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Engineering Electromagnetics

Guided Waves & Radiation

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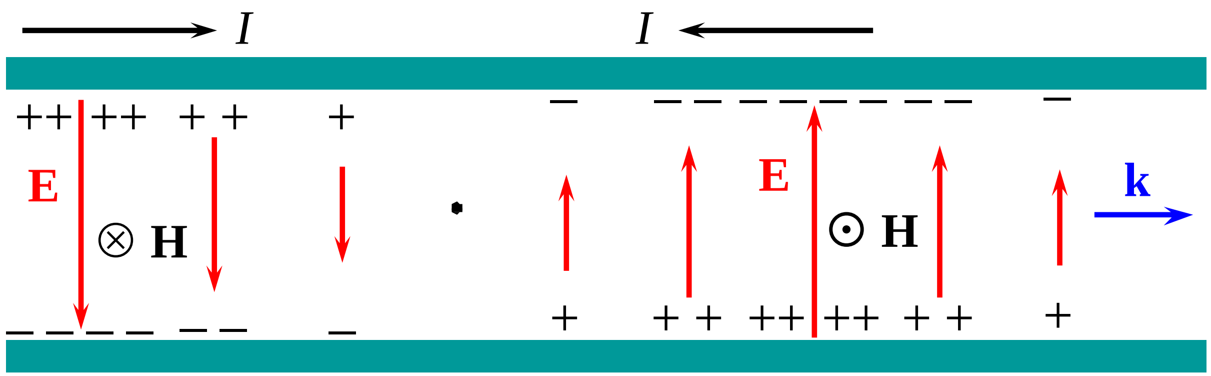
Transmission Line Fields (1)

$$V_s(z) = V_0 e^{-j\beta z}$$

$$I_s(z) = \frac{V_0}{Z_0} e^{-j\beta z}$$

where $Z_0 = \sqrt{L/C}$

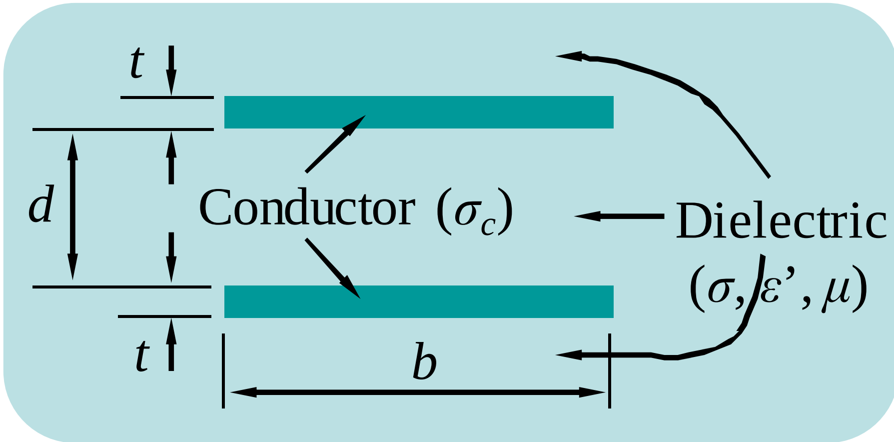


$$\left.\begin{aligned} E_{sx}(z) &= \frac{V_s}{d} = \frac{V_0}{d} e^{-j\beta z} \\ H_{sy}(z) &= K_{sz} = \frac{I_s}{b} = \frac{V_0}{bZ_0} e^{-j\beta z} \end{aligned}\right\} \rightarrow P_z = \int_0^b \int_0^d \frac{1}{2} \mathrm{Re}\{E_{xs}\hat{H}_{ys}\} dxdy$$

$$= \frac{1}{2} \frac{V_0}{d} \frac{\hat{V}_0}{b\hat{Z}_0}(bd)$$

$$= \frac{|V_0|^2}{2\hat{Z}_0} = \frac{1}{2} \mathrm{Re}[V_s \hat{I}_s]$$

Transmission Line Fields (2)



$$C = \frac{\varepsilon' b}{d}$$

$$G = \frac{\sigma}{\varepsilon'} C = \frac{\sigma b}{d}$$

$$L \approx L_{\text{external}} = \frac{\mu d}{b}$$

$$R = \frac{2}{\sigma_c \delta b}$$

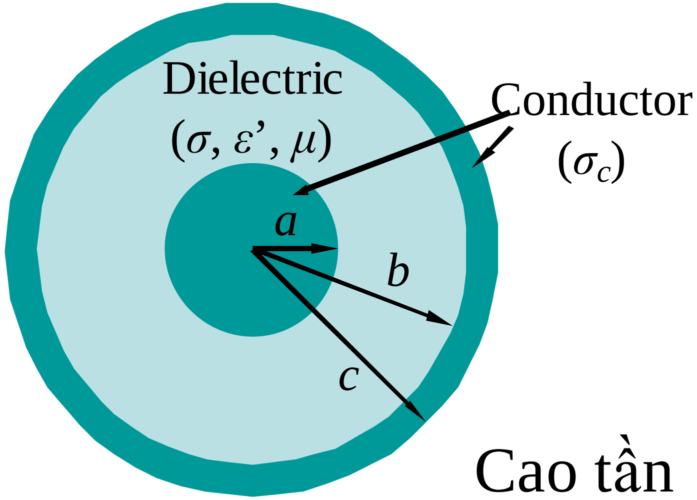
$$Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{b}\sqrt{\frac{\mu}{\varepsilon'}}$$

Transmission Line Fields (3)

$$C = \frac{2\pi\varepsilon'}{\ln(b/a)}$$

$$G = \frac{\sigma}{\varepsilon'}C = \frac{2\pi\sigma}{\ln(b/a)}$$

$$L_{\text{external}} = \frac{\mu}{2\pi}\ln\frac{b}{a}$$



Cao tần

$$R_{\text{internal}} = \frac{1}{2\pi a\delta\sigma_c}, \quad R_{\text{external}} = \frac{1}{2\pi b\delta\sigma_c}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$Z_0 = \sqrt{\frac{L_{\text{external}}}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon'}}\ln\frac{b}{a}$$

Transmission Line Fields (4)

$$C = \frac{2\pi\varepsilon'}{\ln(b/a)}$$

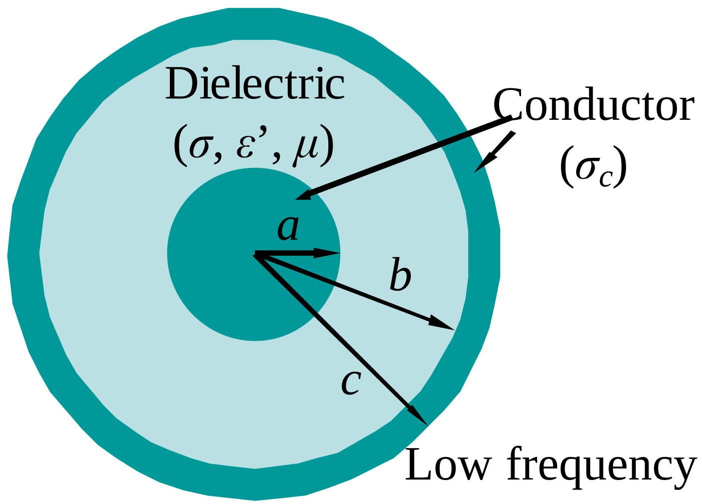
$$G = \frac{\sigma}{\varepsilon'}C = \frac{2\pi\sigma}{\ln(b/a)}$$

$$R_{\text{internal}} = \frac{l}{\sigma_c S} = \frac{1}{\sigma_c(\pi a^2)}$$

$$R_{\text{external}} = \frac{1}{\sigma_c[\pi(c^2 - b^2)]}$$

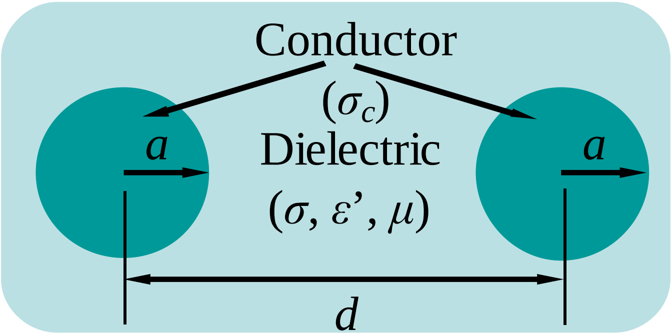
$$R = \frac{1}{\pi\sigma_c}\left(\frac{1}{a^2} + \frac{1}{c^2 - b^2}\right)$$

$$L = \frac{\mu}{2\pi}\left[\ln\frac{b}{a} + \frac{1}{4} + \frac{1}{4(c^2 - b^2)}\left(b^2 - 3c^2 + \frac{4c^2}{c^2 - b^2}\ln\frac{c}{b}\right)\right]$$



Transmission Line Fields (5)

$$C = \frac{\pi\varepsilon'}{\cosh^{-1}(d/2a)} \approx \frac{\pi\varepsilon'}{\ln(d/a)} \quad (a \ll d)$$



High frequency

$$L_{\text{external}} = \frac{\mu}{\pi}\cosh^{-1}(d/2a) \approx \frac{\mu}{\pi}\ln\frac{d}{a} \quad (a \ll d)$$

$$G = \frac{\sigma}{\varepsilon'}C = \frac{\pi\sigma}{\cosh^{-1}(d/2a)}$$

$$R = \frac{1}{\pi a \delta \sigma_c}$$

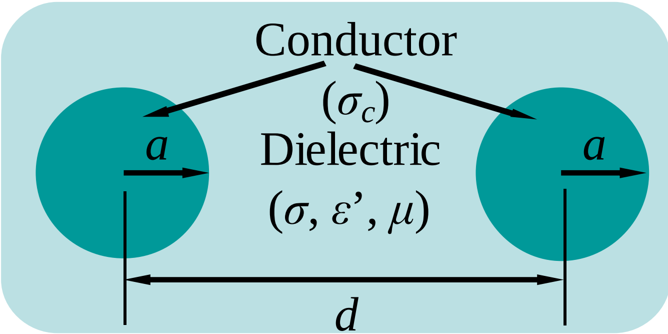
Transmission Line Fields (5)

$$C = \frac{\pi\varepsilon'}{\cosh^{-1}(d/2a)}$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(d/2a)}$$

$$L = \frac{\mu}{\pi}\left[\frac{1}{4} + \cosh^{-1}(d/2a)\right]$$

$$R = \frac{2}{\pi a^2 \sigma_c}$$



Low frequency

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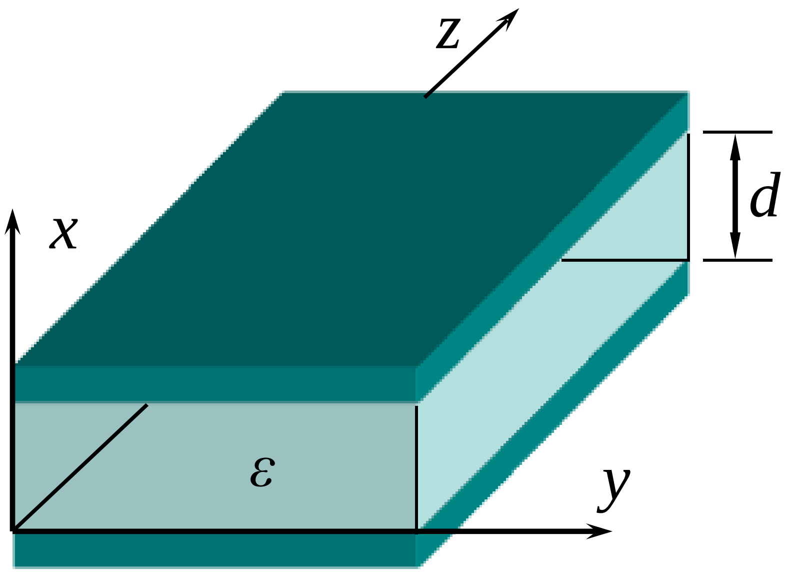
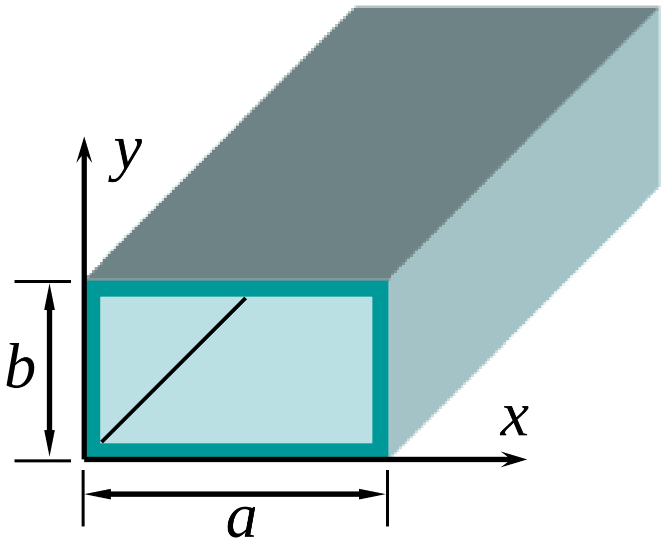
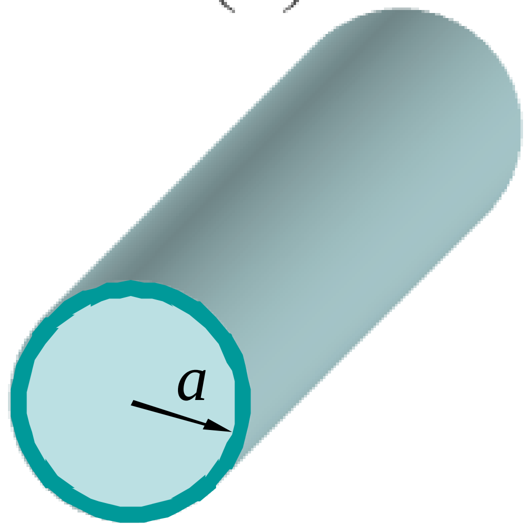
Basic Waveguide Operation (1)

