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Electric Circuit Theory

Two-port Networks

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Two-port Networks

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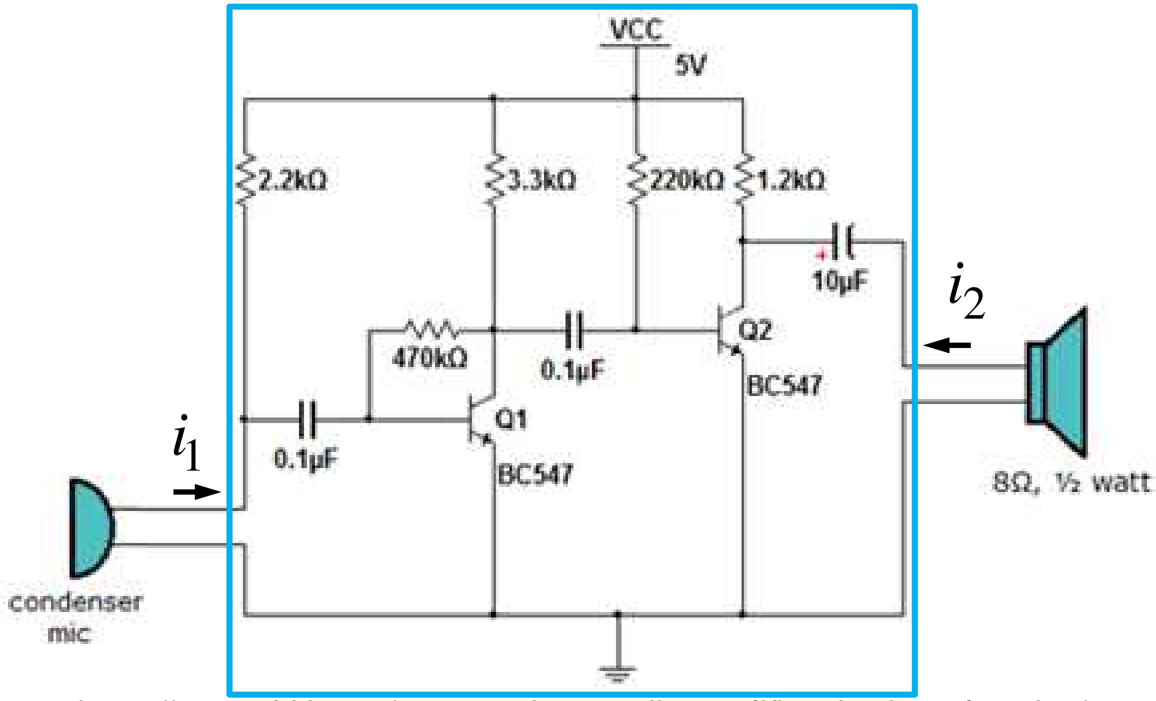
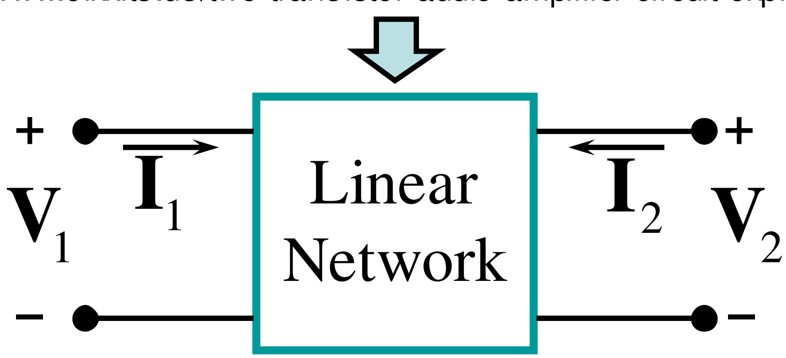
Introduction



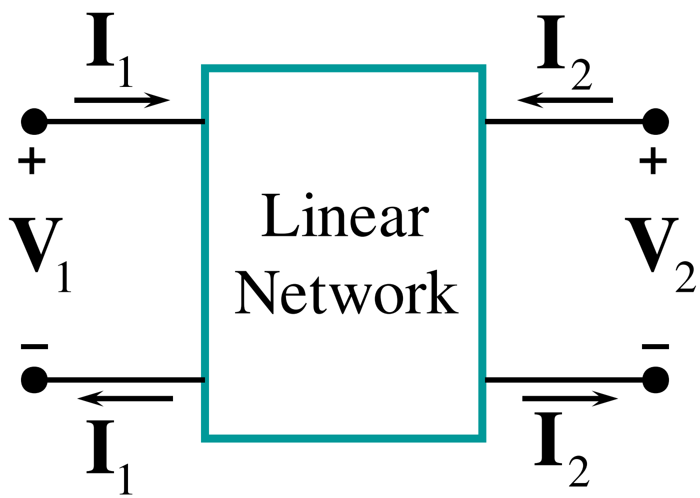
https://www.efxkits.us/two-transistor-audio-amplifier-circuit-explanation/



Two-port Networks

Impedance Parameters (1)

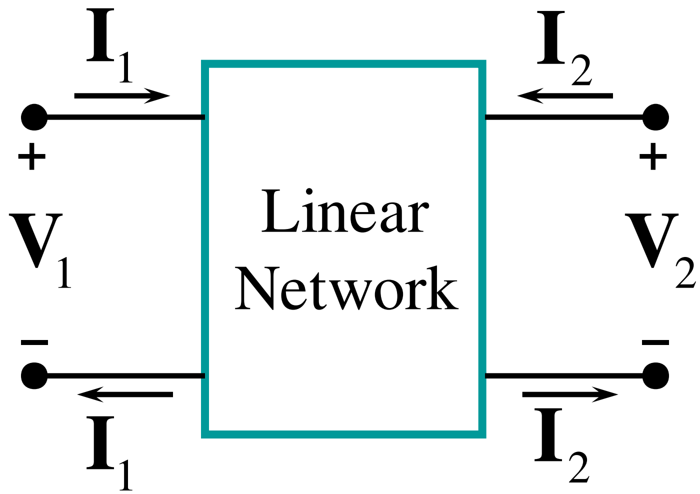
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$



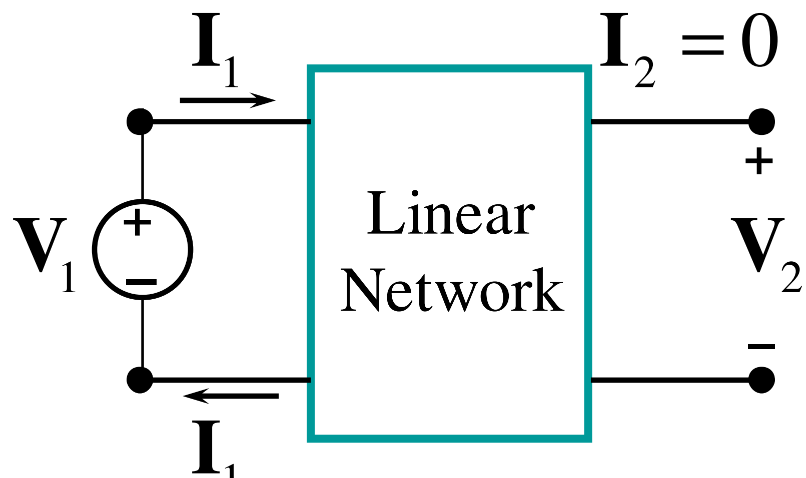
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Impedance Parameters (2)

$$\left.\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \quad \mathbf{I}_2 = 0 \right\}$$

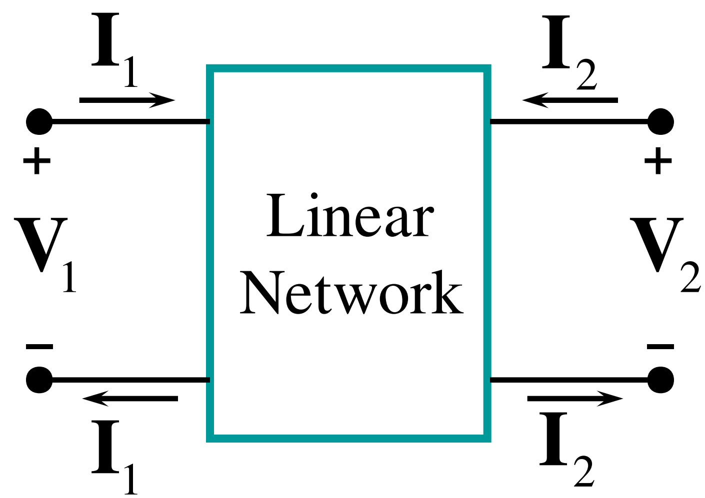


$$\rightarrow \begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 \end{cases} \rightarrow \begin{cases} \mathbf{z}_{11} = \left.\dfrac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{z}_{21} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \end{cases}$$

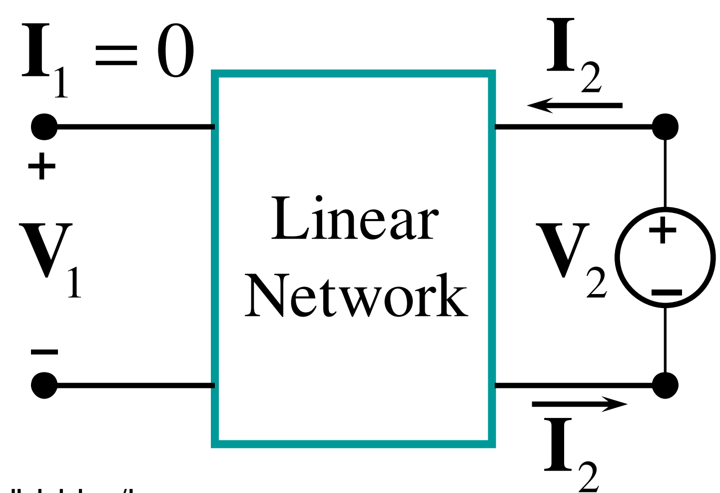


Impedance Parameters (3)

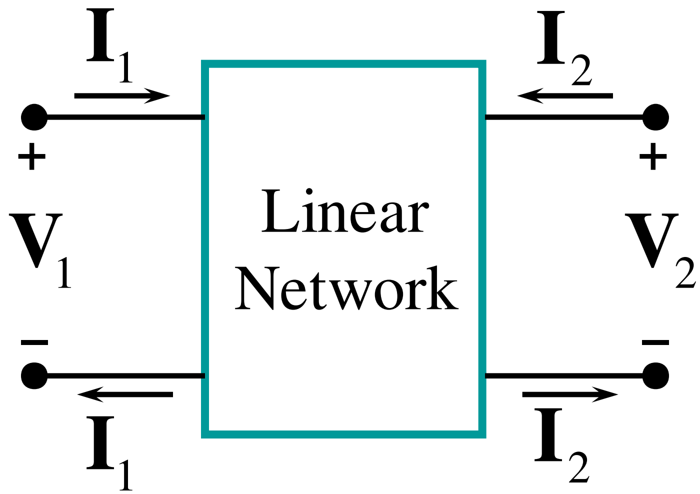
$$\left.\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \\ \mathbf{I}_1 = 0 \right\}$$



$$\to \begin{cases} \mathbf{V}_1 = \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \to \begin{cases} \mathbf{z}_{12} = \left.\dfrac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{22} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{cases}$$



Impedance Parameters (4)



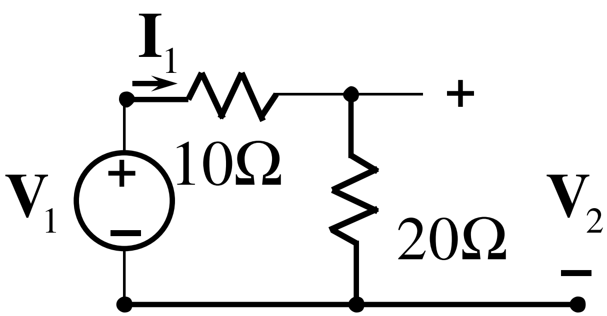
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

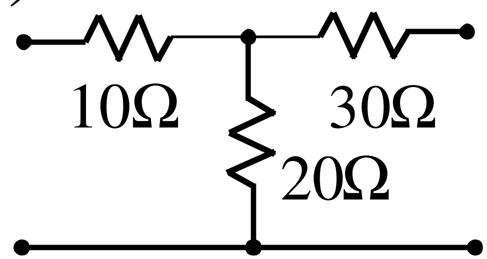
$$\begin{bmatrix} \mathbf{z}_{11} = \left.\dfrac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} & \mathbf{z}_{12} = \left.\dfrac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{21} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} & \mathbf{z}_{22} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{bmatrix}$$

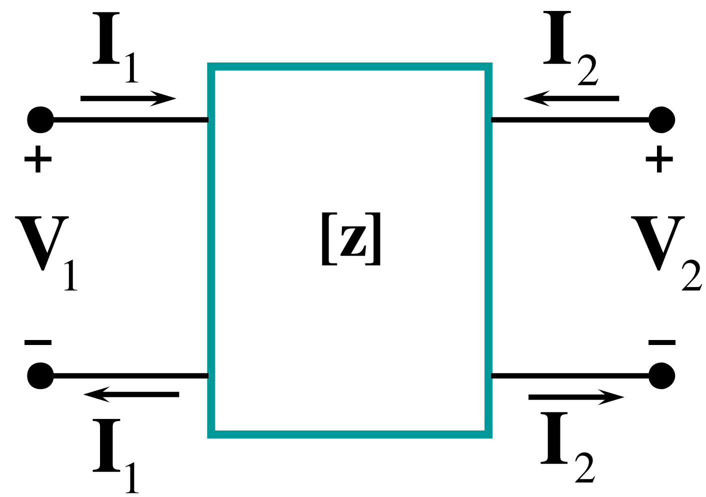
Impedance Parameters (5)

Ex.

Find [z]?







$$\left.\begin{aligned} \mathbf{z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{V}_1 &= (10+20)\mathbf{I}_1 = 30\mathbf{I}_1 \end{aligned}\right\}$$

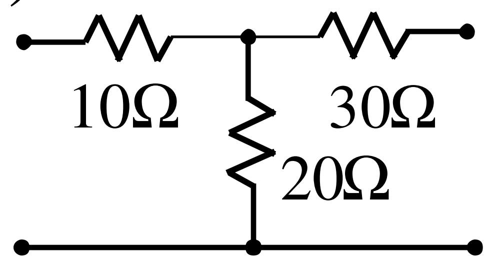
$$\rightarrow \mathbf{z}_{11} = \frac{30\mathbf{I}_1}{\mathbf{I}_1} = \boxed{30\,\Omega}$$

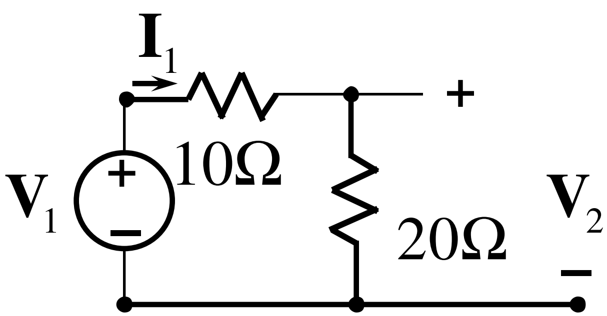
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

Impedance Parameters (6)

Ex. 1

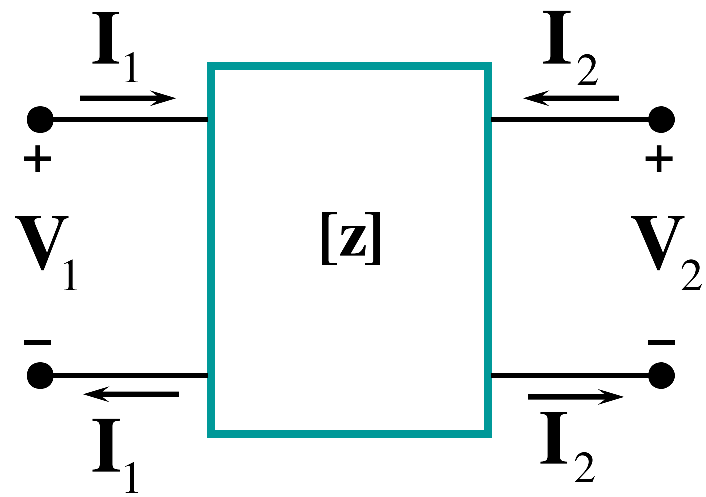
Find [z]?





$$\left.\begin{array}{l} \mathbf{z}_{21} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{V}_2 = 20\mathbf{I}_1 \end{array}\right\}$$

$$\rightarrow \mathbf{z}_{21} = \frac{20\mathbf{I}_1}{\mathbf{I}_1} = \boxed{20\Omega}$$

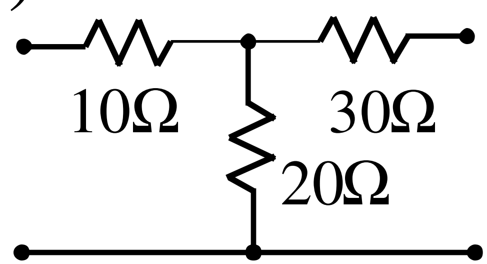


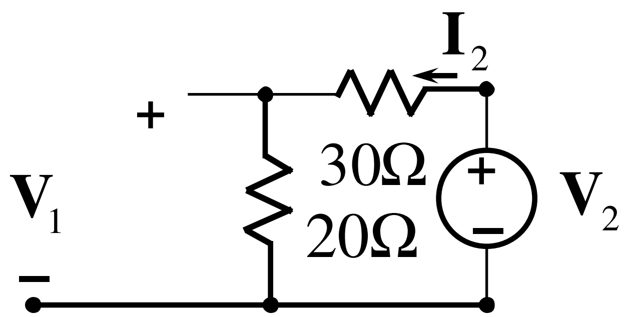
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

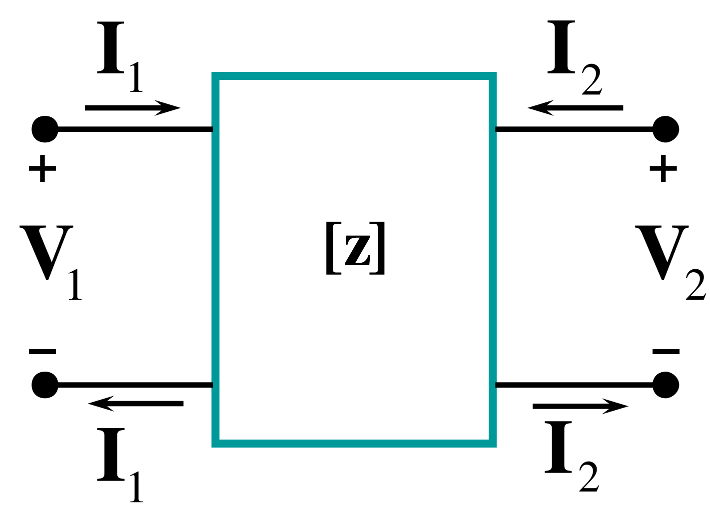
Impedance Parameters (7)

Ex. 1

Find [z]?







$$\left.\begin{aligned} \mathbf{z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{V}_1 &= 20\mathbf{I}_2 \end{aligned}\right\}$$

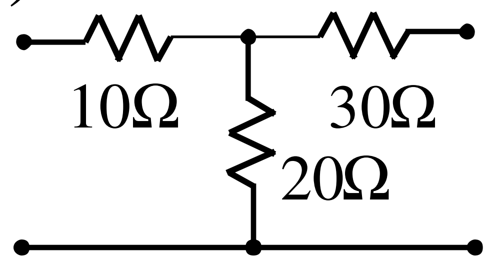
$$\rightarrow \mathbf{z}_{12} = \frac{20\mathbf{I}_2}{\mathbf{I}_2} = \boxed{20\,\Omega}$$

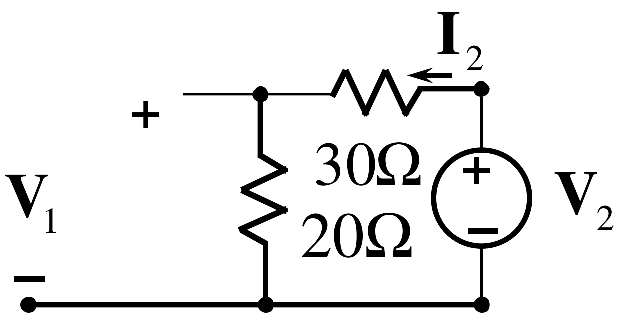
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

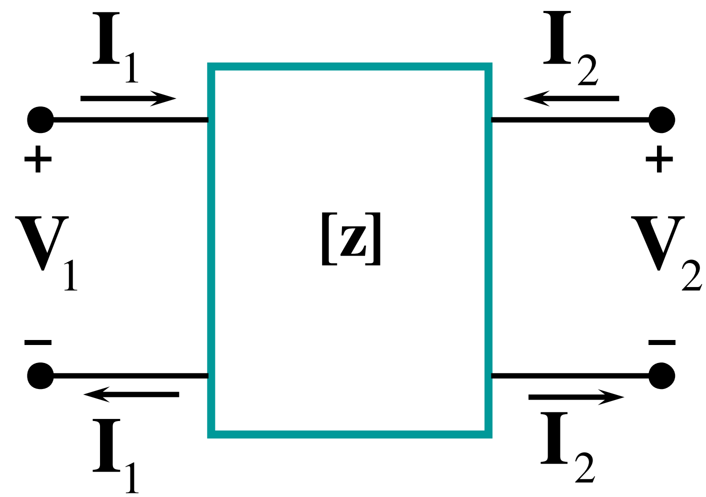
Impedance Parameters (8)

Ex. 1

Find [z]?







$$\left.\begin{array}{l} \mathbf{z}_{22} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{V}_2 = (20+30)\mathbf{I}_2 = 50\mathbf{I}_2 \end{array}\right\}$$

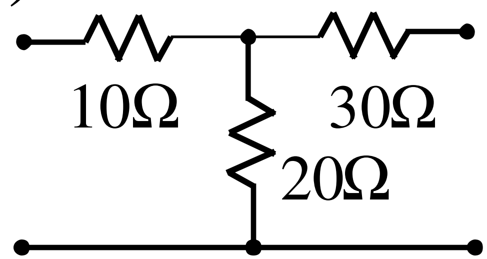
$$\to \mathbf{z}_{22} = \frac{50\mathbf{I}_2}{\mathbf{I}_2} = \boxed{50\,\Omega}$$

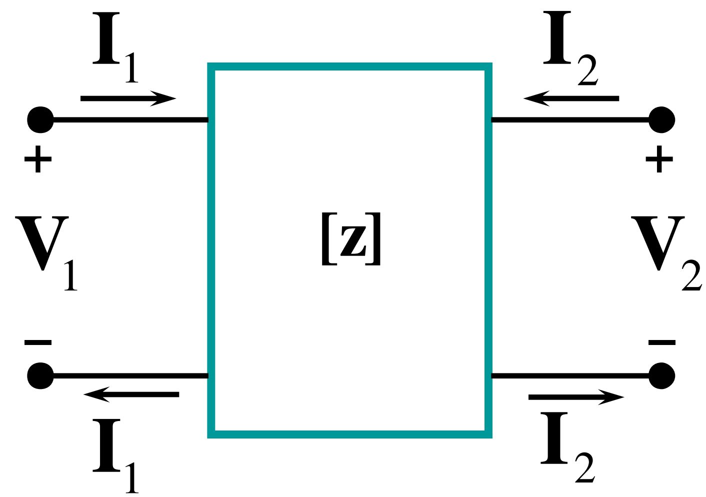
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

Impedance Parameters (9)

Ex. 1

Find [z]?



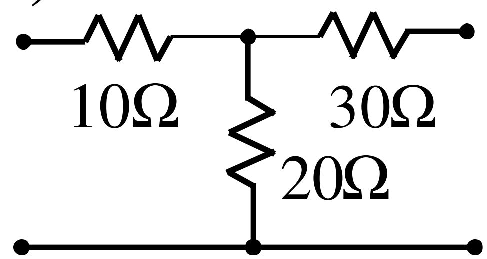


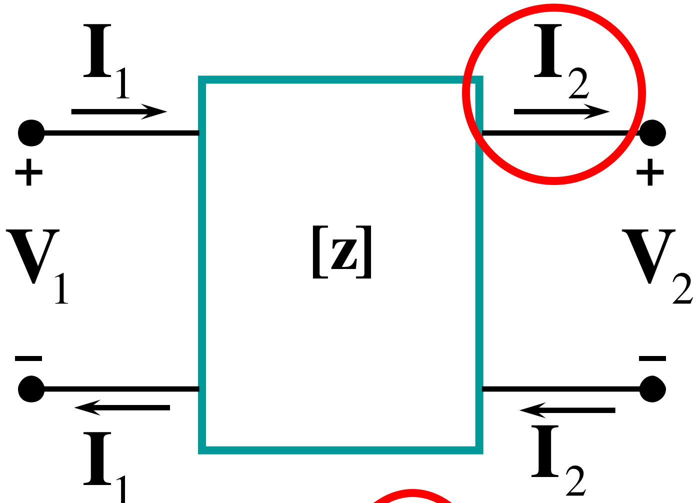
$$\boxed{\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix} \Omega}$$

Impedance Parameters (10)

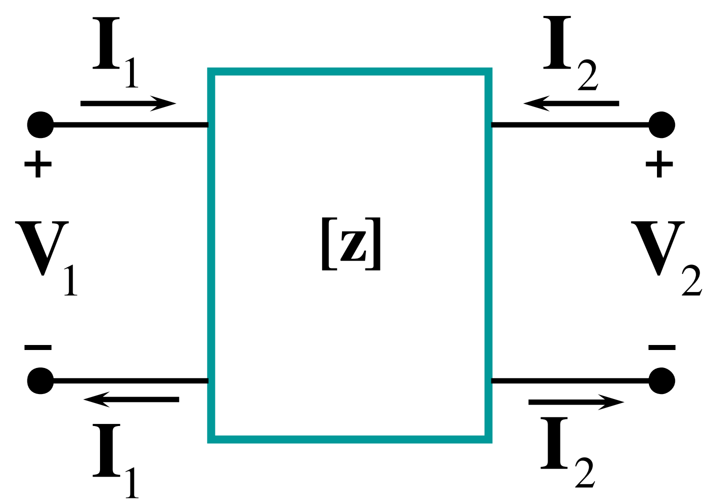
Ex. 1

Find [z]?





$$\mathbf{z} = \begin{bmatrix} 30 & -20 \\ 20 & -50 \end{bmatrix} \Omega$$



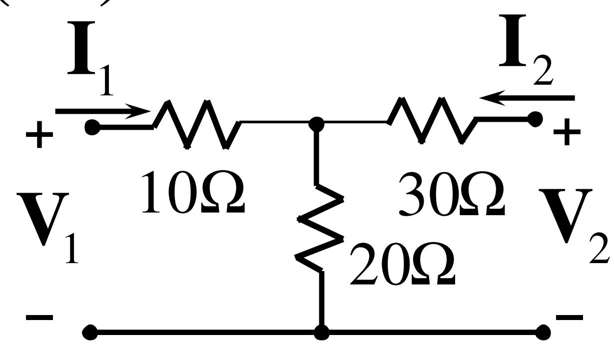
$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix} \Omega$$

Impedance Parameters (11)

Ex. 1

Find [z]?

Method 2



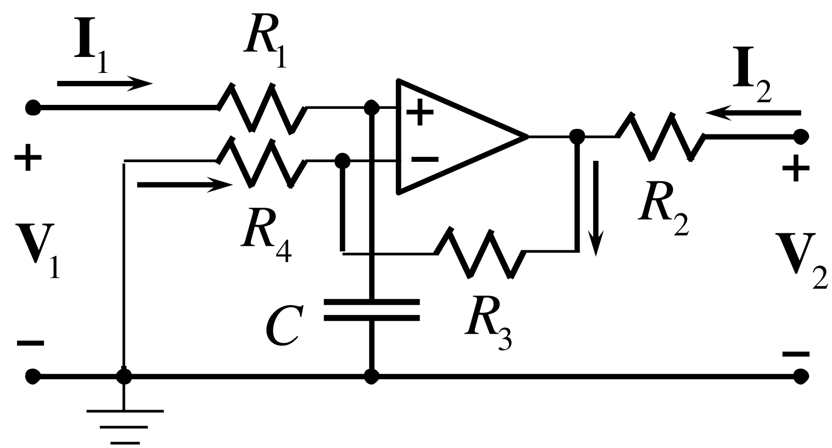
$$\left.\begin{array}{l} \mathbf{V}_1 = \mathbf{V}_{10} + \mathbf{V}_{20} \quad = 10\mathbf{I}_1 + 20(\mathbf{I}_1 + \mathbf{I}_2) \quad = (10+20)\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{V}_{30} + \mathbf{V}_{20} \quad = 30\mathbf{I}_2 + 20(\mathbf{I}_1 + \mathbf{I}_2) \quad = 20\mathbf{I}_1 + (20+30)\mathbf{I}_2 \end{array}\right\}$$

$$\left.\begin{array}{l} \to \begin{cases} \mathbf{V}_1 = \boxed{(10+20)}\mathbf{I}_1 + \boxed{20}\mathbf{I}_2 \\ \mathbf{V}_2 = \boxed{20}\mathbf{I}_1 + \boxed{(20+30)}\mathbf{I}_2 \end{cases} \\ \begin{cases} \mathbf{V}_1 = \boxed{\mathbf{z}_{11}}\mathbf{I}_1 + \boxed{\mathbf{z}_{12}}\mathbf{I}_2 \\ \mathbf{V}_2 = \boxed{\mathbf{z}_{21}}\mathbf{I}_1 + \boxed{\mathbf{z}_{22}}\mathbf{I}_2 \end{cases} \end{array}\right\} \to \begin{cases} \boxed{\mathbf{z}_{11} = 10 + 20} = 30\Omega \\ \boxed{\mathbf{z}_{12} = 20} = 20\Omega \\ \boxed{\mathbf{z}_{21} = 20} = 20\Omega \\ \boxed{\mathbf{z}_{22} = 20 + 30} = 50\Omega \end{cases}$$

Impedance Parameters (12)

Ex. 2

Find [z]?



$$\begin{cases} \mathbf{V}_1 = R_1\mathbf{I}_1 + \dfrac{1}{Cs}\mathbf{I}_1 \\ \mathbf{V}_2 = R_2\mathbf{I}_2 + R_3\mathbf{I}_3 - R_4\mathbf{I}_4 \\ \mathbf{I}_3 = -\mathbf{I}_4 \\ \dfrac{1}{Cs}\mathbf{I}_1 = -R_4\mathbf{I}_4 \end{cases}$$

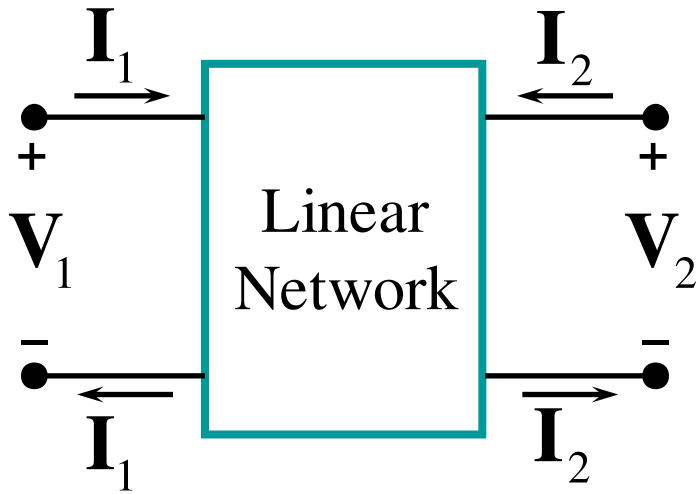
$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{V}_1 = \left(R_1 + \dfrac{1}{Cs}\right)\mathbf{I}_1 \\ \mathbf{V}_2 = \dfrac{R_3 + R_4}{R_4 Cs}\mathbf{I}_1 + R_2\mathbf{I}_2 \end{cases} \qquad \rightarrow [\mathbf{z}] = \begin{bmatrix} R_1 + \dfrac{1}{Cs} & 0 \\ \dfrac{R_3 + R_4}{R_4 Cs} & R_2 \end{bmatrix}$$

Two-port Networks

1. Introduction
2. **Parameters**
   a) Impedance **z**
   b) Admittance **y**
   c) Hybrid **h**
   d) Inverse Hybrid **g**
   e) Transmission **T**
   f) Inverse Transmission **t**
3. Relationships between Parameters
4. Two-port Network Analysis
5. Interconnection of Networks
6. T & Π Networks
7. Equivalent Two-port Networks of Magnetically Coupled Circuits
8. Input Impedance
9. Transfer Function

Admittance Parameters (1)

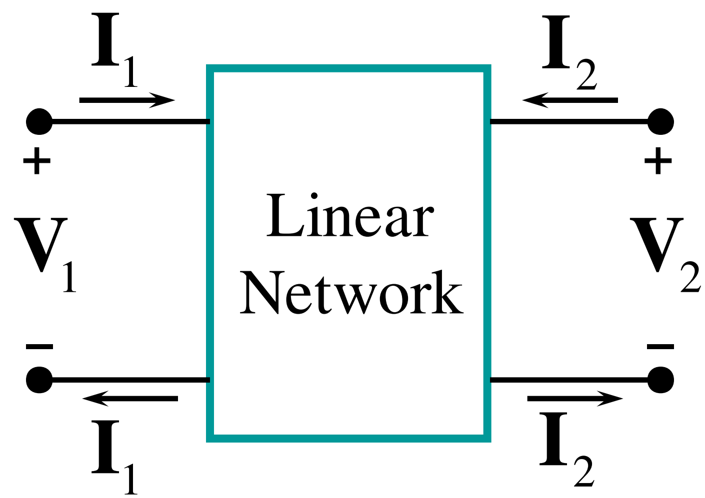


$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$$

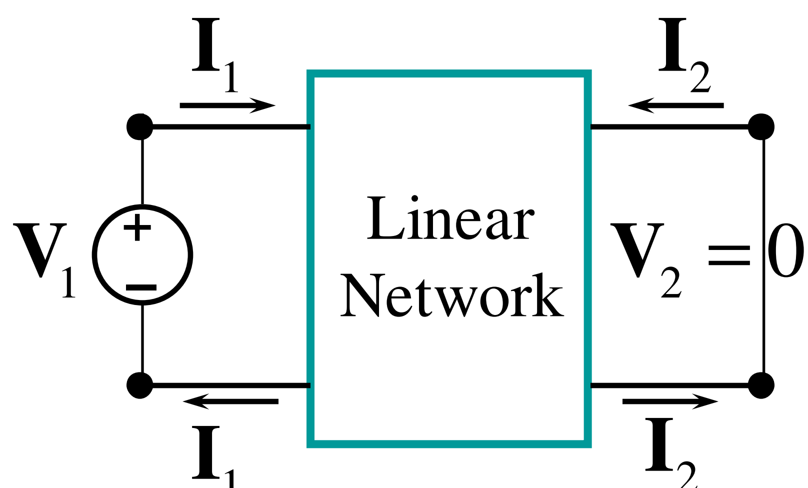
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Admittance Parameters (2)

$$\left\{\begin{array}{l} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{array}\right. \quad \left.\begin{array}{r} \\ \\ \mathbf{V}_2 = 0 \end{array}\right\}$$

