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Engineering Electromagnetics

Coulomb's Law & Electric Field Intensity

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Coulomb’s Law & Electric Field Intensity

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Coulomb's Law (1)

$$\boxed{F = k\frac{Q_1 Q_2}{R^2}}$$

– In free space

– between 2 very small objects (compared to the separation R)

– Q_1 & Q_2 are the positive/negative quantities of charge

– $k = \dfrac{1}{4\pi\varepsilon_0}$

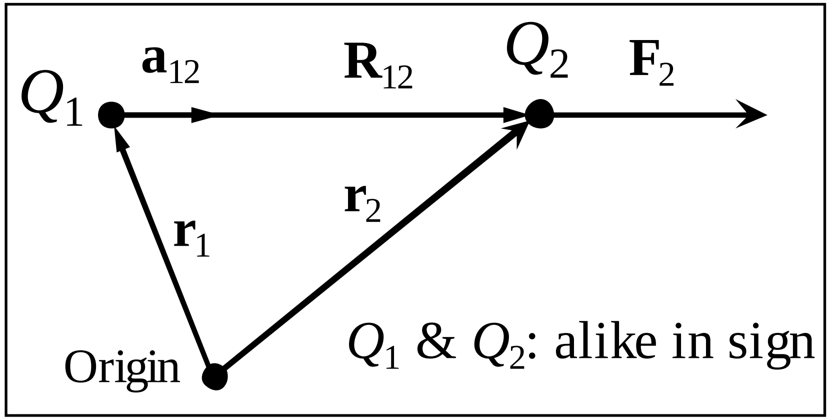
– ε_0: permittivity of free space,

$$\varepsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36\pi} 10^{-9} \text{ F/m}$$

https://www.teylersmuseum.nl/nl/collectie/instrumenten/fk-0556-electrometer-coulomb-balance

Coulomb's Law (2)

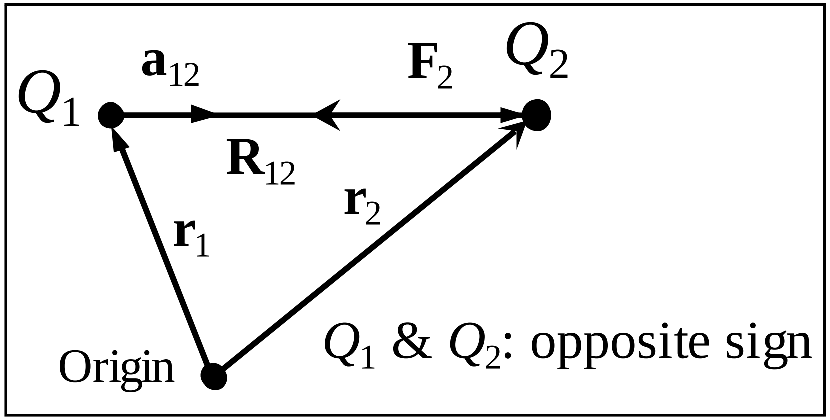
$$\left.\begin{aligned} F &= k\frac{Q_1Q_2}{R^2} \\ k &= \frac{1}{4\pi\varepsilon_0} \end{aligned}\right\} \rightarrow F = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}$$



$$\mathbf{F}_2 = \frac{Q_1Q_2}{4\pi\varepsilon_0 R_{12}^2}\mathbf{a}_{12}$$

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$



Coulomb's Law (3)

Ex.

Given $Q_1 = 4.10^{-4}$ C at $A(3, 2, 1)$ & $Q_2 = -3.10^{-4}$ C at $B(1, 0, 2)$ in vacuum. Find the force exerted on Q_2 by Q_1.

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\mathbf{a}_{12}$$

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = (1-3)\mathbf{a}_x + (0-2)\mathbf{a}_y + (2-1)\mathbf{a}_z = -2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$$

$$\longrightarrow \begin{cases} R_{12} = \sqrt{(-2)^2 + (-2)^2 + 1^2} = 3 \\ \mathbf{a}_{12} = \dfrac{\mathbf{R}_{12}}{R_{12}} = \dfrac{-2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z}{3} \end{cases}$$

$$\rightarrow \mathbf{F}_2 = \frac{4.10^{-4}(-3.10^{-4})}{4\pi\dfrac{1}{36\pi}10^{-9}3^2} \times \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z}{3} = 80\mathbf{a}_x + 80\mathbf{a}_y - 40\mathbf{a}_z \text{ N}$$

Coulomb's Law & Electric Field Intensity

1. Coulomb's Law
2. **Electric Field Intensity**
3. Field Due to a Continuous Volume Charge Distribution
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Electric Field Intensity (1)

- Consider a fixed Q_1 & a test Q_t

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2}\mathbf{a}_{1t} \rightarrow \frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

- *Electric Field Intensity*: the vector force on 1C
- Unit: V/m
- EFI due to a single point charge Q in a vacuum:

$$\boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R}$$

  - **R**: from Q to the point of **E**
  - $\mathbf{a}_R$: unit vector of **R**

Electric Field Intensity (2)

$$\boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R}$$

- If Q is at the center of a spherical coordinate system:

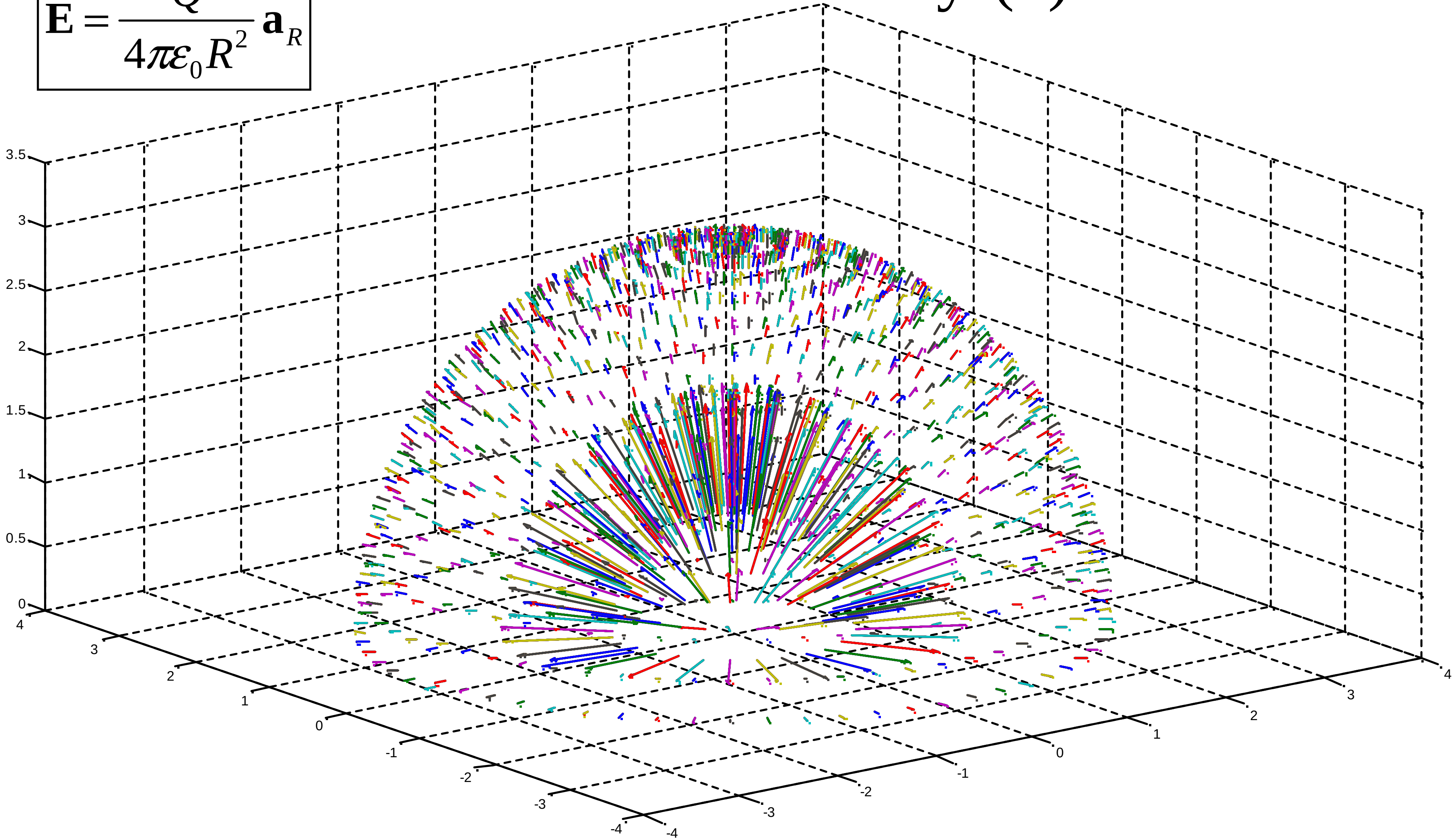
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r$$

- If Q is at the center of a rectangular coordinate system:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)}\left(\frac{x}{\sqrt{x^2 + y^2 + z^2}}\mathbf{a}_x + \frac{y}{\sqrt{x^2 + y^2 + z^2}}\mathbf{a}_y + \frac{z}{\sqrt{x^2 + y^2 + z^2}}\mathbf{a}_z\right)$$

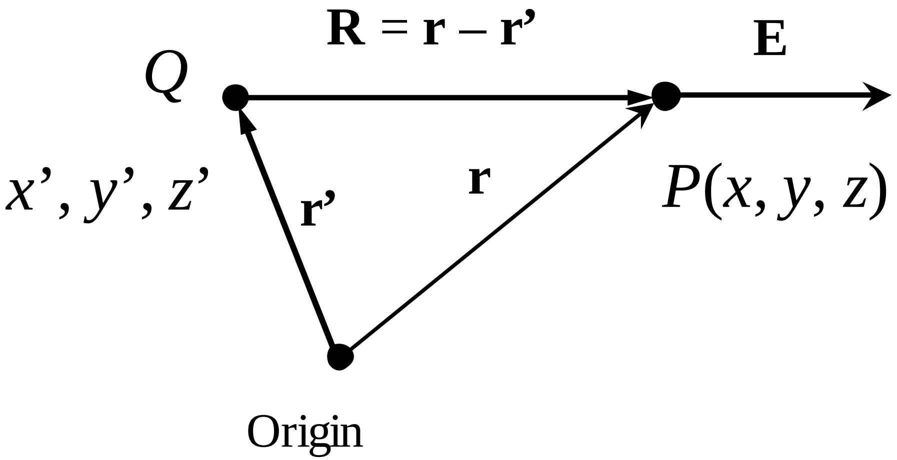
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

Electric Field Intensity (3)



Electric Field Intensity (4)

- If Q is not at the origin:



$$\left.\begin{array}{l}\mathbf{E}(\mathbf{r}) = \dfrac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \\ \mathbf{R} = \mathbf{r} - \mathbf{r}' \rightarrow \begin{cases} R = |\mathbf{r} - \mathbf{r}'| \\ \mathbf{a}_R = \dfrac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \end{cases}\end{array}\right\}$$