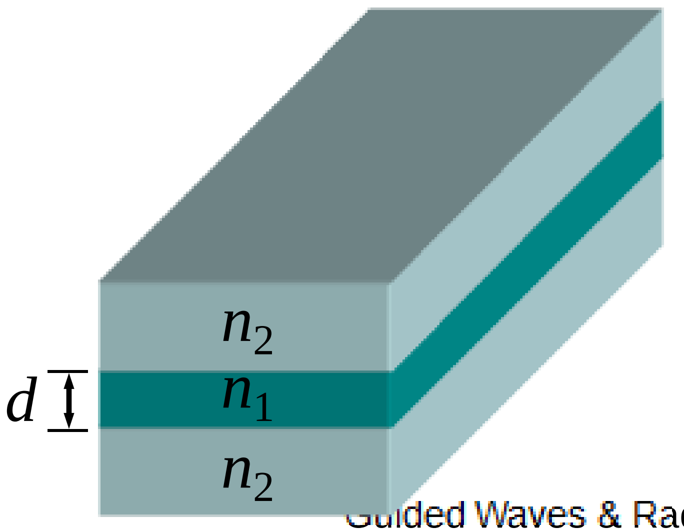


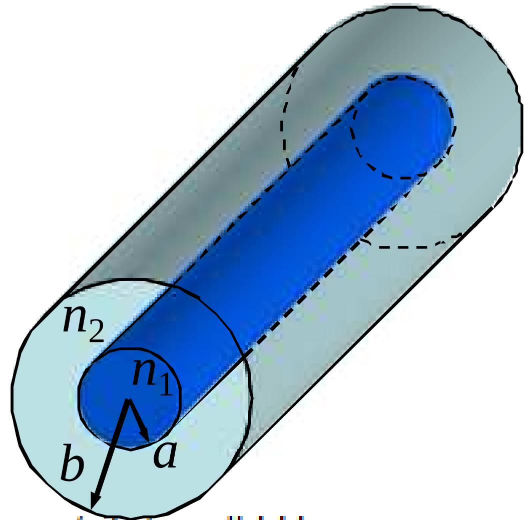
Parallel-plate waveguide

Basic Waveguide Operation (2)

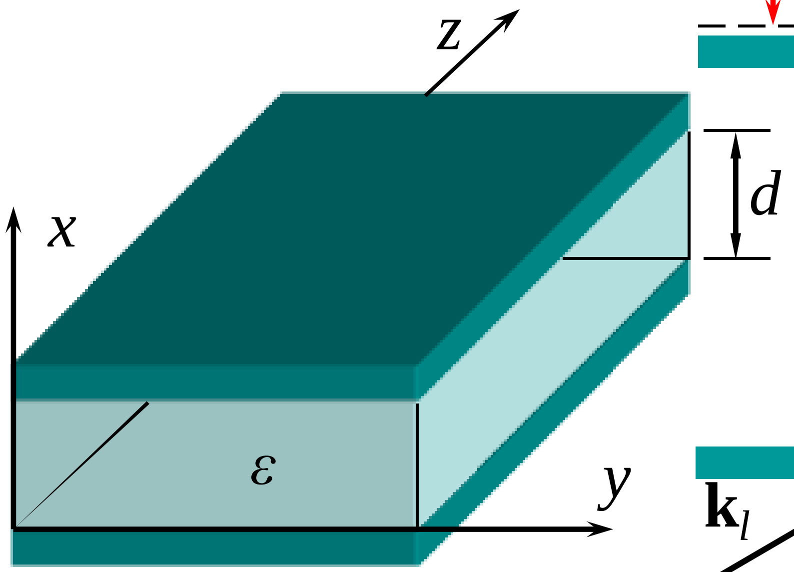


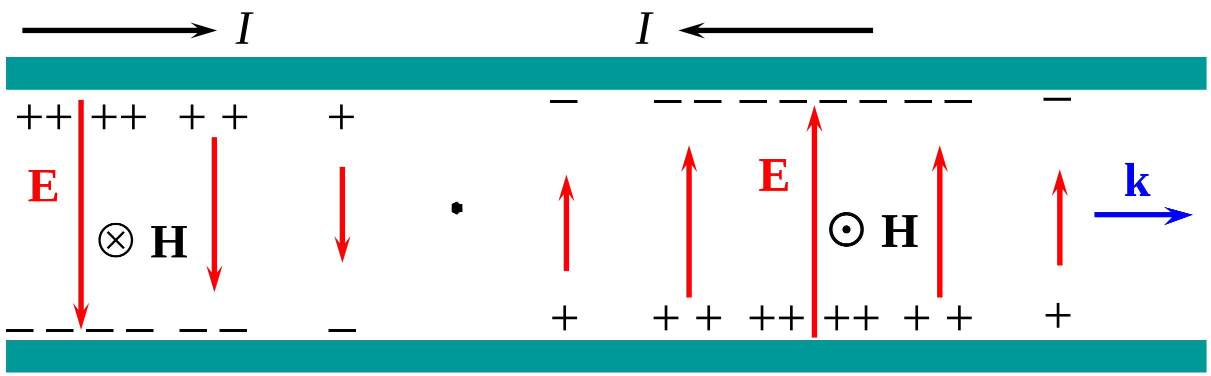




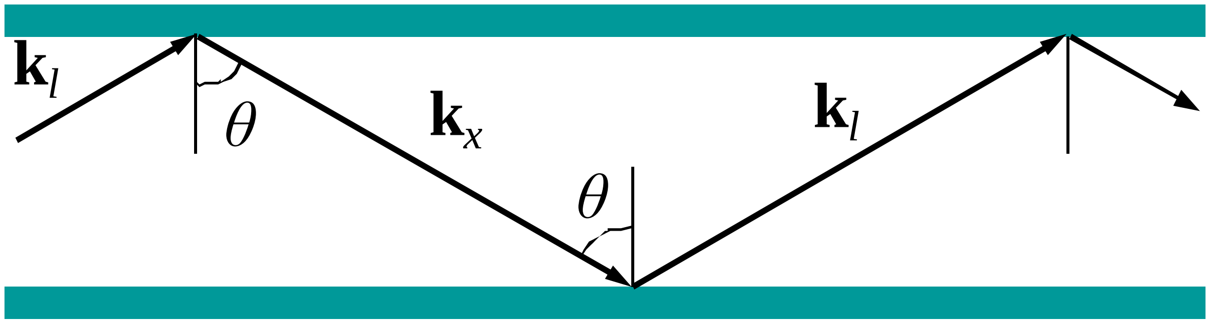


Basic Waveguide Operation (3)

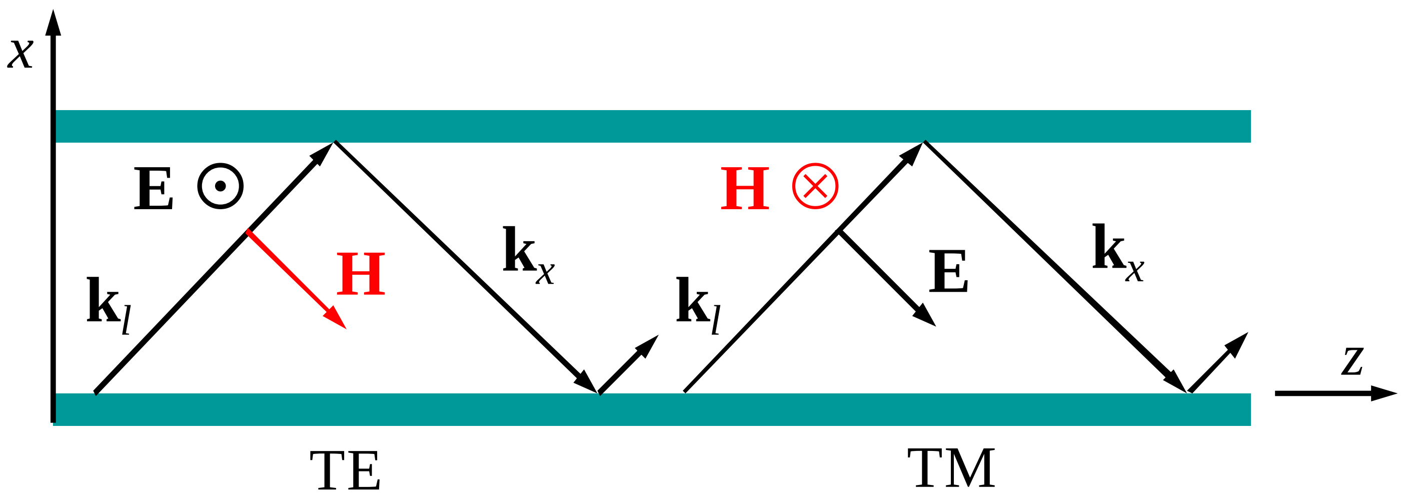


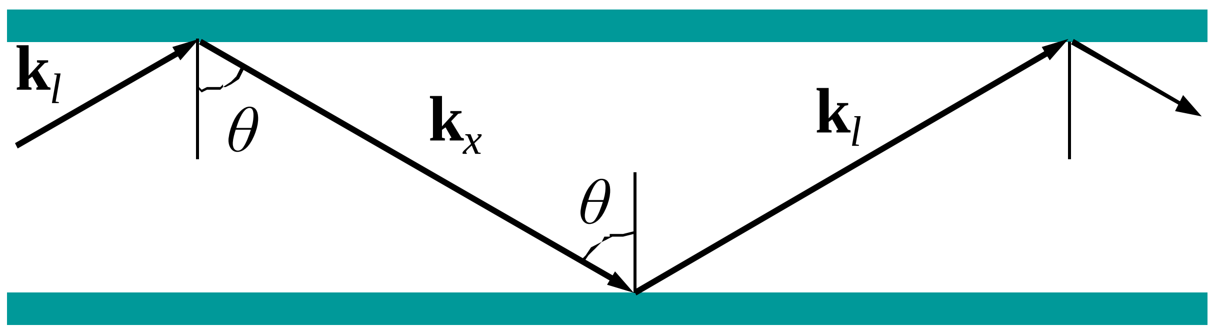


$$\left|\mathbf{k}_l\right| = \left|\mathbf{k}_x\right| = k = \omega\sqrt{\mu\varepsilon}$$



Basic Waveguide Operation (4)





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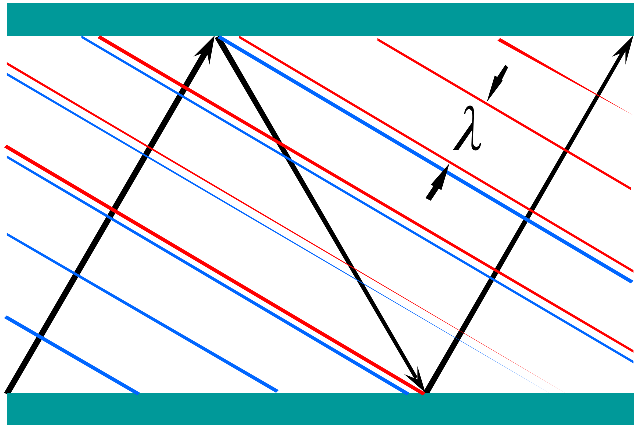
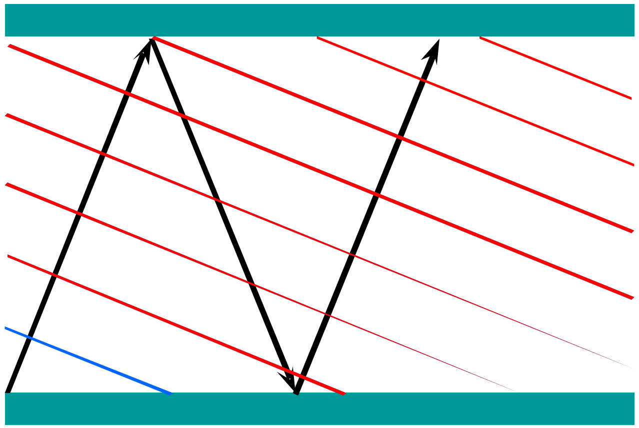
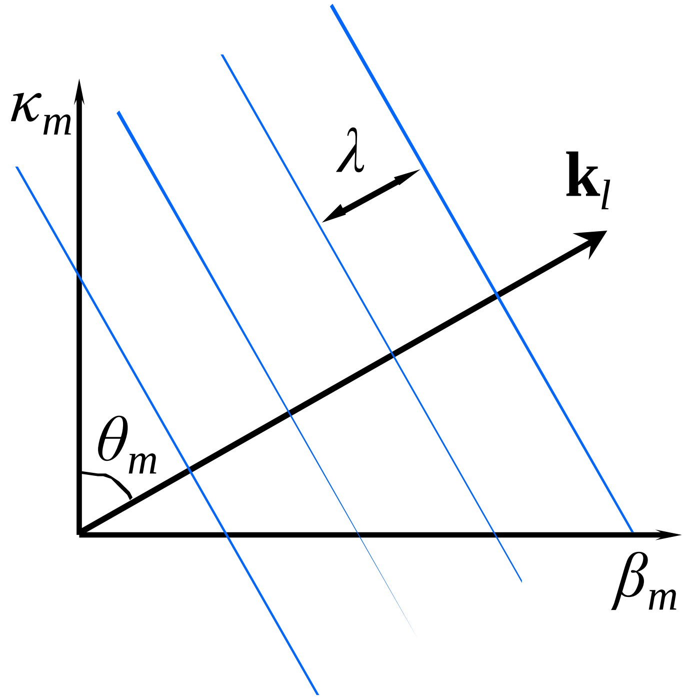
Plane Wave Analysis of the Parallel - Plate Waveguide (1)