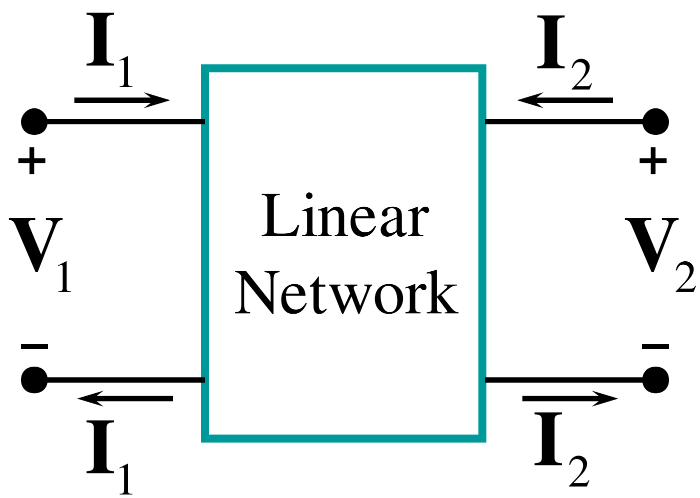


$$\rightarrow \left\{\begin{array}{l} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 \end{array}\right. \rightarrow \left\{\begin{array}{l} \mathbf{y}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \\ \\ \mathbf{y}_{21} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \end{array}\right.$$

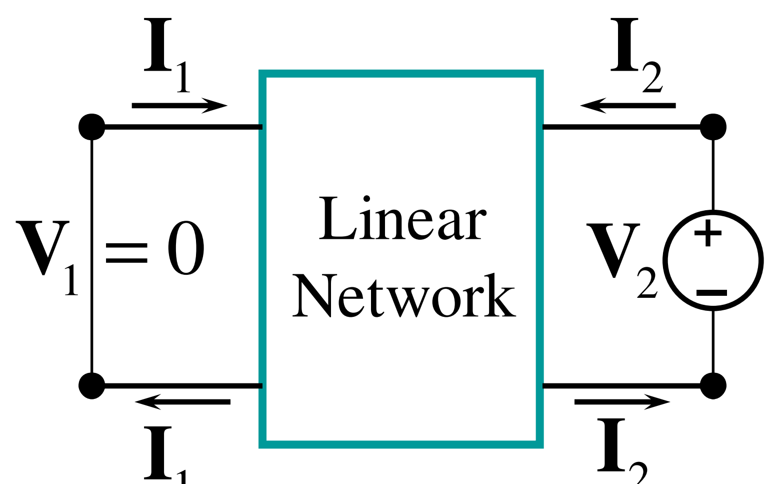


Admittance Parameters (3)

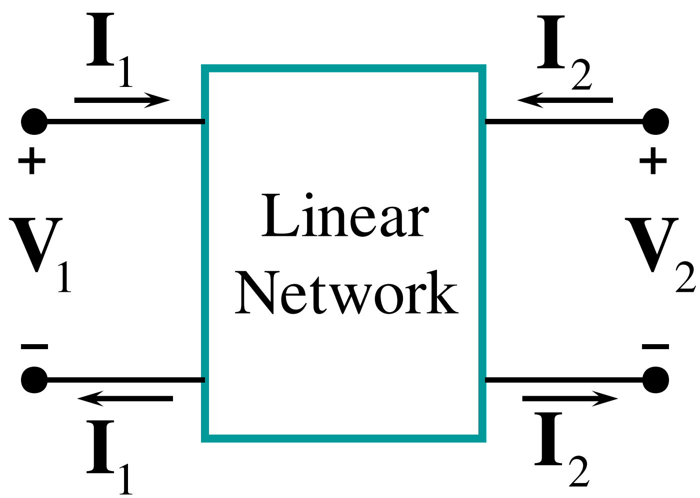
$$\left.\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases} \\ \mathbf{V}_1 = 0 \right\}$$



$$\rightarrow \begin{cases} \mathbf{I}_1 = \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{22}\mathbf{V}_2 \end{cases} \rightarrow \begin{cases} \mathbf{y}_{12} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \\ \mathbf{y}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \end{cases}$$



Admittance Parameters (4)



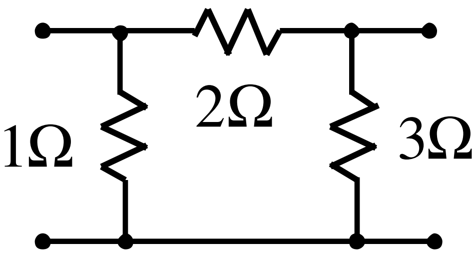
$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$$

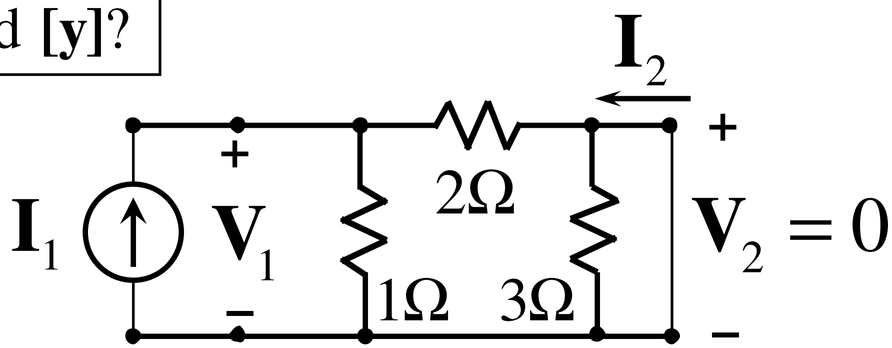
$$\begin{bmatrix} \mathbf{y}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} & \mathbf{y}_{12} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \\ \mathbf{y}_{21} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} & \mathbf{y}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \end{bmatrix}$$

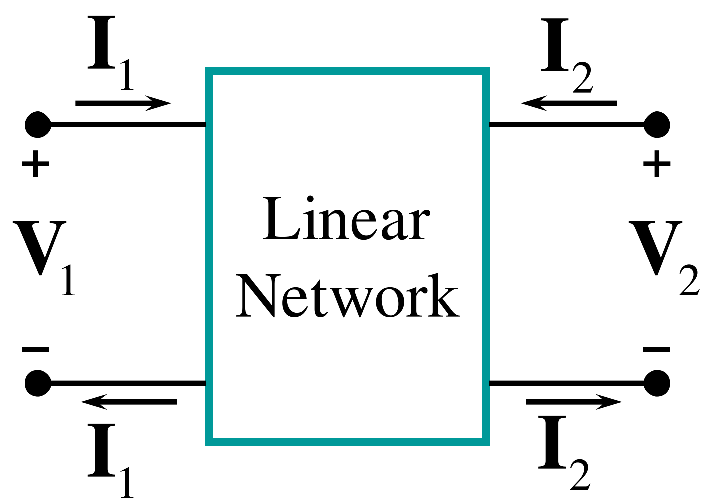
Admittance Parameters (5)

Ex.

Find [y]?







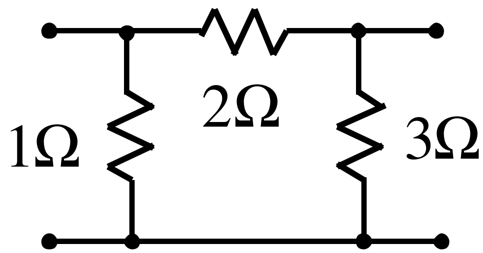
$$\left.\begin{array}{l} \mathbf{y}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \\ \mathbf{V}_1 = (1 // 2)\mathbf{I}_1 = \dfrac{1\times 2}{1+2}\mathbf{I}_1 = 0.67\mathbf{I}_1 \end{array}\right\}$$

$$\rightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{0.67\mathbf{I}_1} = \boxed{1.5\,\mathrm{S}}$$

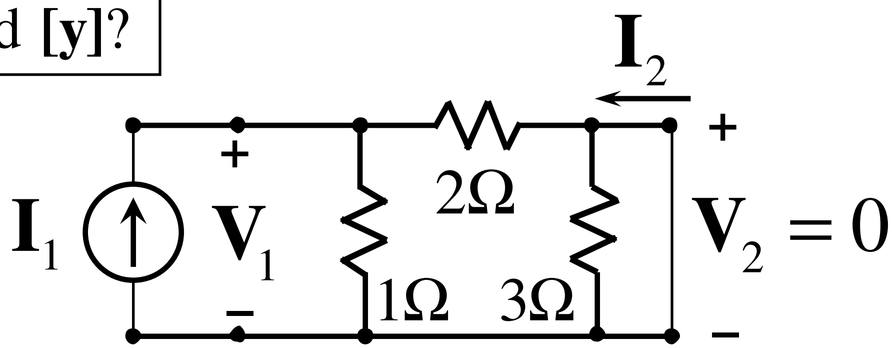
$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$$

Admittance Parameters (6)

Ex.



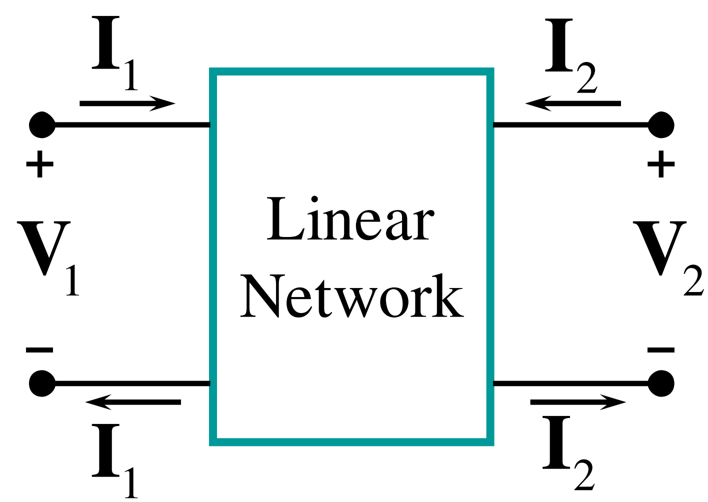
Find [y]?



$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{V}_1 = \mathbf{V}_{1\Omega} = \mathbf{V}_{2\Omega} = -2\mathbf{I}_2$$

$$\rightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{-2\mathbf{I}_2} = \boxed{-0.5\text{S}}$$

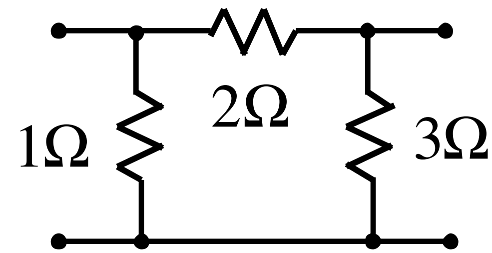


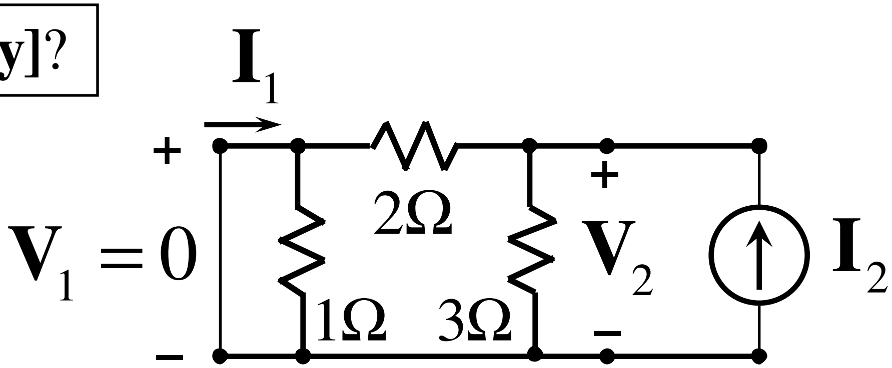
$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$$

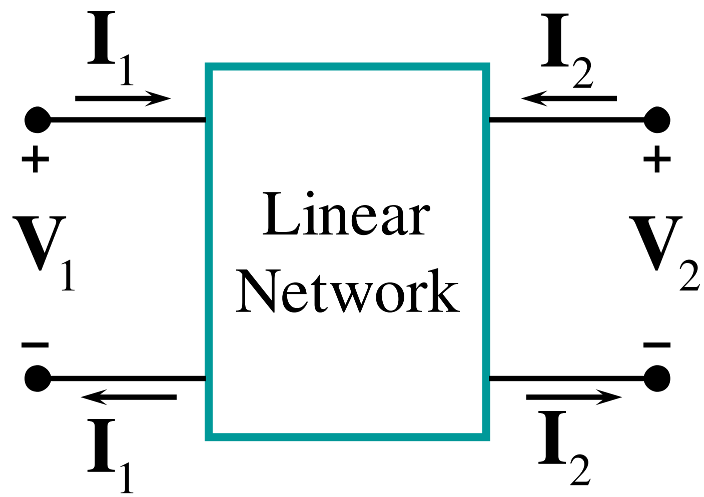
Admittance Parameters (7)

Ex.

Find [y]?







$$\left.\begin{aligned} y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ V_2 &= V_{3\Omega} = V_{2\Omega} = -2I_1 \end{aligned}\right\}$$

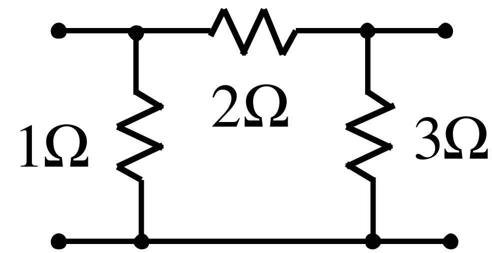
$$\rightarrow y_{12} = \frac{I_1}{-2I_1} = \boxed{-0.5\text{S}}$$

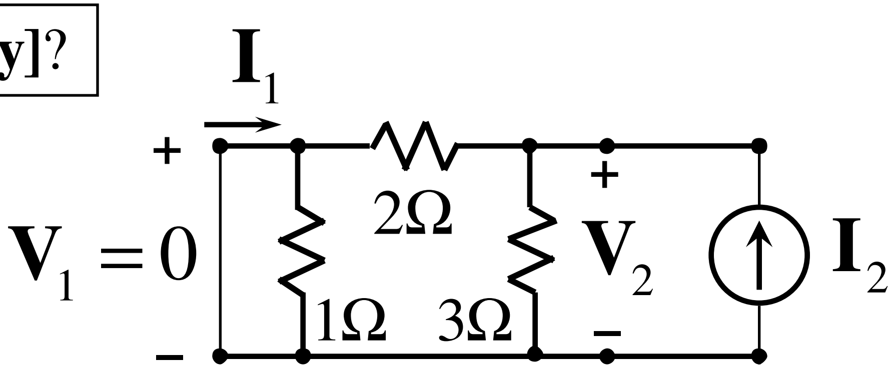
$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases}$$

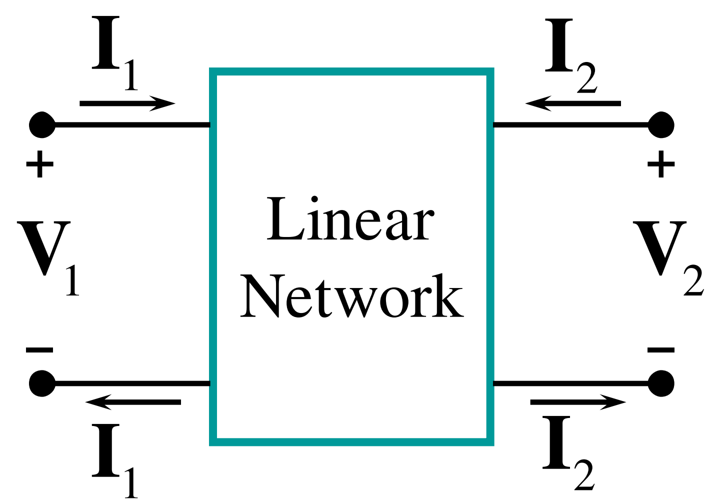
Admittance Parameters (8)

Ex.

Find [y]?







$$\left.\begin{aligned} \mathbf{y}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \\ \mathbf{V}_2 &= (2 // 3)\mathbf{I}_2 = \frac{2\times 3}{2+3}\mathbf{V}_2 = 1.2\mathbf{V}_2 \end{aligned}\right\}$$

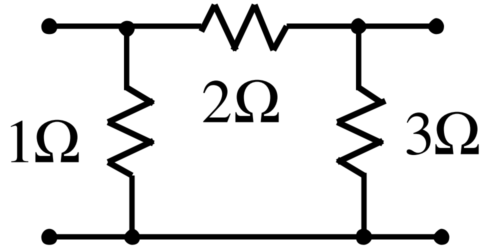
$$\rightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{1.2\mathbf{I}_2} = \boxed{0.83\text{S}}$$

$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$$

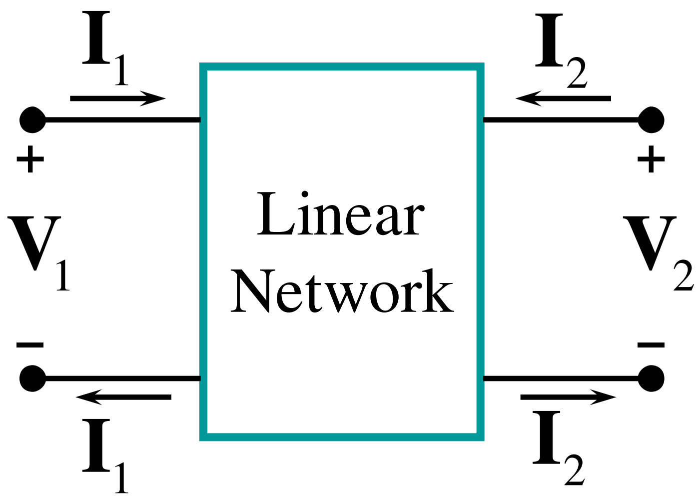
Admittance Parameters (9)

Ex.

Find [y]?



$$\mathbf{y} = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 0.83 \end{bmatrix}$$



Two-port Networks

1. Introduction
2. **Parameters**
   a) Impedance **z**
   b) Admittance **y**
   c) Hybrid **h**
   d) Inverse Hybrid **g**
   e) Transmission **T**
   f) Inverse Transmission **t**
3. Relationships between Parameters
4. Two-port Network Analysis
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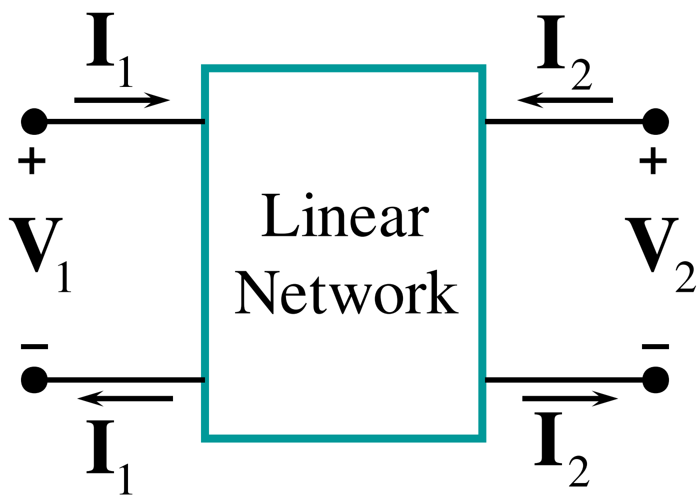
Hybrid Parameters

$$\begin{cases} \mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{cases}$$

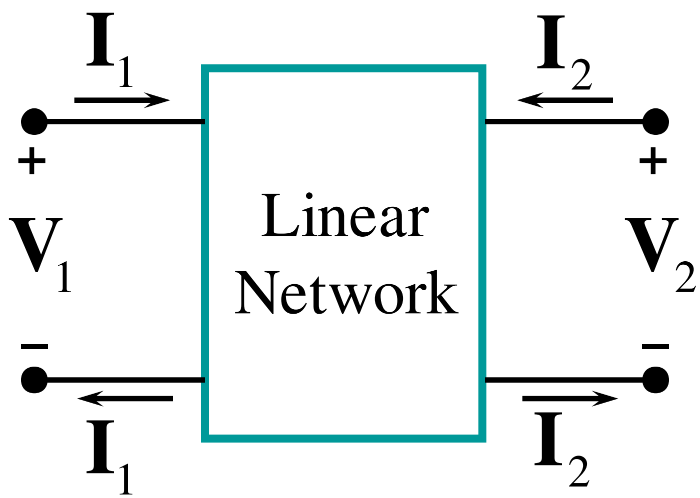


$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{h}_{11} = \left.\dfrac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} & \mathbf{h}_{12} = \left.\dfrac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{h}_{21} = \left.\dfrac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} & \mathbf{h}_{22} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \end{bmatrix}$$

Inverse Hybrid Parameters

$$\begin{cases} \mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{cases}$$

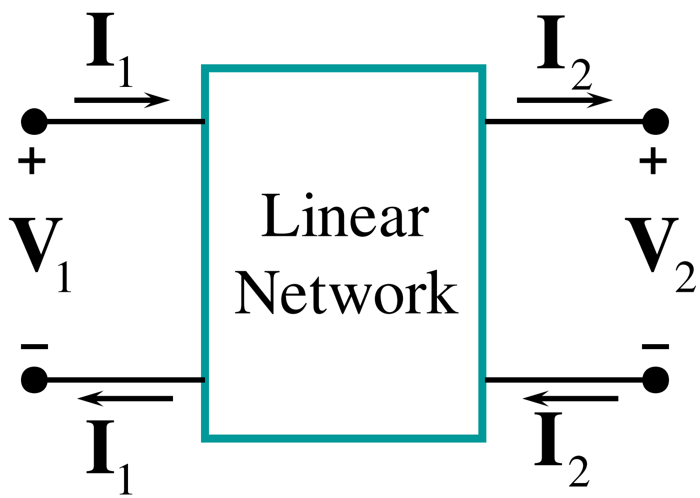


$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{g}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{g}_{11} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0} & \mathbf{g}_{12} = \left.\dfrac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0} \\ \mathbf{g}_{21} = \left.\dfrac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0} & \mathbf{g}_{22} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0} \end{bmatrix}$$

Transmission Parameters

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$

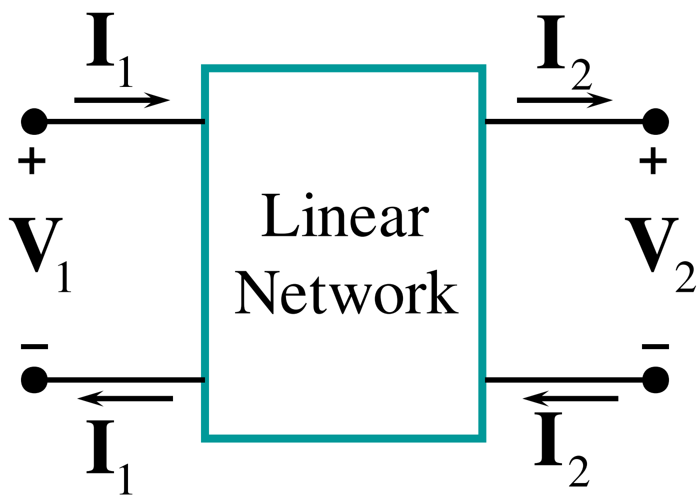


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} & B = \left.\dfrac{V_1}{I_2}\right|_{V_2=0} \\ C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} & D = \left.\dfrac{I_1}{I_2}\right|_{V_2=0} \end{bmatrix}$$

Inverse Transmission Parameters

$$\begin{cases} \mathbf{V}_2 = \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1 \\ \mathbf{I}_2 = \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1 \end{cases}$$



$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{a} = \left.\dfrac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0} & \mathbf{b} = \left.\dfrac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \\ \mathbf{c} = \left.\dfrac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0} & \mathbf{d} = \left.\dfrac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \end{bmatrix}$$

Two-port Networks

$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$	$\begin{cases} \mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{cases}$	$\begin{cases} \mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{cases}$
$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{cases}$	$\begin{cases} \mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{cases}$	$\begin{cases} \mathbf{V}_2 = \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1 \\ \mathbf{I}_2 = \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1 \end{cases}$

Two-port Networks

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Relationships between Parameters (1)

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\left. \begin{aligned} &\rightarrow \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}]^{-1} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \\ &\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \end{aligned} \right\} \rightarrow [\mathbf{y}] = [\mathbf{z}]^{-1}$$

Relationships between Parameters (2)

$$[\mathbf{y}] = [\mathbf{z}]^{-1}$$

$$[\mathbf{g}] = [\mathbf{h}]^{-1}$$

$$[\mathbf{t}] = [\mathbf{T}]^{-1}$$

Relationships between Parameters (3)

$$\begin{cases} \mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{cases} \rightarrow \mathbf{V}_2 = -\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{h}_{22}}\mathbf{I}_2$$

$$\rightarrow \mathbf{V}_1 = \left(\mathbf{h}_{11} - \frac{\mathbf{h}_{12}\mathbf{h}_{21}}{\mathbf{h}_{22}}\right)\mathbf{I}_1 + \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}\mathbf{I}_2$$

$$\rightarrow \begin{cases} \mathbf{V}_1 = \left(\mathbf{h}_{11} - \frac{\mathbf{h}_{12}\mathbf{h}_{21}}{\mathbf{h}_{22}}\right)\mathbf{I}_1 + \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}\mathbf{I}_2 \\ \mathbf{V}_2 = -\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{h}_{22}}\mathbf{I}_2 \end{cases} \rightarrow \begin{bmatrix} \mathbf{z}_{11} = \mathbf{h}_{11} - \frac{\mathbf{h}_{12}\mathbf{h}_{21}}{\mathbf{h}_{22}} & \mathbf{z}_{12} = \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \mathbf{z}_{21} = -\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} & \mathbf{z}_{22} = \frac{1}{\mathbf{h}_{22}} \end{bmatrix}$$

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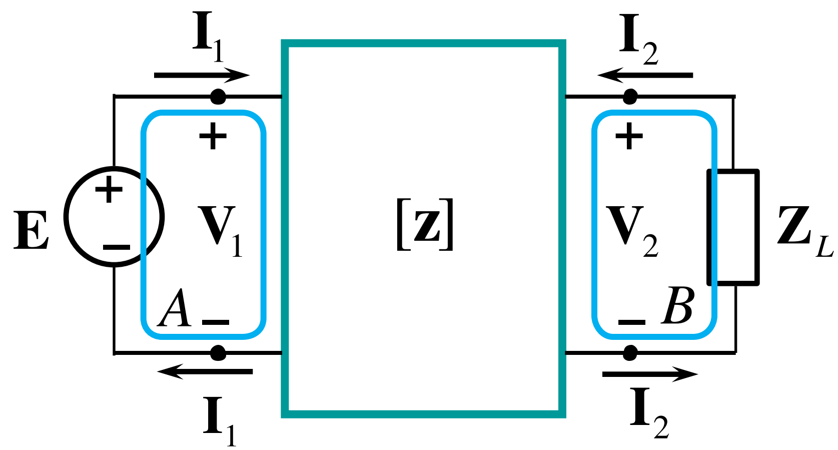
Ex. 1 Two-port Network Analysis (1)

$\mathbf{E} = 220\angle 0^\circ$ V; $\mathbf{Z}_L = j50\,\Omega$;

$$\mathbf{z} = \begin{bmatrix} 10 & j20 \\ j20 & 40 \end{bmatrix};\text{find currents?}$$

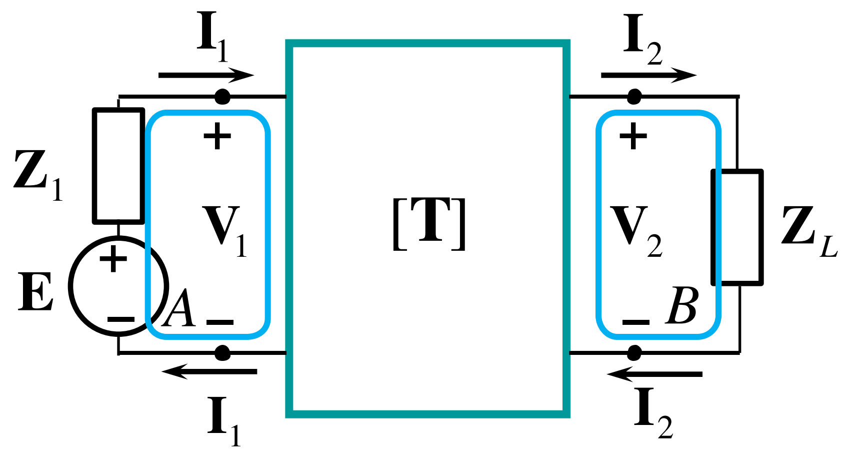


$$\left.\begin{aligned} &\begin{cases} \mathbf{V}_1 = 10\mathbf{I}_1 + j20\mathbf{I}_2 \\ \mathbf{V}_2 = j20\mathbf{I}_1 + 40\mathbf{I}_2 \end{cases} \\ &\mathbf{V}_1 = \mathbf{E} = 220\angle 0^\circ \text{ V} \\ &\mathbf{V}_2 = -\mathbf{Z}_L\mathbf{I}_2 = -j50\mathbf{I}_2 \end{aligned}\right\} \to \begin{cases} 220\angle 0^0 = 10\mathbf{I}_1 + j20\mathbf{I}_2 \\ -j50\mathbf{I}_2 = j20\mathbf{I}_1 + 40\mathbf{I}_2 \end{cases} \to \begin{cases} \boxed{\mathbf{I}_1 = 14.09 + j4.94 \text{ A}} \\ \boxed{\mathbf{I}_2 = -2.47 - j3.96 \text{ A}} \end{cases}$$

Two-port Network Analysis (2)

Ex. 2

$$\mathbf{E} = 220\,\text{V}; \quad \mathbf{Z}_1 = 20\,\Omega; \; \mathbf{Z}_L = j50\;\Omega; \quad \mathbf{T} = \begin{bmatrix} 3 & -200 \\ 0.04 & -3 \end{bmatrix}.$$

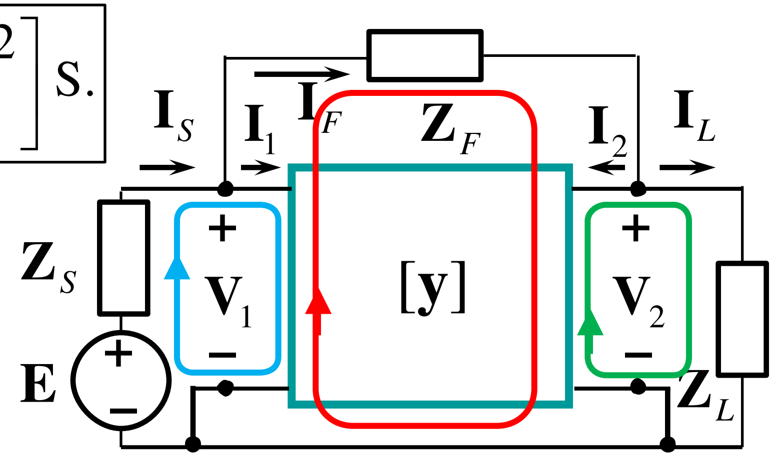


$$\begin{cases} \begin{cases} \mathbf{V}_1 = 3\mathbf{V}_2 + 200\mathbf{I}_2 \\ \mathbf{I}_1 = 0.04\mathbf{V}_2 + 3\mathbf{I}_2 \end{cases} \\ \mathbf{Z}\mathbf{I}_1 + \mathbf{V}_1 = \mathbf{E} \\ \mathbf{Z}_L\mathbf{I}_2 - \mathbf{V}_2 = 0 \end{cases} \rightarrow \begin{cases} \mathbf{V}_1 = 3\mathbf{V}_2 + 200\mathbf{I}_2 \\ \mathbf{I}_1 = 0.04\mathbf{V}_2 + 3\mathbf{I}_2 \\ 20\mathbf{I}_1 + \mathbf{V}_1 = 220 \\ j50\mathbf{I}_2 - \mathbf{V}_2 = 0 \end{cases} \rightarrow \begin{cases} \boxed{\mathbf{I}_1 = 2.46 - j0.11\ \text{A}} \\ \boxed{\mathbf{I}_2 = 0.55 - j0.40\ \text{A}} \end{cases}$$

Two-port Network Analysis (3)

Ex. 3

$$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;\quad \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix} \text{S}.$$



Method 1

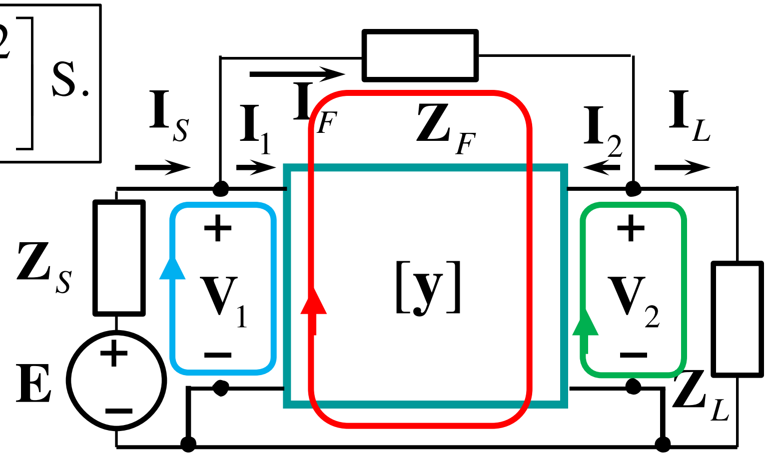
$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \\ \mathbf{I}_S - \mathbf{I}_1 - \mathbf{I}_F = 0 \\ \mathbf{I}_F - \mathbf{I}_2 - \mathbf{I}_L = 0 \\ \mathbf{Z}_S\mathbf{I}_S + \mathbf{V}_1 = \mathbf{E} \\ \mathbf{Z}_F\mathbf{I}_F - \mathbf{V}_1 + \mathbf{V}_2 = 0 \\ \mathbf{V}_2 - \mathbf{Z}_L\mathbf{I}_L = 0 \end{cases} \rightarrow \begin{cases} \mathbf{I}_S = 12.80 + j7.99\ \text{A} \\ \mathbf{I}_L = 7.20 + j10.40\ \text{A} \end{cases}$$

Two-port Network Analysis (4)

Ex. 3

$\mathbf{E} = 200\text{ V};\ \mathbf{Z}_n = 5\ \Omega;$
$\mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;$ $\mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix}$ S.