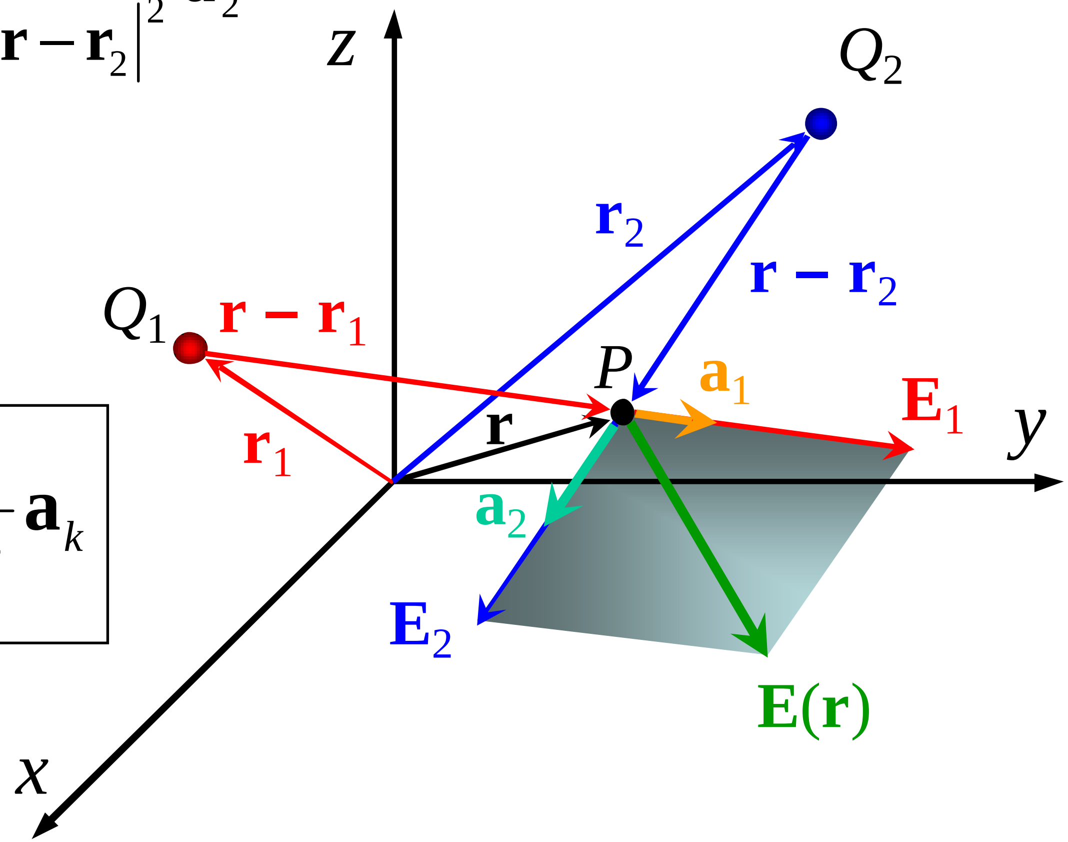
$$\rightarrow \mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \cdot \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} = \frac{Q(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

$$= \frac{Q[(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z]}{4\pi\varepsilon_0 [(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}}$$

Electric Field Intensity (5)

$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\varepsilon_0 \left|\mathbf{r}-\mathbf{r}_1\right|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0 \left|\mathbf{r}-\mathbf{r}_2\right|^2}\mathbf{a}_2$$

$$\boxed{\mathbf{E}(\mathbf{r}) = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 \left|\mathbf{r}-\mathbf{r}_k\right|^2}\mathbf{a}_k}$$



Ex. Electric Field Intensity (6)

Given $Q_1 = 4.10^{-9}$ C at $P_1(3, -2, 1)$, $Q_2 = -3.10^{-9}$ C at $P_2(1, 0, -2)$, $Q_3 = 2.10^{-9}$ C at $P_3(0, 2, 2)$, $Q_4 = -10^{-9}$ C at $P_4(-1, 0, 2)$. Find **E** at $P(1, 1, 1)$.

$$\mathbf{E} = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_k|^2} \mathbf{a}_k$$

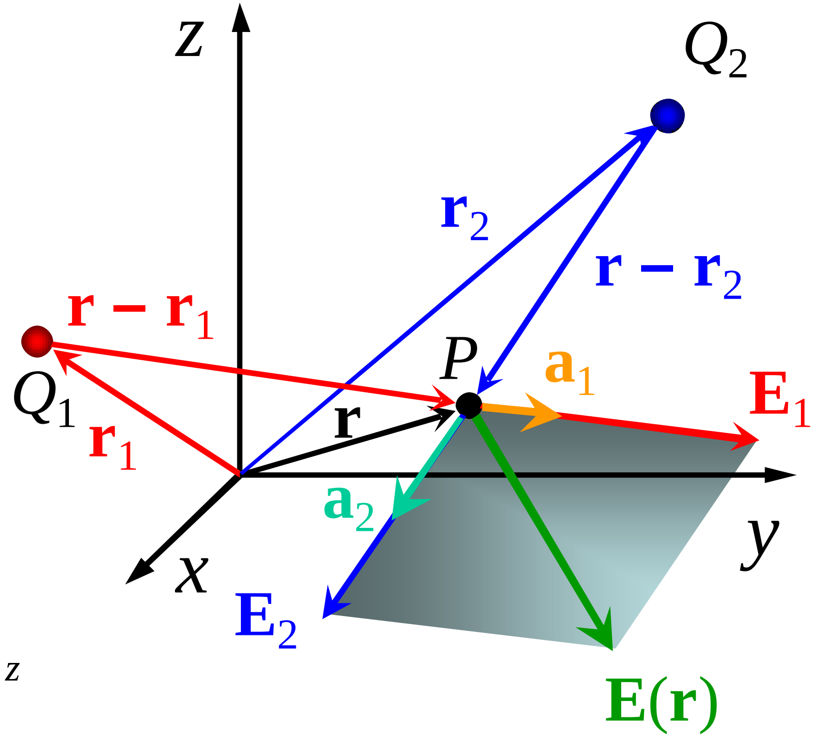
$$= \frac{Q_1}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_1|^2} \mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_2|^2} \mathbf{a}_2 +$$

$$+ \frac{Q_3}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_3|^2} \mathbf{a}_3 + \frac{Q_4}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_4|^2} \mathbf{a}_4$$

$$\mathbf{r} - \mathbf{r}_1 = (x - x_1)\mathbf{a}_x + (y - y_1)\mathbf{a}_y + (z - z_1)\mathbf{a}_z$$

$$= (1-3)\mathbf{a}_x + (1-(-2))\mathbf{a}_y + (1-1)\mathbf{a}_z$$

$$= -2\mathbf{a}_x + 3\mathbf{a}_y$$



Ex. Electric Field Intensity (7)

Given $Q_1 = 4.10^{-9}$ C at $P_1(3, -2, 1)$, $Q_2 = -3.10^{-9}$ C at $P_2(1, 0, -2)$, $Q_3 = 2.10^{-9}$ C at $P_3(0, 2, 2)$, $Q_4 = -10^{-9}$ C at $P_4(-1, 0, 2)$. Find **E** at $P(1, 1, 1)$.

$$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2 + \frac{Q_3}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_3|^2}\mathbf{a}_3 + \frac{Q_4}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}_4|^2}\mathbf{a}_4$$

$$\mathbf{r}-\mathbf{r}_1 = -2\mathbf{a}_x + 3\mathbf{a}_y \rightarrow \begin{cases} |\mathbf{r}-\mathbf{r}_1| = \sqrt{(-2)^2 + 3^2} = 3.32 \\ \mathbf{a}_1 = \dfrac{\mathbf{r}-\mathbf{r}_1}{|\mathbf{r}-\mathbf{r}_1|} = \dfrac{-2}{3.32}\mathbf{a}_x + \dfrac{3}{3.32}\mathbf{a}_y = -0.60\mathbf{a}_x + 0.91\mathbf{a}_y \end{cases}$$

$$|\mathbf{r}-\mathbf{r}_2| = 3.16 \qquad \mathbf{a}_2 = 0.32\mathbf{a}_y + 0.95\mathbf{a}_z$$

$$|\mathbf{r}-\mathbf{r}_3| = 1.73 \qquad \mathbf{a}_3 = 0.58\mathbf{a}_x - 0.58\mathbf{a}_y - 0.58\mathbf{a}_z$$

$$|\mathbf{r}-\mathbf{r}_4| = 2.45 \qquad \mathbf{a}_4 = 0.82\mathbf{a}_x + 0.41\mathbf{a}_y - 0.41\mathbf{a}_z$$

Ex.

Electric Field Intensity (8)

Given Q_1 = 4.10^{-9} C at $P_1(3, -2, 1)$, $Q_2 = -3.10^{-9}$ C at $P_2(1, 0, -2)$, Q_3 = 2.10^{-9} C at $P_3(0, 2, 2)$, $Q_4 = -10^{-9}$ C at $P_4(-1, 0, 2)$. Find **E** at $P(1, 1, 1)$.

$$\mathbf{E} = \frac{4\times10^{-4}}{4\pi\varepsilon_0\times3.32^2}(-0.60\mathbf{a}_x + 0.91\mathbf{a}_y)$$

$$+\frac{-3\times10^{-4}}{4\pi\varepsilon_0\times3.16^2}(0.32\mathbf{a}_y + 0.95\mathbf{a}_z)+$$