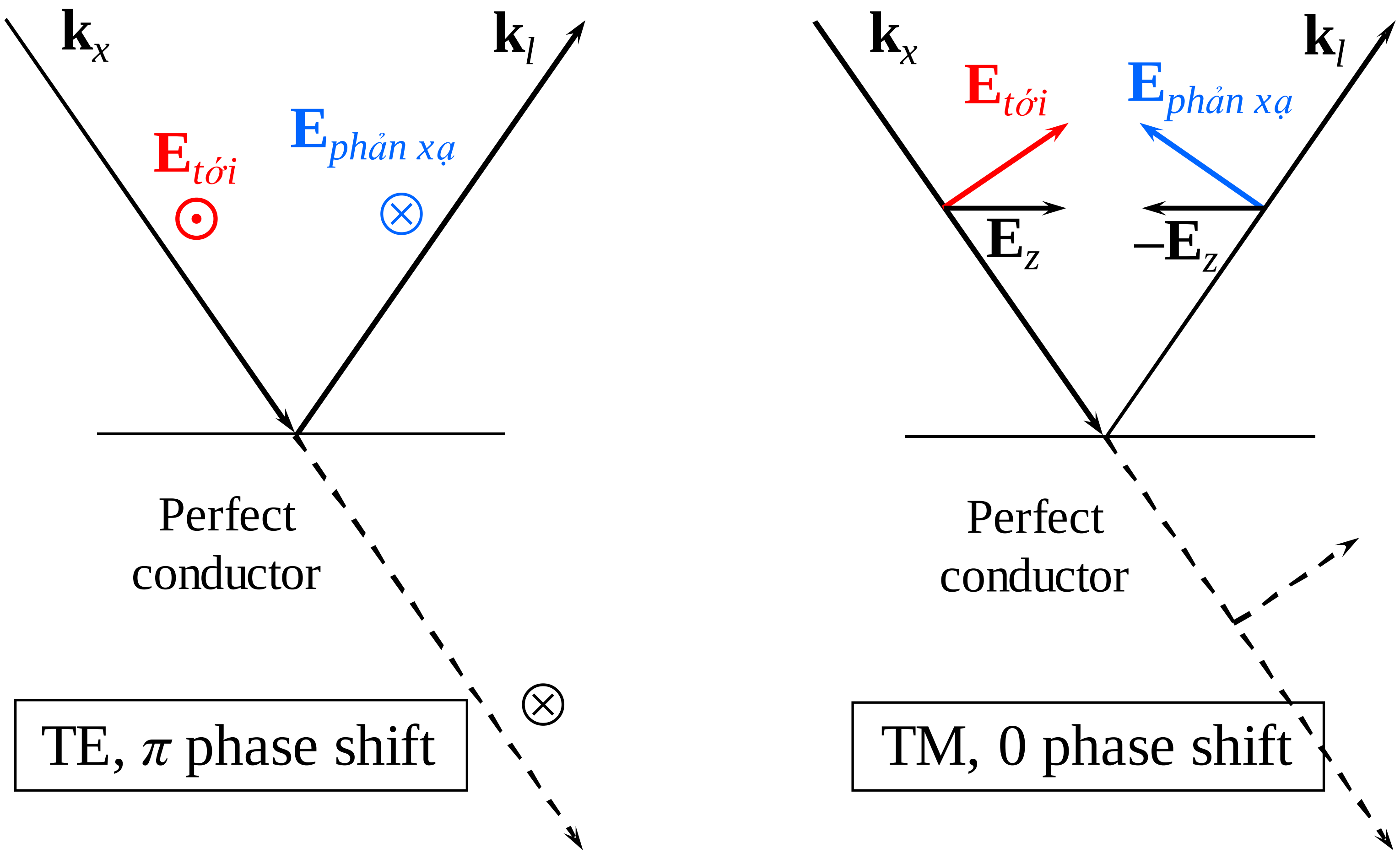




$$\beta_m = \sqrt{k^2 - \kappa_m^2}$$

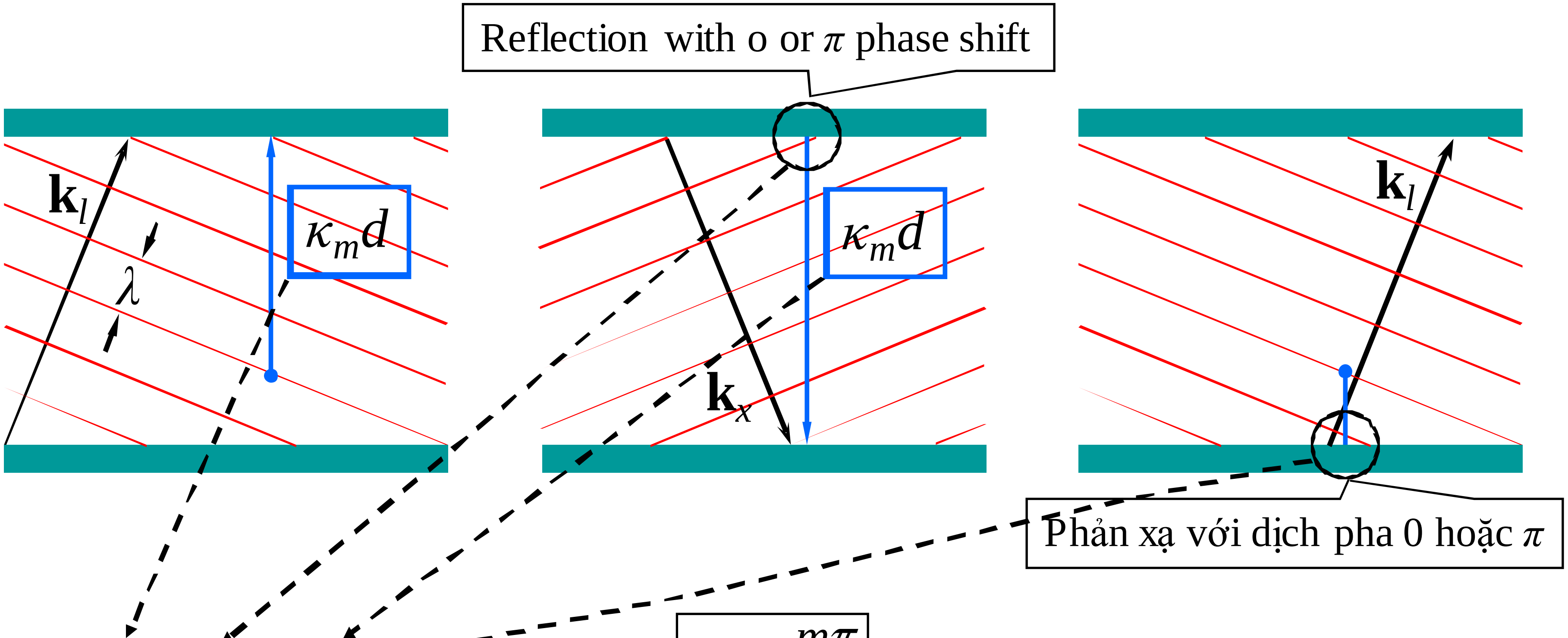
$$k = \omega\sqrt{\mu_0 \varepsilon'} = \frac{\omega\sqrt{\varepsilon_r'}}{c} = \frac{\omega n}{c}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (2)



Plane Wave Analysis of the Parallel - Plate Waveguide (3)

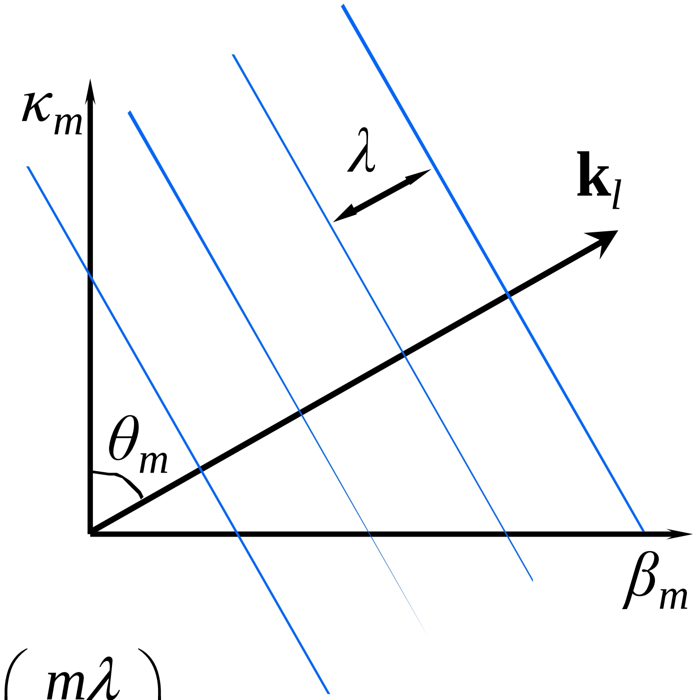
$$\beta_m = \sqrt{k^2 - \kappa_m^2} \qquad k = \omega\sqrt{\mu_0 \varepsilon'} = \frac{\omega\sqrt{\varepsilon_r'}}{c} = \frac{\omega n}{c}$$



$$\kappa_m d + \varphi + \kappa_m d + \varphi = 2m\pi \quad \rightarrow \boxed{\kappa_m = \frac{m\pi}{d}}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (4)

$$\left.\begin{aligned} \beta_m &= \sqrt{k^2 - \kappa_m^2} \\ k &= \omega\sqrt{\mu_0 \varepsilon'} = \frac{\omega\sqrt{\varepsilon_r'}}{c} = \frac{\omega n}{c} \\ \kappa_m &= \frac{m\pi}{d} \\ \kappa_m &= k\cos\theta_m \end{aligned}\right\}$$



$$\rightarrow \begin{cases} \theta_m = \arccos\left(\dfrac{m\pi}{kd}\right) = \arccos\left(\dfrac{m\pi c}{\omega n d}\right) = \arccos\left(\dfrac{m\lambda}{2nd}\right) \\ \beta_m = \sqrt{k^2 - \kappa_m^2} = k\sqrt{1 - \left(\dfrac{m\pi}{kd}\right)^2} = k\sqrt{1 - \left(\dfrac{m\pi c}{\omega n d}\right)^2} \end{cases}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (5)

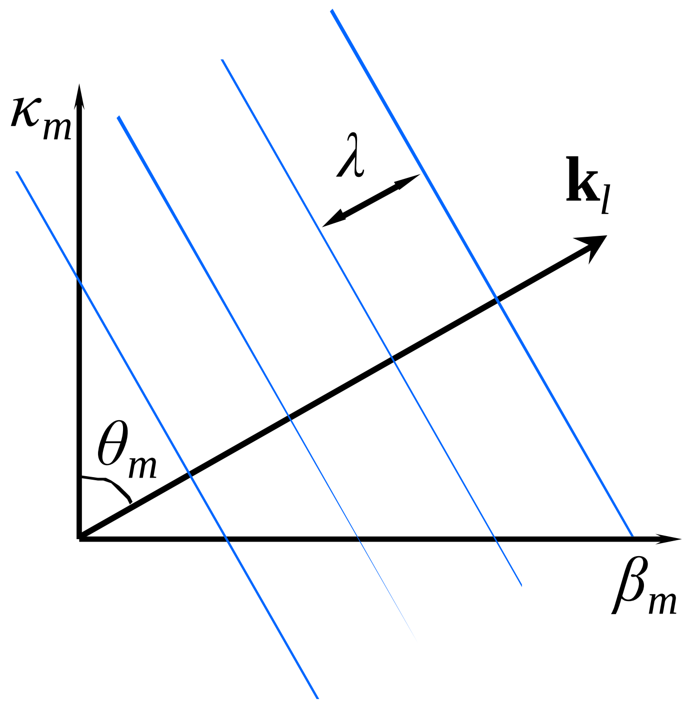
Definition: $\omega_{cm} = \dfrac{m\pi c}{nd}$

$$\beta_m = k\sqrt{1-\left(\frac{m\pi c}{\omega n d}\right)^2}$$

$$\rightarrow \beta_m = \frac{n\omega}{c}\sqrt{1-\left(\frac{\omega_{cm}}{\omega}\right)^2}$$

$$\lambda_{cm} = \frac{2\pi c}{\omega_{cm}} = \frac{2nd}{m}$$

$$\rightarrow \beta_m = \frac{2\pi n}{\lambda}\sqrt{1-\left(\frac{\lambda}{\lambda_{cm}}\right)^2}$$



Plane Wave Analysis of the Parallel - Plate Waveguide (6)

Ex. 1

A parallel-plate transmission lines has plate separation d = 1cm, and is filled with teflon having ε'_r = 2.1. Find the maximum operating frequency such that only the TEM mode will propagate, and find the range of frequencies over which the m = 1 mode will propagate.

$$\omega_{c1} = \frac{m\pi c}{nd} = \frac{1\pi c}{\sqrt{\varepsilon'_r}d} = \frac{\pi \times 3\times 10^8}{\sqrt{2.1}\times 10^{-2}} = \frac{3\pi}{\sqrt{2.1}}10^{10}$$

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = \frac{3\pi\times 10^{10}}{2\pi\sqrt{2.1}} = 1.03\times 10^{10}\text{ Hz} = 10\times 3\text{ GHz}$$

$$10.3\text{ GHz} < f < 20.6\text{ GHz}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (7)

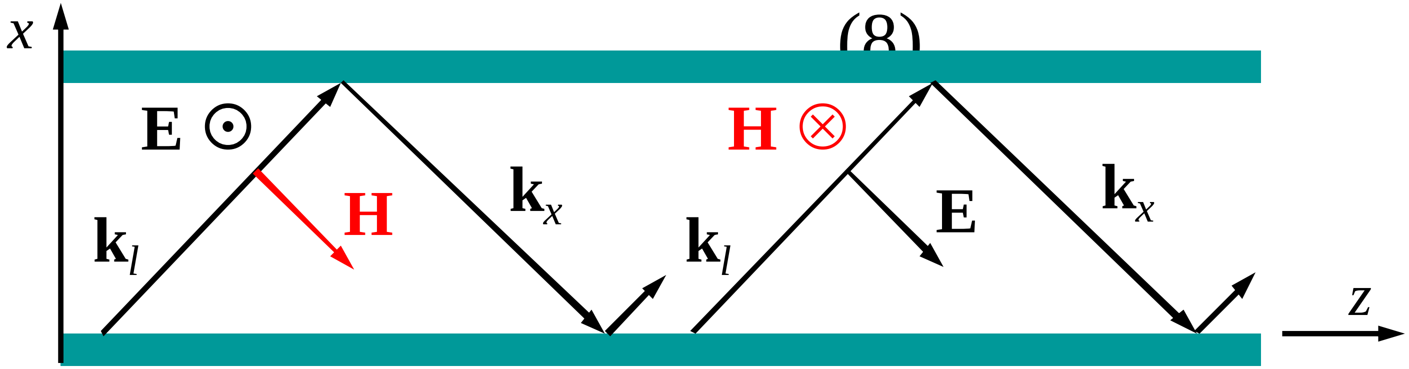
Ex. 2

A parallel-plate transmission lines has plate separation d = 1cm, and is filled with teflon having ε'_r = 2.1. The operating wavelength is λ = 3mm. How many waveguide modes will propagate?

$$\lambda_{cm} = \frac{2nd}{m} \rightarrow 2\times 10^{-3} < \frac{2\sqrt{2.1}\times 10\times 10^{-3}}{m}$$

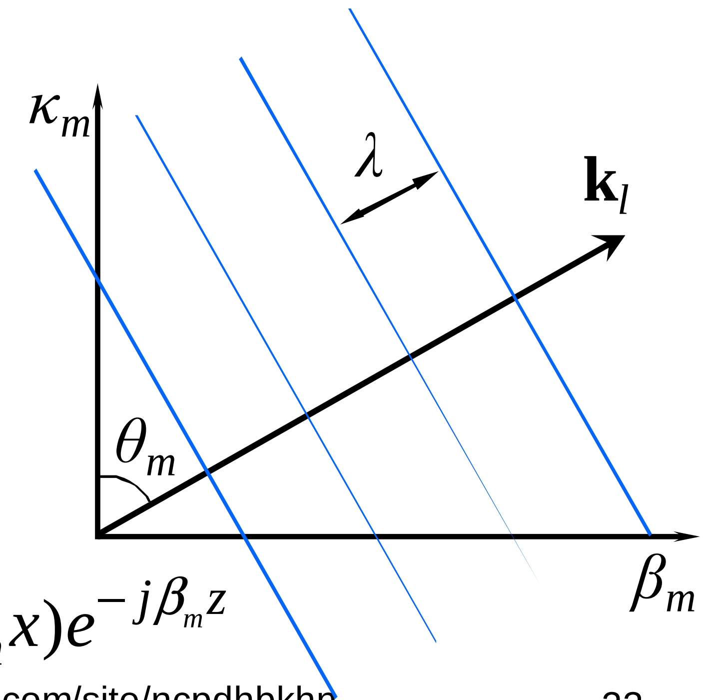
$$\rightarrow m < \frac{2\sqrt{2.1}\times 10}{2} = 14.5$$

Plane Wave Analysis of the Parallel - Plate Waveguide (8)



