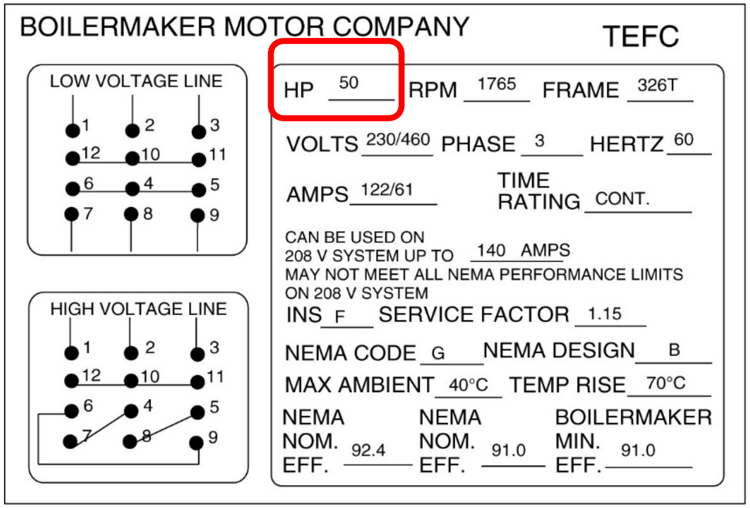
Nguyễn Công Phương

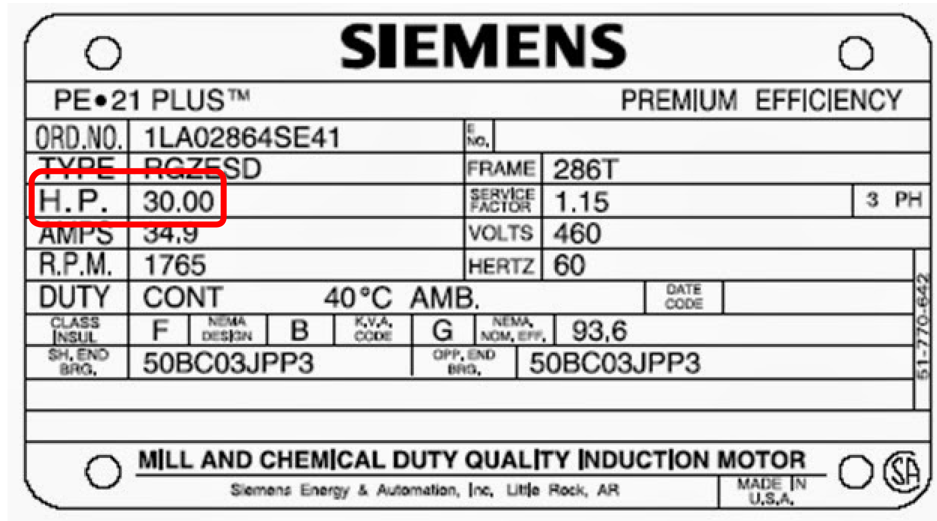
Electric Circuit Theory

AC Power Analysis

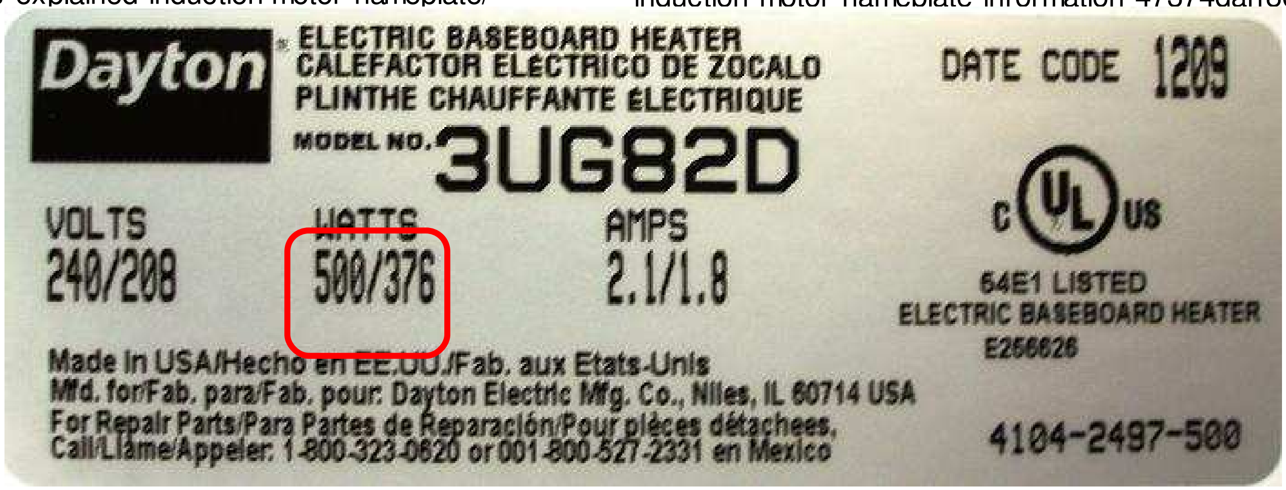
Contents



http://electricalacademia.com/induction-motor/electric-motor-nameplate-details-explained-induction-motor-nameplate/



http://poqynamekyxoqep.oramanageability.com/understanding-induction-motor-nameplate-information-47374dan8099.html



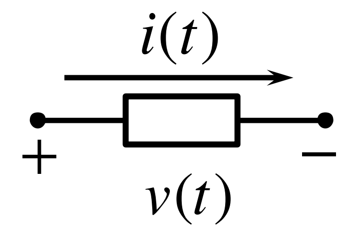
https://www.cpsc.gov/Recalls/2010/marley-engineered-products-recalls-baseboard-heaters-sold-at-grainger-due-to-fire

AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals

Instantaneous Power (1)

$$\left.\begin{aligned} \boxed{p(t) = v(t)i(t)} \\ v(t) = V_m \sin(\omega t + \phi_v) \\ i(t) = I_m \sin(\omega t + \phi_i) \end{aligned}\right\}$$



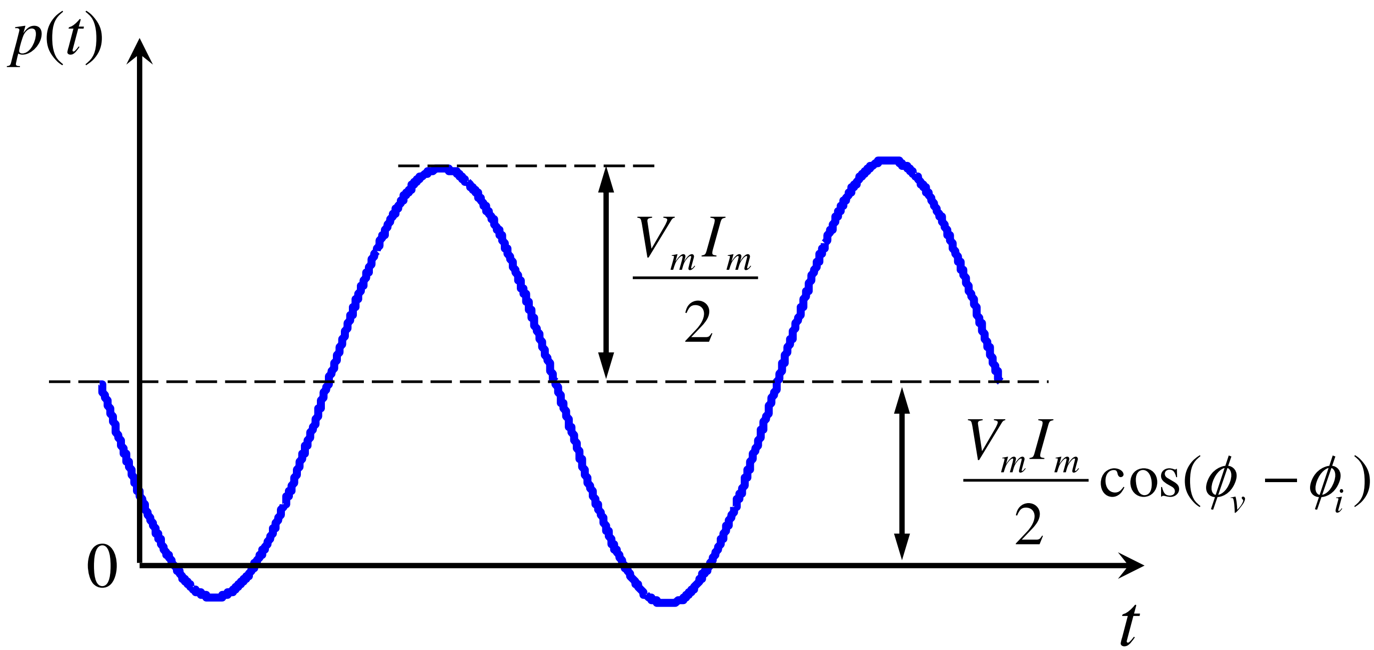
$$\rightarrow p(t) = V_m I_m \sin(\omega t + \phi_v)\sin(\omega t + \phi_i)$$

$$= \frac{V_m I_m}{2}[\cos(\phi_v - \phi_i) - \cos(2\omega t + \phi_v + \phi_i)]$$

$$= \frac{V_m I_m}{2}\cos(\phi_v - \phi_i) - \frac{V_m I_m}{2}\cos(2\omega t + \phi_v + \phi_i)$$