Method 2

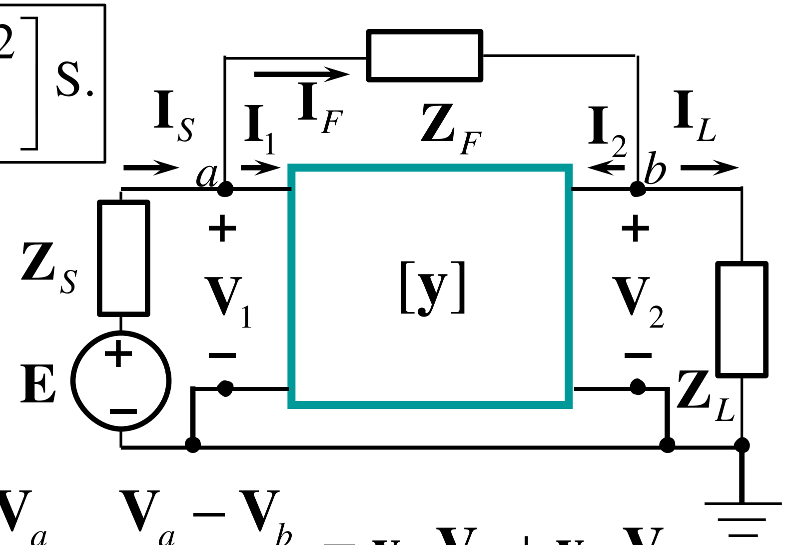


$$\begin{cases} \mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 = \mathbf{I}_b - \mathbf{I}_r \\ \mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 = \mathbf{I}_r - \mathbf{I}_g \\ \mathbf{Z}_S\mathbf{I}_b + \mathbf{V}_1 = \mathbf{E} \\ \mathbf{Z}_F\mathbf{I}_r - \mathbf{V}_1 + \mathbf{V}_2 = 0 \\ \mathbf{Z}_L\mathbf{I}_g - \mathbf{V}_2 = 0 \end{cases} \rightarrow \begin{cases} \mathbf{I}_b = 12.80 + j7.99\text{ A} = \mathbf{I}_S \\ \mathbf{I}_g = 7.20 + j10.40\text{ A} = \mathbf{I}_L \end{cases}$$

Two-port Network Analysis (5)

Ex. 3

$$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;\quad \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix} \text{S.}$$



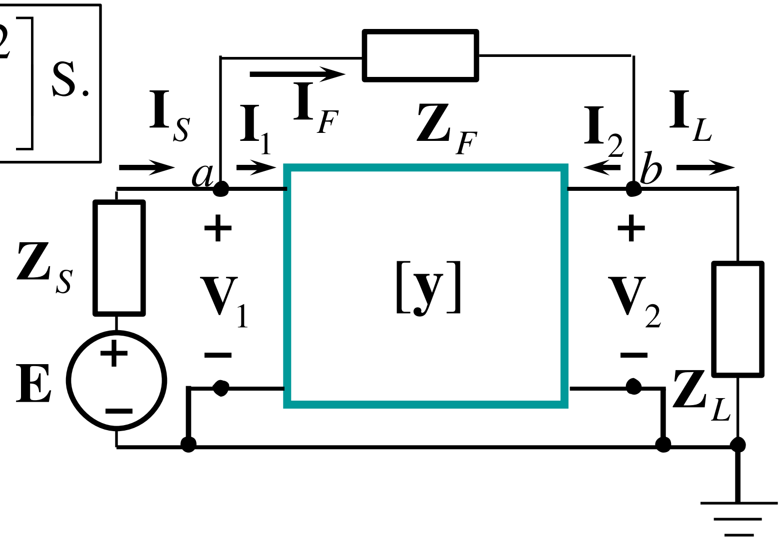
Method 3

$$\left.\begin{aligned}
\mathbf{I}_S - \mathbf{I}_1 - \mathbf{I}_F &= 0 \\
\mathbf{I}_F - \mathbf{I}_2 - \mathbf{I}_L &= 0 \\
\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 &= \mathbf{y}_{11}\mathbf{V}_a + \mathbf{y}_{12}\mathbf{V}_b \\
\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 &= \mathbf{y}_{21}\mathbf{V}_a + \mathbf{y}_{22}\mathbf{V}_b \\
\mathbf{I}_S &= \frac{\mathbf{E} - \mathbf{V}_a}{\mathbf{Z}_S} \\
\mathbf{I}_L &= \frac{\mathbf{V}_b}{\mathbf{Z}_L} \\
\mathbf{I}_F &= \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_F}
\end{aligned}\right\} \rightarrow \left\{\begin{aligned}
\frac{\mathbf{E} - \mathbf{V}_a}{\mathbf{Z}_S} - \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_F} &= \mathbf{y}_{11}\mathbf{V}_a + \mathbf{y}_{12}\mathbf{V}_b \\
\frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_F} - \frac{\mathbf{V}_b}{\mathbf{Z}_L} &= \mathbf{y}_{21}\mathbf{V}_a + \mathbf{y}_{22}\mathbf{V}_b
\end{aligned}\right.$$

Two-port Network Analysis (6)

Ex. 3

$$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;\quad \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix} \text{S}.$$



Method 3

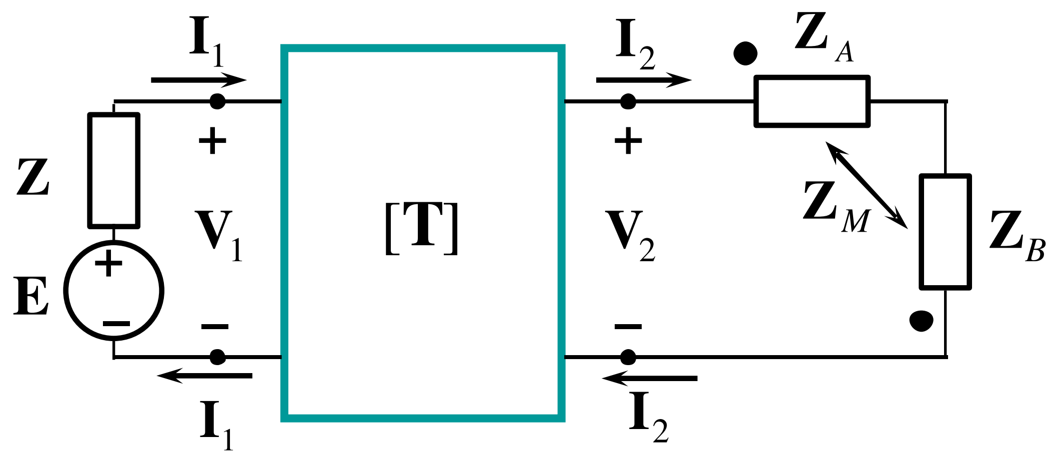
$$\begin{cases} \dfrac{\mathbf{E} - \mathbf{V}_a}{\mathbf{Z}_S} - \dfrac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_F} = \mathbf{y}_{11}\mathbf{V}_a + \mathbf{y}_{12}\mathbf{V}_b \\ \dfrac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_F} - \dfrac{\mathbf{V}_b}{\mathbf{Z}_L} = \mathbf{y}_{21}\mathbf{V}_a + \mathbf{y}_{22}\mathbf{V}_b \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{V}_a = 135.99 - j39.97\ \text{V} \\ \mathbf{V}_b = 207.92 - j143.97\ \text{V} \end{cases} \rightarrow \begin{cases} \mathbf{I}_S = \dfrac{\mathbf{E} - \mathbf{V}_a}{\mathbf{Z}_S} = 12.80 + j7.99\ \text{A} \\ \mathbf{I}_L = \dfrac{\mathbf{V}_b}{\mathbf{Z}_L} = 7.20 + j10.40\ \text{A} \end{cases}$$

Two-port Network Analysis (7)

Ex. 4

Write equations?



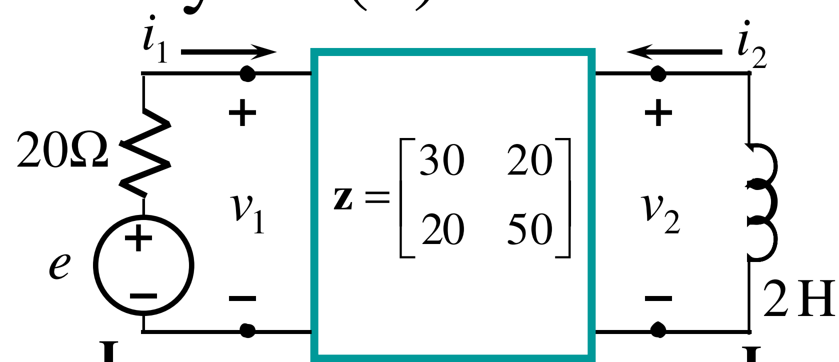
$$\begin{cases} \begin{cases} \mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{cases} \\ \mathbf{Z}\mathbf{I}_1 + \mathbf{V}_1 = \mathbf{E} \\ \mathbf{V}_2 = (\mathbf{Z}_A + \mathbf{Z}_B - 2\mathbf{Z}_M)\mathbf{I}_2 \end{cases}$$

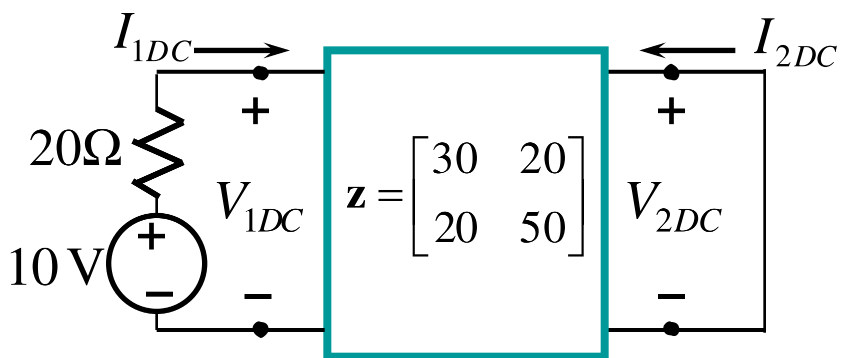
Two-port Network Analysis (8)

Ex. 5

$e = 10 + 20\cos 5t$ V. Find i_1?

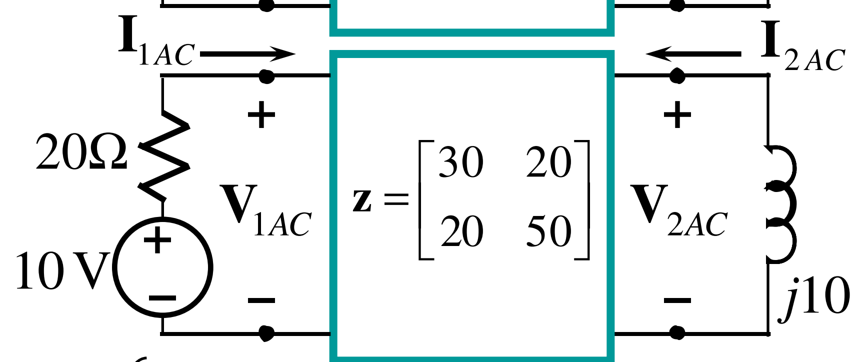
$\rightarrow \boxed{i_1(t) = 0.31 + 0.60\cos(5t - 4.76^\circ)\text{ A}}$





$$\begin{cases} \begin{cases} V_{1DC} = 30I_{1DC} + 20I_{2DC} \\ V_{2DC} = 20I_{1DC} + 50I_{2DC} \end{cases} \\ 10I_{1DC} + V_{1DC} = 10 \\ V_{2DC} = 0 \end{cases}$$

$\rightarrow I_{1DC} = 0.31\text{ A}$



$$\begin{cases} \begin{cases} \mathbf{V}_{1AC} = 30\mathbf{I}_{1AC} + 20\mathbf{I}_{2AC} \\ \mathbf{V}_{2AC} = 20\mathbf{I}_{1AC} + 50\mathbf{I}_{2AC} \end{cases} \\ 10\mathbf{I}_{1AC} + \mathbf{V}_{1AC} = 20 \\ \mathbf{V}_{2AC} + j20\mathbf{I}_{2AC} = 0 \end{cases}$$

$\rightarrow \mathbf{I}_{1AC} = 0.60\angle -4.76^\circ\text{ A}$

Two-port Network Analysis (9)

Ex. 6

Find $i_1(t)$?

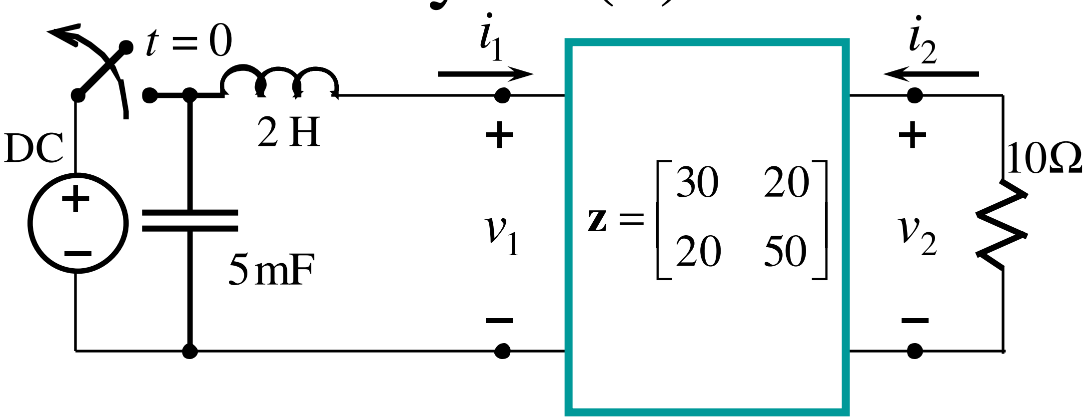
$v_C(0) = 12\text{ V}$

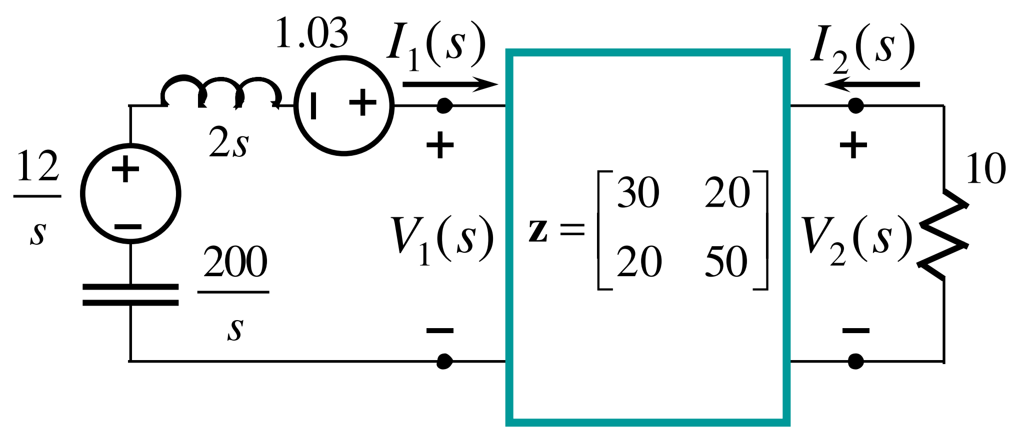
$$\begin{cases} v_1(0) = 30i_1(0) + 20i_2(0) \\ v_2(0) = 20i_1(0) + 50i_2(0) \\ v_1(0) = 12 \\ v_2(0) = -10i_2(0) \end{cases}$$

$\rightarrow i_1(0) = 0.5143\text{ A} = i_L(0)$

$$\begin{cases} V_1(s) = 30I_1(s) + 20I_2(s) \\ V_2(s) = 20I_1(s) + 50I_2(s) \\ \left(2s + \dfrac{200}{s}\right)I_1(s) + V_1(s) = 1.03 + \dfrac{12}{s} \\ V_2(s) = -10I_2(s) \end{cases}$$

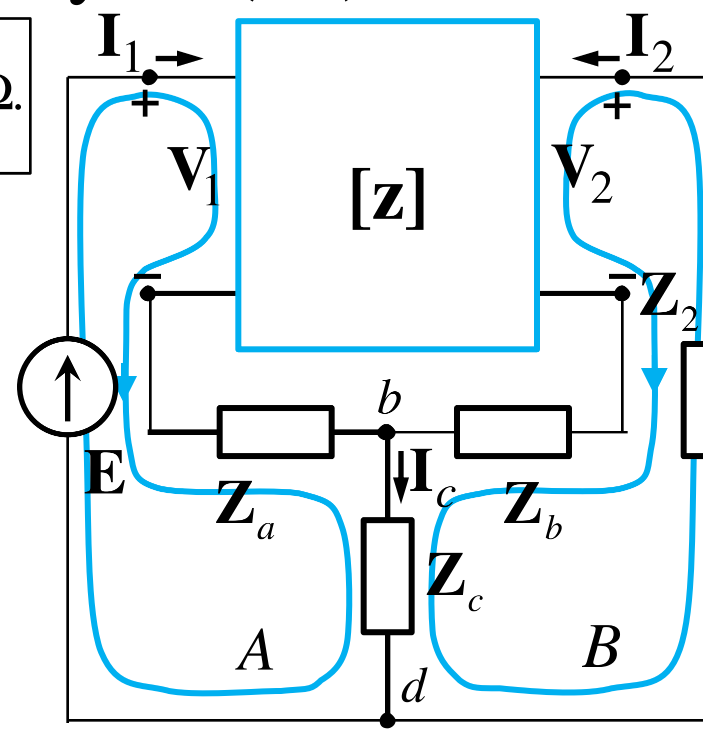
$\rightarrow I_1(s) = \dfrac{0.515s + 6}{s^2 + 11.667s + 100}\text{ A}$ $\rightarrow$ $\boxed{i_1(t) = 0.6334e^{-5.83t}\cos(8.12t - 35.6^\circ)\text{ A}}$





Ex. 7 Two-port Network Analysis (10)

$$\mathbf{E} = 220\text{V};\ \mathbf{Z}_2 = j10\ \Omega;\quad \mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix} \Omega.$$
$$\mathbf{Z}_a = j20\ \Omega;\ \mathbf{Z}_b = -j40\ \Omega;\ \mathbf{Z}_c = 5\ \Omega;$$



Method 1

$$\begin{cases} \begin{cases} \mathbf{V}_1 = 30\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = 20\mathbf{I}_1 + 50\mathbf{I}_2 \end{cases} \\ b:\ \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_c = 0 \\ A:\ \mathbf{V}_1 + \mathbf{Z}_a\mathbf{I}_1 + \mathbf{Z}_c\mathbf{I}_c = \mathbf{E} \\ B:\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 + \mathbf{Z}_b\mathbf{I}_2 + \mathbf{Z}_c\mathbf{I}_c = 0 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = 6.27 - j3.64\ \text{A} \\ \mathbf{I}_2 = -2.89 + j0.076\ \text{A} \\ \mathbf{I}_c = 3.38 - j3.56\ \text{A} \end{cases}$$

Method 2?

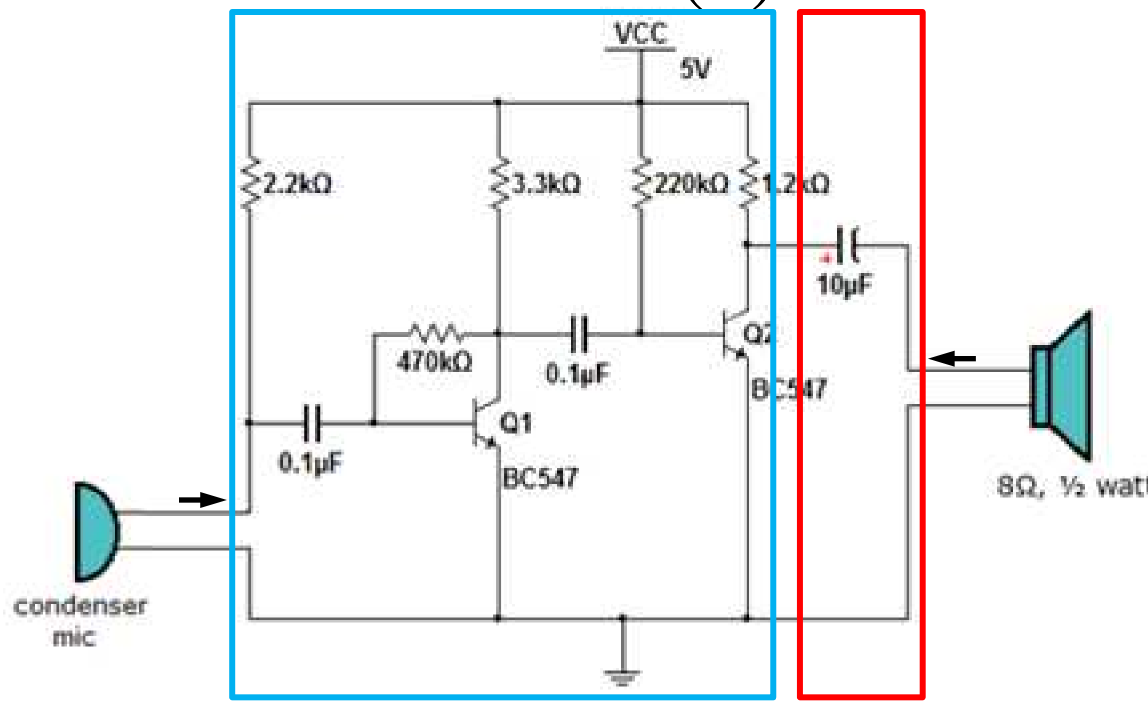
Interconnection of Networks

Two-port Networks

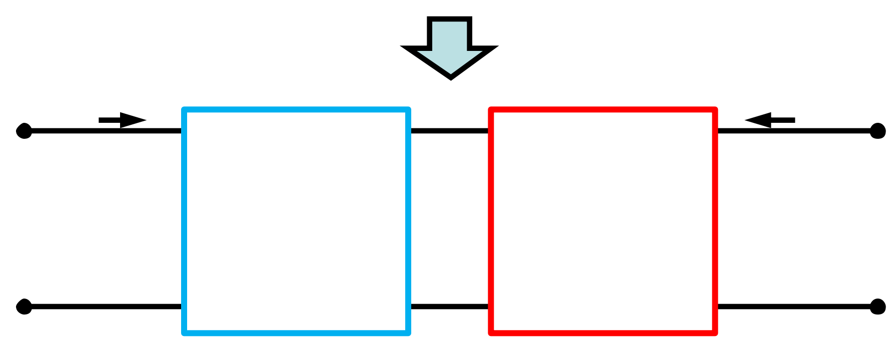
1. Introduction
2. Parameters
3. Relationships between Parameters
4. Two-port Network Analysis
5. **Interconnection of Networks**
6. T & Π Networks
7. Equivalent Two-port Networks of Magnetically Coupled Circuits
8. Input Impedance
9. Transfer Function

Interconnection of Networks (1)

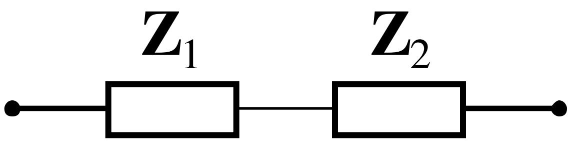
1. Series
2. Parallel
3. Cascade
4. Hybrid 1
5. Hybrid 2



https://www.efxkits.us/two-transistor-audio-amplifier-circuit-explanation/

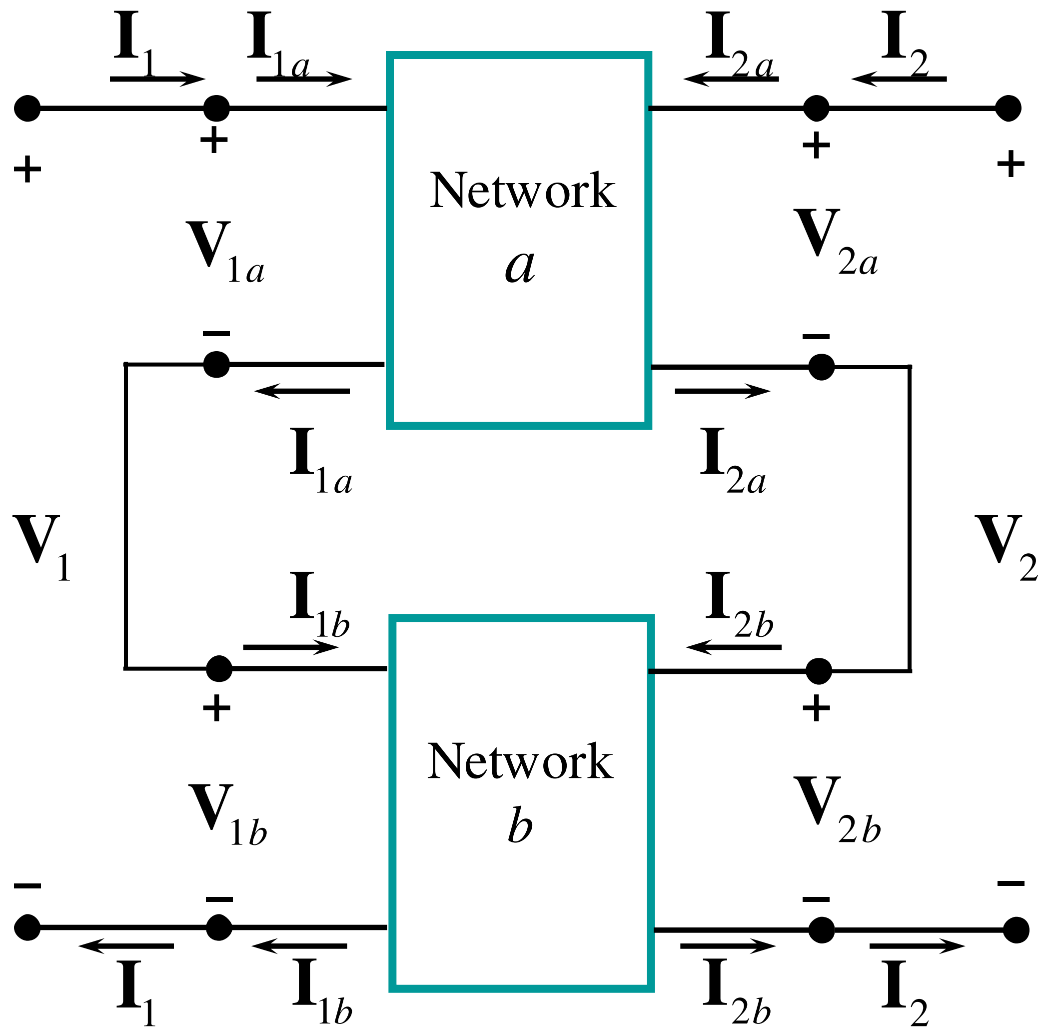


Interconnection of Networks (2), Series



$$\begin{cases} I = I_1 = I_2 \\ V = V_1 + V_2 \end{cases}$$

$$\begin{cases} I_1 = I_{1a} = I_{1b} \\ V_1 = V_{1a} + V_{1b} \\ I_2 = I_{2a} = I_{2b} \\ V_2 = V_{2a} + V_{2b} \end{cases}$$



Interconnection of Networks (3), Series

$$\begin{cases} \mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b} \\ \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \\ \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b} \\ \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \end{cases}$$

$\boxed{\text{Network } a\text{:}}$

$$\begin{cases} \mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a} \\ \mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a} \end{cases}$$

$\boxed{\text{Network } b\text{:}}$

$$\begin{cases} \mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b} \\ \mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b} \end{cases}$$

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$$

$$\rightarrow \begin{cases} \begin{cases} \mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_1 + \mathbf{z}_{12a}\mathbf{I}_2 \\ \mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_1 + \mathbf{z}_{22a}\mathbf{I}_2 \end{cases} \\ \begin{cases} \mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_1 + \mathbf{z}_{12b}\mathbf{I}_2 \\ \mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_1 + \mathbf{z}_{22b}\mathbf{I}_2 \end{cases} \end{cases}$$

Interconnection of Networks (4), Series

$$\begin{cases} \mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b} \\ \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \\ \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b} \\ \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \end{cases}$$

Network a: $\begin{cases} \mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_1 + \mathbf{z}_{12a}\mathbf{I}_2 \\ \mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_1 + \mathbf{z}_{22a}\mathbf{I}_2 \end{cases}$

Network b: $\begin{cases} \mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_1 + \mathbf{z}_{12b}\mathbf{I}_2 \\ \mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_1 + \mathbf{z}_{22b}\mathbf{I}_2 \end{cases}$

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b}$$

$\rightarrow$

$$\rightarrow \begin{cases} \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2 \end{cases}$$

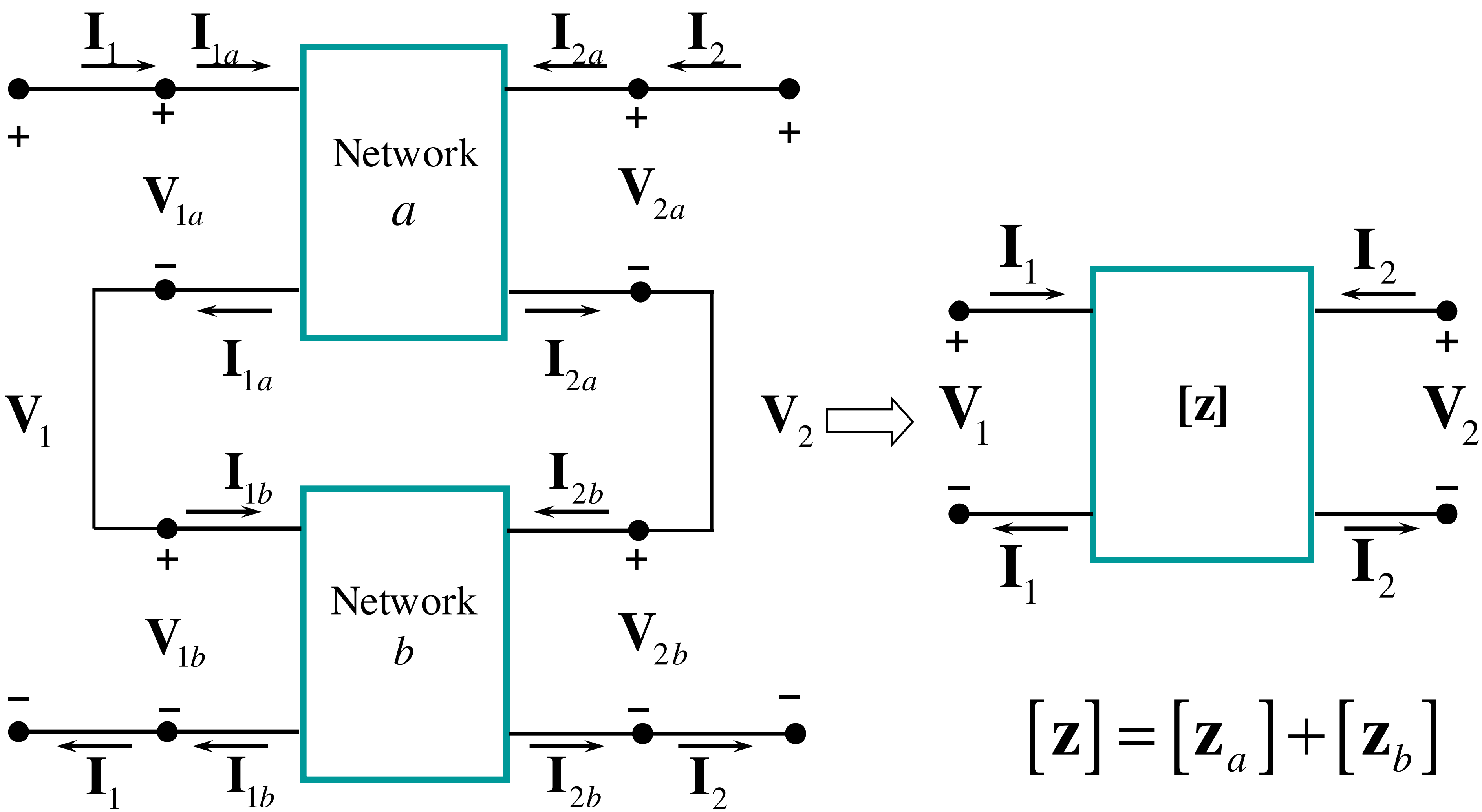
Interconnection of Networks (5), Series

$$\begin{cases} \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2 \end{cases}$$

$$\left.\begin{array}{l} \leftrightarrow \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \\ [\mathbf{z}_a] = \begin{bmatrix} \mathbf{z}_{11a} & \mathbf{z}_{12a} \\ \mathbf{z}_{21a} & \mathbf{z}_{22a} \end{bmatrix}; \ [\mathbf{z}_b] = \begin{bmatrix} \mathbf{z}_{11b} & \mathbf{z}_{12b} \\ \mathbf{z}_{21b} & \mathbf{z}_{22b} \end{bmatrix} \end{array}\right\}$$

