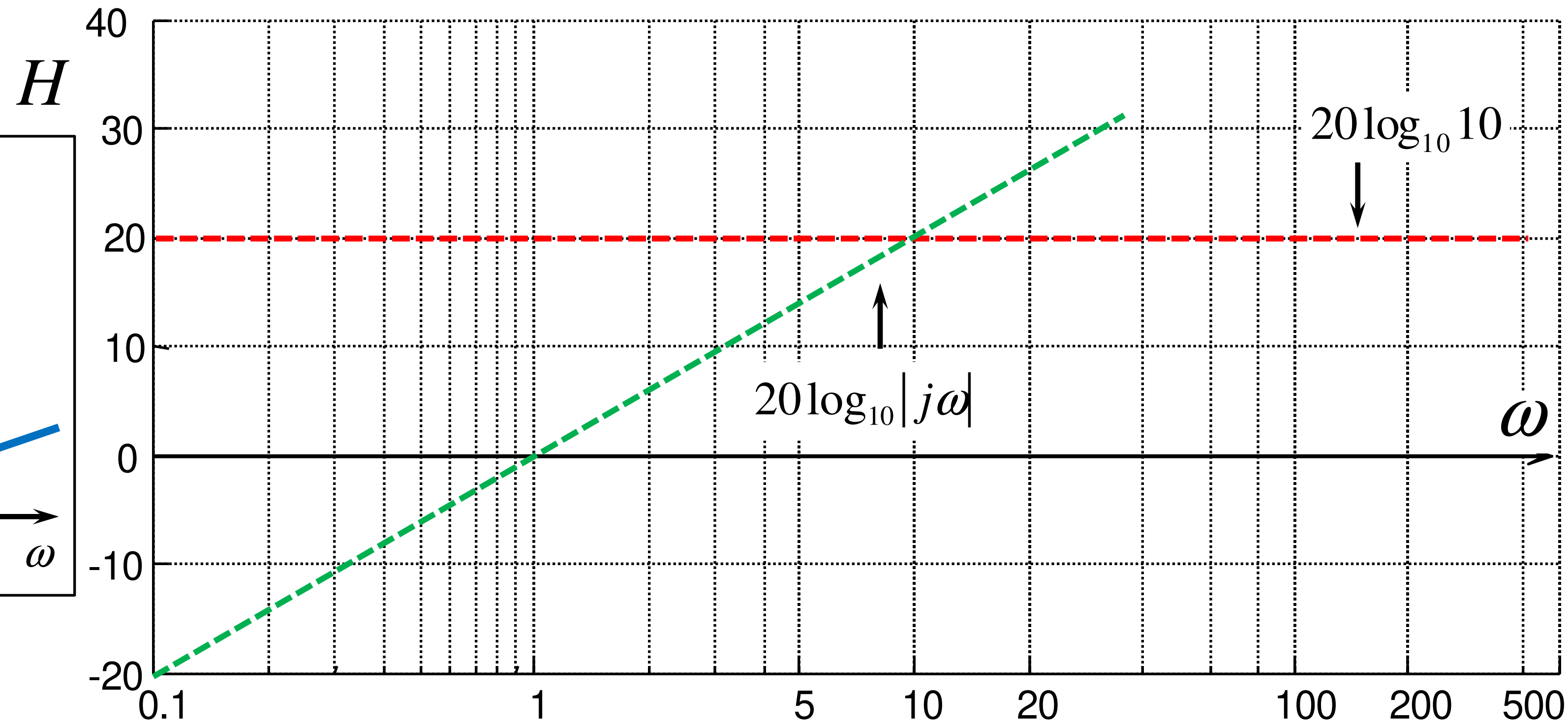
Bode Plots (27)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega)=\dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)}=\dfrac{\boxed{10}\,\boxed{j\omega}}{(1+j\omega/5)(1+j\omega/10)}$

$$H_{dB}=\boxed{20\log_{10}10}+\boxed{20\log_{10}|j\omega|}+20\log_{10}\left|\frac{1}{1+j\omega/5}\right|+20\log_{10}\left|\frac{1}{1+j\omega/10}\right|$$



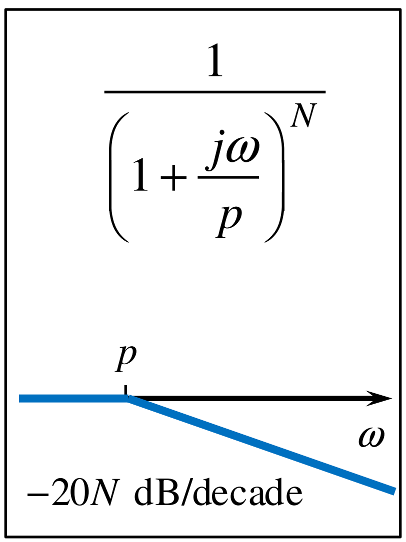


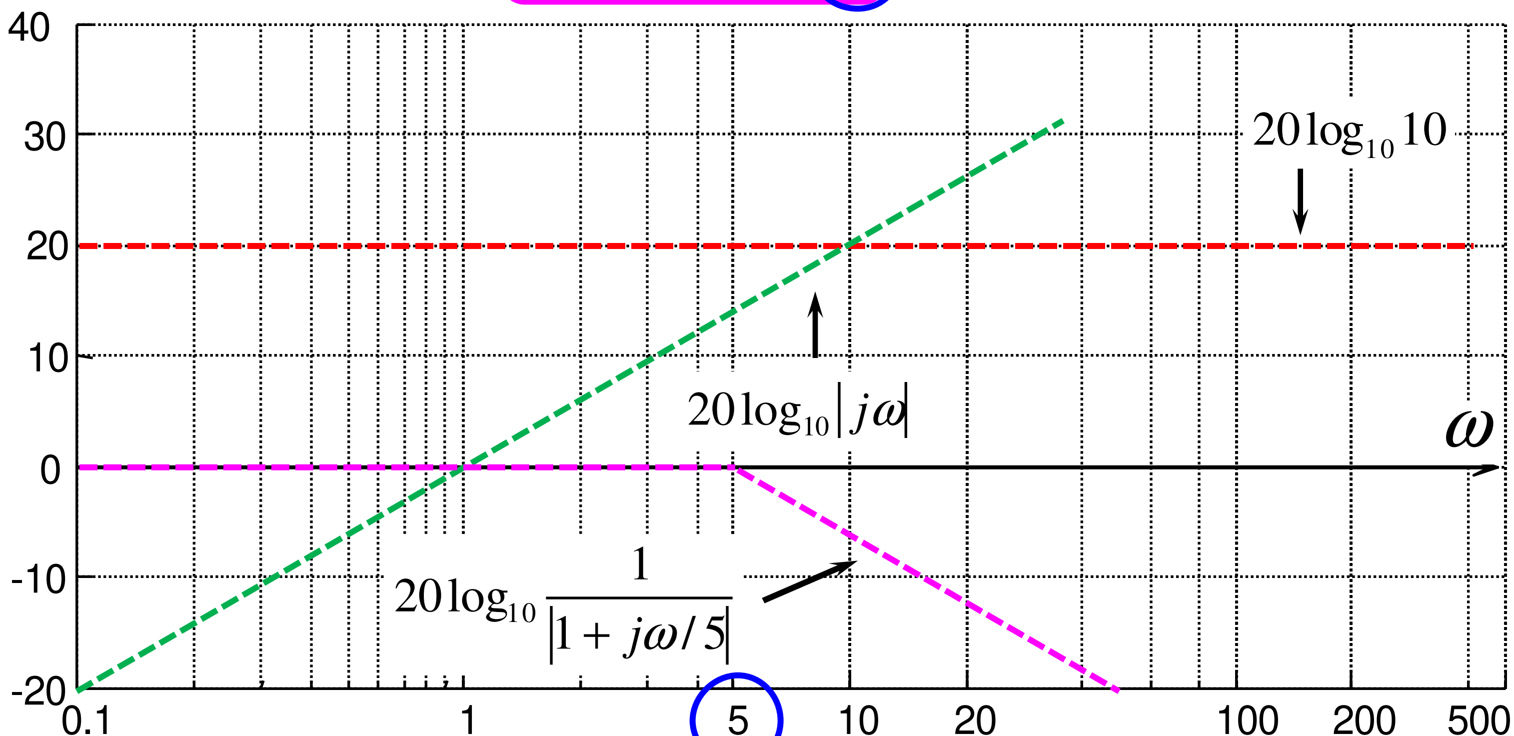
Bode Plots (28)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10\,j\omega}{\boxed{(1+j\omega/5)}(1+j\omega/10)}$

$$H_{dB} = \boxed{20\log_{10}10} + \boxed{20\log_{10}|j\omega|} + \boxed{20\log_{10}\left|\frac{1}{1+j\omega/5}\right|} + 20\log_{10}\left|\frac{1}{1+j\omega/10}\right|$$



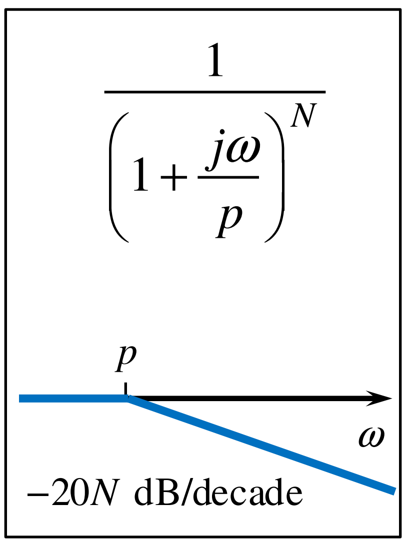


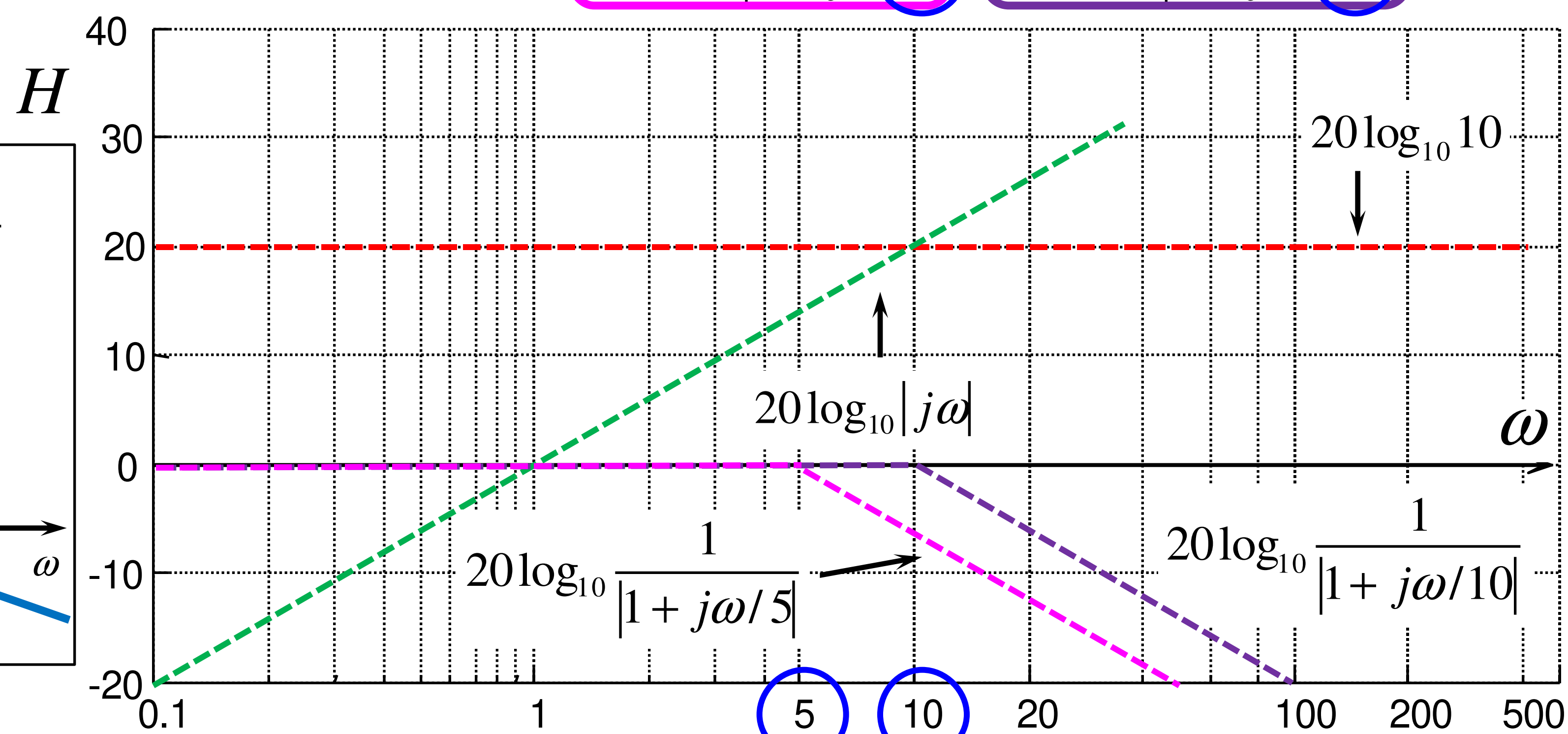
Bode Plots (29)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10\,j\omega}{(1+j\omega/5)(1+j\omega/10)}$

$$H_{dB} = 20\log_{10}10 + 20\log_{10}|j\omega| + 20\log_{10}\left|\frac{1}{1+j\omega/5}\right| + 20\log_{10}\left|\frac{1}{1+j\omega/10}\right|$$



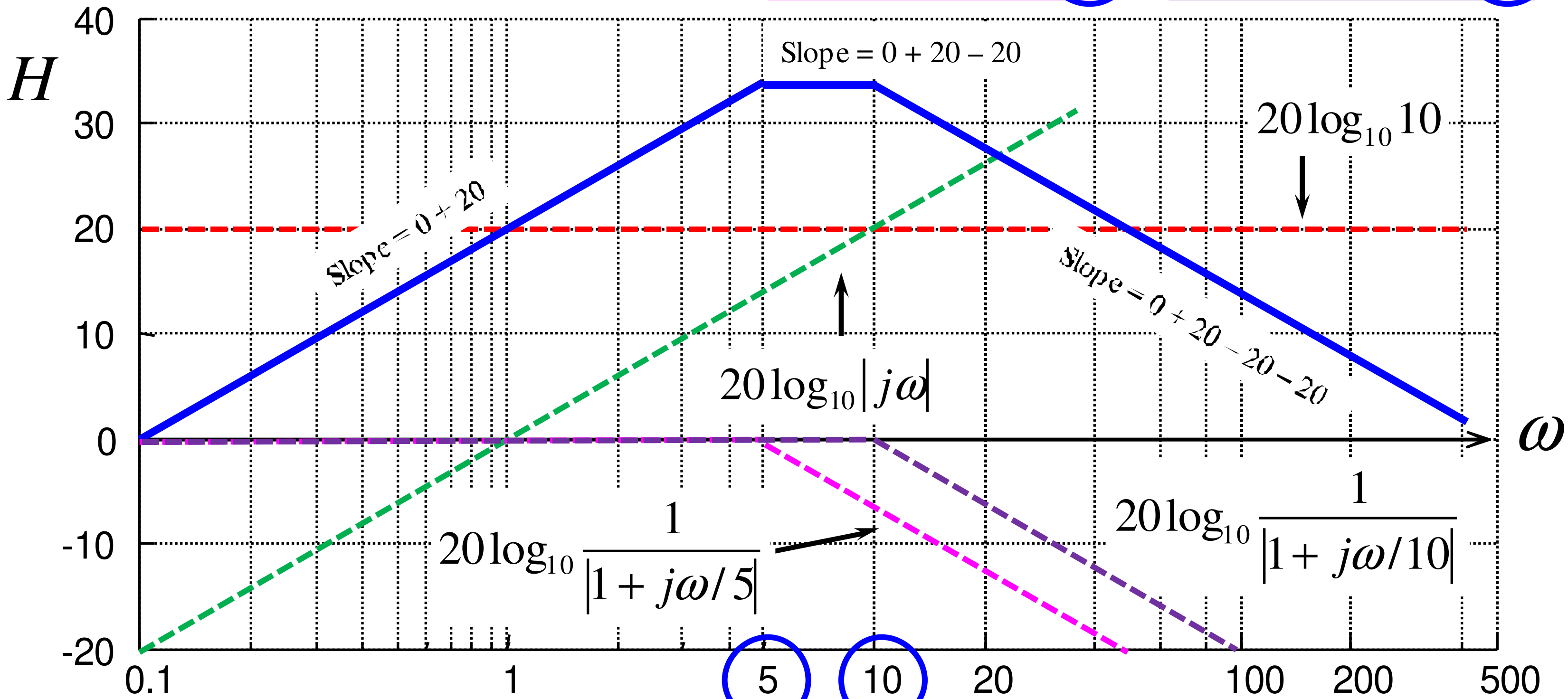


Bode Plots (30)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\, j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10\, j\omega}{(1+j\omega/5)(1+j\omega/10)}$

$$H_{dB} = 20\log_{10} 10 + 20\log_{10}|j\omega| + 20\log_{10}\left|\frac{1}{1+j\omega/5}\right| + 20\log_{10}\left|\frac{1}{1+j\omega/10}\right|$$

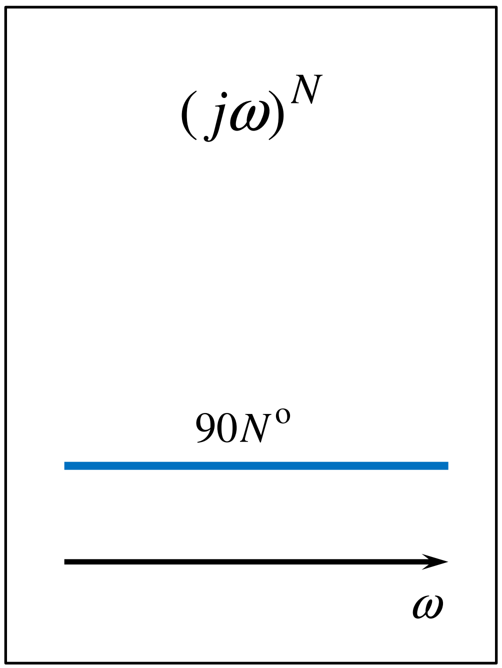


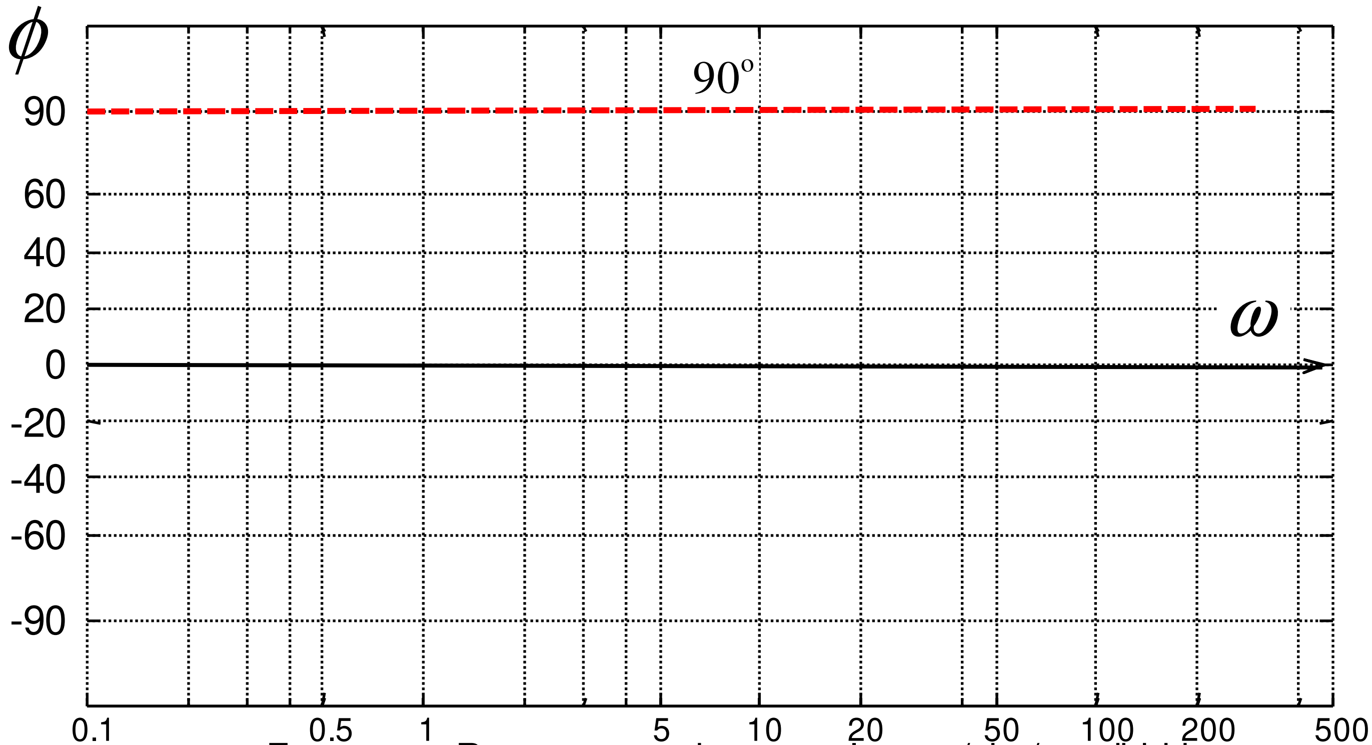
Bode Plots (31)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10\boxed{j\omega}}{(1+j\omega/5)(1+j\omega/10)}$

$$\phi = \boxed{90^{o}} + \tan^{-1}\left(\frac{1}{\omega/5}\right) + \tan^{-1}\left(\frac{1}{\omega/10}\right)$$



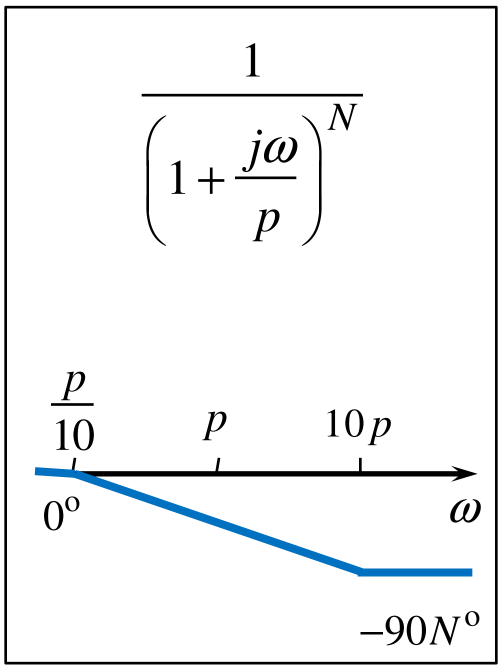


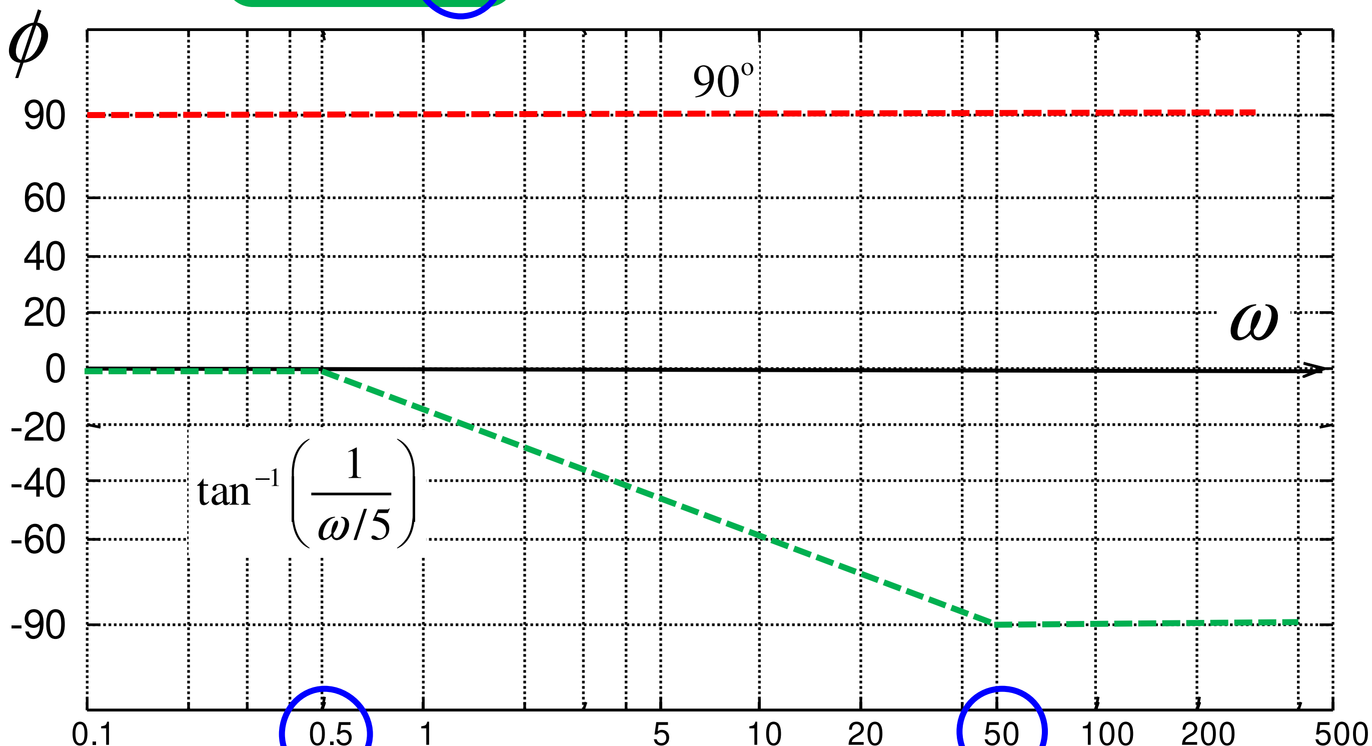
Bode Plots (32)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500 j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10 j\omega}{\boxed{(1+j\omega/5)}(1+j\omega/10)}$

$$\phi = \boxed{90^{\circ}} + \boxed{\tan^{-1}\left(\frac{1}{\omega/5}\right)} + \tan^{-1}\left(\frac{1}{\omega/10}\right)$$



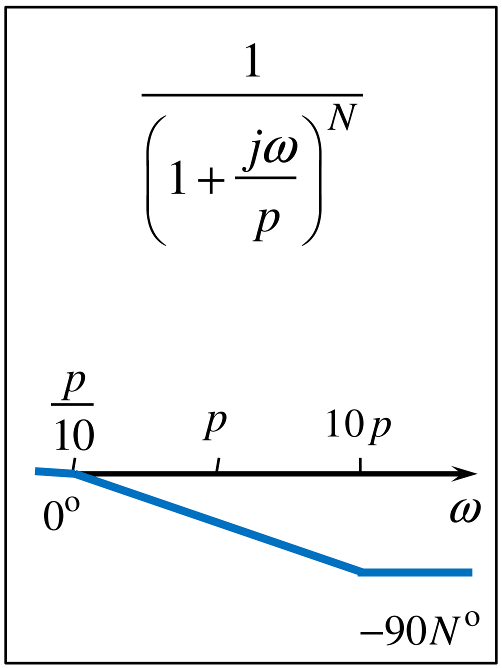


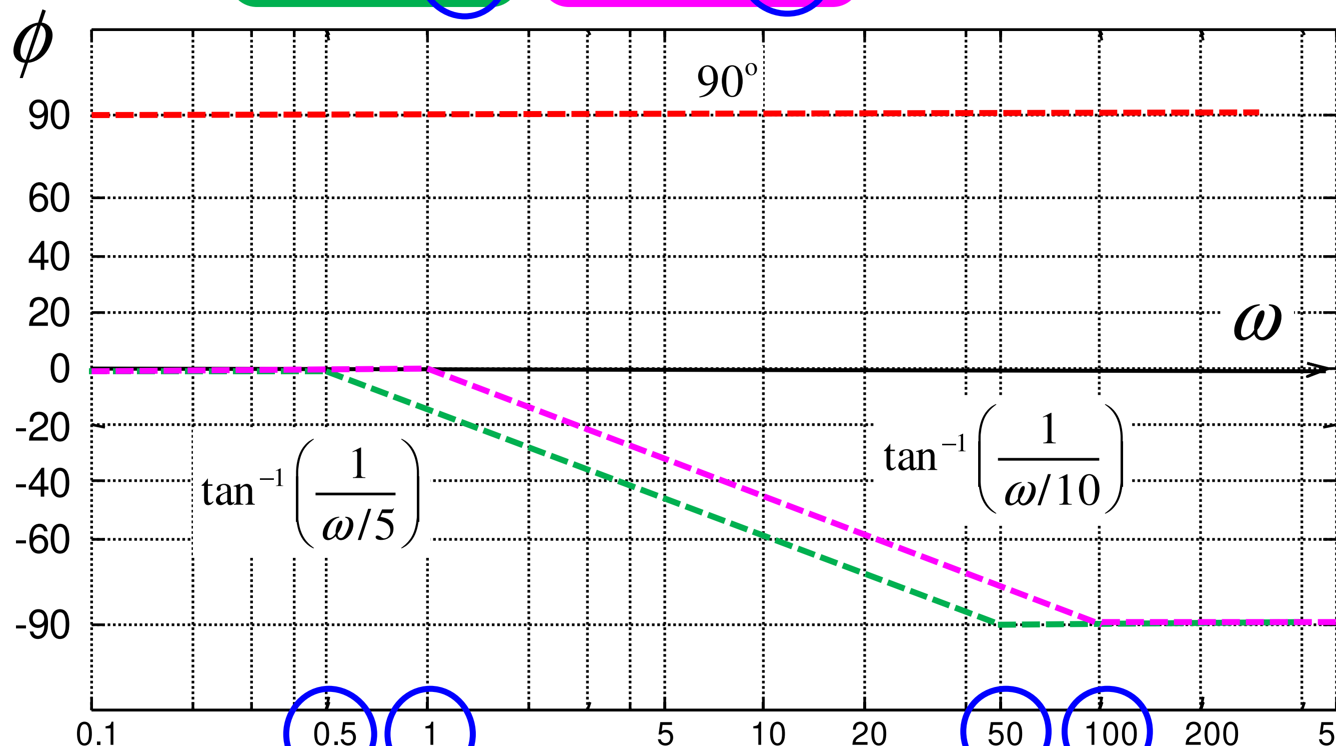
Bode Plots (33)

Ex. 8

Construct the Bode plots for $\mathbf{H}(\omega)=\dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)}=\dfrac{10\,j\omega}{(1+j\omega/5)(1+j\omega/10)}$

$$\phi=\boxed{90^{\circ}}+\boxed{\tan^{-1}\left(\frac{1}{\omega/5}\right)}+\boxed{\tan^{-1}\left(\frac{1}{\omega/10}\right)}$$



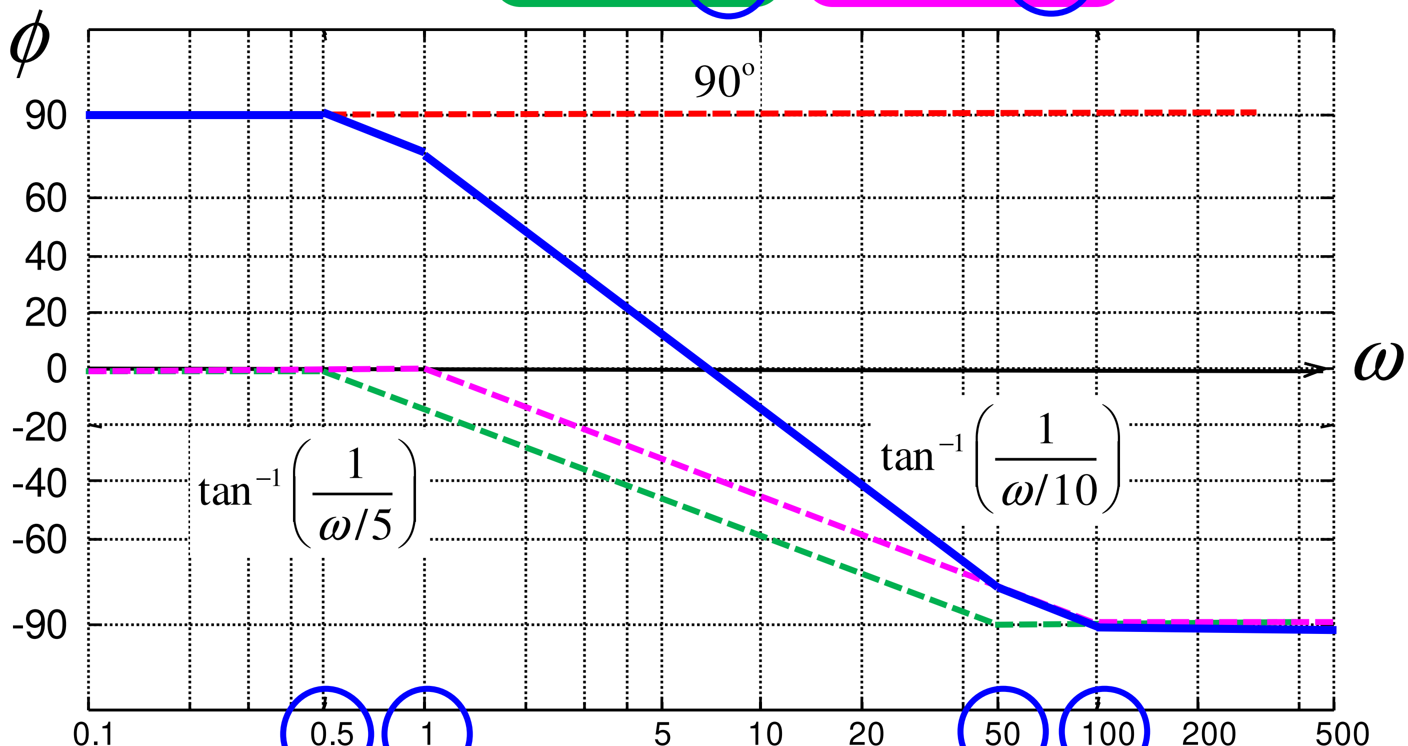


Bode Plots (34)

Ex. 8

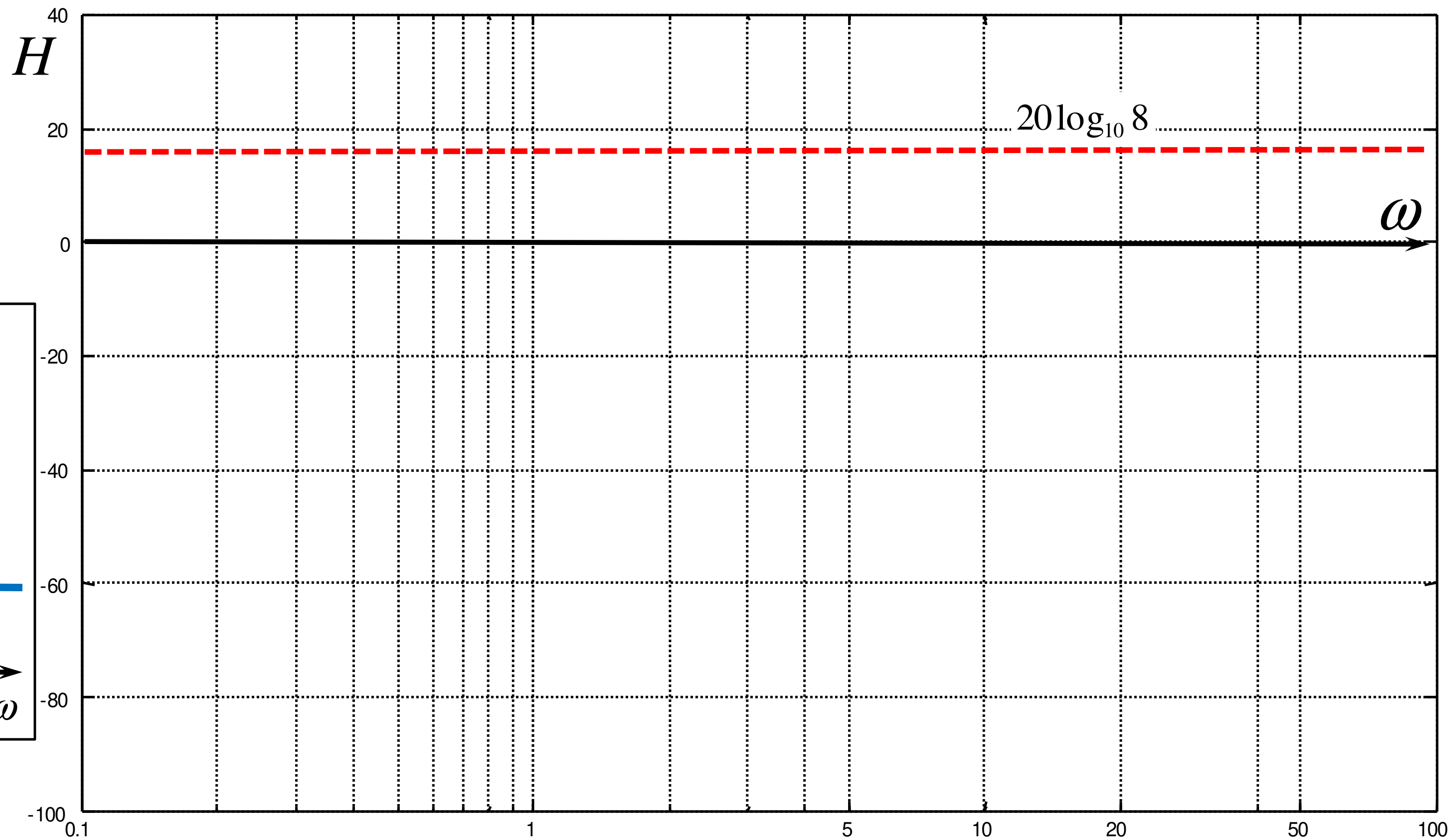
Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{500\,j\omega}{(j\omega+5)(j\omega+10)} = \dfrac{10\,j\omega}{(1+j\omega/5)(1+j\omega/10)}$

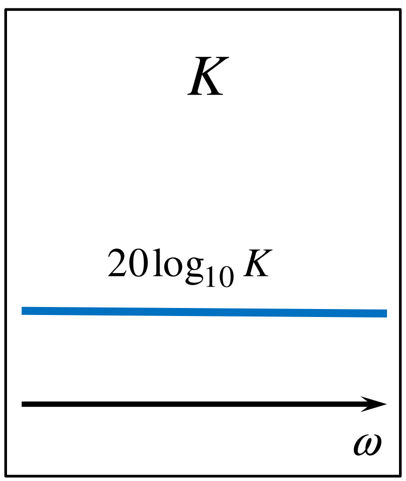
$$\phi = \boxed{90^{o}} + \boxed{\tan^{-1}\left(\frac{1}{\omega/5}\right)} + \boxed{\tan^{-1}\left(\frac{1}{\omega/10}\right)}$$



Ex. 9 Bode Plots (35)

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{\boxed{8}(1+j\omega/20)}{j\omega(1+j\omega/5)^2\left[1+j\omega4/10+(j\omega/10)^2\right]}$