Instantaneous Power (2)

$$p(t) = \frac{V_m I_m}{2}\cos(\phi_v - \phi_i) - \frac{V_m I_m}{2}\cos(2\omega t + \phi_v + \phi_i)$$



Average Power (1)

$$\boxed{P = \frac{1}{T}\int_0^T p(t)dt}$$

$$p(t) = \frac{V_m I_m}{2}\cos(\phi_v - \phi_i) - \frac{V_m I_m}{2}\cos(2\omega t + \phi_v + \phi_i)$$

$$\rightarrow P = \frac{1}{2}V_m I_m \cos(\phi_v - \phi_i)\frac{1}{T}\int_0^T dt - \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \phi_v + \phi_i)dt$$

The average of a sinusoid over its period is zero

$$\rightarrow \boxed{P = \frac{1}{2}V_m I_m \cos(\phi_v - \phi_i)}$$

Average Power (2)

$$\left.\begin{array}{l}\mathbf{V} = V_m \underline{/\phi_v} \\ \mathbf{I} = I_m \underline{/\phi_i} \quad \rightarrow \mathbf{I}^* = I_m \underline{/-\phi_i}\end{array}\right\} \rightarrow \mathbf{VI}^* = V_m I_m \underline{/\phi_v - \phi_i}$$

$$\left.\begin{array}{l}\mathbf{VI}^* = V_m I_m \underline{/\phi_v - \phi_i} = V_m I_m \cos(\phi_v - \phi_i) + jV_m I_m \sin(\phi_v - \phi_i) \\ P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)\end{array}\right\} \rightarrow$$

$$\rightarrow \boxed{P = \frac{1}{2}\mathrm{Re}(\mathbf{VI}^*)}$$

Average Power (3)

$$P = \frac{1}{2}\mathrm{Re}(\mathbf{VI}^*) = \frac{1}{2}V_m I_m \cos(\phi_v - \phi_i)$$

$\phi_v = \phi_i$:

$$P = \frac{1}{2}V_m I_m \cos(0) = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R$$

$\phi_v - \phi_i = \pm 90^{\circ}$:

$$P = \frac{1}{2}V_m I_m \cos(90^{\circ}) = 0$$

Average Power (4)

Ex.

$v(t) = 150\sin(314t - 30^{o})$ V, $i(t) = 10\sin(314t + 45^{o})$ A. Find P?

$$P = \frac{1}{2}V_m I_m \cos(\varphi_u - \varphi_i) = \frac{1}{2}150\times 10\cos(-30^{o} - 45^{o}) = \boxed{194.11\ \text{W}}$$

$$P = \frac{1}{2}\text{Re}(\mathbf{VI}^*)$$

$$\mathbf{V} = V_m \underline{/\varphi_u} = 150\underline{/-30^{o}}$$

$$\mathbf{I} = I_m \underline{/\varphi_i} = 10\underline{/45^{o}} \rightarrow \mathbf{I}^* = 10\underline{/-45^{o}}$$

$$\mathbf{VI}^* = \left(150\underline{/-30^{o}}\right)\left(10\underline{/-45^{o}}\right) = 1500\underline{/-75^{o}} = 388.23 - j1448.9\ \text{VA}$$

$$P = \frac{1}{2}\text{Re}\{388.23 - j1448.9\} = \frac{1}{2}388.23 = \boxed{191.11\text{W}}$$

AC Power Analysis

1. Instantaneous and Average Power
2. **Maximum Average Power Transfer**
3. RMS Value
4. Apparent Power and Power Factor
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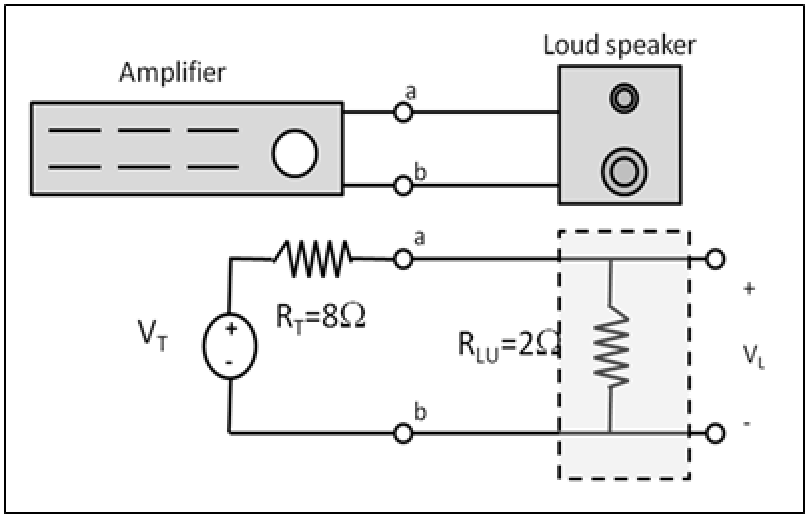
8. Average Power and RMS Value of Periodic Signals

Maximum Average Power Transfer (1)



http://www.chegg.com/homework-help/questions-and-answers/use-maximum-power-transfer-theorem-determine-increase-power-delivered-loudspeaker-resultin-q6983635

Maximum Average Power Transfer (2)

$$P_L = \frac{1}{2} I_{Lm}^2 R_L$$

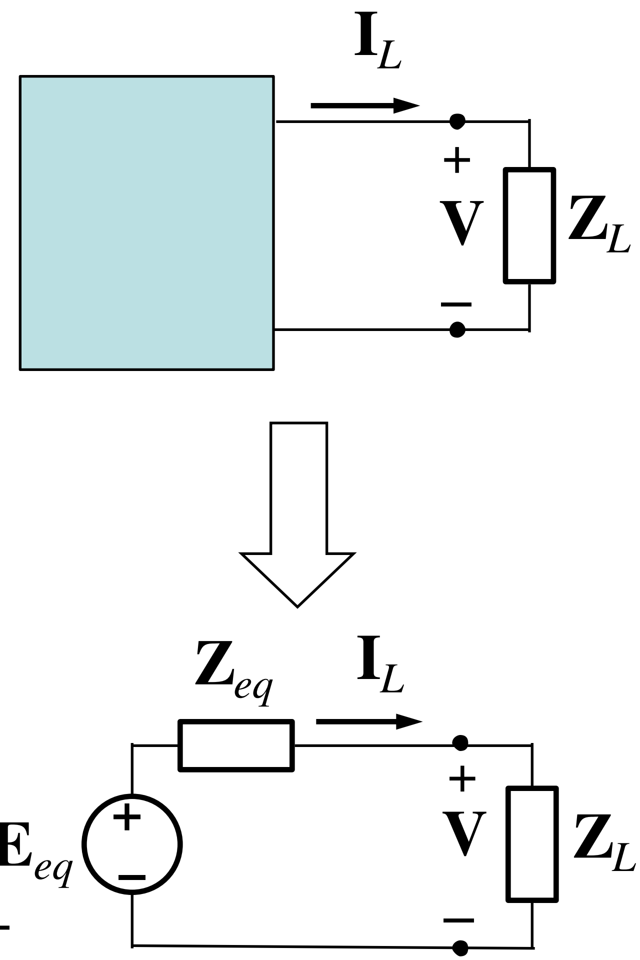
$$\mathbf{I}_L = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_L} \rightarrow I_{Lm} = \frac{|\mathbf{E}_{eq}|}{|\mathbf{Z}_{eq} + \mathbf{Z}_L|}$$

$$\left.\begin{aligned} \mathbf{Z}_{eq} &= R_{eq} + jX_{eq} \\ \mathbf{Z}_L &= R_L + jX_L \end{aligned}\right\} \rightarrow$$

$$\rightarrow \mathbf{Z}_{eq} + \mathbf{Z}_L = R_{eq} + jX_{eq} + R_L + jX_L$$

$$= (R_{eq} + R_L) + j(X_{eq} + X_L)$$

$$\rightarrow |\mathbf{Z}_{eq} + \mathbf{Z}_L| = \sqrt{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