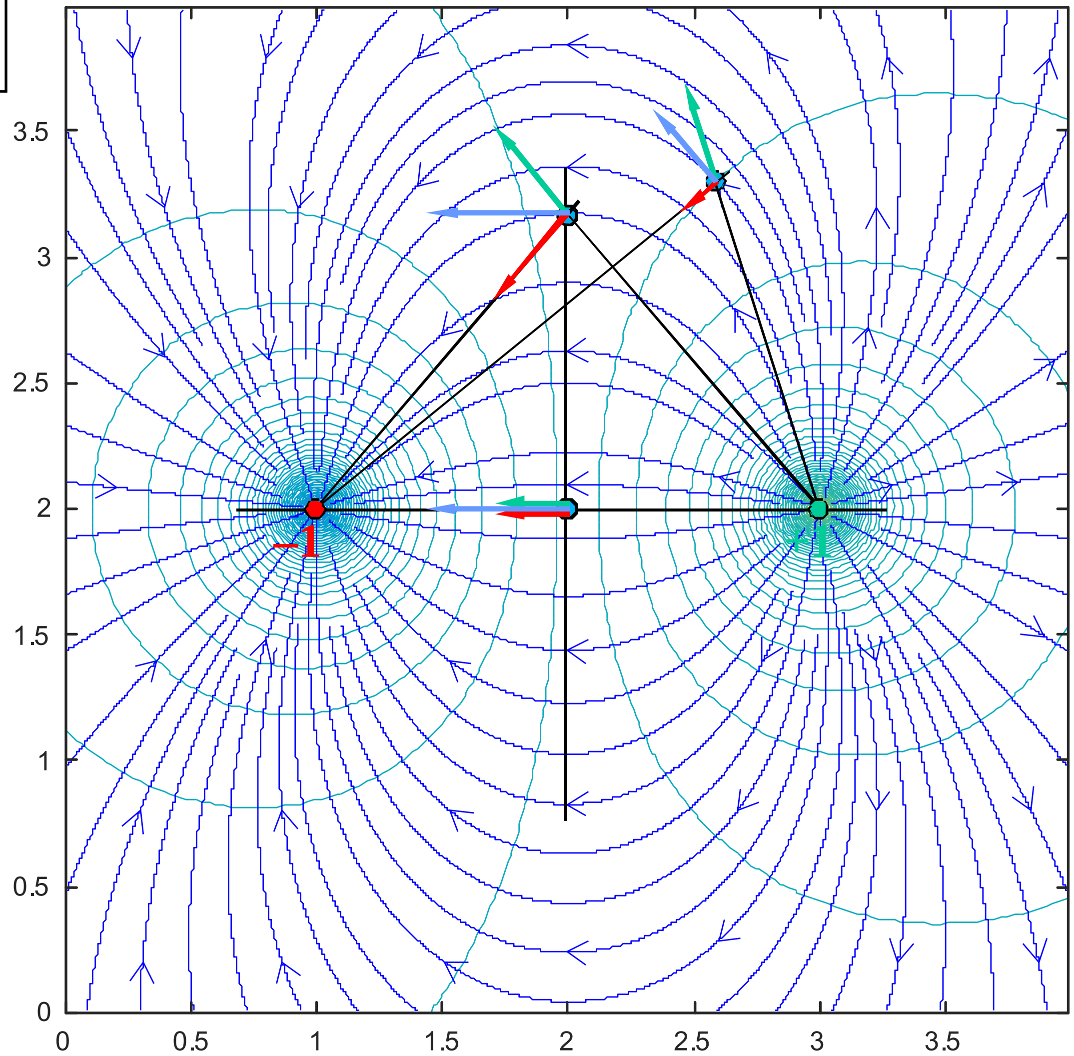
$$+\frac{2\times10^{-4}}{4\pi\varepsilon_0\times1.73^2}(0.58\mathbf{a}_x - 0.58\mathbf{a}_y - 0.58\mathbf{a}_z)+$$

$$+\frac{-10^{-4}}{4\pi\varepsilon_0\times2.45^2}(0.82\mathbf{a}_x + 0.41\mathbf{a}_y - 0.41\mathbf{a}_z)$$

$$= 24.66\mathbf{a}_x + 9.99\mathbf{a}_y - 32.40\mathbf{a}_z \ \text{V/m}$$

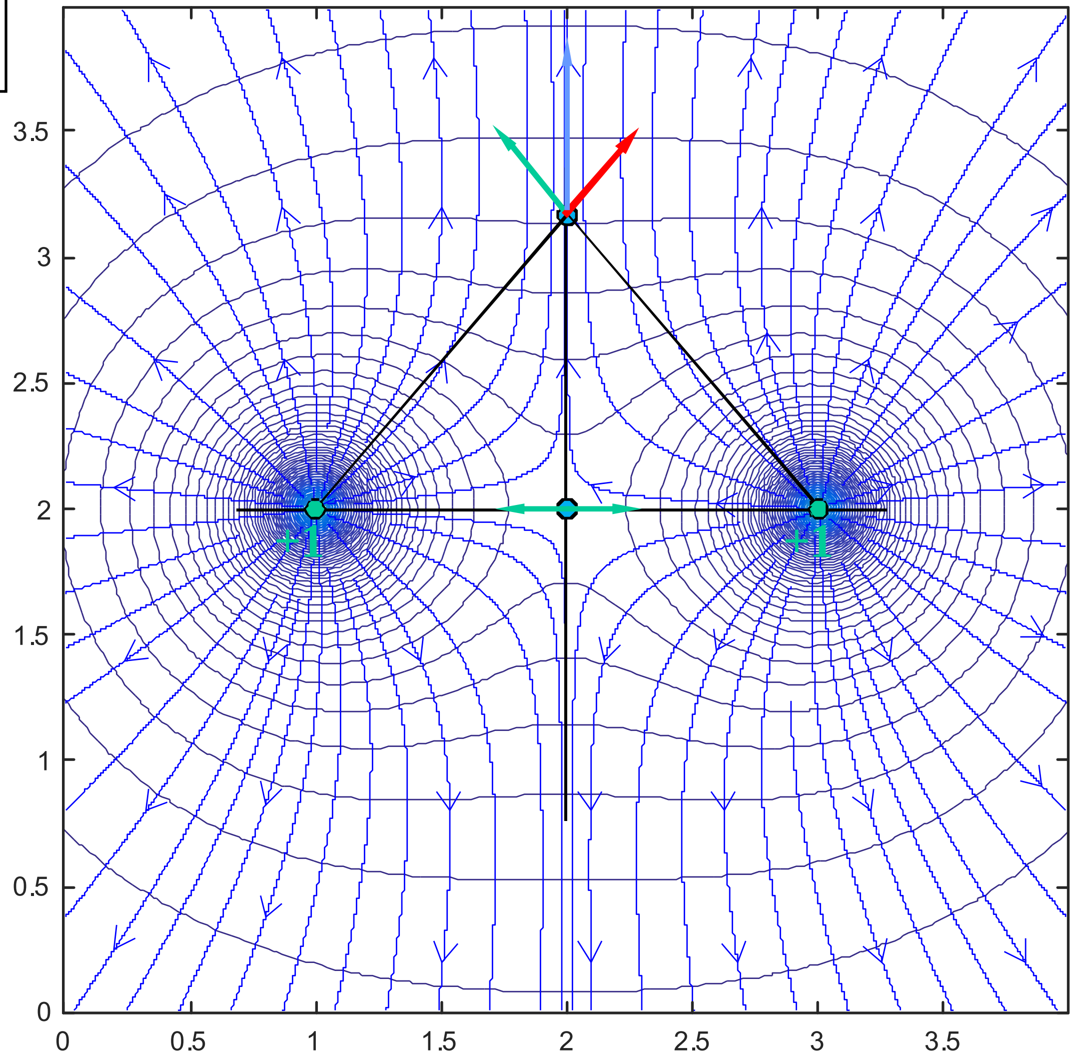
Electric Field Intensity (9)

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$



$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

Electric Field Intensity (10)



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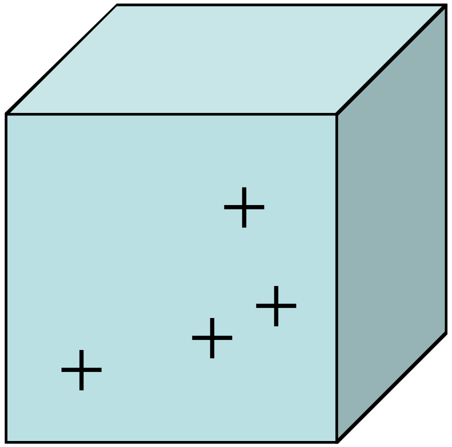
Volume Charge (1)



- Volume charge density (unit C/m^3):

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

$$Q = \int_V \rho_v dv$$

Volume Charge (2)

- EFI at **r** due to ΔQ at **r'** :

$$\left.\begin{array}{r}\mathbf{E}(\mathbf{r}) = \dfrac{Q}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2}.\dfrac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \rightarrow \Delta\mathbf{E}(\mathbf{r}) = \dfrac{\Delta Q}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2}.\dfrac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \\ \Delta Q = \rho_v \Delta v\end{array}\right\}$$

$$\rightarrow \Delta\mathbf{E}(\mathbf{r}) = \frac{\rho_v \Delta v}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2}.\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

- $\rightarrow$ EFI at **r** due to a volume charge:

$$\boxed{\mathbf{E}(\mathbf{r}) = \int_V \frac{\rho_v(\mathbf{r}')dv'}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2}.\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}}$$

Volume Charge (3)

$$\mathbf{E}(\mathbf{r}) = \int_V \frac{\rho_v(\mathbf{r}')dv'}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2} \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

- **r** : the vector locates **E**
- **r**’: the vector locates the volume charge $\rho(\mathbf{r}')dv'$

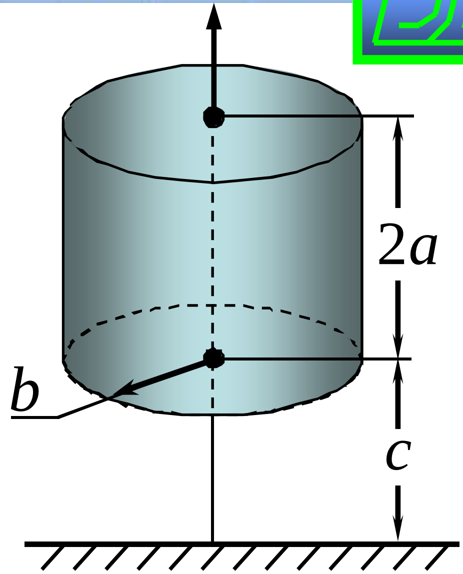
Ex.

Volume Charge (4)

A thundercloud in the form of a cylinder of radius b = 1000 m, height $2a$ = 4000 m, with its bottom c = 1000 m above ground. The cloud has a charge density $\rho_v = 10^{-9}$ C/m^3 uniformly distributed throughout its volume. Find EFI at:

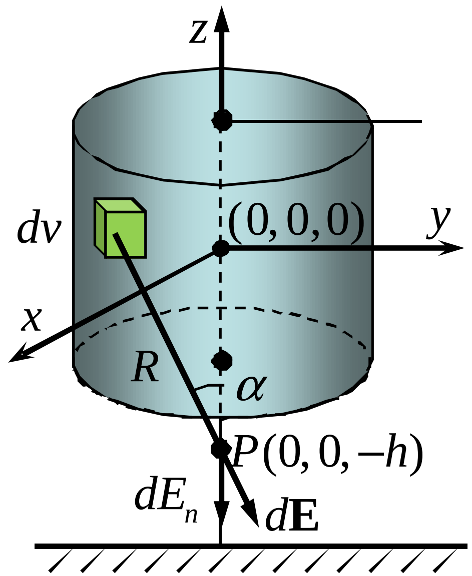
a) Ground level, below the center of the cloud?

b) The bottom of the cloud, on its axis?



$$\left.\begin{aligned} dE &= \frac{dQ}{4\pi\varepsilon_0 R^2} \\ dQ &= \rho_v dv \\ dv &= \rho d\rho d\varphi dz \\ R &= \sqrt{\rho^2 + (h+z)^2} \end{aligned}\right\}$$

$$\rightarrow dE = \frac{\rho_v \rho d\rho d\varphi dz}{4\pi\varepsilon_0 \left[\rho^2 + (h+z)^2\right]}$$



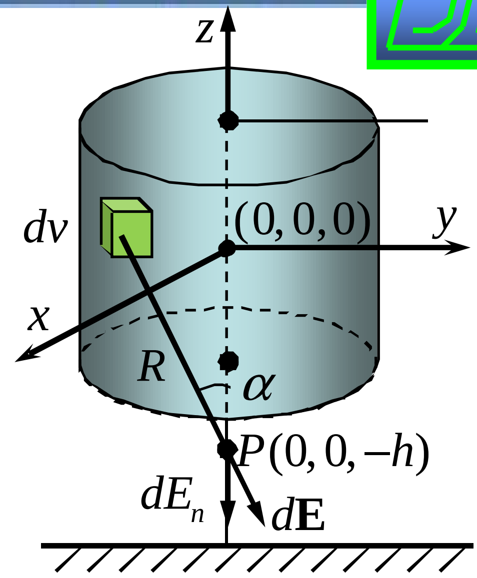
Ex.