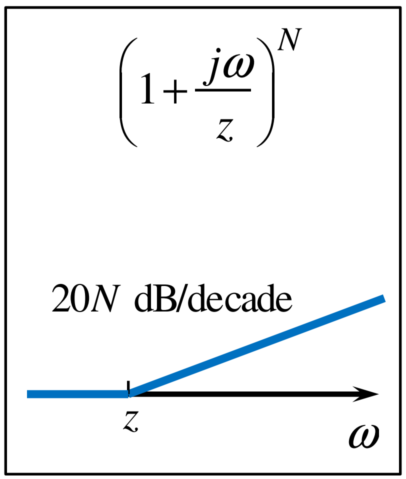


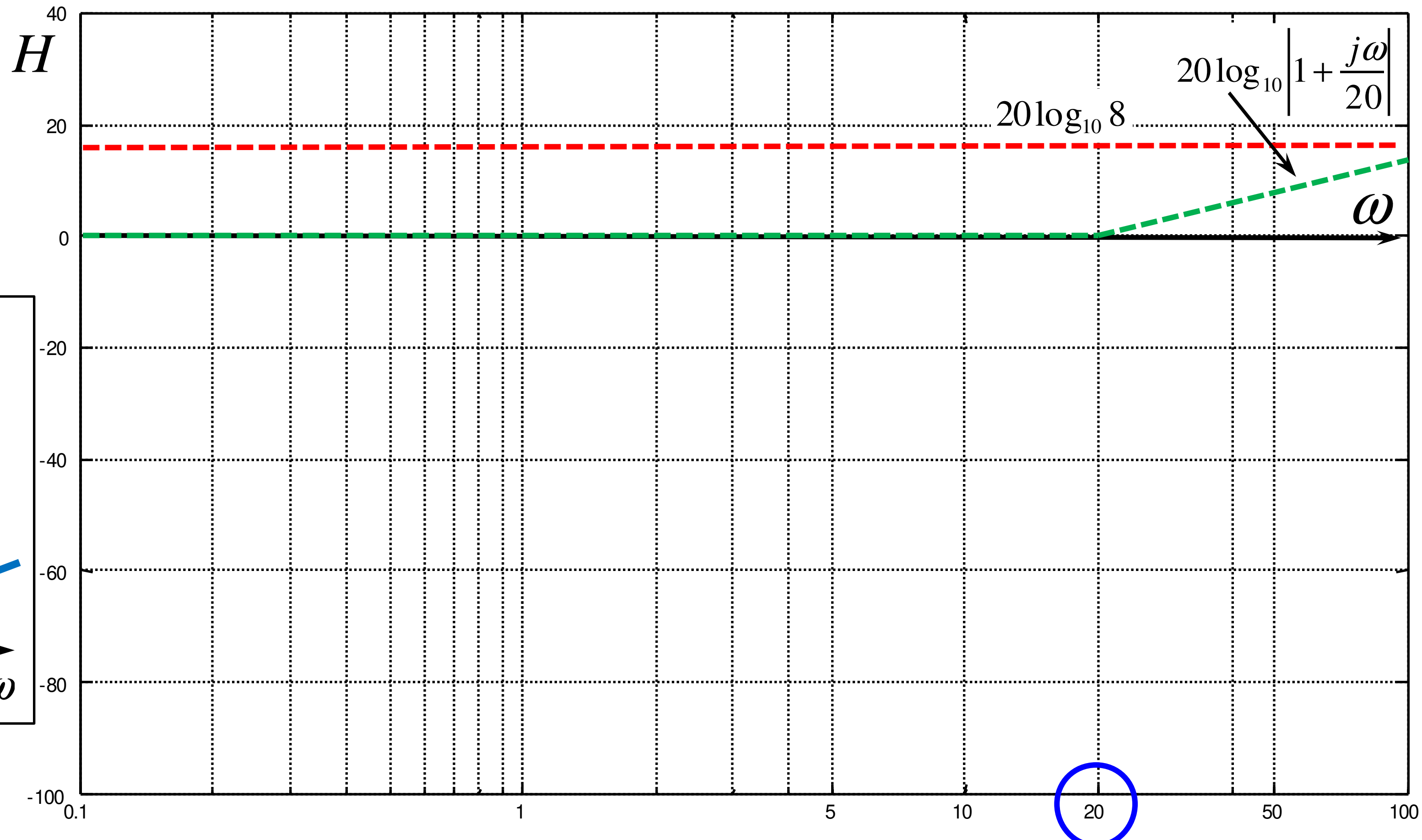


Ex. 9

Bode Plots (36)

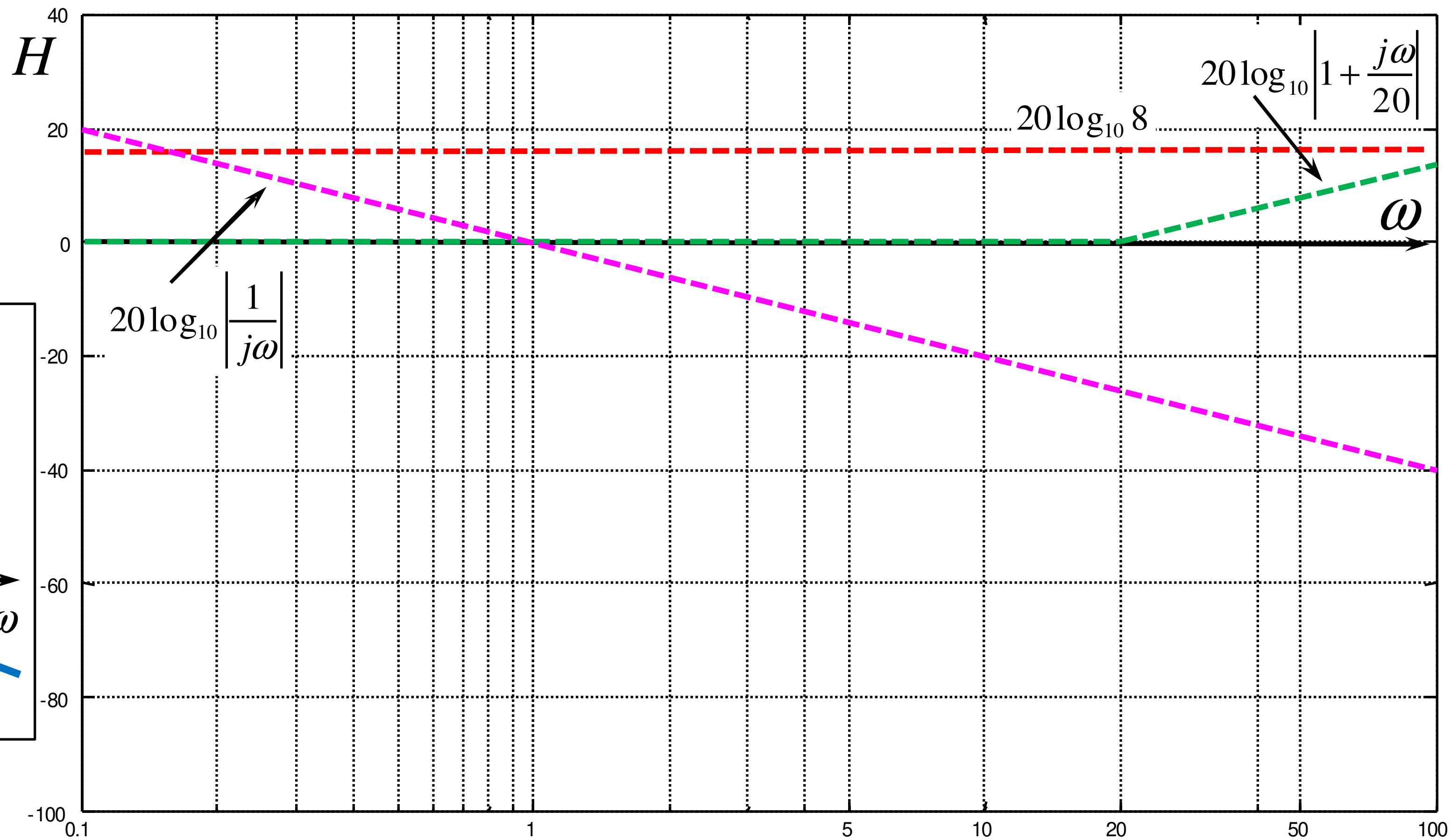
Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{8\boxed{(1+j\omega/20)}}{j\omega(1+j\omega/5)^2\left[1+j\omega 4/10+(j\omega/10)^2\right]}$

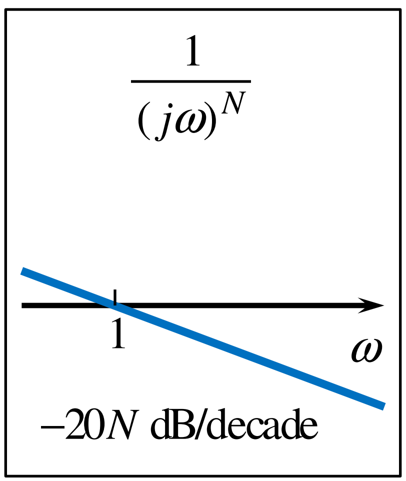




Ex. 9 Bode Plots (37)

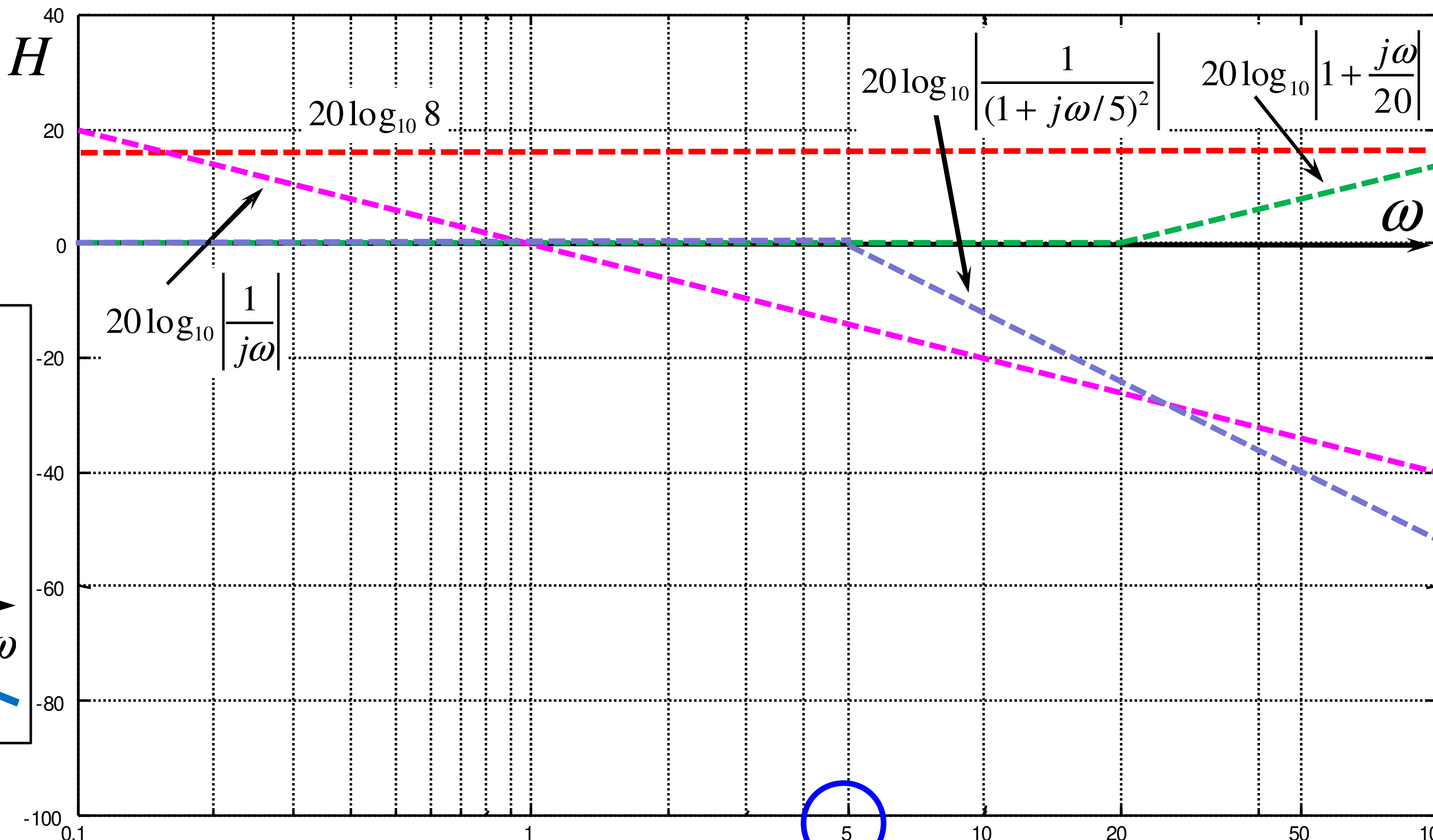
Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{8(1+j\omega/20)}{\boxed{j\omega}(\;+j\omega/5)^2\left[1+j\omega 4/10+(j\omega/10)^2\right]}$

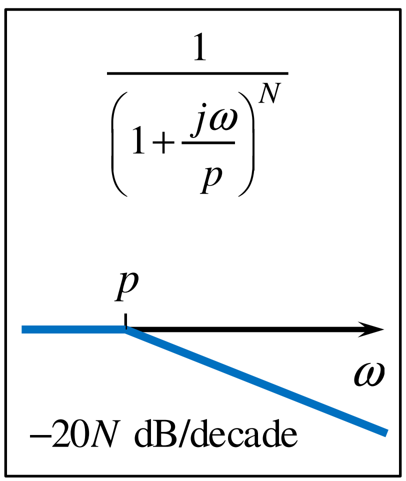




Ex. 9 Bode Plots (38)

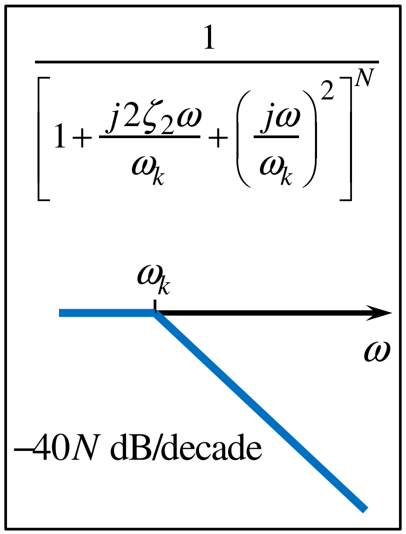
Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{8(1+j\omega/20)}{j\omega\boxed{(1+j\omega/5)^2}\left[1+j\omega4/10+(j\omega/10)^2\right]}$

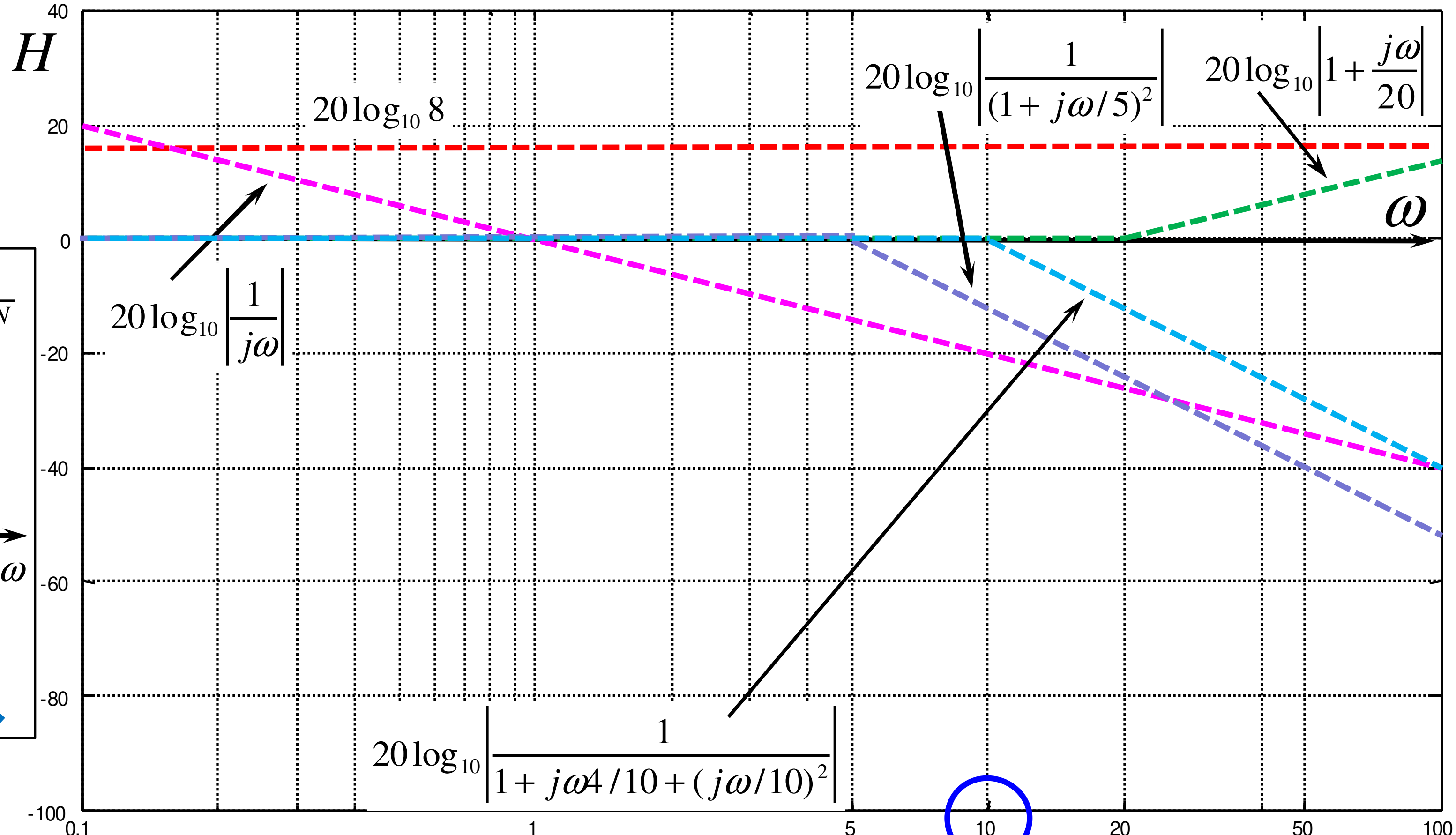




Ex. 9 Bode Plots (39)

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{8(1+j\omega/20)}{j\omega(1+j\omega/5)^2\boxed{\left[1+j\omega 4/10+(j\omega/10)^2\right]}}$

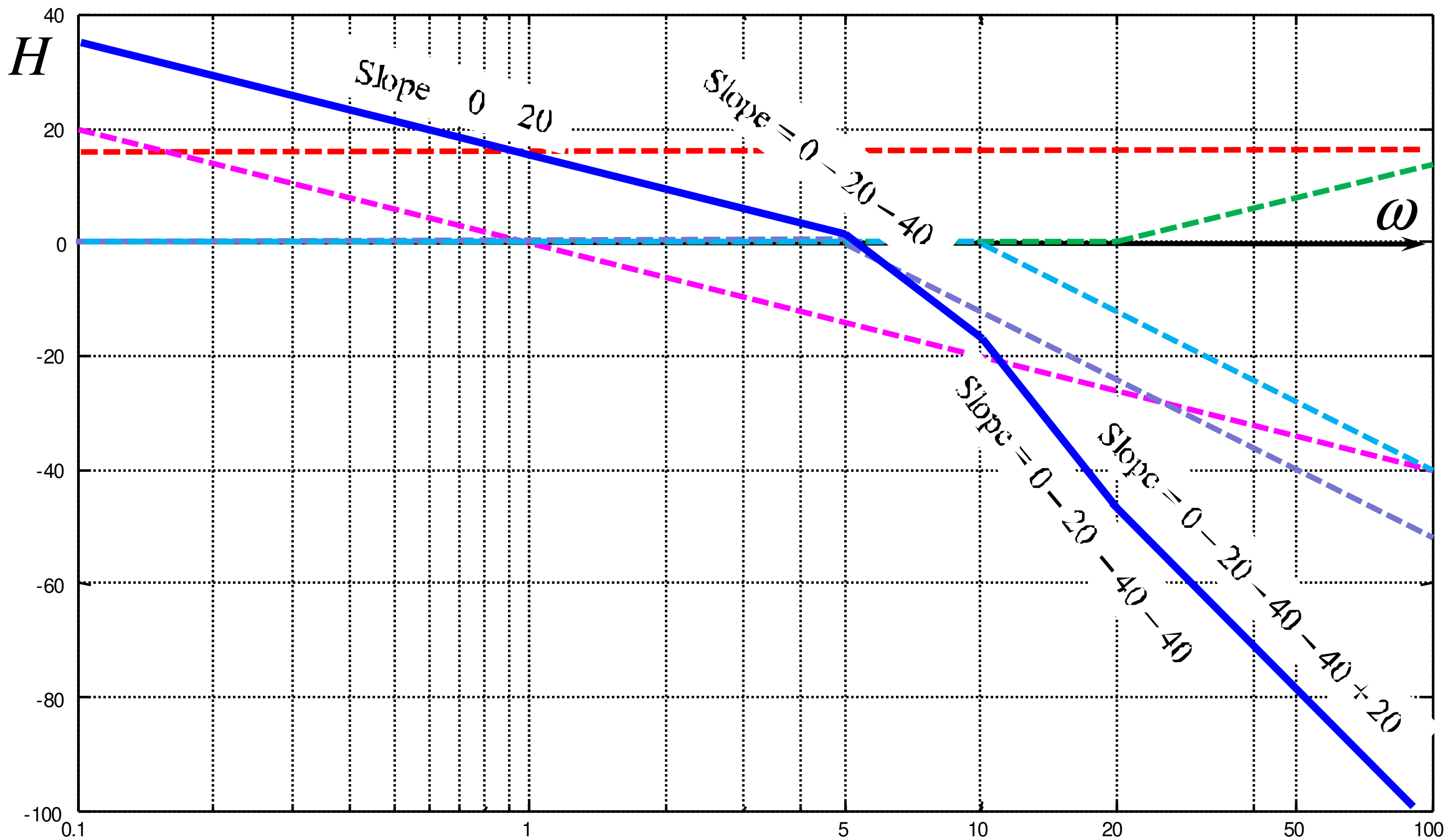




Ex. 9

Bode Plots (40)

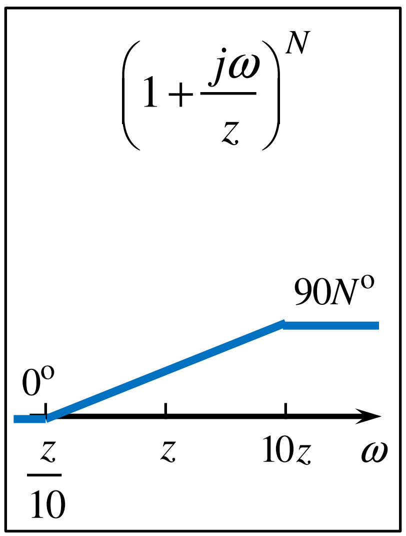
Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega + 20)}{j\omega(j\omega+5)^2\left[(j\omega)^2 + 40j\omega + 100\right]} = \dfrac{8(1 + j\omega/20)}{j\omega(1+j\omega/5)^2\left[1 + j\omega 4/10 + (j\omega/10)^2\right]}$

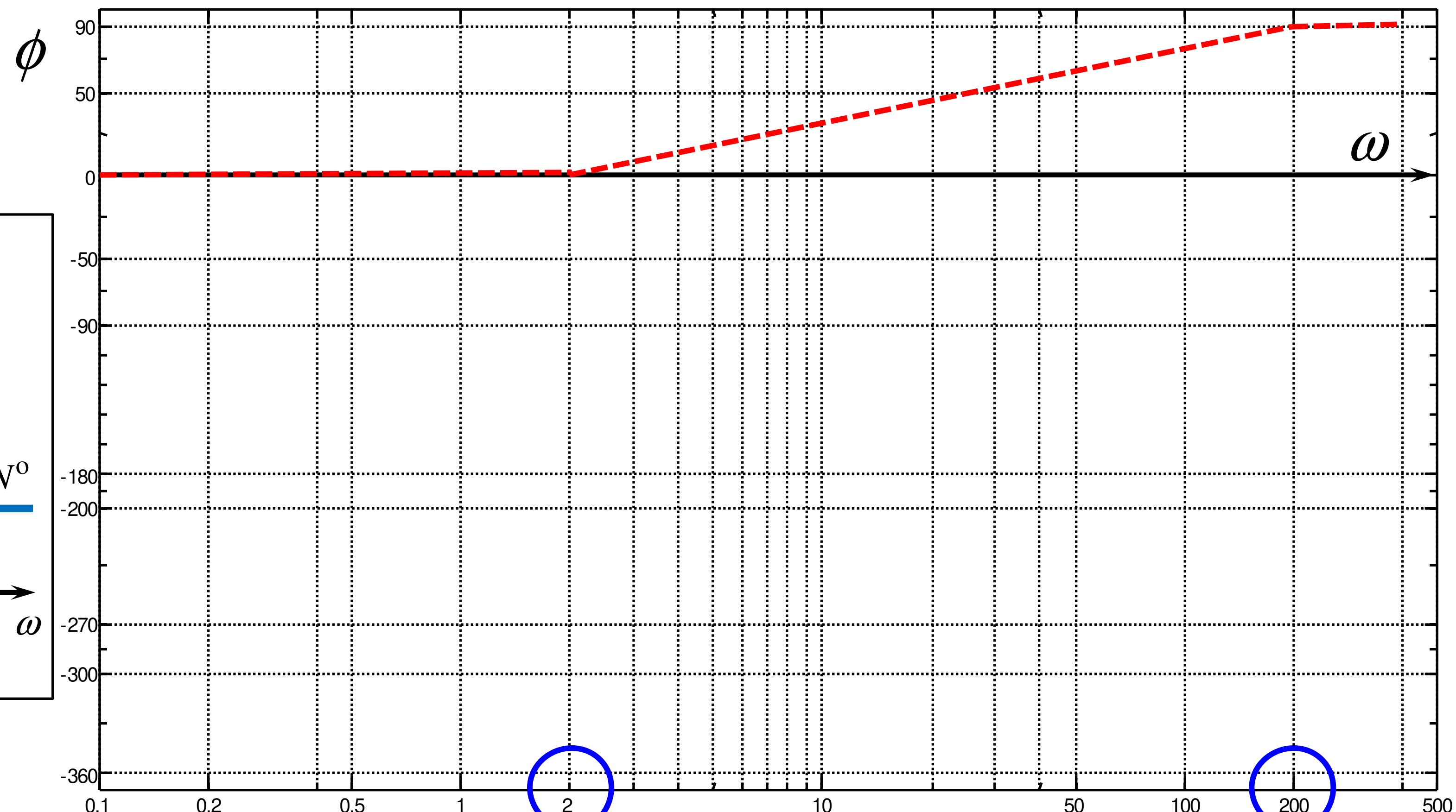


Ex. 9

Bode Plots (41)

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega + 20)}{j\omega(j\omega+5)^2\left[(j\omega)^2 + 40j\omega + 100\right]} = \dfrac{8\boxed{(1 + j\omega/20)}}{j\omega(1+j\omega/5)^2\left[1 + j\omega 4/10 + (j\omega/10)^2\right]}$

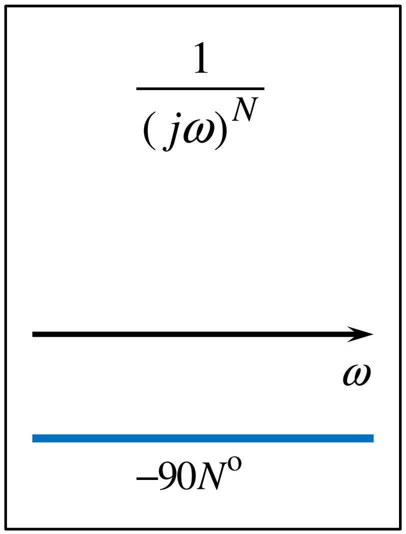


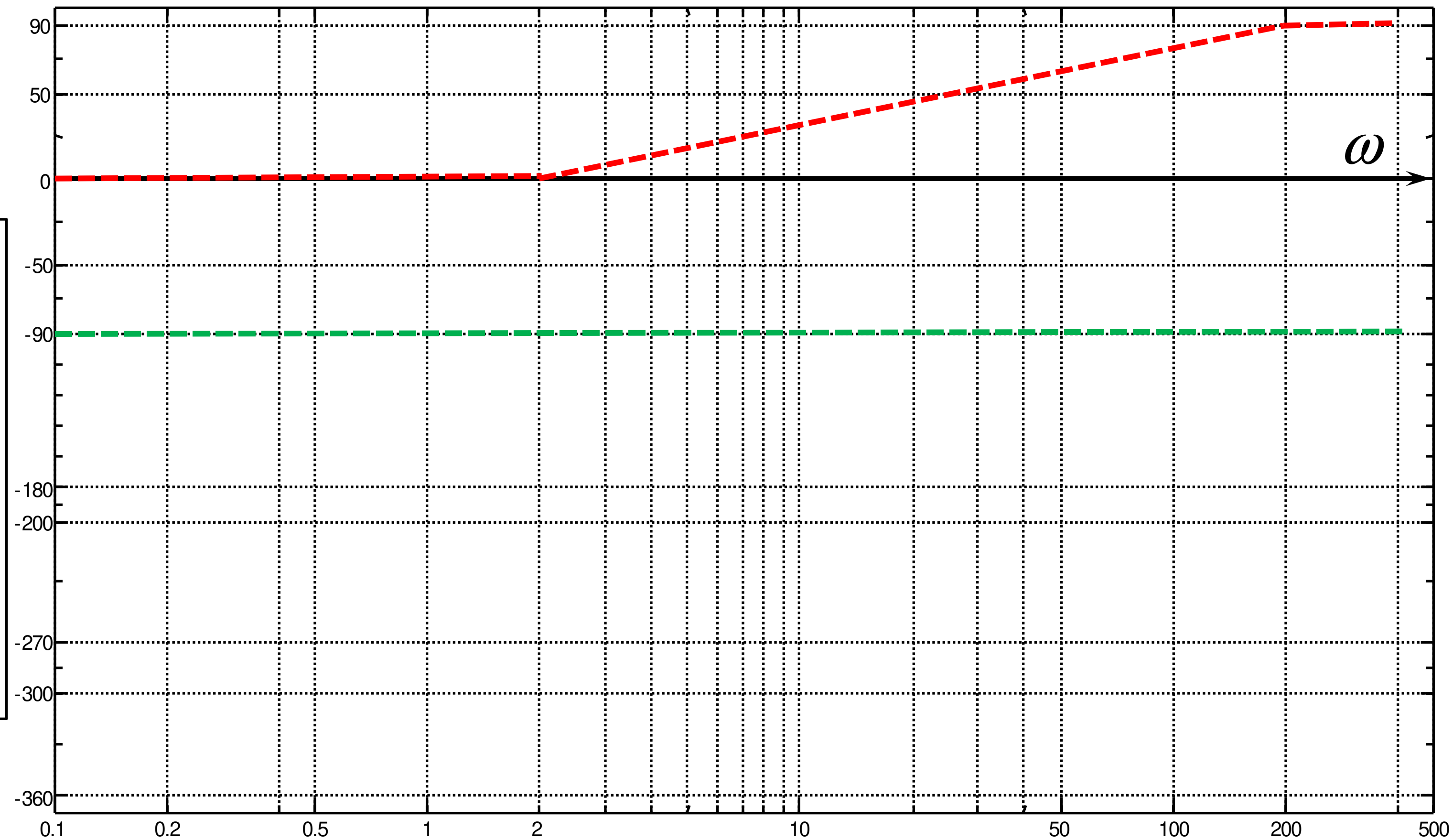


Bode Plots (42)

Ex. 9

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega+20)}{j\omega(j\omega+5)^2\left[(j\omega)^2+40j\omega+100\right]} = \dfrac{8(1+j\omega/20)}{\boxed{j\omega}(1+j\omega/5)^2\left[1+j\omega4/10+(j\omega/10)^2\right]}$



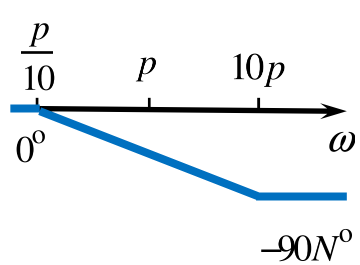


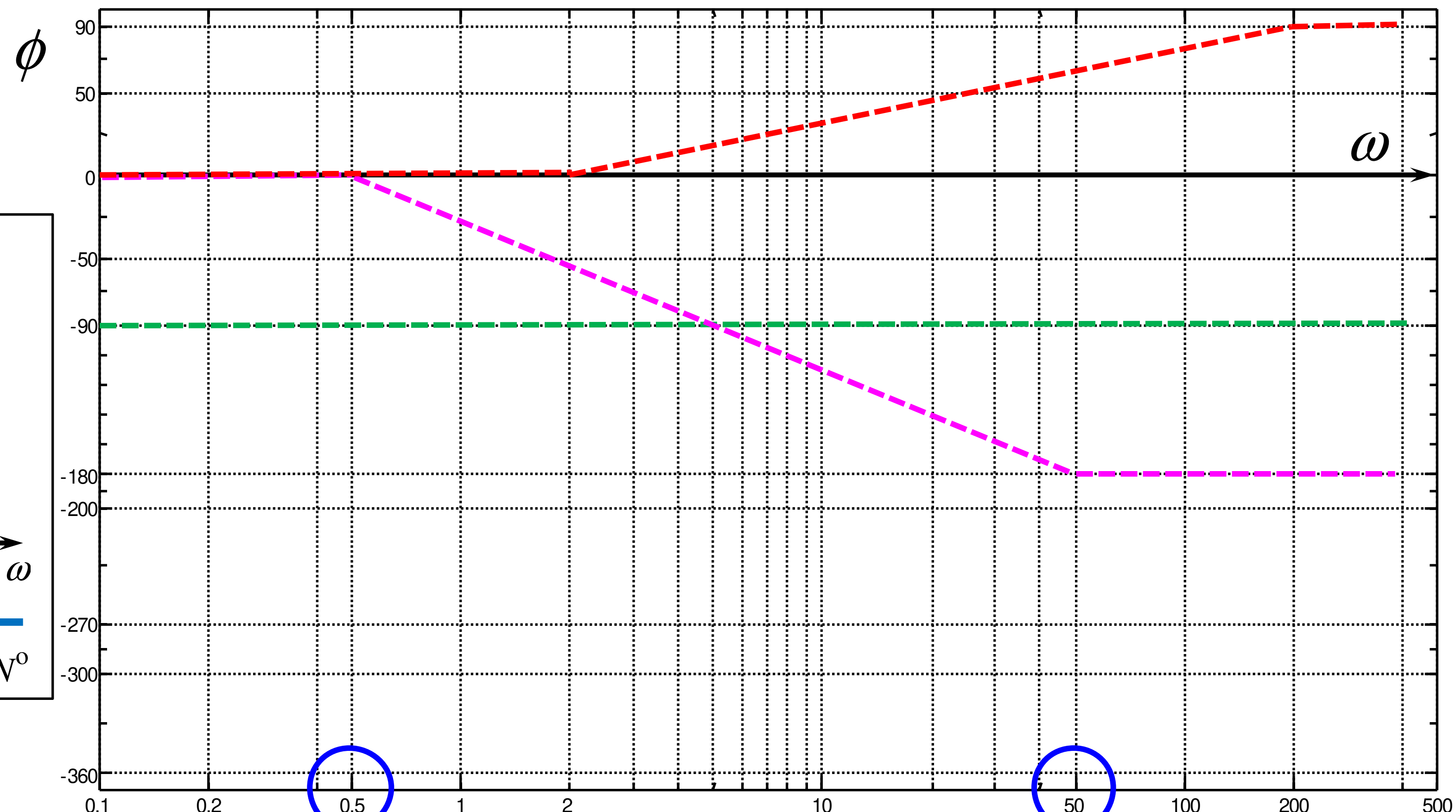
Ex. 9

Bode Plots (43)

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega + 20)}{j\omega(j\omega+5)^2\left[(j\omega)^2 + 40j\omega + 100\right]} = \dfrac{8(1 + j\omega/20)}{j\omega\boxed{(1+j\omega/5)^2}\left[1 + j\omega 4/10 + (j\omega/10)^2\right]}$

$$\frac{1}{\left(1 + \dfrac{j\omega}{p}\right)^N}$$

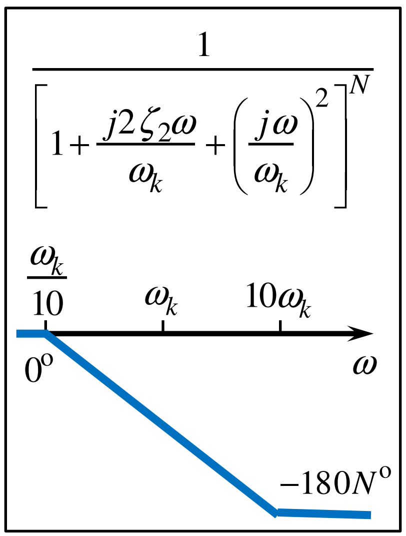


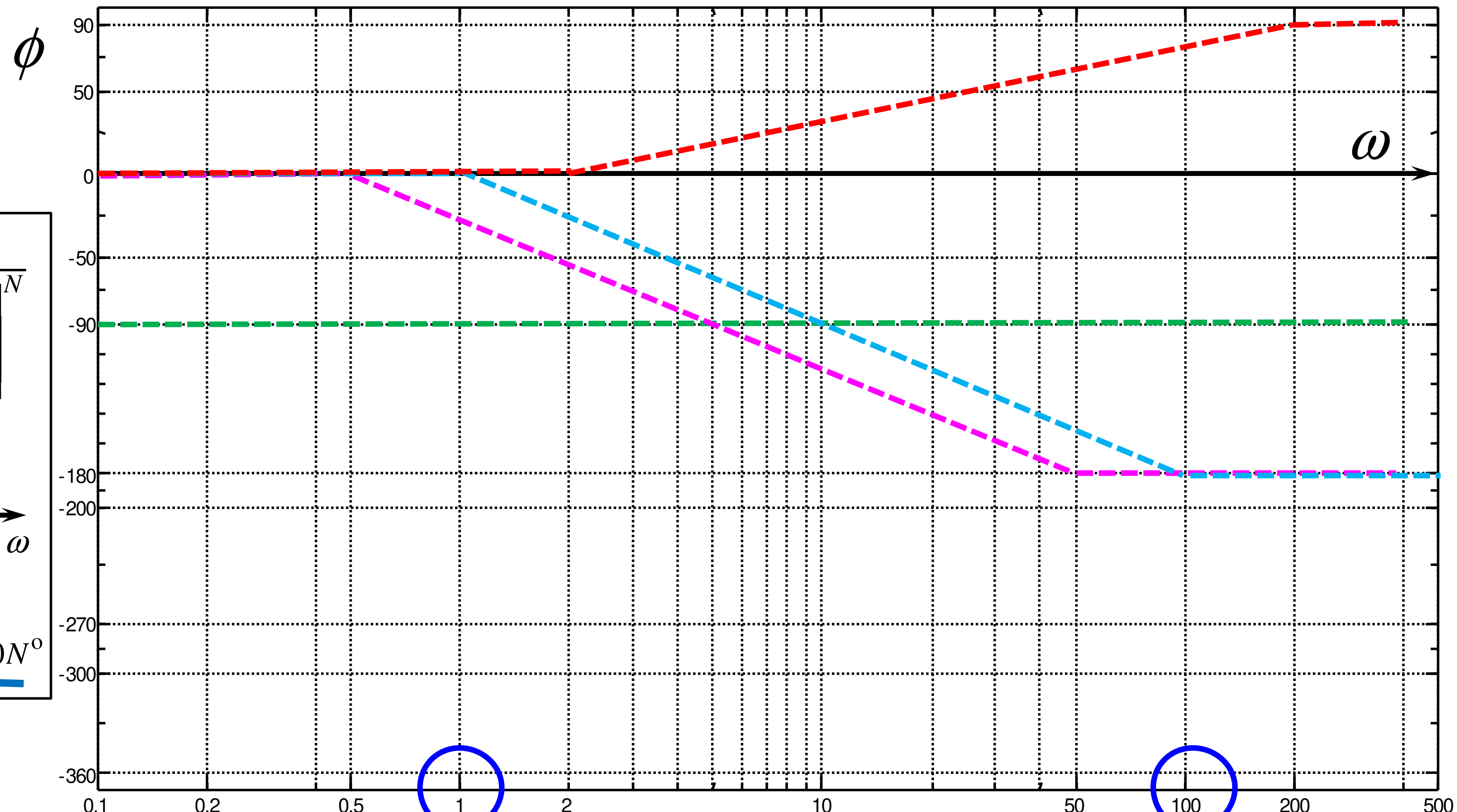


Ex. 9

Bode Plots (44)

Construct the Bode plots for $\mathbf{H}(\omega) = \dfrac{1000(j\omega + 20)}{j\omega(j\omega+5)^2\left[(j\omega)^2 + 40j\omega + 100\right]} = \dfrac{8(1 + j\omega/20)}{j\omega(1 + j\omega/5)^2\boxed{\left[1 + j\omega 4/10 + (j\omega/10)^2\right]}}$

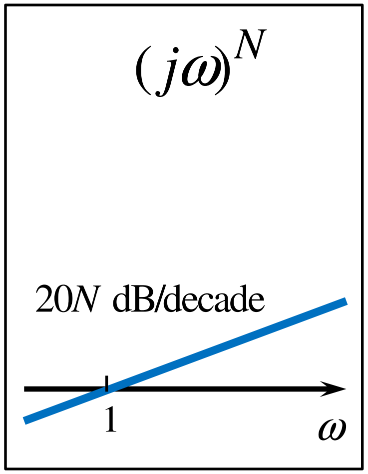


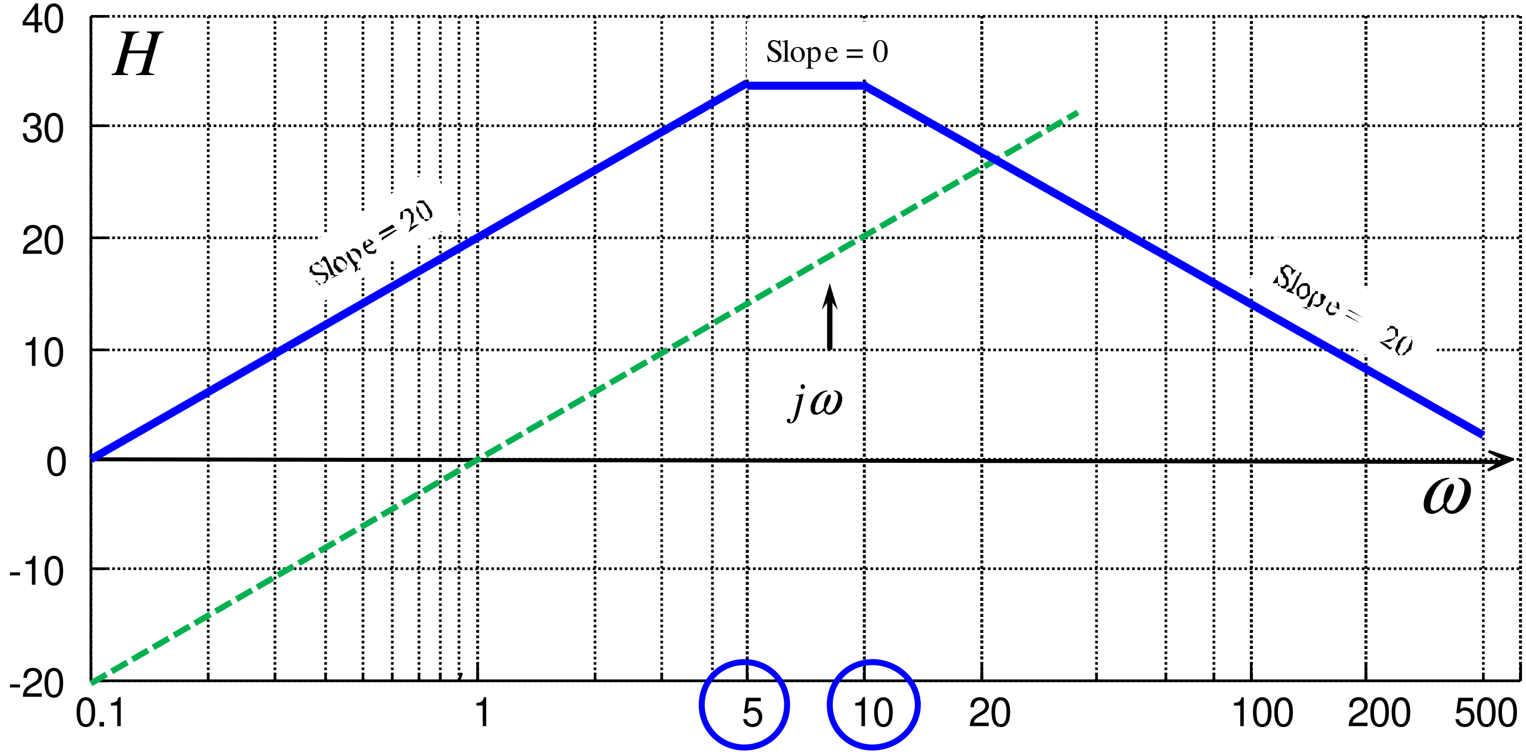


Bode Plots (45)

Ex. 10

Find the transfer function from the Bode plot?