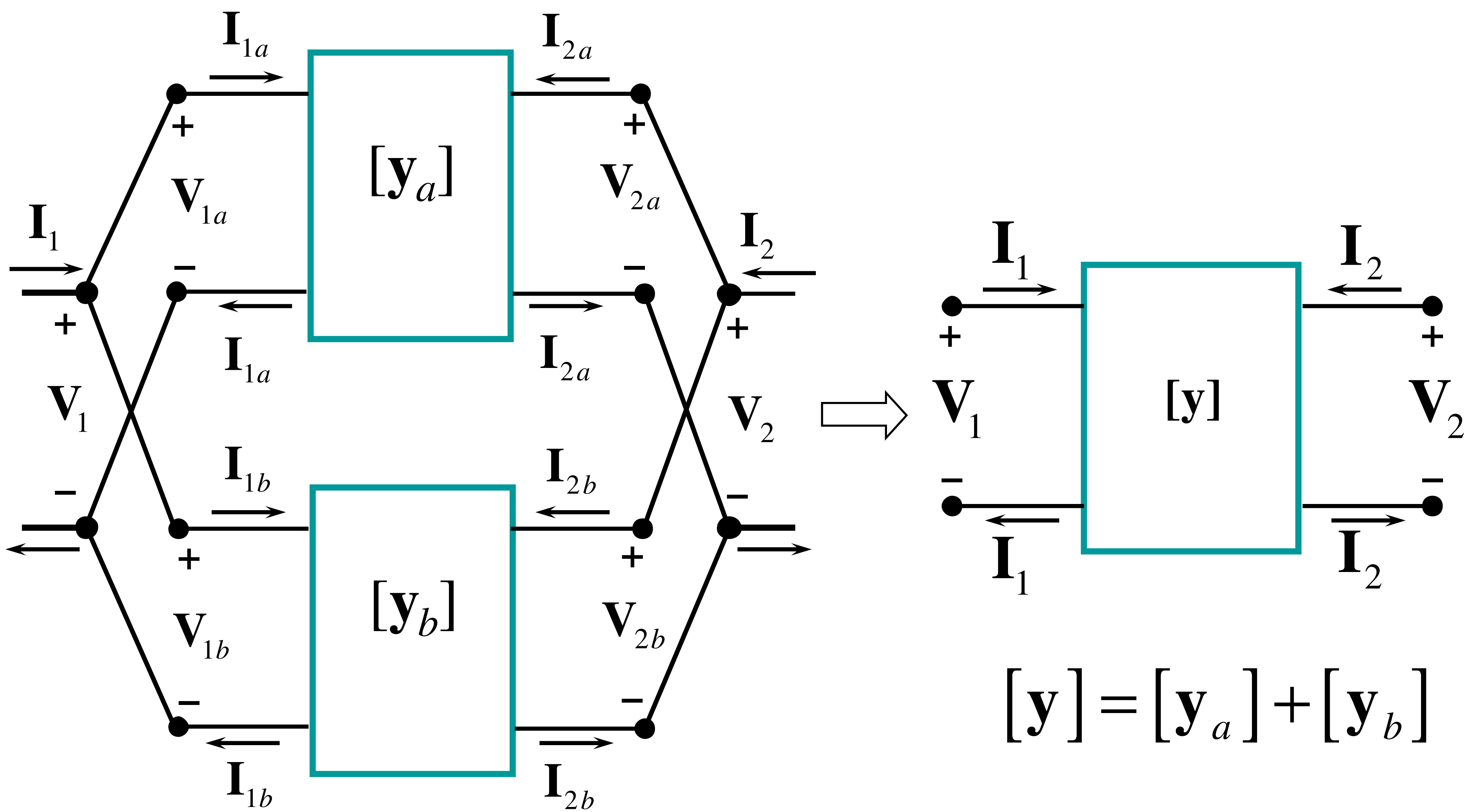
$$\longrightarrow \boxed{[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]}$$

Interconnection of Networks (6), Series



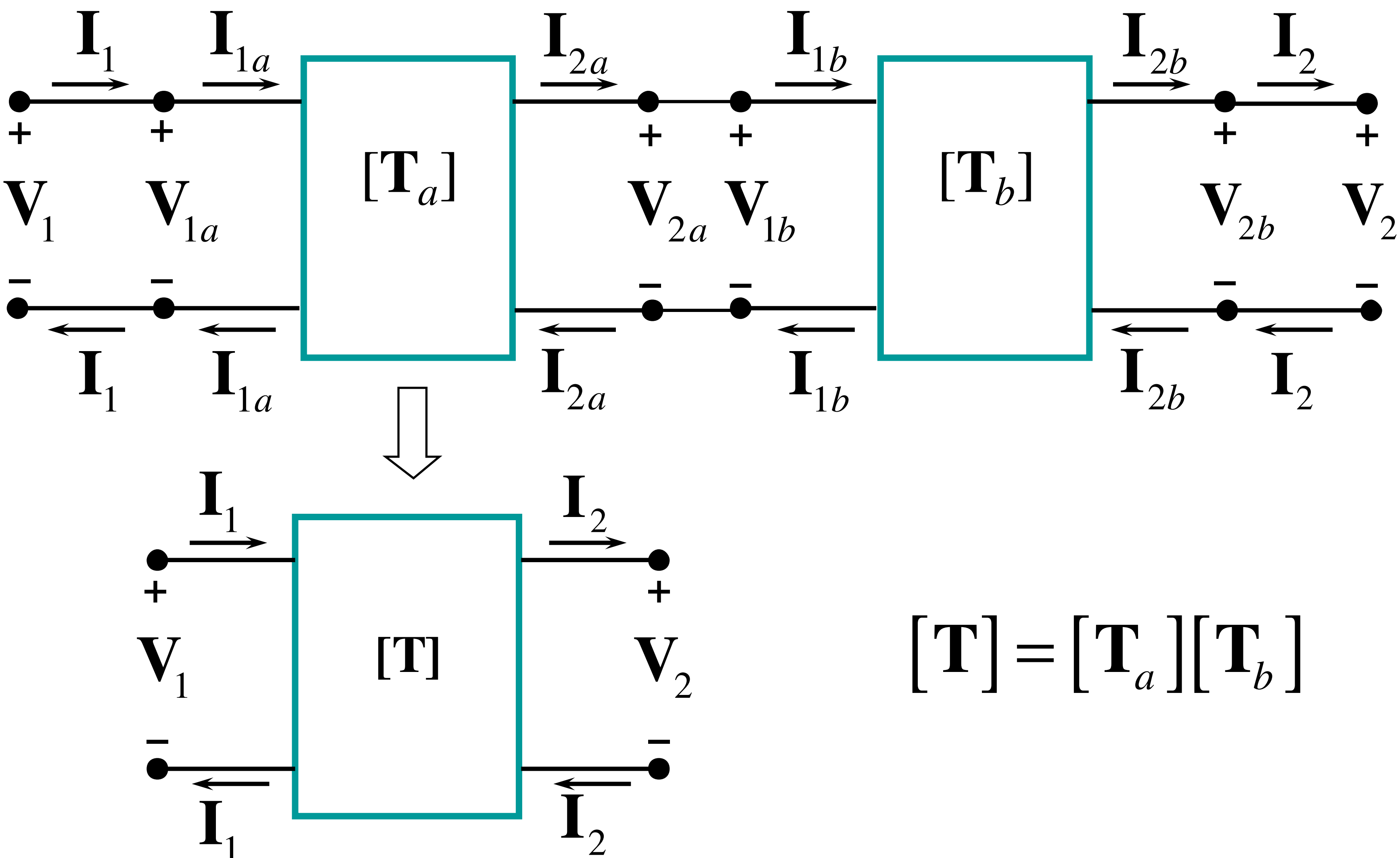
$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$

Interconnection of Networks (7), Parallel



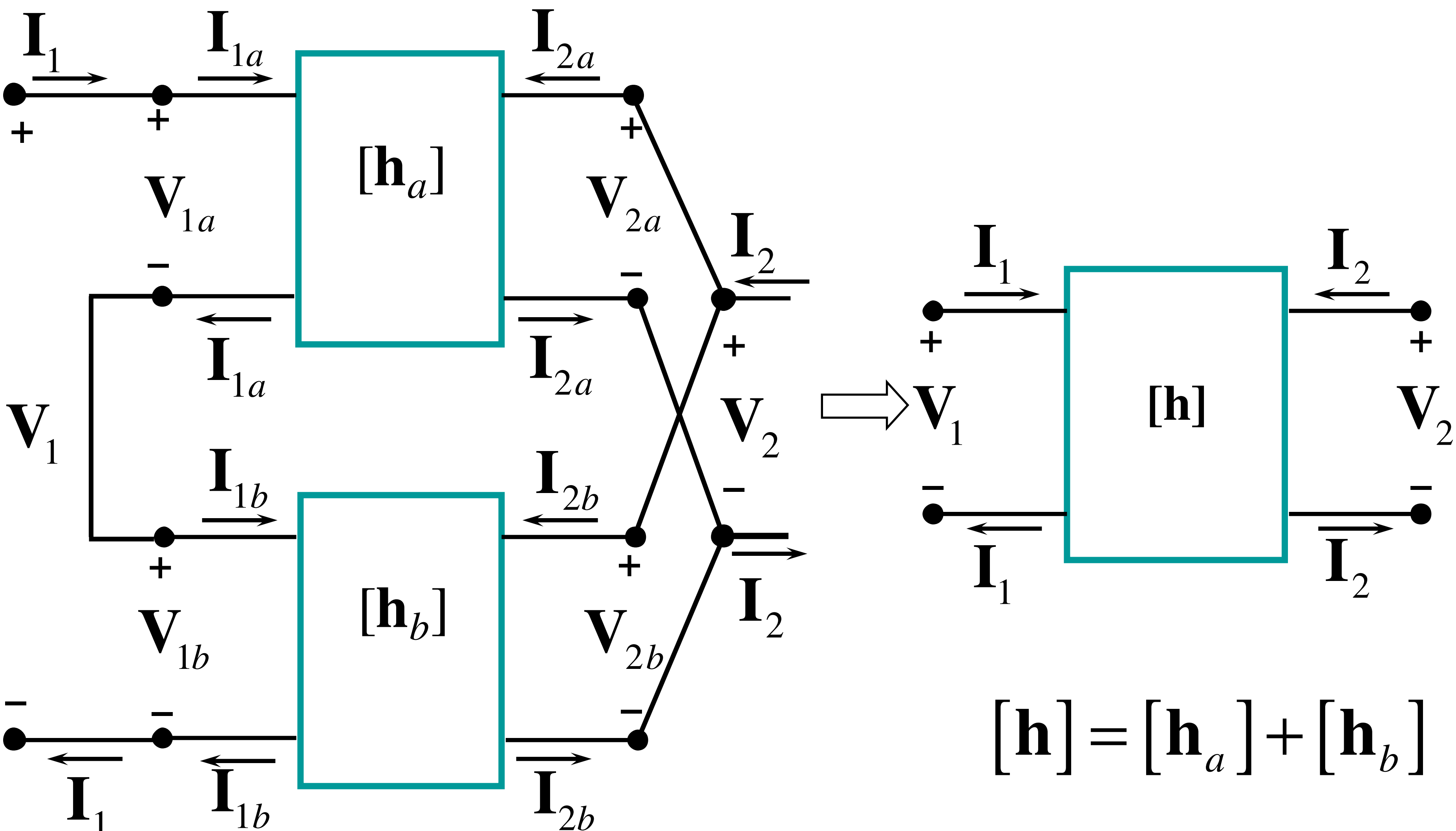
$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]$$

Interconnection of Networks (8), Cascade



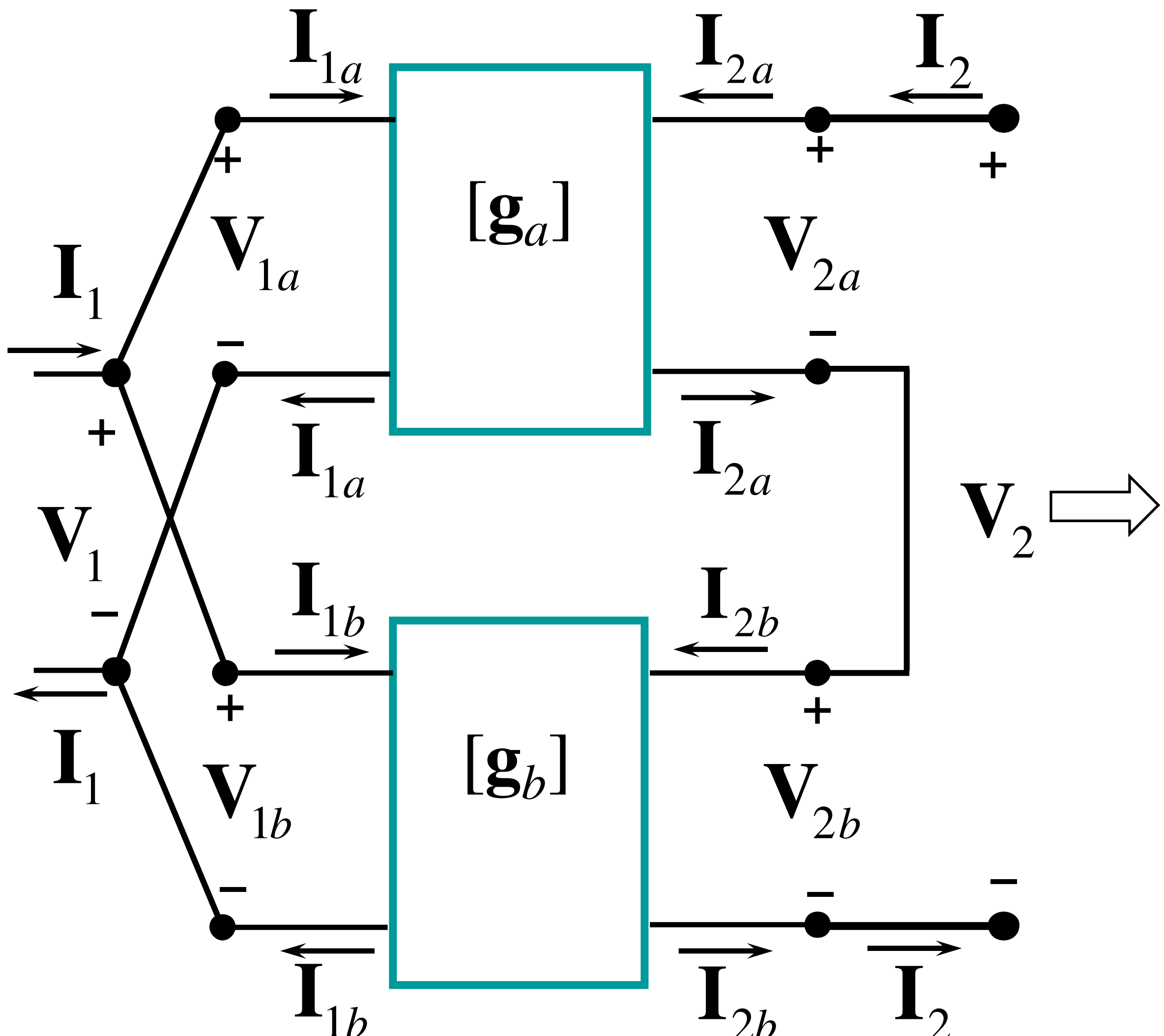
$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b]$$

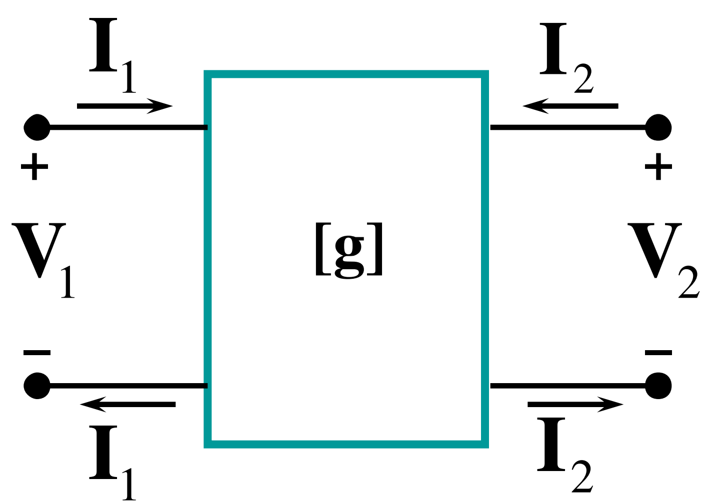
Interconnection of Networks (9), Hybrid 1



$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b]$$

Interconnection of Networks (10), Hybrid 2

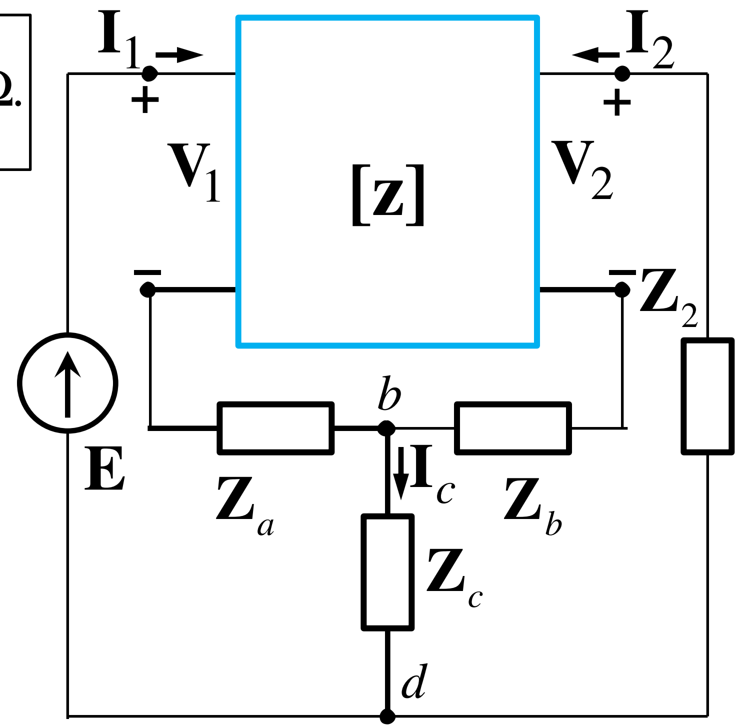




$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b]$$

Ex. 1 Interconnection of Networks (11)

$$\mathbf{E} = 220\text{V};\ \mathbf{Z}_2 = j10\ \Omega;\quad \mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}\Omega.$$
$$\mathbf{Z}_a = j20\ \Omega;\ \mathbf{Z}_b = -j40\ \Omega;\ \mathbf{Z}_c = 5\ \Omega;$$



Method 1

$$\begin{cases} \begin{cases} \mathbf{V}_1 = 30\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = 20\mathbf{I}_1 + 50\mathbf{I}_2 \end{cases} \\ b:\ \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_c = 0 \\ A:\ \mathbf{V}_1 + \mathbf{Z}_a\mathbf{I}_1 + \mathbf{Z}_c\mathbf{I}_c = \mathbf{E} \\ B:\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 + \mathbf{Z}_b\mathbf{I}_2 + \mathbf{Z}_c\mathbf{I}_c = 0 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = 6.27 - j3.64\ \text{A} \\ \mathbf{I}_2 = -2.89 + j0.076\ \text{A} \\ \mathbf{I}_c = 3.38 - j3.56\ \text{A} \end{cases}$$

Ex. 1 Interconnection of Networks (12)

$$\mathbf{E} = 220\text{V};\ \mathbf{Z}_2 = j10\ \Omega;\quad \mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}\Omega.$$
$$\mathbf{Z}_a = j20\ \Omega;\ \mathbf{Z}_b = -j40\ \Omega;\ \mathbf{Z}_c = 5\ \Omega;$$

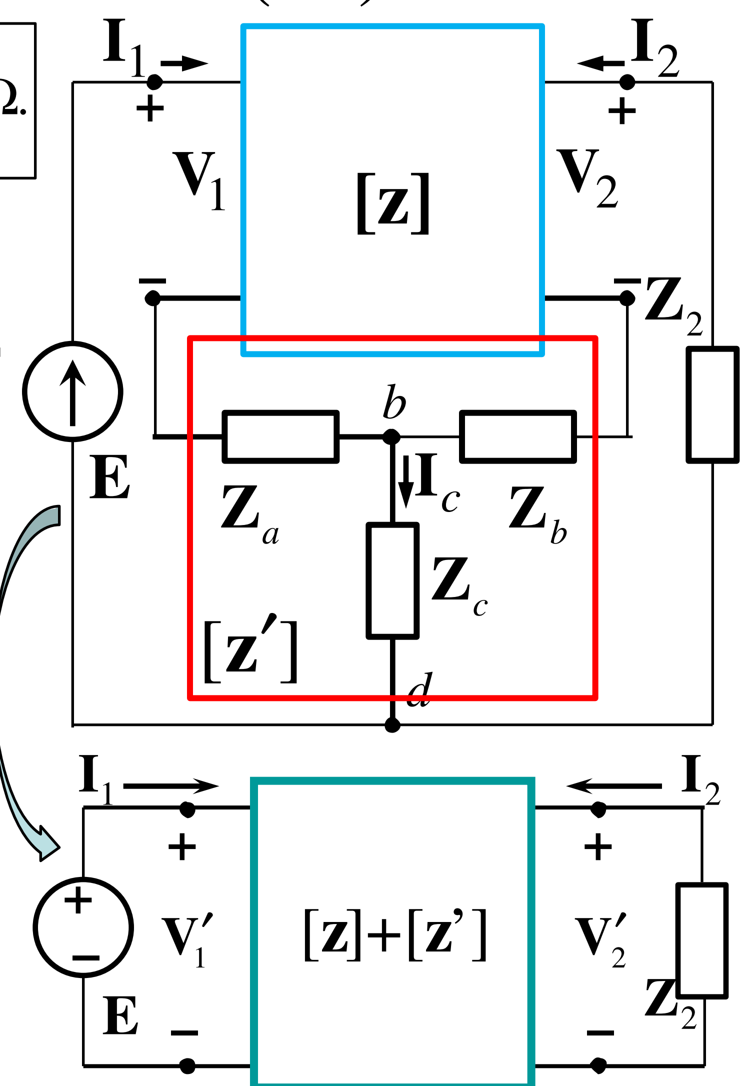
Method 2

$$[\mathbf{z}'] = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} 5 + j20 & 5 \\ 5 & 5 - j40 \end{bmatrix}\Omega$$

$$[\mathbf{z}] + [\mathbf{z}'] = \begin{bmatrix} 35 + j20 & 25 \\ 25 & 55 - j40 \end{bmatrix}\Omega$$

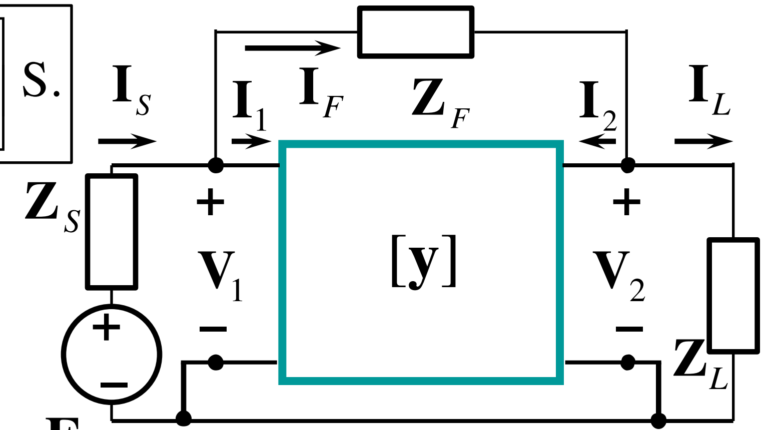
$$\begin{cases} \mathbf{V}_1' = (35 + j20)\mathbf{I}_1 + 25\mathbf{I}_2 = 220 \\ \mathbf{V}_2' = 25\mathbf{I}_1 + (55 - j40)\mathbf{I}_2 = -j10\mathbf{I}_2 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = 6.27 - j3.64\ \text{A} \\ \mathbf{I}_2 = -2.89 + j0.076\ \text{A} \end{cases}$$



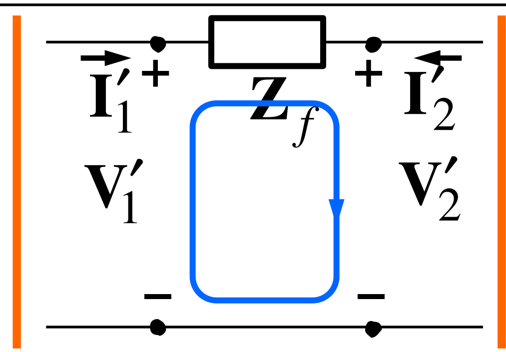
Ex. 2 Interconnection of Networks (13)

$$\mathbf{E}=200\,\text{V};\ \mathbf{Z}_n=5\ \Omega;\ \mathbf{Z}_f=j10\ \Omega;\ \mathbf{Z}_L=-j20\ \Omega;\quad \mathbf{Y}=\begin{bmatrix}0.0455 & -0.0182\\ -0.0182 & 0.0273\end{bmatrix}\text{S.}$$

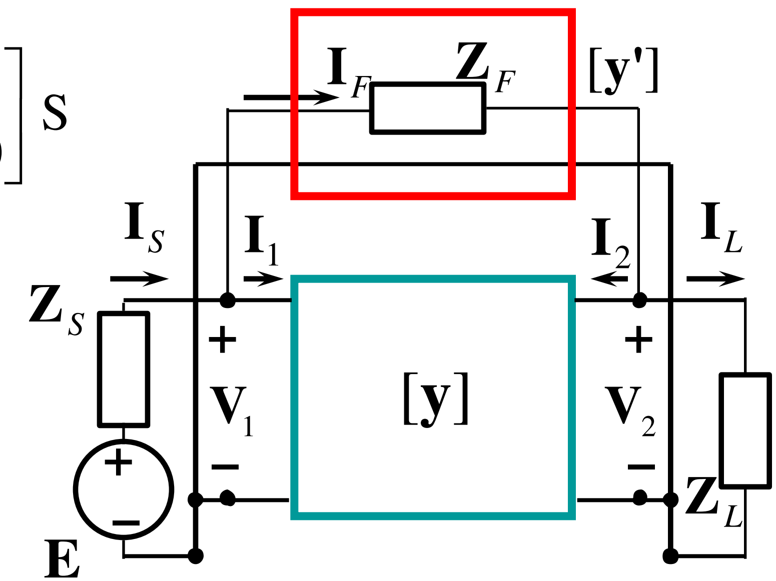


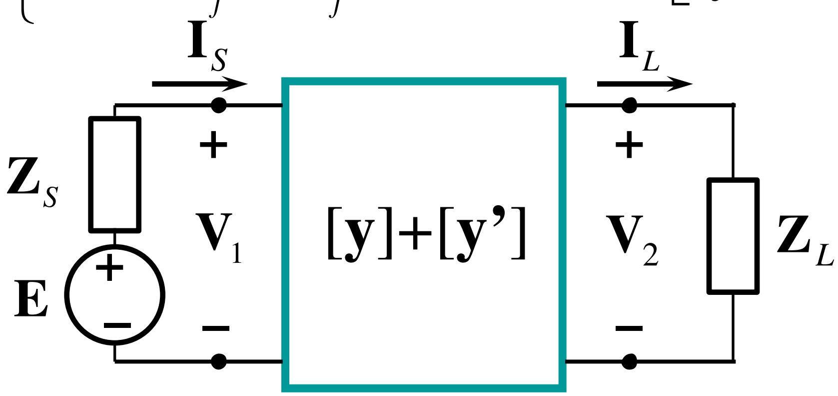
Method 4

$$\begin{cases}-\mathbf{V}_1'+\mathbf{Z}_f\mathbf{I}_1'+\mathbf{V}_2'=0\\ -\mathbf{V}_1'-\mathbf{Z}_f\mathbf{I}_2'+\mathbf{V}_2'=0\end{cases}$$



$$\rightarrow\begin{cases}\mathbf{I}_1'=\dfrac{\mathbf{V}_1'}{\mathbf{Z}_f}-\dfrac{\mathbf{V}_2'}{\mathbf{Z}_f}\\[2ex] \mathbf{I}_2'=-\dfrac{\mathbf{V}_1'}{\mathbf{Z}_f}+\dfrac{\mathbf{V}_2'}{\mathbf{Z}_f}\end{cases}\qquad \rightarrow[\mathbf{y}']=\begin{bmatrix}-j0.10 & j0.10\\ j0.10 & -j0.10\end{bmatrix}\text{S}$$





Ex. 2 Interconnection of Networks (14)

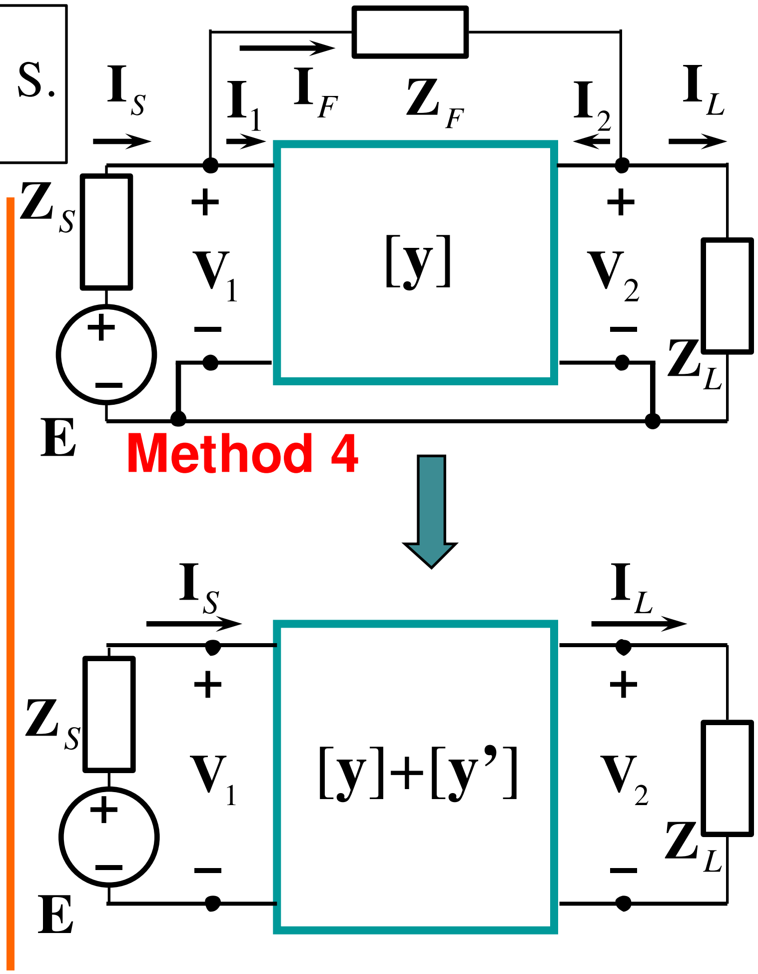
$$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;\ \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix}\text{S}.$$

$$[\mathbf{y}'] = \begin{bmatrix} -j0.10 & j0.10 \\ j0.10 & -j0.10 \end{bmatrix}\text{S}$$

$$[\mathbf{y}]+[\mathbf{y}'] = \begin{bmatrix} 0.0455 - j0.10 & -0.0182 + j0.10 \\ -0.0182 + j0.10 & 0.0273 - j0.10 \end{bmatrix}\text{S}$$

$$\begin{cases} \mathbf{I}_S = (0.0455 - j0.10)\mathbf{V}_1 - (0.0182 - j0.10)\mathbf{V}_2 \\ -\mathbf{I}_L = -(0.0182 - j0.10)\mathbf{V}_1 + (0.0273 - j0.10)\mathbf{V}_2 \\ 5\mathbf{I}_S + \mathbf{V}_1 = 200 \\ \mathbf{V}_2 + j20\mathbf{I}_L = 0 \end{cases}$$

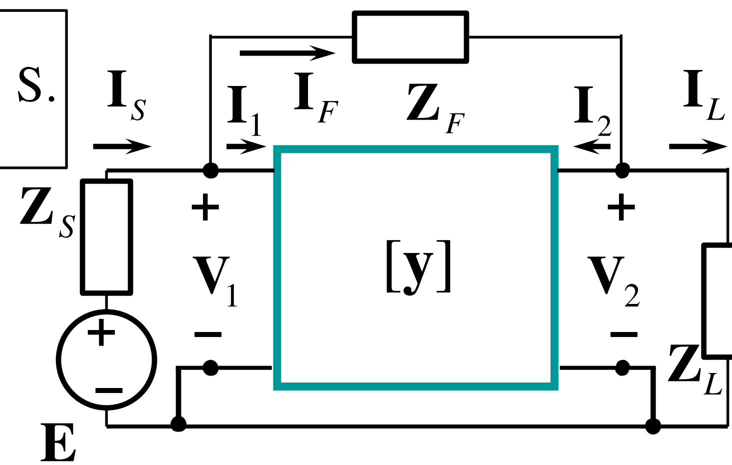
$$\rightarrow \begin{cases} \mathbf{I}_S = 12.76 + j8.02\ \text{A} \\ \mathbf{I}_L = 7.22 + j10.41\ \text{A} \end{cases}$$

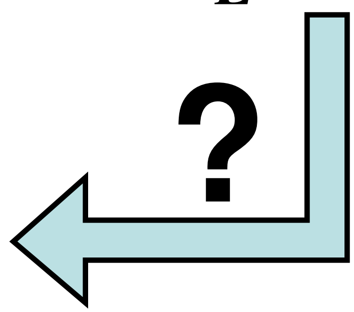


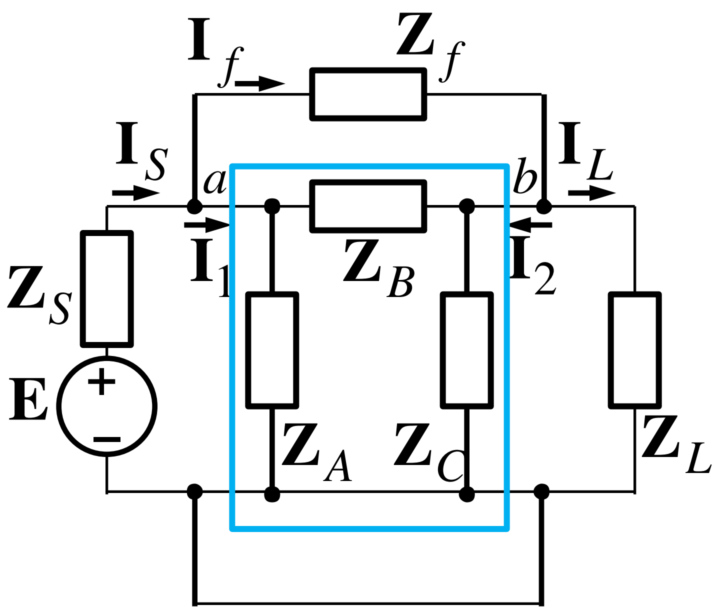
Ex. 2 Interconnection of Networks (15)

$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;$
$\mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;$ $\mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix}\text{S}.$

Method 5





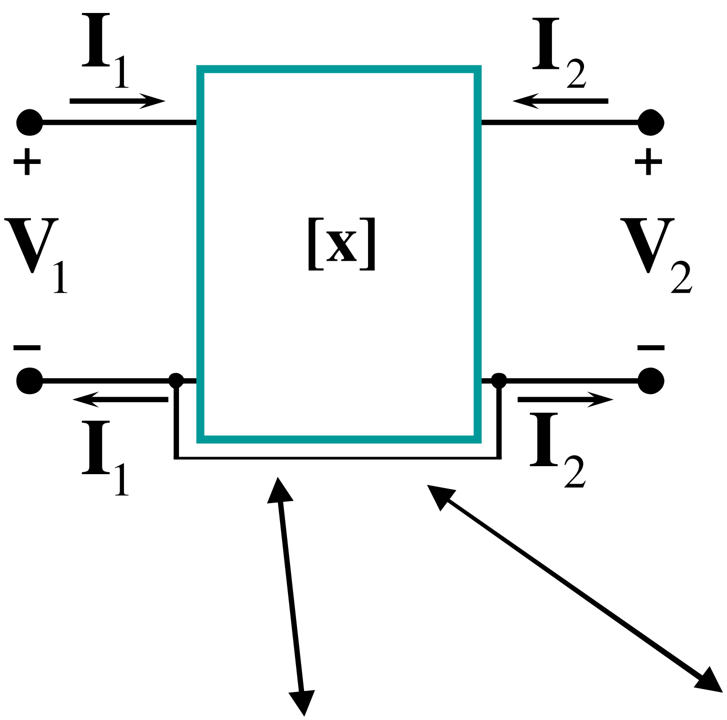


T & Π Networks

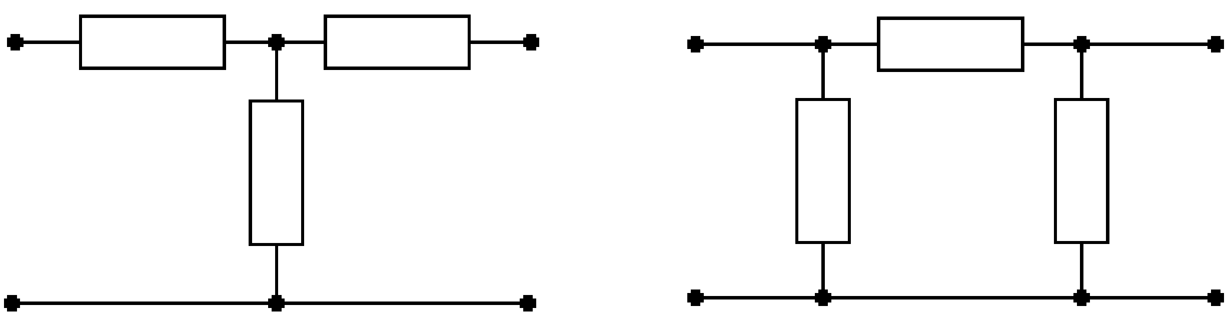
Two-port Networks

1. Introduction
2. Parameters
3. Relationships between Parameters
4. Two-port Network Analysis
5. Interconnection of Networks
6. **T & Π Networks**
7. Equivalent Two-port Networks of Magnetically Coupled Circuits
8. Input Impedance
9. Transfer Function

T & Π Networks (1)



1. Find the [**x**’] of the T (or Π) network,
2. Let [**x**] = [**x**’] (α),
3. Solve for (α) to find impedances of the T (or Π) network.