TE TM

$$\left.\begin{aligned} E_{ys} &= E_0 e^{-j\mathbf{k}_l.\mathbf{r}} - E_0 e^{-j\mathbf{k}_x.\mathbf{r}} \\ \mathbf{k}_l &= \kappa_m \mathbf{a}_x + \beta_m \mathbf{a}_z \\ \mathbf{k}_x &= -\kappa_m \mathbf{a}_x + \beta_m \mathbf{a}_z \\ \mathbf{r} &= x\mathbf{a}_x + z\mathbf{a}_z \end{aligned}\right\}$$



$$\rightarrow E_{ys} = E_0 (e^{-j\kappa_m x} - e^{j\kappa_m x}) e^{-j\beta_m z}$$

$$= 2jE_0 \sin(\kappa_m x) e^{-j\beta_m z} = E_0^{'} \sin(\kappa_m x) e^{-j\beta_m z}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (9)

$$\left.\begin{aligned}
E_{ys} &= E_0\left(e^{-j\kappa_m x} - e^{j\kappa_m x}\right)e^{-j\beta_m z} \\
&= 2jE_0 \sin(\kappa_m x)e^{-j\beta_m z} = E_0' \sin(\kappa_m x)e^{-j\beta_m z} \\
\rightarrow E_y(z,t) &= \mathrm{Re}\left[E_{ys}e^{j\omega t}\right] = E_0' \sin(\kappa_m x)\cos(\omega t - \beta_m z) \\
&\left.\begin{aligned}\beta_m = \frac{n\omega}{c}\sqrt{1-\left(\frac{\omega_{cm}}{\omega}\right)^2} \\ \text{If } \omega < \omega_{cm}\end{aligned}\right\} \rightarrow -j|\beta_m| = -j\alpha_m
\end{aligned}\right\}$$

$$\rightarrow \begin{cases} E_{ys} = E_0' \sin(\kappa_m x)e^{-\alpha_m z} \\ E(z,t) = E_0' \sin(\kappa_m x)e^{-\alpha_m z}\cos\omega t \end{cases}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (10)

$$\left.\begin{array}{r}\beta_m = \dfrac{n\omega}{c}\sqrt{1-\left(\dfrac{\omega_{cm}}{\omega}\right)^2} = \dfrac{n}{c}\sqrt{\omega^2 - \omega_{cm}^2} \\ \omega < \omega_{cm}\end{array}\right\}$$

$$\rightarrow \alpha_m = \frac{n}{c}\sqrt{\omega_{cm}^2 - \omega^2} = \frac{n\omega_{cm}}{c}\sqrt{1-\left(\frac{\omega}{\omega_{cm}}\right)^2} = \frac{2\pi n}{\lambda_{cm}}\sqrt{1-\left(\frac{\lambda_{cm}}{\lambda}\right)^2}$$

$$\left.\begin{array}{r}\theta_m = \arccos\left(\dfrac{m\pi}{kd}\right) = \arccos\left(\dfrac{m\pi c}{\omega n d}\right) = \arccos\left(\dfrac{m\lambda}{2nd}\right) \\ \omega_{cm} = \dfrac{m\pi c}{nd}\end{array}\right\}$$

$$\rightarrow \cos\theta_m = \frac{\omega_{cm}}{\omega} = \frac{\lambda}{\lambda_{cm}}$$

Plane Wave Analysis of the Parallel - Plate Waveguide (11)

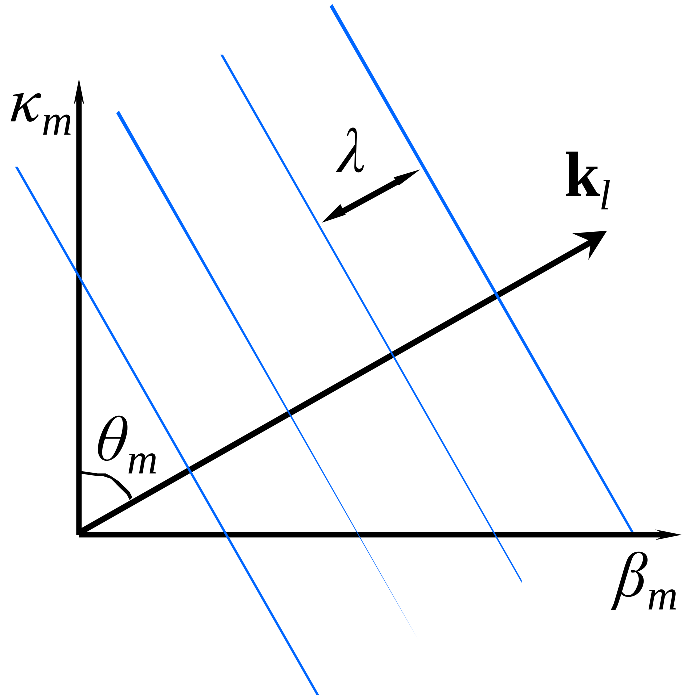
$$\cos\theta_m = \frac{\omega_{cm}}{\omega} = \frac{\lambda}{\lambda_{cm}}$$

$$\beta_m = k\sin\theta_m = \frac{n\omega}{c}\sin\theta_m$$

$$v_{pm} = \frac{\omega}{\beta_m} = \frac{c}{n\sin\theta_m}$$

$$v_{gm}^{-1} = \frac{d\beta_m}{d\omega} = \frac{d}{d\omega}\left[\frac{n\omega}{c}\sqrt{1-\left(\frac{\omega_{cm}}{\omega}\right)^2}\right]$$

$$\rightarrow v_{gm} = \frac{c}{n}\sqrt{1-\left(\frac{\omega_{cm}}{\omega}\right)^2} = \frac{c}{n}\sin\theta_m$$



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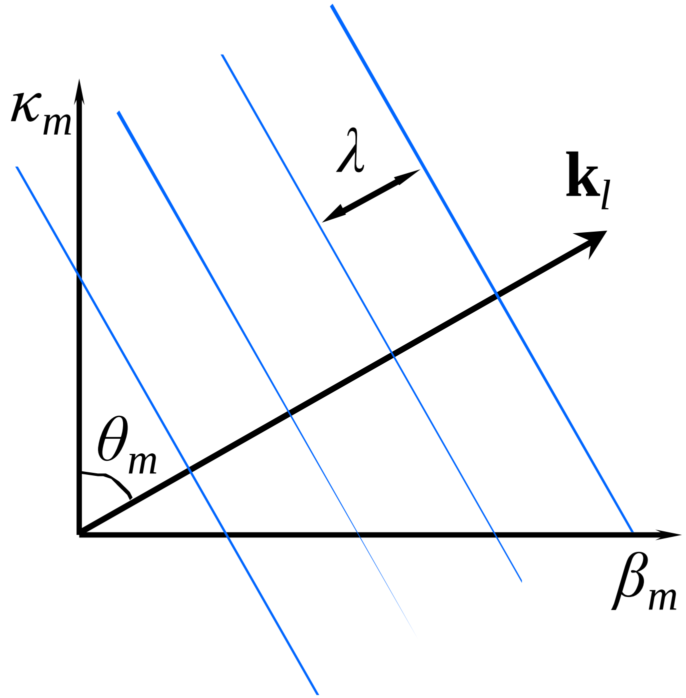
Parallel - Plate Guide Analysis Using the Wave Equation (1)

$$\nabla^2 \mathbf{E}_s = -k_0^2 \mathbf{E}_s$$

$$\rightarrow \nabla^2 \mathbf{E}_s = -k^2 \mathbf{E}_s, \qquad k = n\omega/c$$

$$\rightarrow \left.\begin{aligned} &\frac{\partial^2 E_{ys}}{\partial x^2} + \frac{\partial^2 E_{ys}}{\partial y^2} + \frac{\partial^2 E_{ys}}{\partial z^2} + k^2 E_{ys} = 0 \\ &\frac{\partial^2 E_{ys}}{\partial y^2} = 0 \\ &E_{ys} = E_0 f_m(x) e^{-j\beta_m z} \end{aligned}\right\}$$



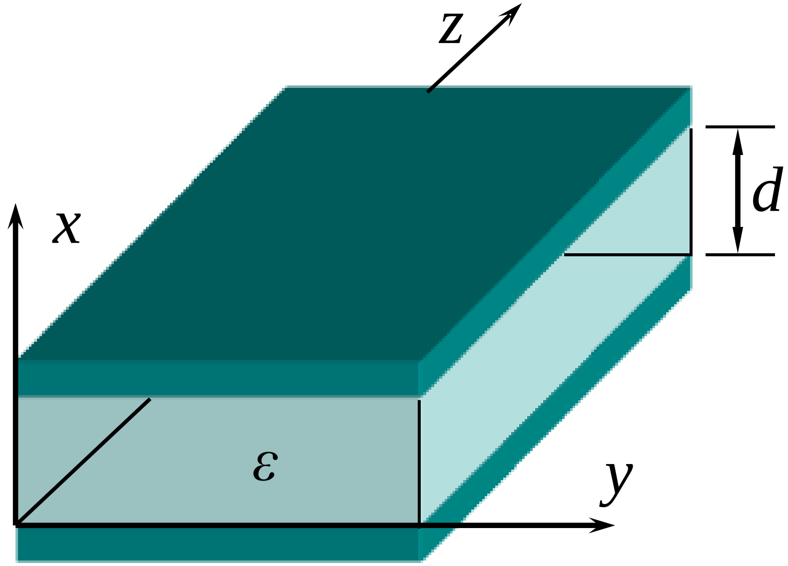
$$\rightarrow \left.\begin{aligned} &\frac{d^2 f_m(x)}{dx^2} + (k^2 - \beta_m^2) f_m(x) = 0 \\ &k^2 - \beta_m^2 = \kappa_m^2 \end{aligned}\right\} \rightarrow \frac{d^2 f_m(x)}{dx^2} + \kappa_m^2 f_m(x) = 0$$

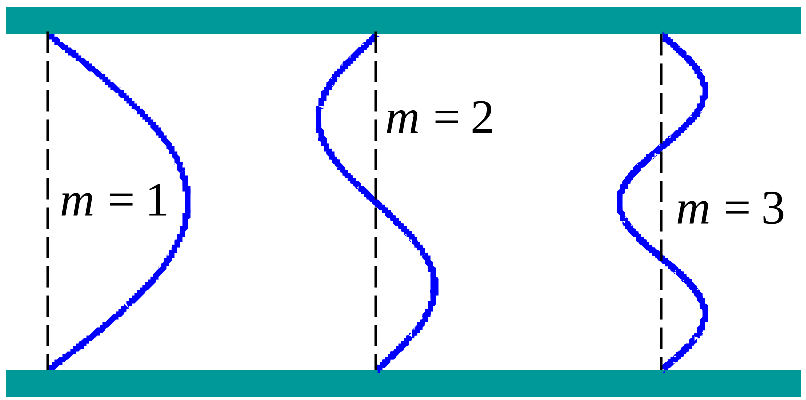
$$\rightarrow f_m(x) = \cos(\kappa_m x) + \sin(\kappa_m x)$$

Parallel - Plate Guide Analysis Using the Wave Equation (2)

$$\left.\begin{array}{l}
E_{ys} = E_0 f_m(x) e^{-j\beta_m z} \\
f_m(x) = \cos(\kappa_m x) + \sin(\kappa_m x) \\
E_y\big|_{x=0} = 0 \rightarrow f_m(x) = \sin(\kappa_m x) \\
E_y\big|_{x=d} = 0 \rightarrow \kappa_m = \dfrac{m\pi}{d}
\end{array}\right\}$$

$$\rightarrow \boxed{E_{ys} = E_0 \sin\left(\frac{m\pi x}{d}\right) e^{-j\beta_m z}}$$





Parallel - Plate Guide Analysis Using the Wave Equation (3)

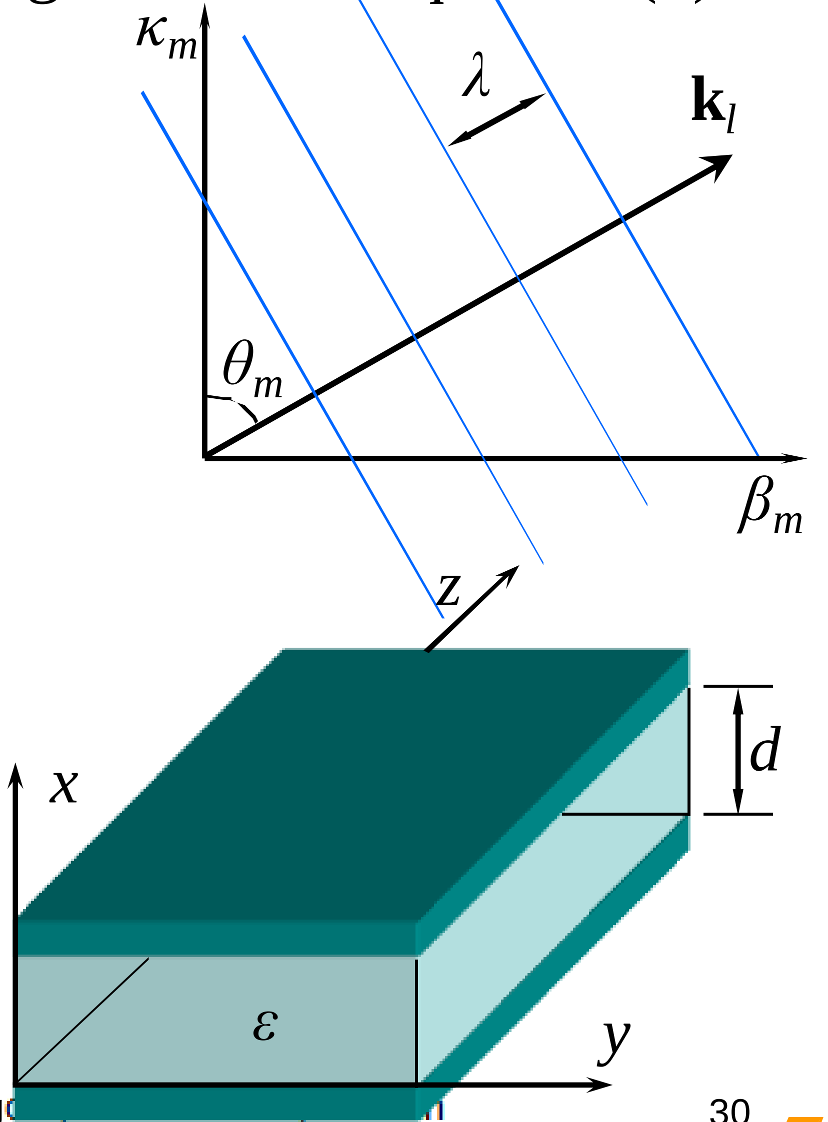
$$E_{ys} = E_0 \sin\left(\frac{m\pi x}{d}\right) e^{-j\beta_m z}$$

$$\left.\begin{aligned} \cos\theta_m &= \frac{\omega_{cm}}{\omega} \\ \text{If } \omega &= \omega_{cm} \end{aligned}\right\} \rightarrow \beta_m = 0$$

$$\left.\begin{aligned} &\rightarrow \kappa_m = k = \frac{2n\pi}{\lambda_{cm}} \\ &\kappa_m = \frac{m\pi}{d} \end{aligned}\right\}$$