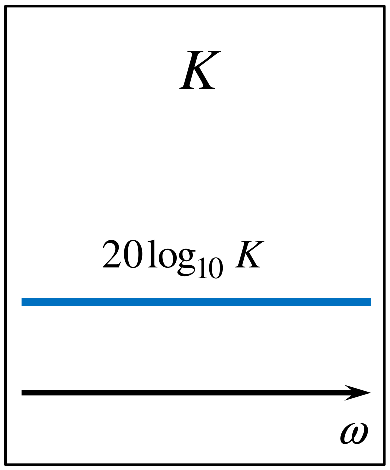


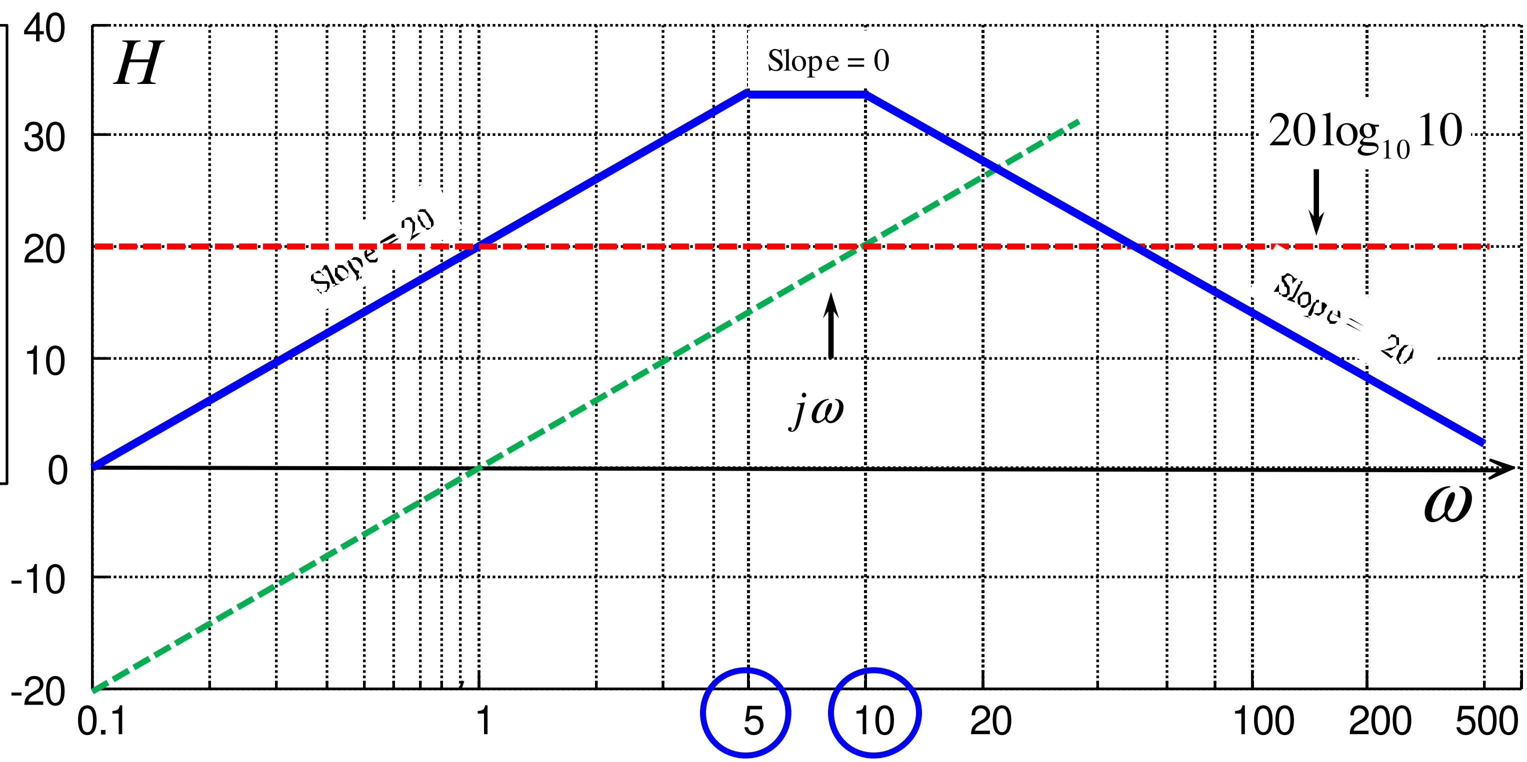


Bode Plots (46)

Ex. 10

Find the transfer function from the Bode plot?

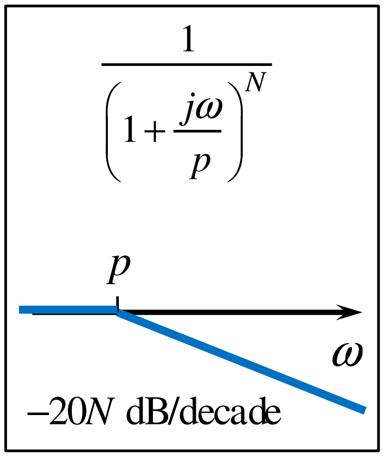


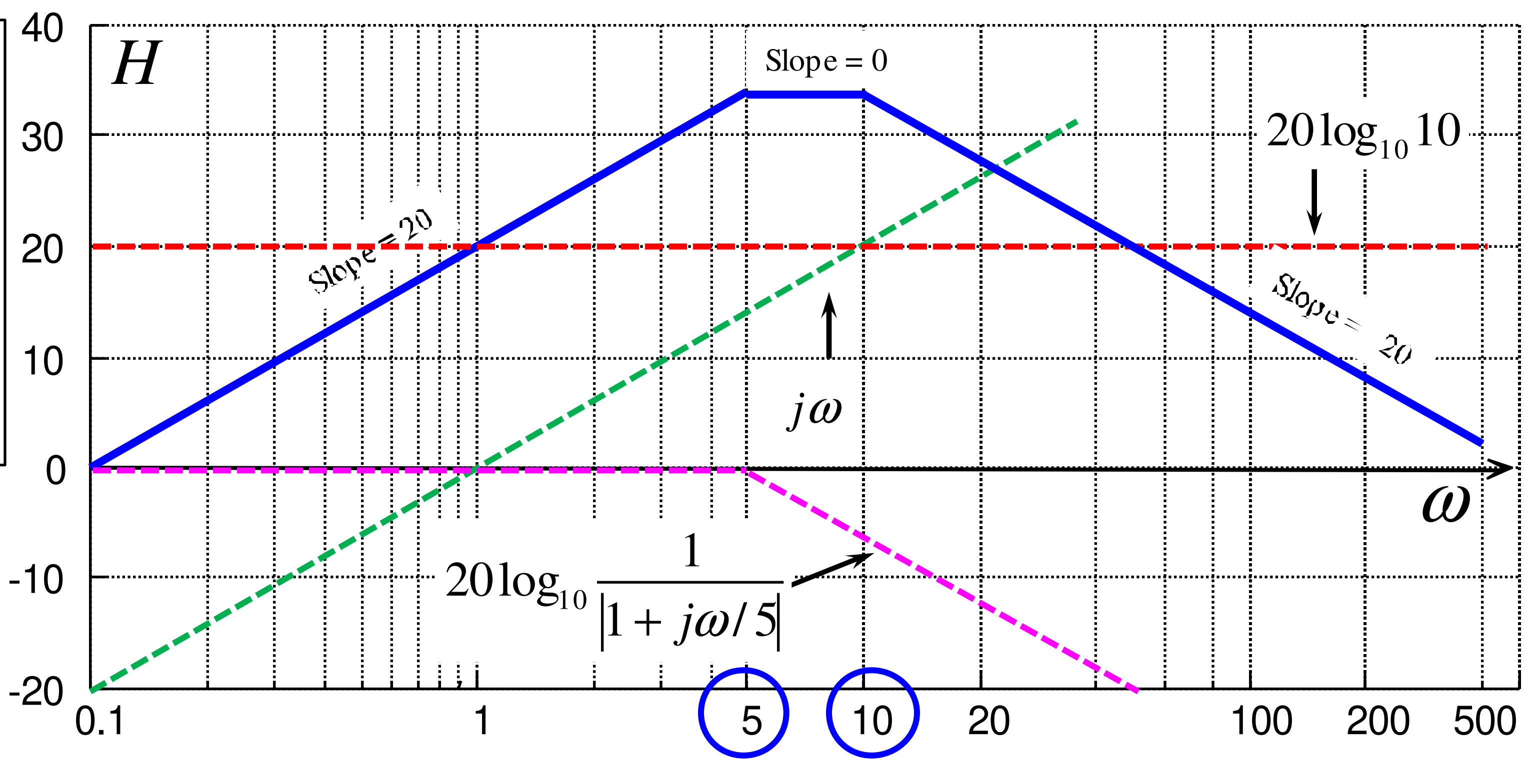


Bode Plots (47)

Ex. 10

Find the transfer function from the Bode plot?



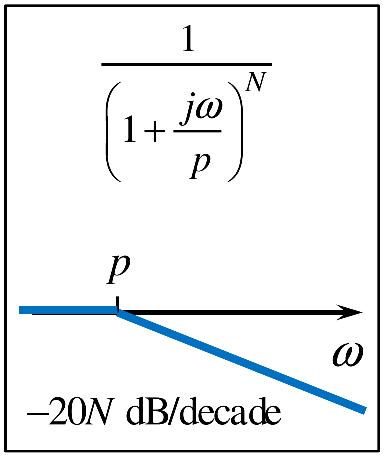


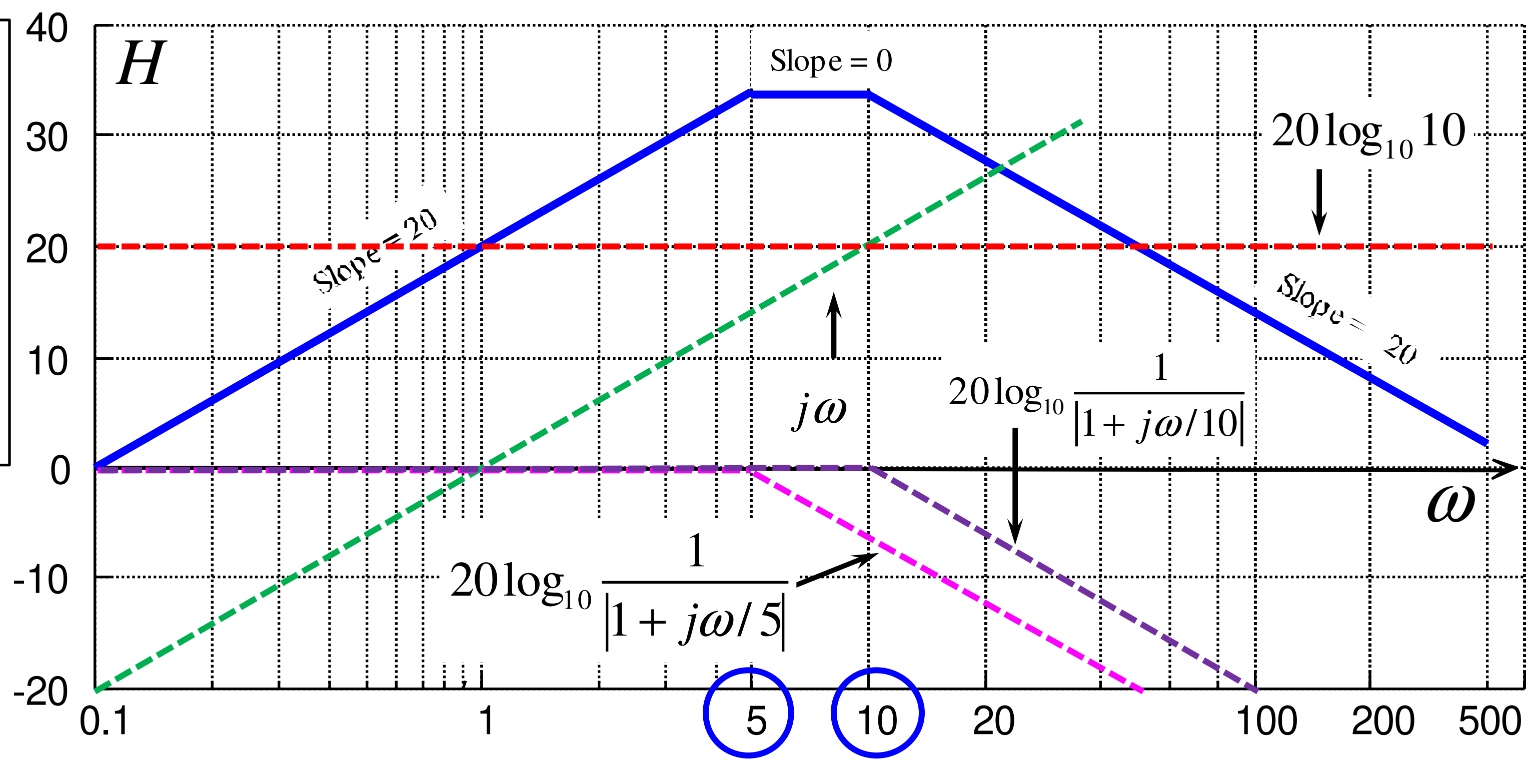
Bode Plots (48)

Ex. 10

Find the transfer function from the Bode plot?

$$\mathbf{H}(\omega) = \frac{10j\omega}{(1+j\omega/5)(1+j\omega/10)}$$





Bode Plots (49)

Ex. 11

$$\mathbf{H}(\omega) = \frac{10j\omega}{(1 + j\omega/5)(1 + j\omega/10)} = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

$$x = \sin(0.1t)$$

$$\to \mathbf{X} = 1$$

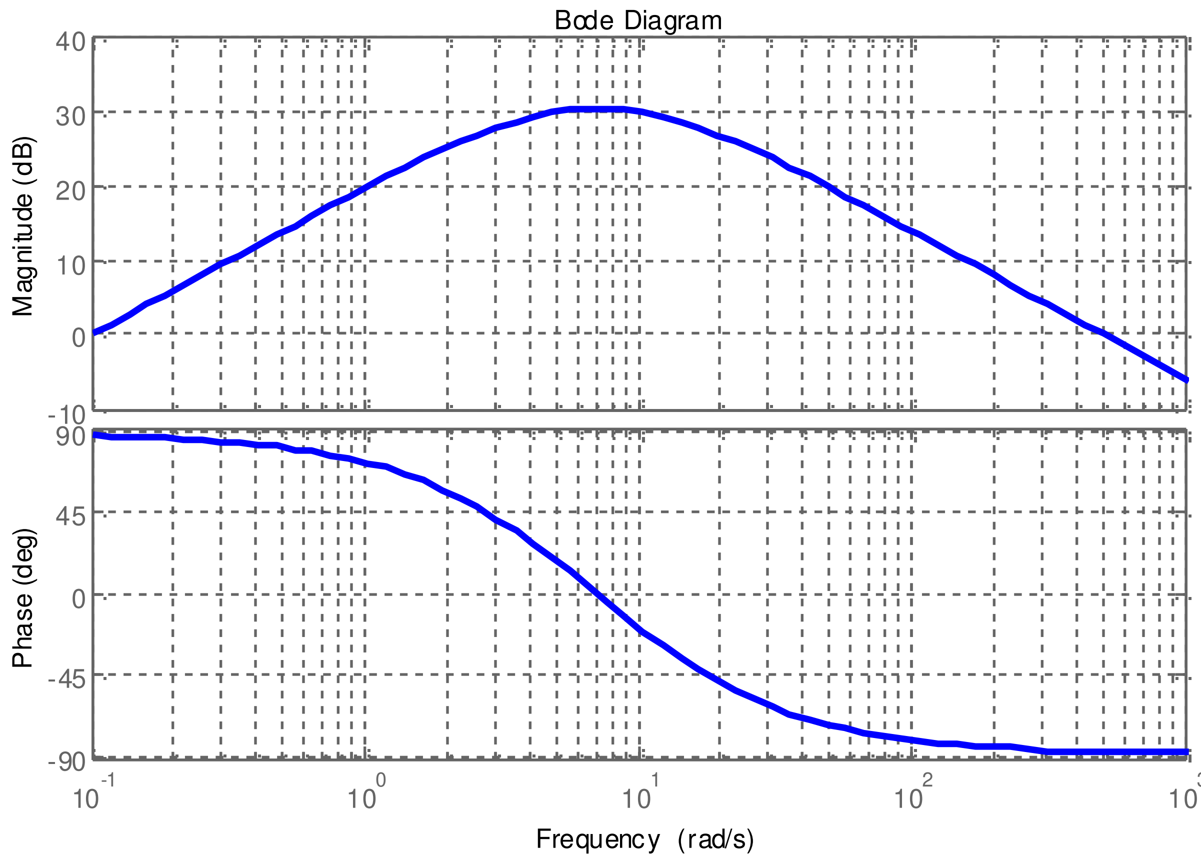
$$\to \mathbf{Y} = \frac{10j\omega}{(1 + j\omega/5)(1 + j\omega/10)}\mathbf{X}$$

$$= \frac{10j\omega}{(1 + j\omega/5)(1 + j\omega/10)} \times 1$$

$$= 0.030 + j1.00$$

$$= 1.00\underline{/88.3^{\circ}}$$

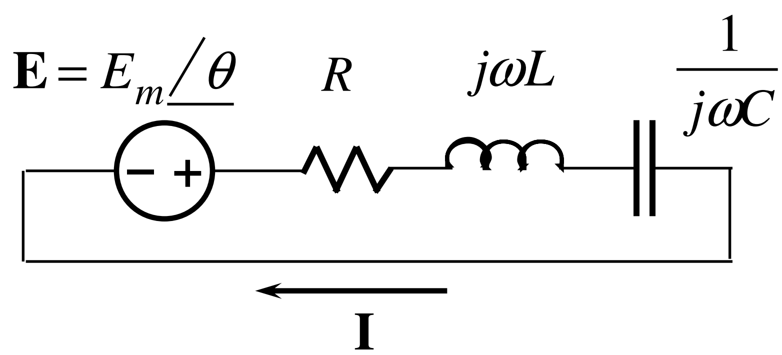
$$\to y = 1.00\sin(0.1t + 88.3^{\circ})$$



Frequency Response

1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. **Series Resonance**
5. Parallel Resonance
6. Passive Filters
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. Narrowband Bandpass & Banstop Filters

Series Resonance (1)



$$\mathbf{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) = \mathbf{H}(\omega)$$

$$\text{Resonance: } \operatorname{Im}(\mathbf{Z}) = 0 \rightarrow \omega L - \frac{1}{\omega C} = 0$$

$$\text{Resonant frequency: } \omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s; } f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Series Resonance (2)

$$I = \frac{E_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

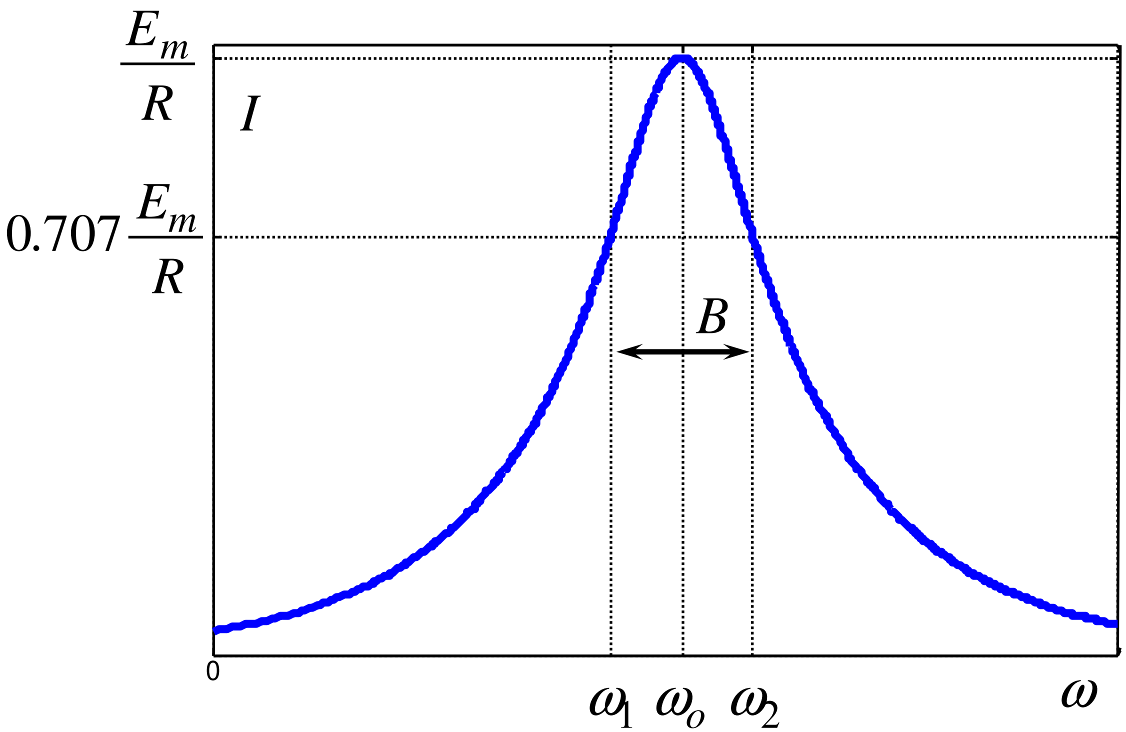
$$\omega_o = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_2}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

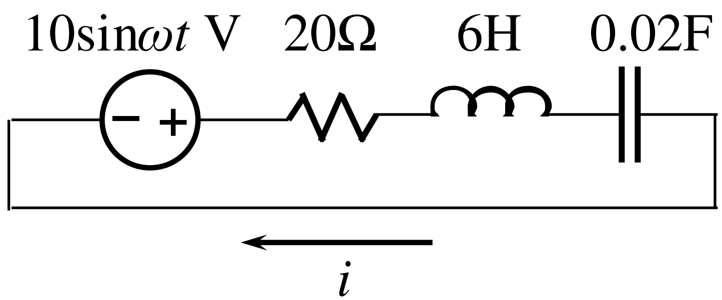
$$Q = \frac{\omega_o}{B}$$



Series Resonance (3)

Ex. 1

Find ω_o, ω_1, ω_2, B, Q?



$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{6\times0.02}} = 2.89 \text{ rad/s}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = -\frac{20}{2\times6} + \sqrt{\left(\frac{20}{2\times6}\right)^2 + \frac{1}{6\times0.02}} = 1.67 \text{ rad/s}$$

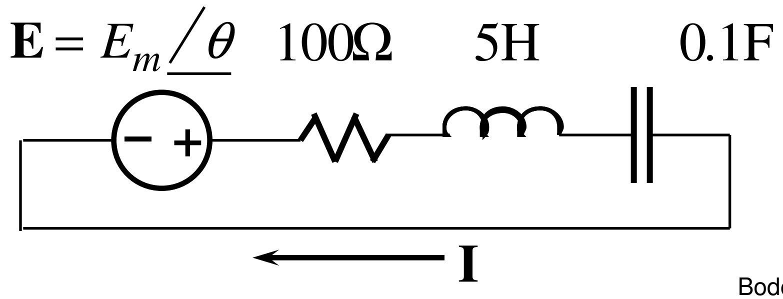
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \frac{20}{2\times6} + \sqrt{\left(\frac{20}{2\times6}\right)^2 + \frac{1}{6\times0.02}} = 5.00 \text{ rad/s}$$

$$B = \omega_2 - \omega_1 = 5.00 - 1.67 = 3.33 \text{ rad/s}$$

$$Q = \frac{\omega_o}{B} = \frac{2.89}{3.33} = 0.87$$

Series Resonance (4)

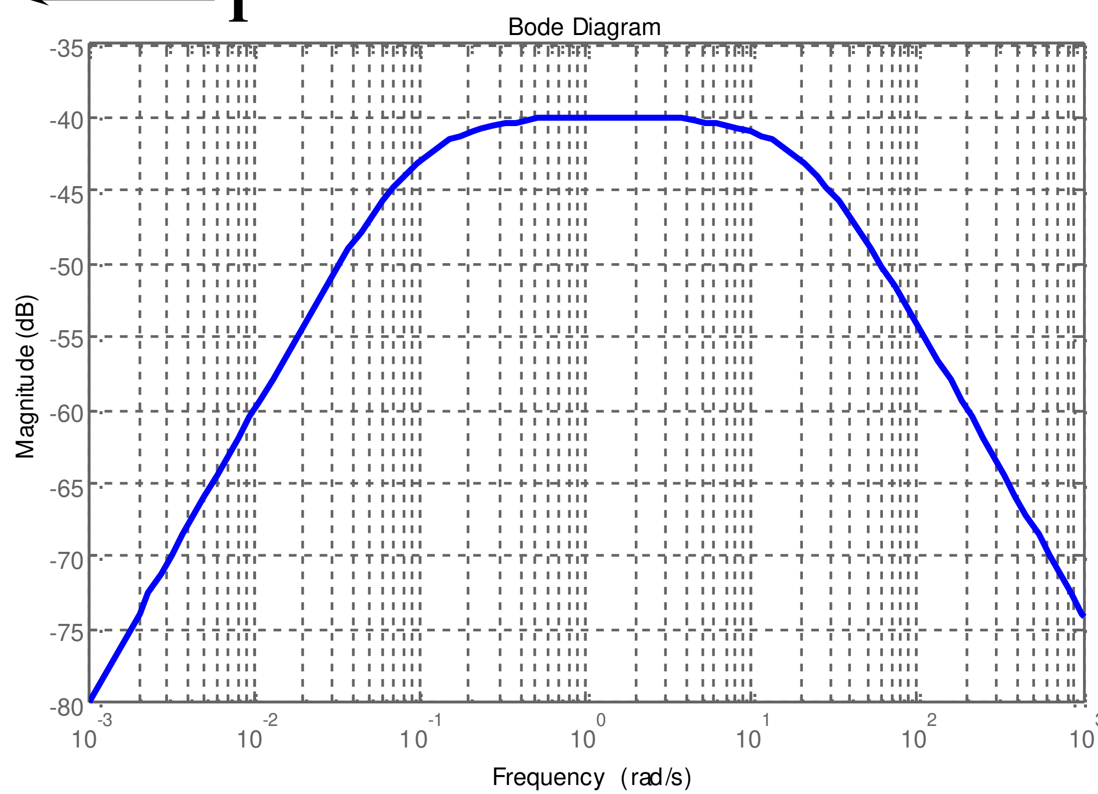
Ex. 2



$$\mathbf{H} = \frac{\mathbf{I}}{\mathbf{E}} = \frac{1}{\mathbf{Z}}$$

$$= \frac{1}{100 + j5\omega + \dfrac{1}{j0.1\omega}}$$

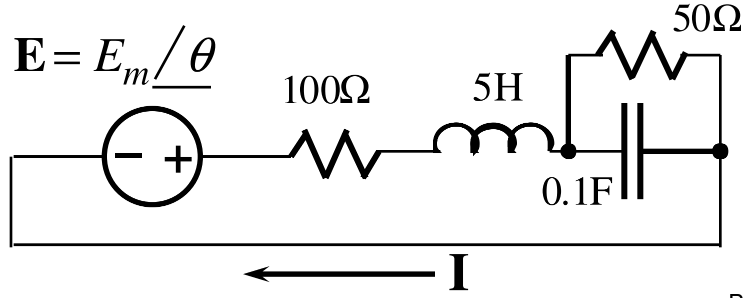
$$= \frac{0.2\,j\omega}{(j\omega)^2 + 20\,j\omega + 2}$$



Series Resonance (5)

Ex. 3

$\mathbf{E} = E_m \angle \theta$



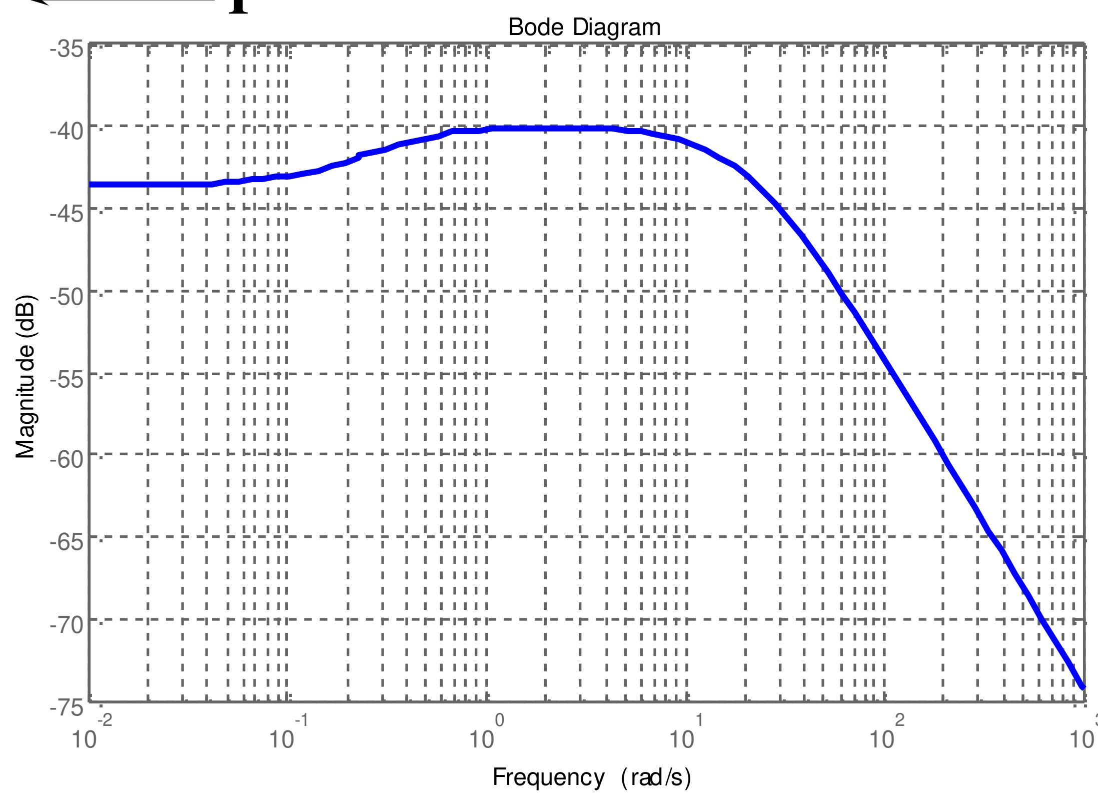
$$\mathbf{H} = \frac{\mathbf{I}}{\mathbf{E}} = \frac{1}{\mathbf{Z}}$$

$$\mathbf{Z} = 100 + j5\omega + \frac{50\dfrac{1}{j0.1\omega}}{50 + \dfrac{1}{j0.1\omega}}$$

$$= \frac{25(j\omega)^2 + 505\,j\omega + 150}{5\,j\omega + 1}$$

$$= \frac{2525\omega^2 + 150}{25\omega^2 + 1} + j\frac{125\omega^3 - 245\omega}{25\omega^2 + 1}$$

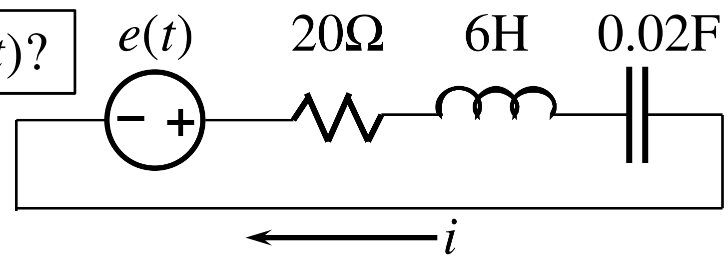
$$\mathrm{Im}\{\mathbf{Z}\} = 0 \rightarrow \omega_o = 1.40$$



Series Resonance (6)

Ex. 4

Given $e(t)$ = sin0.03t + sin3t + sin300t (V). Find $i(t)$?



$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{6\times 0.02}} = 2.89 \text{ rad/s}$$

$$\mathbf{I}_{0.03} = \frac{1}{20 + j0.18 + \dfrac{1}{j0.0006}} = 0.00060\angle 89.3^o \text{ A} \quad \rightarrow i_{0.03}(t) = 0.00060\sin(0.03t + 89.3^o) \text{ A}$$

$$\mathbf{I}_{3} = \frac{1}{20 + j18 + \dfrac{1}{j0.06}} = 0.050\angle -3.8^o \text{ A} \quad \rightarrow i_{3}(t) = 0.050\sin(3t - 3.8^o) \text{ A}$$

$$\mathbf{I}_{300} = \frac{1}{20 + j1800 + \dfrac{1}{j6}} = 0.00055\angle -89.4^o \text{ A} \quad \rightarrow i_{300}(t) = 0.00056\sin(300t - 89.4^o) \text{ A}$$

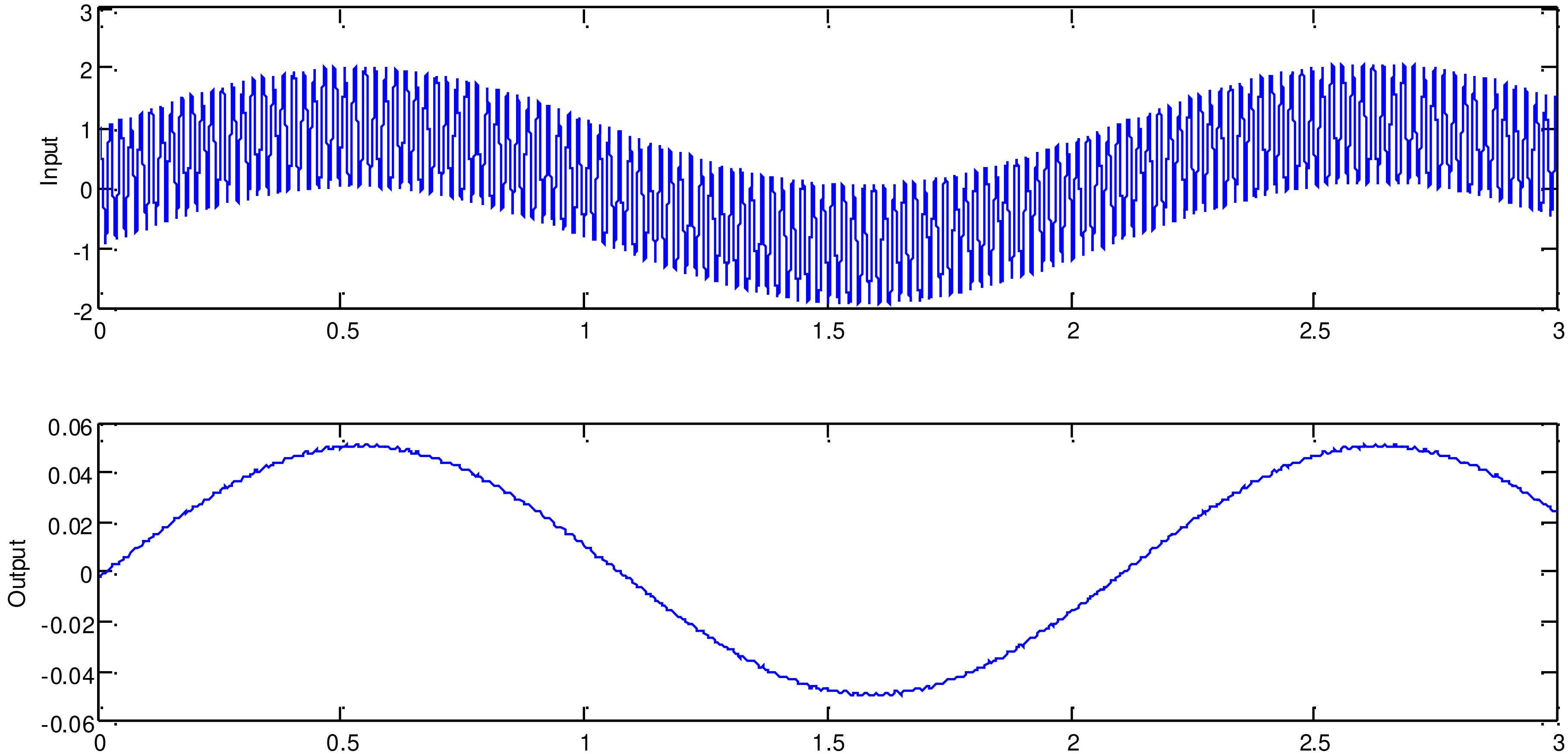
$$\rightarrow i(t) = 0.00060\sin(0.03t + 89.3^o) + 0.050\sin(3t - 3.8^o) + 0.00056\sin(300t - 89.4^o) \text{ A}$$

Series Resonance (7)

Ex. 4

Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find $i(t)$?