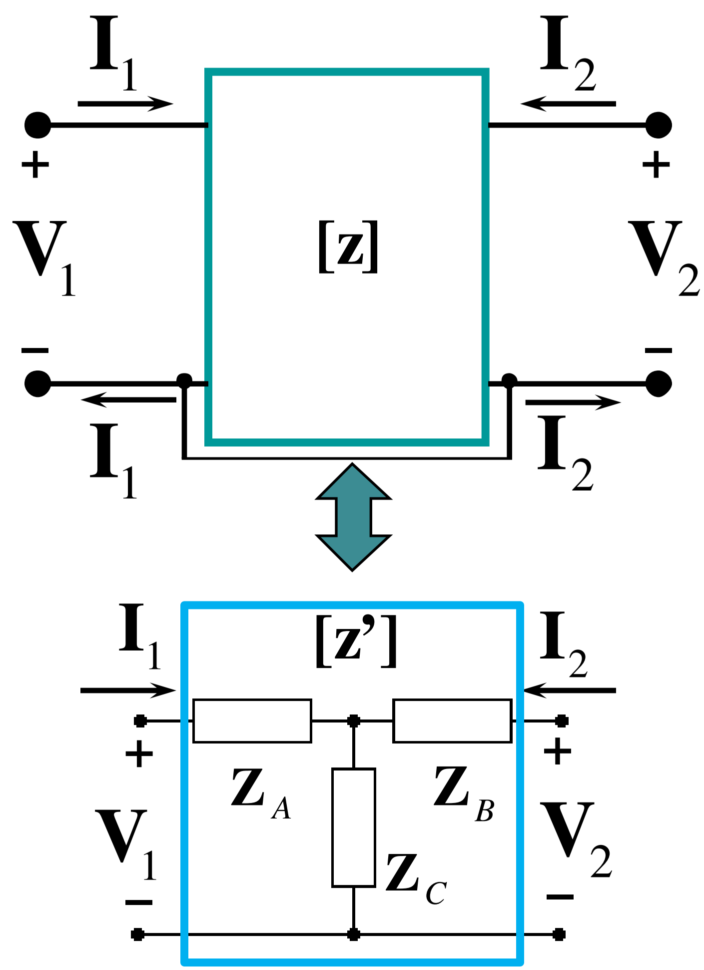
T & Π Networks (2)

$$\boxed{[\mathbf{z}'] = \begin{bmatrix} \mathbf{Z}_A + \mathbf{Z}_B & \mathbf{Z}_B \\ \mathbf{Z}_B & \mathbf{Z}_B + \mathbf{Z}_C \end{bmatrix}}$$

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}']$$

$$\rightarrow \begin{cases} \mathbf{Z}_A + \mathbf{Z}_B = \mathbf{z}_{11} \\ \mathbf{Z}_B = \mathbf{z}_{12} \\ \mathbf{Z}_B = \mathbf{z}_{21} \\ \mathbf{Z}_B + \mathbf{Z}_C = \mathbf{z}_{22} \end{cases} \rightarrow \begin{cases} \boxed{\begin{aligned} \mathbf{Z}_A &= \mathbf{z}_{11} - \mathbf{z}_{12} \\ \mathbf{Z}_B &= \mathbf{z}_{12} \\ \mathbf{Z}_C &= \mathbf{z}_{22} - \mathbf{z}_{12} \end{aligned}} \end{cases}$$

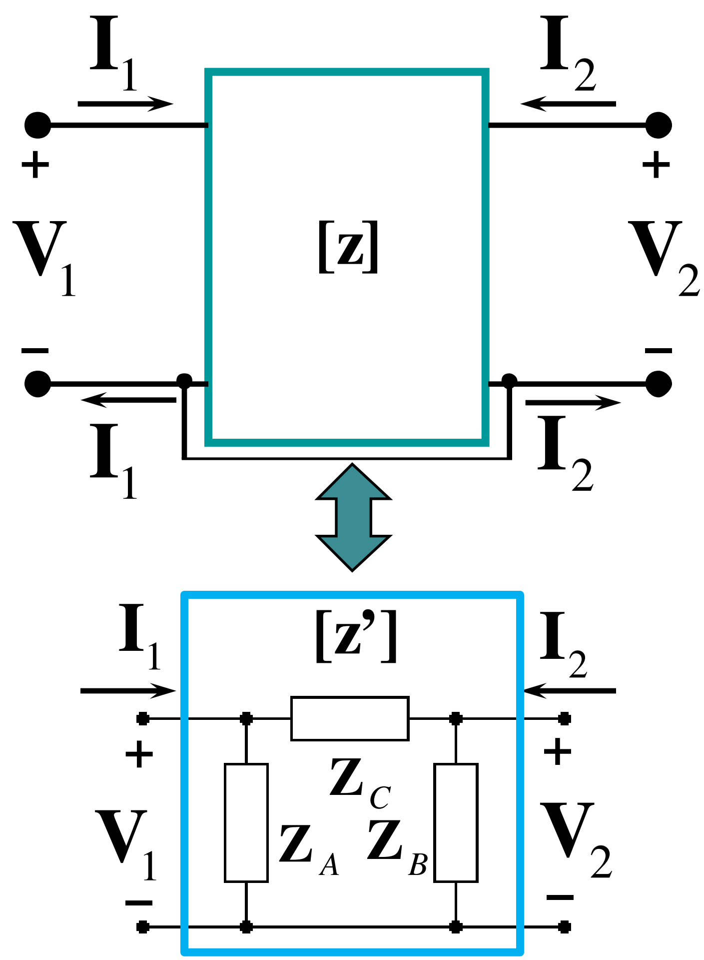


T & Π Networks (3)

$$[\mathbf{z}'] = \begin{bmatrix} \dfrac{Z_A(Z_B+Z_C)}{Z_A+Z_B+Z_C} & \dfrac{Z_A Z_C}{Z_A+Z_B+Z_C} \\ \dfrac{Z_A Z_C}{Z_A+Z_B+Z_C} & \dfrac{Z_C(Z_A+Z_B)}{Z_A+Z_B+Z_C} \end{bmatrix}$$

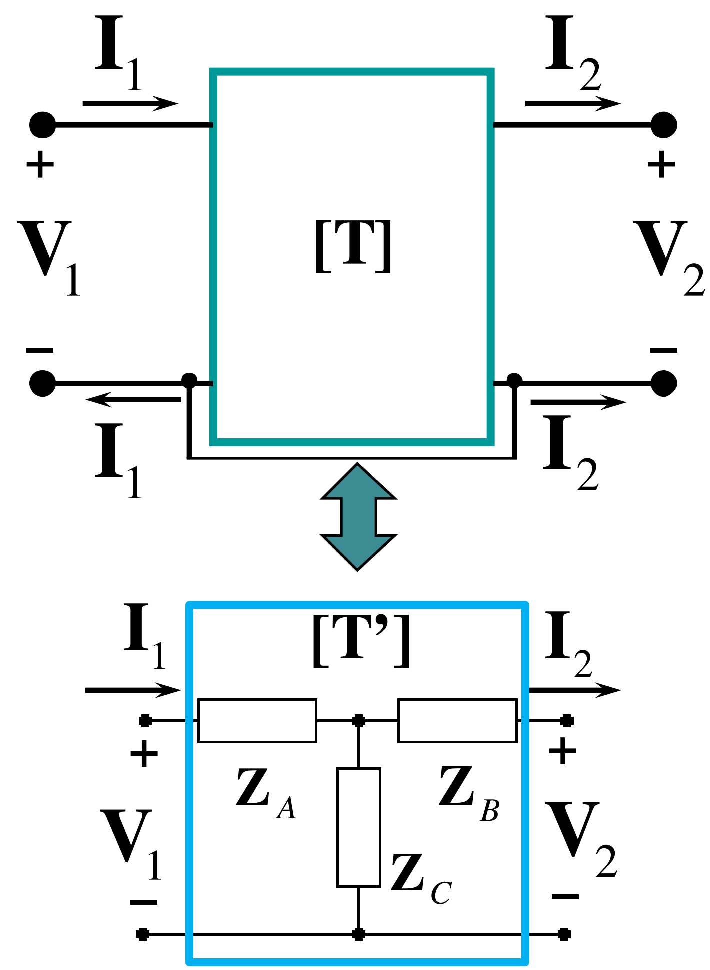
$$[\mathbf{z}] = [\mathbf{z}']$$

$$\rightarrow \begin{cases} Z_A = \dfrac{z_{11}z_{22} - z_{12}^2}{z_{22} - z_{12}} \\ Z_B = \dfrac{z_{11}z_{22} - z_{12}^2}{z_{12}} \\ Z_C = \dfrac{z_{11}z_{22} - z_{12}^2}{z_{11} - z_{12}} \end{cases}$$



T & Π Networks (4)

$$[\mathbf{T}'] = \begin{bmatrix} 1 + \dfrac{\mathbf{Z}_A}{\mathbf{Z}_B} & -\left(\mathbf{Z}_A + \mathbf{Z}_C + \dfrac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_B}\right) \\ \dfrac{1}{\mathbf{Z}_B} & -\left(1 + \dfrac{\mathbf{Z}_C}{\mathbf{Z}_B}\right) \end{bmatrix}$$



$$\mathbf{Z}_A = \frac{\mathbf{A} - 1}{\mathbf{C}}$$

$$\mathbf{Z}_B = \frac{1}{\mathbf{C}}$$

$$\mathbf{Z}_C = \frac{-\mathbf{D} - 1}{\mathbf{C}}$$

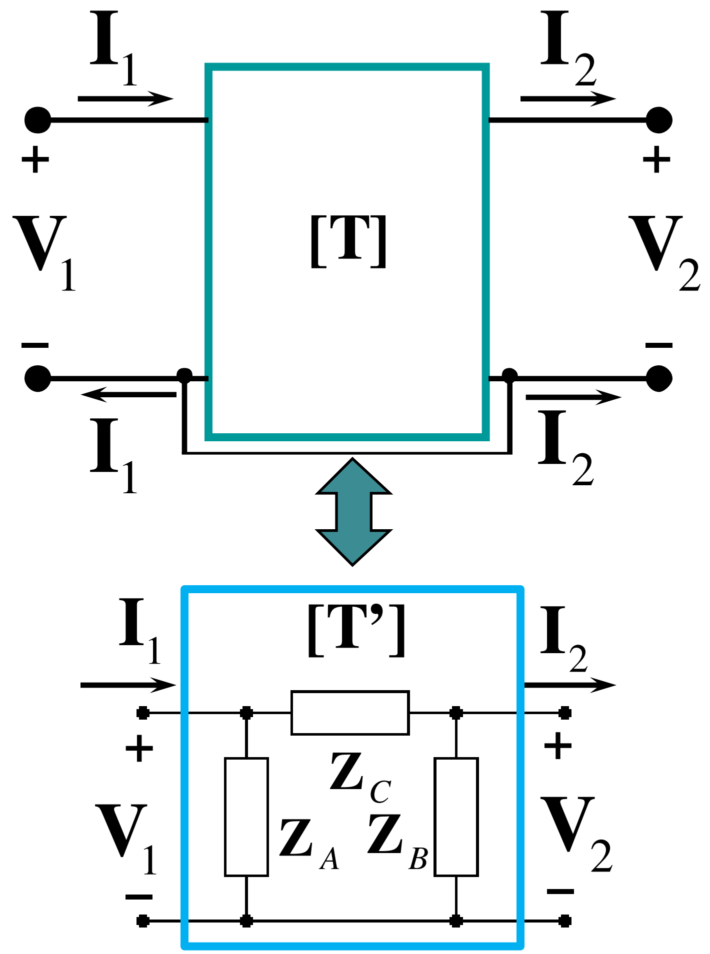
T & Π Networks (5)

$$[\mathbf{T}'] = \begin{bmatrix} 1+\dfrac{\mathbf{Z}_B}{\mathbf{Z}_C} & -\mathbf{Z}_B \\ \dfrac{\mathbf{Z}_A+\mathbf{Z}_B+\mathbf{Z}_C}{\mathbf{Z}_A\mathbf{Z}_C} & -\left(1+\dfrac{\mathbf{Z}_B}{\mathbf{Z}_A}\right) \end{bmatrix}$$

$$\mathbf{Z}_A = -\frac{\mathbf{B}}{\mathbf{D}+1}$$

$$\mathbf{Z}_B = -\mathbf{B}$$

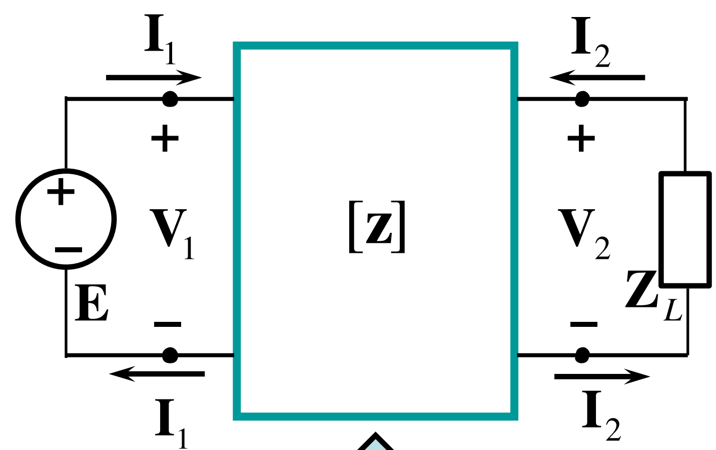
$$\mathbf{Z}_C = \frac{-\mathbf{B}}{\mathbf{A}-1}$$



T & Π Networks (6)

Ex. 1

$$\mathbf{E} = 220\,\text{V};\ \ \mathbf{Z}_L = j50\ \Omega;\ \ \mathbf{Z} = \begin{bmatrix} 10 & j20 \\ j20 & 40 \end{bmatrix}\Omega.$$

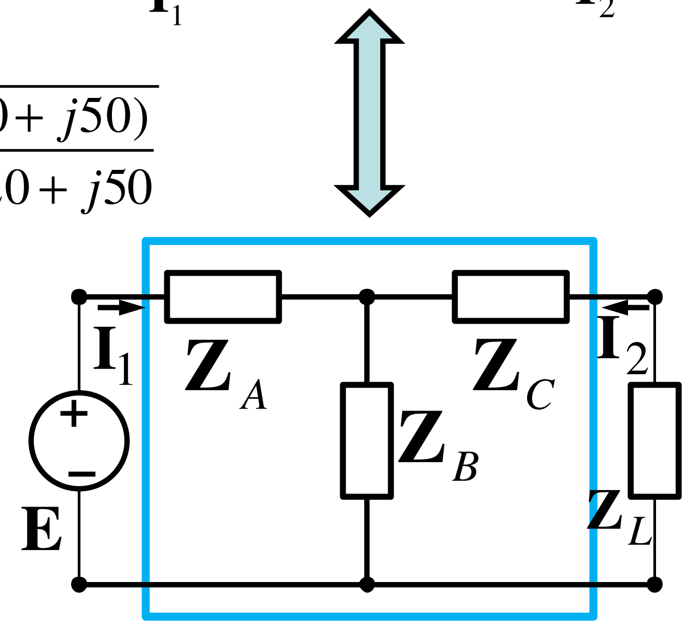


$$\begin{cases} \mathbf{Z}_A = \mathbf{z}_{11} - \mathbf{z}_{12} = 10 - j20\ \Omega \\ \mathbf{Z}_B = \mathbf{z}_{12} = j20\ \Omega \\ \mathbf{Z}_C = \mathbf{z}_{22} - \mathbf{z}_{12} = 40 - j20\ \Omega \end{cases}$$

Method 2

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_A + \mathbf{Z}_B // (\mathbf{Z}_C + \mathbf{Z}_L)} = \frac{220}{(10 - j20) + \dfrac{j20(40 - j20 + j50)}{j20 + 40 - j20 + j50}}$$

$$= \boxed{14.09 + j4.94\ \text{A}}$$



$$\mathbf{I}_2 = \frac{-\mathbf{I}_1\mathbf{Z}_B}{\mathbf{Z}_B + \mathbf{Z}_C + \mathbf{Z}_L} = \frac{-(14.09 + j4.94)\,j20}{j20 + 40 - j20 + j50}$$

$$= \boxed{-2.47 - j3.96\ \text{A}}$$

T & Π Networks (7)

Ex. 1

$$\boxed{\mathbf{E} = 220\,\text{V};\ \ \mathbf{Z}_L = j50\ \Omega;\ \ \mathbf{Z} = \begin{bmatrix} 10 & j20 \\ j20 & 40 \end{bmatrix} \Omega.}$$

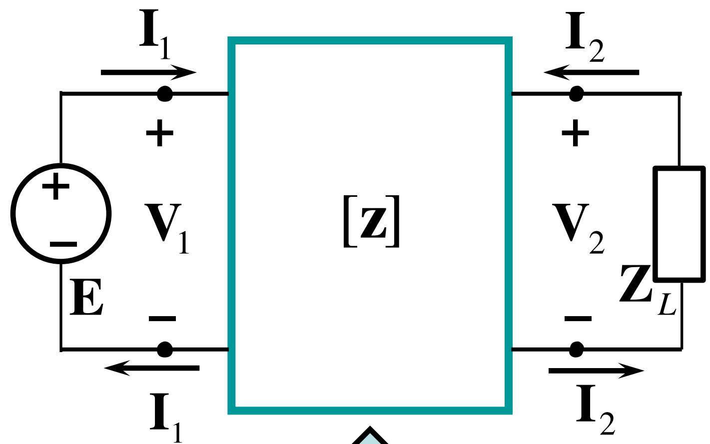
Method 3

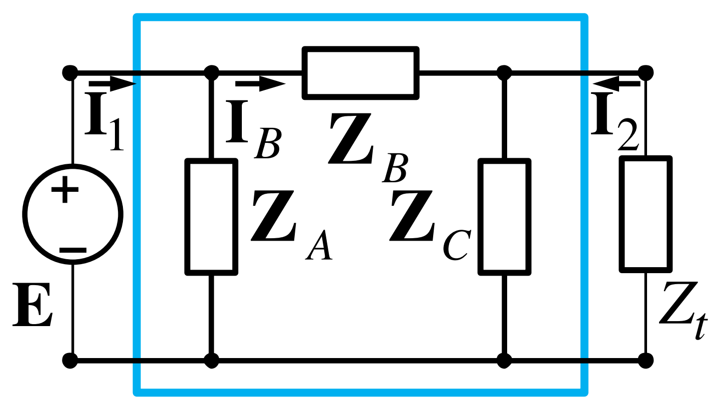
$$\begin{cases} \mathbf{Z}_A = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{22} - \mathbf{z}_{12}} = 16 + j8\,\Omega \\ \mathbf{Z}_B = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{12}} = -j40\Omega \\ \mathbf{Z}_C = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{11} - \mathbf{z}_{12}} = 16 + j32\,\Omega \end{cases}$$

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_A // [\mathbf{Z}_B + (\mathbf{Z}_C // \mathbf{Z}_L)]} = \boxed{14.09 + j4.94\ \text{A}}$$

$$\mathbf{I}_B = \frac{\mathbf{E}}{\mathbf{Z}_B + (\mathbf{Z}_C // \mathbf{Z}_L)} = 3.09 + j10.44\ \text{A}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_B \mathbf{Z}_C}{\mathbf{Z}_C + \mathbf{Z}_L} = \boxed{-2.47 - j3.96\ \text{A}}$$



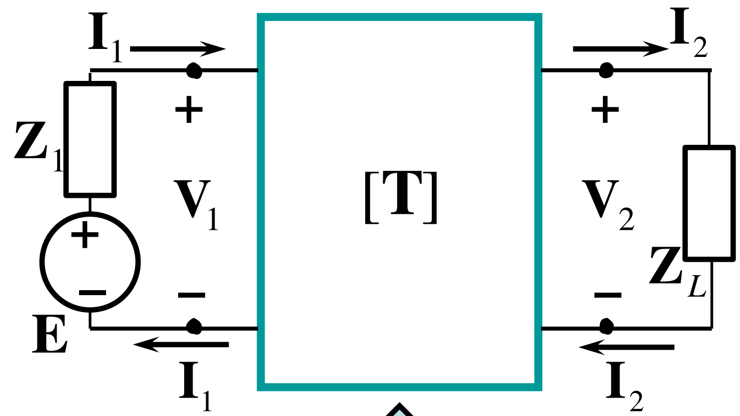


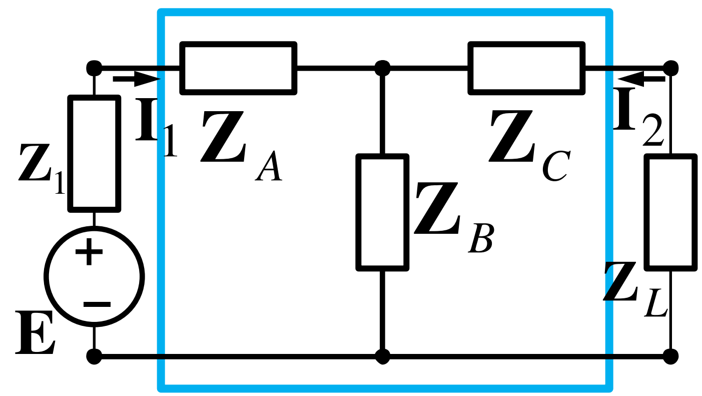
T & Π Networks (8)

Ex. 2

$$\mathbf{E} = 220\,\text{V}; \quad \mathbf{Z}_1 = 20\,\Omega;\ \ \mathbf{Z}_L = j50\ \Omega; \quad \mathbf{T} = \begin{bmatrix} 3 & -200 \\ 0.04 & -3 \end{bmatrix}.$$

Method 2





$$\begin{cases} \mathbf{Z}_A = \dfrac{\mathbf{A}-1}{\mathbf{C}} = \dfrac{3-1}{0.04} = 50\,\Omega \\ \mathbf{Z}_B = \dfrac{1}{\mathbf{C}} = \dfrac{1}{0.04} = 25\,\Omega \\ \mathbf{Z}_C = \dfrac{-\mathbf{D}-1}{\mathbf{C}} = \dfrac{3-1}{0.04} = 50\,\Omega \end{cases}$$

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_A + \mathbf{Z}_B // (\mathbf{Z}_C + \mathbf{Z}_L)} = \boxed{2.46 - j0.11\ \text{A}}$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1 \mathbf{Z}_B}{\mathbf{Z}_B + \mathbf{Z}_C + \mathbf{Z}_L} = \boxed{0.55 - j0.40\ \text{A}}$$

T & Π Networks (9)

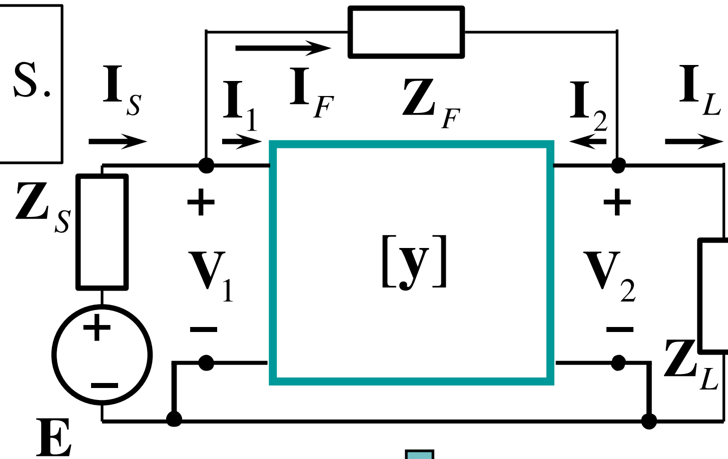
Ex. 3

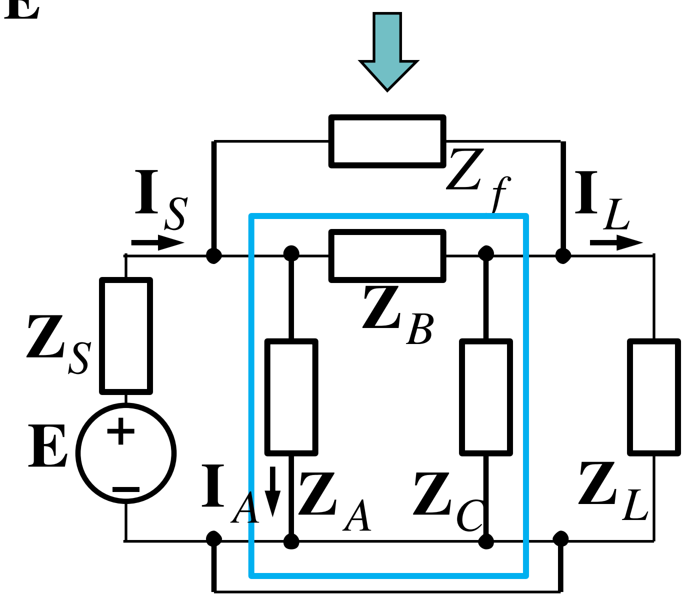
$$\mathbf{E} = 200\,\text{V};\ \mathbf{Z}_n = 5\ \Omega;\ \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix}\text{S}.$$

Method 5

$$[\mathbf{y}] = [\mathbf{z}]^{-1} = \begin{bmatrix} 0,0455 & -0,0182 \\ -0,0182 & 0,0273 \end{bmatrix}^{-1} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}\Omega$$

$$\begin{cases} \mathbf{Z}_A = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{22} - \mathbf{z}_{12}} = 36.67\,\Omega \\ \mathbf{Z}_B = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{12}} = 55.00\,\Omega \\ \mathbf{Z}_C = \dfrac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}^2}{\mathbf{z}_{11} - \mathbf{z}_{12}} = 110.00\,\Omega \end{cases}$$

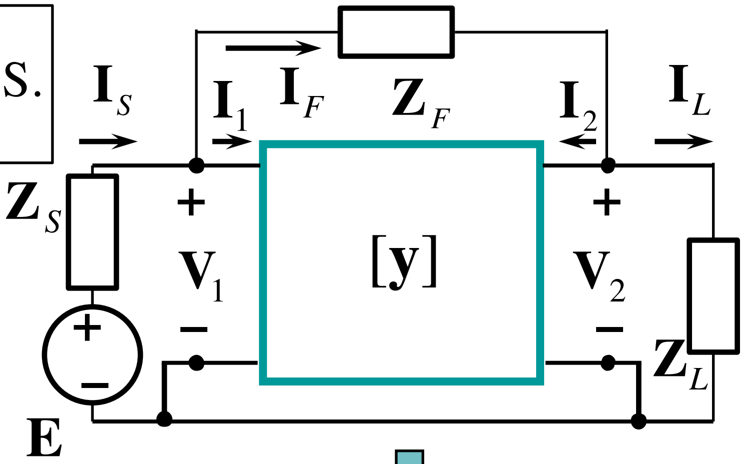




T & Π Networks (10)

Ex. 3

$$\mathbf{E} = 200\text{V};\ \mathbf{Z}_n = 5\ \Omega;\ \mathbf{Z}_f = j10\ \Omega;\ \mathbf{Z}_L = -j20\ \Omega;\quad \mathbf{Y} = \begin{bmatrix} 0.0455 & -0.0182 \\ -0.0182 & 0.0273 \end{bmatrix} \text{S.}$$



Method 5

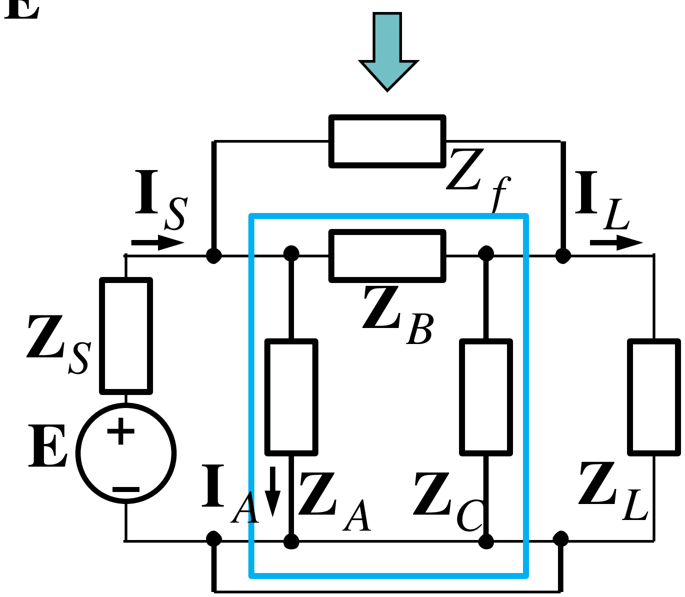
$$\mathbf{Z}_A = 36.67\ \Omega;\ \mathbf{Z}_B = 55.00\ \Omega;\ \mathbf{Z}_C = 110.00\ \Omega$$

$$\mathbf{I}_n = \frac{\mathbf{E}}{\mathbf{Z}_S + \{\mathbf{Z}_A // [(\mathbf{Z}_f // \mathbf{Z}_B) + (\mathbf{Z}_L // \mathbf{Z}_C)]\}}$$

$$= \boxed{12.80 + j8.00\ \text{A}}$$

$$\mathbf{I}_A = \frac{\mathbf{E} - \mathbf{Z}_S \mathbf{I}_S}{\mathbf{Z}_A} = 3.71 - j1.09\ \text{A}$$

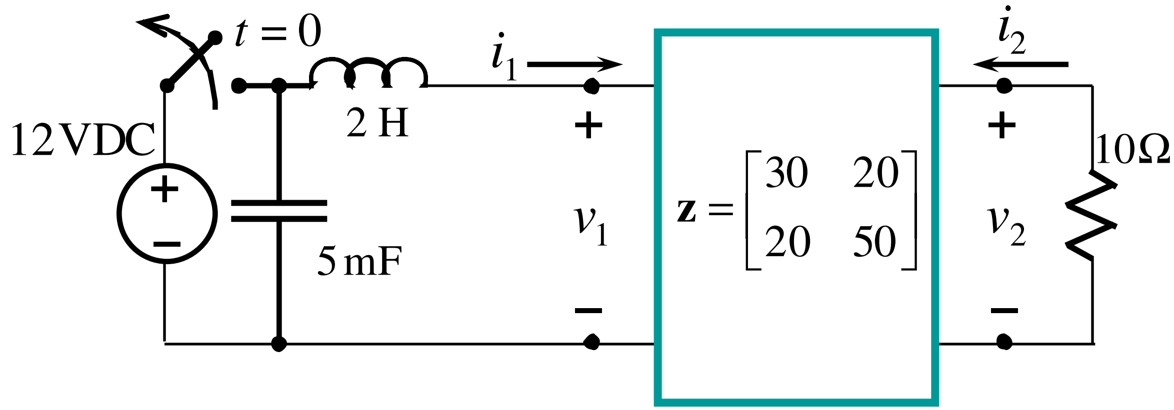
$$\mathbf{I}_L = \frac{(\mathbf{I}_S - \mathbf{I}_A)\mathbf{Z}_C}{\mathbf{Z}_C + \mathbf{Z}_L} = \boxed{7.20 + j10.40\ \text{A}}$$