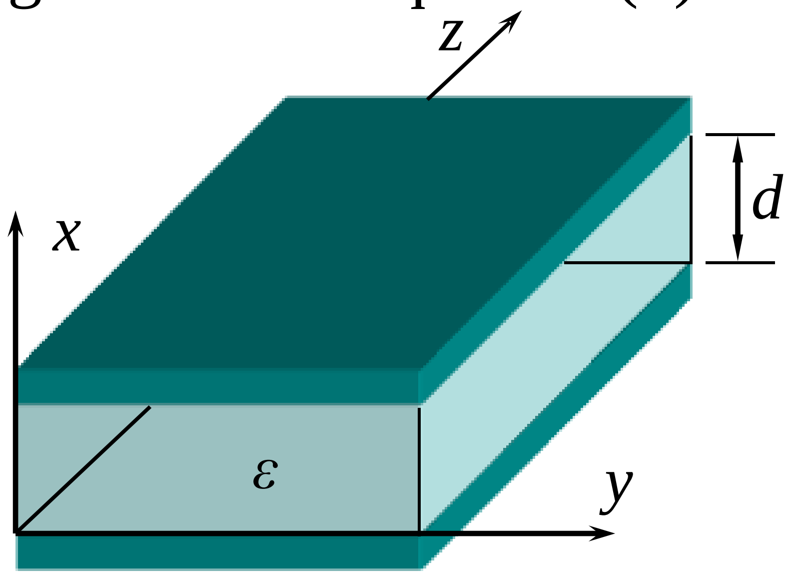
$$\rightarrow \frac{m\pi}{d} = \frac{2n\pi}{\lambda_{cm}} \rightarrow d = \frac{m\lambda_{cm}}{2n}$$

$$\rightarrow E_{ys} = E_0 \sin\left(\frac{m\pi x}{d}\right) = E_0 \sin\left(\frac{2n\pi x}{\lambda_{cm}}\right)$$



Parallel - Plate Guide Analysis Using the Wave Equation (4)

$$\left.\begin{aligned}\nabla\times\mathbf{E}_s &= -j\omega\mu\mathbf{H}_s \\ E_{ys} &= E_0 \sin\left(\frac{m\pi x}{d}\right)e^{-j\beta_m z}\end{aligned}\right\}$$



$$\rightarrow \nabla\times\mathbf{E}_s = \frac{\partial E_{ys}}{\partial x}\mathbf{a}_z - \frac{\partial E_{ys}}{\partial z}\mathbf{a}_x$$

$$= \kappa_m E_0 \cos(\kappa_m x)e^{-j\beta_m z}\mathbf{a}_z + j\beta_m E_0 \sin(\kappa_m x)e^{-j\beta_m z}\mathbf{a}_x$$

$$\nabla\times\mathbf{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\mathbf{a}_z$$

Parallel - Plate Guide Analysis Using the Wave Equation (5)

$$\left.\begin{aligned}\nabla\times\mathbf{E}_s &= \kappa_m E_0\cos(\kappa_m x)e^{-j\beta_m z}\mathbf{a}_z + j\beta_m E_0\sin(\kappa_m x)e^{-j\beta_m z}\mathbf{a}_x\\ \nabla\times\mathbf{E}_s &= -j\omega\mu\mathbf{H}_s\end{aligned}\right\}$$

$$\left.\begin{aligned}&\rightarrow\left\{\boxed{\begin{aligned}H_{xs} &= \frac{\beta_m}{\omega\mu}E_0\sin(\kappa_m x)e^{-j\beta_m z}\\ H_{zs} &= j\frac{\kappa_m}{\omega\mu}E_0\cos(\kappa_m x)e^{-j\beta_m z}\end{aligned}}\right.\\ &|\mathbf{H}_s| = \sqrt{\mathbf{H}_s.\hat{\mathbf{H}}_s} = \sqrt{H_{xs}\hat{H}_{xs} + H_{zs}\hat{H}_{zs}}\end{aligned}\right\}$$

$$\left.\begin{aligned}&\rightarrow|\mathbf{H}_s| = \frac{E_0}{\omega\mu}\left(\kappa_m^2+\beta_m^2\right)^{1/2}\left[\sin^2(\kappa_m x)+\cos^2\kappa_m x\right]^{1/2}\\ &\kappa_m^2+\beta_m^2 = k^2, \qquad \sin^2(\kappa_m x)+\cos^2\kappa_m x = 1\end{aligned}\right\}$$

$$\rightarrow|\mathbf{H}_s| = \frac{kE_0}{\omega\mu} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu} = \frac{E_0}{\eta}$$

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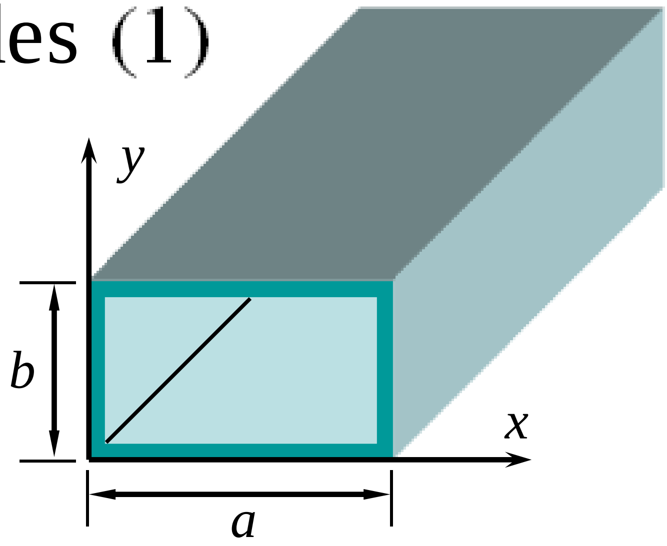
Rectangular Waveguides (1)

$$\frac{\partial^2 E_{ys}}{\partial x^2}+\frac{\partial^2 E_{ys}}{\partial y^2}+\frac{\partial^2 E_{ys}}{\partial z^2}+k^2 E_{ys}=0$$

$$\rightarrow \frac{\partial^2 E_{ys}}{\partial x^2}+\frac{\partial^2 E_{ys}}{\partial y^2}+(k^2-\beta_{mp}^2)E_{ys}=0$$

$$\rightarrow E_{ys}=E_0 f_m(x) f_p(y) e^{-j\beta_{mp}z}$$



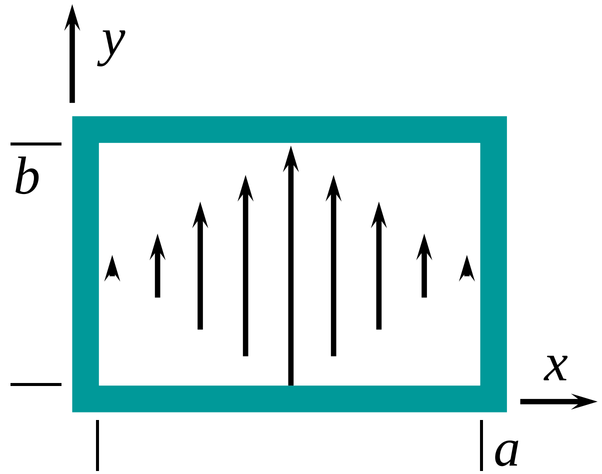
$$E_{ys}=E_0\sin(\kappa_{m0}x)e^{-j\beta_{m0}z}, \qquad \kappa_{m0}=\frac{m\pi}{a}$$

$$H_{xs}=-\frac{\beta_{m0}}{\omega\mu}E_0\sin(\kappa_{m0}x)e^{-j\beta_{m0}z}$$

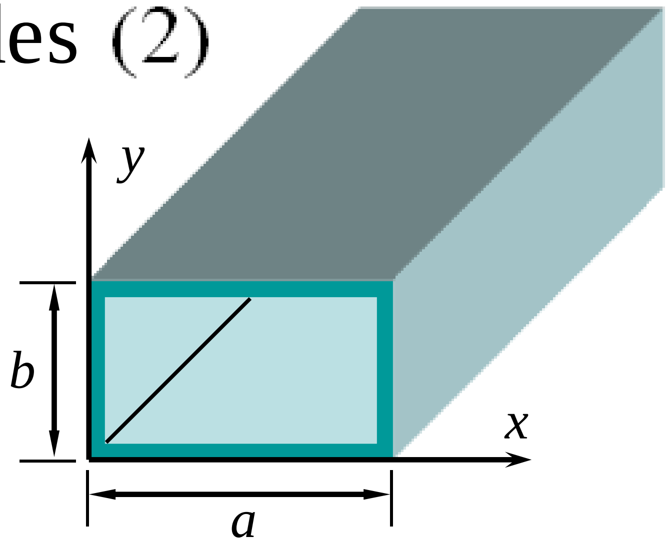
$$H_{zs}=j\frac{\kappa_{m0}}{\omega\mu}E_0\cos(\kappa_{m0}x)e^{-j\beta_{m0}z}$$

$$\kappa_{m0}^2+\beta_{m0}^2=k^2$$

$$\omega_c(m0)=\frac{m\pi c}{na}$$



Rectangular Waveguides (2)

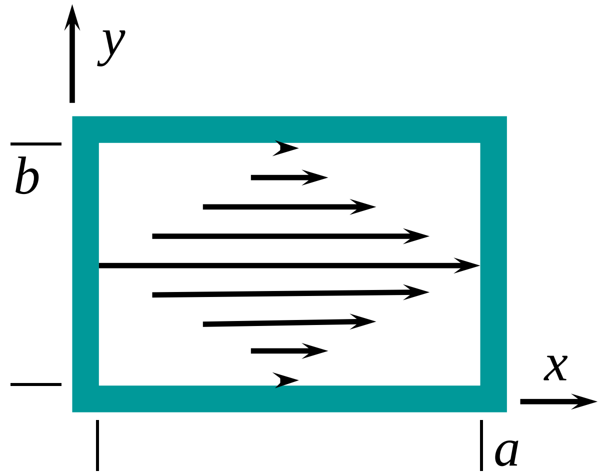


$$E_{xs} = E_0 \sin(\kappa_{0p} y) e^{-j\beta_{0p} z}, \qquad \kappa_{0p} = \frac{p\pi}{b}$$

$$H_{ys} = \frac{\beta_{0p}}{\omega\mu} E_0 \sin(\kappa_{0p} y) e^{-j\beta_{0p} z}$$

$$H_{zs} = -j \frac{\kappa_{0p}}{\omega\mu} E_0 \cos(\kappa_{0p} y) e^{-j\beta_{0p} z}$$

$$\omega_c(0p) = \frac{p\pi c}{nb}$$



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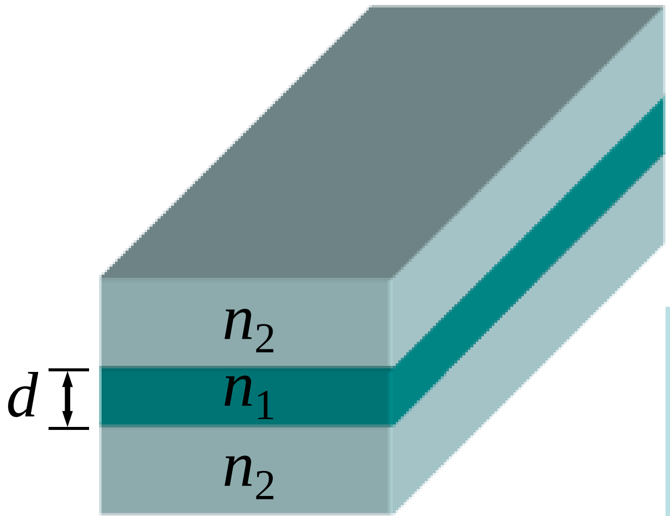
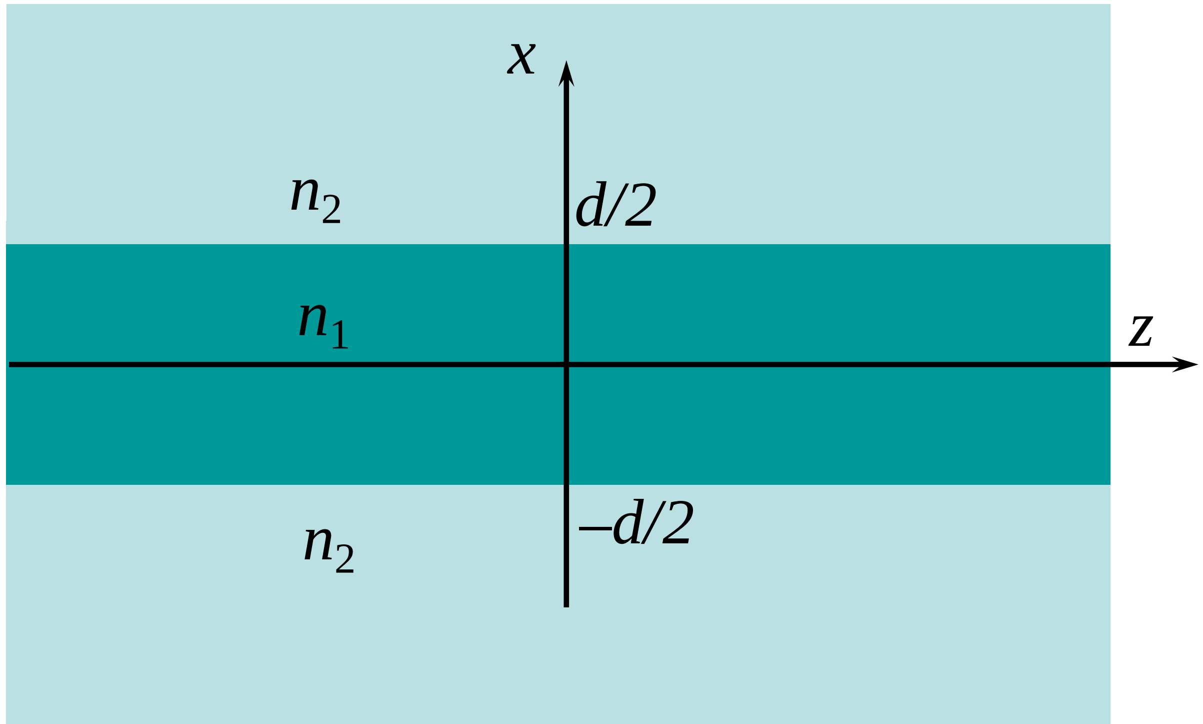
Planar Dielectric Waveguides (1)