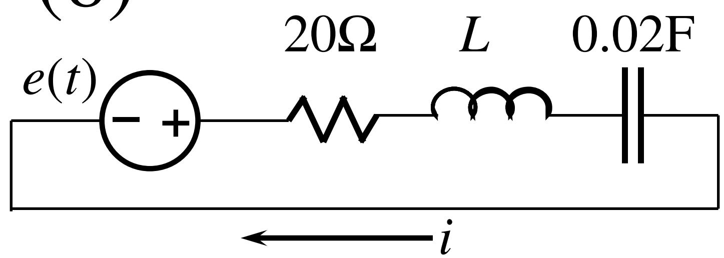
$$i(t) = 0.00060\sin(0.03t + 89.3^{\circ}) + 0.050\sin(3t - 3.8^{\circ}) + 0.00056\sin(300t - 89.4^{\circ})\ \text{A}$$



Series Resonance (8)

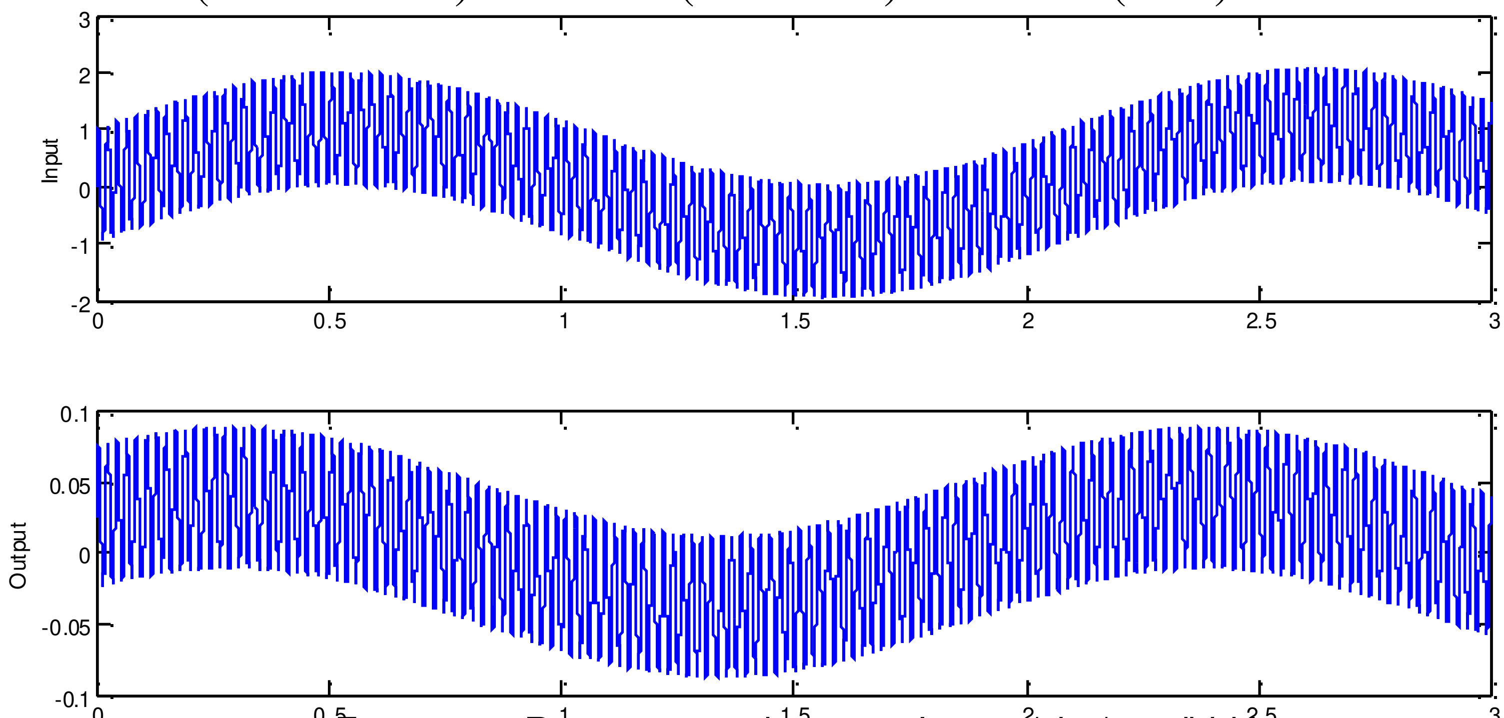
Ex. 5

Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find L to extract the highest frequency component?



$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.02L}} = 300 \rightarrow L = 5.56\times10^{-4} \text{ H}$$

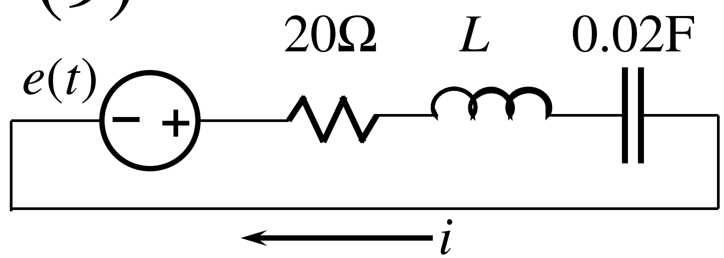
$$i(t) = 0.00060\sin(0.03t + 89.3^o) + 0.038\sin(3t + 39.8^o) + 0.050\sin(300t) \text{ A}$$



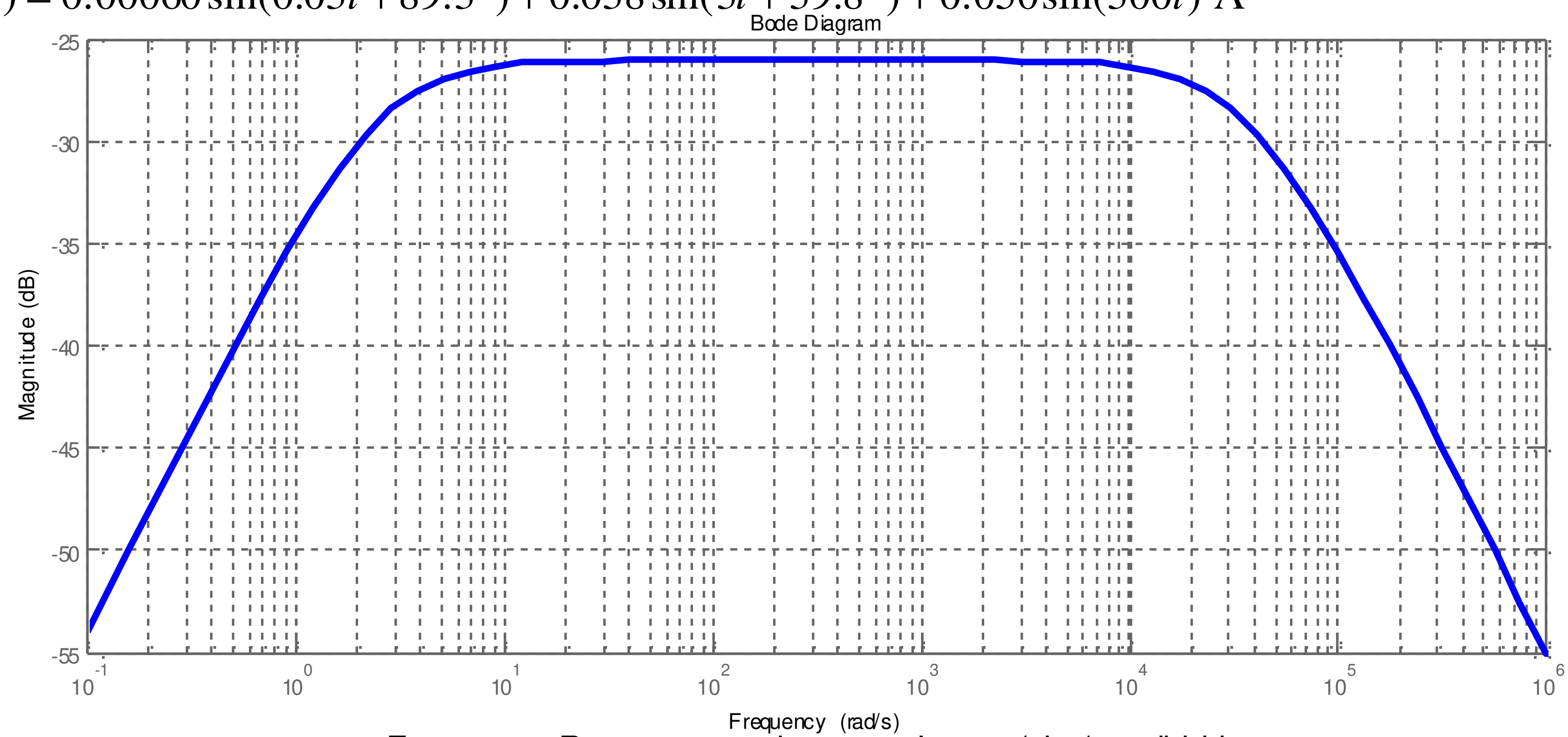
Series Resonance (9)

Ex. 5

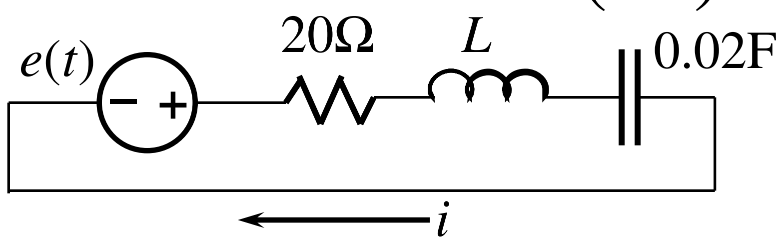
Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find L to extract the highest frequency component?

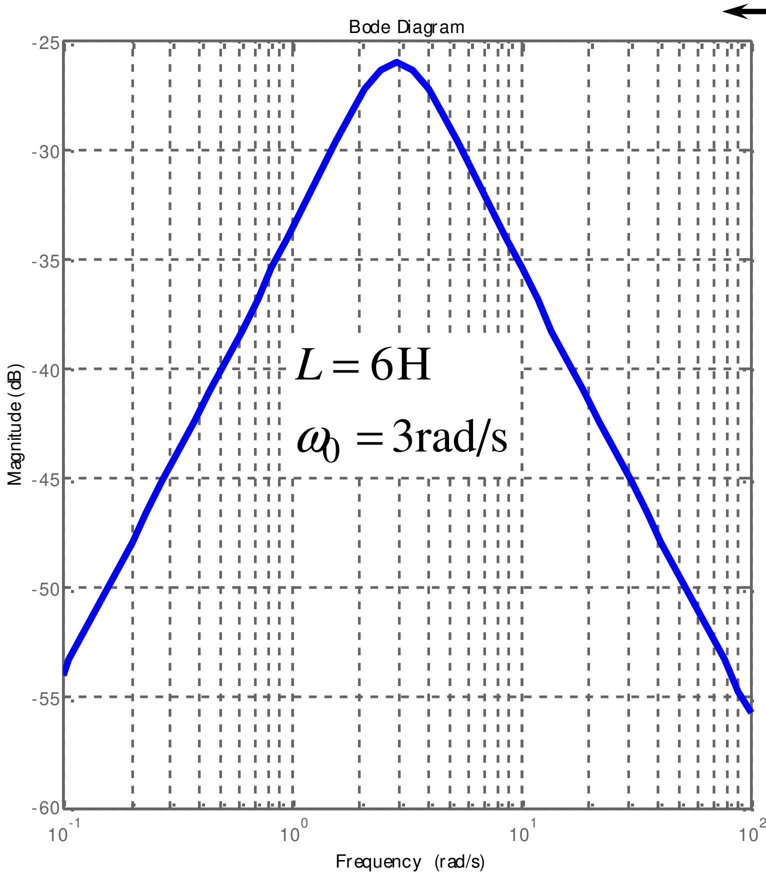


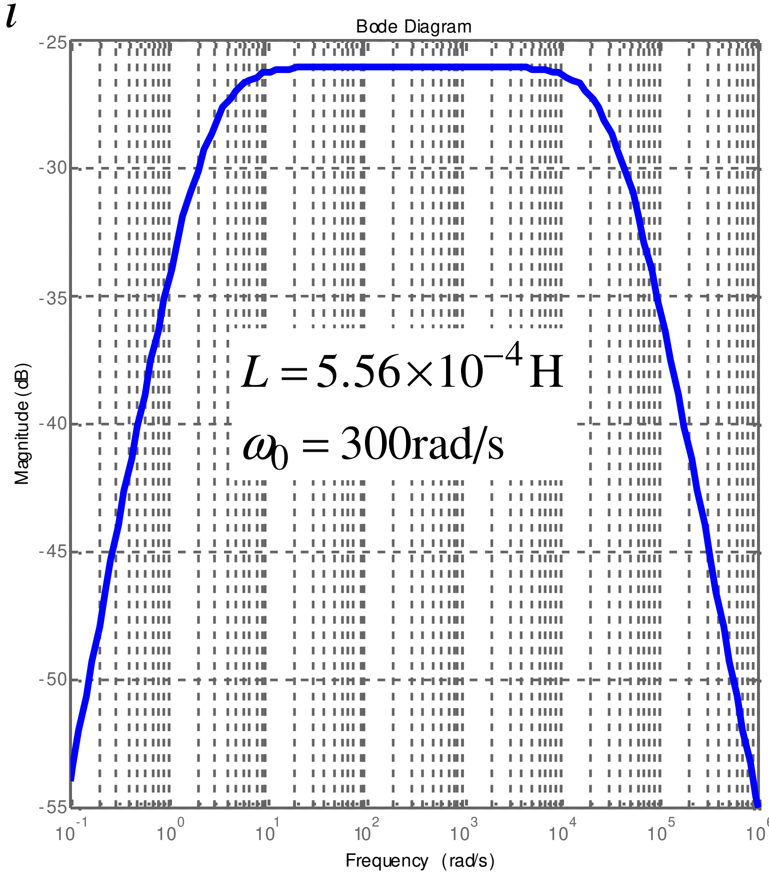
$$i(t) = 0.00060\sin(0.03t + 89.3^o) + 0.038\sin(3t + 39.8^o) + 0.050\sin(300t)\ \text{A}$$



Series Resonance (10)



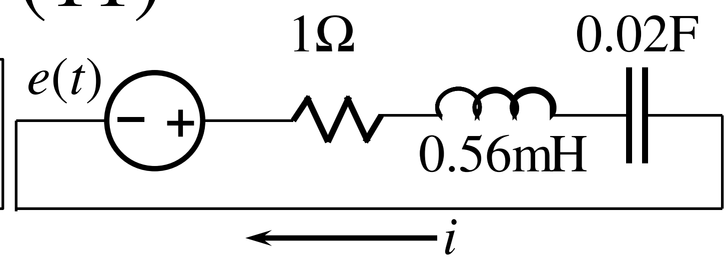




Series Resonance (11)

Ex. 5

Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find L to extract the highest frequency component?



$$i(t) = 0.00060\sin(0.03t + 89.3^{\circ}) + 0.038\sin(3t + 39.8^{\circ}) + 0.050\sin(300t) \text{ A}$$

$$\mathbf{Z}_3 = 20 + j3\times 5.56\times 10^{-4} + \frac{1}{j3\times 0.02} = 20 - j16.67\Omega$$

$$\rightarrow |\mathbf{Z}_3| = \sqrt{20^2 + 16.67^2} = 26.03\Omega$$

$$\mathbf{Z}_{300} = 20 + j300\times 5.56\times 10^{-4} + \frac{1}{j300\times 0.02} = 20 + j0.0001\Omega$$

$$\rightarrow |\mathbf{Z}_{300}| = 20.00\Omega$$

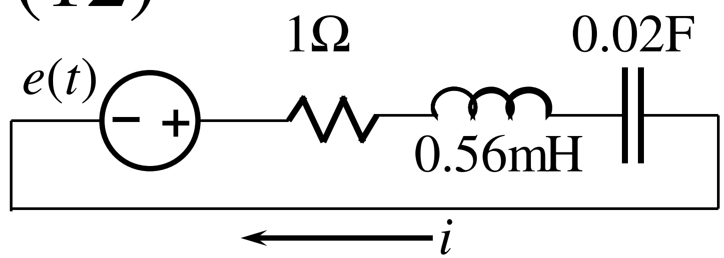
$$R = 1\Omega \rightarrow |\mathbf{Z}_3| = 16.70\Omega;\ |\mathbf{Z}_{300}| = 1.00\Omega$$

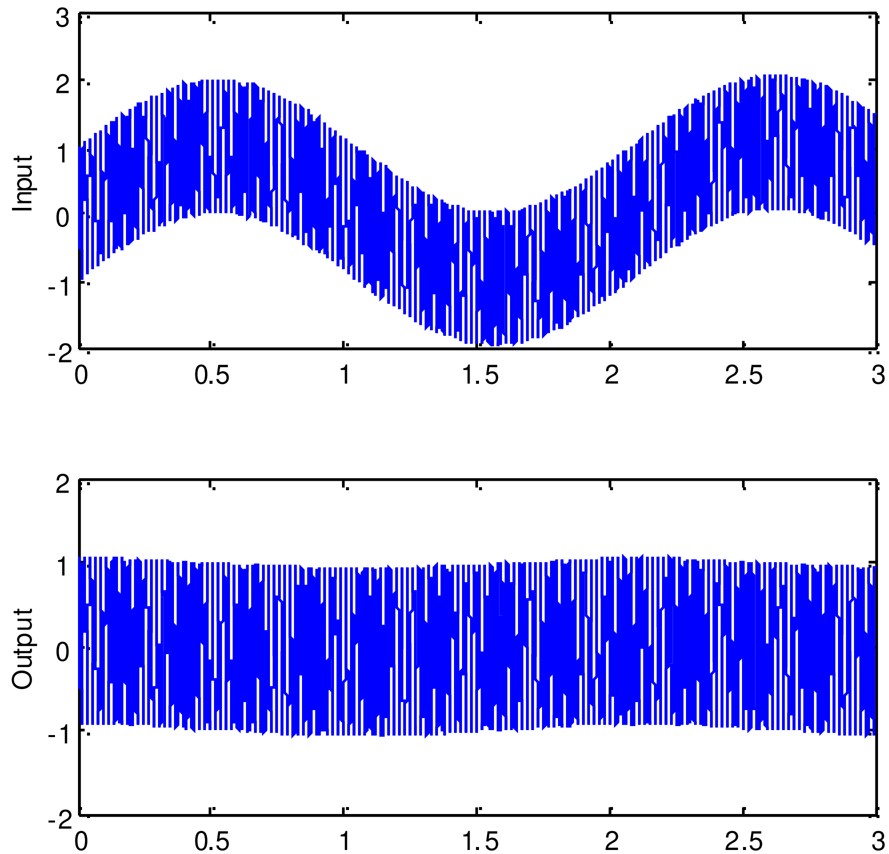
$$i(t) = 0.00060\sin(0.03t + 90,0^{\circ}) + 0.060\sin(3t + 86.6^{\circ}) + 1.00\sin(300t) \text{ A}$$

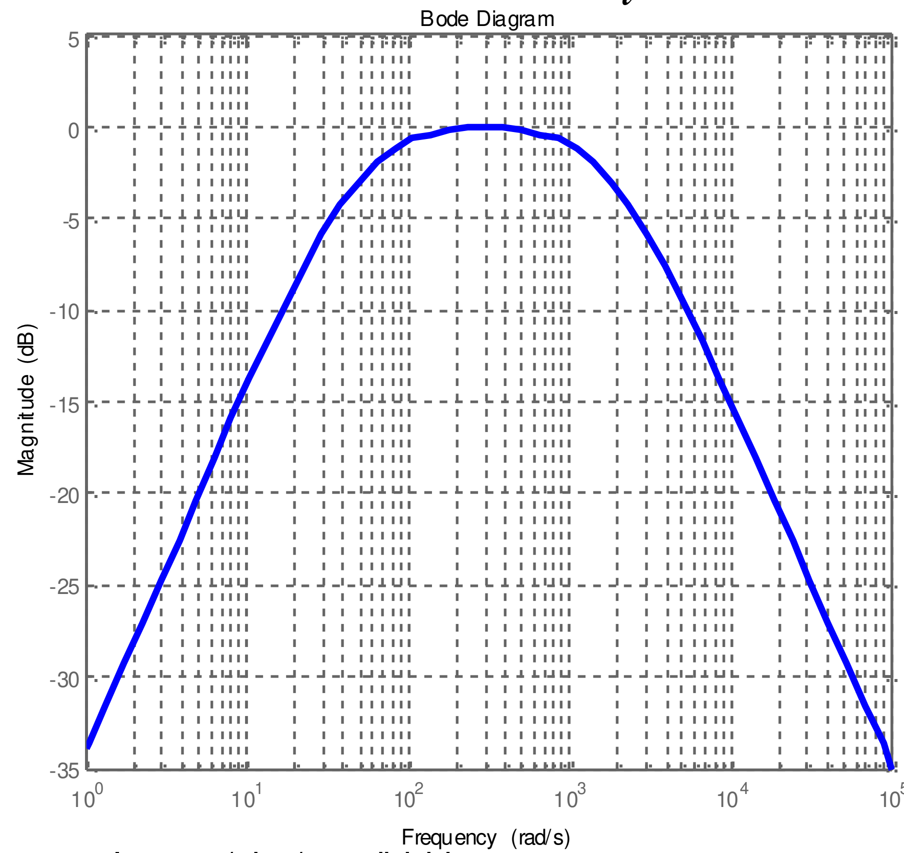
Series Resonance (12)

Ex. 5

Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find L to extract the highest frequency component?



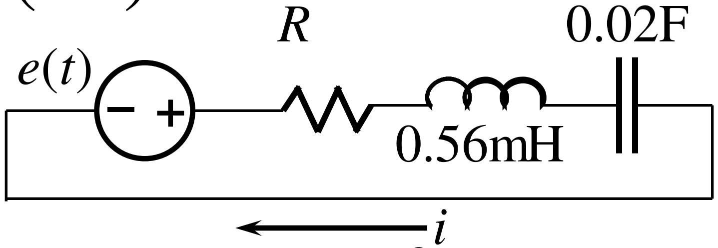


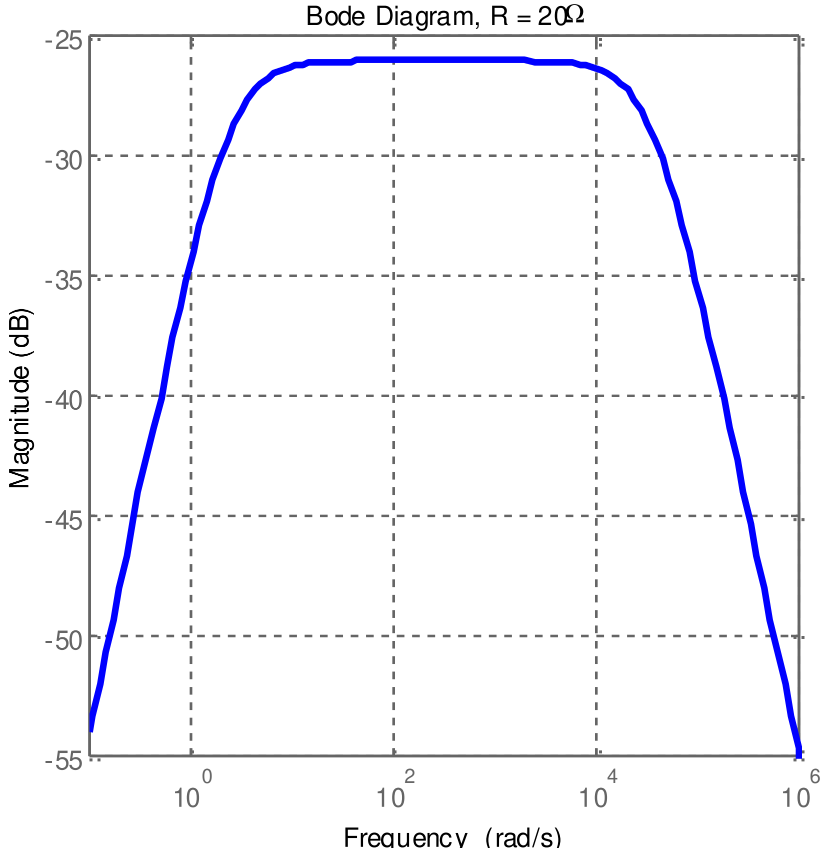


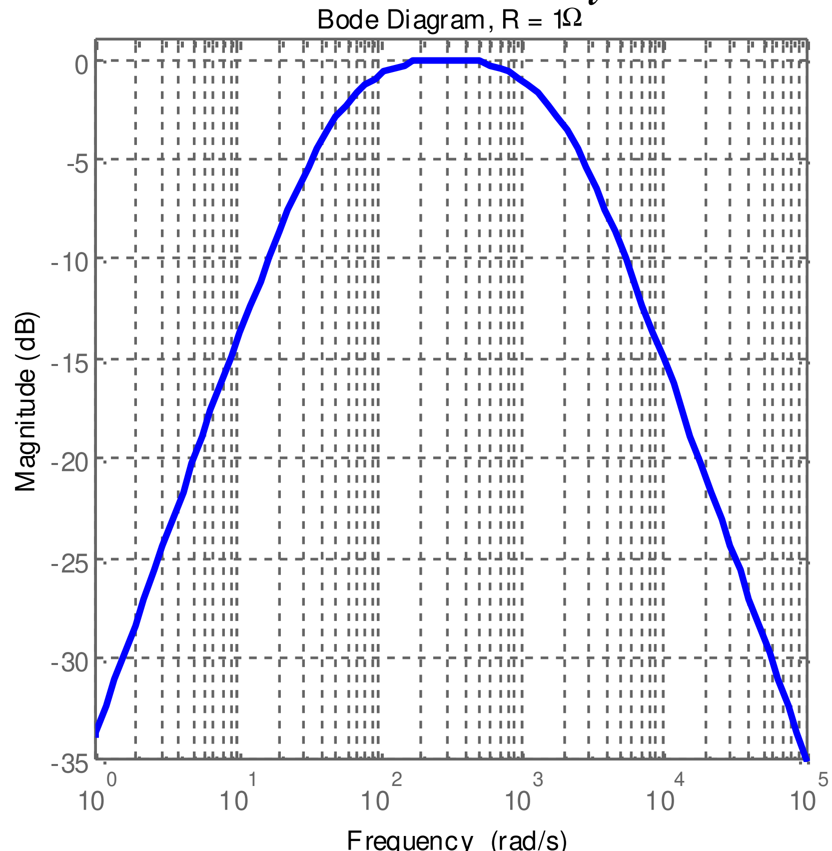
Series Resonance (13)

Ex. 5

Given $e(t) = \sin 0.03t + \sin 3t + \sin 300t$ (V). Find L to extract the highest frequency component?

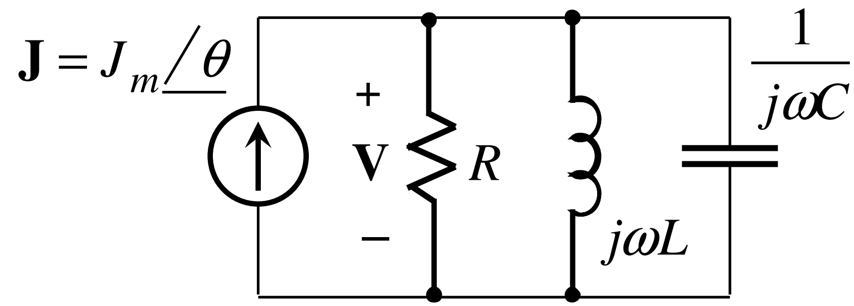






Frequency Response

Parallel Resonance (1)



$$\mathbf{Y} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \mathbf{H}(\omega)$$

$$\text{Resonance:}\quad \text{Im}(\mathbf{Y}) = 0 \rightarrow \omega C - \frac{1}{\omega L} = 0$$

$$\text{Resonant frequency:}\quad \omega_o = \frac{1}{\sqrt{LC}}\ \text{rad/s};\ \ f_o = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz}$$

Parallel Resonance (2)

$$V = \frac{J_m}{\sqrt{R^2 + (\omega C - 1/\omega L)^2}}$$

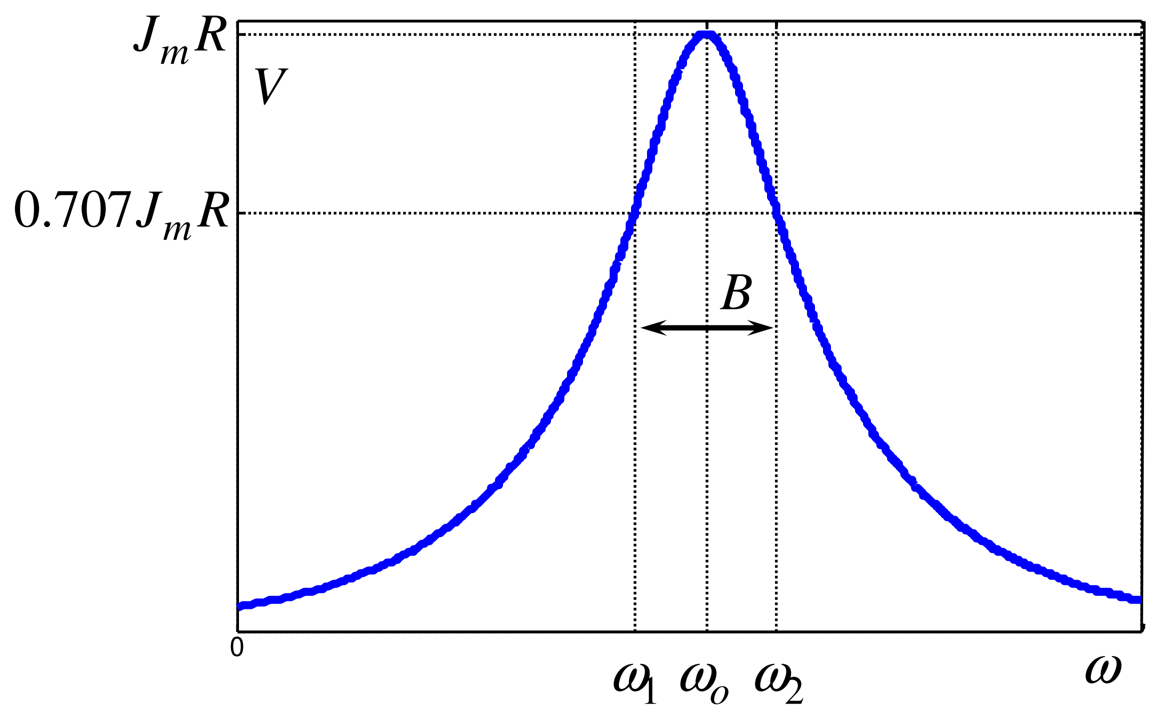
$$\omega_o = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_2}$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

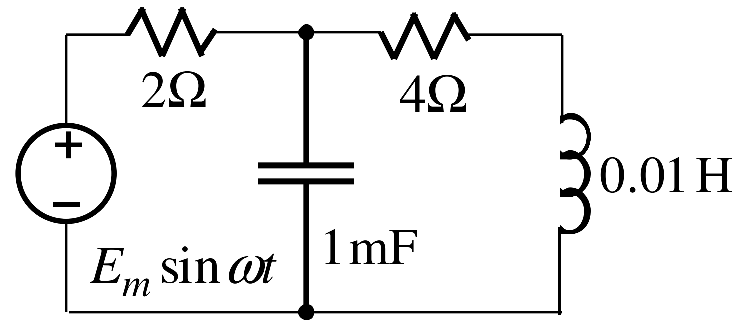
$$Q = \frac{\omega_o}{B}$$



Parallel Resonance (3)

Ex.

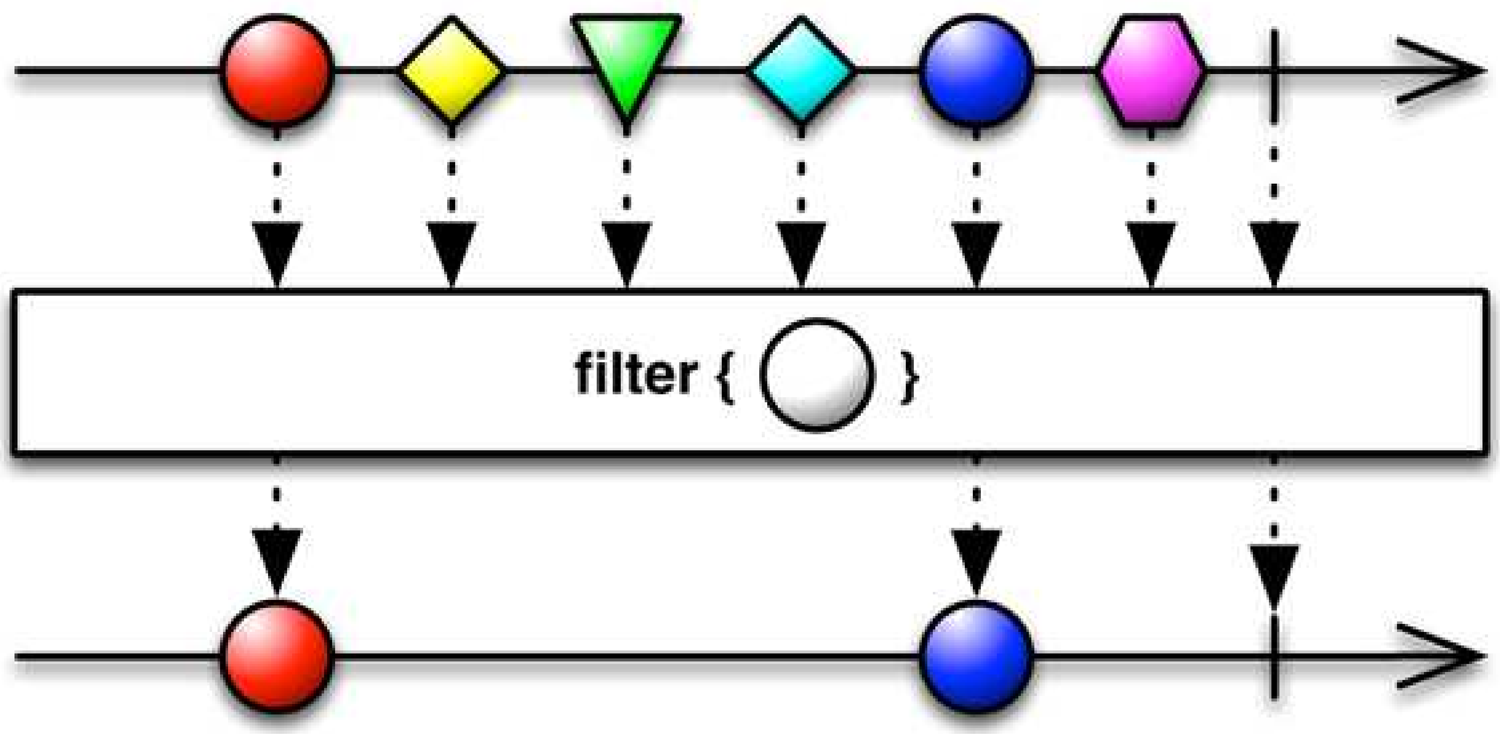
Find the resonant frequency?