Volume Charge (5)

A thundercloud in the form of a cylinder of radius b = 1000 m, height $2a$ = 4000 m, with its bottom c = 1000 m above ground. The cloud has a charge density $\rho_v = 10^{-9}$ C/m^3 uniformly distributed throughout its volume. Find EFI at:

a) Ground level, below the center of the cloud?

b) The bottom of the cloud, on its axis?



$$dE = \frac{\rho_v \rho d\rho d\varphi dz}{4\pi\varepsilon_0\left[\rho^2 + (h+z)^2\right]}$$

$$dE_n = dE\cos\alpha = \frac{\rho_v \rho d\rho d\varphi dz}{4\pi\varepsilon_0\left[\rho^2 + (h+z)^2\right]}\frac{h+z}{R} = \frac{\rho_v \rho (h+z) d\rho d\varphi dz}{4\pi\varepsilon_0\left[\rho^2 + (h+z)^2\right]^{3/2}}$$

$$\mathbf{E} = -\mathbf{a}_z\int_V dE_n = -\mathbf{a}_z\int_{\rho=0}^{b}\int_{\varphi=0}^{2\pi}\int_{z=-a}^{a}\frac{\rho_v \rho (h+z) d\rho d\varphi dz}{4\pi\varepsilon_0\left[\rho^2 + (h+z)^2\right]^{3/2}}$$

$$= -\mathbf{a}_z\frac{\rho_v}{2\varepsilon_0}\left[2a + \sqrt{b^2 + (h-a)^2} - \sqrt{b^2 + (h+a)^2}\right] \text{V/m}$$

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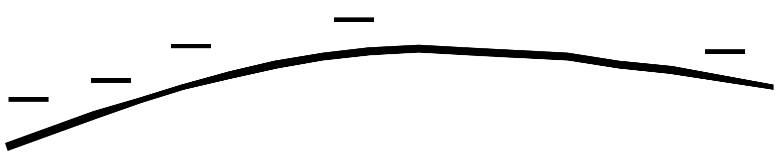
Line Charge (1)

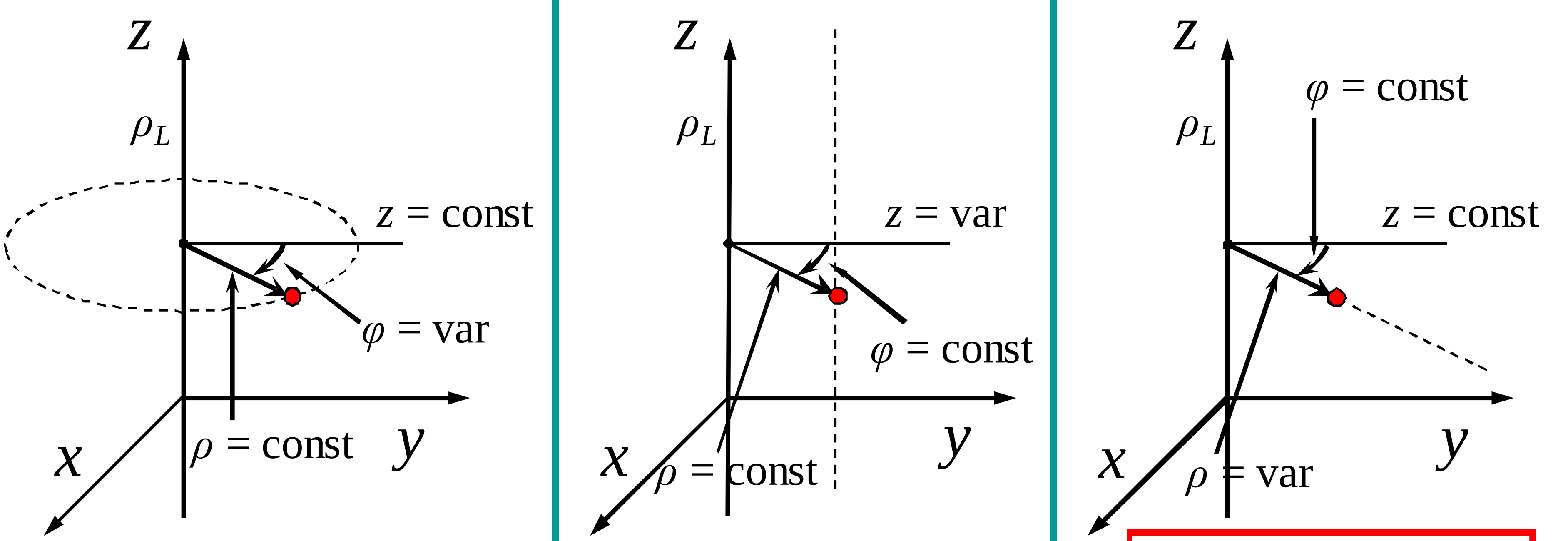


- Line charge density ρ_L (unit: C/m)
- Cylindrical coordinate system
- EFI of an infinite uniform line charge has only an E_ρ component & it varies only with ρ

Line Charge (2)

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \qquad\qquad \mathbf{E} = E_\rho(\rho)\mathbf{a}_\rho$$

EFI of an infinite uniform line charge has only an E_ρ component & it varies only with ρ

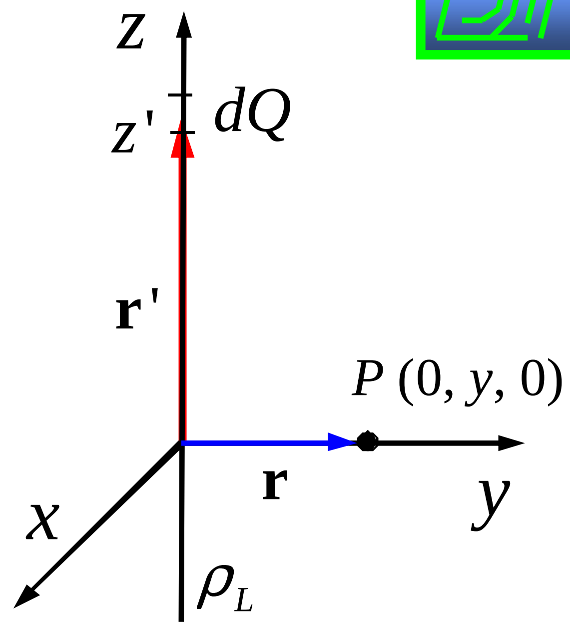


$$\left.\begin{array}{l}\rho = \text{const}\\ \varphi = \text{var}\\ z = \text{const}\end{array}\right\} \rightarrow E = \text{const}$$

$$\left.\begin{array}{l}\rho = \text{const}\\ \varphi = \text{const}\\ z = \text{var}\end{array}\right\} \rightarrow E = \text{const}$$

$$\boxed{\left.\begin{array}{l}\rho = \text{var}\\ \varphi = \text{const}\\ z = \text{const}\end{array}\right\} \rightarrow \mathbf{E} = \text{var}}$$

Line Charge (3)



$$\mathbf{E}(\mathbf{r}) = \frac{Q(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^3} \rightarrow d\mathbf{E} = \frac{dQ(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^3}$$

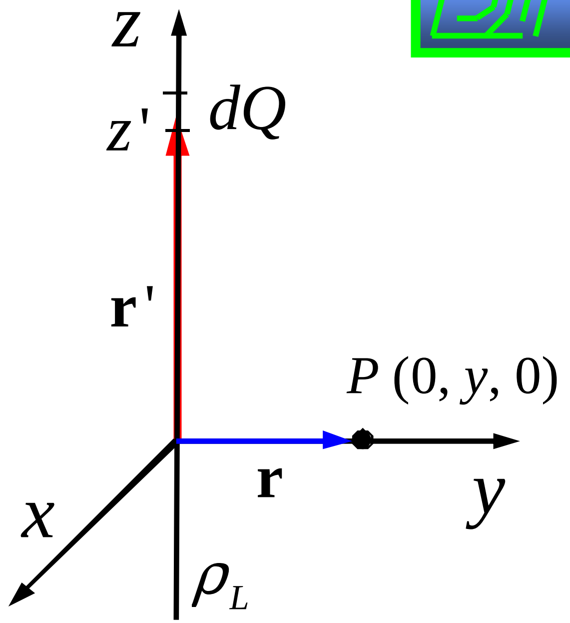
$$\rho_L = \frac{dQ}{dz'} \rightarrow dQ = \rho_L dz'$$

$$\rightarrow d\mathbf{E} = \frac{\rho_L dz'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^3}$$

$$\left.\begin{array}{l} \mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho \\ \mathbf{r}' = z'\mathbf{a}_z \end{array}\right\} \rightarrow \left\{\begin{array}{l} \mathbf{r}-\mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z \\ |\mathbf{r}-\mathbf{r}'| = \sqrt{\rho^2 + z'^2} \end{array}\right.$$

$$\rightarrow d\mathbf{E} = \frac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\varepsilon_0 (\rho^2 + z'^2)^{3/2}}$$

Line Charge (4)



$$\left.\begin{aligned} d\mathbf{E} &= \frac{\rho_L dz'(\rho \mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\varepsilon_0(\rho^2 + z'^2)^{3/2}} \\ &\mathbf{E} \text{ is not a function of } z \end{aligned}\right\}$$