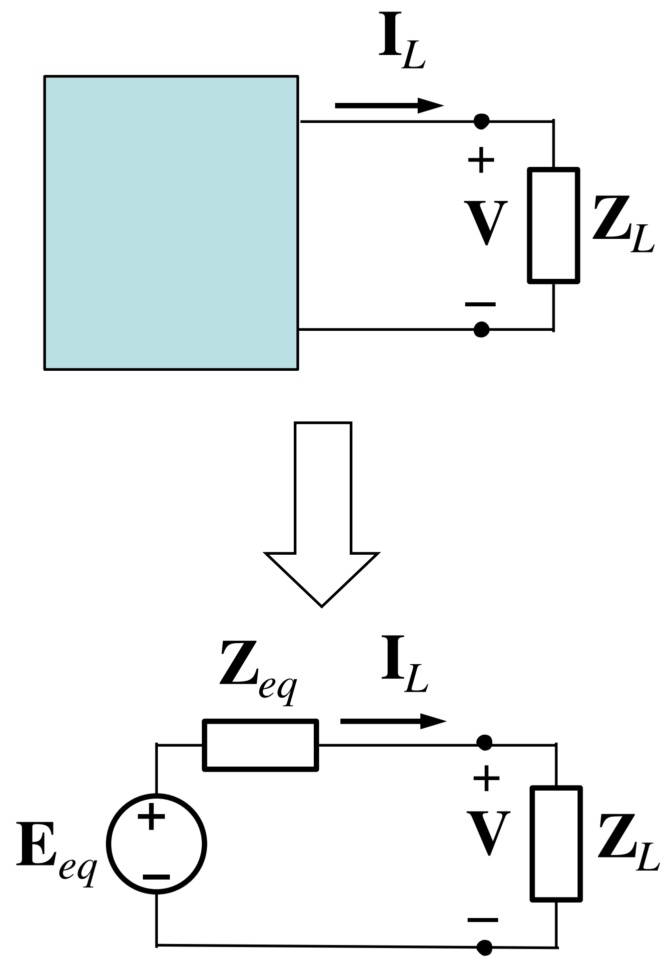


Maximum Average Power Transfer (3)

$$\left.\begin{aligned}
P_L &= \frac{1}{2} I_{Lm}^2 R_L \\
I_{Lm} &= \frac{\left|\mathbf{E}_{eq}\right|}{\left|\mathbf{Z}_{eq} + \mathbf{Z}_L\right|} \\
\left|\mathbf{Z}_{eq} + \mathbf{Z}_L\right| &= \sqrt{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}
\end{aligned}\right\}$$

$$\rightarrow P_L = \frac{1}{2} \times \frac{\left|\mathbf{E}_{eq}\right|^2 R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

$$P_L \text{ is maximum if: } \begin{cases} \dfrac{\partial P_L}{\partial R_L} = 0 \\[2ex] \dfrac{\partial P_L}{\partial X_L} = 0 \end{cases}$$



Maximum Average Power Transfer (4)

$$P_L = \frac{1}{2} \times \frac{|\mathbf{E}_{eq}|^2 R_L}{(R_{eq}+R_L)^2 + (X_{eq}+X_L)^2}$$

$$\begin{cases} \dfrac{\partial P_L}{\partial X_L} = 0 \rightarrow \dfrac{\partial P_L}{\partial X_L} = |\mathbf{E}_{eq}|^2 \dfrac{R_L(X_{eq}+X_L)}{[(R_{eq}+R_L)^2+(X_{eq}+X_L)^2]^2} = 0 \\ \dfrac{\partial P_L}{\partial R_L} = 0 \rightarrow \dfrac{\partial P_L}{\partial R_L} = |\mathbf{E}_{eq}|^2 \dfrac{(R_{eq}+R_L)^2+(X_{eq}+X_L)^2 - 2R_L(R_{eq}+R_L)}{2[(R_{eq}+R_L)^2+(X_{eq}+X_L)^2]^2} = 0 \end{cases}$$

$$\rightarrow \begin{cases} X_L = -X_{eq} \\ R_L = \sqrt{R_{eq}^2 + (X_{eq}+X_L)^2} \end{cases} \qquad \rightarrow \begin{cases} X_L = -X_{eq} \\ R_L = R_{eq} \end{cases}$$

$$\rightarrow \boxed{\mathbf{Z}_L = \mathbf{Z}_{eq}^*}$$

For maximum average power transfer, the load impedance must be equal to the complex conjugate of the equivalent impedance

Maximum Average Power Transfer (5)

$$\left.\begin{array}{l} P_L = \dfrac{1}{2} \times \dfrac{\left|\mathbf{E}_{eq}\right|^2 R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2} \\ \begin{cases} X_L = -X_{eq} \\ R_L = R_{eq} \end{cases} \end{array}\right\} \rightarrow P_{L\max} = \frac{\left|\mathbf{E}_{eq}\right|^2}{8R_{eq}}$$

Maximum Average Power Transfer (6)

For maximum average power transfer, the load impedance must be equal to the complex conjugate of the equivalent impedance

$$\mathbf{Z}_L = \mathbf{Z}_{eq}^*$$

$$\left.\begin{aligned} &\text{If } Z_L = R_L \,? \quad \rightarrow X_L = 0 \\ &\frac{\partial P_L}{\partial R_L} = 0 \rightarrow R_L = \sqrt{R_{eq}^2 + (X_{eq} + X_L)^2} \end{aligned}\right\} \rightarrow$$

$$\rightarrow R_L = \sqrt{R_{eq}^2 + X_{eq}^2} = \left|\mathbf{Z}_{eq}\right|$$

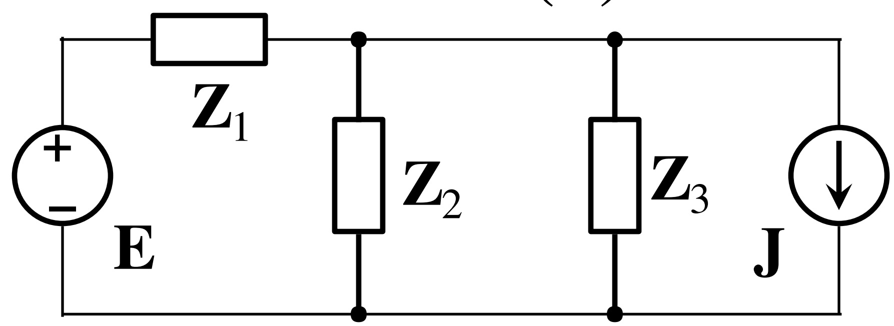
Ex. 1

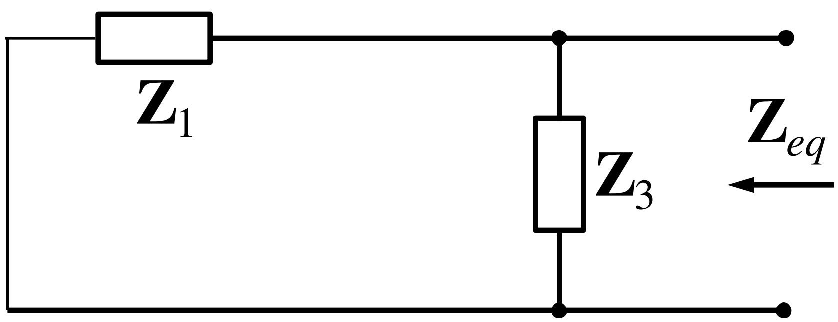
Maximum Average Power Transfer (7)

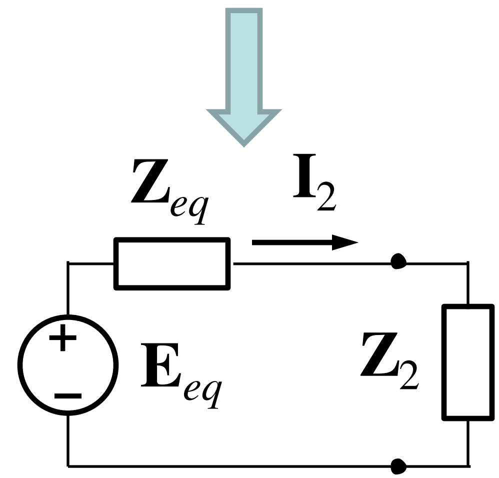
$\mathbf{E} = 20\angle -45^\circ$ V; $\mathbf{J} = 5\angle 60^\circ$ A

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$

Determine the load impedance $\mathbf{Z}_2$ that maximize the average power. What is the maximum average power?







$$\mathbf{Z}_{eq} = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_3} = \frac{12(-j16)}{12 - j16} = 7.68 - j5.76\,\Omega$$

$$\rightarrow \mathbf{Z}_2 = \boxed{7.68 + j5.76\,\Omega}$$

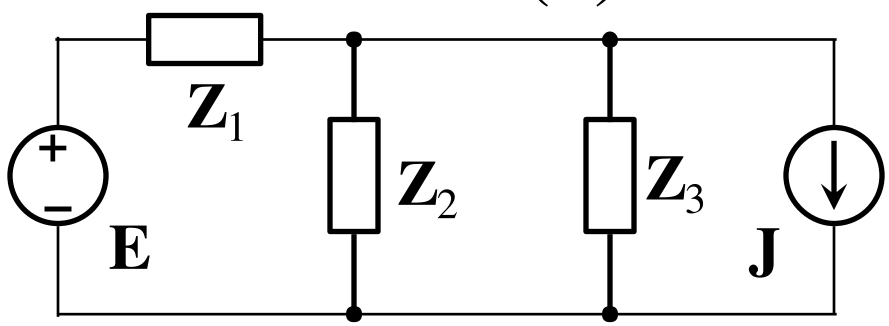
$$\boxed{\mathbf{Z}_2 = \mathbf{Z}_{eq}^*}$$

Ex. 1 Maximum Average Power Transfer (8)