Frequency Response

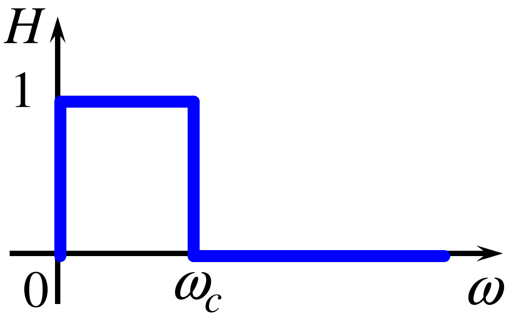
1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. Series Resonance
5. Parallel Resonance
6. **Passive Filters**
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. Narrowband Bandpass & Banstop Filters

Filters



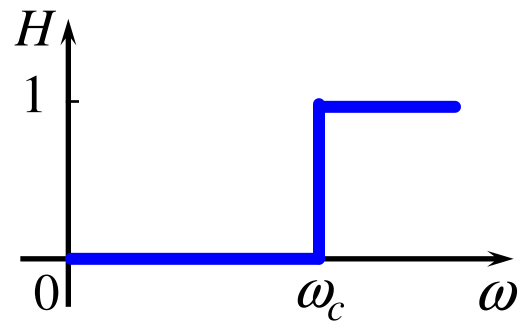
http://reactivex.io/documentation/operators/filter.html

Passive Filters (1)



$H(0) = 1; H(\infty) = 0$

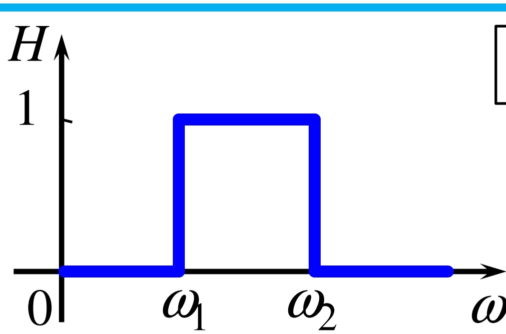
Lowpass



$H(0) = 0; H(\infty) = 1$

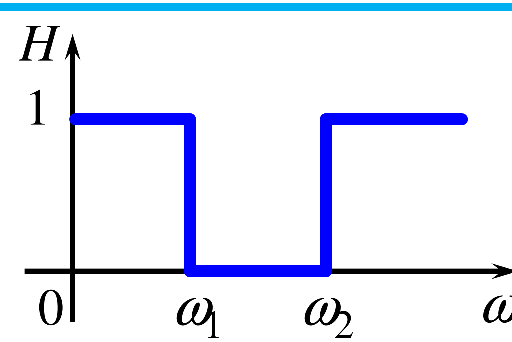
Highpass

Bandpass



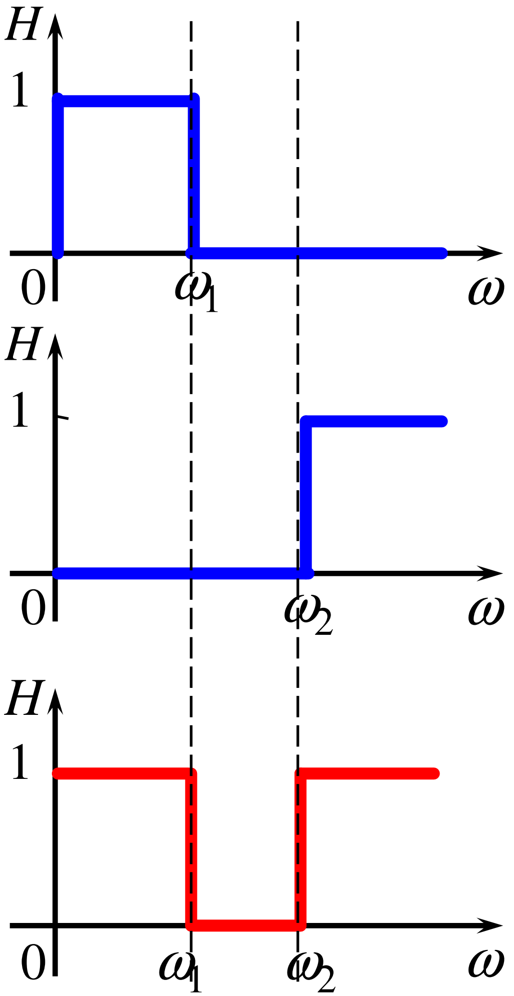
$H(0) = 0; H(\infty) = 0$

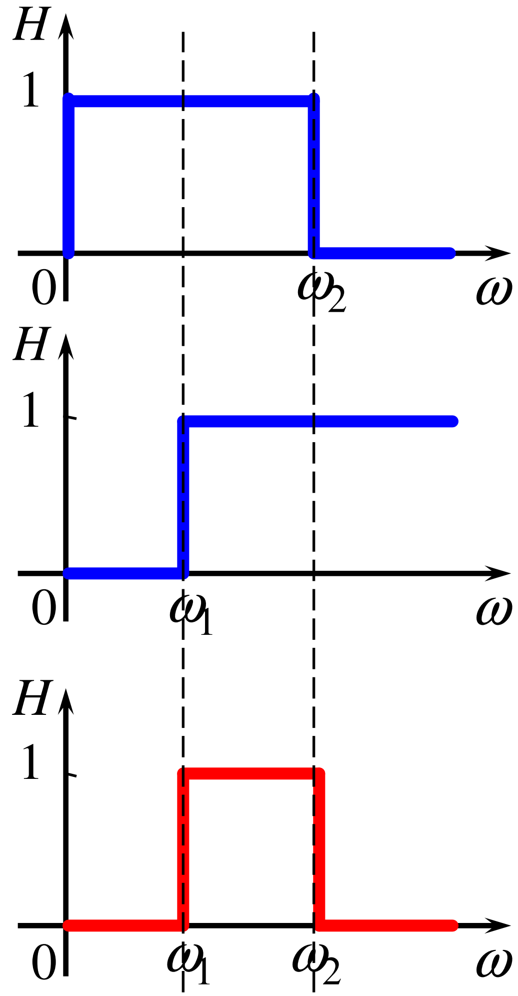
Bandstop

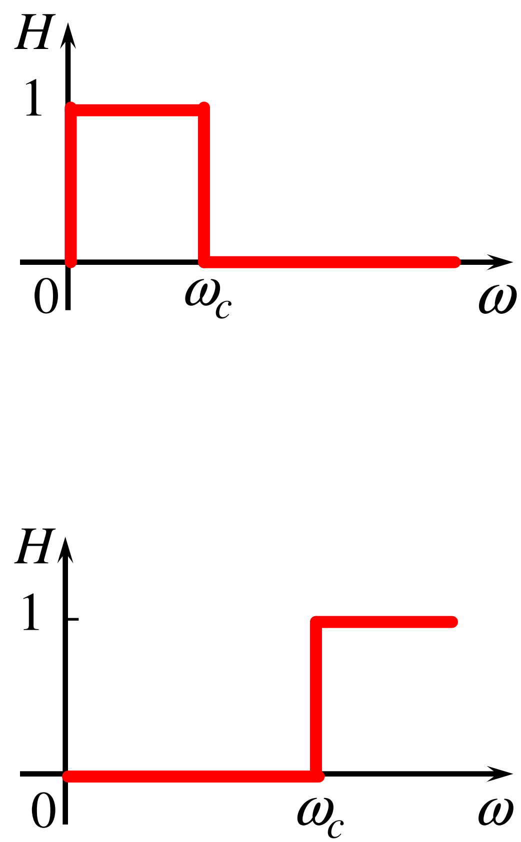


$H(0) = 1; H(\infty) = 1$

Passive Filters (2)

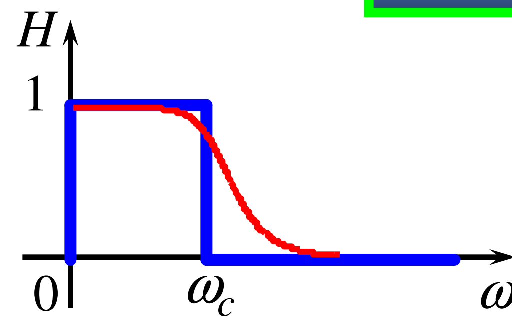






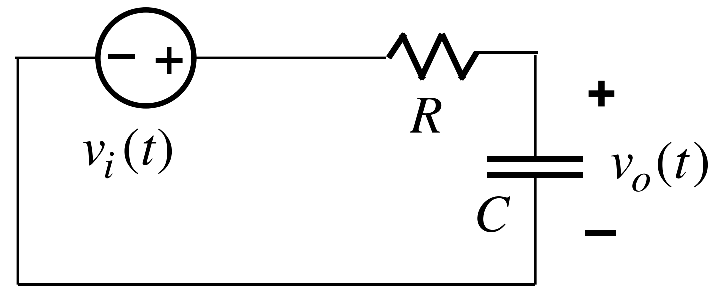
Frequency Response

Lowpass Filters (1)



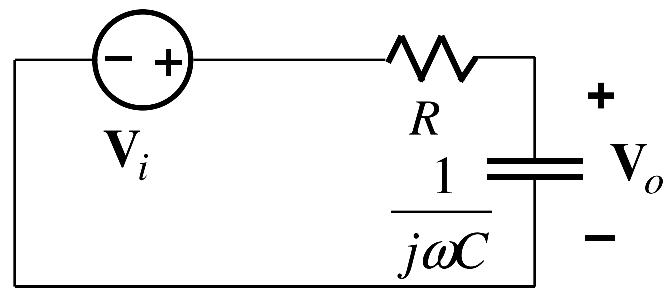
$$\mathbf{V}_o = \frac{1}{j\omega C} \times \frac{\mathbf{V}_i}{R + 1/j\omega C}$$

$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$



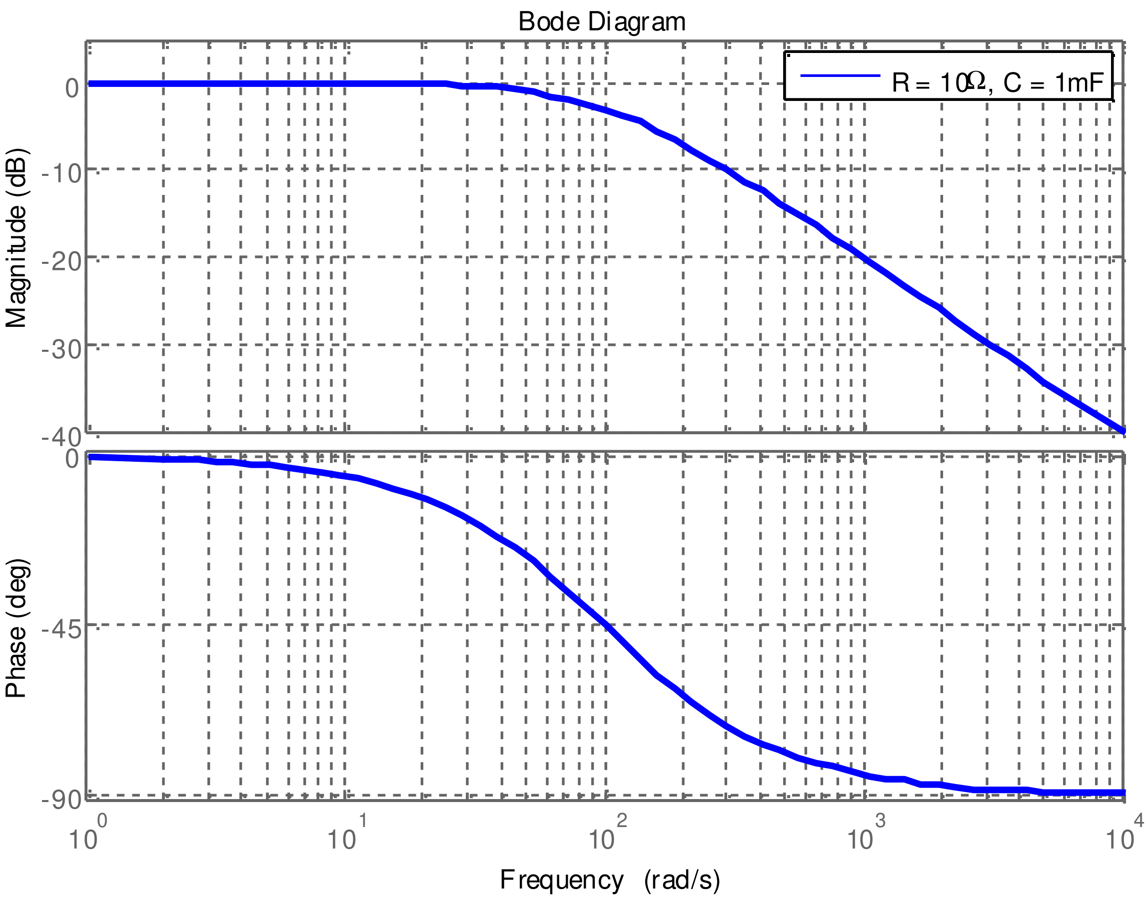
$$\frac{P_{resonant}}{P_{average}} = \frac{1}{2} \rightarrow |\mathbf{H}(\omega_c)| = \frac{1}{\sqrt{2}}$$

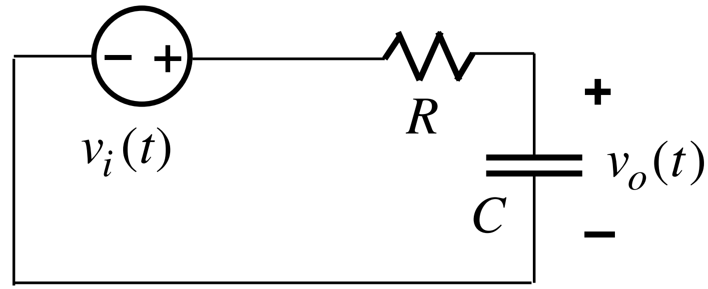
$$\mathbf{H}(\omega_c) = \frac{1}{1 + j\omega_c RC} \rightarrow |\mathbf{H}(\omega_c)| = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}}$$



$$\rightarrow \omega_c = \frac{1}{RC}$$

Lowpass Filters (2)





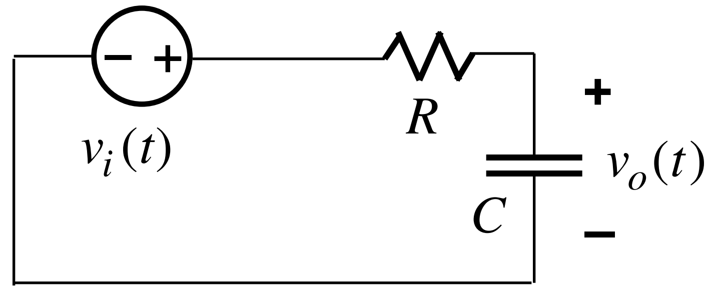
$$\mathbf{H}(\omega) = \frac{1/\,j\omega C}{R + 1/\,j\omega C} = \frac{1}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

Lowpass Filters (3)

Ex. 1

Choose values for R & C that will yield a lowpass filter with a cutoff frequency of 4kHz.



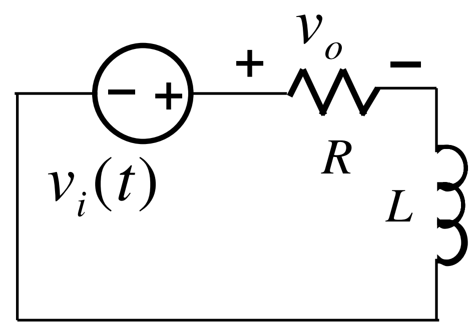
$$\omega_c = \frac{1}{RC} = 2\pi f_c = (2\pi)4000$$

$$C = 1\mu\text{F} \rightarrow R = \frac{1}{\omega_c C}$$

$$= \frac{1}{(2\pi)(4000)(1\times10^{-6})}$$

$$= 39.79\Omega$$

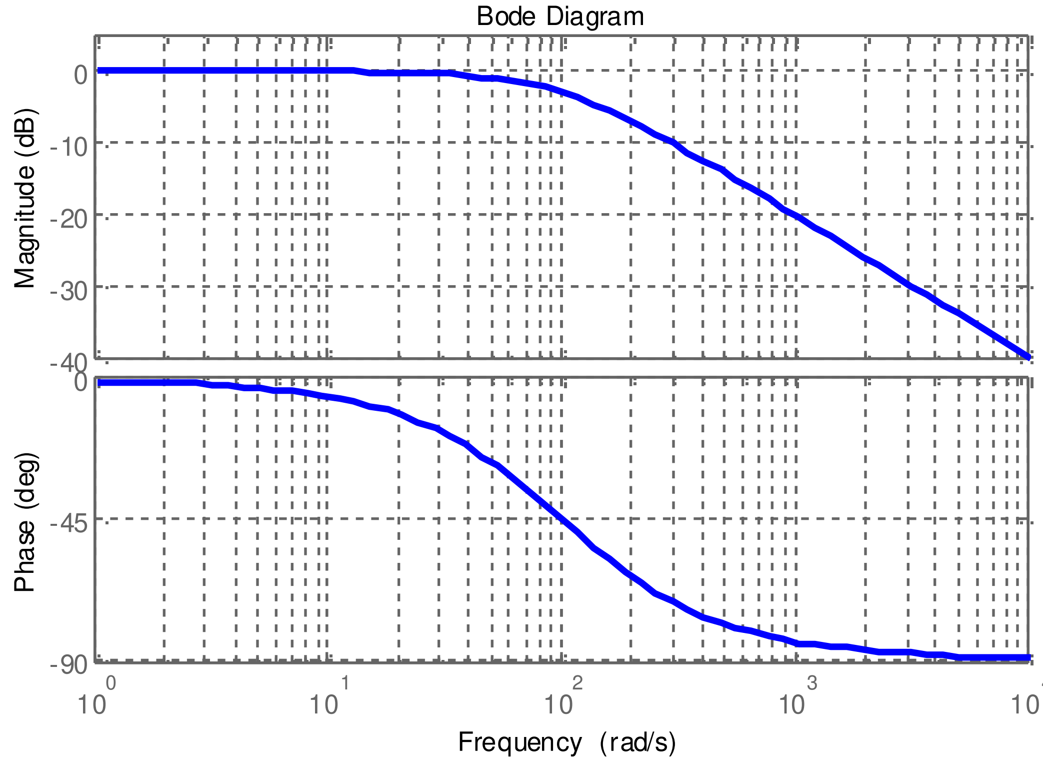
Lowpass Filters (4)



$$\mathbf{V}_o = R \times \frac{\mathbf{V}_i}{R + j\omega L} \rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega(L/R)}$$

$$\left|\mathbf{H}(\omega_c)\right| = \frac{1}{\sqrt{2}} = \frac{R/L}{\sqrt{\omega_c^2 + (R/L)^2}}$$

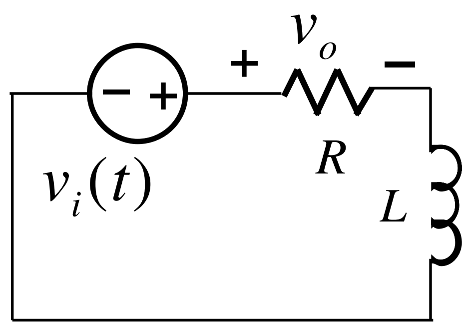
$$\rightarrow \omega_c = \frac{R}{L}$$



Lowpass Filters (4)

Ex. 2

Choose values for R & L that will yield a lowpass filter with a cutoff frequency of 10Hz.



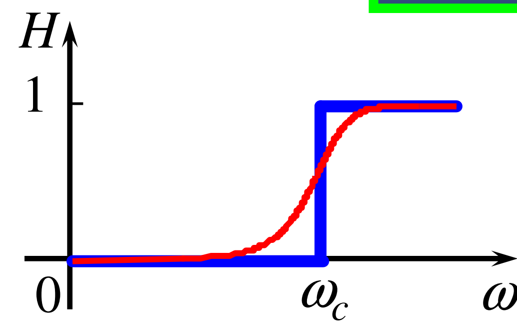
$$\omega_c = \frac{R}{L} = 2\pi f_c = (2\pi)10$$

$$L = 100\text{mH} \rightarrow R = \omega_c L$$
$$= (2\pi)10(0.1)$$
$$= 6.28\Omega$$

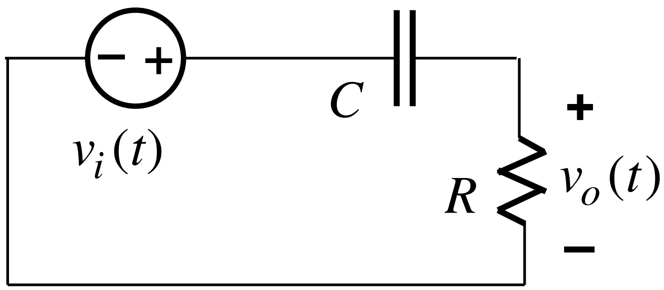
Frequency Response

1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. Series Resonance
5. Parallel Resonance
6. **Passive Filters**
    a) Lowpass Filters
    b) Highpass Filters
    c) Bandpass Filters
    d) Bandstop Filters
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. Narrowband Bandpass & Banstop Filters

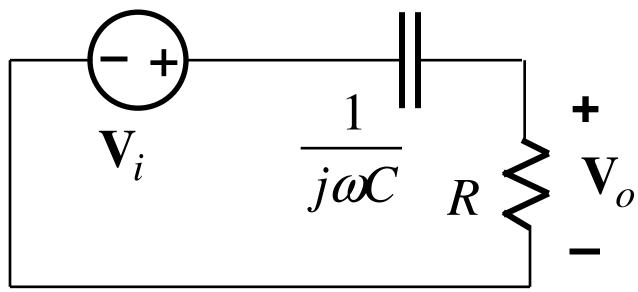
Highpass Filters (1)



$$\mathbf{V}_o = R\frac{\mathbf{V}_i}{R + 1/\,j\omega C}$$

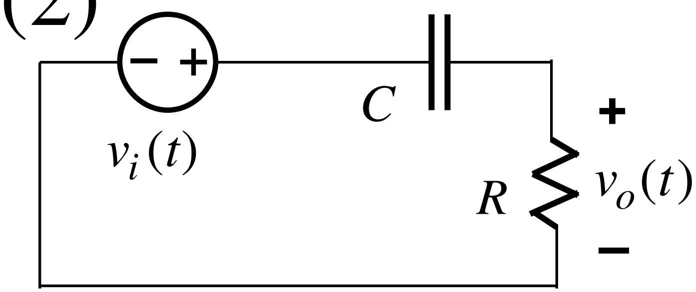


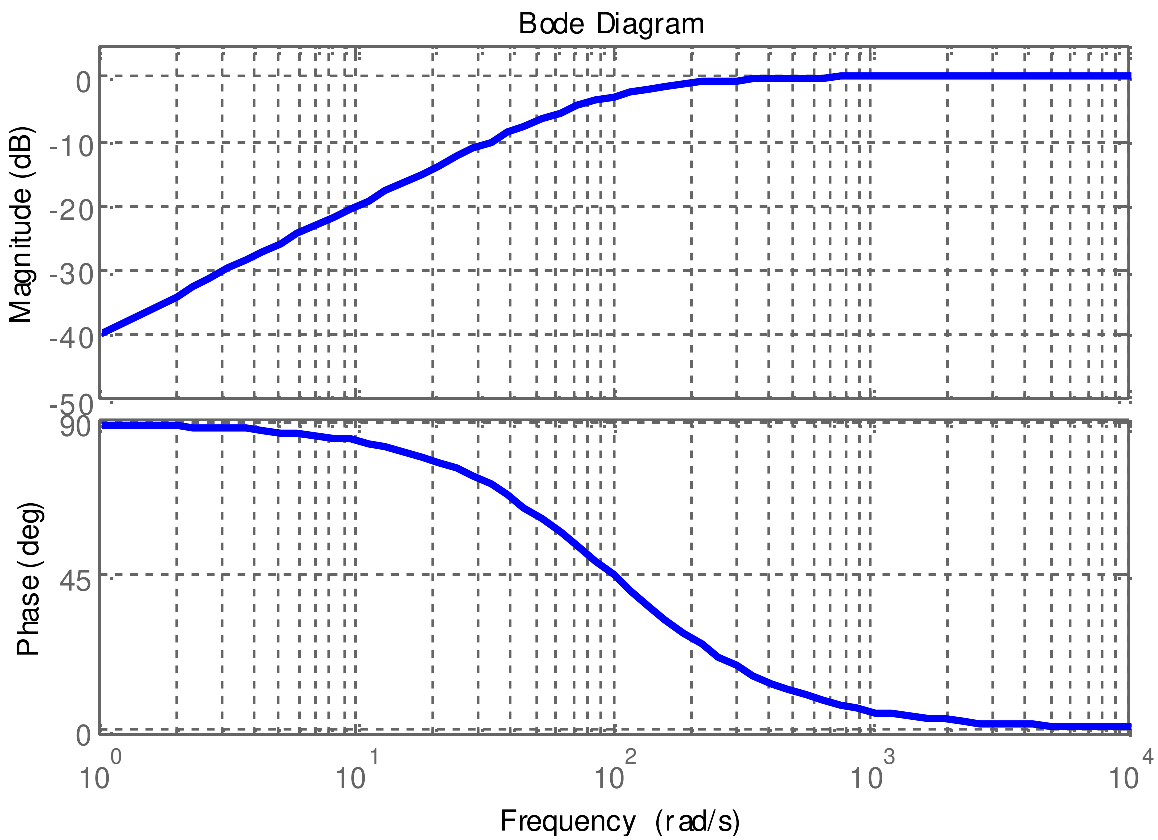
$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/\,j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$



$$\left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{2}} \rightarrow \omega_c = \frac{1}{RC}$$

Highpass Filters (2)

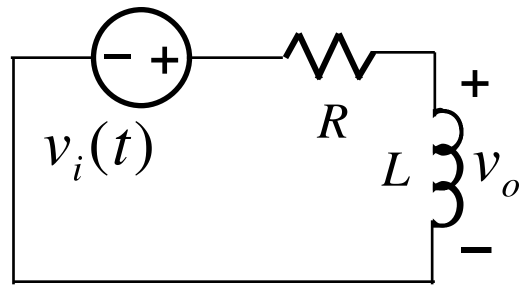




$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega_c = \frac{1}{RC}$$

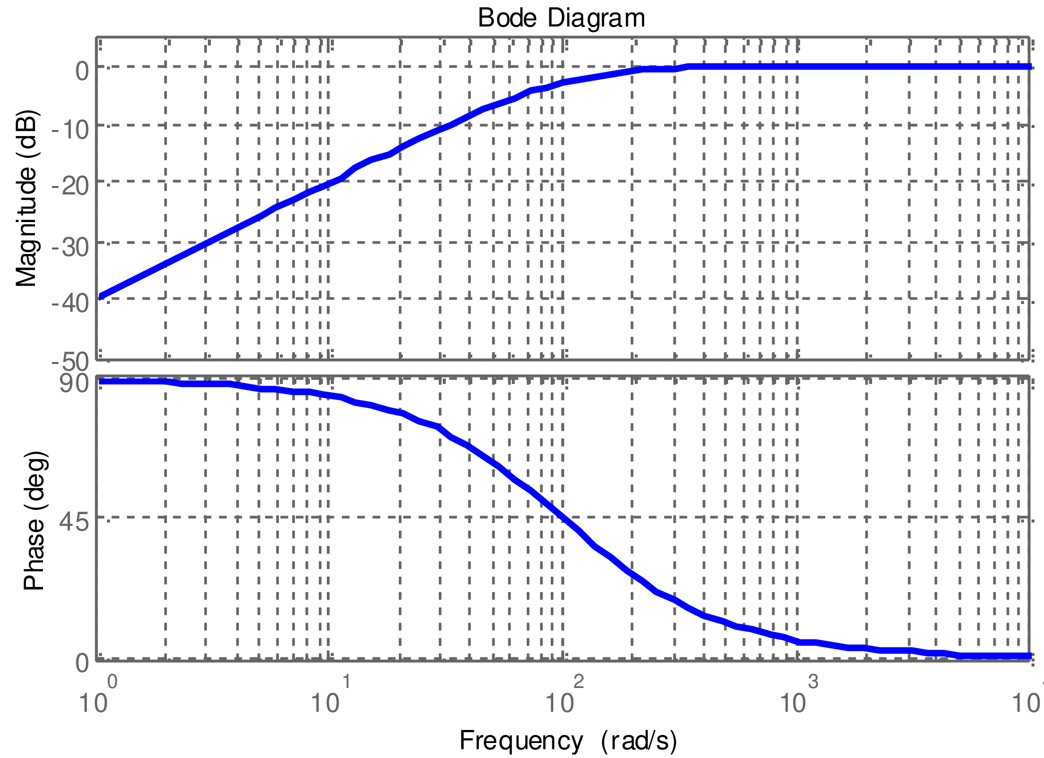
Highpass Filters (3)



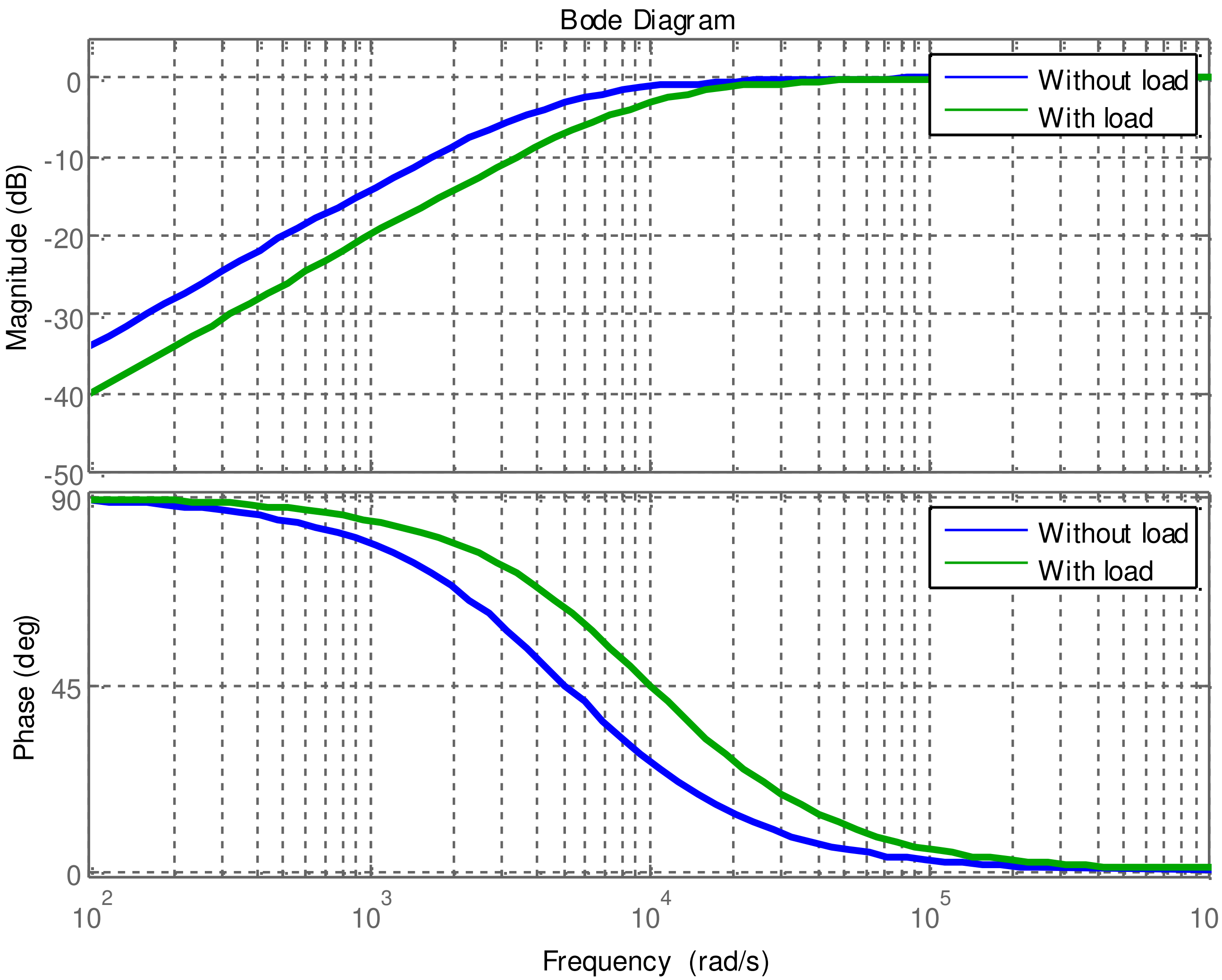
$$\mathbf{V}_o = j\omega L \frac{\mathbf{V}_i}{R + j\omega L}$$

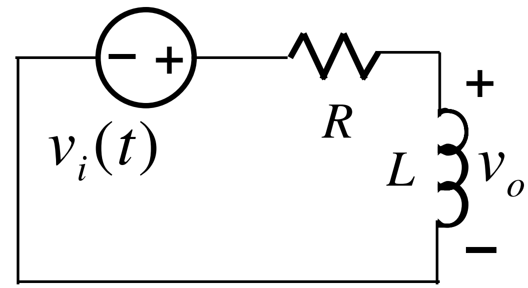
$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$$\left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{2}} \rightarrow \omega_c = \frac{R}{L}$$

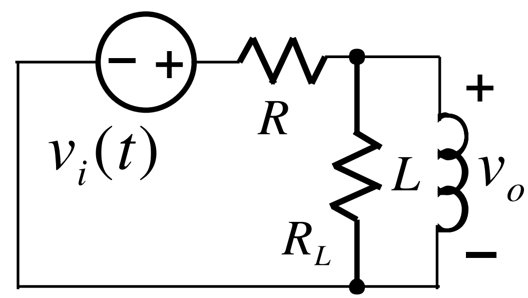


Highpass Filters (4)





$$\mathbf{H}(\omega) = \frac{j\omega L}{R + j\omega L}$$

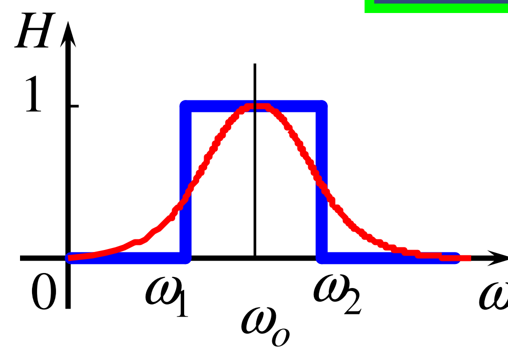


$$\mathbf{H}(\omega) = \frac{j\omega \dfrac{R_L L}{R + R_L}}{R + j\omega \dfrac{R_L L}{R + R_L}}$$

Frequency Response

1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. Series Resonance
5. Parallel Resonance
6. **Passive Filters**
   a) Lowpass Filters
   b) Highpass Filters
   c) **Bandpass Filters**
   d) Bandstop Filters
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. Narrowband Bandpass & Banstop Filters

Bandpass Filters (1)



$$\mathbf{V}_o = R\frac{\mathbf{V}_i}{R + j\omega L + 1/j\omega C}$$

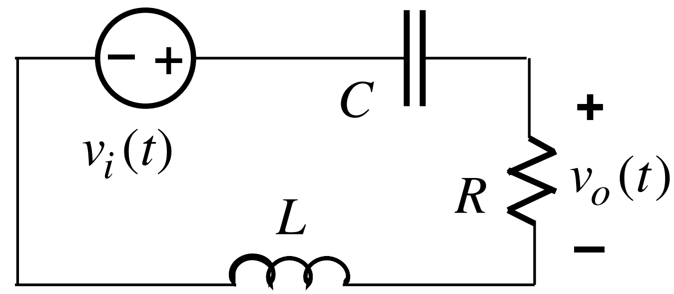
$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L + 1/j\omega C}$$

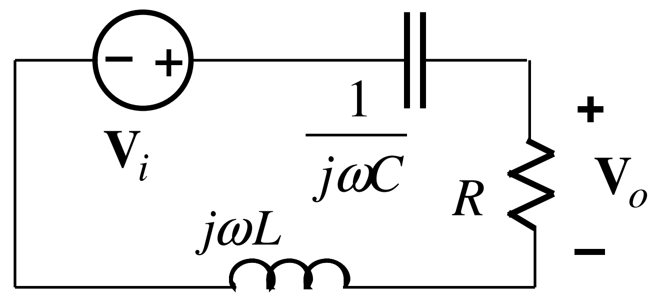
$$= \frac{R}{R + j(\omega L - 1/\omega C)}$$

$$j\omega_o L + \frac{1}{j\omega_o C} = 0 \rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

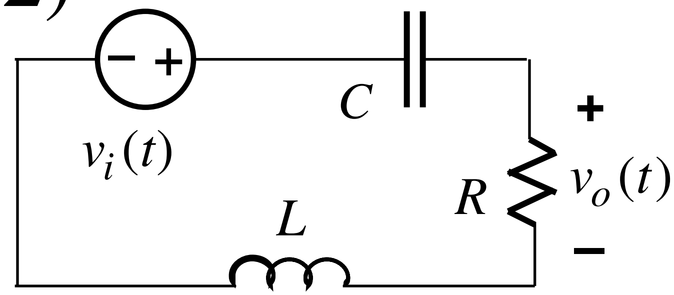
$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{2}} \rightarrow \omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

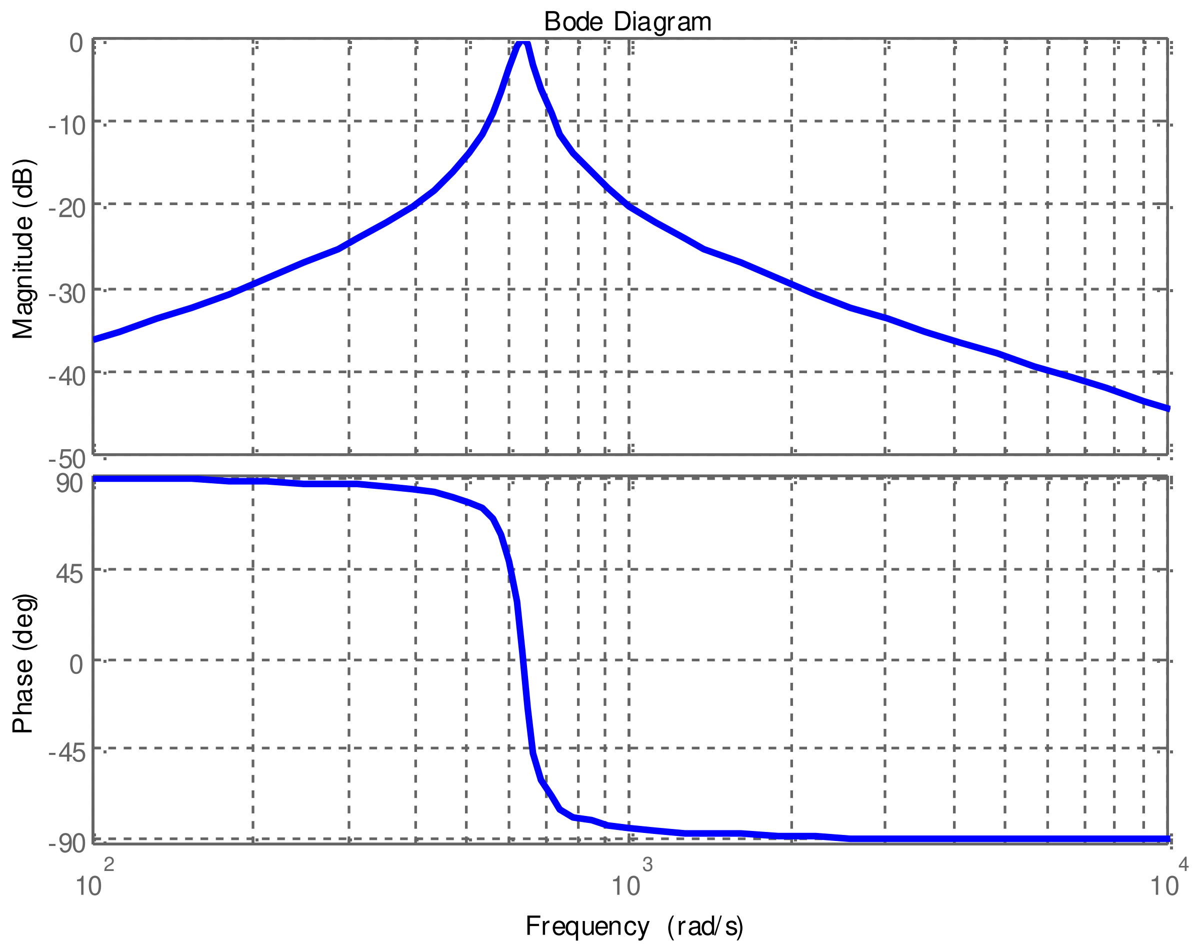
$$\omega_o = \sqrt{\omega_1\omega_2}$$





Bandpass Filters (2)





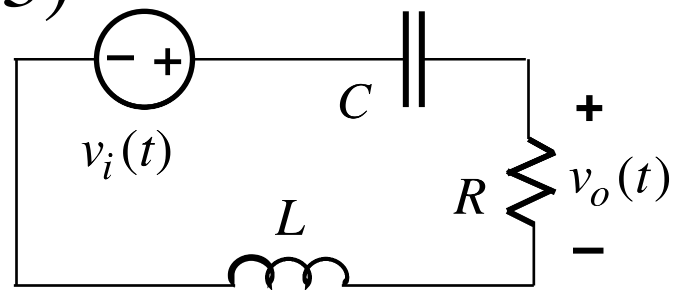
$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Bandpass Filters (3)

Ex. 1

Choose values for R, L & C that will yield a lowpass filter able to select inputs within the 1 – 10kHz frequency band.



$$f_o = \sqrt{f_1 f_2} = \sqrt{1000 \times 10{,}000} = 3162.28\text{Hz}$$

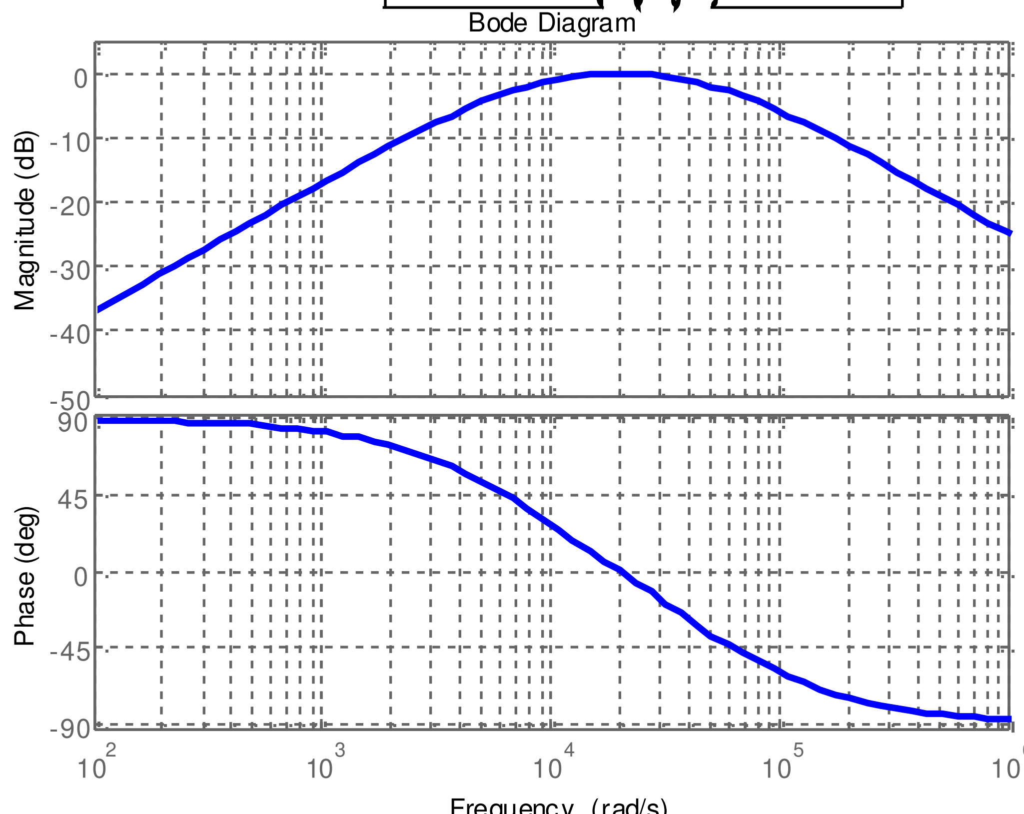
$$C = 1\mu\text{F} \rightarrow L = \frac{1}{\omega_o^2 C}$$

$$= \frac{1}{2\pi(3162.28)^2 10^{-6}}$$

$$= 2.533\text{mH}$$

$$\omega_2 - \omega_1 = \frac{R}{L} \rightarrow R = L(\omega_2 - \omega_1)$$

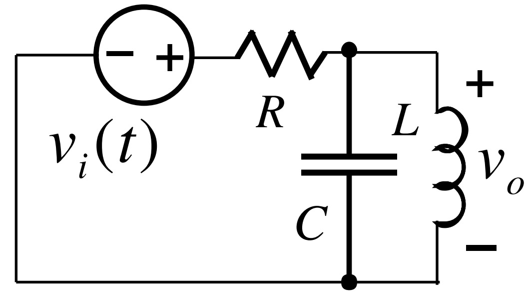
$$= 143.24\Omega$$



Bandpass Filters (4)

Ex. 2

Choose values for R & L that will yield a lowpass filter with a center frequency of 5kHz and a bandwidth of 200Hz, using a 2μF capacitor.