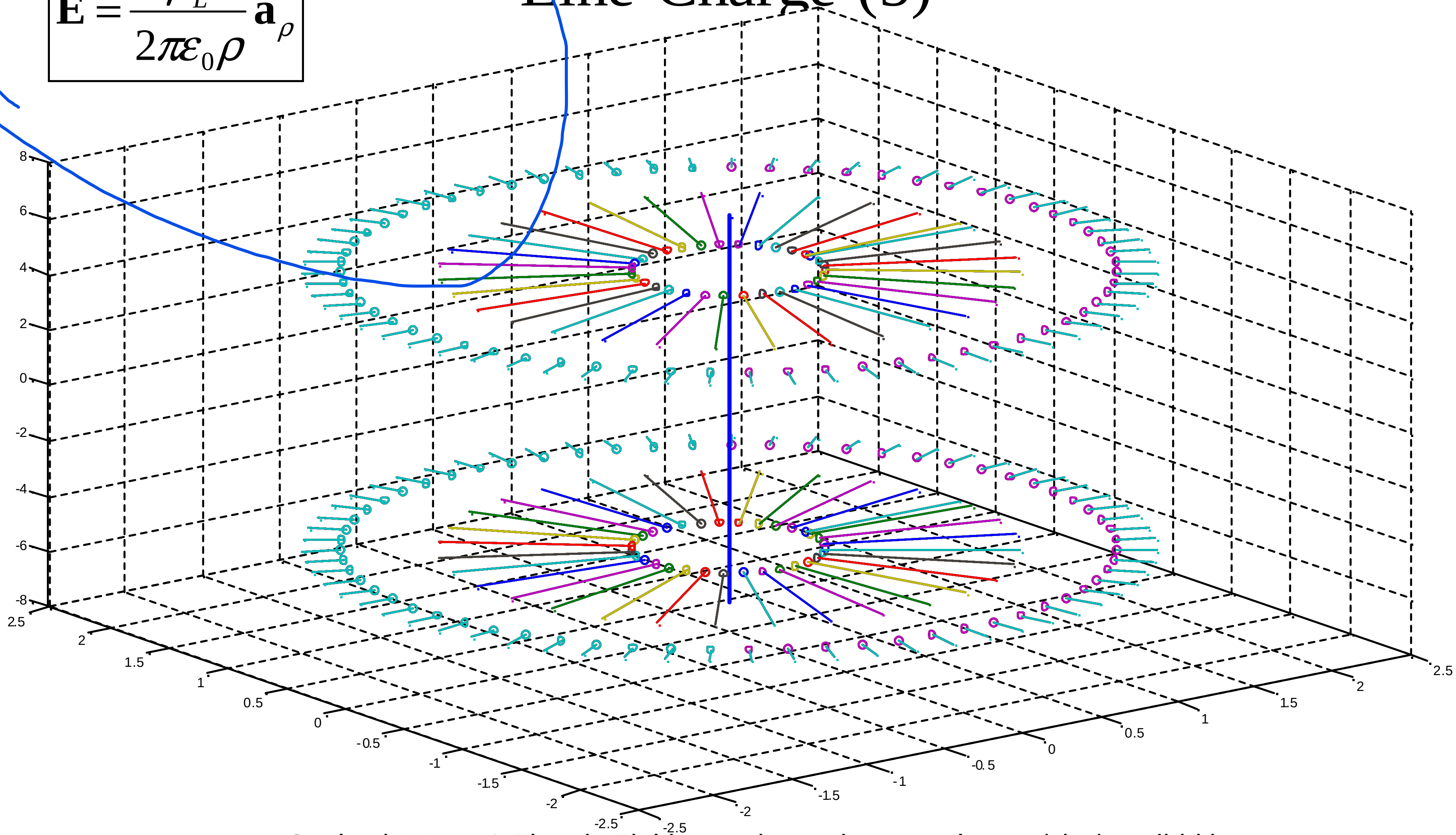
$$\to dE_\rho = \frac{\rho_L \rho dz'}{4\pi\varepsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$\to E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho dz'}{4\pi\varepsilon_0(\rho^2 + z'^2)^{3/2}} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}$$

$$\to \boxed{\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\mathbf{a}_\rho}$$

Line Charge (5)

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho$$



Line Charge (6)

Ex. 1

Infinite uniform line charge of 10 nC/m lie along the x & y axes in free space. Find **E** at (0, 0, 3).

Line Charge (7)

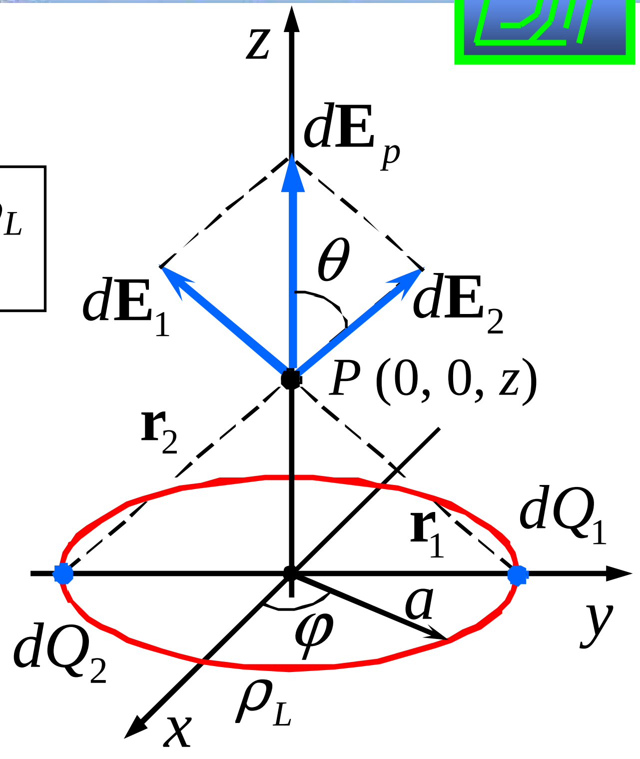
Ex. 2

The *x* & *y* axes are charged with uniform line charge of 10 nC/m. A point charge of 20nC is located at (3, 3, 0). The whole system is in free space. Find **E** at (0, 0, 3).

Ex. 3

Line Charge (8)

Given a circular hoop of radius a with uniform line charge ρ_L centered about the origin in the z = 0 plane. Find EFI at P?



$$\left.\begin{aligned} dE_1 &= \frac{dQ_1}{4\pi\varepsilon_0 r_1^2} \\ dQ_1 &= \rho_L dL = \rho_L a d\varphi \\ r_1 &= \sqrt{a^2+z^2} \end{aligned}\right\} \to dE_1 = \frac{\rho_L a d\varphi}{4\pi\varepsilon_0 (a^2+z^2)}$$

$$\left.\begin{aligned} dE_1 &= \frac{\rho_L a d\varphi}{4\pi\varepsilon_0 (a^2+z^2)} \\ dE_{Pz} &= 2dE_{1z} = 2dE_1\cos\theta \\ \cos\theta &= \frac{z}{\sqrt{a^2+z^2}} \end{aligned}\right\}$$

$$\to dE_{Pz} = \frac{\rho_L azd\varphi}{2\pi\varepsilon_0 (a^2+z^2)^{3/2}} \to E_{Pz} = \int_{\varphi=0}^{\pi} \frac{\rho_L azd\varphi}{2\pi\varepsilon_0 (a^2+z^2)^{3/2}} = \frac{\rho_L az}{2\varepsilon_0 (a^2+z^2)^{3/2}}$$

$$\to \mathbf{E}_P = \frac{\rho_L az}{2\varepsilon_0 (a^2+z^2)^{3/2}} \mathbf{a}_z$$

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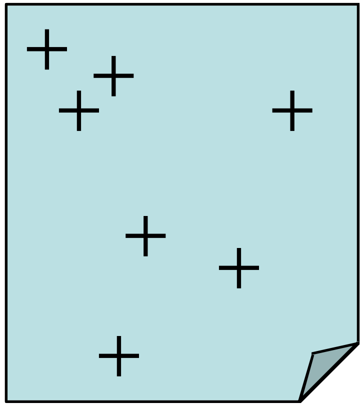
7. Applications

Sheet Charge (1)



- Charge is distributed on the *surface* of a plate (e.g. of a parallel-plate capacitor)
- Sheet/surface charge density ρ_S (unit: C/m^2)

$$\rho_S = \frac{dQ}{dS}$$

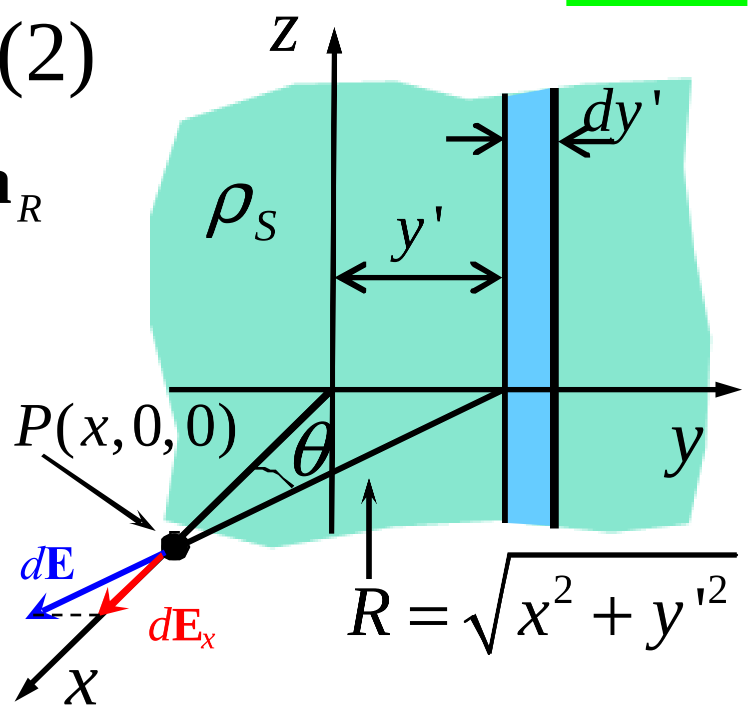
Sheet Charge (2)

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\mathbf{a}_R \rightarrow d\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\sqrt{x^2 + y'^2}}\mathbf{a}_R$$

$$dQ = \rho_S dS = \rho_S L dy'$$

$$L \rightarrow \infty$$

$$\rightarrow \rho_L = \frac{dQ}{L} = \frac{\rho_S L dy'}{L} = \rho_S dy'$$



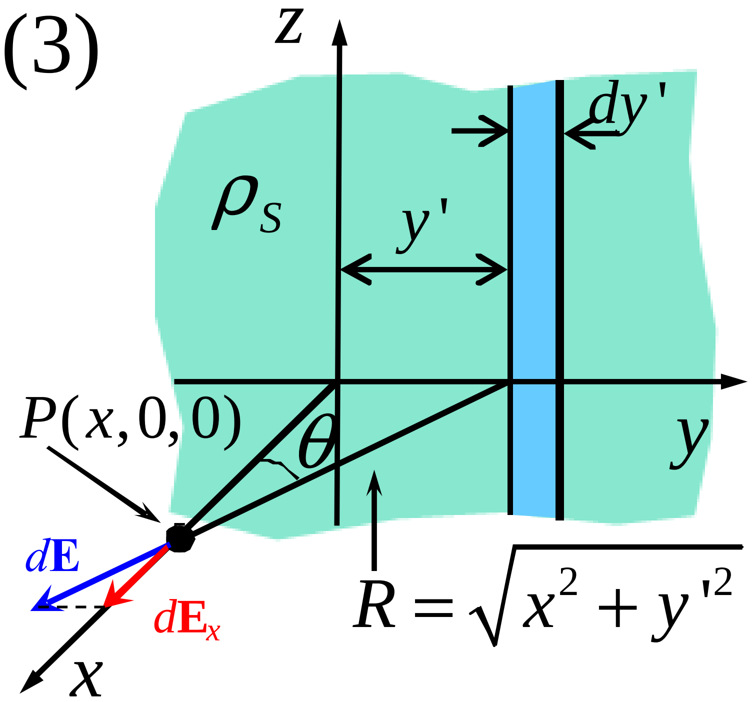
$$\left.\begin{array}{l} \rightarrow d\mathbf{E} = \dfrac{\rho_S dy' \mathbf{a}_R}{2\pi\varepsilon_0\sqrt{x^2 + y'^2}} \\ dE_x = dE\cos\theta \end{array}\right\} \rightarrow dE_x = \frac{\rho_S dy' \cos\theta}{2\pi\varepsilon_0\sqrt{x^2 + y'^2}}$$