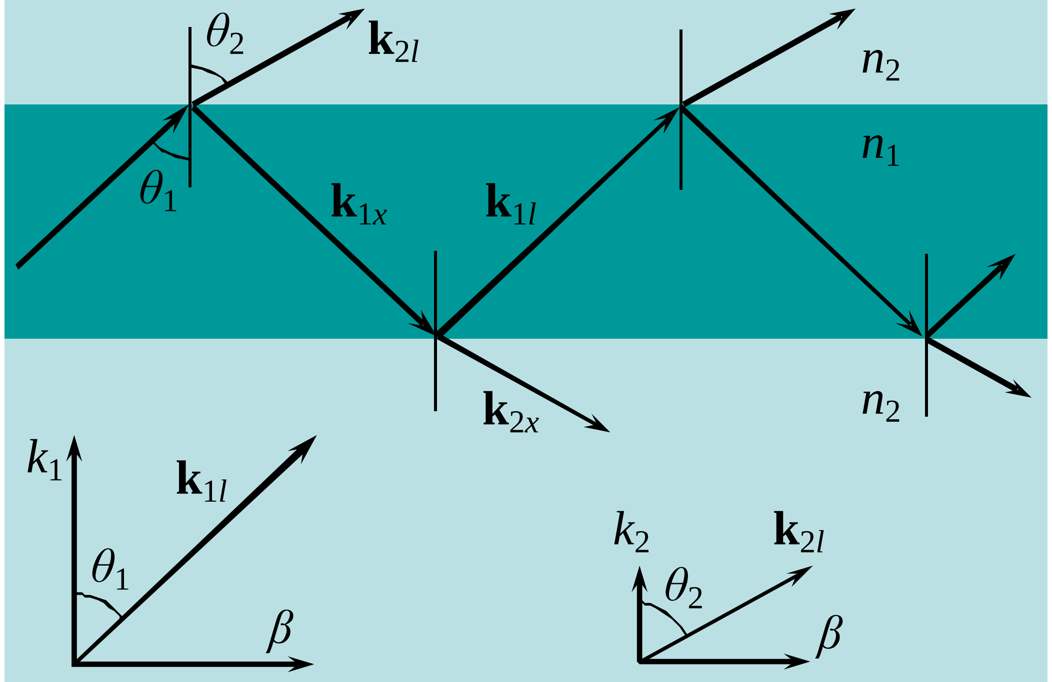




Planar Dielectric Waveguides (2)

$$\theta_1 \geq \theta_c = \arcsin\frac{n_2}{n_1}$$

$$\left.\begin{array}{l} E_{y1s} = E_0 e^{-j\mathbf{k}_{1l}.\mathbf{r}} \pm E_0 e^{-j\mathbf{k}_{1x}.\mathbf{r}} \\ \text{where} \left| \begin{array}{l} -\dfrac{d}{2} < x < \dfrac{d}{2} \\ \mathbf{k}_{1l} = \kappa_1 \mathbf{a}_x + \beta \mathbf{a}_z \\ \mathbf{k}_{1x} = -\kappa_1 \mathbf{a}_x + \beta \mathbf{a}_z \end{array} \right. \\ \mathbf{r} = x\mathbf{a}_x + z\mathbf{a}_z \end{array}\right\}$$



$$\rightarrow \left[\begin{array}{l} E_{y1s} = E_0[e^{j\kappa_1 x} + e^{-j\kappa_1 x}]e^{-j\beta z} = 2E_0\cos(\kappa_1 x)e^{-j\beta z} \\ E_{y1s} = E_0[e^{j\kappa_1 x} - e^{-j\kappa_1 x}]e^{-j\beta z} = 2E_0\sin(\kappa_1 x)e^{-j\beta z} \end{array}\right.$$

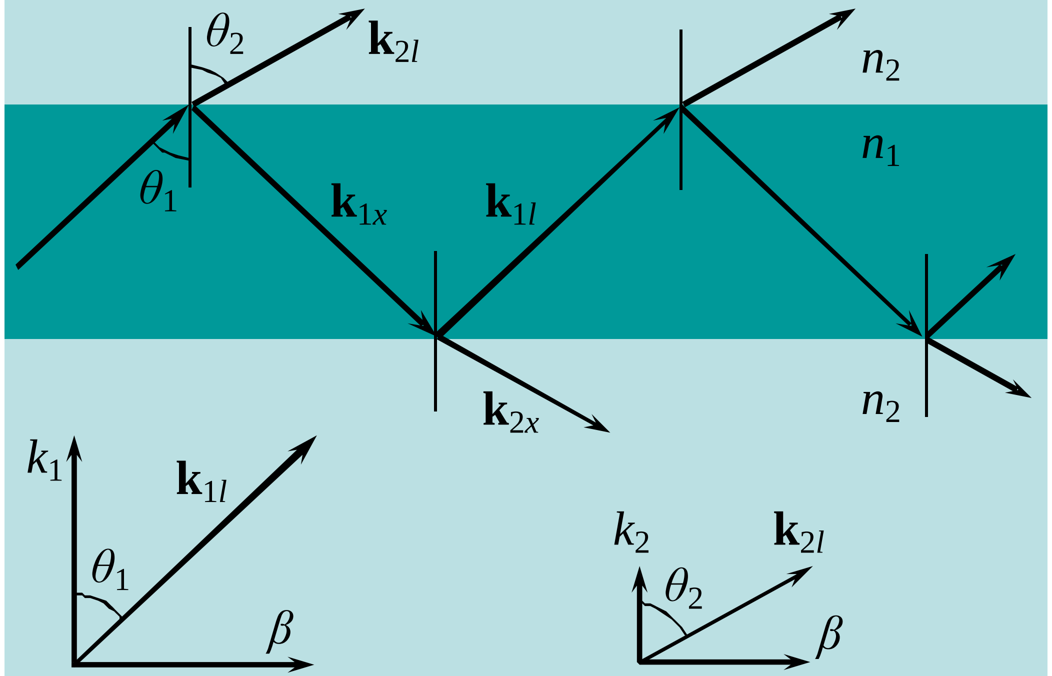
Planar Dielectric Waveguides (3)

$$E_{y2s} = E_{02} e^{-j\mathbf{k}_2 . \mathbf{r}} = E_{02} e^{-j\kappa_2 x} e^{-j\beta z}$$

$$\kappa_2 = -j\gamma_2$$

$$\gamma_2 = j\kappa_2 = jn_2 k_0 \cos\theta_2$$

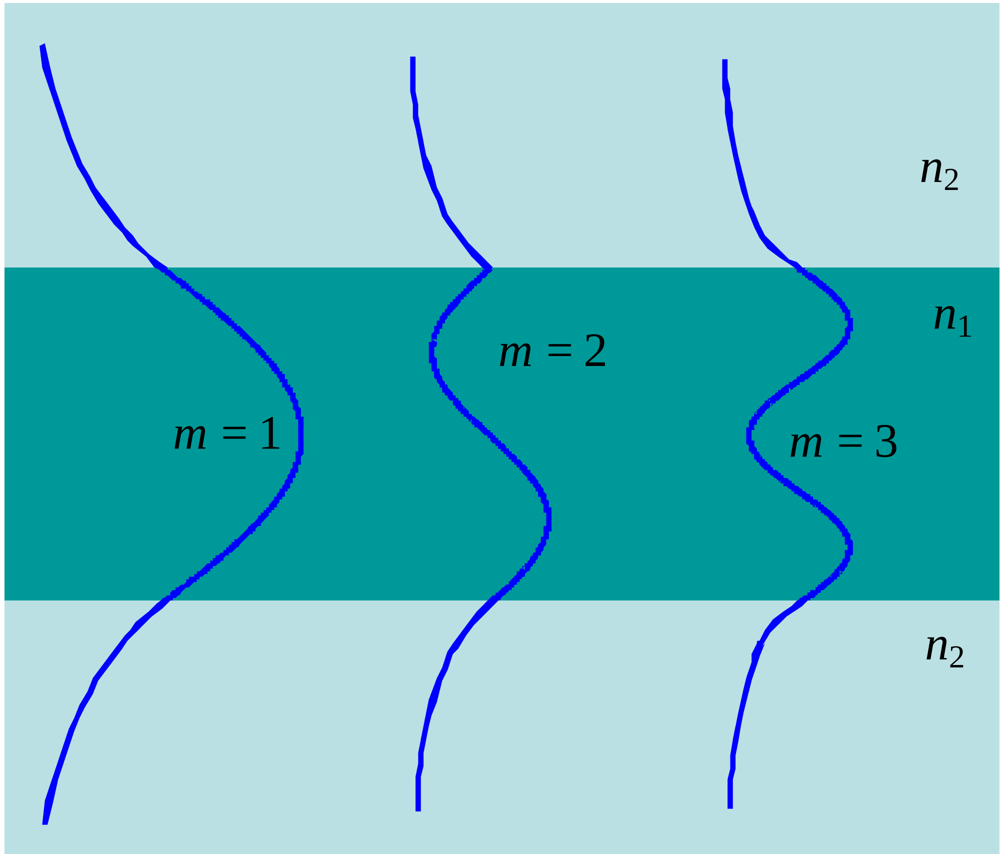
$$= jn_2 k_0 (-j) \left[\left(\frac{n_1}{n_2} \right)^2 \sin^2\theta_1 - 1 \right]^{1/2}$$



$$E_{y2s} = E_{02} e^{-\gamma_2 (x - d/2)} e^{-j\beta z} \qquad \left(x > \frac{d}{2} \right)$$

$$E_{y2s} = E_{02} e^{\gamma_2 (x + d/2)} e^{-j\beta z} \qquad \left(x < -\frac{d}{2} \right)$$

Planar Dielectric Waveguides (4)



Planar Dielectric Waveguides (5)

$$E_{sc}(\text{even TE}) = \begin{cases} E_{0c}\cos(\kappa_1 x)e^{-j\beta z} & \left(-\dfrac{d}{2} < x < \dfrac{d}{2}\right) \\ E_{0c}\cos(\kappa_1 \dfrac{d}{2})e^{-\gamma_2(x-d/2)}e^{-j\beta z} & \left(x > \dfrac{d}{2}\right) \\ E_{0c}\cos(\kappa_1 \dfrac{d}{2})e^{\gamma_2(x+d/2)}e^{-j\beta z} & \left(x < -\dfrac{d}{2}\right) \end{cases}$$

$$E_{sl}(\text{odd TE}) = \begin{cases} E_{0l}\sin(\kappa_1 x)e^{-j\beta z} & \left(-\dfrac{d}{2} < x < \dfrac{d}{2}\right) \\ E_{0l}\sin(\kappa_1 \dfrac{d}{2})e^{-\gamma_2(x-d/2)}e^{-j\beta z} & \left(x > \dfrac{d}{2}\right) \\ -E_{0l}\cos(\kappa_1 \dfrac{d}{2})e^{\gamma_2(x+d/2)}e^{-j\beta z} & \left(x < -\dfrac{d}{2}\right) \end{cases}$$

Planar Dielectric Waveguides (5)

$$k_0 d\sqrt{n_1^2 - n_2^2} \geq (m-1)\pi \qquad (m = 1, 2, 3, \ldots)$$

$$k_0 d\sqrt{n_1^2 - n_2^2} < \pi \rightarrow \lambda > 2d\sqrt{n_1^2 - n_2^2}$$

Planar Dielectric Waveguides (6)

Ex. 1

A symmetric dielectric slab waveguide is to guide light at wavelength λ = 1.30 μm; the slab thickness is d = 5,00 μm; the refractive index of the surrounding material is n_2 = 1.450. Determine the maximum allowable refractive index of the slab material that will allow single TE and TM mode operation.

$$\lambda > 2d\sqrt{n_1^2 - n_2^2}$$

$$\rightarrow n_1 < \sqrt{\left(\frac{\lambda}{2d}\right)^2 + n_2^2} = \sqrt{\left(\frac{1.30}{2\times 5.00}\right)^2 + 1.450^2} = 1.456$$

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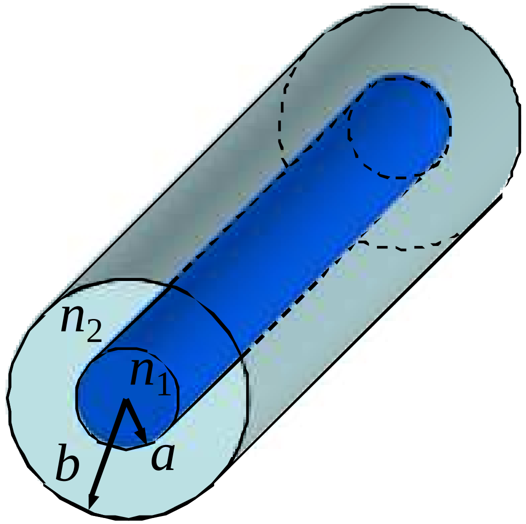
Optical Fiber (1)



$$E_{xs}(\rho,\varphi,z)=\sum_i R_i(\rho)\Phi_i(\varphi)\exp(-j\beta_i z)$$

$$\nabla^2\mathbf{E}_s=-k^2\mathbf{E}_s$$

$$\rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial^2 E_{xs}}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 E_{xs}}{\partial\varphi^2}+(k^2-\beta^2)E_{xs}=0$$

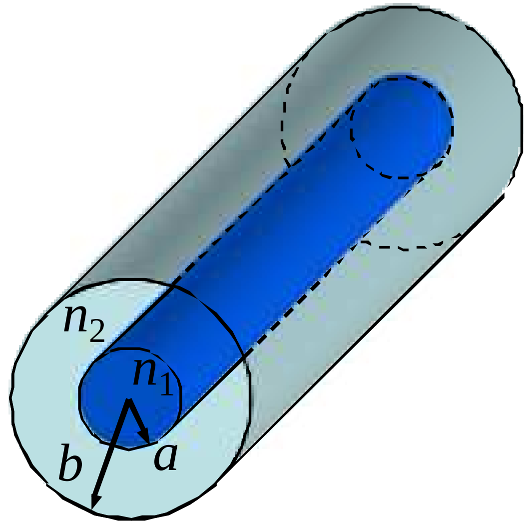
$$\rightarrow \underbrace{\frac{\rho^2}{R}\frac{d^2R}{d\rho^2}+\frac{\rho}{R}\frac{dR}{d\rho}+\rho^2(k^2-\beta^2)}_{\ell^2}=\underbrace{-\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}}_{\ell^2}$$

$$\rightarrow \begin{cases}\dfrac{d^2\Phi}{d\varphi^2}+\ell^2\Phi=0 \longrightarrow \Phi(\varphi)=\left[\begin{array}{l}\cos(\ell\varphi+\alpha)\\ \sin(\ell\varphi+\alpha)\end{array}\right.\\[2ex] \dfrac{d^2R}{d\rho^2}+\dfrac{1}{\rho}\dfrac{dR}{d\rho}+\left[k^2-\beta^2-\dfrac{\ell^2}{\rho^2}\right]R=0\end{cases}$$

$$\downarrow$$

$$\Phi(\varphi)=\cos(\ell\varphi)$$

Optical Fiber (2)



$$\left\{\begin{array}{l} \dfrac{d^2\Phi}{d\varphi^2} + \ell^2\Phi = 0 \rightarrow \Phi(\varphi) = \cos(\ell\varphi) \\ \dfrac{d^2R}{d\rho^2} + \dfrac{1}{\rho}\dfrac{dR}{d\rho} + \left[k^2 - \beta^2 - \dfrac{\ell^2}{\rho^2}\right]R = 0 \end{array}\right.$$