$$\mathbf{V}_o = \mathbf{Z}_{LC}\frac{\mathbf{V}_i}{R+\mathbf{Z}_{LC}} = \frac{j\omega L\dfrac{1}{j\omega C}}{j\omega L+\dfrac{1}{j\omega C}} \times \frac{\mathbf{V}_i}{R+\dfrac{j\omega L\dfrac{1}{j\omega C}}{j\omega L+\dfrac{1}{j\omega C}}} = \mathbf{V}_i\frac{j\omega L}{(j\omega)^2 RLC + j\omega L + R}$$

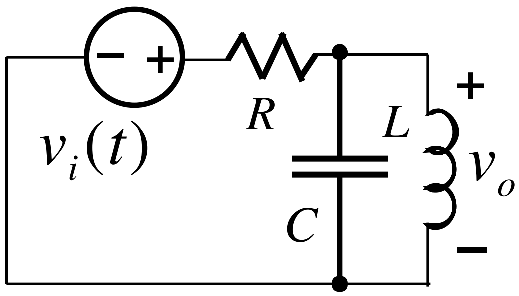
$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{(j\omega)^2 RLC + j\omega L + R} \qquad \rightarrow \left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{1+\left(RC\omega - \dfrac{R}{L\omega}\right)^2}}$$

$$RC\omega - \frac{R}{L\omega} = 0 \rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

Bandpass Filters (5)

Ex. 2

Choose values for R & L that will yield a lowpass filter with a center frequency of 5kHz and a bandwidth of 200Hz, using a 2μF capacitor.



$$\mathbf{H}(\omega) = \frac{j\omega L}{(j\omega)^2 RLC + j\omega L + R}, \quad \left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{1 + \left(RC\omega - \frac{R}{L\omega}\right)^2}}, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

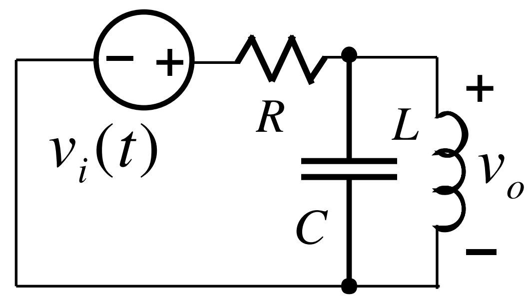
$$\left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{2}} \rightarrow RC\omega - \frac{R}{L\omega} = \pm 1 \qquad \rightarrow \omega_{1,2} = \mp\frac{1}{2RC} + \sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_o}{B} = \sqrt{\frac{R^2 C}{L}}$$

Bandpass Filters (6)

Ex. 2



Choose values for R & L that will yield a lowpass filter with a center frequency of 5kHz and a bandwidth of 200Hz, using a 2μF capacitor.

$$\mathbf{H}(\omega) = \frac{j\omega L}{(j\omega)^2 RLC + j\omega L + R}, \quad \omega_o = \frac{1}{\sqrt{LC}}, \quad B = \frac{1}{RC}$$

$$R = \frac{1}{BC} = \frac{1}{(2\pi)(200)(2\times 10^{-6})} = 397.89\Omega$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{[2\pi(5000)]^2(2\times 10^{-6})} = 50.66\text{mH}$$

Frequency Response

1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. Series Resonance
5. Parallel Resonance
6. **Passive Filters**
    a) Lowpass Filters
    b) Highpass Filters
    c) Bandpass Filters
    d) **Bandstop Filters**
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. Narrowband Bandpass & Banstop Filters

Bandstop Filters (1)

$$\mathbf{V}_o = \frac{(j\omega L + 1/j\omega C)\mathbf{V}_i}{R + j\omega L + 1/j\omega C}$$

$$\rightarrow \mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C}$$

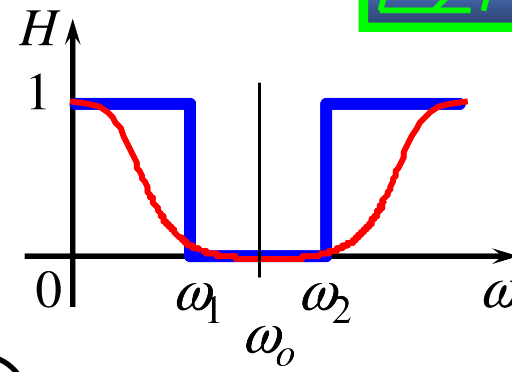
$$= \frac{(j\omega)^2 + 1/(LC)}{(j\omega)^2 + (R/L)j\omega + 1/(LC)}$$

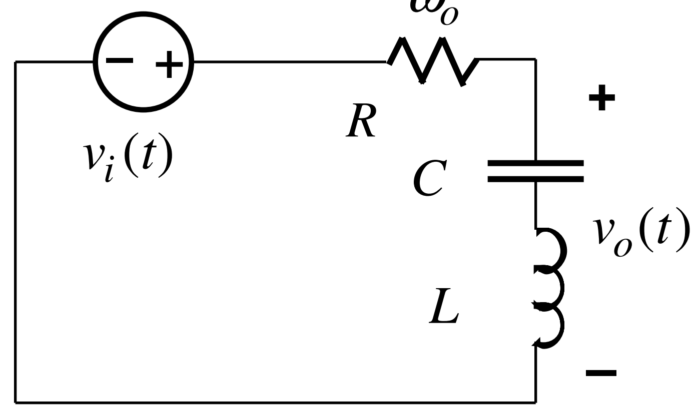
$$\omega_o = \frac{1}{\sqrt{LC}}$$

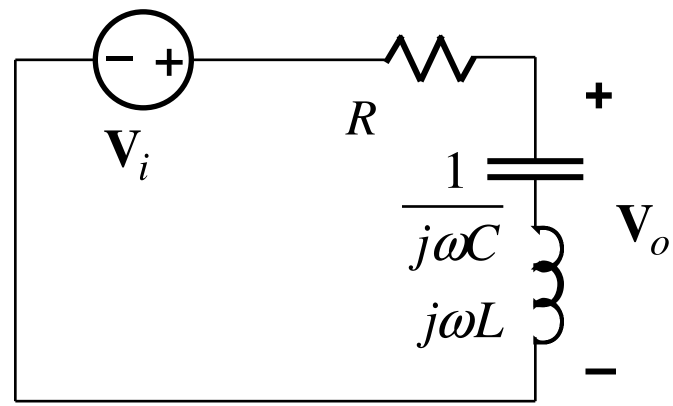
$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$B = \frac{R}{L}$$

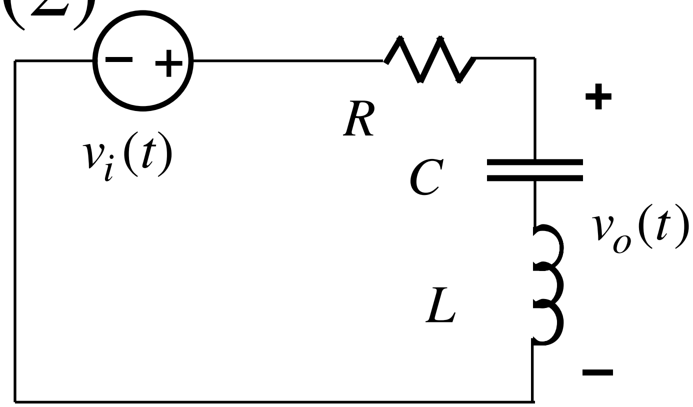
$$Q = \sqrt{\frac{L}{R^2 C}}$$







Bandstop Filters (2)



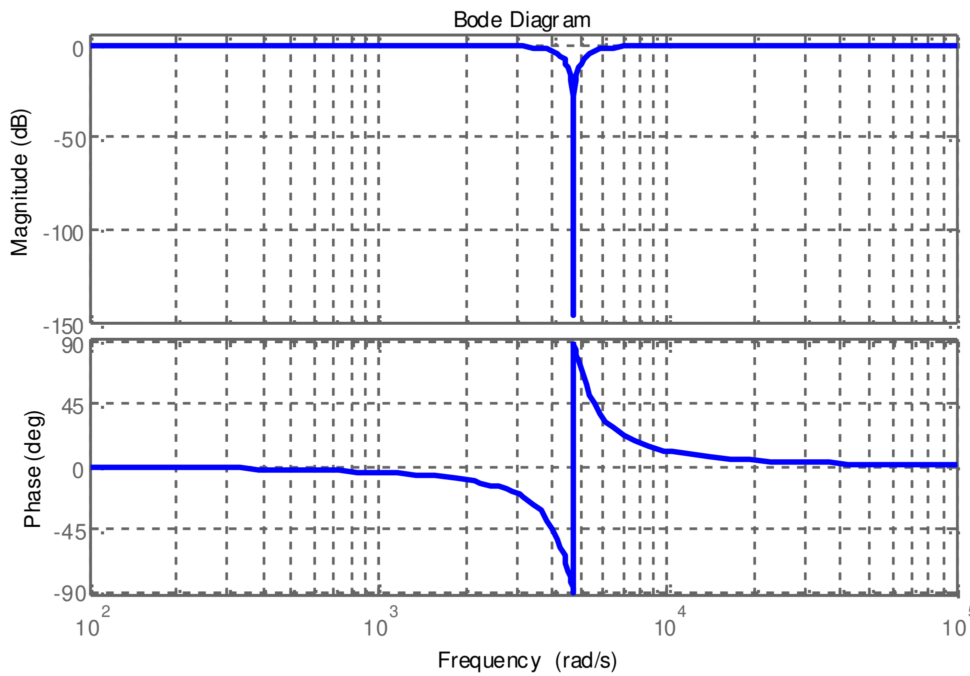
$$\mathbf{H}(\omega) = \frac{(j\omega)^2 + 1/(LC)}{(j\omega)^2 + (R/L)j\omega + 1/(LC)}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$B = \frac{R}{L}$$

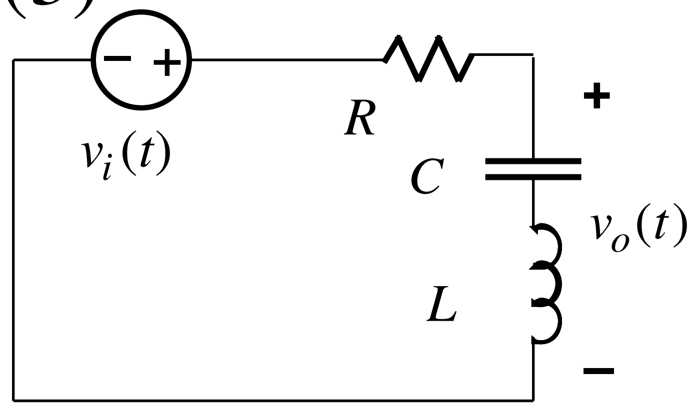
$$Q = \sqrt{\frac{L}{R^2C}}$$



Bandstop Filters (3)

Ex.

Given a series RLC circuit, compute the component values that yield a bandstop filter with a bandwidth of 250Hz and a center frequency of 750Hz, using a 150nF capacitor.



$$Q = \frac{\omega_o}{B} = \frac{750}{250} = 3$$

$$\omega_o = \frac{1}{\sqrt{LC}} \rightarrow L = \frac{1}{\omega_o^2 C} = \frac{1}{(2\pi \times 750)^2 (150 \times 10^{-9})} = 300\text{mH}$$

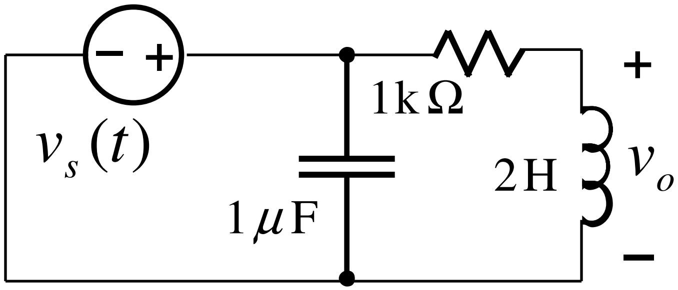
$$B = \frac{R}{L} \rightarrow R = BL = 2\pi \times 750 \times 0.3 = 1415\Omega$$

Frequency Response

Passive Filters (3)

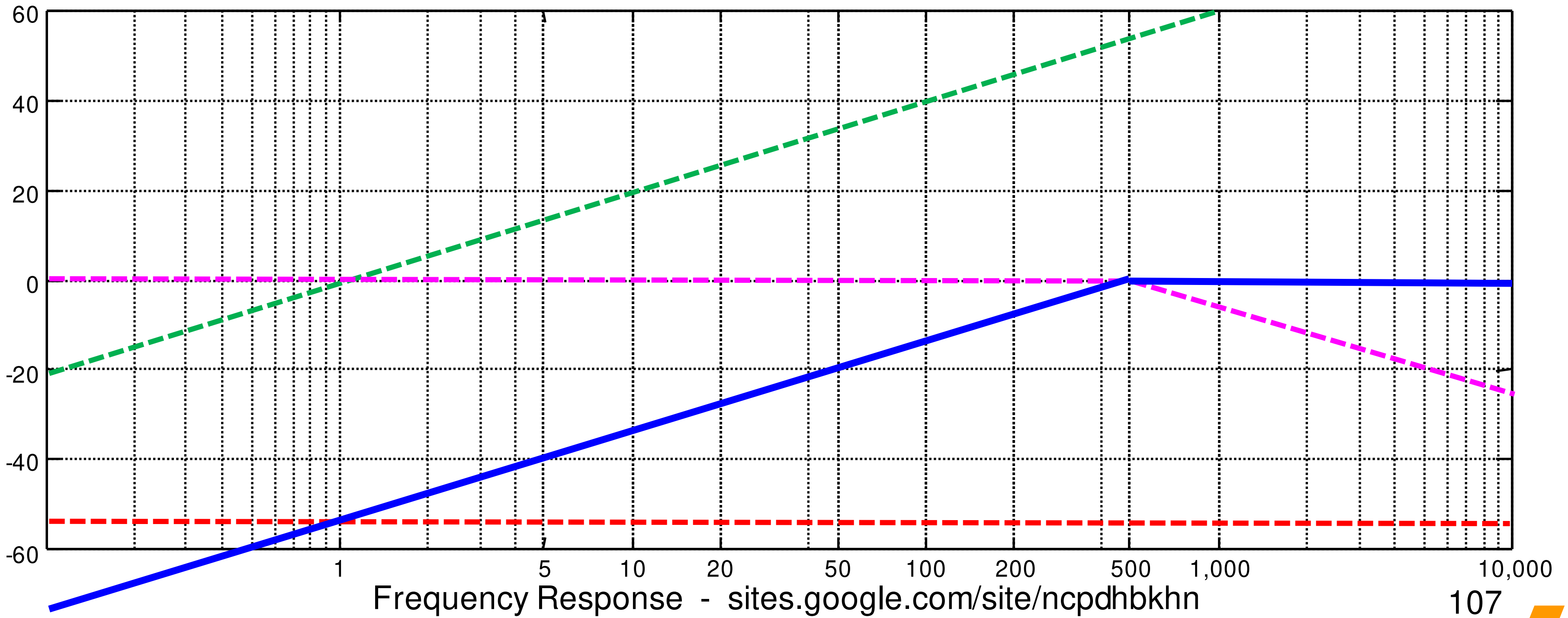
Ex. 1

What is the type of this filter?



$$\mathbf{V}_o = j\omega L \frac{\mathbf{V}_s}{R + j\omega L} \rightarrow \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j\omega L}{R + j\omega L} = \mathbf{H}(\omega)$$

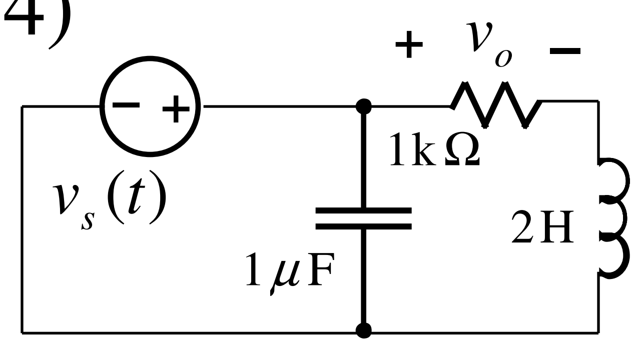
$$\rightarrow \mathbf{H}(\omega) = \frac{j\omega 2}{1000 + j\omega 2} = \frac{j\omega 2/1000}{1 + j\omega 2/1000} = \frac{j\omega}{500(1 + j\omega/500)}$$



Passive Filters (4)

Ex. 2

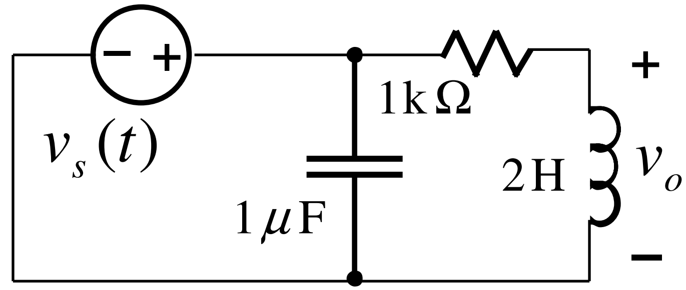
What is the type of this filter?



Passive Filters (5)

Ex. 3

Find the cutoff frequency?



$$\mathbf{V}_o = j\omega L\frac{\mathbf{V}_s}{R + j\omega L} \rightarrow \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j\omega L}{R + j\omega L} = \mathbf{H}(\omega)$$

$$\rightarrow \mathbf{H}(\omega) = \frac{j\omega 2}{1000 + j\omega 2} = \frac{(1000 - j\omega 2)\,j\omega 2}{1000^2 + 4\omega^2} = \frac{4\omega^2}{1000^2 + 4\omega^2} + j\frac{2000\omega}{1000^2 + 4\omega^2}$$

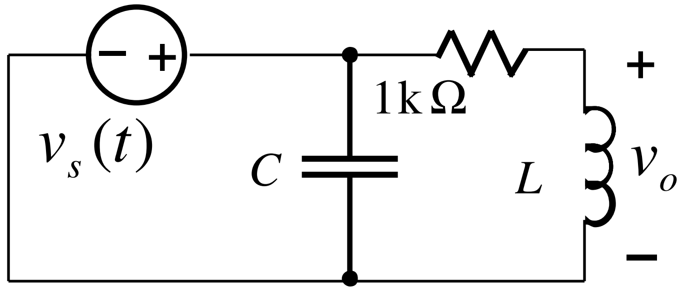
$$\rightarrow \left|\mathbf{H}(\omega)\right| = \frac{\sqrt{16\omega^4 + 4.10^6\,\omega^2}}{4\omega^2 + 10^6}$$

$$\left|\mathbf{H}(\omega_c)\right| = \frac{1}{\sqrt{2}} \rightarrow \frac{\sqrt{16\omega^4 + 4.10^6\,\omega^2}}{4\omega^2 + 10^6} = \frac{1}{\sqrt{2}} \rightarrow \boxed{\omega_c = 500 \text{ rad/s}}$$

Passive Filters (6)

Ex. 4

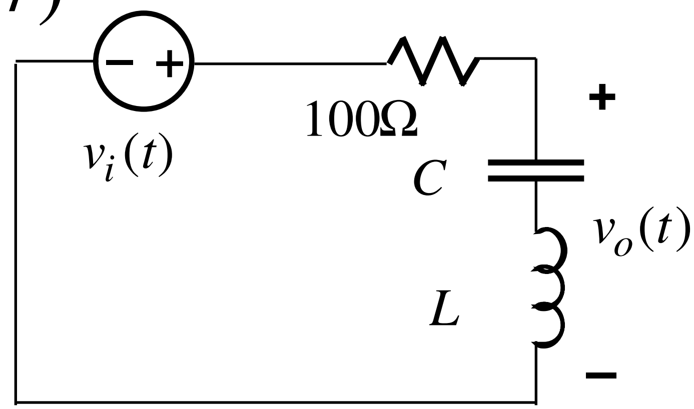
Find L if $\omega_c = 400$ rad/s?



Ex. 5

Passive Filters (7)

The filter is to reject a 200-Hz sinusoid while passing other frequencies, its bandwidth is 100 Hz. Find L and C?



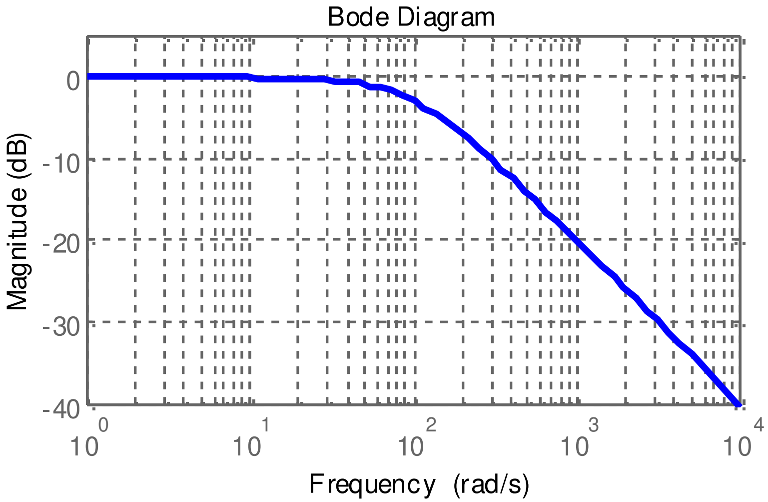
$$B = 2\pi f = 2\pi \times 100 = 200\pi \text{ rad/s}$$

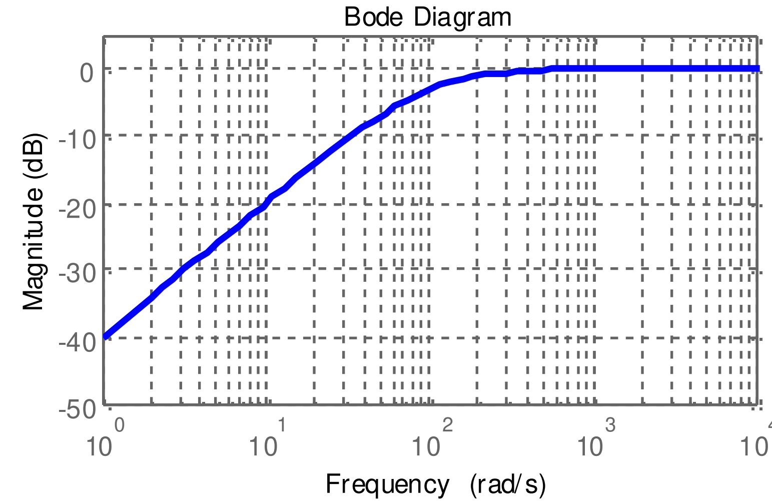
$$B = \frac{R}{L} \rightarrow L = \frac{R}{B} = \frac{100}{200\pi} = \boxed{0.1592\,\text{H}}$$

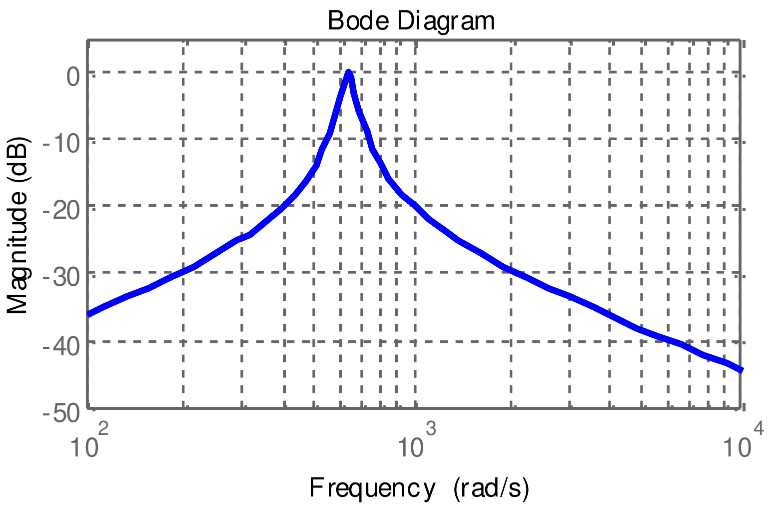
$$\omega_0 = 2\pi f_0 = 2\pi \times 200 = 400\pi \text{ rad/s}$$

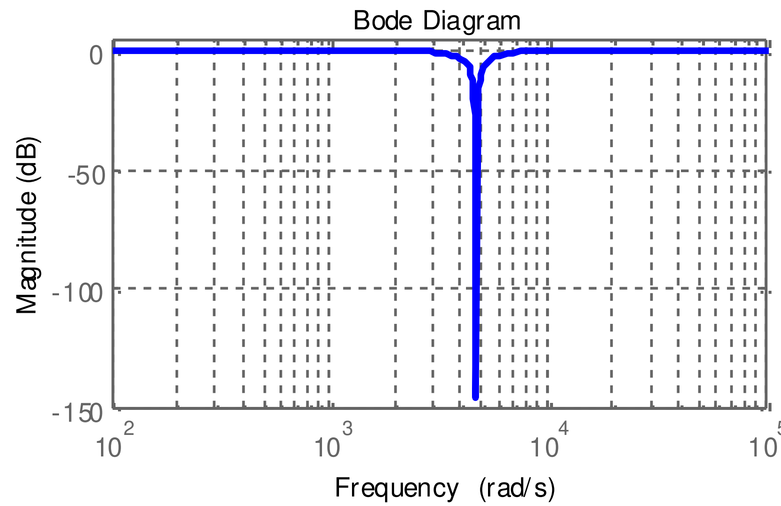
$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2 \times 0.1592} = \boxed{3.9777\,\mu\text{F}}$$

Passive Filters (8)









Passive Filters (9)

- *Passive filters*:
  - Gain is always less than 1,
  - May require expensive inductors, therefore bulky and expensive.
- *Active filters*:
  - Consist of resistors, capacitors, and op amps,
  - Smaller and less expensive, because they don't need inductors, hence can be integrated to IC,
  - Gain can be greater than 1.

Frequency Response

First-Order Lowpass Filters (1)

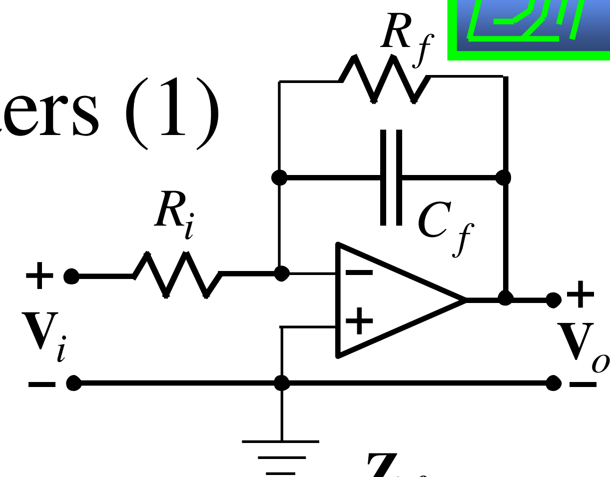
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i}$$

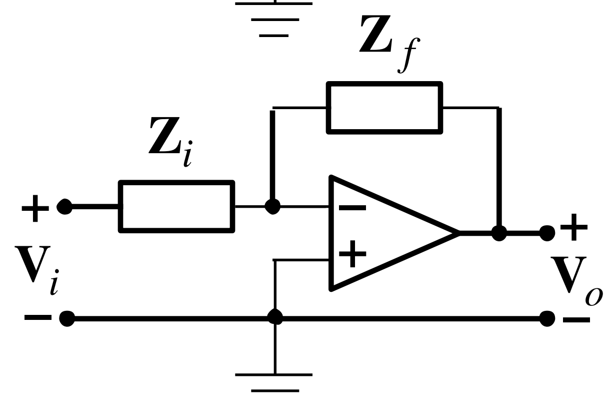
$$\mathbf{Z}_i = R_i$$

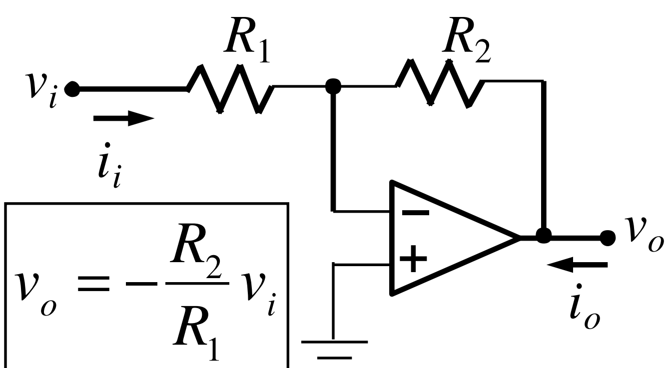
$$\mathbf{Z}_f = \frac{R_f \dfrac{1}{j\omega C_f}}{R_f + \dfrac{1}{j\omega C_f}} = \frac{R_f}{1 + j\omega R_f C_f}$$

$$\rightarrow \mathbf{H}(\omega) = -\frac{R_f}{R_i} \times \frac{1}{1 + j\omega R_f C_f}$$

$$\omega_c = \frac{1}{R_f C_f}$$



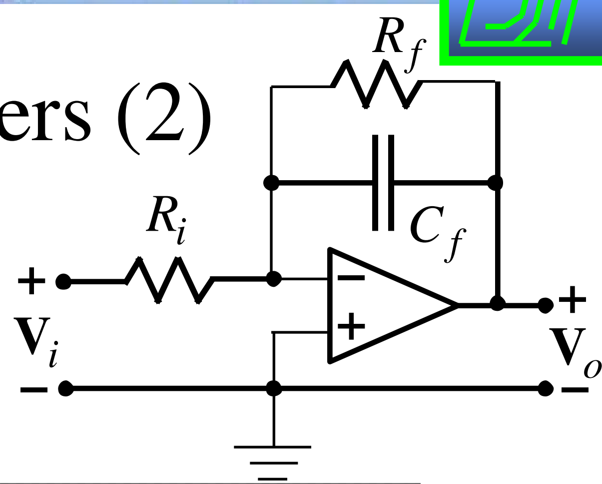


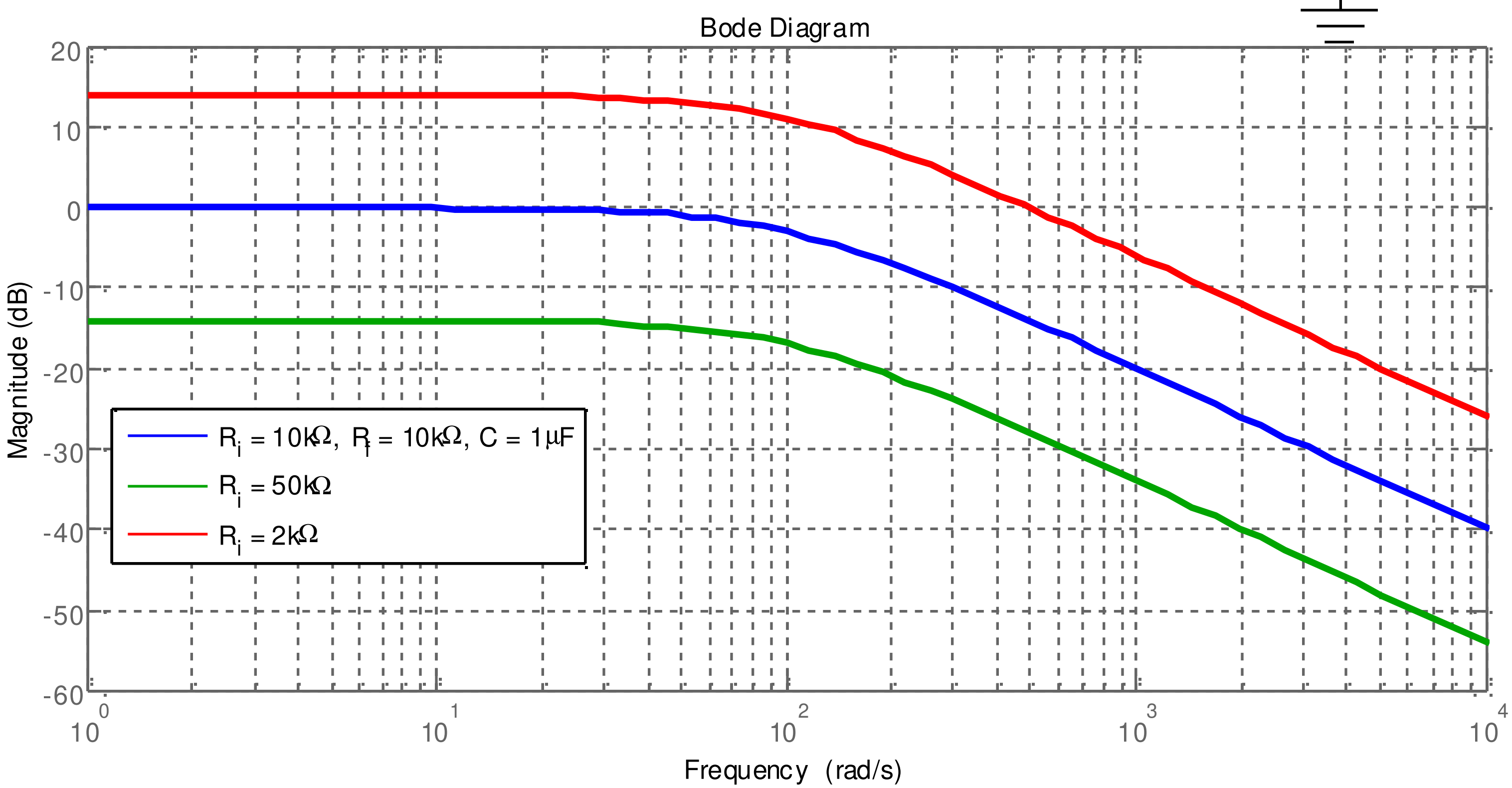


$$\boxed{v_o = -\frac{R_2}{R_1} v_i}$$

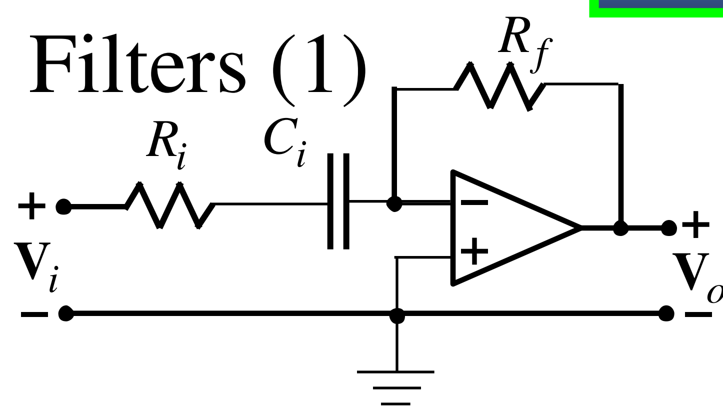
First-Order Lowpass Filters (2)

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \times \frac{1}{1 + j\omega R_f C_f}, \quad \omega_c = \frac{1}{R_f C_f}$$





First-Order Highpass Filters (1)



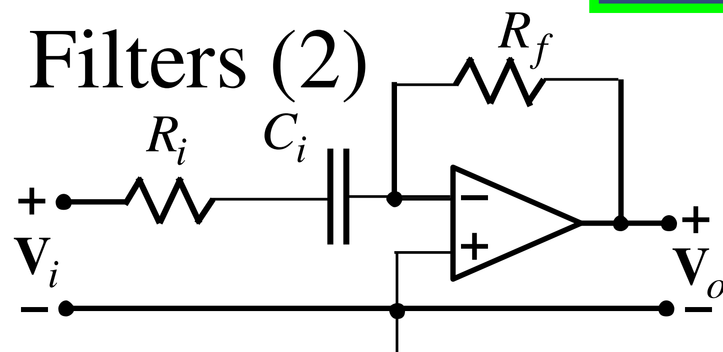
$$\left.\begin{aligned}
\mathbf{H}(\omega) &= \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \\
\mathbf{Z}_i &= R_i + \frac{1}{j\omega C_i} \\
\mathbf{Z}_f &= R_f
\end{aligned}\right\}$$

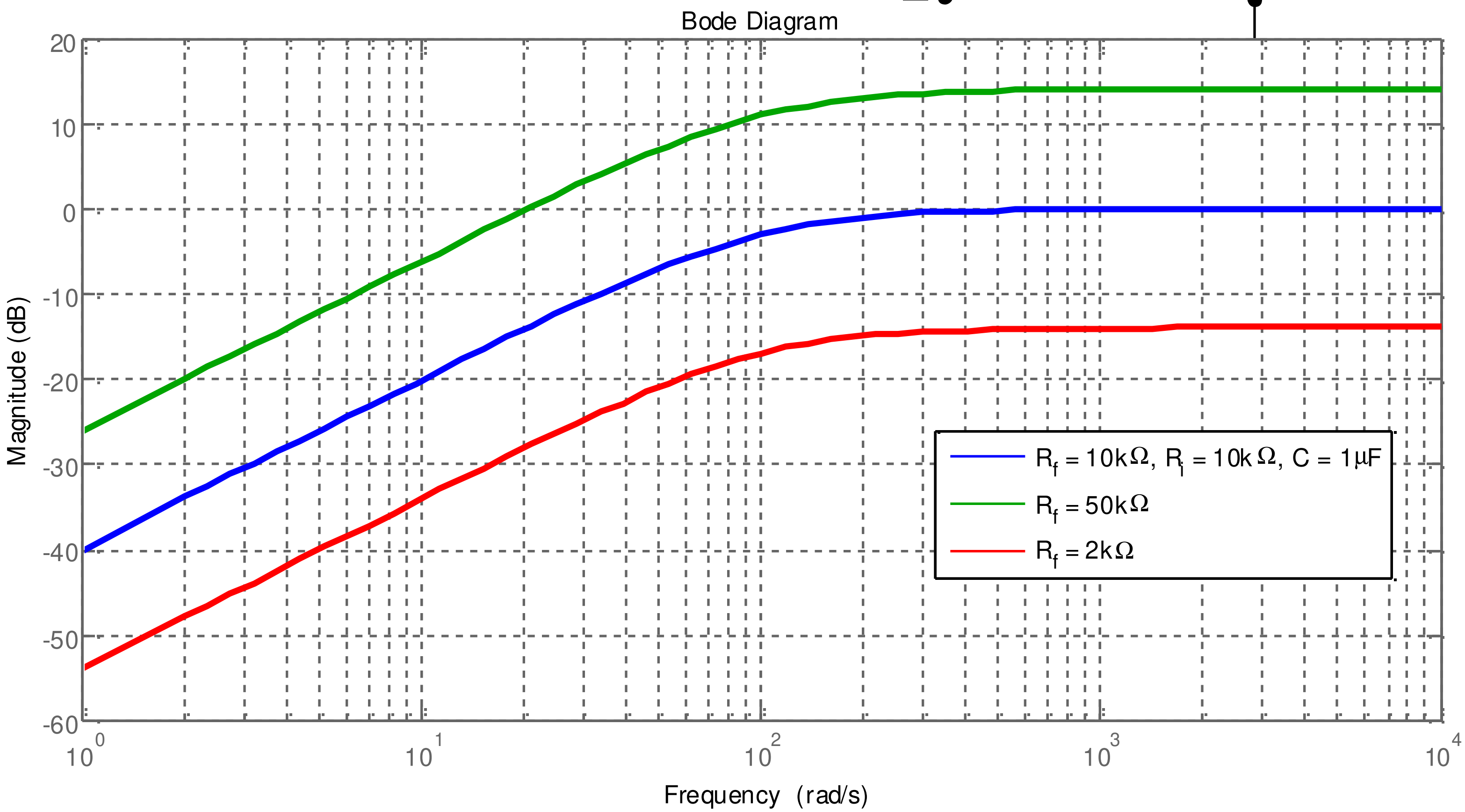
$$\rightarrow \mathbf{H}(\omega) = -\frac{R_f}{R_i + \dfrac{1}{j\omega C_i}} = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i}$$

$$\omega_c = \frac{1}{R_i C_i}$$

First-Order Highpass Filters (2)