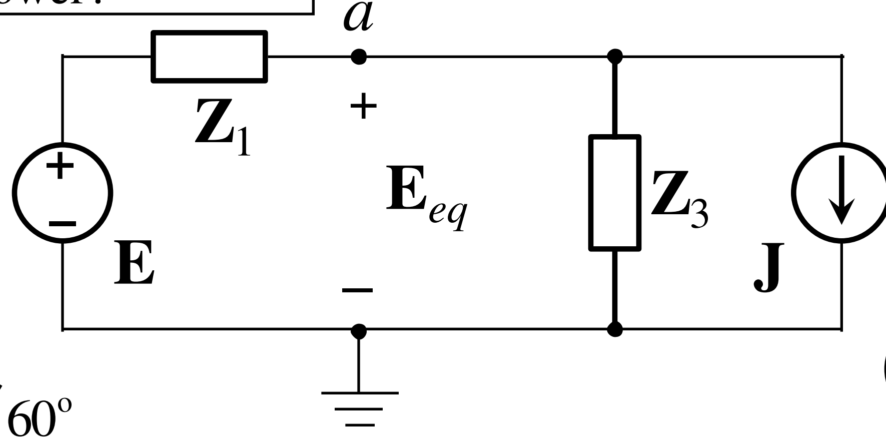
$\mathbf{E} = 20\angle -45^\circ$ V; $\mathbf{J} = 5\angle 60^\circ$ A

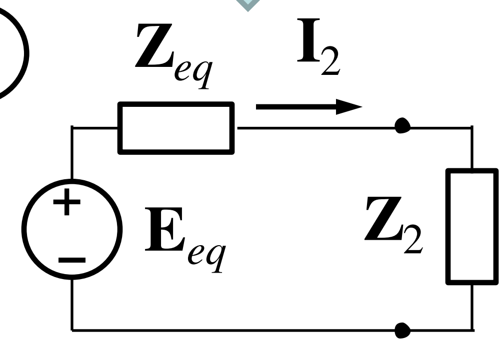
$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$

Determine the load impedance $\mathbf{Z}_2$ that maximize the average power. What is the maximum average power?



$$\mathbf{E}_{eq} = \mathbf{V}_a = \frac{\dfrac{\mathbf{E}}{\mathbf{Z}_1} - \mathbf{J}}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_3}}$$





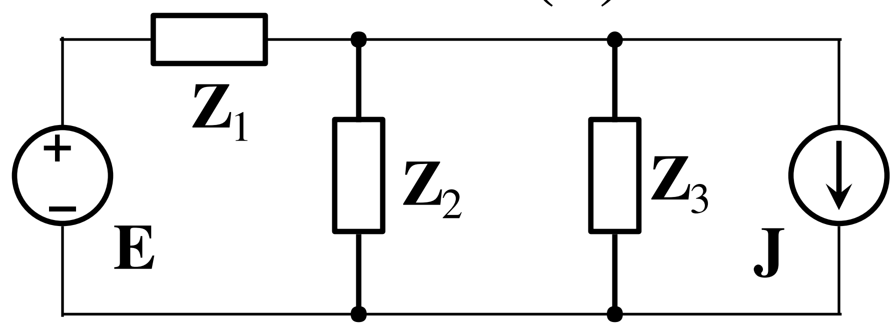
$$= \frac{\dfrac{20\angle -45^\circ}{12} - 5\angle 60^\circ}{\dfrac{1}{12} + \dfrac{1}{-j16}} = 54.38\angle -140.4^\circ \text{ V}$$

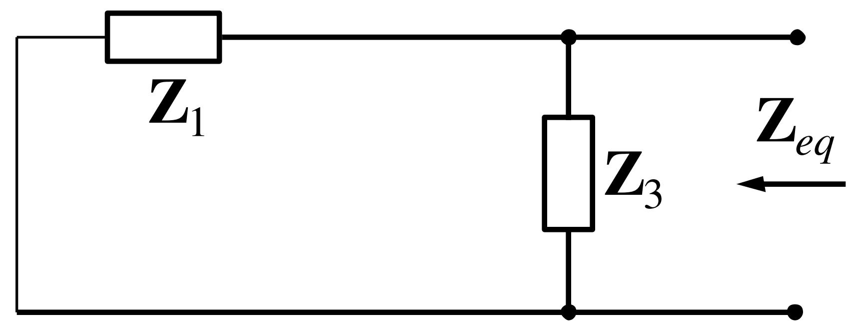
$$\to |\mathbf{E}_{eq}| = 54.38\text{ V} \to P_{2\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}} = \frac{54.38^2}{8\times 7.68} = \boxed{48.13\text{ W}}$$

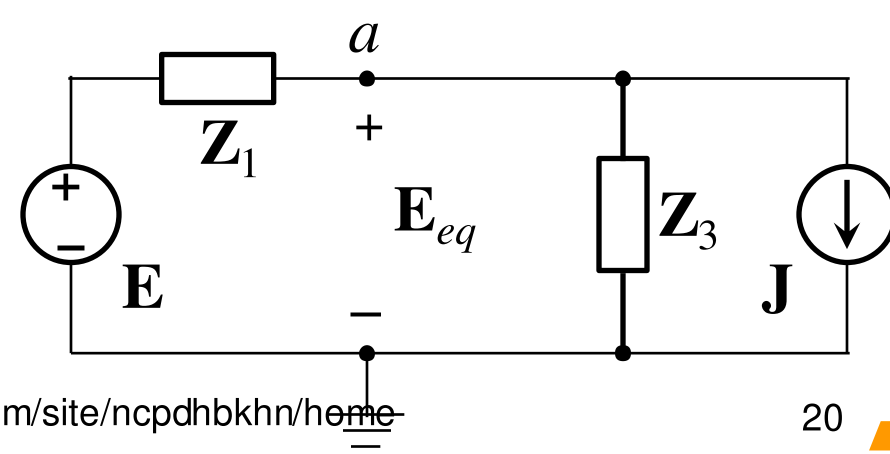
$$\boxed{P_{2\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}}$$

Maximum Average Power Transfer (9)

1. Find the Thevenin equivalent
a. $\mathbf{Z}_{eq}$
b. $\mathbf{E}_{eq}$
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$
3. $P_{\max} = \dfrac{\lvert \mathbf{E}_{eq} \rvert^2}{8R_{eq}}$







1a. $\mathbf{Z}_{eq} = 7.68 - j5.76\,\Omega$

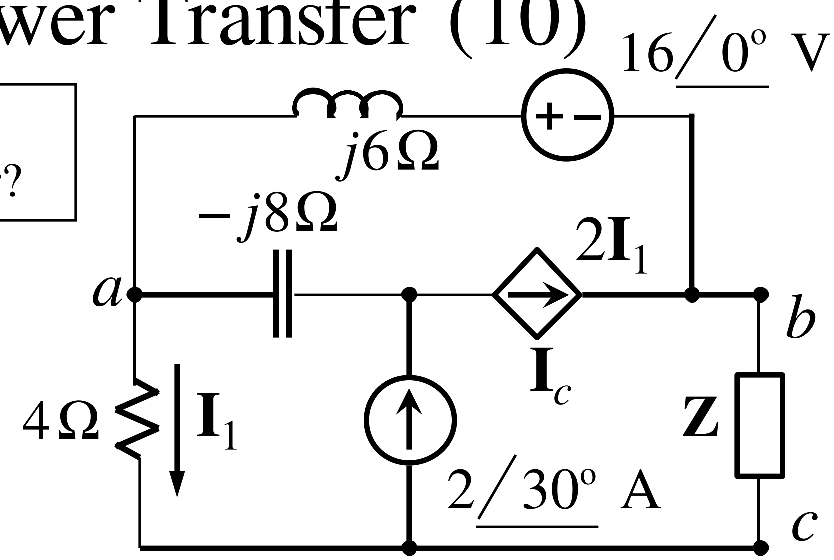
1b. $\mathbf{E}_{eq} = 54.38\underline{/-140.4^\circ}\text{ V}$

2. $\mathbf{Z}_2 = 7.68 + j5.76\,\Omega$

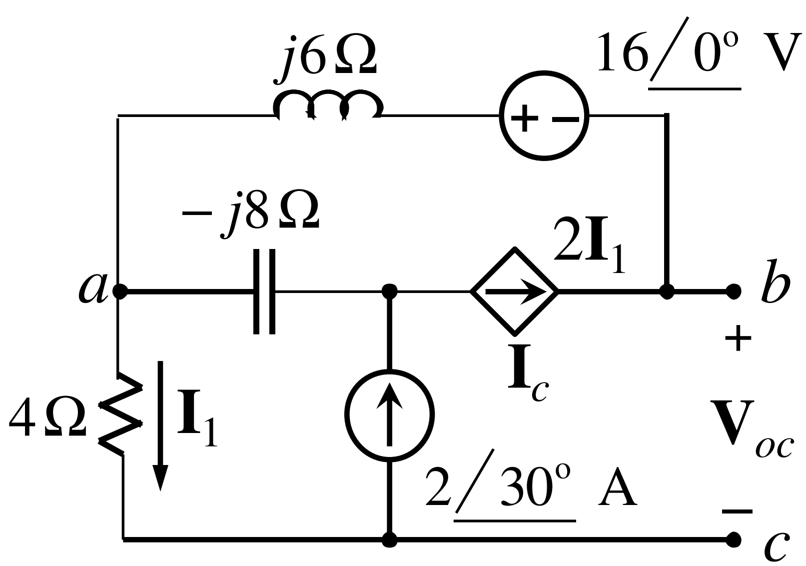
3. $P_{2\max} = 48.13\text{W}$

Ex. 2 Maximum Average Power Transfer (10)

Determine the load impedance **Z** that maximize the average power. What is the maximum average power?