Sheet Charge (3)

$$\left.\begin{aligned} dE_x &= \frac{\rho_S dy'\cos\theta}{2\pi\varepsilon_0\sqrt{x^2+y'^2}} \\ \cos\theta &= \frac{x}{\sqrt{x^2+y'^2}} \end{aligned}\right\}$$

$$\rightarrow dE_x = \frac{\rho_S}{2\pi\varepsilon_0}\cdot\frac{xdy'}{x^2+y'^2}$$

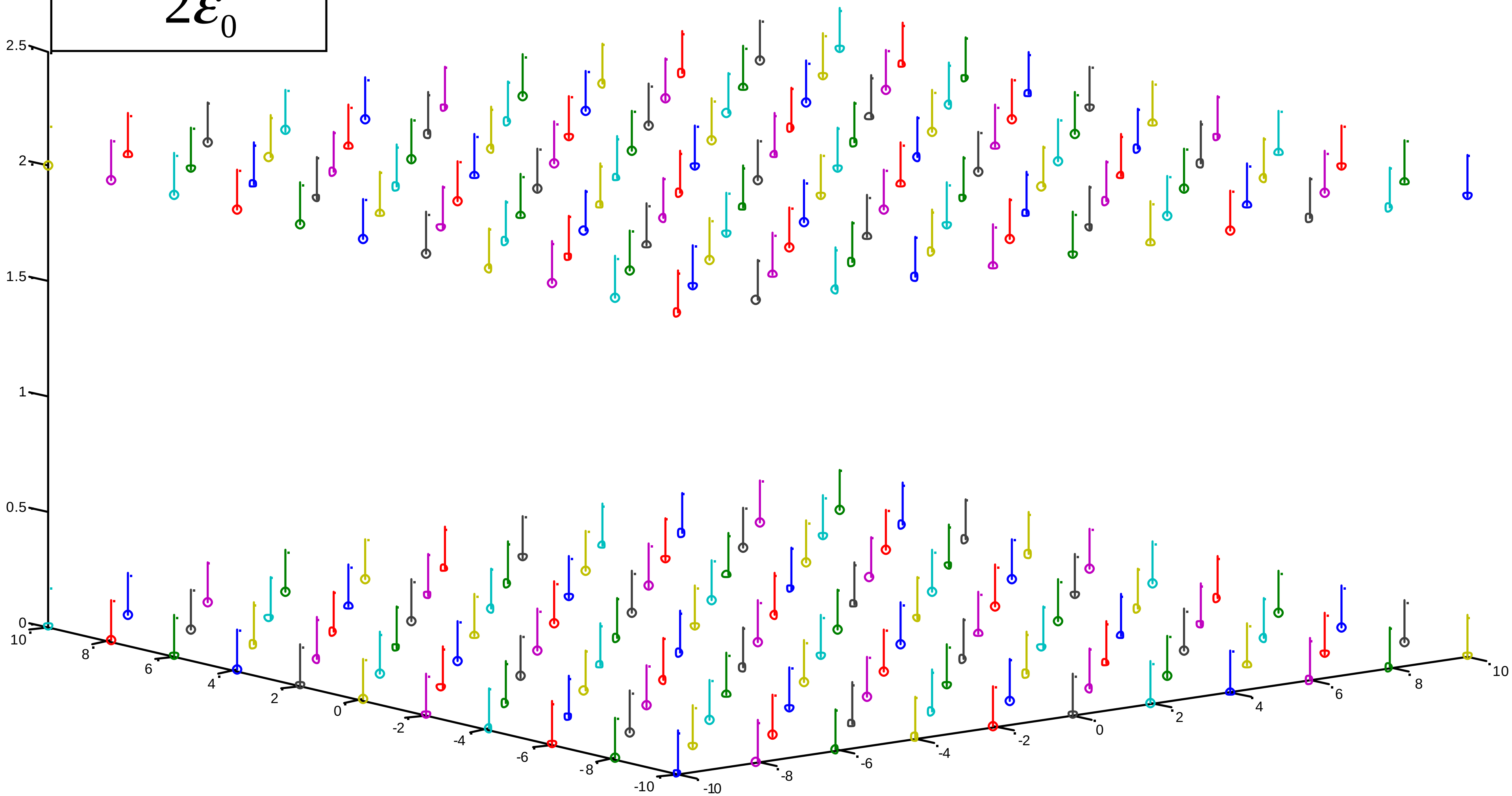
$$\rightarrow E_x = \frac{\rho_S}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{xdy'}{x^2+y'^2} = \frac{\rho_S}{2\varepsilon_0} \rightarrow \boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_N}$$



($\mathbf{a}_N$: vector perpendicular to the sheet)

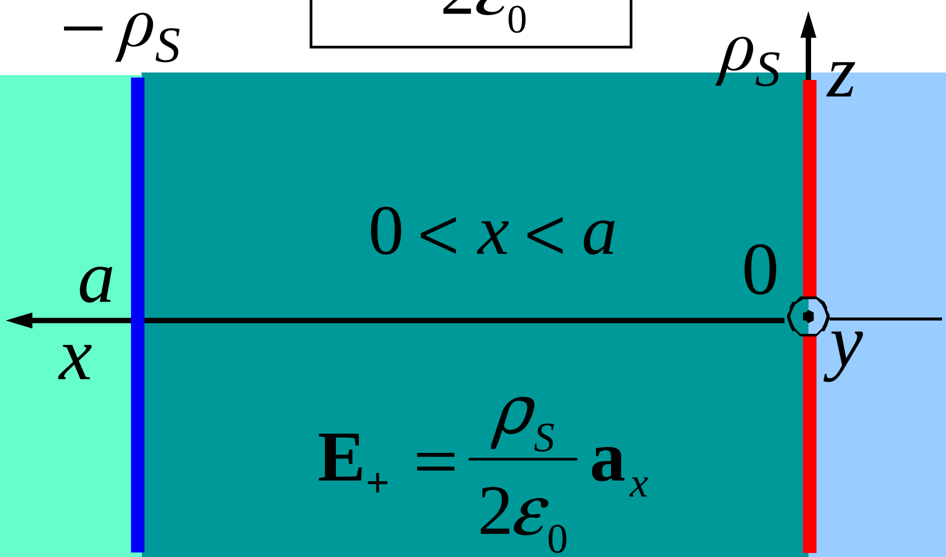
$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_N$$

Sheet Charge (4)



Sheet Charge (5)

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_N}$$



$x > a$

$$\mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\mathbf{E}_- = -\frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\rightarrow \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$$

$0 < x < a$

$$\mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\rightarrow \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho_S}{\varepsilon_0}\mathbf{a}_x$$

$x < 0$

$$\mathbf{E}_+ = -\frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x$$

$$\rightarrow \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$$

Sheet Charge (6)

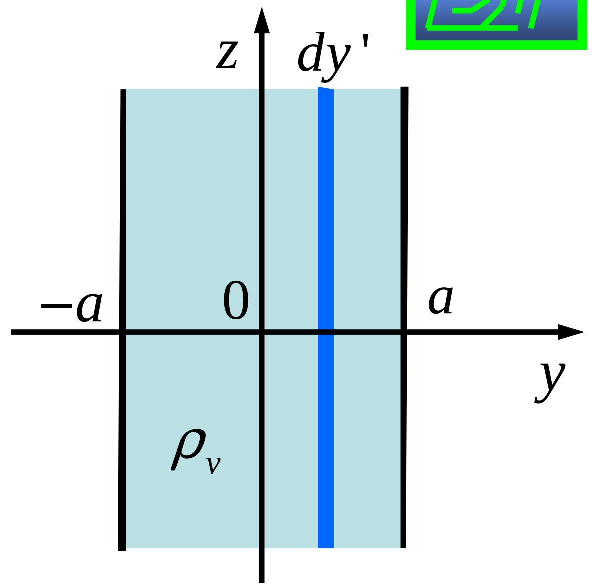
Ex. 1

Given 3 infinite uniform sheets (all parallel to $x0y$) at $z = -3$, $z = 2$ & $z = 3$. Their surface charge density are 4 nC/m^2, 6 nC/m^2 & –9 nC/m^2 respectively. Find **E** at $P(5, 5, 5)$.

Ex. 2

Sheet Charge (7)

A uniformly volume charge density charge ρ_v of infinite extent in the x & z directions & of width $2a$ is centered about the y axis. Find EFI?



$$\rho_S = \rho_v dy'$$

$$y \le -a: \quad dE_y = -\frac{\rho_S}{2\varepsilon_0} = -\frac{\rho_v dy'}{2\varepsilon_0} \rightarrow E_y = -\int_{-a}^{a} \frac{\rho_v dy'}{2\varepsilon_0} = -\frac{a\rho_v}{\varepsilon_0}$$

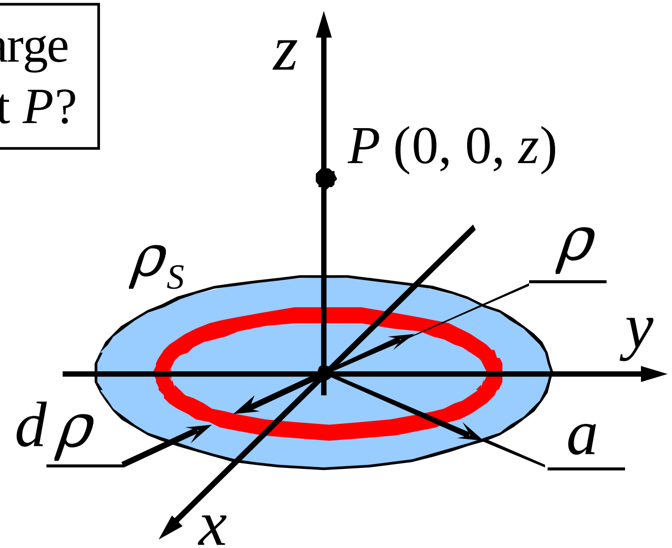
$$y \ge a: \quad dE_y = \frac{\rho_S}{2\varepsilon_0} = \frac{\rho_v dy'}{2\varepsilon_0} \rightarrow E_y = \int_{-a}^{a} \frac{\rho_v dy'}{2\varepsilon_0} = \frac{a\rho_v}{\varepsilon_0}$$

$$-a \le y \le a: \quad \rightarrow E_y = \int_{-a}^{y} \frac{\rho_v dy'}{2\varepsilon_0} - \int_{y}^{a} \frac{\rho_v dy'}{2\varepsilon_0} = \frac{\rho_v}{\varepsilon_0} y$$

Ex. 3

Sheet Charge (8)

Given a circular disk of radius *a* with uniform surface charge ρ_S centered about the origin in the $z = 0$ plane. Find EFI at *P*?