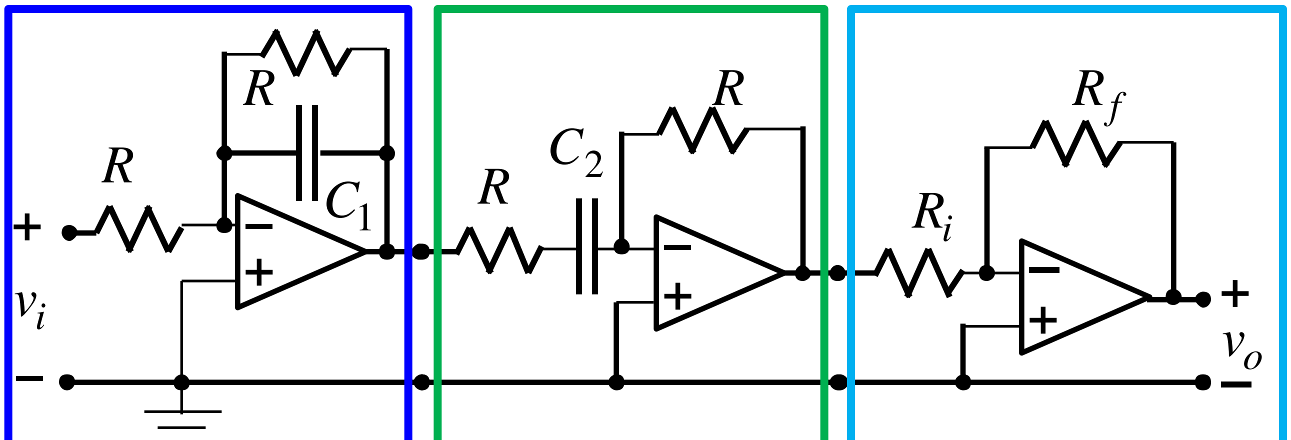
$$\mathbf{H}(\omega) = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i}, \quad \omega_c = \frac{1}{R_i C_i}$$





Frequency Response

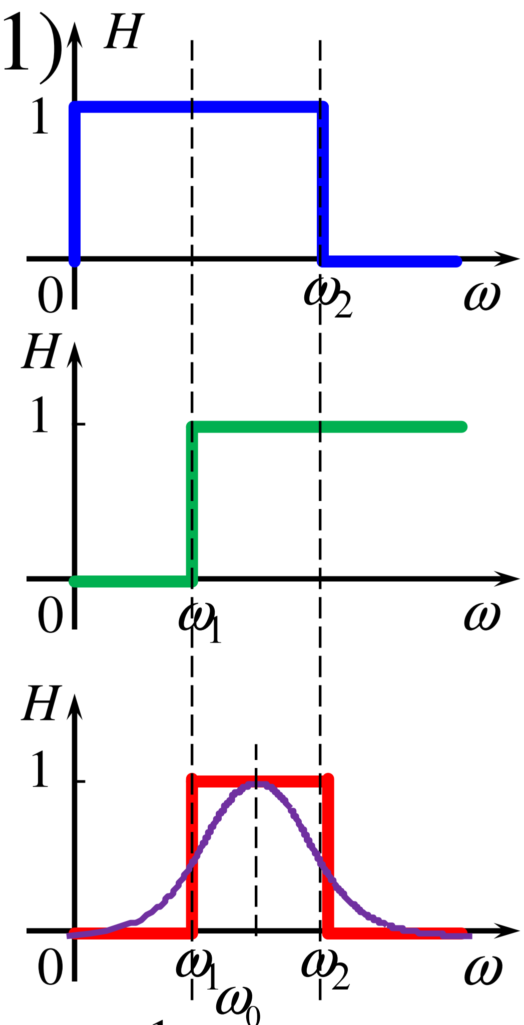
Op Amp Bandpass Filters (1)



$$-\frac{R_f}{R_i}\times\frac{1}{1+j\omega R_f C_f}$$

$$-\frac{j\omega R_f C_i}{1+j\omega R_i C_i}$$

$$-\frac{R_f}{R_i}$$



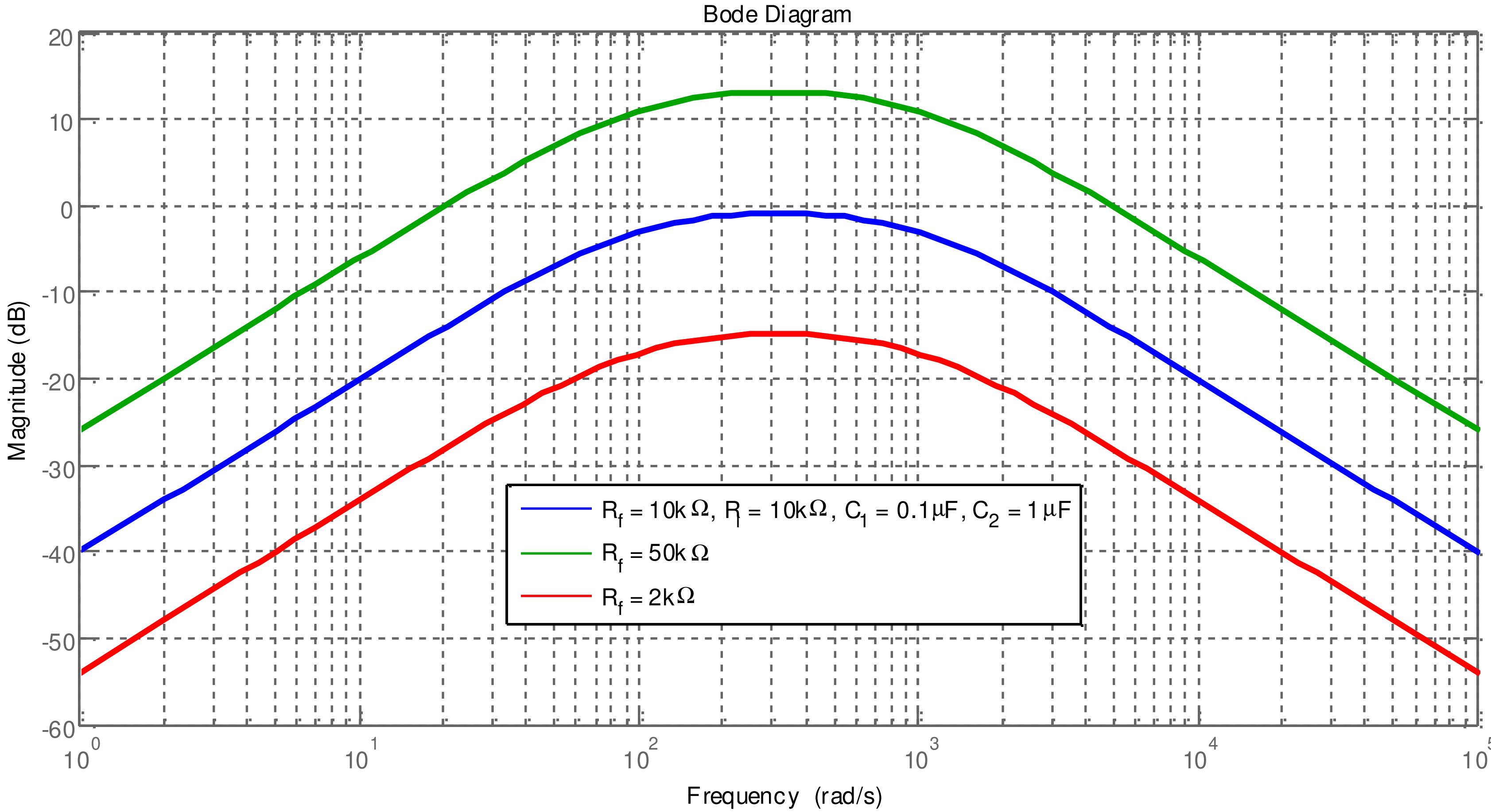
$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o}{\mathbf{V}_i}=\left(-\frac{1}{1+j\omega RC_1}\right)\left(-\frac{j\omega RC_2}{1+j\omega RC_2}\right)\left(-\frac{R_f}{R_i}\right)$$

$$=-\frac{R_f}{R_i}\times\frac{1}{1+j\omega RC_1}\times\frac{j\omega RC_2}{1+j\omega RC_2}$$

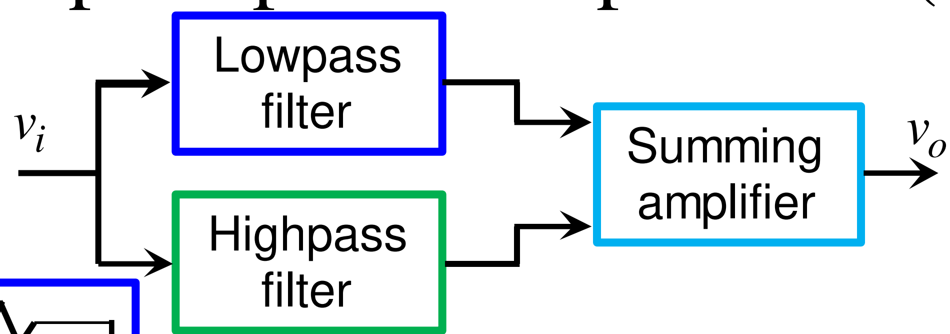
$$\omega_2=\frac{1}{RC_1};\ \omega_1=\frac{1}{RC_2};\ \omega_0=\sqrt{\omega_1\omega_2}$$

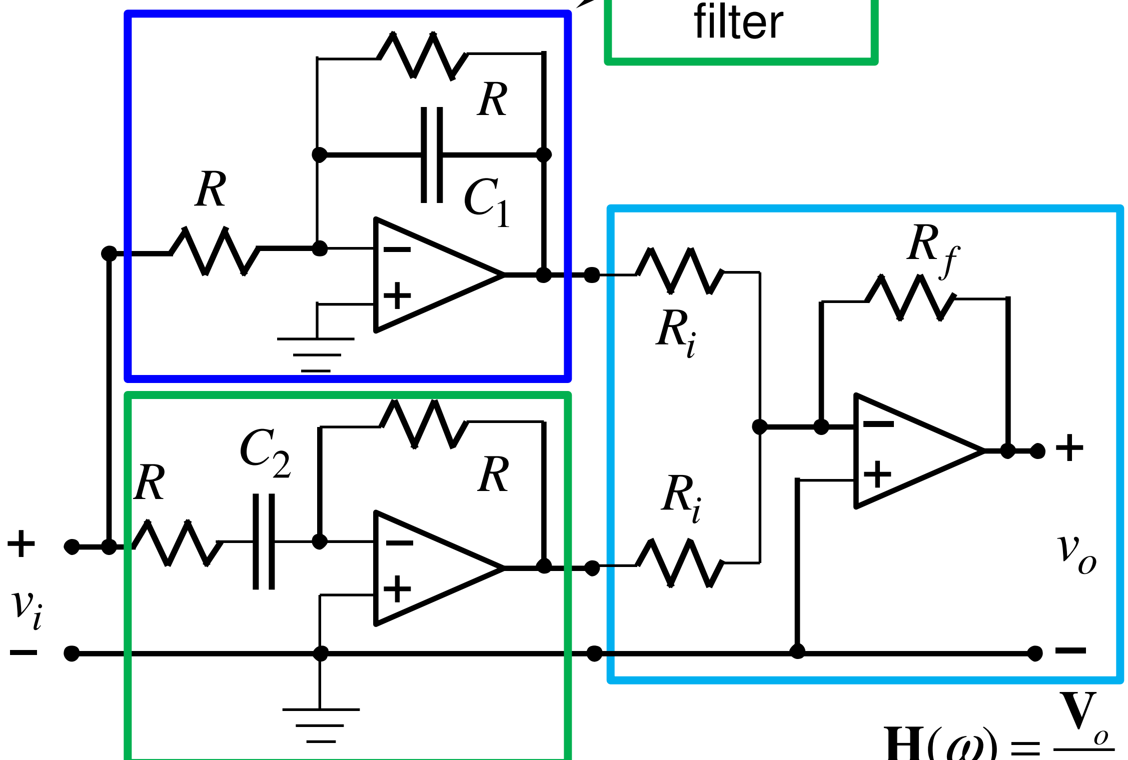
Op Amp Bandpass Filters (2)

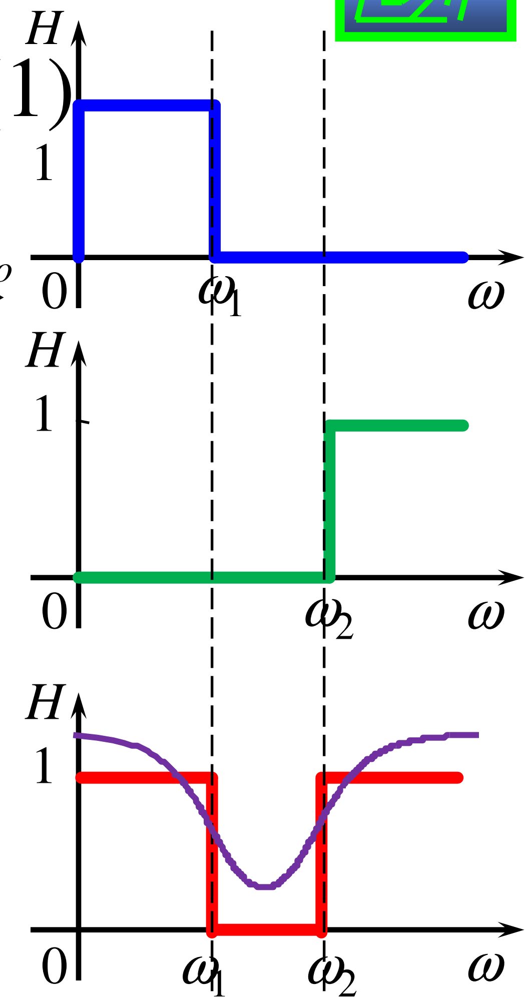
$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \times \frac{1}{1 + j\omega RC_1} \times \frac{j\omega RC_2}{1 + j\omega RC_2}, \quad \omega_2 = \frac{1}{RC_1}, \omega_1 = \frac{1}{RC_2}, \omega_0 = \sqrt{\omega_1 \omega_2}$$



Op Amp Bandstop Filters (1)



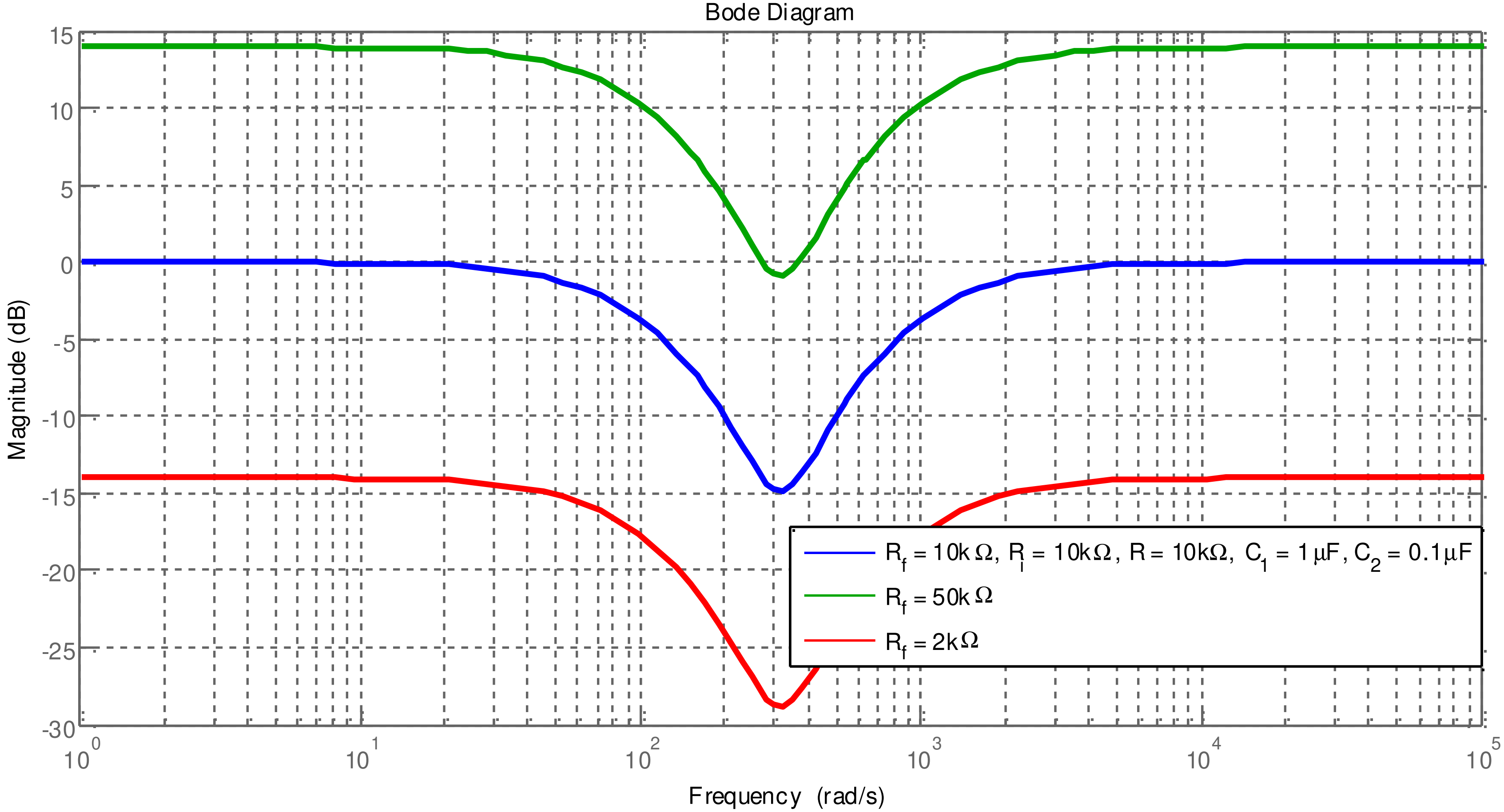




$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1 + j\omega RC_1} - \frac{j\omega RC_2}{1 + j\omega RC_2}\right)$$

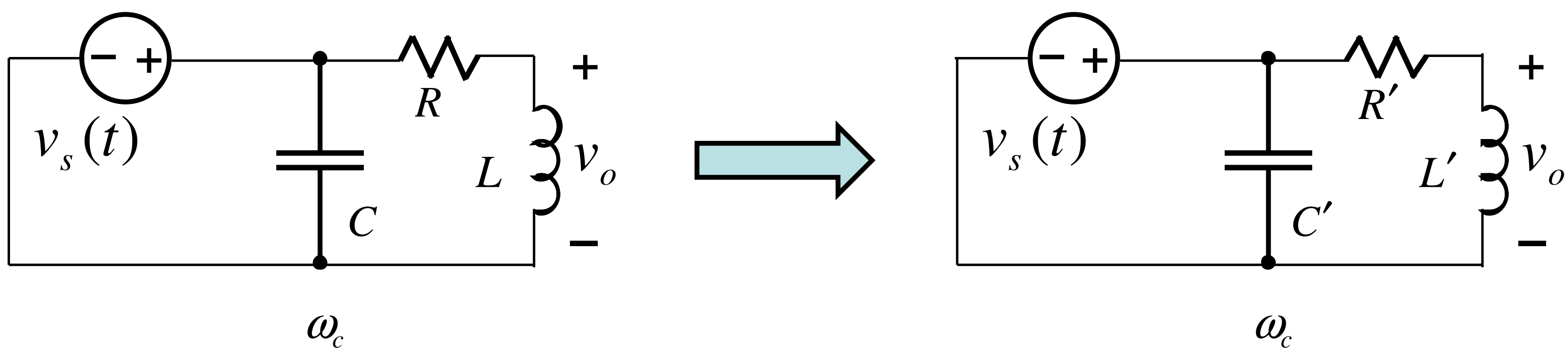
Op Amp Bandstop Filters (2)

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\left(-\frac{1}{1+j\omega RC_1} - \frac{j\omega RC_2}{1+j\omega RC_2}\right)$$



Frequency Response

Scaling (1)



$$L' = f(R, L, C, \omega_c, R')$$
$$C' = g(R, L, C, \omega_c, R')$$

Scaling (2)

$$\left.\begin{aligned} \omega' &= \omega \\ R' &= K_m R \end{aligned}\right\} \rightarrow \left\{\begin{aligned} L' &= K_m L \\ C' &= \frac{C}{K_m} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ R' &= R \end{aligned}\right\} \rightarrow \left\{\begin{aligned} L' &= \frac{L}{K_f} \\ C' &= \frac{C}{K_f} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ R' &= K_m R \end{aligned}\right\} \rightarrow \left\{\begin{aligned} L' &= \frac{K_m}{K_f} L \\ C' &= \frac{C}{K_m K_f} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= \omega \\ L' &= K_m L \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= K_m R \\ C' &= \frac{C}{K_m} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ L' &= \frac{L}{K_f} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= R \\ C' &= \frac{C}{K_f} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ L' &= \frac{K_m}{K_f} L \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= K_m R \\ C' &= \frac{C}{K_m K_f} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= \omega \\ C' &= \frac{C}{K_m} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= K_m R \\ L' &= K_m L \end{aligned}\right.$$

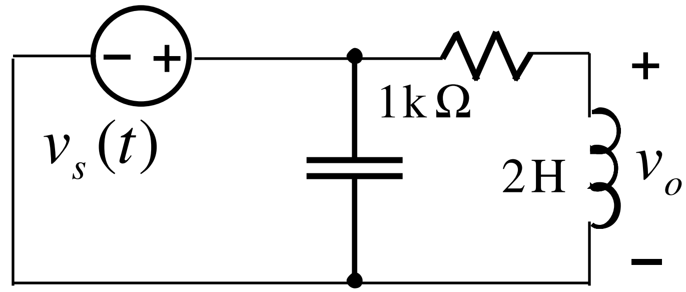
$$\left.\begin{aligned} \omega' &= K_f \omega \\ C' &= \frac{C}{K_f} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= R \\ L' &= \frac{L}{K_f} \end{aligned}\right.$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ C' &= \frac{C}{K_m K_f} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} R' &= K_m R \\ L' &= \frac{K_m}{K_f} L \end{aligned}\right.$$

Scaling (3)

Ex. 1

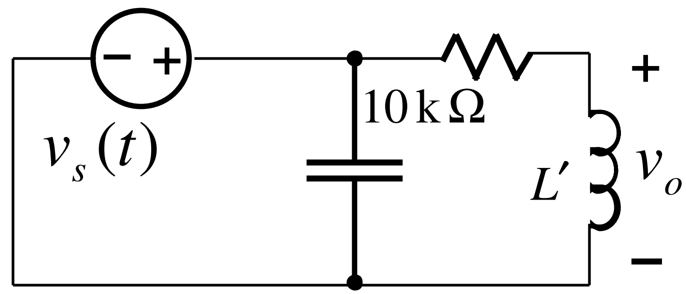
The cutoff frequency is 500 rad/s. Scale the circuit for a cutoff frequency of 25 kHz using a 10-kΩ resistor?



$$\omega' = 2\pi \times 25 \times 10^3 = 5\pi \times 10^4 \ \text{ rad/s}$$

$$K_f = \frac{\omega'}{\omega} = \frac{5\pi \times 10^4}{500} = 100\pi$$

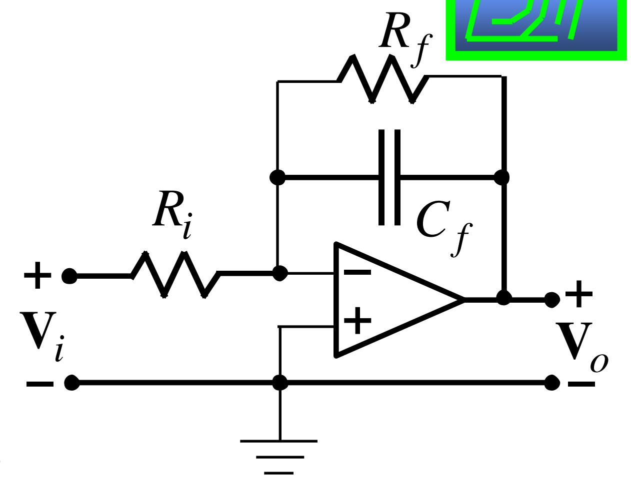
$$K_m = \frac{R'}{R} = \frac{10}{1} = 10$$



$$\left.\begin{aligned} \omega' &= K_f \omega \\ R' &= K_m R \end{aligned}\right\} \rightarrow L' = \frac{K_m}{K_f} L = \frac{10}{100\pi} 2 = \boxed{0.064\,\text{H}}$$

Scaling (4)

Ex. 2



$R_i = R_f = 1\Omega$, $C_f = 1$F. Use a new capacitor of 0.01 μF to redesign the filter with a gain of 5 & a cutoff frequency of 1kHz.

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i} \times \frac{1}{1 + j\omega R_f C_f}, \quad \omega_c = \frac{1}{R_f C_f} = \frac{1}{1\times 1} = 1 \text{ rad/s}$$

$$\left.\begin{aligned} \omega' &= K_f \omega \\ C' &= \frac{C}{K_m K_f} \end{aligned}\right\} \to \left\{\begin{aligned} R' &= K_m R \\ L' &= \frac{K_m}{K_f} L \end{aligned}\right.$$

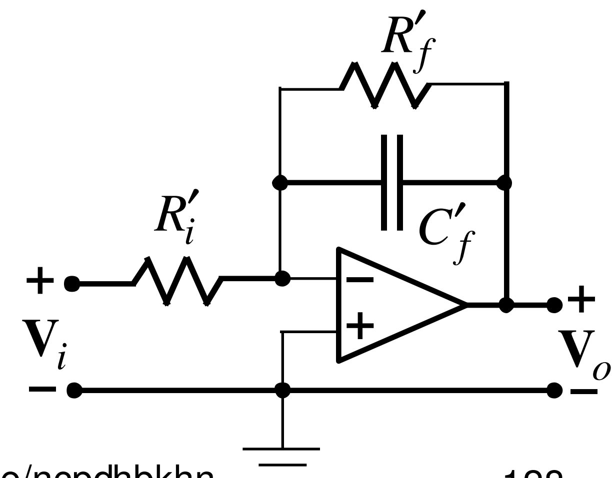
$$K_f = \frac{\omega_c'}{\omega_c} = \frac{2\pi \times 1000}{1} = 6283.19$$

$$K_m = \frac{C}{K_f C_f'} = \frac{1}{6283.19 \times 10^{-8}} = 15916$$

$$R_i' = R_f' = K_m R_i = 15916 \times 1 = 15916\Omega$$

$$K_{gain} = 5 = \frac{R_f'}{R_i''} \to R_i'' = \frac{R_f'}{5} = \frac{15916}{5} = 3183\Omega$$

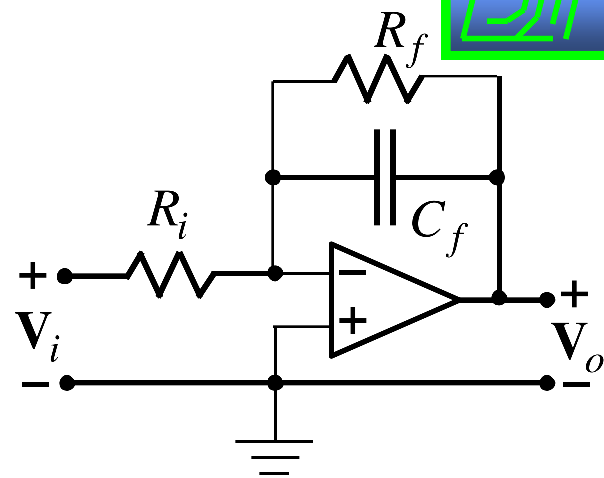
$$\boxed{R_i' = 3183\Omega,\ R_f' = 15916\Omega,\ C_f' = 10^{-8}\text{ F}}$$



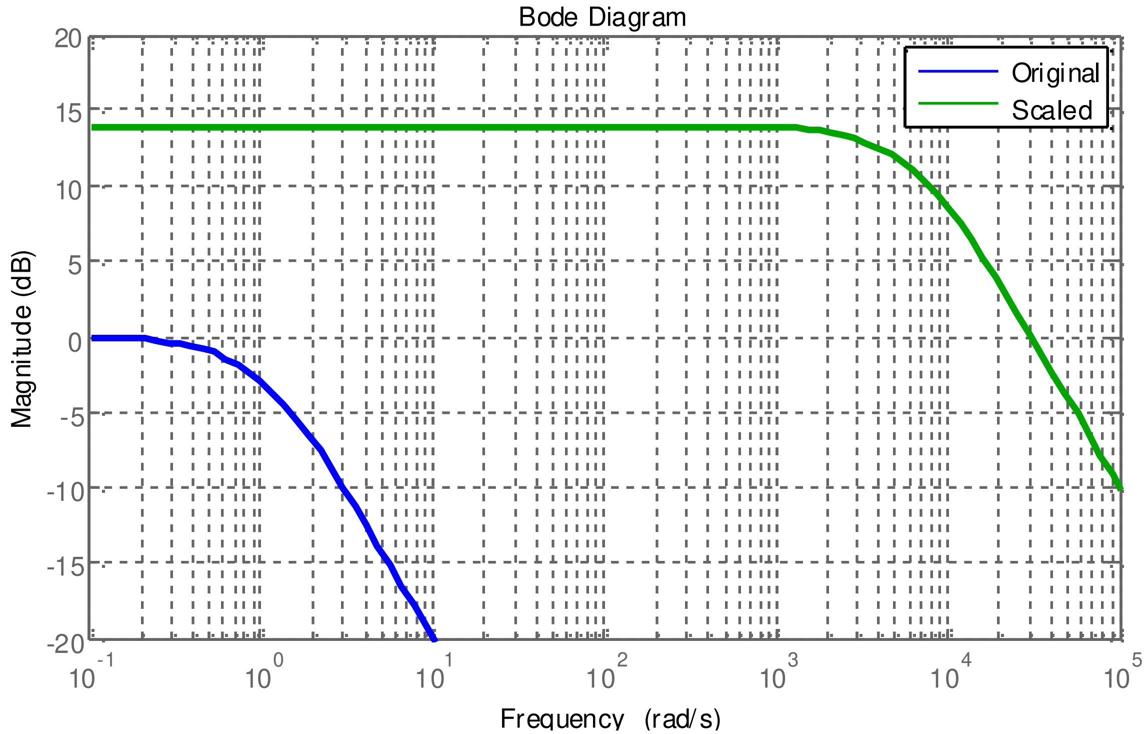
Scaling (5)

Ex. 2

$R_i = R_f = 1\Omega$, $C_f = 1$F. Use a new capacitor of 0.01 μF to redesign the filter with a gain of 5 & a cutoff frequency of 1kHz.



$$R_i = 8183\Omega,\ R_f = 15916\Omega,\ C_f = 10^{-8}\ \text{F}$$



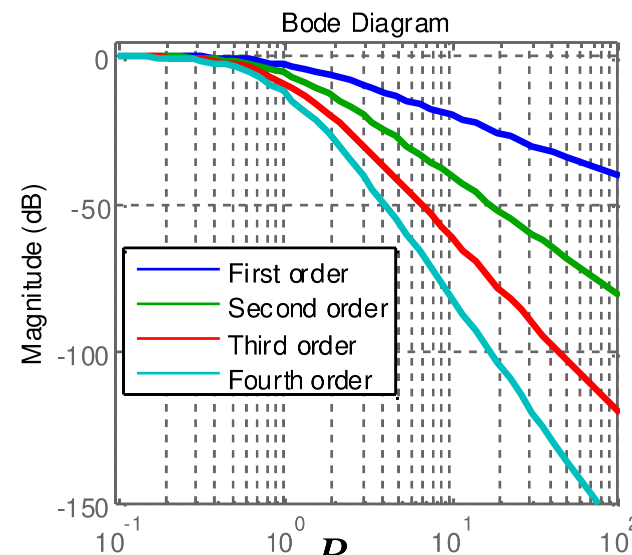
Frequency Response

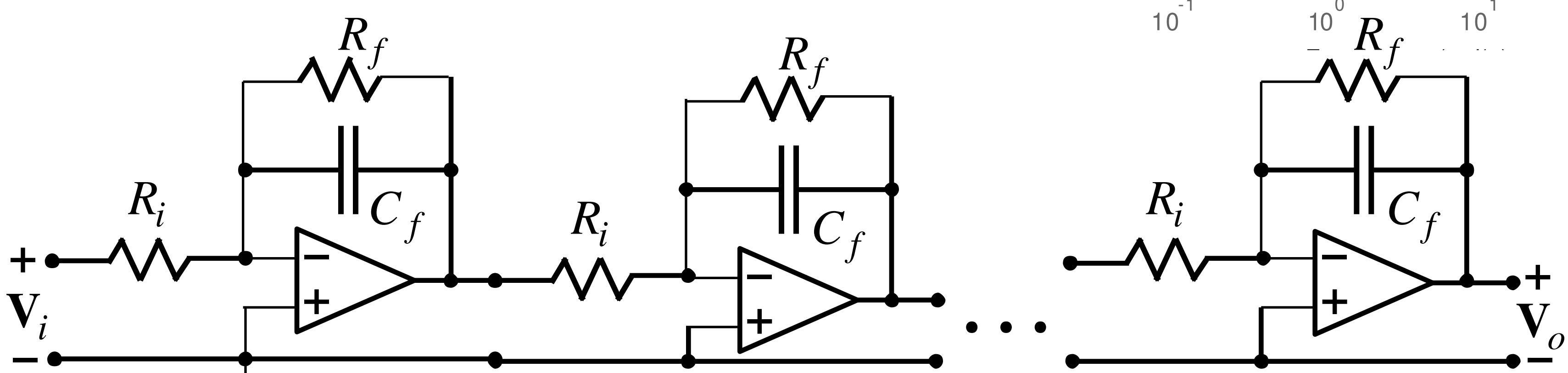
Cascading Identical Filters (1)

$$R_i = R_f = 1\Omega,\ C_f = 1\text{F}$$

$$\rightarrow \mathbf{H}(\omega) = \left(\frac{-1}{1+j\omega}\right)\left(\frac{-1}{1+j\omega}\right)\cdots\left(\frac{-1}{1+j\omega}\right) = \frac{(-1)^n}{(1+j\omega)^n}$$

$$\left|\mathbf{H}(\omega)\right| = \frac{1}{\sqrt{2}} \rightarrow \omega_{cn} = \sqrt{\sqrt[n]{2}-1}$$





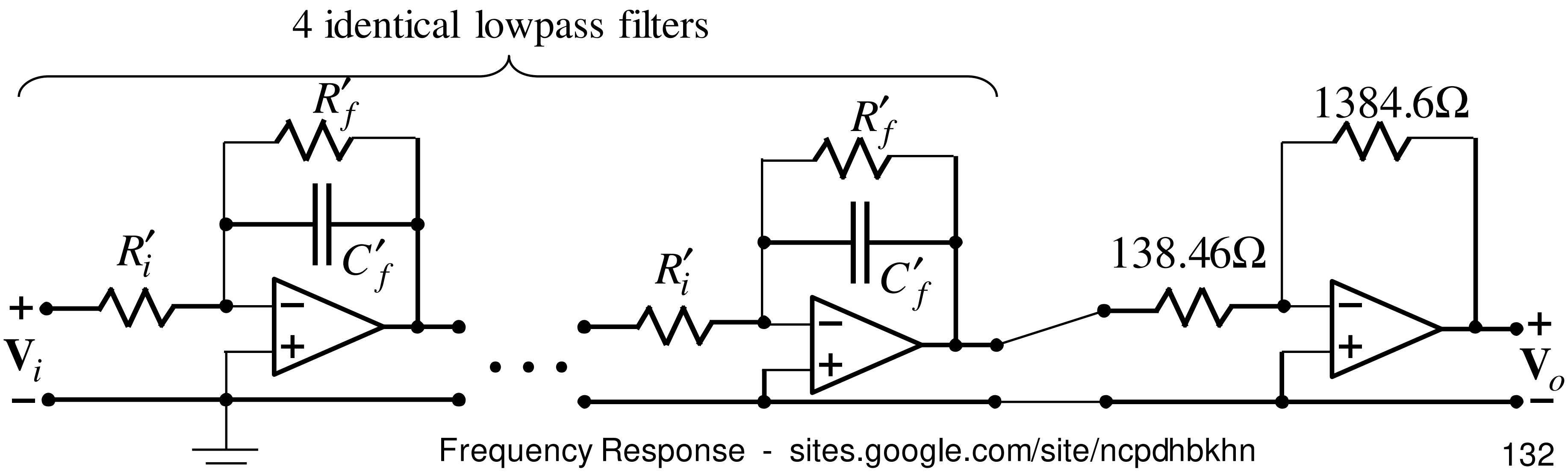
Ex. Cascading Identical Filters (2)

Design a fourth-order lowpass filter with a cutoff frequency of 500Hz & a passband gain of 10. Use 1 μF capacitors.

$$R_i = R_f = 1\Omega,\ C_f = 1\,\text{F} \rightarrow \omega_{cn} = \sqrt{\sqrt[n]{2} - 1} \rightarrow \omega_{c4} = \sqrt{\sqrt[4]{2} - 1} = 0.435 \text{ rad/s}$$

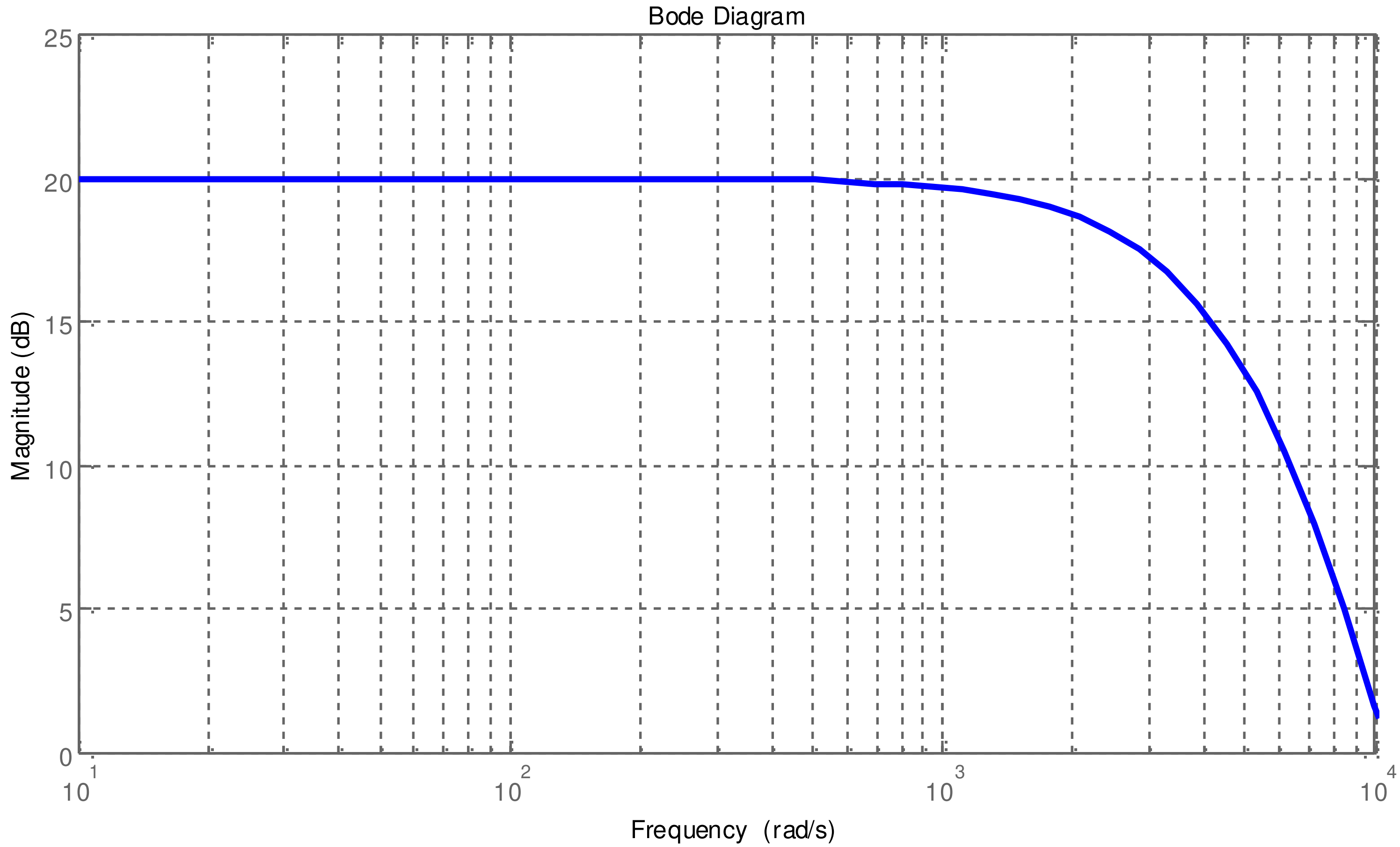
$$K_f = \frac{\omega'_c}{\omega_c} = \frac{2\pi \times 500}{0.435} = 7222.4$$

$$K_m = \frac{C_f}{K_f C'_f} = \frac{1}{7222.4 \times 10^{-6}} = 138.46 \rightarrow R'_i = R'_f = K_m R_i = 138.46 \times 1 = \boxed{138.46\Omega}$$



Ex. Cascading Identical Filters (3)

Design a fourth-order lowpass filter with a cutoff frequency of 500Hz & a passband gain of 10. Use 1 μF capacitors.