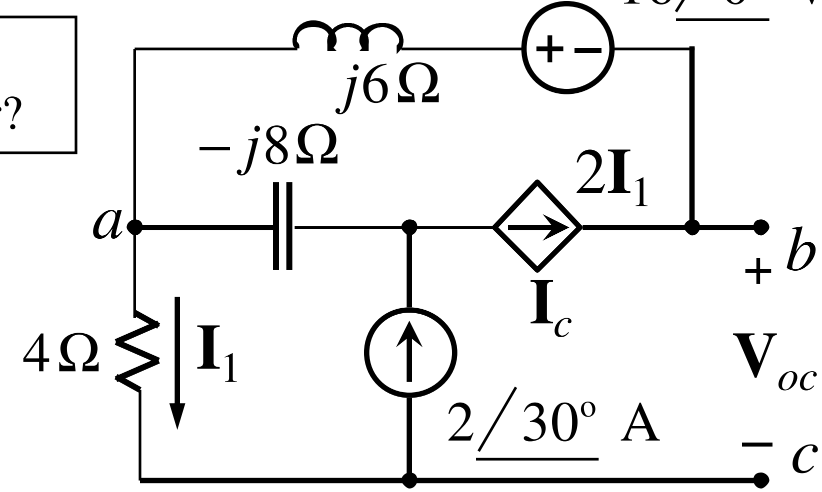
$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}}$$



1. Find the Thevenin equivalent
   a. $\mathbf{Z}_{eq}$
   b. $\mathbf{E}_{eq}$
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$
3. $P_{\max} = \dfrac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (11)

Determine the load impedance **Z** that maximize the average power. What is the maximum average power?



$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$

$$\left.\begin{aligned}(\mathbf{V}_c - \mathbf{V}_b) - 16 + j6\mathbf{I}_2 + 4\mathbf{I}_1 &= 0\\ \mathbf{V}_{oc} &= \mathbf{V}_b - \mathbf{V}_c\end{aligned}\right\}$$

$$\left.\begin{aligned}&\rightarrow \mathbf{V}_{oc} = -16 + j6\mathbf{I}_2 + 4\mathbf{I}_1\\ &\mathbf{I}_1 = 2\angle 30^{o}\\ &\mathbf{I}_2 = \mathbf{I}_c = 2\mathbf{I}_1 = 2\times 2\angle 30^{o}\end{aligned}\right\}$$

$$\rightarrow \mathbf{V}_{oc} = -16 + j6\times 2\times 2\angle 30^{o} + 4\times 2\angle 30^{o}$$

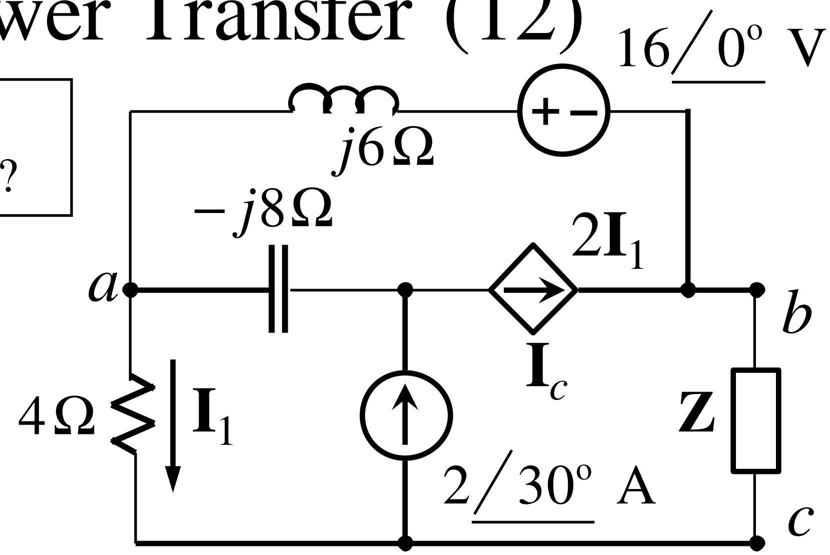
$$= \boxed{-21.07 + j24.78 \text{ V}}$$

1. Find the Thevenin equivalent
   a. $\mathbf{Z}_{eq}$
   b. ✓ $\mathbf{E}_{eq}$
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^{*}$
3. $P_{\max} = \dfrac{\left|\mathbf{E}_{eq}\right|^2}{8R_{eq}}$

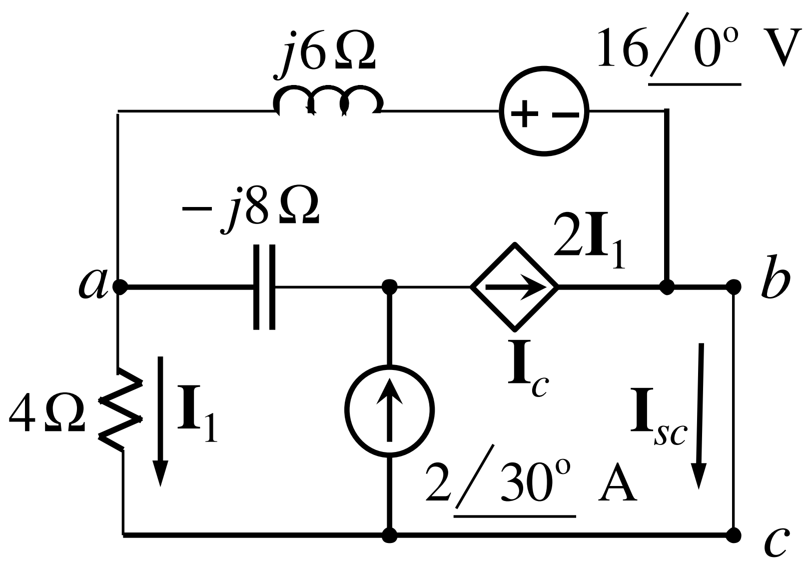
TRƯỜNG ĐẠI HỌC BÁCH KHOA HÀ NỘI

Ex. 2 Maximum Average Power Transfer (12)

Determine the load impedance **Z** that maximize the average power. What is the maximum average power?



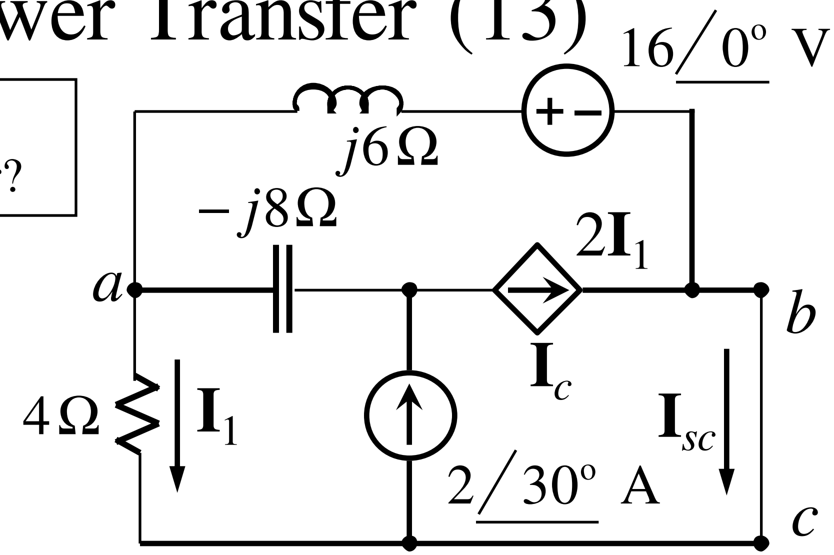
$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$



1. Find the Thevenin equivalent
   a. $\mathbf{Z}_{eq}$
   b. ✓ $\mathbf{E}_{eq}$
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$
3. $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (13)

Determine the load impedance **Z** that maximize the average power. What is the maximum average power?