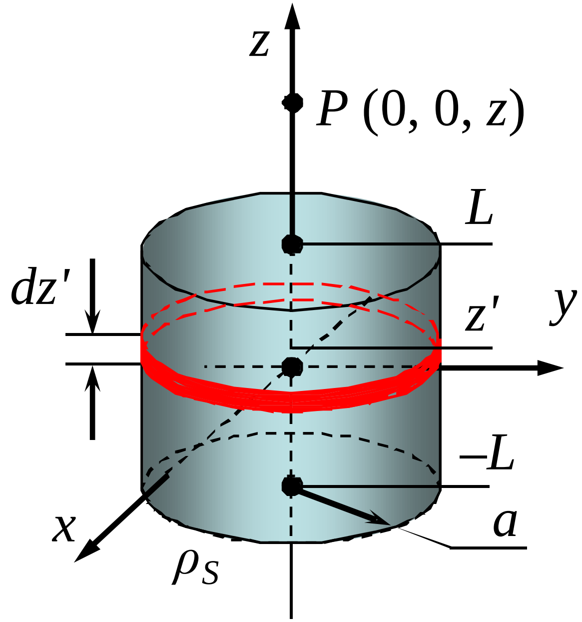
$$\left.\begin{aligned} \rho_L &= \rho_S d\rho \\ dE_{Pz} &= \frac{\rho_L \rho z}{2\varepsilon_0(\rho^2 + z^2)^{3/2}} \end{aligned}\right\}$$

$$\rightarrow dE_{Pz} = \frac{(\rho_S d\rho)\rho z}{2\varepsilon_0(\rho^2 + z^2)^{3/2}} \rightarrow E_{Pz} = \int_0^a \frac{(\rho_S d\rho)\rho z}{2\varepsilon_0(\rho^2 + z^2)^{3/2}} = -\frac{\rho_S}{2\varepsilon_0}\left(\frac{z}{\sqrt{a^2 + z^2}} - \frac{z}{|z|}\right)$$

Ex. 4

Sheet Charge (9)

A hollow cylinder of radius a & length $2L$ with uniform surface charge ρ_S on its outer surface. Find EFI at P?



$$\left.\begin{aligned} \rho_L &= \rho_S dz' \\ dE_{Pz} &= \frac{\rho_L az}{2\varepsilon_0(a^2+z^2)^{3/2}} \end{aligned}\right\}$$

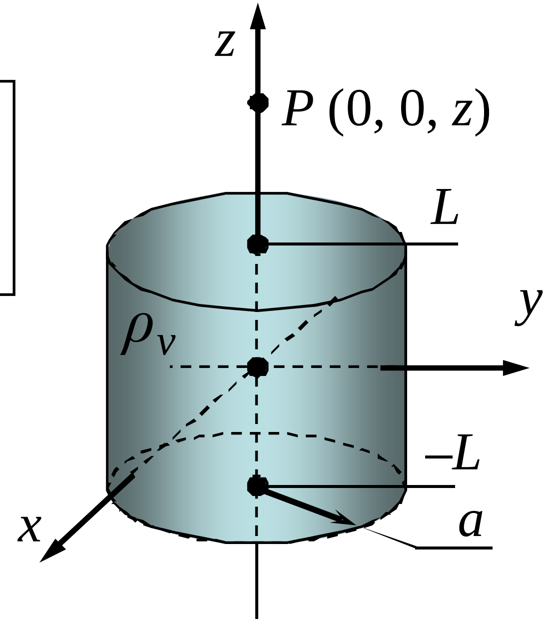
$$\rightarrow dE_{Pz} = \frac{(\rho_S dz')a(z-z')}{2\varepsilon_0[a^2+(z-z')^2]^{3/2}} \rightarrow E_{Pz} = \int_{-L}^{L} \frac{(\rho_S dz')a(z-z')}{2\varepsilon_0[a^2+(z-z')^2]^{3/2}}$$

$$= \frac{\rho_S a}{2\varepsilon_0}\left[\frac{1}{\sqrt{a^2+(z-L)^2}} - \frac{1}{\sqrt{a^2+(z+L)^2}}\right]$$

Ex. 5

Sheet Charge (10)

A cylinder (of radius a & length $2L$) is uniformly charged throughout the volume with volume charge density ρ_v. Find EFI at P?



Basic Charge Configurations

Point Charge

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2} \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Line Charge

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \mathbf{a}_\rho$$

Sheet Charge

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0} \mathbf{a}_N$$

Volume Charge

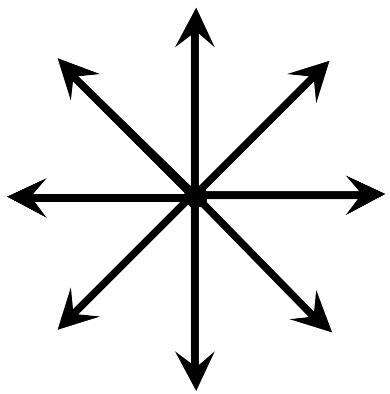
$$\mathbf{E} = \int_V \frac{\rho_v(\mathbf{r}')dV'}{4\pi\varepsilon_0 |\mathbf{r}-\mathbf{r}'|^2} \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Coulomb’s Law & Electric Field Intensity

1. Coulomb’s Law
2. Electric Field Intensity
3. Field Due to a Continuous Volume Charge Distribution
4. Field of a Line Charge
5. Field of a Sheet Charge
6. **Sketches of Fields**
7. Applications

Sketches of Fields

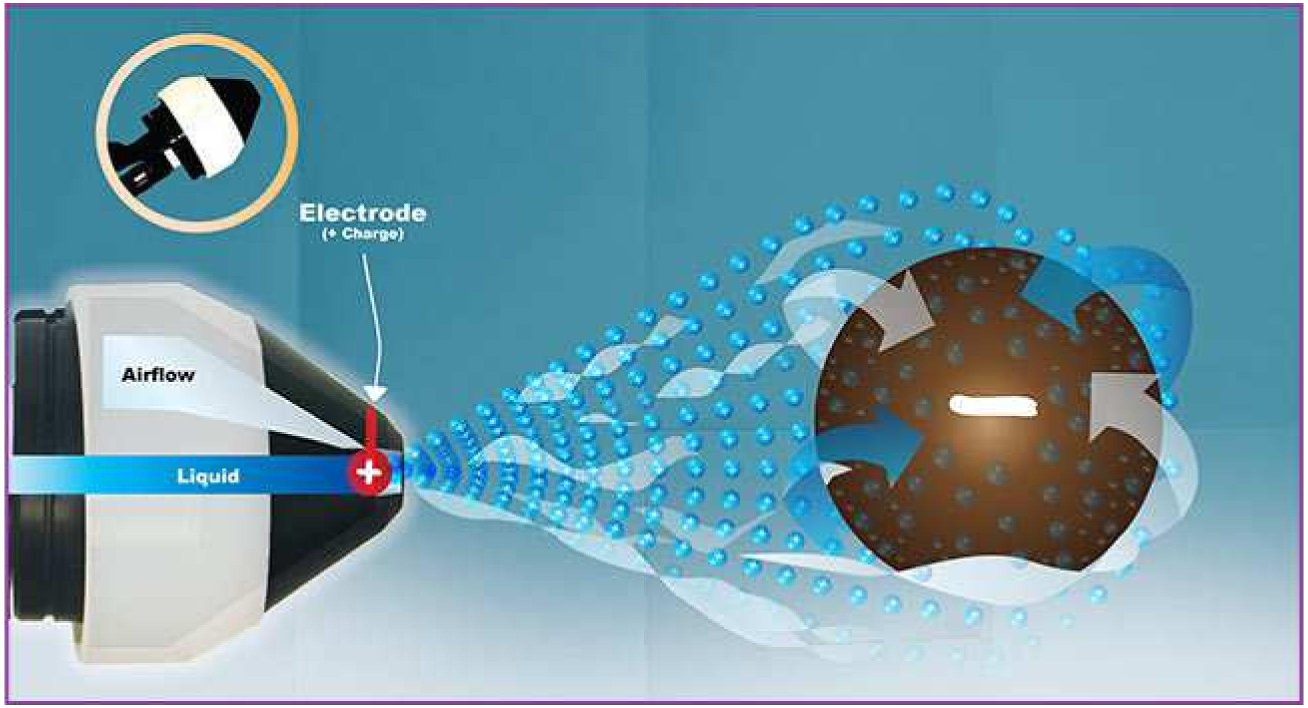
- To “picture” a field
- A set of vectors of a field



Coulomb's Law & Electric Field Intensity

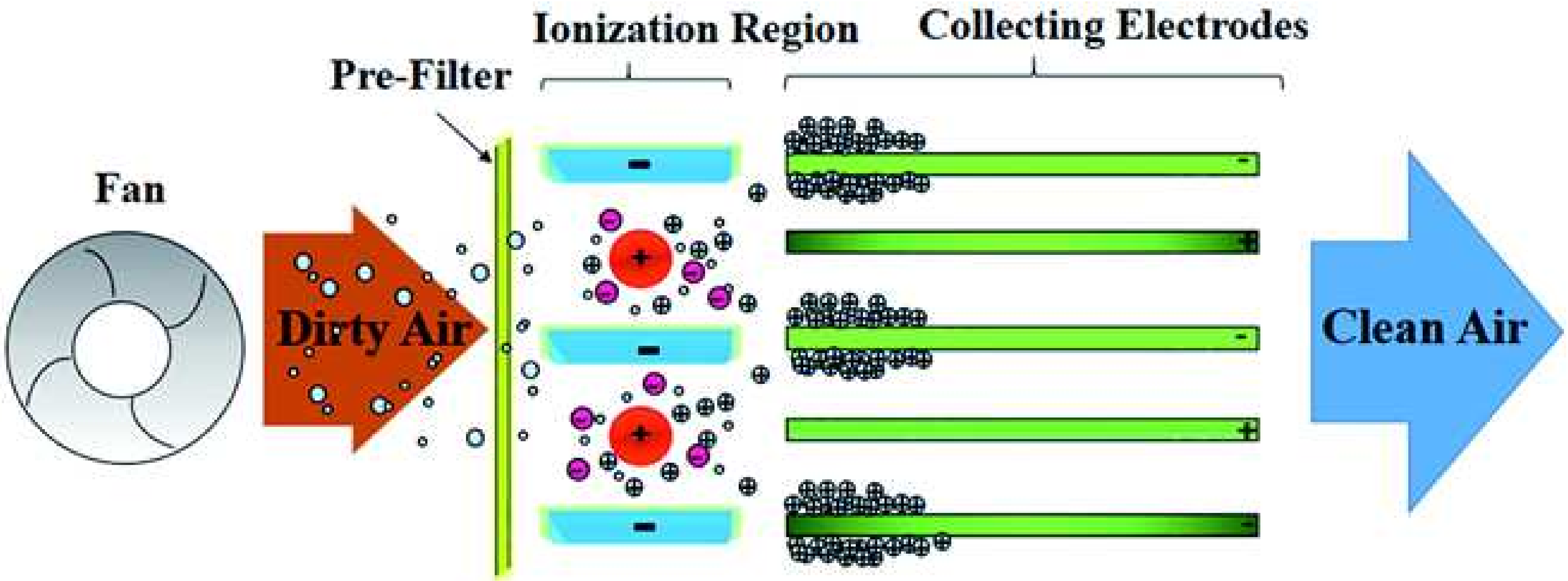
1. Coulomb's Law
2. Electric Field Intensity
3. Field Due to a Continuous Volume Charge Distribution
4. Field of a Line Charge
5. Field of a Sheet Charge
6. Sketches of Fields
7. **Applications**