Frequency Response

Butterworth Filters (1)

A unity-gain Butterworth lowpass filter has a transfer function whose magnitude is given by:

$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_c}\right)^{2n}}}$$

$$\omega_c = 1\,\text{rad/s} \rightarrow |\mathbf{H}(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

$$\left.\begin{aligned} &\rightarrow |\mathbf{H}(\omega)|^2 = \frac{1}{1+\omega^{2n}} = \frac{1}{1+(-1)^n(j\omega)^{2n}} \\ &|\mathbf{H}(\omega)|^2 = \mathbf{H}(j\omega)\mathbf{H}(-j\omega) \end{aligned}\right\}$$

$$\rightarrow \mathbf{H}(j\omega)\mathbf{H}(-j\omega) = \frac{1}{1+(-1)^n(j\omega)^{2n}}$$

The procedure for finding $\mathbf{H}(j\omega)$ for a given value of n:

1. Find the roots of the polynomial:

$$1 + (-1)^n(j\omega)^{2n} = 0$$

2. Assign the left-half plane roots to $\mathbf{H}(j\omega)$ & the right-half plane roots to $\mathbf{H}(-j\omega)$.
3. Combine terms in the denominator of $\mathbf{H}(j\omega)$ to form first- and second-order factors.

Butterworth Filters (2)

Ex. 1

Find the Butterworth transfer function for $n = 2$ and $n = 3$?

$$1+(-1)^2(j\omega)^4=0$$

$$\rightarrow \begin{cases} (j\omega)_1 = 1/\sqrt{2} + j/\sqrt{2} \\ \boxed{\begin{aligned} (j\omega)_2 &= -1/\sqrt{2} + j/\sqrt{2} \\ (j\omega)_3 &= -1/\sqrt{2} - j/\sqrt{2} \end{aligned}} \\ (j\omega)_4 = 1/\sqrt{2} - j/\sqrt{2} \end{cases}$$

The procedure for finding $\mathbf{H}(j\omega)$ for a given value of n:

1. Find the roots of the polynomial:
   $$1 + (-1)^n(j\omega)^{2n} = 0$$
2. Assign the left-half plane roots to $\mathbf{H}(j\omega)$ & the right-half plane roots to $\mathbf{H}(-j\omega)$.
3. Combine terms in the denominator of $\mathbf{H}(j\omega)$ to form first- and second-order factors.

$$\mathbf{H}(j\omega) = \frac{1}{[j\omega-(-1/\sqrt{2}+j/\sqrt{2})][j\omega-(-1/\sqrt{2}-j/\sqrt{2})]}$$

$$= \frac{1}{(j\omega)^2+\sqrt{2}\,j\omega+1}$$

Ex. 1 Butterworth Filters (3)

Find the Butterworth transfer function for $n = 2$ and $n = 3$?

$$1+(-1)^3(j\omega)^6 = 0$$

$$\rightarrow \begin{cases} (j\omega)_1 = 1 \\ (j\omega)_2 = 1/2 + j\sqrt{3}/2 \\ (j\omega)_3 = -1/2 + j\sqrt{3}/2 \\ (j\omega)_4 = -1 \\ (j\omega)_5 = -1/2 - j\sqrt{3}/2 \\ (j\omega)_6 = 1/2 - j\sqrt{3}/2 \end{cases}$$

$$\mathbf{H}(j\omega) = \frac{1}{[j\omega-(-1/2+j\sqrt{3}/2)][j\omega-(-1)][j\omega-(-1/2-j\sqrt{3}/2)]}$$

$$= \frac{1}{[j\omega+1][(j\omega)^2+j\omega+1]}$$

The procedure for finding $\mathbf{H}(j\omega)$ for a given value of n:

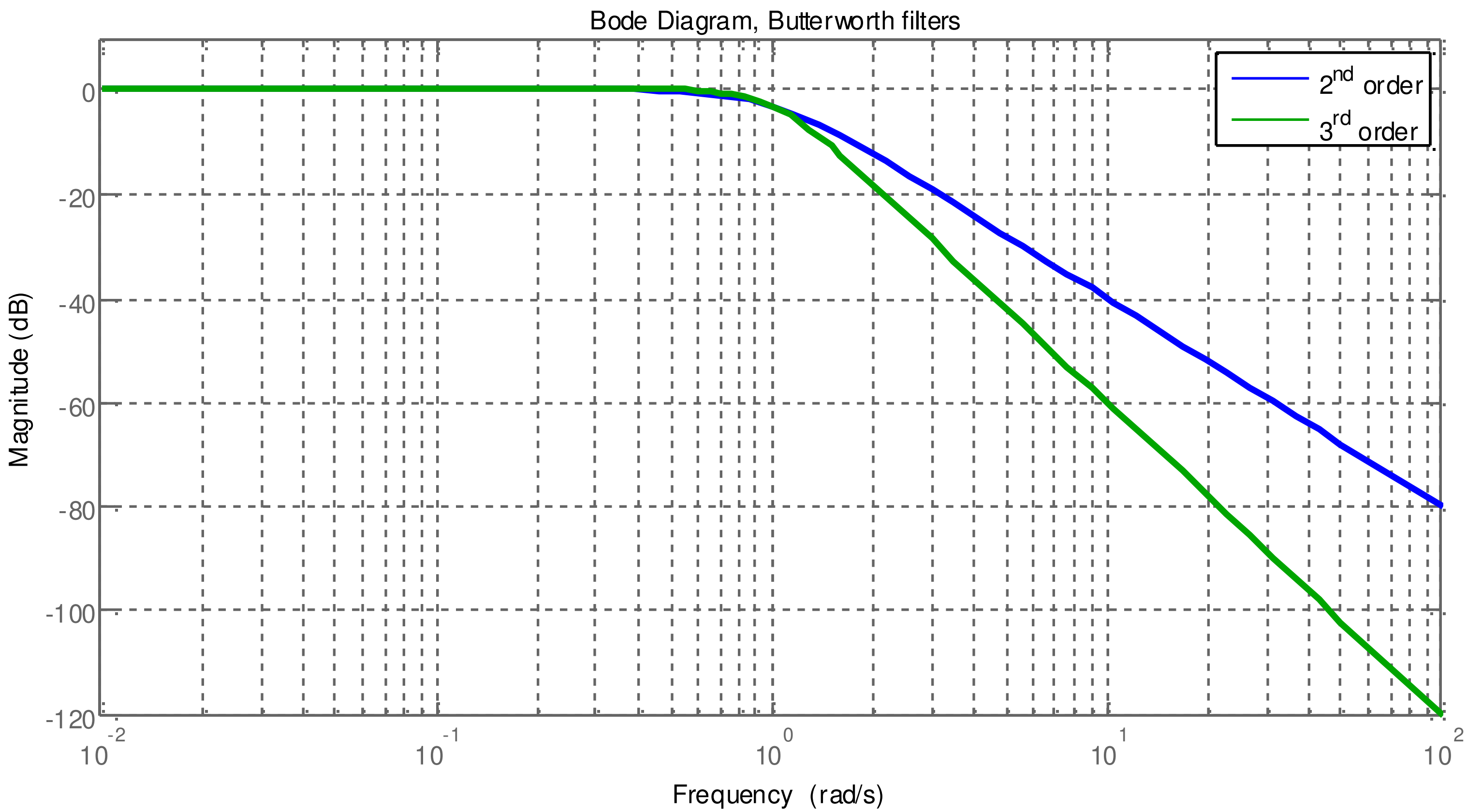
1. Find the roots of the polynomial:
$$1 + (-1)^n(j\omega)^{2n} = 0$$
2. Assign the left-half plane roots to $\mathbf{H}(j\omega)$ & the right-half plane roots to $\mathbf{H}(-j\omega)$.
3. Combine terms in the denominator of $\mathbf{H}(j\omega)$ to form first- and second-order factors.

Butterworth Filters (4)

Ex. 1

Find the Butterworth transfer function for $n = 2$ and $n = 3$?

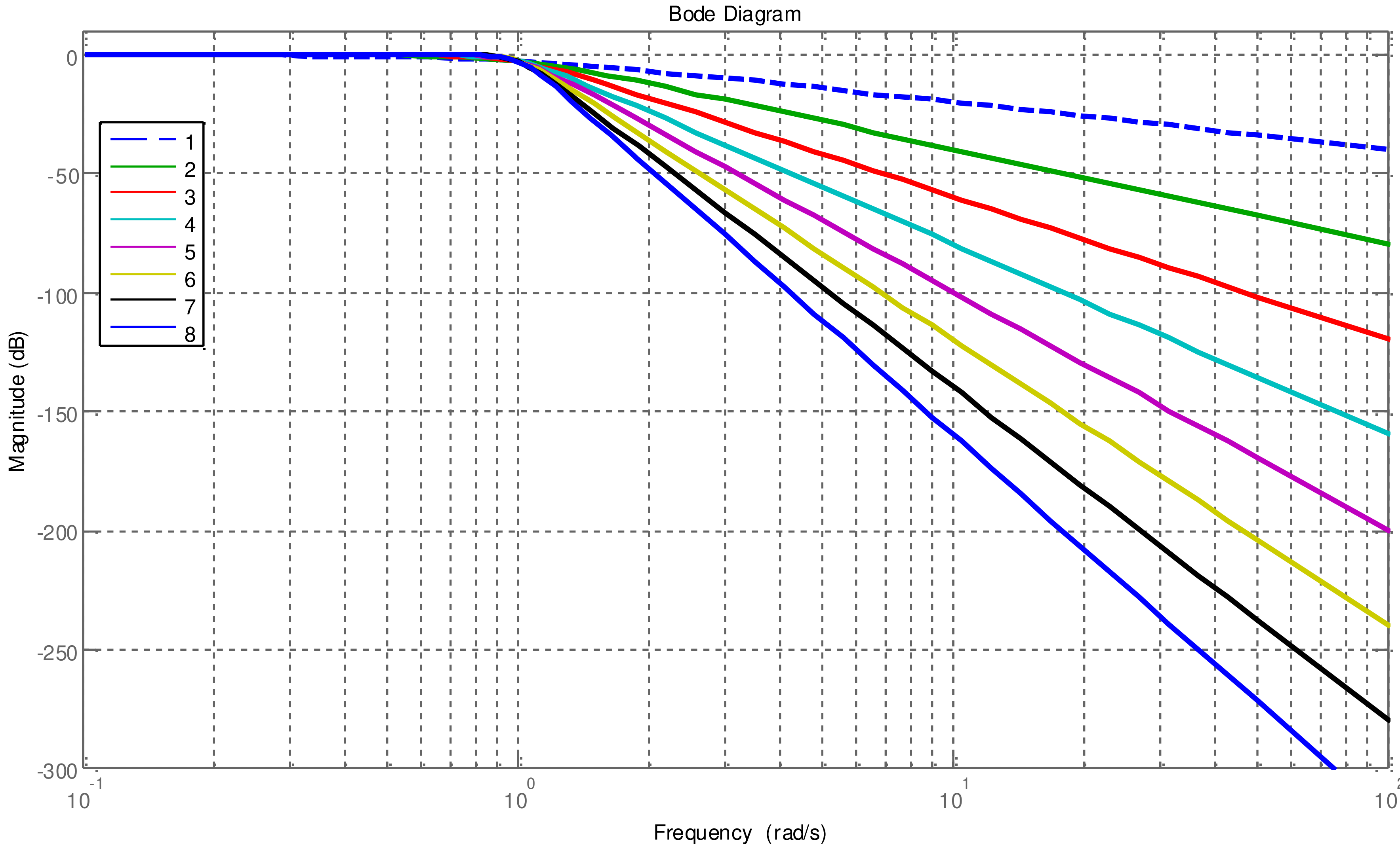
$$\mathbf{H}_2(j\omega) = \frac{1}{(j\omega)^2 + \sqrt{2}\, j\omega + 1}, \quad \mathbf{H}_3(j\omega) = \frac{1}{[j\omega + 1][(j\omega)^2 + j\omega + 1]}$$



Butterworth Filters (5)

1	$j\omega+1$
2	$(j\omega)^2+\sqrt{2}\,j\omega+1$
3	$(j\omega+1)[(j\omega)^2+j\omega+1]$
4	$[(j\omega)^2+0.765\,j\omega+1][(j\omega)^2+1.848\,j\omega+1]$
5	$(j\omega+1)[(j\omega)^2+0.618\,j\omega+1][(j\omega)^2+1.618\,j\omega+1]$
6	$[(j\omega)^2+0.518\,j\omega+1][(j\omega)^2+\sqrt{2}\,j\omega+1][(j\omega)^2+1.932\,j\omega+1]$
7	$(j\omega+1)[(j\omega)^2+0.445\,j\omega+1][(j\omega)^2+1.247\,j\omega+1][(j\omega)^2+1.802\,j\omega+1]$
8	$[(j\omega)^2+0.390\,j\omega+1][(j\omega)^2+1.111\,j\omega+1][(j\omega)^2+1.6663\,j\omega+1][(j\omega)^2+1.962\,j\omega+1]$

Butterworth Filters (6)



Butterworth Filters (7)

$$\mathbf{H}_5(j\omega) = \frac{1}{(j\omega+1)[(j\omega)^2+0.618j\omega+1][(j\omega)^2+1.618j\omega+1]}$$

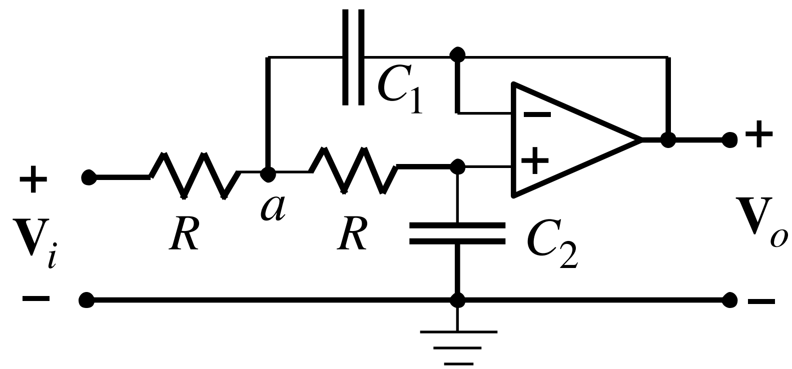
$$\mathbf{V}_i \longrightarrow \boxed{\frac{1}{j\omega+1}} \longrightarrow \boxed{\frac{1}{(j\omega)^2+0.618j\omega+1}} \longrightarrow \boxed{\frac{1}{(j\omega)^2+1.618j\omega+1}} \longrightarrow \mathbf{V}_o$$

$$\mathbf{H}(j\omega) = \frac{1}{(j\omega)^2+b_1 j\omega+1}$$

Butterworth Filters (8)

$$\begin{cases} \mathbf{I}_{Rleft} + \mathbf{I}_{C1} + \mathbf{I}_{Rright} = 0 \\ \mathbf{I}_{C2} + \mathbf{I}_{Rright} = 0 \end{cases}$$

$$\rightarrow \begin{cases} \dfrac{\mathbf{V}_i - \mathbf{V}_a}{R} + j\omega C_1(\mathbf{V}_o - \mathbf{V}_a) + \dfrac{\mathbf{V}_o - \mathbf{V}_a}{R} = 0 \\ j\omega C_2 \mathbf{V}_o + \dfrac{\mathbf{V}_a - \mathbf{V}_o}{R} = 0 \end{cases}$$



$$\rightarrow \mathbf{V}_0 = \frac{\mathbf{V}_i}{R^2 C_1 C_2 (j\omega)^2 + 2RC_2 j\omega + 1}$$

$$\rightarrow \mathbf{H}(j\omega) = \frac{\mathbf{V}_0}{\mathbf{V}_i} = \frac{\dfrac{1}{R^2 C_1 C_2}}{(j\omega)^2 + \dfrac{2}{RC_1} j\omega + \dfrac{1}{R^2 C_1 C_2}}$$

$$= \frac{1}{(j\omega)^2 + b_1 j\omega + 1}, \quad b_1 = \frac{2}{C_1}, \; 1 = \frac{1}{C_1 C_2}, \quad R = 1\Omega$$

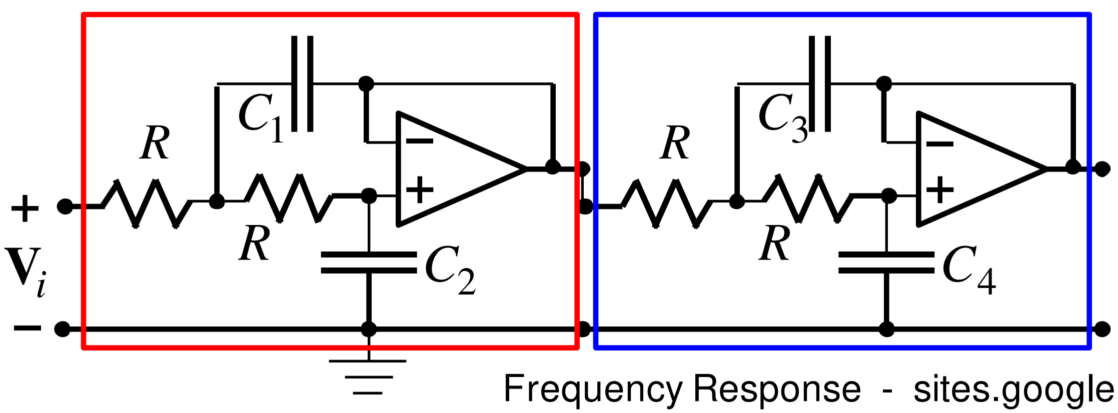
Butterworth Filters (9)

Ex. 2

Design a fourth-order Butterworth lowpass filter with a cutoff frequency of 500Hz & a passband gain of 10. Use as many 1kΩ resistors as possible.

$$\mathbf{H}(j\omega) = \frac{1}{\boxed{[(j\omega)^2 + 0.765\,j\omega + 1]}\boxed{[(j\omega)^2 + 1.848\,j\omega + 1]}}$$

$$\mathbf{H}_2(j\omega) = \frac{1}{(j\omega)^2 + b_1 j\omega + 1}, \quad b_1 = \frac{2}{C_1}, \quad 1 = \frac{1}{C_1 C_2}, \quad R = 1\Omega \rightarrow \begin{cases} \frac{2}{C_1} = 0.765 \\ \frac{1}{C_1 C_2} = 1 \\ \frac{2}{C_3} = 1.848 \\ \frac{1}{C_3 C_4} = 1 \end{cases}$$



$$\rightarrow \begin{cases} C_1 = 2.6144\text{F} \\ C_2 = 0.3825\text{F} \\ C_3 = 1.0823\text{F} \\ C_4 = 0.9240\text{F} \end{cases}$$

Butterworth Filters (10)

Ex. 2

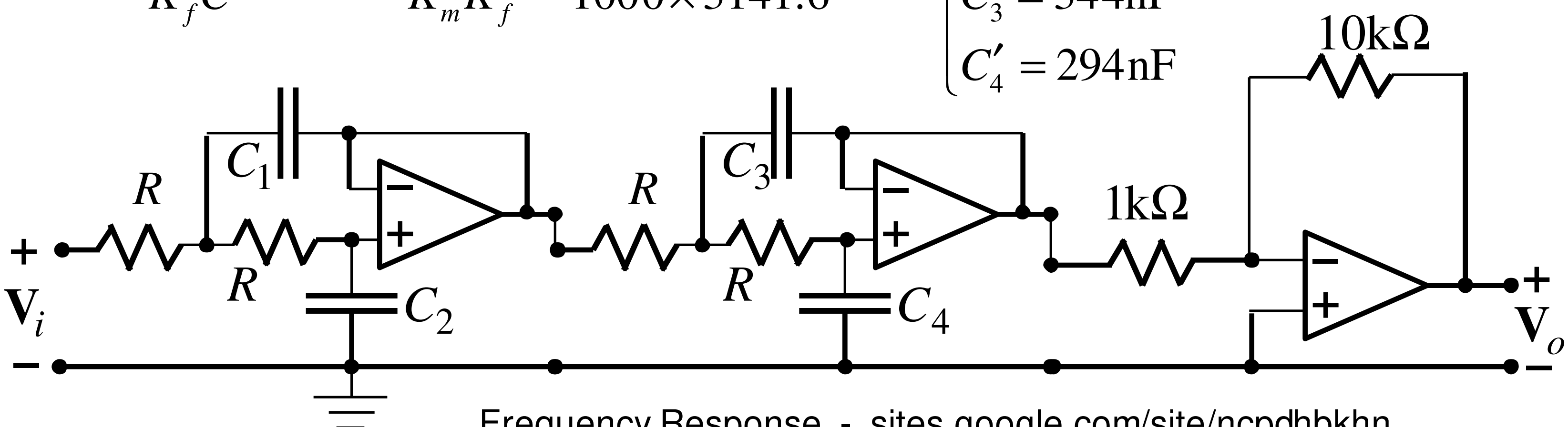
Design a fourth-order Butterworth lowpass filter with a cutoff frequency of 500Hz & a passband gain of 10. Use as many 1kΩ resistors as possible.

$$R = 1\Omega,\ C_1 = 2.61\text{F},\ C_2 = 0.38\text{F},\ C_3 = 1.08\text{F},\ C_4 = 0.92\text{F},\ \omega_c = 1\text{rad/s}$$

$$K_f = \frac{\omega_c'}{\omega_c} = \frac{2\pi \times 500}{1} = 3141.6$$

$$K_m = \frac{R'}{R} = \frac{1000}{1} = 1000$$

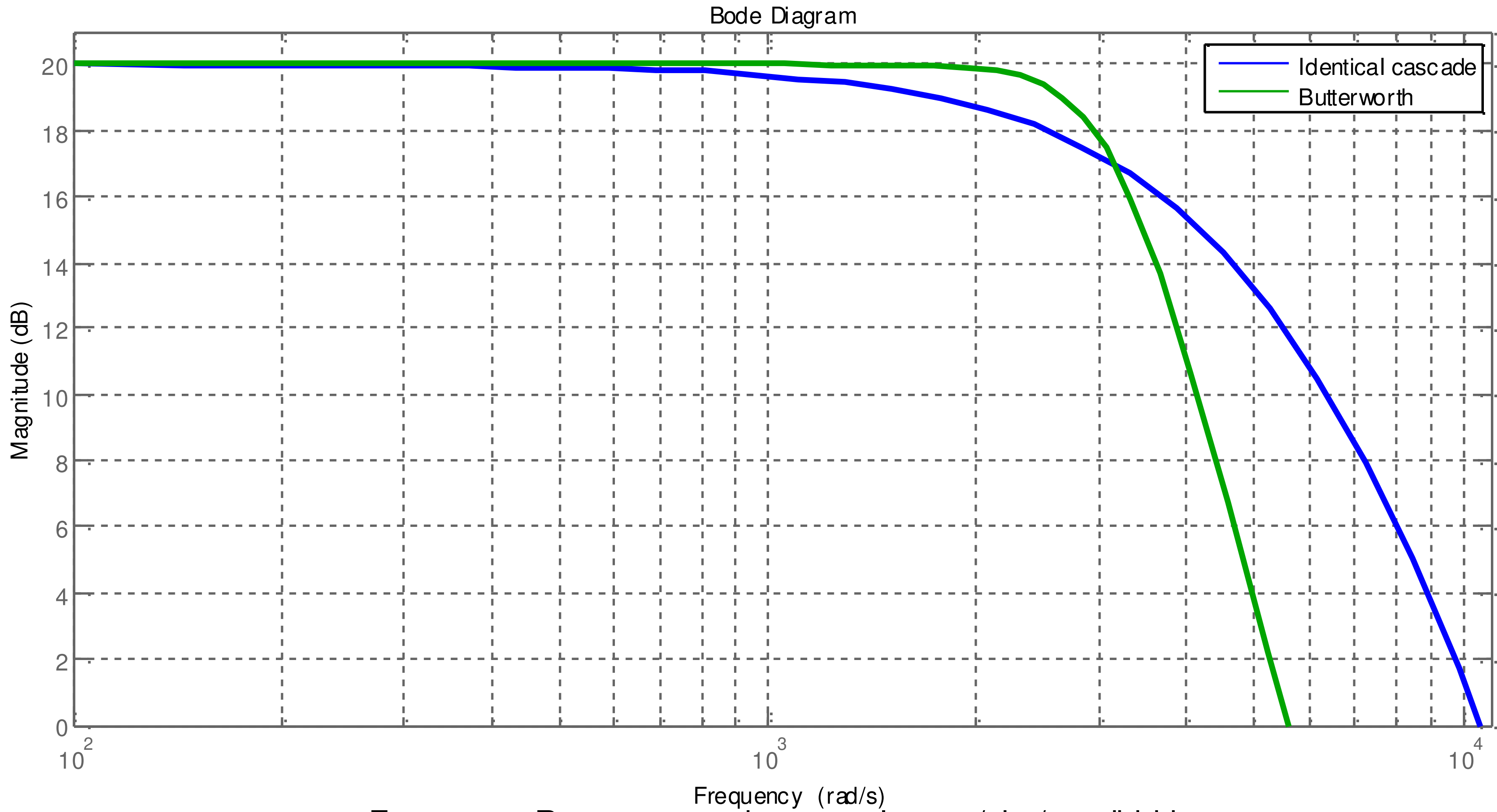
$$K_m = \frac{C}{K_f C'} \to C' = \frac{C}{K_m K_f} = \frac{C}{1000 \times 3141.6} \to \begin{cases} C_1' = 831\text{nF} \\ C_2' = 121\text{nF} \\ C_3' = 344\text{nF} \\ C_4' = 294\text{nF} \end{cases}$$



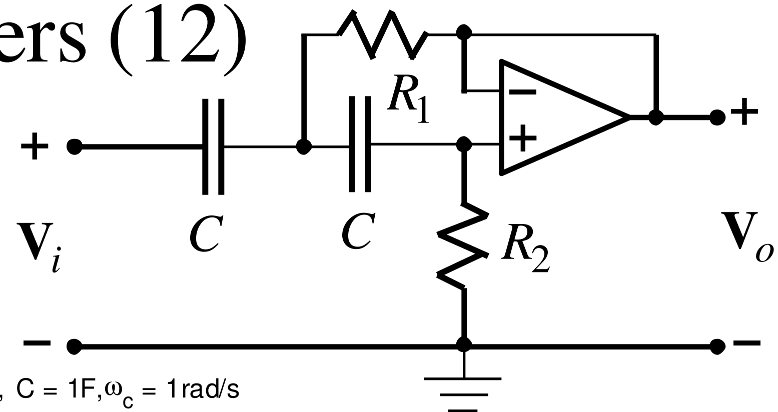
Ex. 2

Butterworth Filters (11)

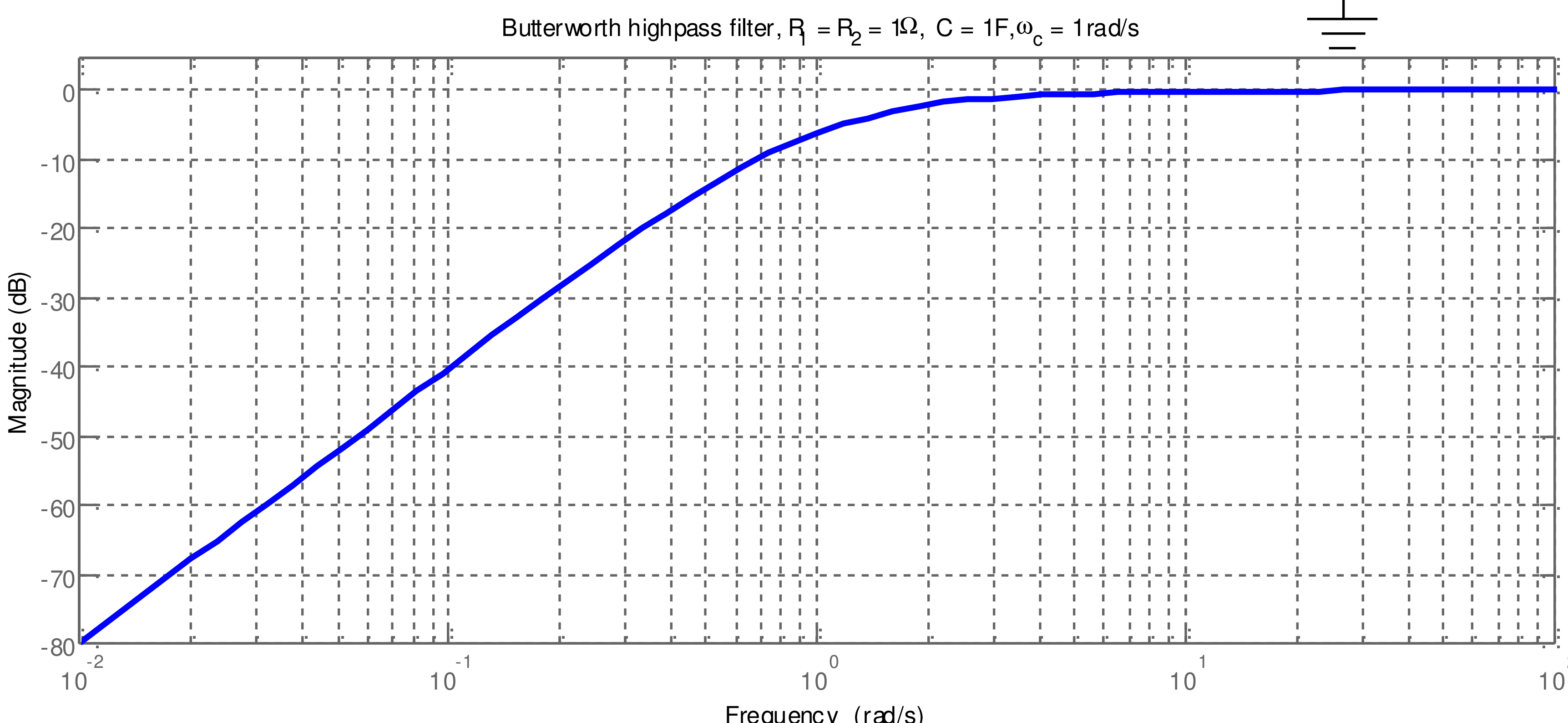
Design a fourth-order Butterworth lowpass filter with a cutoff frequency of 500Hz & a passband gain of 10. Use as many 1kΩ resistors as possible.



Butterworth Filters (12)



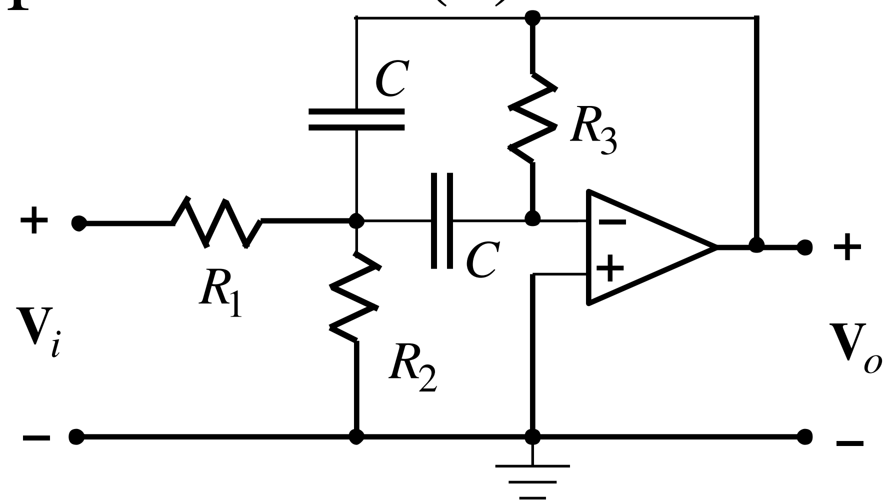
$$\mathbf{H}(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + \dfrac{2}{R_2 C} j\omega + \dfrac{1}{R_1 R_2 C^2}}$$



Frequency Response

1. Transfer Function
2. The Decibel Scale
3. Bode Plots
4. Series Resonance
5. Parallel Resonance
6. Passive Filters
7. Active Filters
8. Scaling
9. Higher Order Op Amp Filters
10. **Narrowband Bandpass & Banstop Filters**

Narrowband Bandpass Filters (1)



$$\mathbf{H}(j\omega) = \frac{\dfrac{-j\omega}{R_1 C}}{(j\omega)^2 + \dfrac{2}{R_3 C} j\omega + \dfrac{1}{R_{eq} R_3 C^2}}$$

$$= \frac{K\beta j\omega}{(j\omega)^2 + \beta j\omega + \omega_o^2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \quad \beta = \frac{2}{R_3 C}, \quad \omega_o^2 = \frac{1}{R_{eq} R_3 C^2}$$

$$R_1 = \frac{Q}{K}, \quad Q = \frac{\omega_o}{B} = \frac{\omega_o}{\omega_2 - \omega_1}$$

$$R_2 = \frac{Q}{2Q^2 - K}$$

$$R_3 = 2Q$$

Ex. 1 Narrowband Bandpass Filters (2)

Design a bandpass filter with a center frequency of 3000Hz, a quality factor of 10, & a passband gain of 2. Use 0.01μF capacitors.

$$R_1 = \frac{Q}{K} = \frac{10}{2} = 5\Omega$$

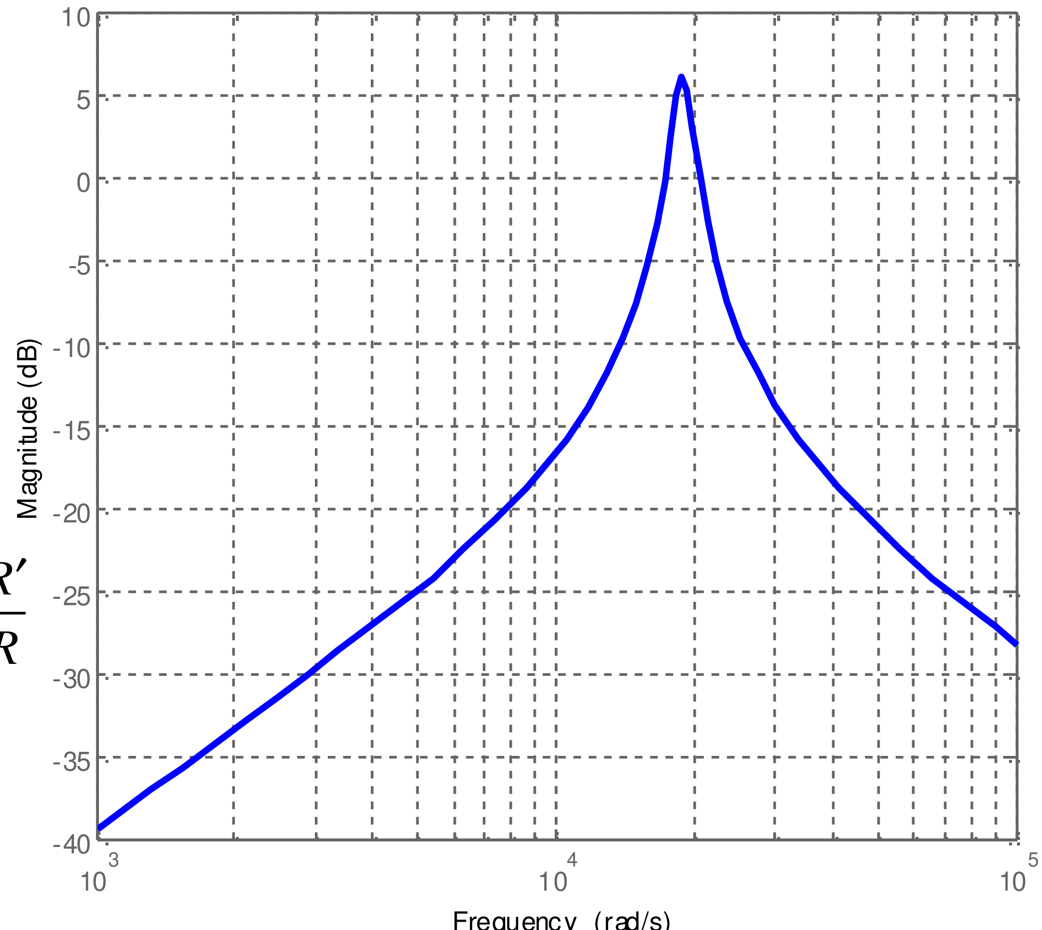
$$R_2 = \frac{Q}{2Q^2 - K} = \frac{10}{2\times10^2 - 2} = 0.0505\Omega$$

$$R_3 = 2Q = 2\times10 = 20\Omega$$

$$K_f = \frac{\omega_c'}{\omega_c} = \frac{2\pi\times3000}{1} = 6000\pi$$

$$K_m = \frac{C}{K_f C'} = \frac{1}{6000\pi\times10^{-8}} = 5305.2 = \frac{R'}{R}$$

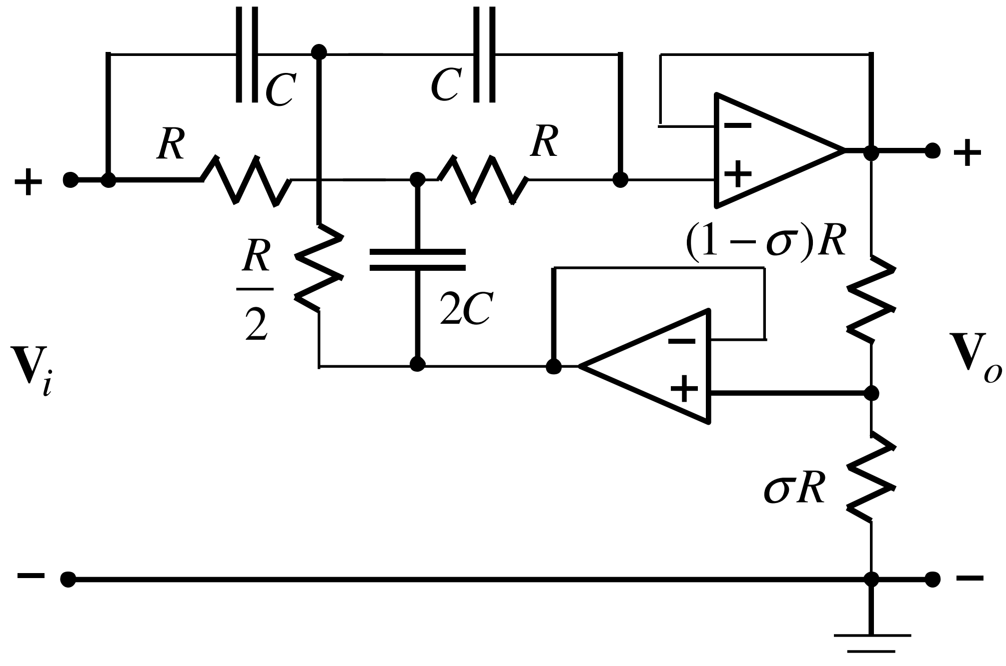
$$\rightarrow \begin{cases} R_1 = K_m R_1 = 26.5\,\text{k}\Omega \\ R_2 = K_m R_2 = 268.0\,\Omega \\ R_2 = K_m R_3 = 106.1\,\text{k}\Omega \end{cases}$$



Narrowband Bandstop Filters (3)

$$\mathbf{H}(j\omega) = \frac{(j\omega)^2 + \dfrac{1}{R^2C^2}}{(j\omega)^2 + \dfrac{4(1-\sigma)}{RC} j\omega + \dfrac{1}{R^2C^2}}$$

$$= \frac{(j\omega)^2 + \omega_o^2}{(j\omega)^2 + \beta j\omega + \omega_o^2}$$



$$\omega_o^2 = \frac{2}{R^2C^2}, \quad \beta = \frac{4(1-\sigma)}{RC}$$

$$R = \frac{1}{\omega_o C}, \quad \sigma = 1 - \frac{1}{4Q}$$

Ex. 2 Narrowband Bandstop Filters (4)

Design a bandstop filter with a center frequency of 5000 rad/s & a bandwidth of 1000 rad/s. Use 1μF capacitors.

$$R = \frac{1}{\omega_o C} = \frac{1}{5000 \times 10^{-6}} = 200\Omega$$

$$\sigma = 1 - \frac{1}{4Q}$$
$$= 1 - \frac{1}{4(\omega_o / B)}$$
$$= 1 - \frac{1000}{4 \times 5000}$$
$$= 0.95$$