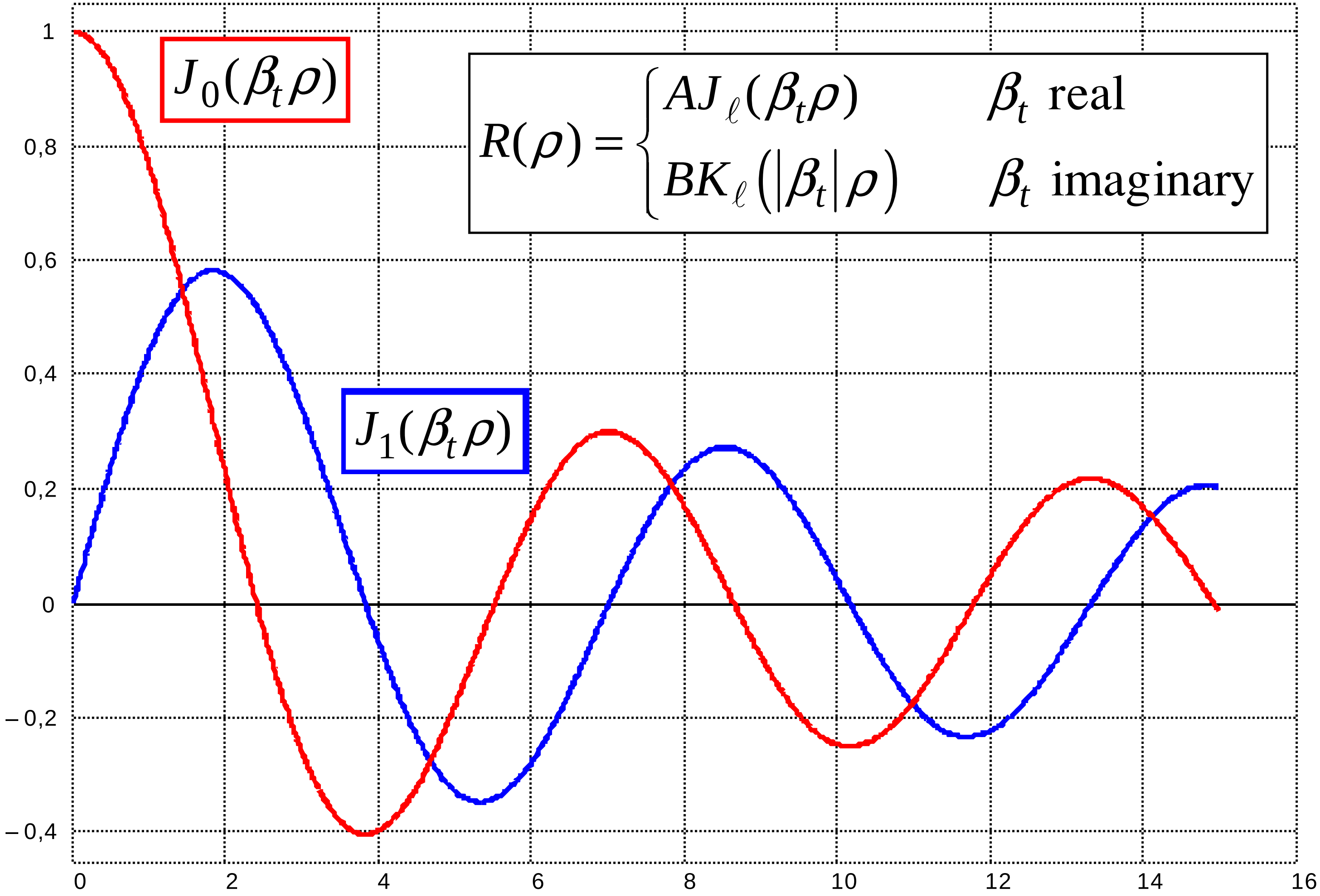
Define $\beta_t = \sqrt{k^2 - \beta^2}$

$$\beta_t = \left[\begin{array}{ll} \beta_{t1} = \sqrt{n_1^2 k_0^2 - \beta^2} & (\rho < a) \\ \beta_{t2} = \sqrt{n_2^2 k_0^2 - \beta^2} & (\rho > a) \end{array}\right.$$

$$\rightarrow R(\rho) = \left\{\begin{array}{ll} AJ_\ell(\beta_t\rho) & \beta_t \text{ real} \\ BK_\ell\left(|\beta_t|\rho\right) & \beta_t \text{ imaginary} \end{array}\right.$$

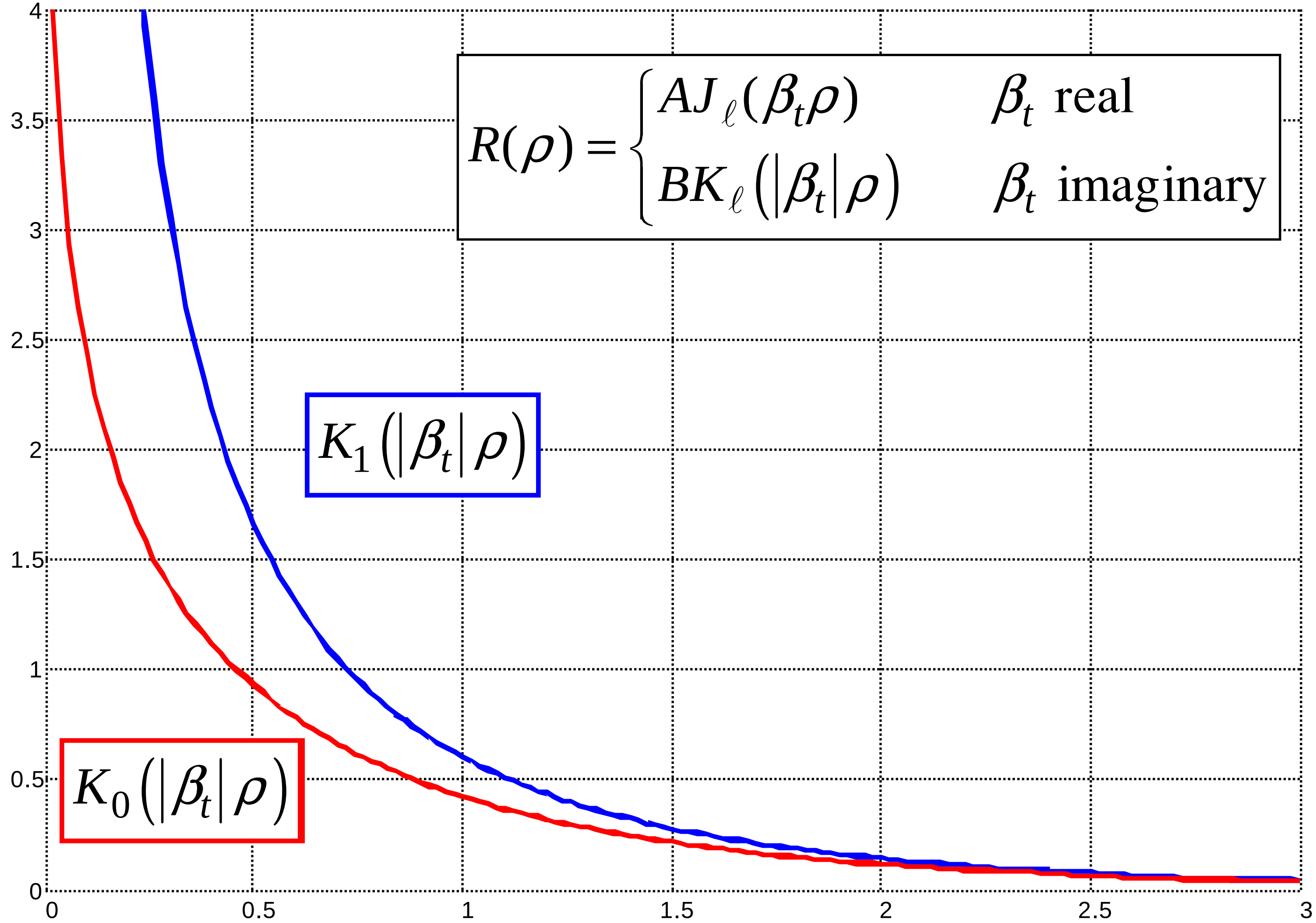
Optical Fiber (3)

$$R(\rho)=\begin{cases} AJ_\ell(\beta_t\rho) & \beta_t \text{ real} \\ BK_\ell(|\beta_t|\rho) & \beta_t \text{ imaginary} \end{cases}$$

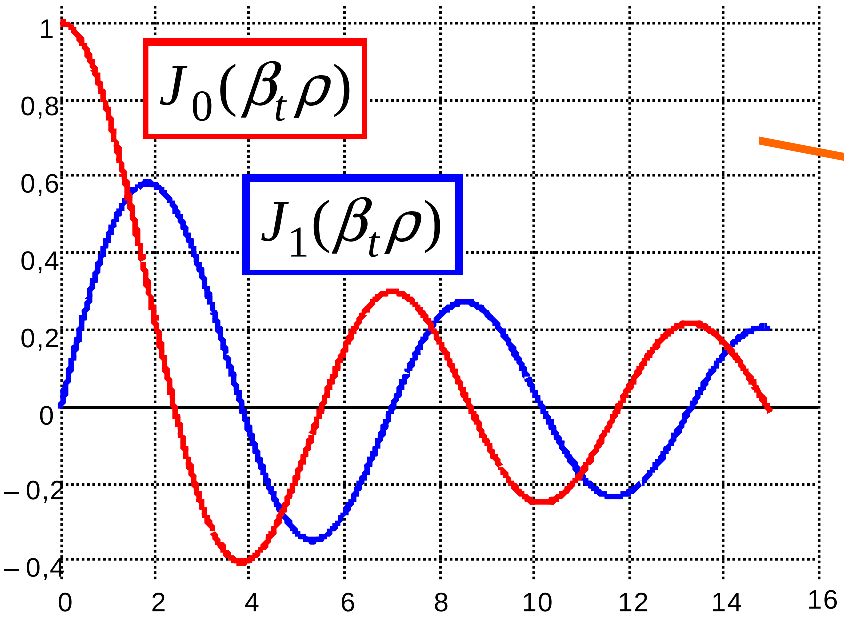


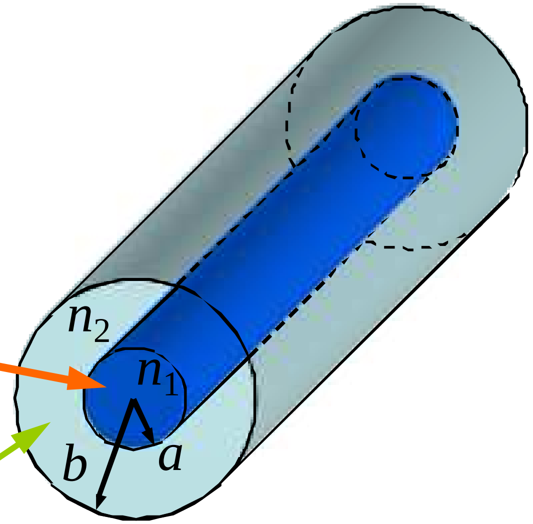
Optical Fiber (4)

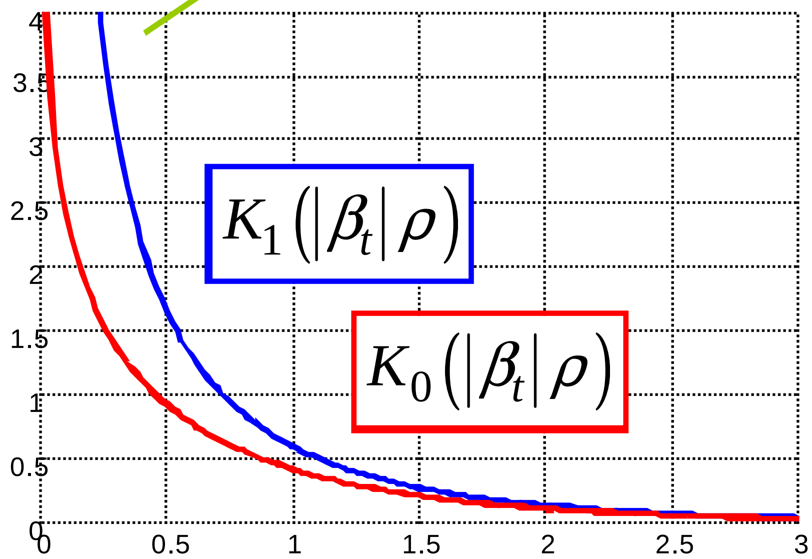
$$R(\rho) = \begin{cases} AJ_\ell(\beta_t \rho) & \beta_t \text{ real} \\ BK_\ell\left(|\beta_t|\rho\right) & \beta_t \text{ imaginary} \end{cases}$$



Optical Fiber (5)



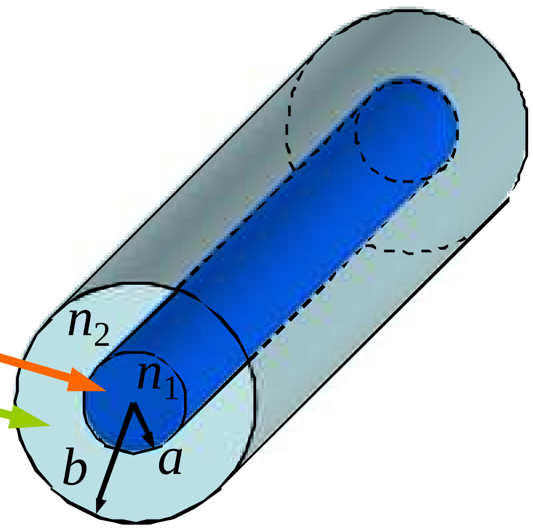




$$R(\rho)=\begin{cases} AJ_\ell(\beta_t\rho) & \beta_t \text{ real} \\ BK_\ell\left(|\beta_t|\rho\right) & \beta_t \text{ imaginary} \end{cases}$$

Optical Fiber (6)

$$R(\rho)=\begin{cases} AJ_\ell(\beta_t\rho) & \beta_t \text{ real} \\ BK_\ell(|\beta_t|\rho) & \beta_t \text{ imaginary} \end{cases}$$



Define $u = a\beta_{t1} = a\sqrt{n_1^2k_0^2 - \beta^2}$

Define $w = a|\beta_{t2}| = a\sqrt{\beta^2 - n_2^2k_0^2}$

$$\rightarrow \boxed{E_{xs} = \begin{cases} E_0 J_\ell(u\rho/a)\cos(\ell\varphi)e^{-j\beta z} & \rho \le a \\ E_0\left[J_\ell(u)/K_\ell(w)\right]K_\ell(w\rho/a)\cos(\ell\varphi)e^{-j\beta z} & \rho \ge a \end{cases}}$$