$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$

$$\mathbf{I}_1 - 2\angle 30^o + \mathbf{I}_{sc} = 0 \rightarrow \mathbf{I}_{sc} = 2\angle 30^o - \mathbf{I}_1$$

$$\left.\begin{aligned} & j6\mathbf{I}_2 + 4\mathbf{I}_1 = 16\angle 0^o \\ & \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^o - \mathbf{I}_c = 0 \\ & \rightarrow \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^o - 2\mathbf{I}_1 = 0 \\ & \rightarrow 3\mathbf{I}_1 - \mathbf{I}_2 = 2\angle 30^o \end{aligned}\right\}$$

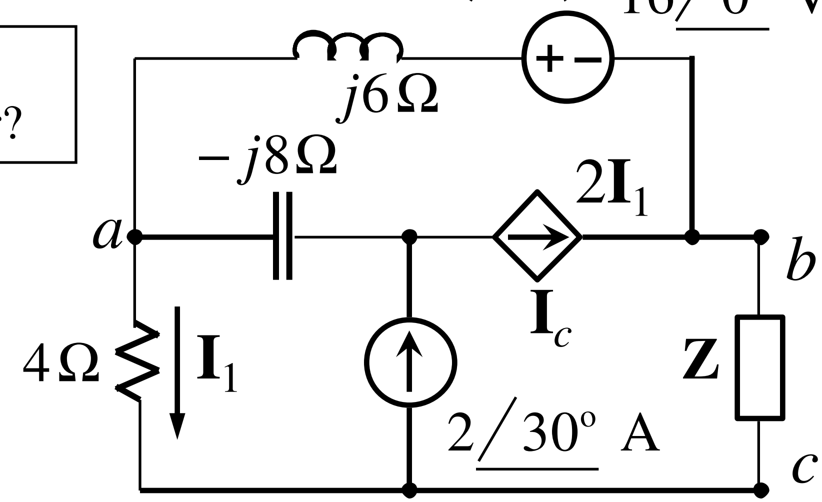
$$\rightarrow \mathbf{I}_1 = 0.67 - j0.41 \text{ A}$$

$$\rightarrow \mathbf{I}_{sc} = 2\angle 30^o - (0.67 - j0.41) = \boxed{1.06 + j1.41 \text{ A}}$$

1. Find the Thevenin equivalent
   a. $\mathbf{Z}_{eq}$
   b. ✓ $\mathbf{E}_{eq}$
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$
3. $P_{\max} = \dfrac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (14)

Determine the load impedance **Z** that maximize the average power. What is the maximum average power?



$$\left.\begin{aligned}\mathbf{Z}_{eq} &= \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}} \\ \mathbf{V}_{oc} &= -21.07 + j24.78 \text{ V} \\ \mathbf{I}_{sc} &= 1.06 + j1.41 \text{ A}\end{aligned}\right\}$$

$$\rightarrow \mathbf{Z}_{eq} = \frac{-21.07 + j24.78}{1.06 + j1.41} = \boxed{4.00 + j18.00\ \Omega}$$

$$\rightarrow \boxed{\mathbf{Z} = 4.00 - j18.00\ \Omega}$$

$$P_{\max} = \frac{21.07^2 + 24.78^2}{8 \times 4} = \boxed{33.06\text{ W}}$$

1. Find the Thevenin equivalent
   a. ✓ $\mathbf{Z}_{eq}$
   b. ✓ $\mathbf{E}_{eq}$
2. ✓ $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$
3. ✓ $P_{\max} = \dfrac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

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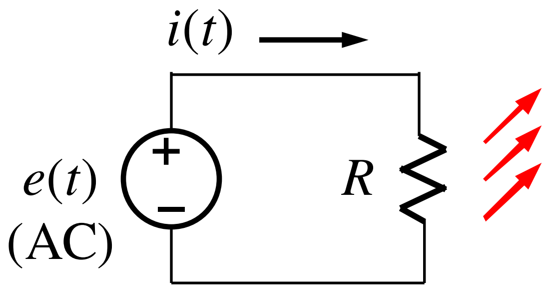
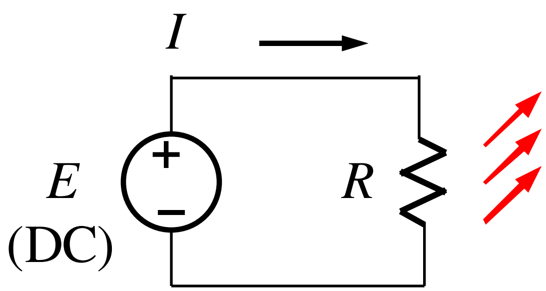
RMS Value (1)



$$\rightarrow P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt$$



$$\rightarrow P = I^2 R$$

$$\rightarrow I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

I is the effective/RMS value of *i*(*t*)

$$\boxed{X_{eff} = X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2 dt}}$$

RMS Value (2)

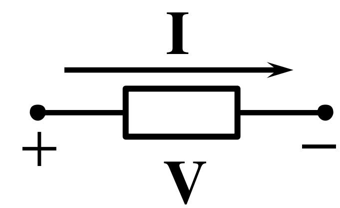
$$\left.\begin{array}{l} I_{rms} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^T i^2 dt} \\ i(t) = I_m \sin\omega t \end{array}\right\} \to I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{1}{T}\int_0^T [I_m \sin\omega t]^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T I_m^2 \frac{1-\cos 2\omega t}{2} dt}$$

$$= \sqrt{\frac{I_m^2}{2T}\int_0^T dt} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}} \qquad \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

RMS Value (3)



$$\boxed{P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\rightarrow \boxed{P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)}$$

$$\boxed{P = \frac{1}{2} \mathrm{Re}(\mathbf{VI}^*)}$$

RMS Value (4)

Ex.

$v(t) = 150\sin(314t - 30^o)$ V, $i(t) = 10\sin(314t + 45^o)$ A. Find V_{rms} & I_{rms}?

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{150}{\sqrt{2}} = 106.07 \text{ V}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

AC Power Analysis

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2. Maximum Average Power Transfer
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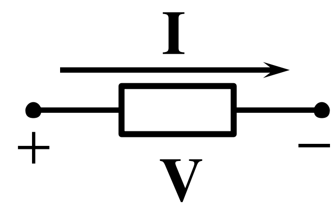
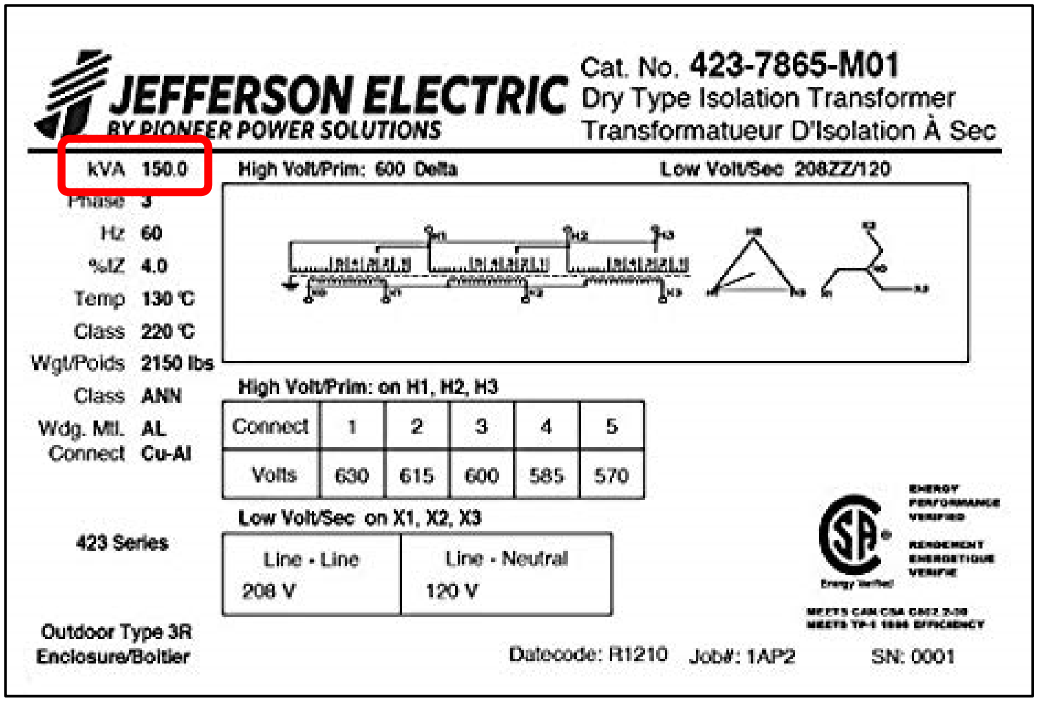
Apparent Power (1)



$$\boxed{S = V_{rms} I_{rms}} \quad \text{(in volt-ampere, VA)}$$



https://www.amazon.com/Ventilated-Transformer-Enclosure-Nameplate-Details/dp/B07G3DNTXN

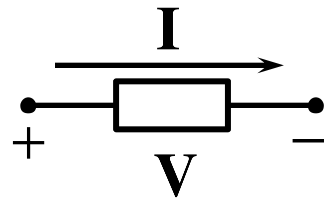
Apparent Power (2)

Ex.