Applications (1)
Electrostatic Spraying



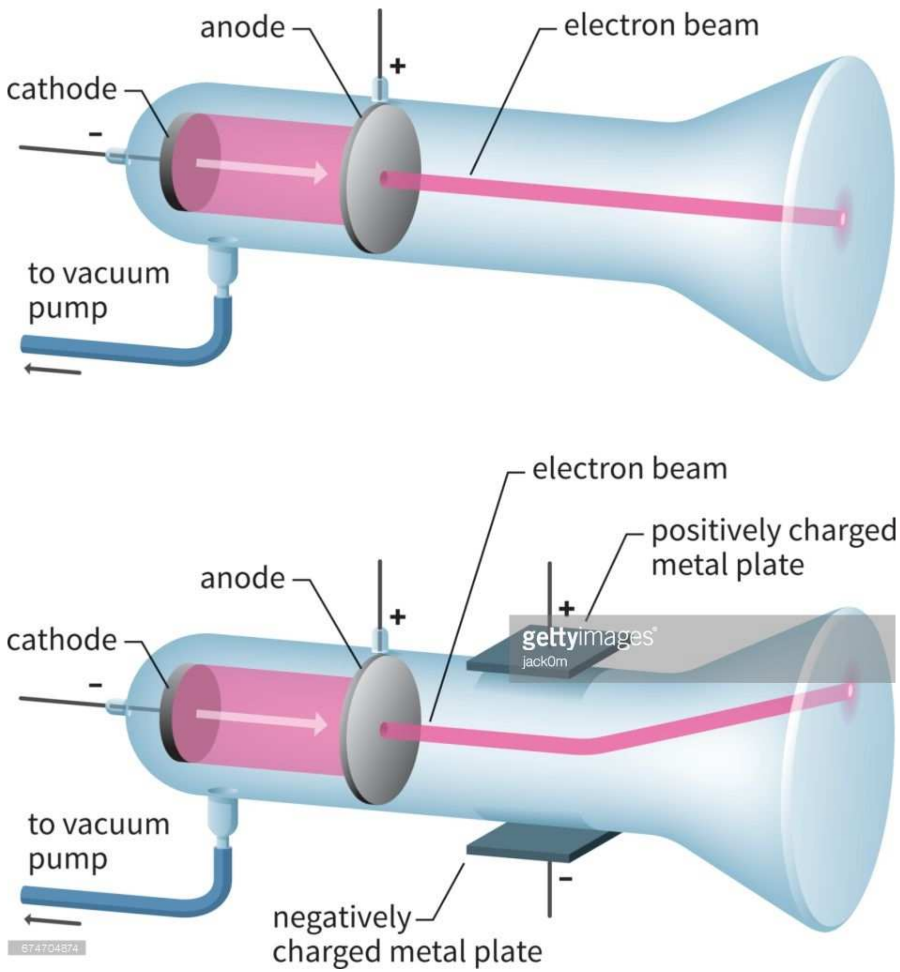
http://haisolutionsllc.com/index.php/resources/78-germ-buster-cart

Applications (2) – Electrostatic Cleaners/Separators/Scrubbers/Precipitators



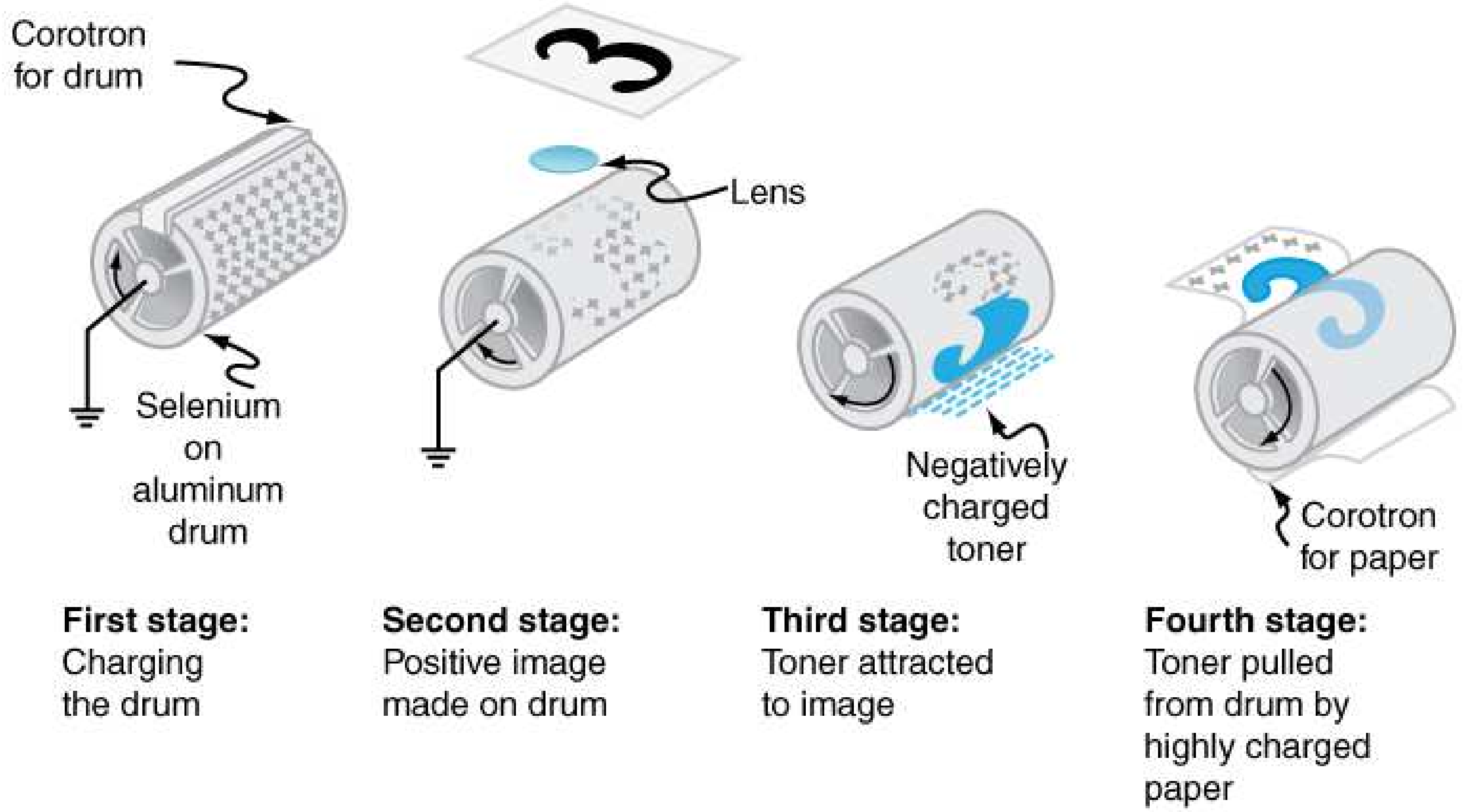
http://pubs.rsc.org/en/content/articlelanding/2016/ra/c6ra13542k/unauth#!divAbstract

Applications (3)
Cathode Ray Tube



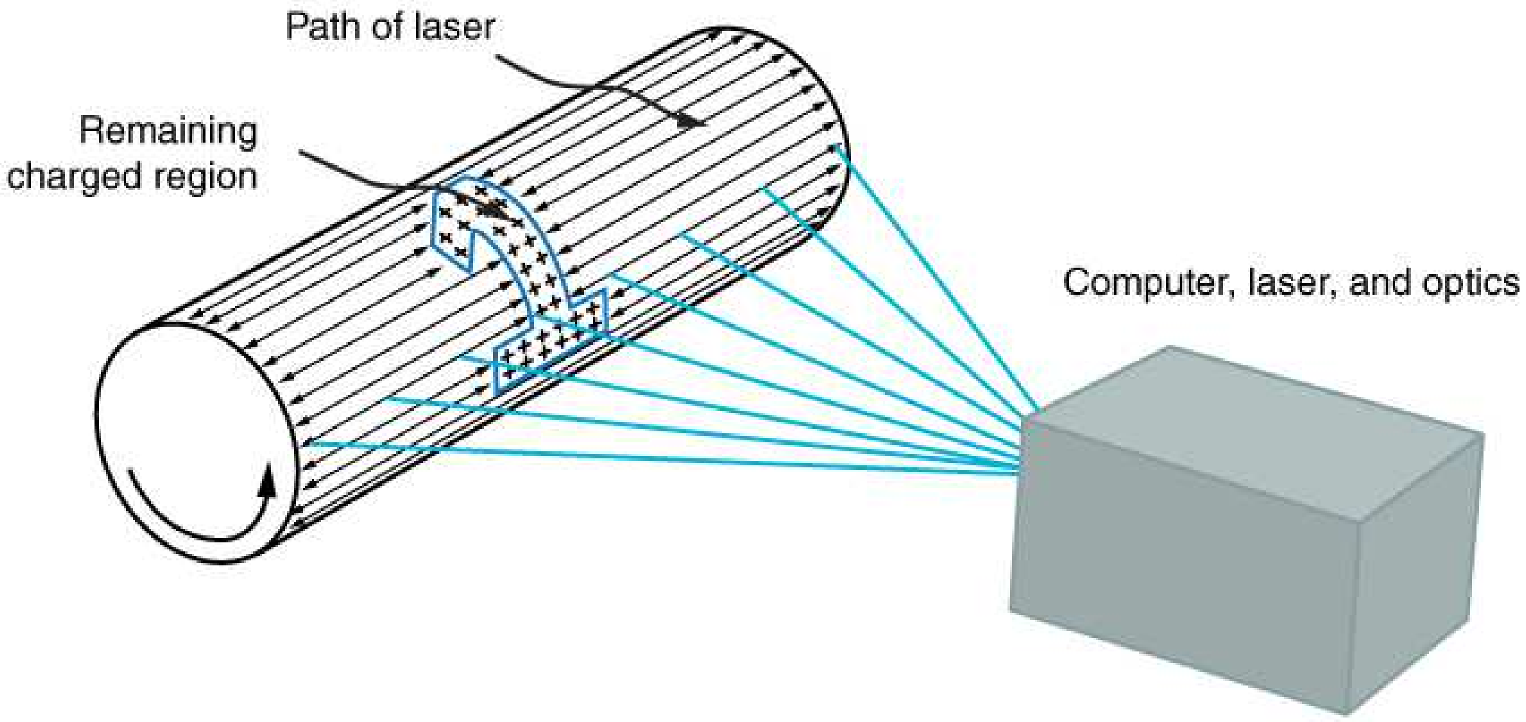
http://www.gettyimages.com/detail/illustration/cathode-ray-tube-royalty-free-illustration/674704874

Applications (4)
Xerography



http://archive.cnx.org/contents/b76ece9b-3fb0-4701-bb7a-b92b7941e4c5@1/18-9-applications-of-electrostatics

Applications (4)
Laser Printers



http://archive.cnx.org/contents/b76ece9b-3fb0-4701-bb7a-b92b7941e4c5@1/18-9-applications-of-electrostatics