T & Π Networks (11)

Ex. 4

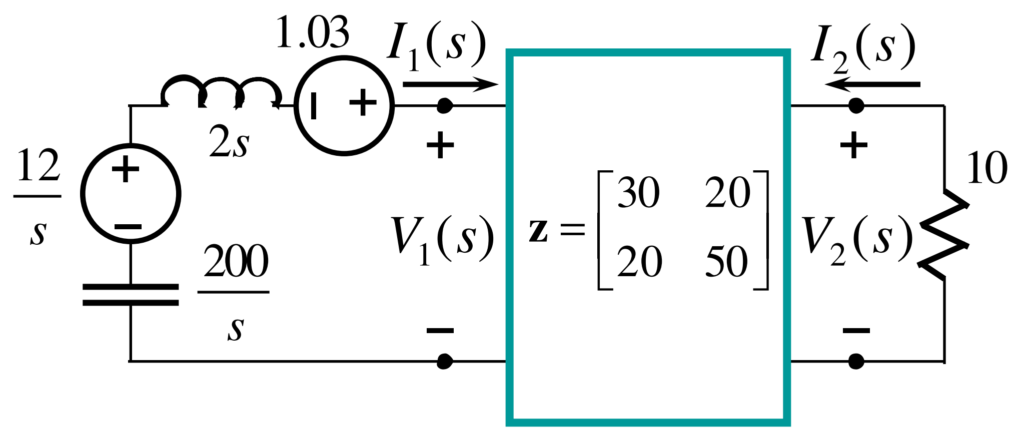
$\boxed{\text{Find } i_1(t)?}$ **Method 1**



$v_C(0) = 12\text{ V}$

$$\begin{cases} v_1(0) = 30i_1(0) + 20i_2(0) \\ v_2(0) = 20i_1(0) + 50i_2(0) \\ v_1(0) = 12 \\ v_2(0) = -10i_2(0) \end{cases}$$

$\rightarrow i_1(0) = 0.5143\text{ A} = i_L(0)$



$$\begin{cases} V_1(s) = 30I_1(s) + 20I_2(s) \\ V_2(s) = 20I_1(s) + 50I_2(s) \\ \left(2s + \dfrac{200}{s}\right) I_1(s) + V_1(s) = 1.03 + \dfrac{12}{s} \\ V_2(s) = -10I_2(s) \end{cases}$$

$\rightarrow I_1(s) = \dfrac{0.515s + 6}{s^2 + 11.667s + 100}\text{ A}$ $\rightarrow \boxed{i_1(t) = 0.6334e^{-5.83t}\cos(8.12t - 35.6^{\circ})\text{ A}}$

T & Π Networks (12)

Ex. 4

$\boxed{\text{Find } i_1(t)?}$ **Method 2**

$v_C(0) = 12\text{ V}$

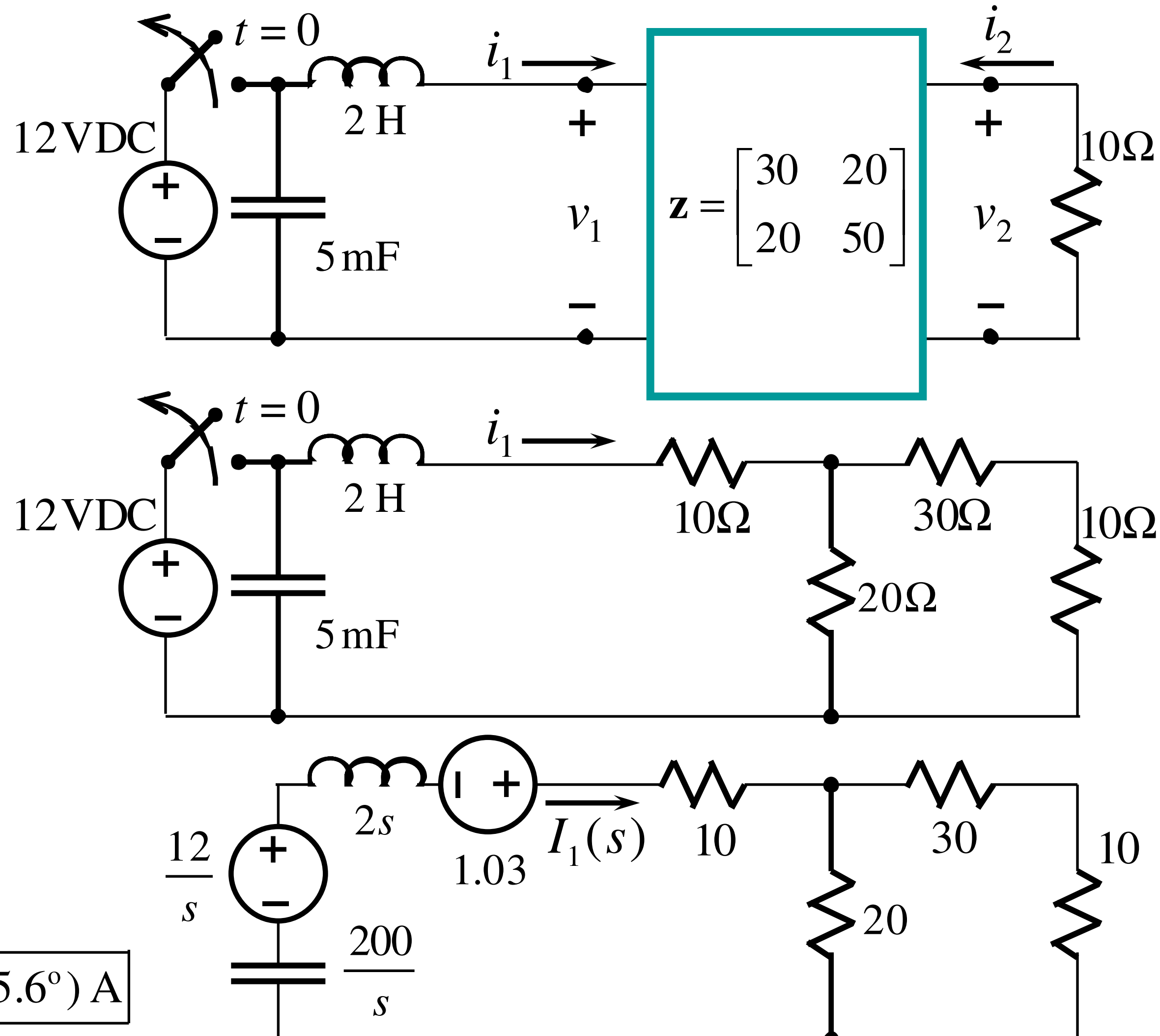
$$R_{eq} = 10 + \frac{(30+10)20}{30+10+20} = 23.33\Omega$$

$$i_1(0) = \frac{12}{23.33} = 0.5143\text{ A} = i_L(0)$$

$$I_1(s) = \frac{1.03 + \frac{12}{s}}{\frac{200}{s} + 2s + 23.33}$$

$$= \frac{0.515s + 6}{s^2 + 11.667s + 100}\text{ A}$$

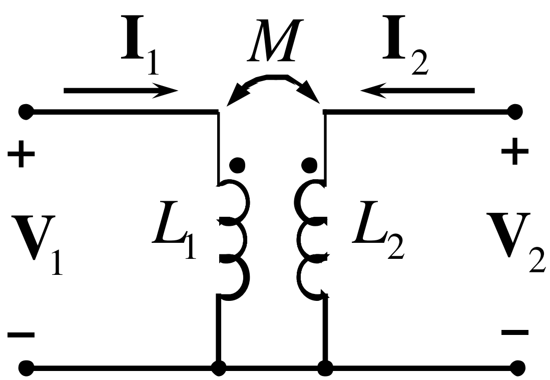
$$\rightarrow \boxed{i_1(t) = 0.6334e^{-5.83t}\cos(8.12t - 35.6^\circ)\text{ A}}$$



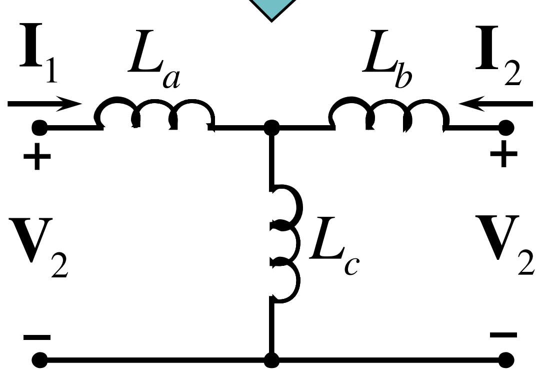
Two-port Networks

1. Introduction
2. Parameters
3. Relationships between Parameters
4. Two-port Network Analysis
5. Interconnection of Networks
6. T & Π Networks
7. **Equivalent Two-port Networks of Magnetically Coupled Circuits**
8. Input Impedance
9. Transfer Function

Equivalent Two-port Networks of Magnetically Coupled Circuits (1)



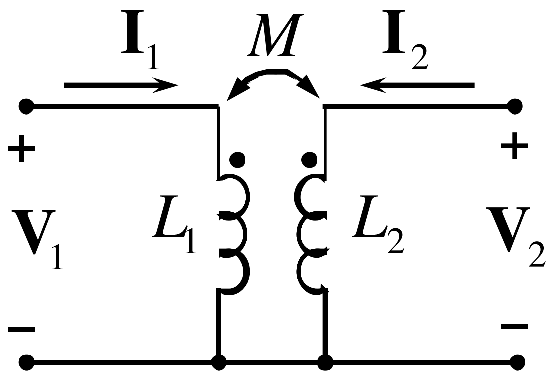
$$\rightarrow \begin{cases} \boxed{\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2} \\ \boxed{\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2} \end{cases}$$

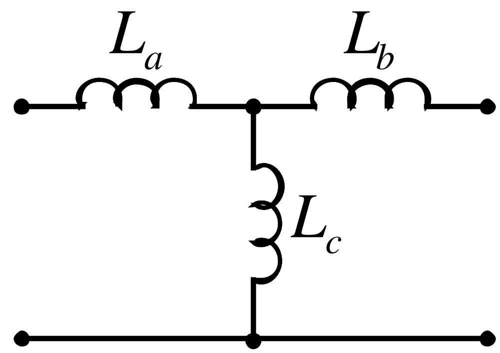


$$\rightarrow \begin{cases} \mathbf{V}_1 = j\omega L_a\mathbf{I}_1 + j\omega L_c(\mathbf{I}_1 + \mathbf{I}_2) \\ \quad = \boxed{j\omega(L_a + L_c)\mathbf{I}_1 + j\omega L_c\mathbf{I}_2} \\ \mathbf{V}_2 = j\omega L_c(\mathbf{I}_1 + \mathbf{I}_2) + j\omega L_b\mathbf{I}_2 \\ \quad = \boxed{j\omega L_c\mathbf{I}_1 + j\omega(L_c + L_b)\mathbf{I}_2} \end{cases}$$

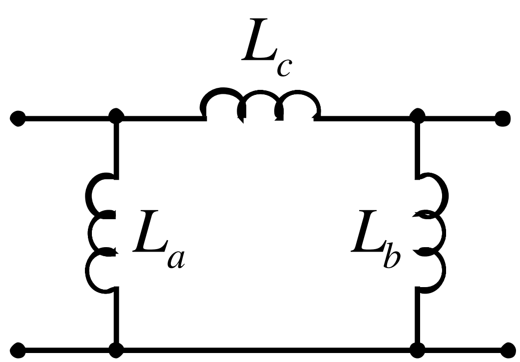
$$\rightarrow \begin{cases} L_a = L_1 - M \\ L_b = L_2 - M \\ L_c = M \end{cases}$$

Equivalent Two-port Networks of Magnetically Coupled Circuits (2)





$$L_a = L_1 - M$$
$$L_b = L_2 - M$$
$$L_c = M$$

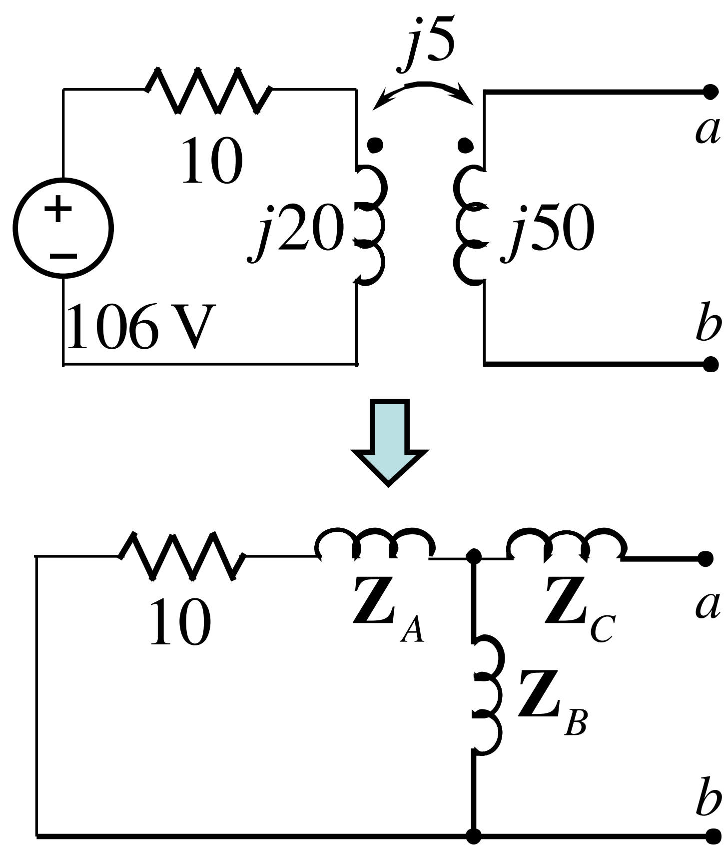


$$L_a = \frac{L_1 L_2 - M^2}{L_2 - M}$$
$$L_b = \frac{L_1 L_2 - M^2}{L_1 - M}$$
$$L_c = \frac{L_1 L_2 - M^2}{M}$$

Equivalent Two-port Networks of Magnetically Coupled Circuits (3)

Ex. 1

$\boxed{\mathbf{Z}_{ab} = ?}$ **Method 3**



$$\mathbf{Z}_A = j20 - j5 = j15\,\Omega$$

$$\mathbf{Z}_C = j50 - j5 = j45\,\Omega$$

$$\mathbf{Z}_B = j5\,\Omega$$

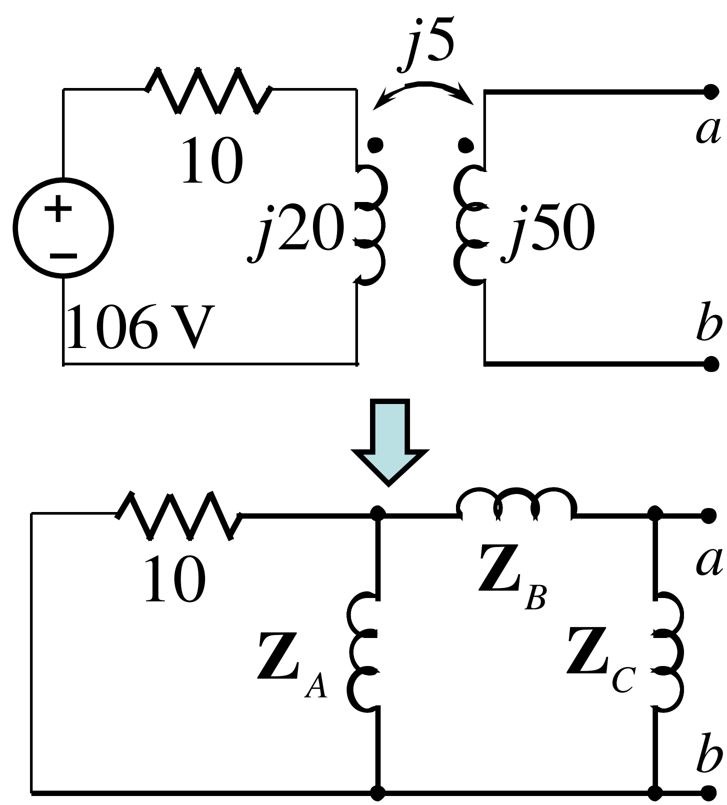
$$\mathbf{Z}_{ab} = \frac{\mathbf{Z}_B(10 + \mathbf{Z}_A)}{\mathbf{Z}_B + 10 + \mathbf{Z}_A} + \mathbf{Z}_C$$

$$= \boxed{0.50 + j49\,\Omega}$$

Equivalent Two-port Networks of Magnetically Coupled Circuits (4)

Ex. 1

$\boxed{\mathbf{Z}_{ab} = ?}$ **Method 4**



$$\mathbf{Z}_A = \frac{j20 \times j50 - (j5)^2}{j50 - j5} = j21.67\ \Omega$$

$$\mathbf{Z}_B = \frac{j20 \times j50 - (j5)^2}{j5} = j195\ \Omega$$

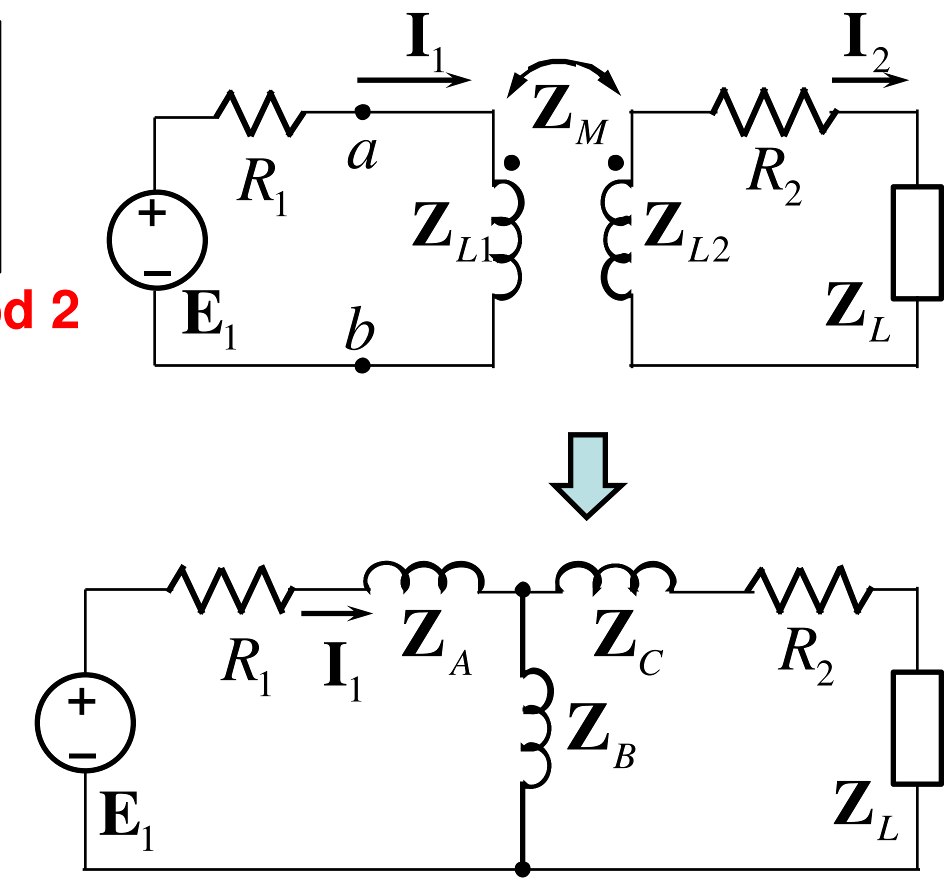
$$\mathbf{Z}_C = \frac{j20 \times j50 - (j5)^2}{j20 - j5} = j65\ \Omega$$

$$\mathbf{Z}_{ab} = \frac{\left(\dfrac{10\mathbf{Z}_A}{10+\mathbf{Z}_A} + \mathbf{Z}_B\right)\mathbf{Z}_C}{\dfrac{10\mathbf{Z}_A}{10+\mathbf{Z}_A} + \mathbf{Z}_B + \mathbf{Z}_C} = \boxed{0.50 + j49\ \Omega}$$

Equivalent Two-port Networks of Magnetically Coupled Circuits (5)

Ex. 2

$\mathbf{E}_1 = 100\underline{/30^\circ}$ V; $R_1 = 60\,\Omega$; $R_2 = 40\,\Omega$;
$\mathbf{Z}_L = 80 + j10\,\Omega$; $\mathbf{Z}_{L1} = j20\,\Omega$; $\mathbf{Z}_{L2} = j40\,\Omega$;
$\mathbf{Z}_M = j5\,\Omega$. Find $\mathbf{I}_1$?



Method 2

$$\mathbf{Z}_A = j20 - j5 = j15\,\Omega$$

$$\mathbf{Z}_B = j5\,\Omega$$

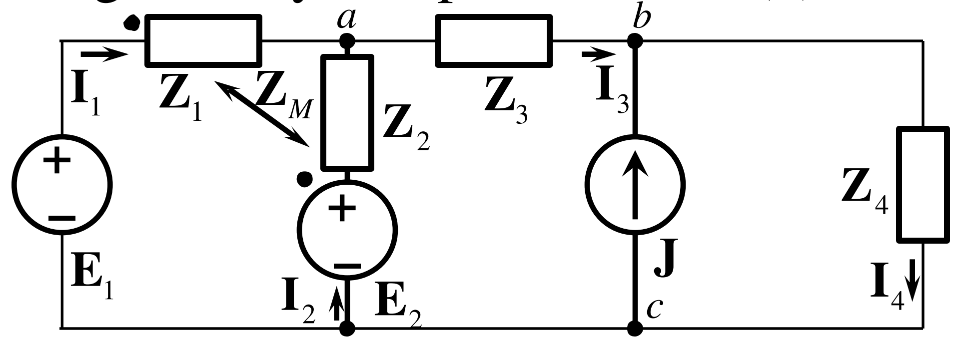
$$\mathbf{Z}_C = j40 - j5 = j35\,\Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{E}_1}{R_1 + \mathbf{Z}_A + \dfrac{\mathbf{Z}_B(\mathbf{Z}_C + R_2 + \mathbf{Z}_L)}{\mathbf{Z}_B + \mathbf{Z}_C + R_2 + \mathbf{Z}_L}}$$

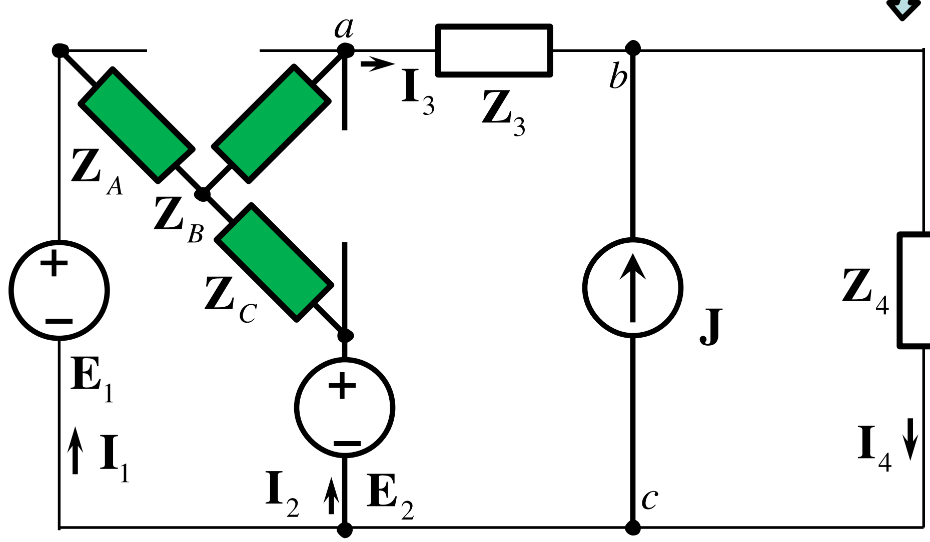
$$= \boxed{1.54 + j0.32\,\text{A}}$$

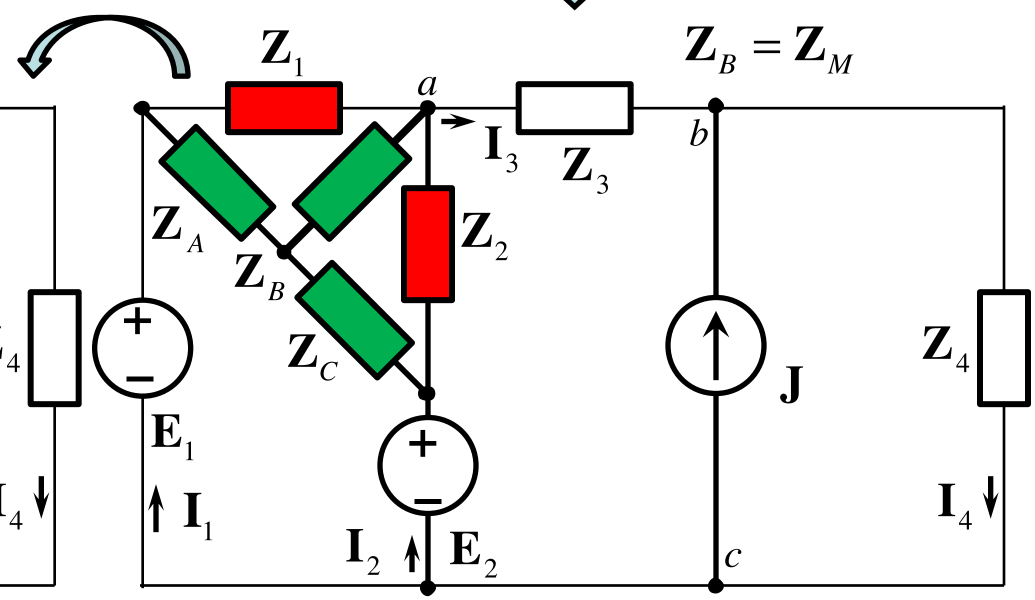
Ex. 3 Equivalent Two-port Networks of Magnetically Coupled Circuits (6)

$\mathbf{Z}_1 = 10 + j15\,\Omega$; $\mathbf{Z}_2 = 20 + j10\,\Omega$; $\mathbf{Z}_M = j2\,\Omega$;
$\mathbf{Z}_3 = -j20\,\Omega$; $\mathbf{Z}_4 = 25\,\Omega$; $\mathbf{E}_1 = 100\,\text{V}$;
$\mathbf{E}_2 = 150\underline{/30^\circ}\ \text{V}$; $\mathbf{J} = 5\underline{/45^\circ}\ \text{A}$.



$$\mathbf{Z}_A = \mathbf{Z}_1 - \mathbf{Z}_M$$
$$\mathbf{Z}_C = \mathbf{Z}_2 - \mathbf{Z}_M$$
$$\mathbf{Z}_B = \mathbf{Z}_M$$

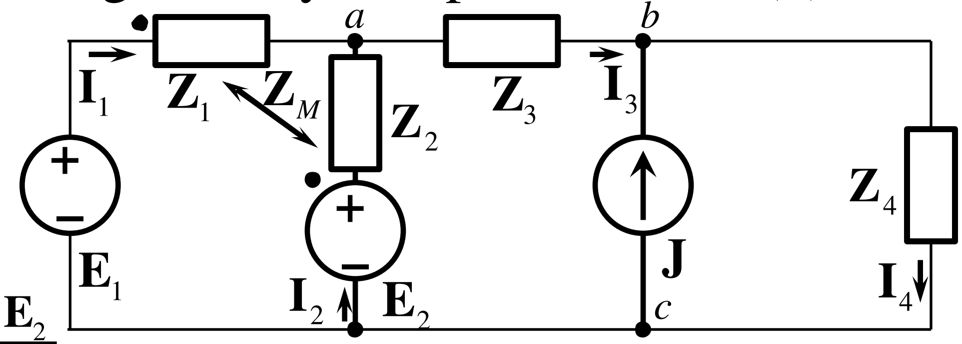




Equivalent Two-port Networks of Magnetically Coupled Circuits (7)

Ex. 3

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\,\Omega;\ \mathbf{Z}_M = j2\,\Omega;$
$\mathbf{Z}_3 = -j20\,\Omega;\ \mathbf{Z}_4 = 25\,\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\underline{/30^\circ}\ \text{V};\ \mathbf{J} = 5\underline{/45^\circ}\ \text{A}.$



$\mathbf{Z}_A = \mathbf{Z}_1 - \mathbf{Z}_M$

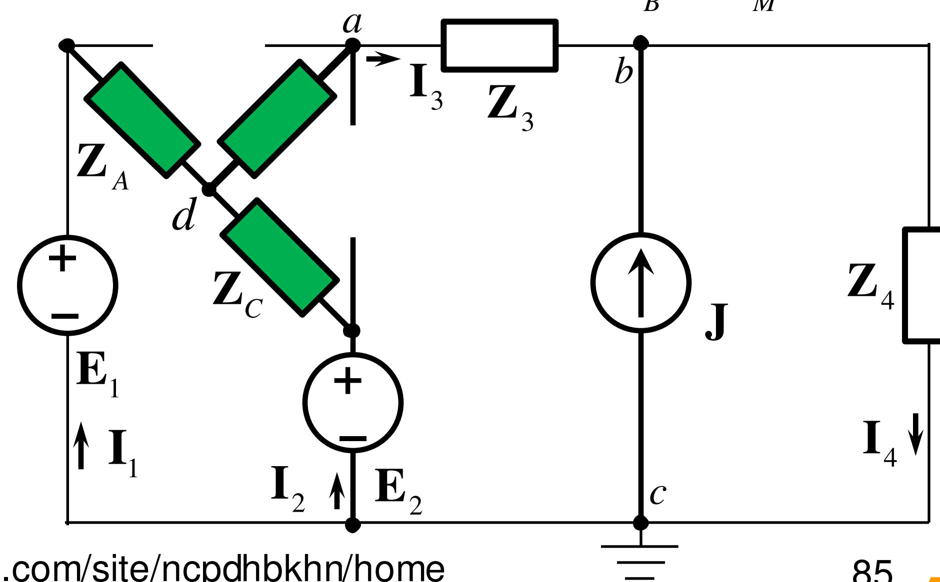
$\mathbf{Z}_C = \mathbf{Z}_2 - \mathbf{Z}_M$

$\mathbf{Z}_B = \mathbf{Z}_M$

$$\begin{cases} \left(\dfrac{1}{\mathbf{Z}_A} + \dfrac{1}{\mathbf{Z}_C} + \dfrac{1}{\mathbf{Z}_3 + \mathbf{Z}_B}\right)\mathbf{V}_d - \dfrac{1}{\mathbf{Z}_3 + \mathbf{Z}_B}\mathbf{V}_b = \dfrac{\mathbf{E}_1}{\mathbf{Z}_A} + \dfrac{\mathbf{E}_2}{\mathbf{Z}_C} \\ -\dfrac{1}{\mathbf{Z}_3 + \mathbf{Z}_B}\mathbf{V}_d + \left(\dfrac{1}{\mathbf{Z}_3 + \mathbf{Z}_B} + \dfrac{1}{\mathbf{Z}_4}\right)\mathbf{V}_b = \mathbf{J} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{V}_d = 88.11 + j40.06\ \text{V} \\ \mathbf{V}_b = 111.12 + j56.43\ \text{V} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = (\mathbf{E}_1 - \mathbf{V}_d)/\mathbf{Z}_A = \boxed{-1.49 - j2.06\ \text{A}} \\ \mathbf{I}_2 = (\mathbf{E}_2 - \mathbf{V}_d)/\mathbf{Z}_C = \boxed{2.40 + j0.79\ \text{A}} \\ \mathbf{I}_3 = (\mathbf{V}_d - \mathbf{V}_b)/(\mathbf{Z}_B + \mathbf{Z}_3) = \boxed{0.91 - j1.28\ \text{A}} \\ \mathbf{I}_4 = \mathbf{V}_b/\mathbf{Z}_4 = \boxed{4.44 + j2.26\ \text{A}} \end{cases}$$

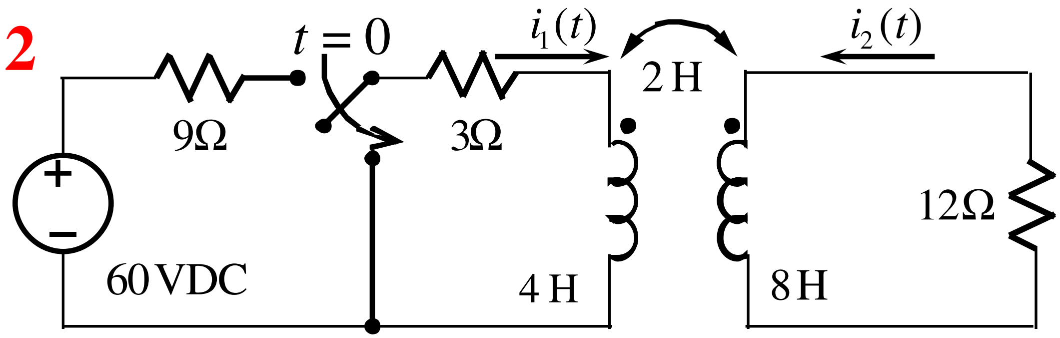


Equivalent Two-port Networks of Magnetically Coupled Circuits (8)

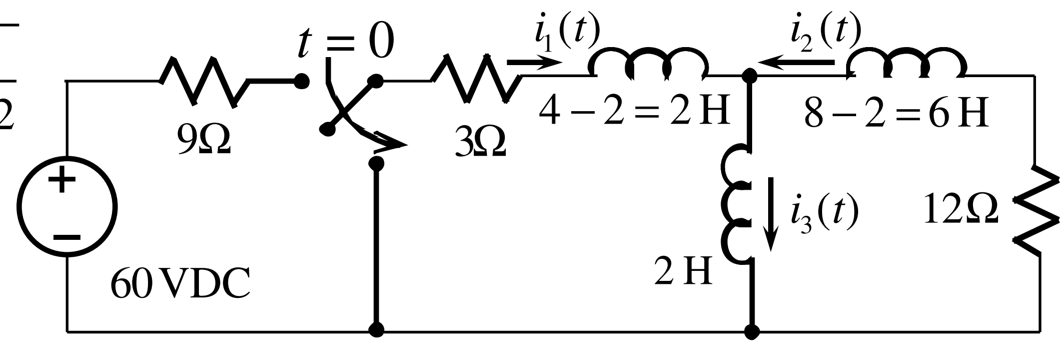
Ex. 4

Find the current $i_2(t)$?