$v(t) = 150\sin(314t - 30^o)$ V, $i(t) = 10\sin(314t + 45^o)$ A. Find S?

$$S = V_{rms} I_{rms} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = \frac{150}{\sqrt{2}} \times \frac{10}{\sqrt{2}} = 750 \text{ VA}$$

Power Factor (1)



$$P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)$$

$$S = V_{rms} I_{rms}$$

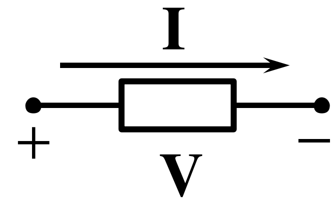
$$\boxed{pf = \frac{P}{S} = \cos(\phi_v - \phi_i)}$$

- $\phi_v - \phi_i = 0 \rightarrow pf = 1 \rightarrow P = S = V_{rms} I_{rms}$
- $\phi_v - \phi_i = \pm 90^{\circ} \rightarrow pf = 0 \rightarrow P = 0$

Power Factor (2)

Ex.

$v(t) = 150\sin(314t - 30^{o})$ V, $i(t) = 10\sin(314t + 45^{o})$ A. Find pf?



$$pf = \cos(-30^{o} - 45^{o}) = 0.2588$$

AC Power Analysis

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2. Maximum Average Power Transfer
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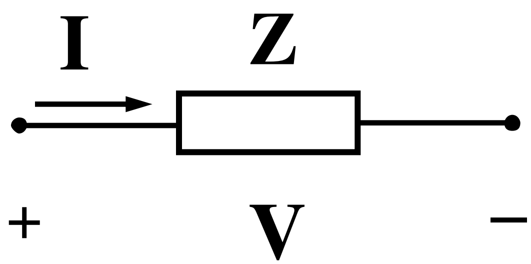
Complex Power (1)



$$\boxed{\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*}$$

$$\mathbf{V} = V_m \underline{/\phi_v}$$

$$\mathbf{I} = I_m \underline{/\phi_i} \rightarrow \mathbf{I}^* = I_m \underline{/-\phi_i}$$

$$\rightarrow \mathbf{S} = \frac{1}{2} V_m I_m \underline{/\phi_v - \phi_i} = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) + j\frac{1}{2} V_m I_m \sin(\phi_v - \phi_i)$$

Complex Power (2)

$$\left.\begin{array}{l}\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* \\ \mathbf{V} = \mathbf{Z}\mathbf{I}\end{array}\right\}$$

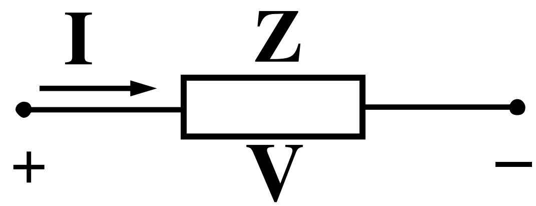
$$\rightarrow \begin{cases} \mathbf{S} = \frac{1}{2}\mathbf{V}\left(\frac{\mathbf{V}}{\mathbf{Z}}\right)^* = \frac{1}{2}\frac{\mathbf{V}\mathbf{V}^*}{\mathbf{Z}^*} = \frac{1}{2}\frac{\left(V_m \angle \phi_v\right)\left(V_m \angle -\phi_v\right)}{\mathbf{Z}^*} = \boxed{\frac{V_{rms}^2}{\mathbf{Z}^*}} \\ \mathbf{S} = \frac{1}{2}\mathbf{Z}\mathbf{I}\mathbf{I}^* = \frac{1}{2}\mathbf{Z}\left(I_m \angle \phi_i\right)\left(I_m \angle -\phi_i\right) = \frac{1}{2}\mathbf{Z}I_m^2 = \boxed{\mathbf{Z}I_{rms}^2} \end{cases}$$

$$\left.\begin{array}{r}\mathbf{S} = \mathbf{Z}I_{rms}^2 \\ \mathbf{Z} = R + jX\end{array}\right\}$$

$$\rightarrow \mathbf{S} = (R + jX)I_{rms}^2 = RI_{rms}^2 + jXI_{rms}^2 \rightarrow \begin{cases} \boxed{P = \mathrm{Re}(\mathbf{S}) = RI_{rms}^2} \\ \boxed{Q = \mathrm{Im}(\mathbf{S}) = XI_{rms}^2} \end{cases}$$

Reactive power (in volt-ampere reactive, VAR)

Complex Power (3)



$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = P + jQ = \frac{1}{2}V_m I_m \underline{/\phi_v - \phi_i} = V_{rms} I_{rms} \underline{/\phi_v - \phi_i} = \mathbf{Z} I_{rms}^2 = \frac{V_{rms}^2}{\mathbf{Z}^*}$$

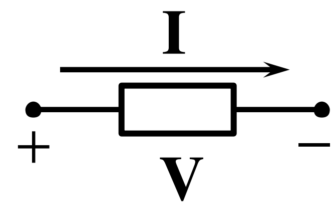
$$S = |\mathbf{S}| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$$

$$P = \mathrm{Re}(\mathbf{S}) = V_{rms} I_{rms} \cos(\phi_v - \phi_i) = S\cos(\phi_v - \phi_i) = R I_{rms}^2$$

$$Q = \mathrm{Im}(\mathbf{S}) = S\sin(\phi_v - \phi_i) = X I_{rms}^2$$

$$pf = \frac{P}{S} = \cos(\phi_v - \phi_i)$$

Complex Power (4)



Ex.

$v(t) = 150\sin(314t - 30^{o})$ V, $i(t) = 10\sin(314t + 45^{o})$ A.

$$\mathbf{V} = 150\angle{-30^{o}} \text{ V}, \ \mathbf{I} = 10\angle{45^{o}} \text{ A}$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^{*} = \frac{1}{2}\left(150\angle{-30^{o}}\right)\left(10\angle{-45^{o}}\right) = 750\angle{-75^{o}} \text{ VA}$$

$$S = |\mathbf{S}| = 750 \text{ VA}$$

$$P = S\cos(\varphi_u - \varphi_i) = 750\cos(-75^{o}) = 194.11 \text{ W}$$

$$Q = S\sin(\varphi_u - \varphi_i) = 750\sin(-75^{o}) = -724.44 \text{ VAR}$$

$$pf = \cos(\varphi_u - \varphi_i) = \cos(-75^{o}) = 0.26$$

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Conservation of AC Power (1)

$$\boxed{\sum_{i=1}^{M} \mathbf{S}_{source,i} = \sum_{i=1}^{N} \mathbf{S}_{load,i}}$$

$$\rightarrow \begin{cases} \sum_{i=1}^{M} P_{source,i} = \sum_{i=1}^{N} P_{load,i} \\ \sum_{i=1}^{M} Q_{source,i} = \sum_{i=1}^{N} Q_{load,i} \end{cases}$$

$$\sum_{i=1}^{M} S_{source,i} \neq \sum_{i=1}^{N} S_{load,i}$$

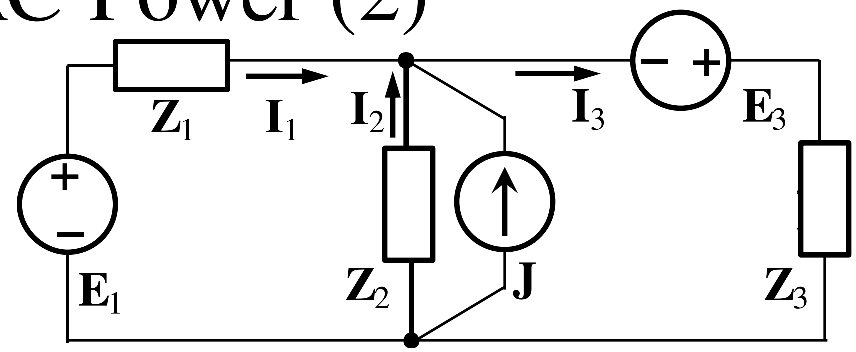
Conservation of AC Power (2)

Ex.

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\underline{/15^o}\ \text{V};\ \mathbf{J} = 2\underline{/-30^o}\ \text{A};$

Find currents?



$$\mathbf{I}_1 = 4.09\underline{/75.2^o}\ \text{A},\ \mathbf{I}_2 = 2.20\underline{/26.4^o}\ \text{A},\ \mathbf{I}_3 = 6.16\underline{/39.6^o}\ \text{A}$$

$$\left.\begin{aligned} \mathbf{S}_{Z1} &= \mathbf{Z}_1 I_{1rms}^2 = 10(4.09)^2/2 = 83.64\ \text{VA} \\ \mathbf{S}_{Z2} &= \mathbf{Z}_2 I_{2rms}^2 = j20(2.20)^2/2 = j48.40\ \text{VA} \\ \mathbf{S}_{Z3} &= \mathbf{Z}_3 I_{3rms}^2 = (5 - j10)(6.16^2)/2 = 94.86 - j189.73\ \text{VA} \end{aligned}\right\} \rightarrow \boxed{\sum \mathbf{S}_{load} = 178.50 - j141.33\ \text{VA}}$$

$$\left.\begin{aligned} \mathbf{S}_{E1} &= \frac{1}{2}\mathbf{E}_1\mathbf{I}_1^* = \frac{1}{2}30.4.09\underline{/-75,2^o} = 61.35\underline{/-75,2^o}\ \text{VA} \\ \mathbf{S}_{E3} &= \frac{1}{2}\mathbf{E}_3\mathbf{I}_3^* = 138.60\underline{/-24.6^o}\ \text{VA} \\ \mathbf{S}_J &= \frac{1}{2}\mathbf{U}_J\mathbf{J}^* = (-\mathbf{Z}_2\mathbf{I}_2)\mathbf{J}^* = 36.65 - j24.35\ \text{VA} \end{aligned}\right\} \rightarrow \boxed{\sum \mathbf{S}_{source} = 178.34 - j141.36\ \text{VA}}$$