Method 2

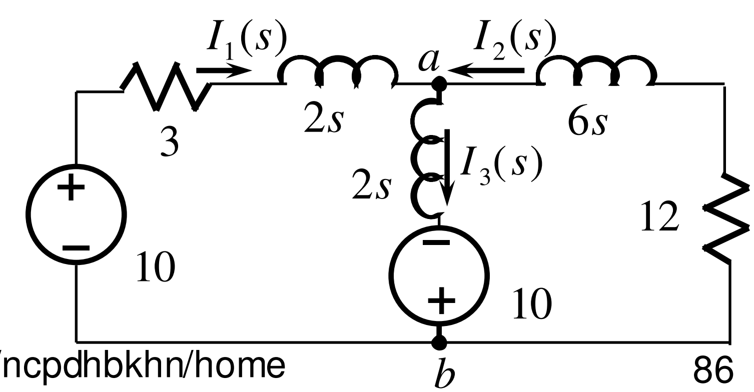


$$i_1(0) = i_3(0) = \frac{60}{12} = 5\text{A};\ i_2(0) = 0$$



$$V_b(s) = 0 \rightarrow V_a(s) = \frac{\dfrac{10}{2s+3} - \dfrac{10}{2s}}{\dfrac{1}{2s+3} + \dfrac{1}{2s} + \dfrac{1}{6s+12}}$$

$$I_2(s) = \frac{-V_a(s)}{6s+12} = \frac{15}{2(7s^2+18s+9)}\ \text{A}$$

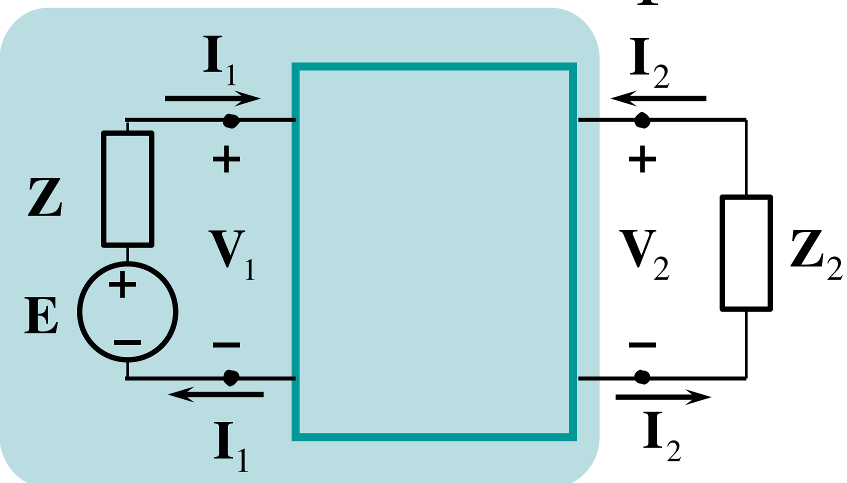


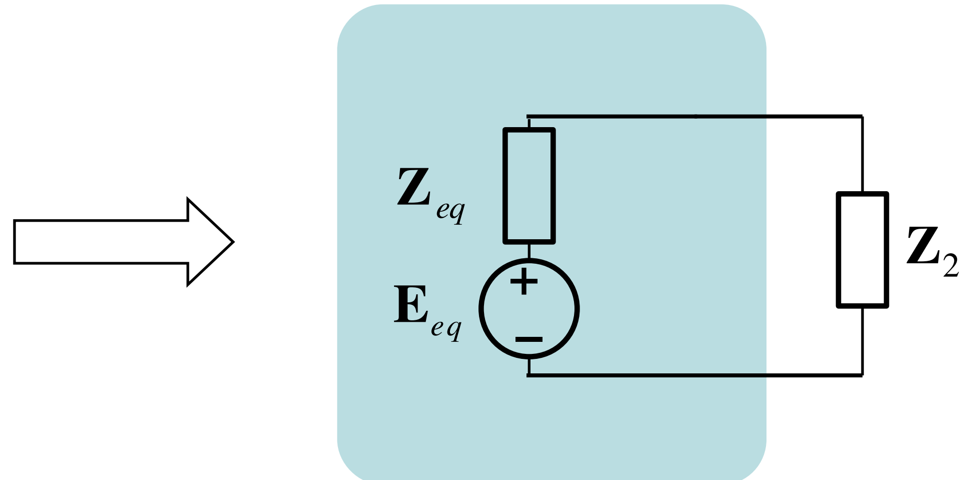
$$\rightarrow \boxed{i_2(t) = 0.8838(e^{-0.6796t} - e^{-1.8918t})\ \text{A}}$$

Two-port Networks

1. Introduction
2. Parameters
3. Relationships between Parameters
4. Two-port Network Analysis
5. Interconnection of Networks
6. T & Π Networks
7. Equivalent Two-port Networks of Magnetically Coupled Circuits
8. **Input Impedance**
9. Transfer Function

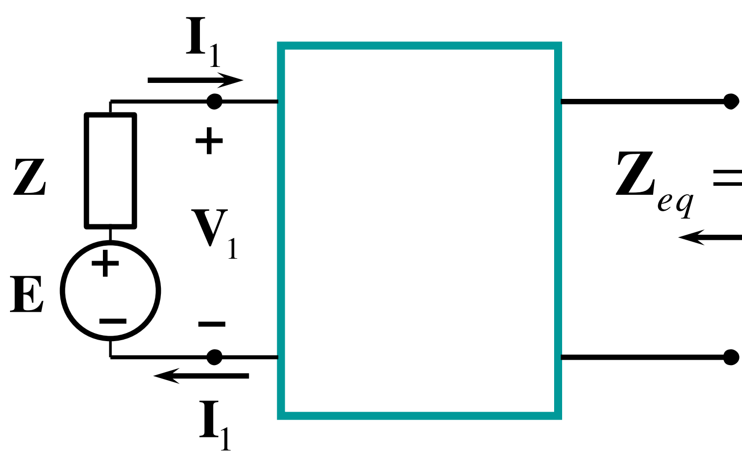
Input Impedance (1)





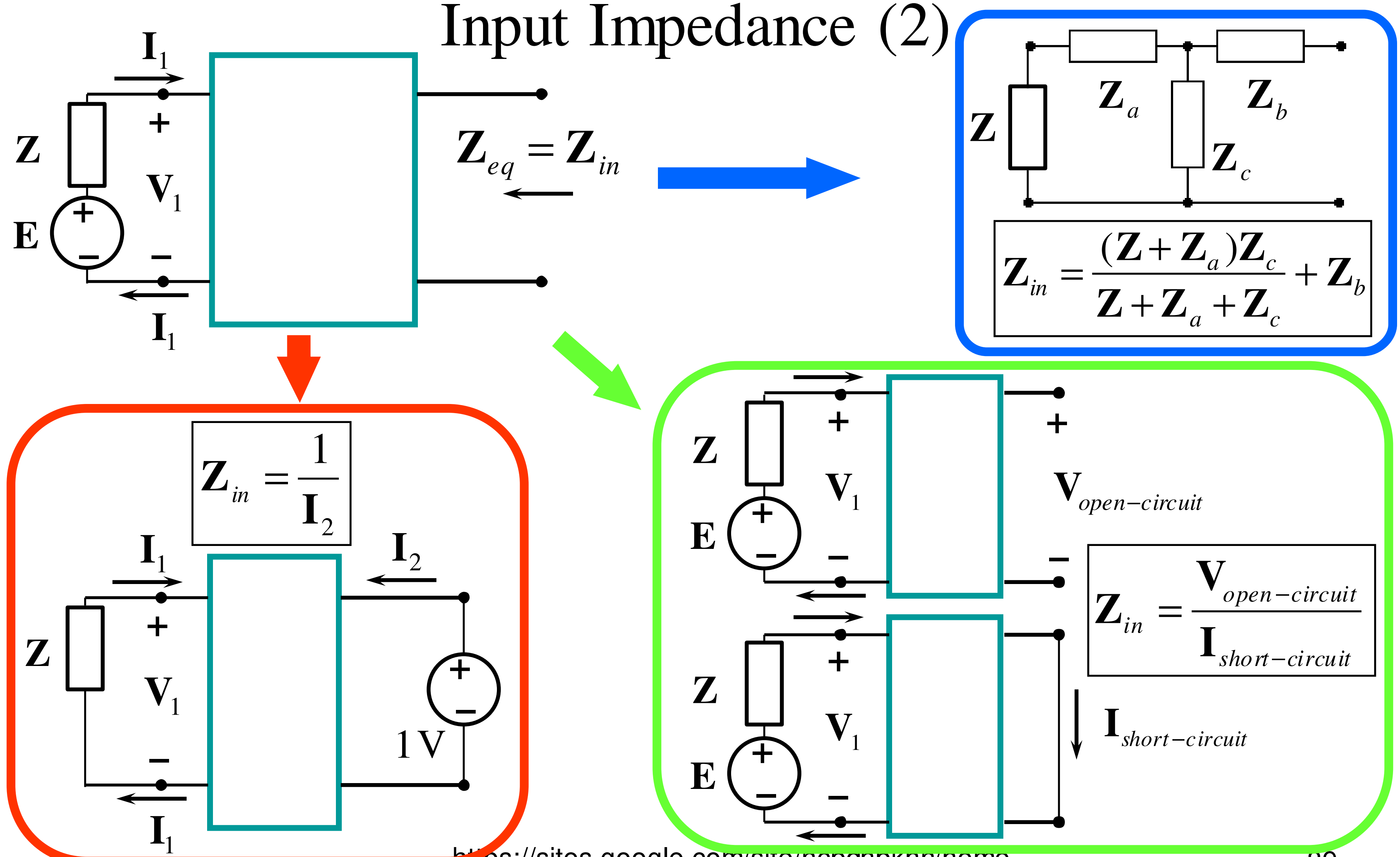
Maximum power for $\mathbf{Z}_2$?

$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$



$$\mathbf{Z}_{eq} = \mathbf{Z}_{in}$$

Input Impedance (2)

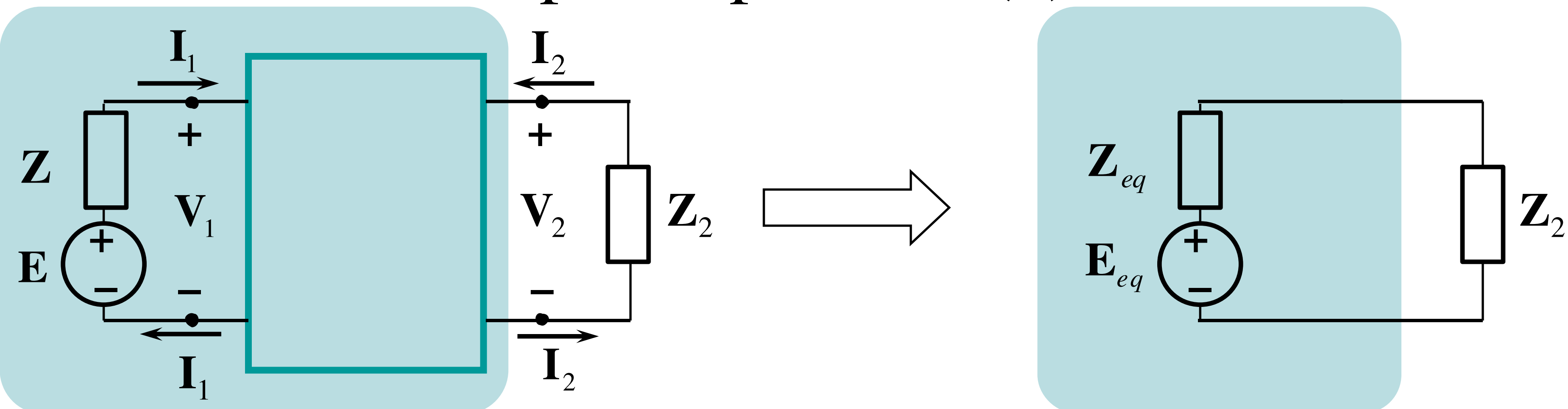


$$Z_{in} = \frac{(Z+Z_a)Z_c}{Z+Z_a+Z_c} + Z_b$$

$$Z_{in} = \frac{1}{I_2}$$

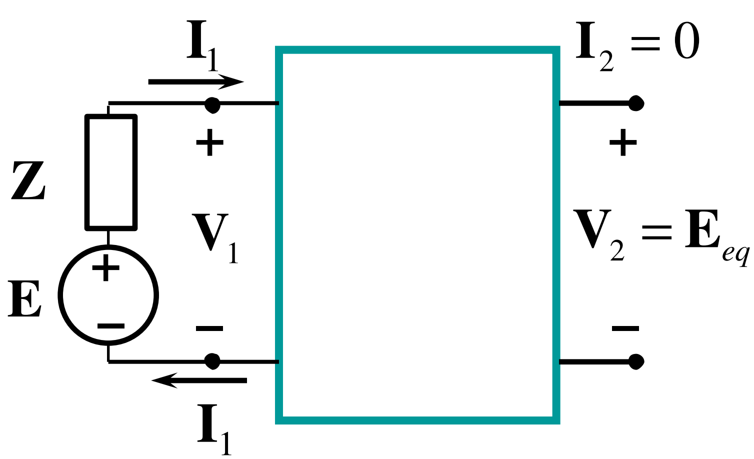
$$Z_{in} = \frac{V_{open-circuit}}{I_{short-circuit}}$$

Input Impedance (3)



$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$

Maximum power for $\mathbf{Z}_2$?



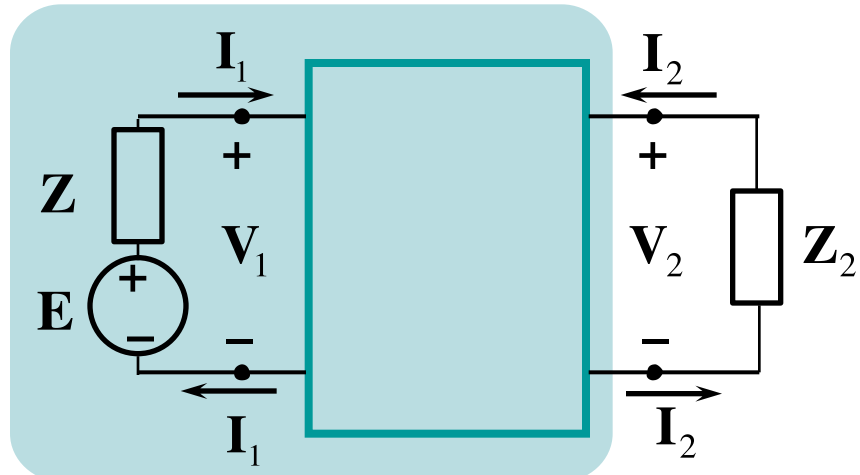
$$\begin{cases} \mathbf{Z}_1\mathbf{I}_1 + \mathbf{V}_1 = \mathbf{E} \\ \mathbf{I}_2 = 0 \\ \begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \end{cases} \rightarrow \mathbf{V}_2$$

Input Impedance (4)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \qquad \mathbf{E} = 220\,\text{V} \qquad \mathbf{Z} = 15 + j25\,\Omega$$

What $\mathbf{Z}_2$ will absorb maximum power from the circuit?



$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$

Method 1

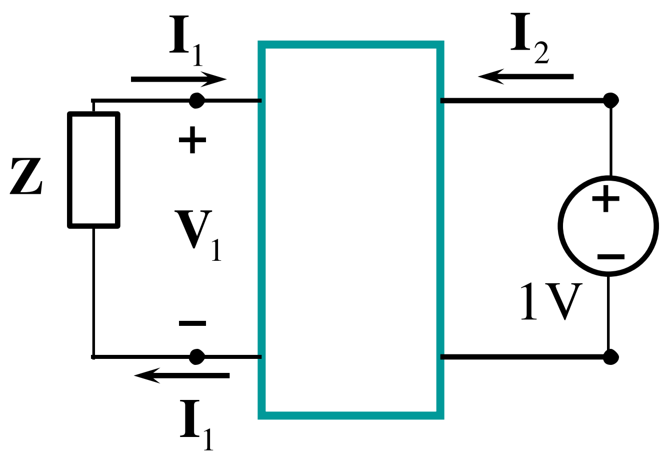
$$\mathbf{Z}_{eq} = \frac{1}{\mathbf{I}_2}$$

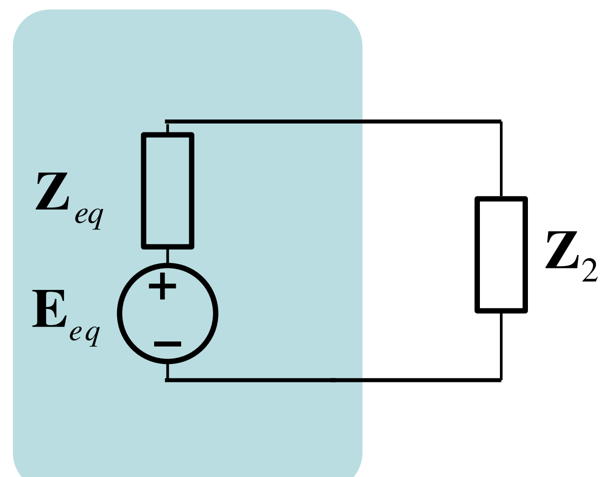
$$(15 + j25)\mathbf{I}_1 + \mathbf{V}_1 = 0$$

$$\mathbf{V}_2 = 1$$

$$\begin{cases} \mathbf{V}_1 = 30\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = 20\mathbf{I}_1 + 50\mathbf{I}_2 \end{cases}$$

$$\rightarrow \mathbf{I}_2 = 0.023 - j0.002\ \text{A}$$

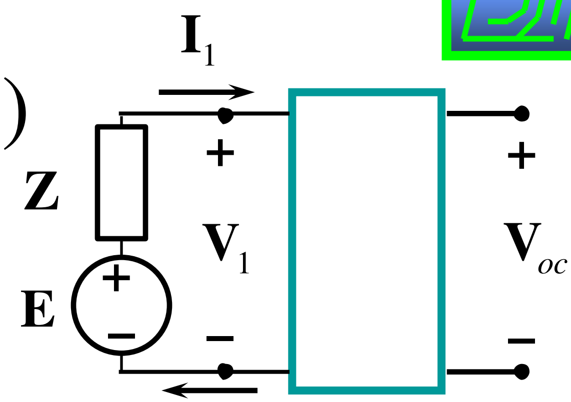




$$\rightarrow \mathbf{Z}_{eq} = 43.15 + j3.75\,\Omega \;\rightarrow \boxed{\mathbf{Z}_2 = 43.15 - j3.75\,\Omega}$$

Input Impedance (5)

Ex. 1



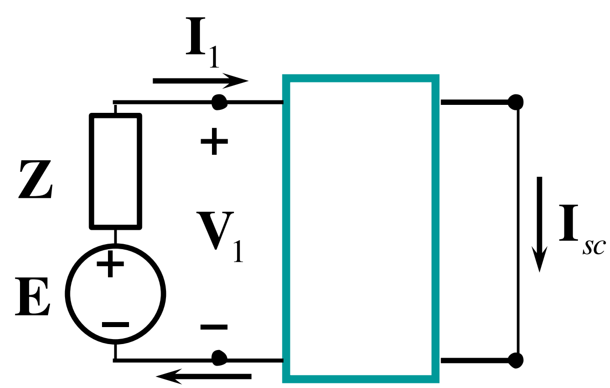
$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \qquad \mathbf{E} = 220\,\text{V} \qquad \mathbf{Z} = 15 + j25\,\Omega$$

What $\mathbf{Z}_2$ will absorb maximum power from the circuit?

$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$

Method 2

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}$$



$$\left.\begin{array}{l} (15 + j25)\mathbf{I}_1 + \mathbf{V}_1 = 220 \\ \mathbf{I}_2 = 0 \\ \begin{cases} \mathbf{V}_1 = 30\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = 20\mathbf{I}_1 + 50\mathbf{I}_2 \end{cases} \end{array}\right\}$$

$$\rightarrow \mathbf{V}_2 = 74.72 - j41.51\,\text{V} = \mathbf{V}_{oc}$$

$$\left.\begin{array}{l} (15 + j25)\mathbf{I}_1 + \mathbf{V}_1 = 220 \\ \mathbf{V}_2 = 0 \\ \begin{cases} \mathbf{V}_1 = 30\mathbf{I}_1 + 20\mathbf{I}_2 \\ \mathbf{V}_2 = 20\mathbf{I}_1 + 50\mathbf{I}_2 \end{cases} \end{array}\right\}$$

$$\rightarrow \mathbf{I}_2 = -1.63 + j1.10\,\text{A} = -\mathbf{I}_{sc}$$

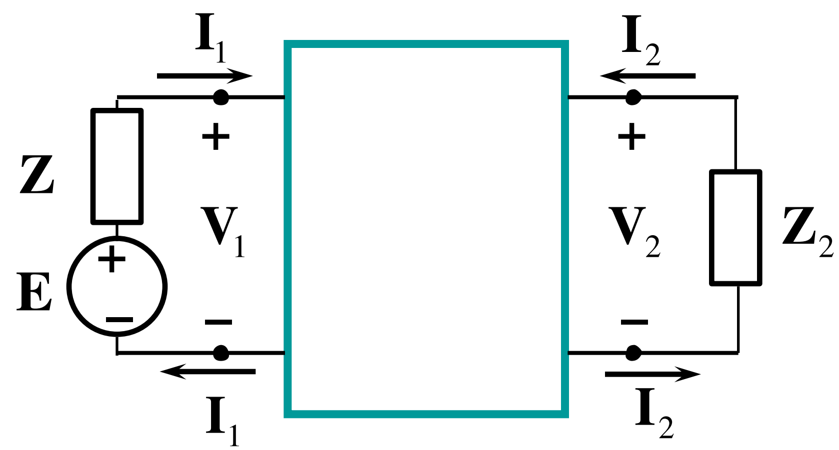
$$\rightarrow \mathbf{Z}_{eq} = \frac{74.72 - j41.51}{1.63 - j1.10} = 43.31 + j3.77\,\Omega \rightarrow \boxed{\mathbf{Z}_2 = 43.31 - j3.77\,\Omega}$$

Input Impedance (5)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \quad \mathbf{E} = 220\,\text{V} \quad \mathbf{Z} = 15 + j25\,\Omega$$

What $\mathbf{Z}_2$ will absorb maximum power from the circuit?



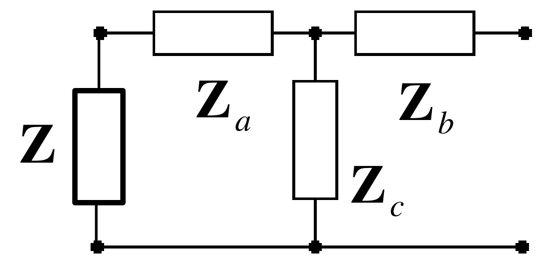
$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$

Method 3 $$\mathbf{Z}_{eq} = \frac{(\mathbf{Z}+\mathbf{Z}_a)\mathbf{Z}_c}{\mathbf{Z}+\mathbf{Z}_a+\mathbf{Z}_c} + \mathbf{Z}_b$$

$$\mathbf{Z}_a = 10\,\Omega$$

$$\mathbf{Z}_c = 20\,\Omega$$

$$\mathbf{Z}_b = 30\,\Omega$$



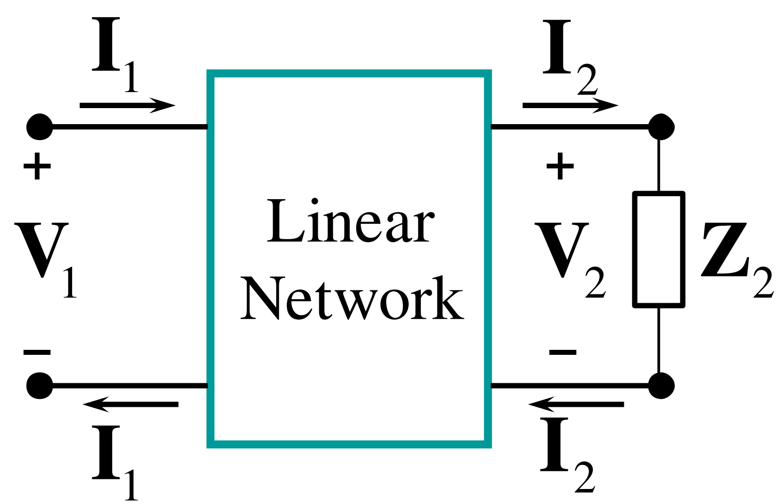
$$\rightarrow \mathbf{Z}_{eq} = \frac{(15+j25+10)20}{15+j25+10+20} + 30 = 43.21 + j3.77\,\Omega \rightarrow \boxed{\mathbf{Z}_2 = 43.21 - j3.77\,\Omega}$$

Input Impedance (6)

$$\begin{cases} \mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{cases}$$

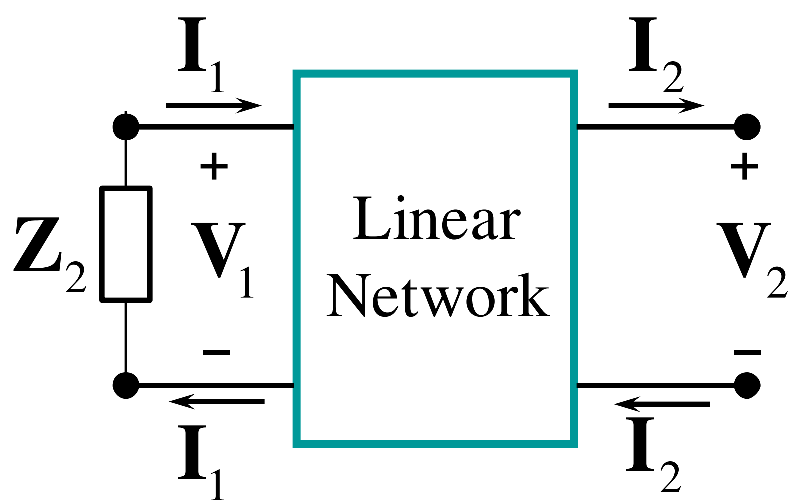
$$\left.\begin{aligned} \mathbf{Z}_{1in} &= \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2}{\mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2} \\ \mathbf{V}_2 &= \mathbf{Z}_2\mathbf{I}_2 \end{aligned}\right\}$$

$$\rightarrow \boxed{\mathbf{Z}_{1in} = \frac{\mathbf{A}\mathbf{Z}_2 - \mathbf{B}}{\mathbf{C}\mathbf{Z}_2 - \mathbf{D}}}$$



$$\left.\begin{aligned} \mathbf{Z}_{2in} &= \frac{\mathbf{V}_2}{-\mathbf{I}_2} = \frac{\mathbf{D}\mathbf{V}_1 - \mathbf{B}\mathbf{I}_1}{-\mathbf{C}\mathbf{V}_1 + \mathbf{A}\mathbf{I}_1} \\ \mathbf{V}_1 &= -\mathbf{Z}_1\mathbf{I}_1 \end{aligned}\right\}$$

$$\rightarrow \boxed{\mathbf{Z}_{2in} = \frac{-\mathbf{D}\mathbf{Z}_1 - \mathbf{B}}{\mathbf{C}\mathbf{Z}_1 + \mathbf{A}}}$$



Input Impedance (7)

$$\left.\begin{array}{l} \mathbf{Z}_{1in} = \dfrac{\mathbf{AZ}_2 - \mathbf{B}}{\mathbf{CZ}_2 - \mathbf{D}} \\ \mathbf{Z}_2 = 0 \ \text{(short-circuit)} \end{array}\right\} \rightarrow \boxed{\mathbf{Z}_{1sc} = \frac{\mathbf{B}}{\mathbf{D}}}$$

$$\left.\begin{array}{l} \mathbf{Z}_{1in} = \dfrac{\mathbf{AZ}_2 - \mathbf{B}}{\mathbf{CZ}_2 - \mathbf{D}} \\ \mathbf{Z}_2 \rightarrow \infty \text{(open-circuit)} \end{array}\right\} \rightarrow \boxed{\mathbf{Z}_{1oc} = \frac{\mathbf{A}}{\mathbf{C}}}$$

$$\left.\begin{array}{l} \mathbf{Z}_{2in} = \dfrac{-\mathbf{DZ}_1 - \mathbf{B}}{\mathbf{CZ}_1 + \mathbf{A}} \\ \mathbf{Z}_1 = 0 \ \text{(short-circuit)} \end{array}\right\} \rightarrow \boxed{\mathbf{Z}_{2sc} = \frac{-\mathbf{B}}{\mathbf{A}}}$$

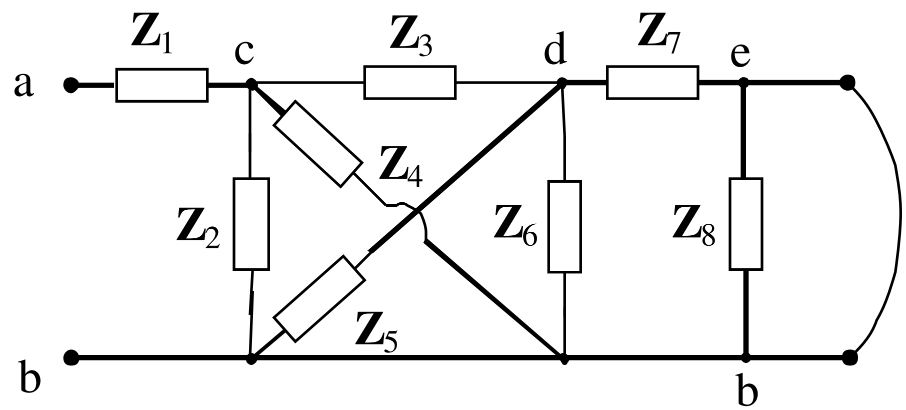
$$\left.\begin{array}{l} \mathbf{Z}_{2in} = \dfrac{-\mathbf{DZ}_1 - \mathbf{B}}{\mathbf{CZ}_1 + \mathbf{A}} \\ \mathbf{Z}_1 \rightarrow \infty \text{(open-circuit)} \end{array}\right\} \rightarrow \boxed{\mathbf{Z}_{2oc} = \frac{-\mathbf{D}}{\mathbf{C}}}$$

Input Impedance (8)

$$\left.\begin{aligned} \mathbf{Z}_{1sc} &= \frac{\mathbf{B}}{\mathbf{D}} \\ \mathbf{Z}_{1oc} &= \frac{\mathbf{A}}{\mathbf{C}} \\ \mathbf{Z}_{2sc} &= \frac{-\mathbf{B}}{\mathbf{A}} \\ \mathbf{Z}_{2oc} &= \frac{-\mathbf{D}}{\mathbf{C}} \end{aligned}\right\} \rightarrow \begin{cases} \mathbf{A} = \sqrt{\dfrac{\mathbf{Z}_{1sc}\mathbf{Z}_{1oc}}{\mathbf{Z}_{2sc}(\mathbf{Z}_{1oc} - \mathbf{Z}_{1sc})}} \\ \mathbf{B} = -\mathbf{A}\mathbf{Z}_{2sc} \\ \mathbf{C} = \dfrac{\mathbf{A}}{\mathbf{Z}_{1oc}} \\ \mathbf{D} = -\dfrac{\mathbf{B}}{\mathbf{Z}_{1sc}} \end{cases}$$

Input Impedance (9)

Ex. 2 Find T?