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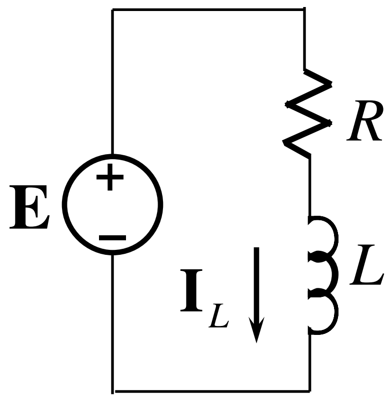
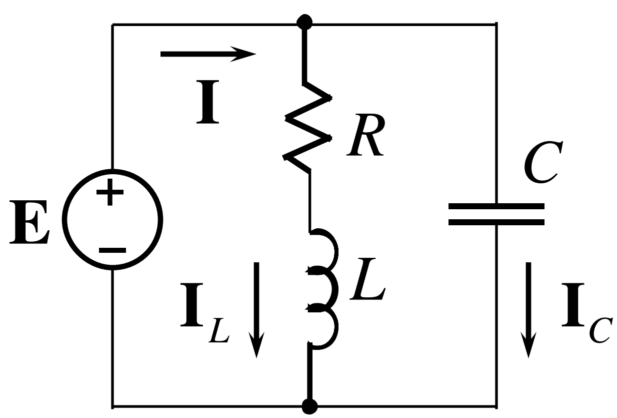
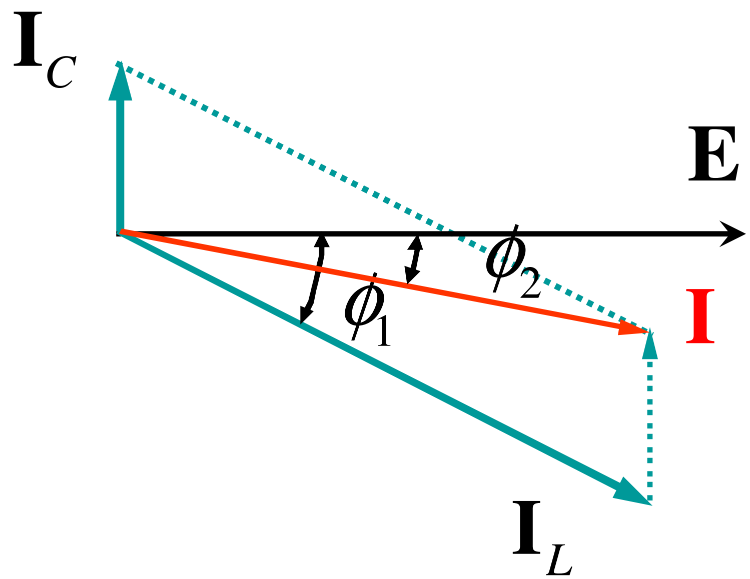
Power Factor Improvement (1)



$$pf = \cos\phi$$

$$pf_2 > pf_1 \rightarrow \phi_2 < \phi_1$$





$$\phi_2 < \phi_1 \quad \rightarrow C = ? \;\; (P = \text{const})$$

Power Factor Improvement (2)

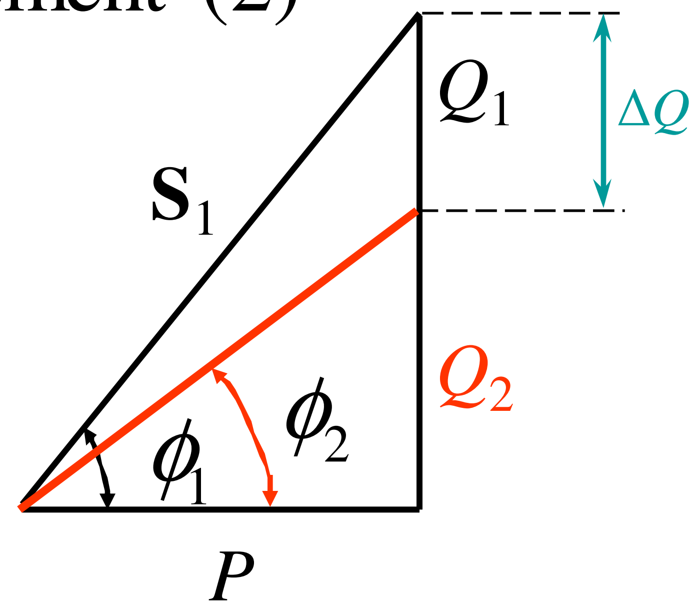
$$Q_1 = P\tan\phi_1,\ \ Q_2 = P\tan\phi_2$$

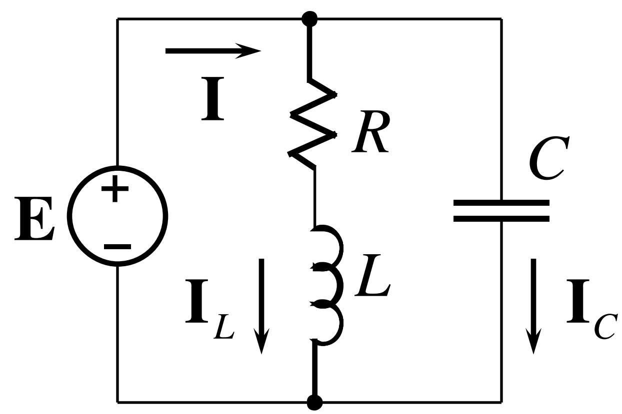
$$\Delta Q = Q_1 - Q_2$$

$$\Delta Q = \frac{E_{rms}^2}{X_C} = \omega C E_{rms}^2 \rightarrow C = \frac{\Delta Q}{\omega E_{rms}^2}$$

$$C = \frac{Q_1 - Q_2}{\omega E_{rms}^2} = \frac{P\tan\phi_1 - P\tan\phi_2}{\omega E_{rms}^2}$$

$$= \boxed{P\frac{\tan\phi_1 - \tan\phi_2}{\omega E_{rms}^2}}$$





Ex

Power Factor Improvement (3)

A load is connected to a 220 V (rms), 50 Hz power line. This load absorbs a power of 1000kW. Its power factor is 0.8. Find the capacitor required to raise the pf to 0.9?

$$C = P\frac{\tan\phi_1 - \tan\phi_2}{\omega E_{rms}^2}$$

$$pf_1 = 0.8 \rightarrow \cos\phi_1 = 0.8 \rightarrow \phi_1 = 36.9^{\circ} \rightarrow \tan\phi_1 = 0.75$$

$$pf_2 = 0.9 \rightarrow \cos\phi_2 = 0.9 \rightarrow \phi_2 = 25.8^{\circ} \rightarrow \tan\phi_2 = 0.48$$

$$\rightarrow C = 1000\times10^3\frac{0.75-0.48}{314\times(220)^2} = \boxed{0.0178\,\mathrm{F}}$$

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Average Power and RMS Value of Periodic Signals (1)

$$v(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \sin(n\omega_0 t - \theta_n)$$

$$i(t) = I_{dc} + \sum_{m=1}^{\infty} I_n \sin(m\omega_0 t - \phi_m)$$

$$P = V_{dc} I_{dc} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

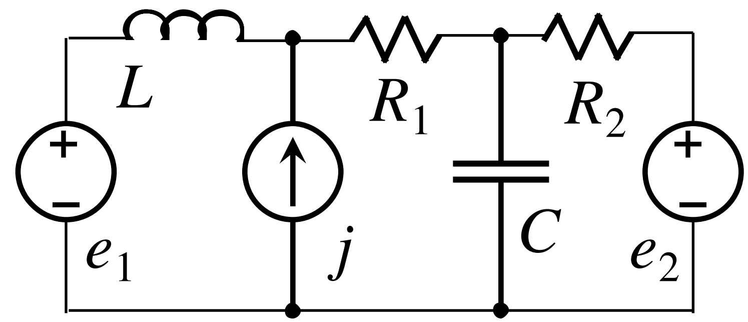
$$V_{rms} = \sqrt{V_{dc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} V_n^2}$$

$$I_{rms} = \sqrt{I_{dc}^2 + \frac{1}{2} \sum_{m=1}^{\infty} I_m^2}$$

Average Power and RMS Value of Periodic Signals (2)

Ex.

$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^o)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; find the RMS value of the voltage across R_1?



$$v_{R1} = -1 + 1.06\sin(10t - 58^o) + 4.14\sin(50t + 32^o)\,\text{V}$$

$$V_{rms} = \sqrt{V_{dc}^2 + \frac{1}{2}\sum_{n=1}^{\infty} V_n^2}$$

$$= \sqrt{(-1)_{dc}^2 + \frac{1}{2}(1.06^2 + 4.14^2)} = \boxed{3.18\,\text{V}}$$