$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}$$

$$\mathbf{Z}_{1sc} = ?$$

$$\mathbf{A} = \sqrt{\frac{\mathbf{Z}_{1sc}\mathbf{Z}_{1oc}}{\mathbf{Z}_{2sc}(\mathbf{Z}_{1oc} - \mathbf{Z}_{1sc})}}$$

$$\mathbf{B} = -\mathbf{A}\mathbf{Z}_{2sc}$$

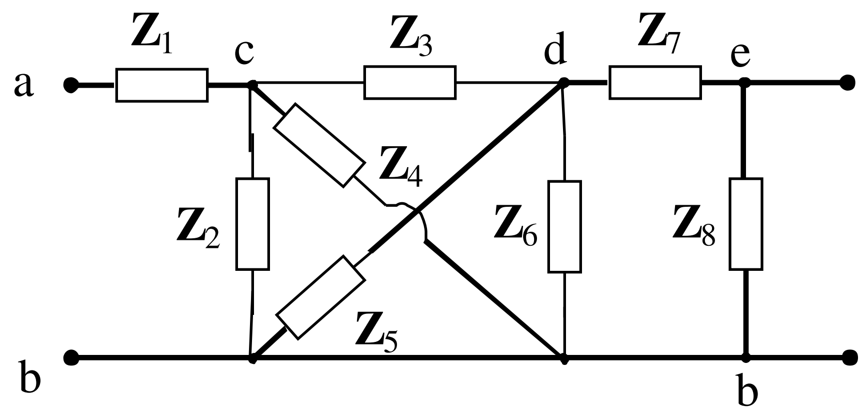
$$\mathbf{C} = \frac{\mathbf{A}}{\mathbf{Z}_{1oc}}$$

$$\mathbf{Z}_{1sc} = \mathbf{Z}_{ab} = \{[(\mathbf{Z}_7//\mathbf{Z}_6//\mathbf{Z}_5)+\mathbf{Z}_3]//\mathbf{Z}_4//\mathbf{Z}_2\}+\mathbf{Z}_1$$

$$\mathbf{D} = -\frac{\mathbf{B}}{\mathbf{Z}_{1sc}}$$

Input Impedance (10)

Ex. 2 Find **T**?



$$\mathbf{A} = \sqrt{\frac{\mathbf{Z}_{1sc}\mathbf{Z}_{1oc}}{\mathbf{Z}_{2sc}(\mathbf{Z}_{1oc} - \mathbf{Z}_{1sc})}}$$

$$\mathbf{Z}_{1oc} = ?$$

$$\mathbf{B} = -\mathbf{A}\mathbf{Z}_{2sc}$$

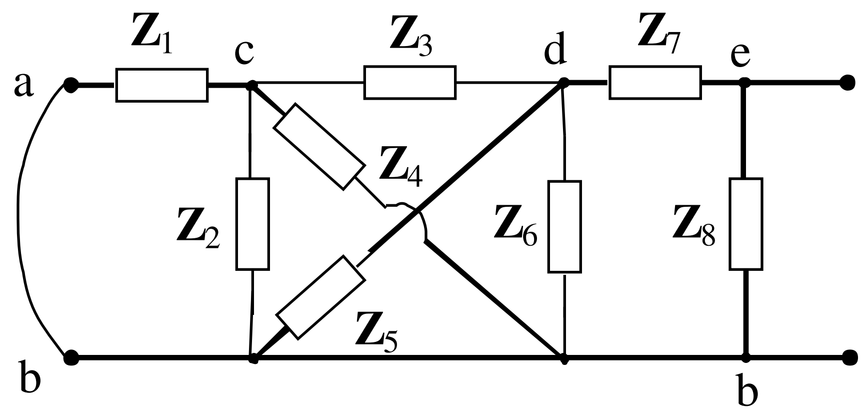
$$\mathbf{C} = \frac{\mathbf{A}}{\mathbf{Z}_{1oc}}$$

$$\mathbf{Z}_{1oc} = \mathbf{Z}_{ab} = \left[\left\{[(\mathbf{Z}_7+\mathbf{Z}_8)//\mathbf{Z}_6//\mathbf{Z}_5]+\mathbf{Z}_3\right\}//\mathbf{Z}_4//\mathbf{Z}_2\right]+\mathbf{Z}_1$$

$$\mathbf{D} = -\frac{\mathbf{B}}{\mathbf{Z}_{1sc}}$$

Input Impedance (11)

Ex. 2 Find **T**?



$$\mathbf{Z}_{2sc} = ?$$

$$\mathbf{A} = \sqrt{\frac{\mathbf{Z}_{1sc}\mathbf{Z}_{1oc}}{\mathbf{Z}_{2sc}(\mathbf{Z}_{1oc} - \mathbf{Z}_{1sc})}}$$

$$\mathbf{B} = -\mathbf{A}\mathbf{Z}_{2sc}$$

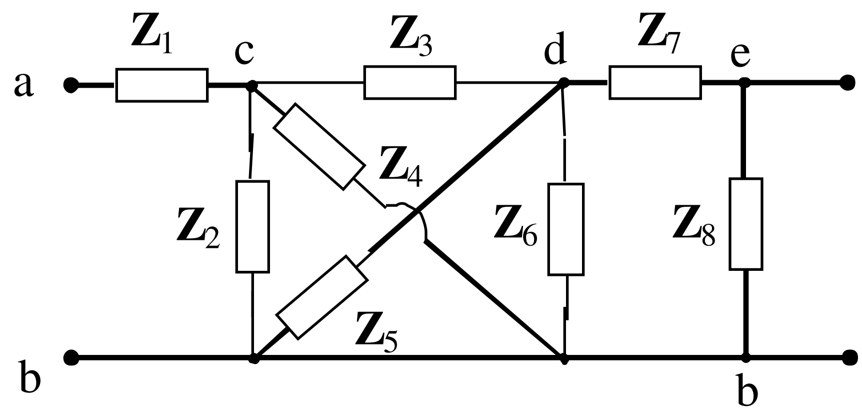
$$\mathbf{C} = \frac{\mathbf{A}}{\mathbf{Z}_{1oc}}$$

$$\mathbf{D} = -\frac{\mathbf{B}}{\mathbf{Z}_{1sc}}$$

$$\mathbf{Z}_{2sc} = \mathbf{Z}_{eb} = [\{[(\mathbf{Z}_1//\mathbf{Z}_2//\mathbf{Z}_4)+\mathbf{Z}_3]//\mathbf{Z}_5//\mathbf{Z}_6\}+\mathbf{Z}_7]//\mathbf{Z}_8$$

Input Impedance (12)

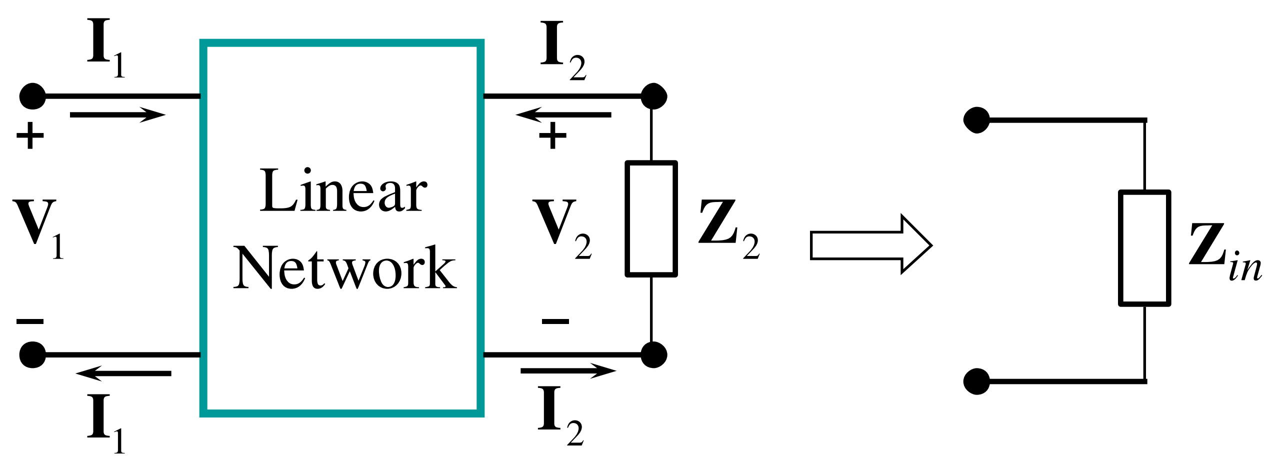
Ex. 2 Find **T**?



$$\left.\begin{array}{l}\mathbf{Z}_{1sc} \\ \\ \mathbf{Z}_{1oc} \\ \\ \mathbf{Z}_{2sc}\end{array}\right\} \rightarrow \left\{\begin{array}{l}\mathbf{A} = \sqrt{\dfrac{\mathbf{Z}_{1sc}\mathbf{Z}_{1oc}}{\mathbf{Z}_{2sc}(\mathbf{Z}_{1oc} - \mathbf{Z}_{1sc})}} \\ \mathbf{B} = -\mathbf{A}\mathbf{Z}_{2sc} \\ \mathbf{C} = \dfrac{\mathbf{A}}{\mathbf{Z}_{1oc}} \\ \mathbf{D} = -\dfrac{\mathbf{B}}{\mathbf{Z}_{1sc}}\end{array}\right.$$

Input Impedance (13)

$$\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases}$$

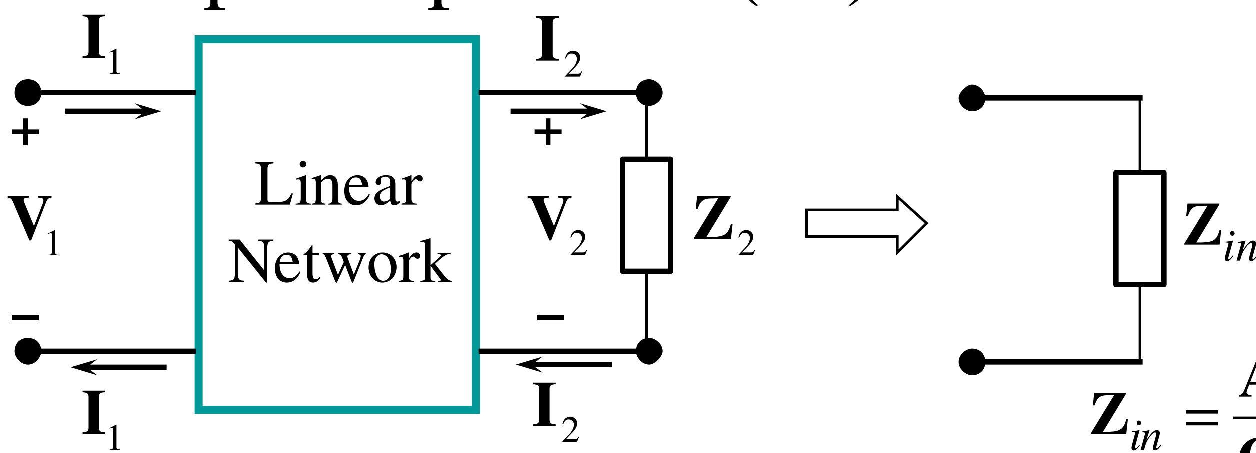


$$\rightarrow \mathbf{I}_1 = \frac{\mathbf{V}_2 - z_{22}\mathbf{I}_2}{\mathbf{Z}_{21}}$$

$$\left.\begin{array}{r} \rightarrow \mathbf{Z}_{in} = \dfrac{\mathbf{V}_1}{\mathbf{I}_1} = \dfrac{\mathbf{z}_{11}\dfrac{\mathbf{V}_2 - \mathbf{z}_{22}\mathbf{I}_2}{\mathbf{Z}_{21}} + \mathbf{z}_{12}\mathbf{I}_2}{\dfrac{\mathbf{V}_2 - z_{22}\mathbf{I}_2}{\mathbf{Z}_{21}}} \\ \mathbf{V}_2 = -\mathbf{Z}_2\mathbf{I}_2 \end{array}\right\} \rightarrow \mathbf{Z}_{in} = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_2}{\mathbf{z}_{22} + \mathbf{Z}_2}$$

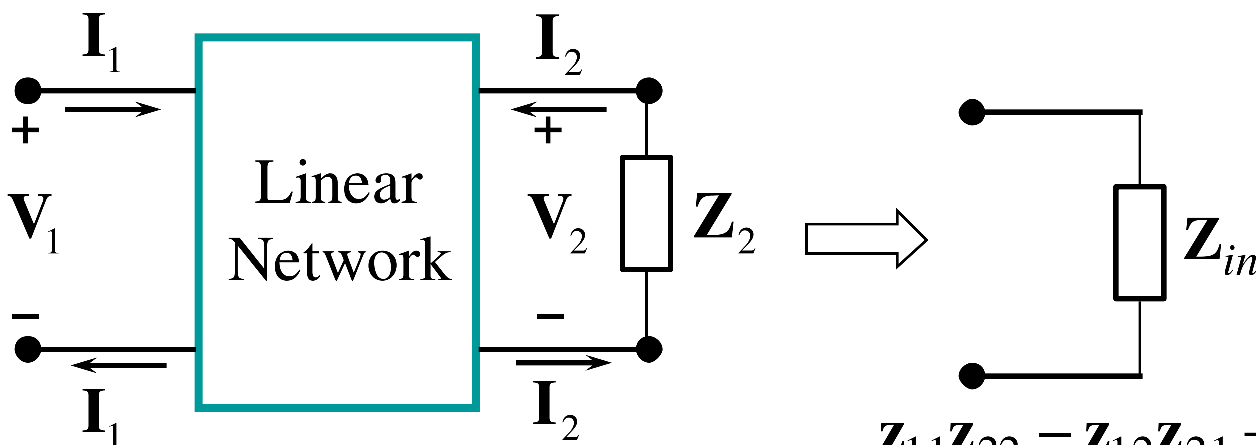
Input Impedance (14)

$$\begin{cases} V_1 = AV_2 - BI_2 \\ I_1 = CV_2 - DI_2 \end{cases}$$



$$Z_{in} = \frac{AZ_2 - B}{CZ_2 - D}$$

$$\begin{cases} V_1 = z_{11}I_1 + z_{12}I_2 \\ V_2 = z_{21}I_1 + z_{22}I_2 \end{cases}$$



$$Z_{in} = \frac{z_{11}z_{22} - z_{12}z_{21} + z_{11}Z_2}{z_{22} + Z_2}$$

Input Impedance (15)

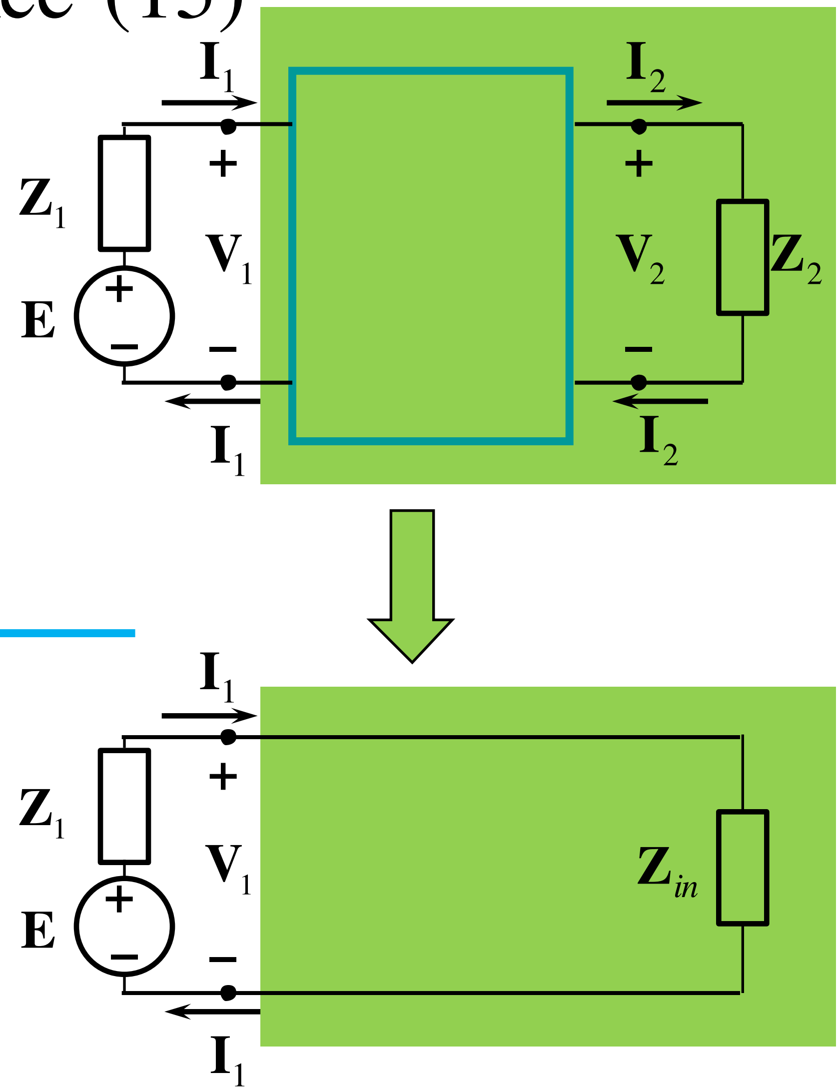
Ex. 3

$$\mathbf{E} = 220\,\text{V};\ \mathbf{Z}_1 = 20\Omega;\ \mathbf{Z}_2 = j50\Omega;\ \mathbf{T} = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}.$$

Solve for $\mathbf{I}_1$?

$$\left.\begin{aligned} &\begin{cases} \mathbf{V}_1 = 3\mathbf{V}_2 + 4\mathbf{I}_2 \\ \mathbf{I}_1 = 2\mathbf{V}_2 + 3\mathbf{I}_2 \end{cases} \\ &20\mathbf{I}_1 + \mathbf{V}_1 = 220 \\ &\mathbf{V}_2 = j50\mathbf{I}_2 \end{aligned}\right\} \rightarrow \mathbf{I}_1 = 10.23 - j0.0024\,\text{A}$$

Method 1



$$\mathbf{Z}_{in} = \frac{\mathbf{A}\mathbf{Z}_2 - \mathbf{B}}{\mathbf{C}\mathbf{Z}_2 - \mathbf{D}} = \frac{3(j50) + 4}{2(j50) + 3} = 1.50 + j0.0050\Omega$$

Method 2

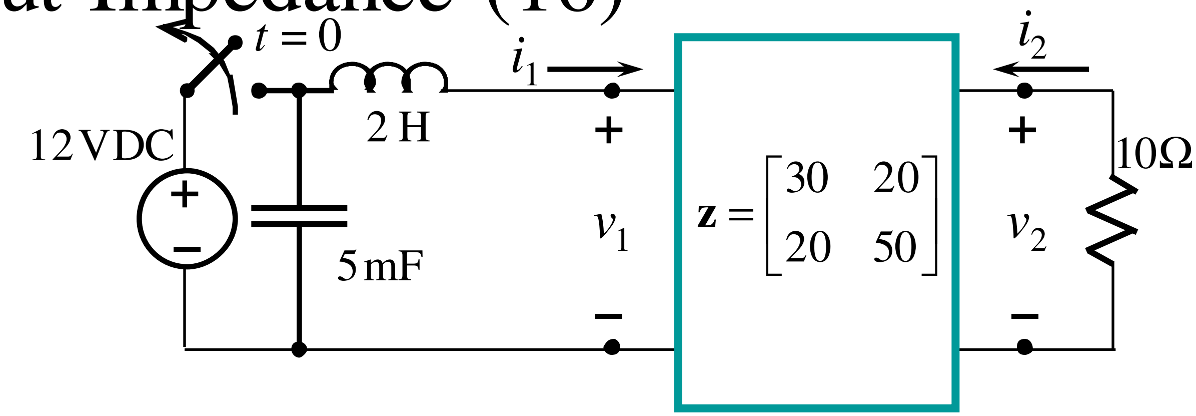
$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_{in}} = \frac{220}{20 + 1.50 + j0.0050}$$

$$= 10.23 - j0.0024\,\text{A}$$

Input Impedance (16)

Ex. 4

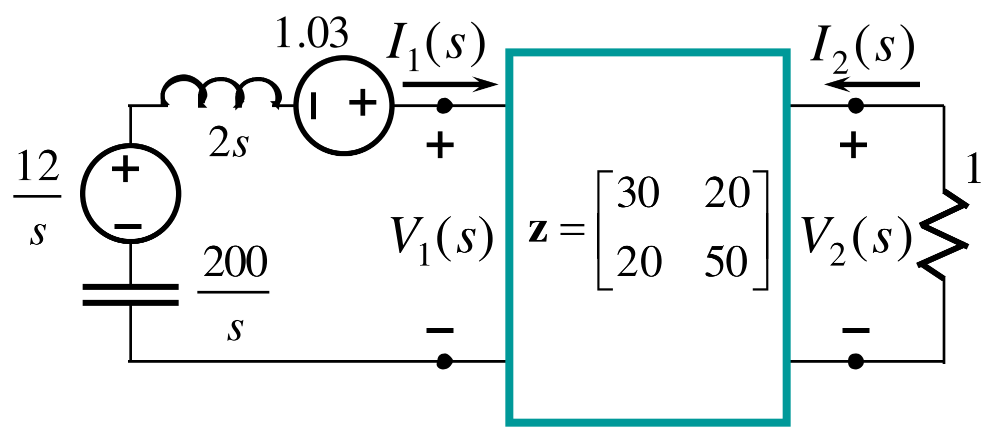
Find $i_1(t)$? Method 1



$v_C(0) = 12\text{ V}$

$$\begin{cases} v_1(0) = 30i_1(0) + 20i_2(0) \\ v_2(0) = 20i_1(0) + 50i_2(0) \\ v_1(0) = 12 \\ v_2(0) = -10i_2(0) \end{cases}$$

$\rightarrow i_1(0) = 0.5143\text{ A} = i_L(0)$



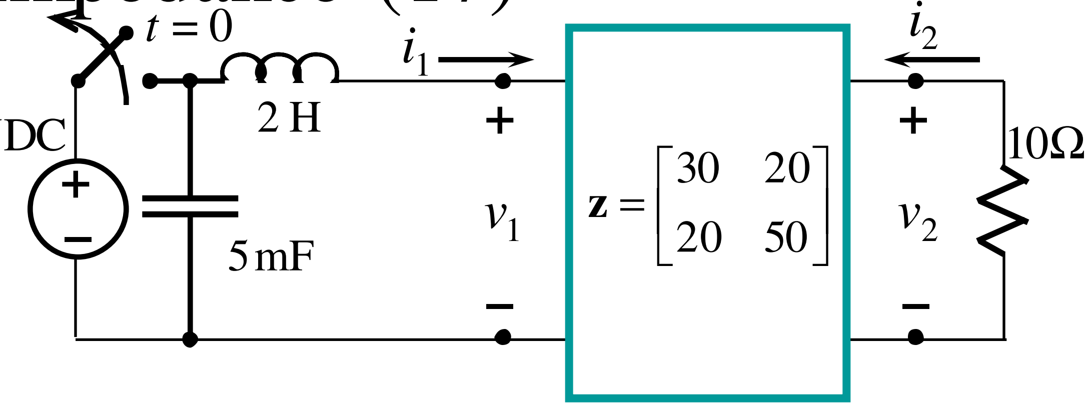
$$\begin{cases} V_1(s) = 30I_1(s) + 20I_2(s) \\ V_2(s) = 20I_1(s) + 50I_2(s) \\ \left(2s + \dfrac{200}{s}\right)I_1(s) + V_1(s) = 1.10 + \dfrac{12}{s} \\ V_2(s) = -10I_2(s) \end{cases}$$

$\rightarrow I_1(s) = \dfrac{0.515s + 6}{s^2 + 11.667s + 100}\text{ A} \quad \rightarrow \boxed{i_1(t) = 0.6334e^{-5.83t}\cos(8.12t - 35.6^\circ)\text{ A}}$

Input Impedance (17)

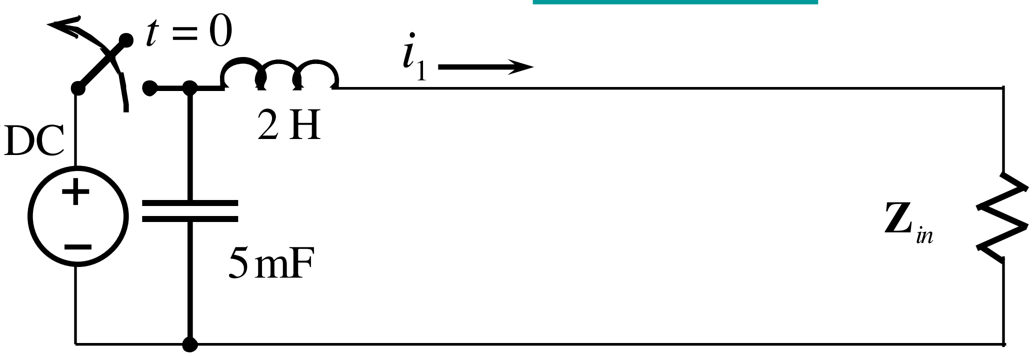
Ex. 4

Find $i_1(t)$? Method 3



$v_C(0) = 12\,\text{V}$

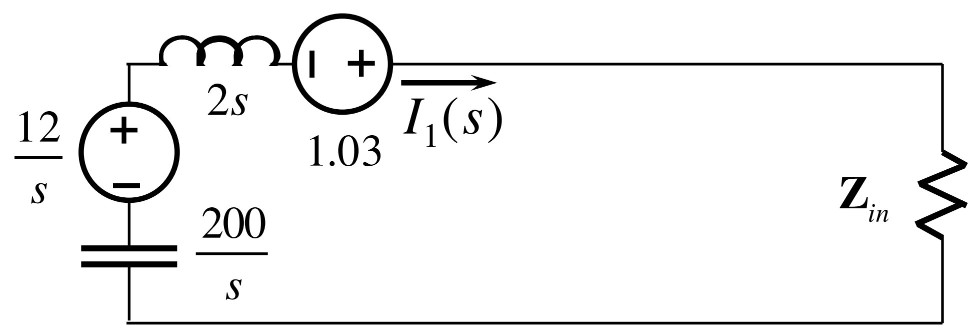
$$\mathbf{z}_{in} = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_2}{\mathbf{z}_{22} + \mathbf{Z}_2} = 23.33\Omega$$



$$i_1(0) = \frac{12}{23.33} = 0.5143\,\text{A} = i_L(0)$$

$$I_1(s) = \frac{1.03 + \dfrac{12}{s}}{\dfrac{200}{s} + 2s + 23.33}$$

$$= \frac{0.515s + 6}{s^2 + 11.667s + 100}\,\text{A}$$



$$\rightarrow \boxed{i_1(t) = 0.6334e^{-5.83t}\cos(8.12t - 35.6^\circ)\,\text{A}}$$

Two-port Networks

1. Introduction
2. Parameters
3. Relationships between Parameters
4. Two-port Network Analysis
5. Interconnection of Networks
6. T & Π Networks
7. Equivalent Two-port Networks of Magnetically Coupled Circuits
8. Input Impedance
9. **Transfer Function**

Transfer Function (1)

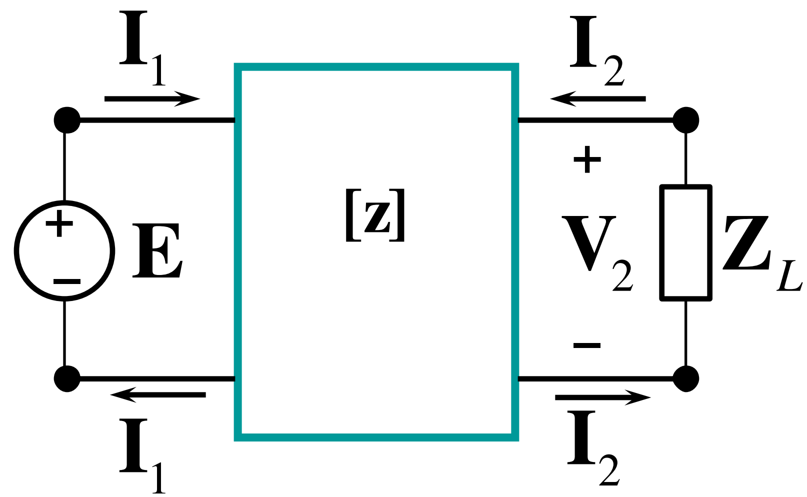
- Voltage transfer function: $$\mathbf{K}_v = \frac{\mathbf{V}_2}{\mathbf{V}_1}$$
- Current transfer function: $$\mathbf{K}_i = \frac{\mathbf{I}_2}{\mathbf{I}_1}$$
- Voltage – current transfer function: $$\mathbf{K}_{vi} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

Transfer Function (2)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \quad \mathbf{E} = 220\,\text{V}, \quad \mathbf{Z}_L = 15 + j25\,\Omega$$

Find $\mathbf{K}_v$, $\mathbf{K}_i$, $\mathbf{K}_{vi}$?



$$\left.\begin{cases} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \begin{matrix} \\ \\ \mathbf{V}_1 = \mathbf{E} \\ \mathbf{V}_2 = -\mathbf{Z}_L\mathbf{I}_2 \end{matrix}\right\}$$

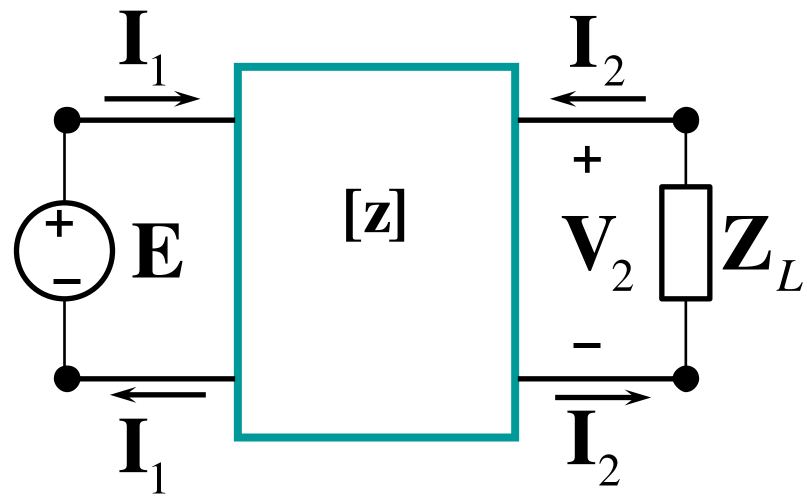
$$\rightarrow \begin{cases} \mathbf{E} = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ -\mathbf{Z}_L\mathbf{I}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{cases} \rightarrow \begin{cases} \mathbf{I}_1 = \dfrac{\mathbf{z}_{22} + \mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{I}_2 = \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \end{cases}$$

Transfer Function (3)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \quad \mathbf{E} = 220\,\text{V}; \quad \mathbf{Z}_L = 15 + j25\,\Omega$$

Find $\mathbf{K}_v$, $\mathbf{K}_i$, $\mathbf{K}_{vi}$?



$$\mathbf{I}_1 = \frac{\mathbf{z}_{22} + \mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E}$$

$$\left.\begin{aligned} \mathbf{I}_2 &= \frac{-\mathbf{z}_{21}}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{V}_2 &= -\mathbf{Z}_L\mathbf{I}_2 \end{aligned}\right\} \rightarrow \mathbf{V}_2 = \frac{\mathbf{z}_{21}\mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E}$$

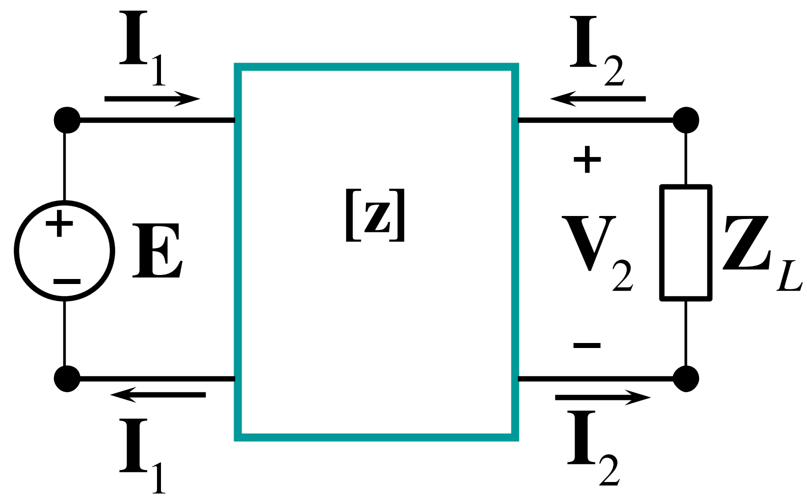
$$\rightarrow \mathbf{K}_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{z}_{21}\mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L} = \boxed{0.28 + j0.19}$$

Transfer Function (4)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \quad \mathbf{E} = 220\,\text{V}, \quad \mathbf{Z}_L = 15 + j25\,\Omega$$

Find $\mathbf{K}_v$, $\mathbf{K}_i$, $\mathbf{K}_{vi}$?



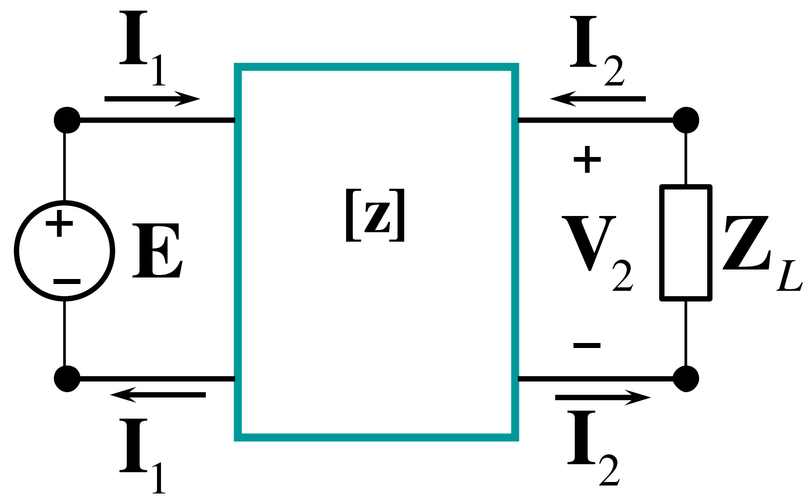
$$\left.\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{z}_{22} + \mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{I}_2 &= \frac{-\mathbf{z}_{21}}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{K}_i &= \frac{\mathbf{I}_2}{\mathbf{I}_1} \end{aligned}\right\} \rightarrow \mathbf{K}_i = \frac{-\mathbf{z}_{21}}{\mathbf{Z}_{22} + \mathbf{Z}_L} = \boxed{-0.27 + j0.10}$$

Transfer Function (5)

Ex. 1

$$\mathbf{z} = \begin{bmatrix} 30 & 20 \\ 20 & 50 \end{bmatrix}; \quad \mathbf{E} = 220\,\text{V}; \quad \mathbf{Z}_L = 15 + j25\,\Omega$$

Find $\mathbf{K}_v$, $\mathbf{K}_i$, $\mathbf{K}_{vi}$?

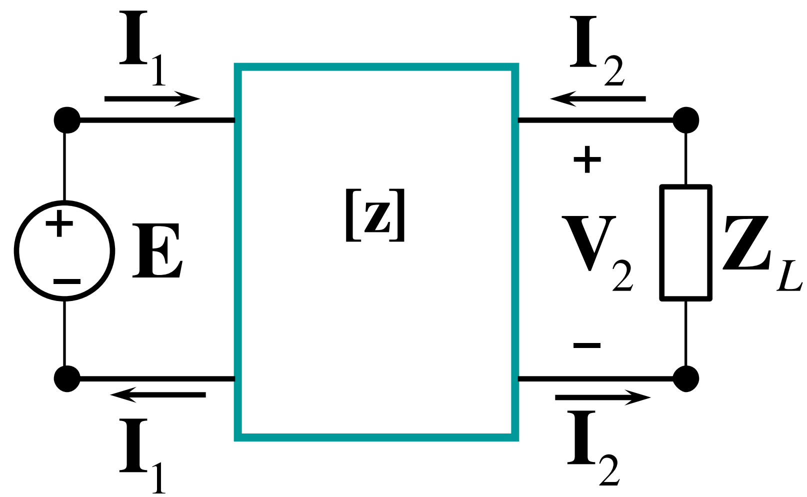


$$\left.\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{z}_{22} + \mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{V}_2 &= \frac{\mathbf{z}_{21}\mathbf{Z}_L}{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} + \mathbf{z}_{11}\mathbf{Z}_L}\mathbf{E} \\ \mathbf{K}_{vi} &= \frac{\mathbf{V}_2}{\mathbf{I}_1} \end{aligned}\right\} \rightarrow \mathbf{K}_{vi} = \frac{\mathbf{z}_{21}\mathbf{Z}_L}{\mathbf{z}_{22} + \mathbf{Z}_L} = \boxed{6.60 + j5.15\ \Omega}$$

Transfer Function (6)

Ex. 2

$\mathbf{E} = 380\,\text{V}; \mathbf{Z}_L = 15 + j25\,\Omega;$
$\mathbf{K}_v = 0.28 + j0.19;$ Find V_2?



$$\left.\begin{array}{l} \mathbf{K}_v = \dfrac{\mathbf{V}_2}{\mathbf{V}_1} \\ \mathbf{V}_1 = \mathbf{E} \end{array}\right\} \rightarrow \mathbf{V}_2 = \mathbf{K}_v\mathbf{E} = (0.28 + j0.19) \times 380$$

$$= 107.7 + j70.5\text{ V}$$

$$\rightarrow \boxed{V_2 = 128.7\,\text{V}}$$