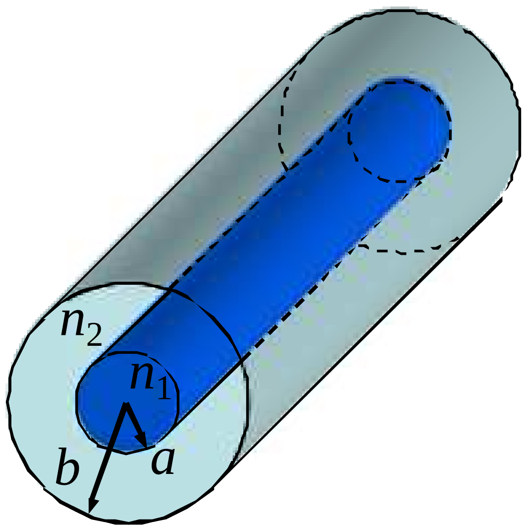
$$\left|S_{z,avr}\right| = \left|\frac{1}{2}\mathrm{Re}[\mathbf{E}_s \times \hat{\mathbf{H}}_s]\right| = \frac{1}{2}\mathrm{Re}[E_{xs}\times \hat{H}_{ys}] = \frac{1}{2\eta}\left|E_{xs}\right|^2$$

Optical Fiber (7)

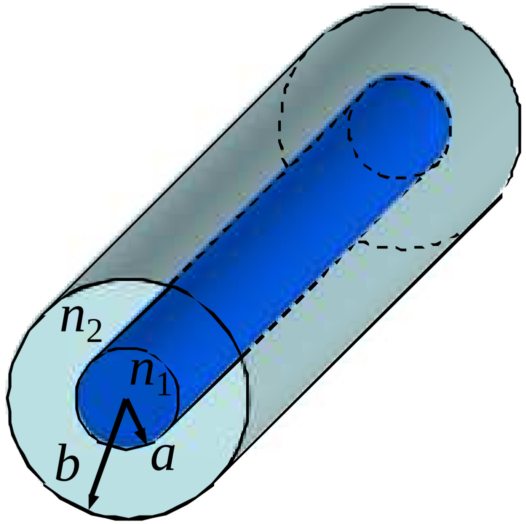
$$\left|S_{z,\mathrm{avr}}\right| = \left|\frac{1}{2}\mathrm{Re}[\mathbf{E}_s \times \hat{\mathbf{H}}_s]\right| = \frac{1}{2}\mathrm{Re}[E_{xs} \times \hat{H}_{ys}] = \frac{1}{2\eta}\left|E_{xs}\right|^2$$

$$I_{\ell m} = I_0 J_\ell^2\left(\frac{u\rho}{a}\right)\cos^2(\ell\varphi) \qquad \rho \le a$$

$$I_{\ell m} = I_0\left(\frac{J_\ell^2(u)}{K_\ell(w)}\right)^2 K_\ell^2\left(\frac{w\rho}{a}\right)\cos^2(\ell\varphi) \qquad \rho \ge a$$



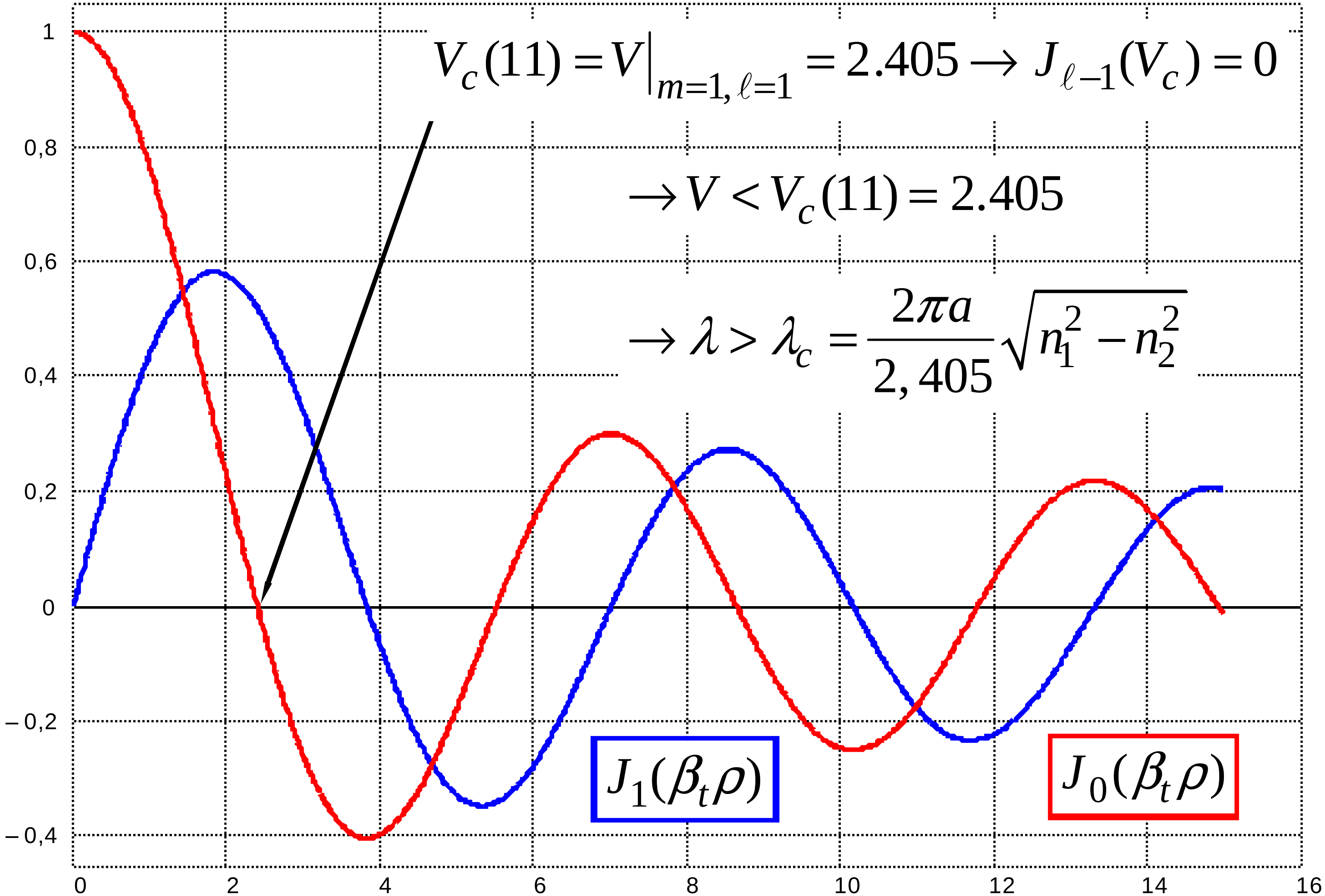
Optical Fiber (8)



$$\left.\begin{array}{l}(\nabla \times \mathbf{E}_{s1})_z\big|_{\rho=a} = (\nabla \times \mathbf{E}_{s2})_z\big|_{\rho=a} \\ E_{xs} = \begin{cases} E_0 J_\ell(u\rho/a)\cos(\ell\varphi)e^{-j\beta z} \\ E_0\left[J_\ell(u)/K_\ell(w)\right]K_\ell(w\rho/a)\cos(\ell\varphi)e^{-j\beta z} \end{cases}\end{array}\right\}$$

$$\left.\begin{array}{r} \rightarrow \dfrac{J_{\ell-1}(u)}{J_\ell(u)} = -\dfrac{w}{u}\dfrac{K_{\ell-1}(w)}{K_\ell(w)} \\ \left.\begin{array}{r}\text{Define } V = \sqrt{u^2 + w^2} \\ u = a\sqrt{n_1^2 k_0^2 - \beta^2} \\ w = a\sqrt{\beta^2 - n_2^2 k_0^2}\end{array}\right\} \rightarrow V = ak_0\sqrt{n_1^2 - n_2^2} \end{array}\right\} \rightarrow \boxed{J_{\ell-1}(V_c) = 0}$$

Optical Fiber (9)



$$V_c(11) = V\big|_{m=1,\ell=1} = 2.405 \rightarrow J_{\ell-1}(V_c) = 0$$

$$\rightarrow V < V_c(11) = 2.405$$

$$\rightarrow \lambda > \lambda_c = \frac{2\pi a}{2,405}\sqrt{n_1^2 - n_2^2}$$

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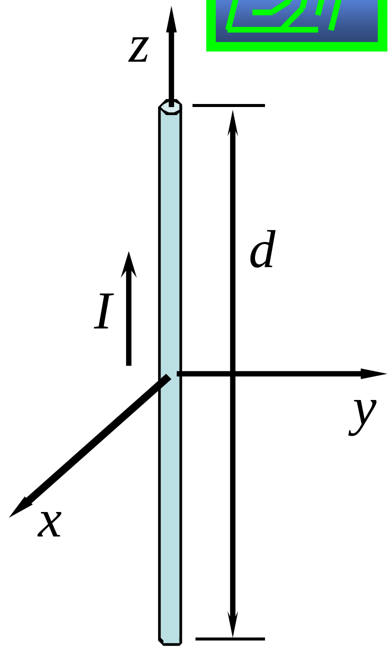
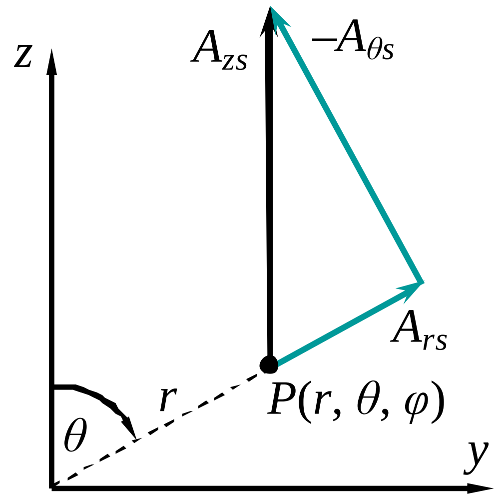
Basic Antenna Principles (1)

$$\left.\begin{aligned} I &= I_0 \cos \omega t \\ \mathbf{A} &= \int_V \frac{\mu[\mathbf{J}]}{4\pi R} dv = \int \frac{\mu[I]d\mathbf{L}}{4\pi R} = \frac{\mu[I]d}{4\pi R}\mathbf{a}_z \\ [I] &= I_0 \cos\left[\omega\left(t - \frac{R}{v}\right)\right] \\ \rightarrow [I_s] &= I_0 e^{-j\omega R/v} \end{aligned}\right\} \rightarrow A_{zs} = \frac{\mu I_0 d}{4\pi R} e^{-j\omega R/v}$$



$$\begin{cases} A_{rs} = A_{zs} \cos\theta \\ A_{\theta s} = -A_{zs} \sin\theta \\ A_{\varphi s} = 0 \end{cases} \rightarrow \begin{cases} A_{rs} = \dfrac{\mu I_0 d}{4\pi R} \cos\theta e^{-j\omega r/v} \\ A_{\theta s} = -\dfrac{\mu I_0 d}{4\pi R} \sin\theta e^{-j\omega r/v} \end{cases}$$



Basic Antenna Principles (2)

$$\left.\begin{aligned}
A_{rs} &= \frac{\mu I_0 d}{4\pi R}\cos\theta e^{-j\omega r/v} \\
A_{\theta s} &= -\frac{\mu I_0 d}{4\pi R}\sin\theta e^{-j\omega r/v} \\
A_{\varphi s} &= 0 \\
\mathbf{B}_s &= \mu \mathbf{H}_s = \nabla\times\mathbf{A}_s \\
\nabla\times\mathbf{A} &= \frac{1}{r\sin\theta}\left(\frac{\partial(A_\varphi\sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\varphi}\right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial r}\right)\mathbf{a}_\theta + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\mathbf{a}_\varphi
\end{aligned}\right\}$$

$$\rightarrow \begin{cases} H_{\varphi s} = \dfrac{1}{\mu r}\dfrac{\partial}{\partial r}(rA_{\theta s}) - \dfrac{1}{\mu r}\dfrac{\partial A_{rs}}{\partial\theta} \\ H_{rs} = H_{\theta s} = 0 \end{cases} \rightarrow H_{\varphi s} = \frac{I_0 d}{4\pi}\sin\theta e^{-j\omega r/v}\left(j\frac{\omega}{vr} + \frac{1}{r^2}\right)$$

Basic Antenna Principles (3)

$$\left.\begin{array}{l} H_{\varphi s} = \dfrac{I_0 d}{4\pi}\sin\theta e^{-j\omega r/v}\left(j\dfrac{\omega}{vr}+\dfrac{1}{r^2}\right) \\ H_{rs} = H_{\theta s} = 0 \\ \nabla\times\mathbf{H} = \dfrac{\partial\mathbf{D}}{\partial t} \to \nabla\times\mathbf{H}_s = j\omega\varepsilon\mathbf{E}_s \\ \nabla\times\mathbf{H} = \dfrac{1}{r\sin\theta}\left(\dfrac{\partial(H_\varphi\sin\theta)}{\partial\theta}-\dfrac{\partial H_\theta}{\partial\varphi}\right)\mathbf{a}_r + \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial H_r}{\partial\varphi}-\dfrac{\partial(rH_\varphi)}{\partial r}\right)\mathbf{a}_\theta + \dfrac{1}{r}\left(\dfrac{\partial(rH_\theta)}{\partial r}-\dfrac{\partial H_r}{\partial\theta}\right)\mathbf{a}_\varphi \end{array}\right\}$$

$$\to\begin{cases} E_{rs} = \dfrac{1}{j\omega\varepsilon}\dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\theta}(H_{\varphi s}\sin\theta) \\ E_{\theta s} = \dfrac{1}{j\omega\varepsilon}\left(-\dfrac{1}{r}\right)\dfrac{\partial}{\partial\theta}(rH_{\varphi s}) \end{cases} \to\begin{cases} E_{rs} = \dfrac{I_0 d}{2\pi}\cos\theta e^{-j\omega r/v}\left(\dfrac{1}{\varepsilon v r^2}+\dfrac{1}{j\omega\varepsilon r^3}\right) \\ E_{\theta s} = \dfrac{I_0 d}{4\pi}\sin\theta e^{-j\omega r/v}\left(\dfrac{j\omega}{\varepsilon v^2 r}+\dfrac{1}{\varepsilon v r^2}+\dfrac{1}{j\omega\varepsilon r^3}\right) \end{cases}$$

Basic Antenna Principles (4)

$$\left.\begin{array}{l}
H_{\varphi s} = \dfrac{I_0 d}{4\pi} \sin\theta e^{-j\omega r/v} \left(j\dfrac{\omega}{vr} + \dfrac{1}{r^2} \right) \\[2ex]
E_{rs} = \dfrac{I_0 d}{2\pi} \cos\theta e^{-j\omega r/v} \left(\dfrac{1}{\varepsilon v r^2} + \dfrac{1}{j\omega\varepsilon r^3} \right) \\[2ex]
E_{\theta s} = \dfrac{I_0 d}{4\pi} \sin\theta e^{-j\omega r/v} \left(\dfrac{j\omega}{\varepsilon v^2 r} + \dfrac{1}{\varepsilon v r^2} + \dfrac{1}{j\omega\varepsilon r^3} \right) \\[2ex]
\omega = 2\pi f, \;\; f\lambda = v, \;\; v = 1/\sqrt{\mu\varepsilon}, \;\; \eta = \sqrt{\mu/\varepsilon}
\end{array}\right\}$$

$$\rightarrow \left\{\begin{array}{l}
H_{\varphi s} = \dfrac{I_0 d}{4\pi} \sin\theta e^{-j2\pi r/\lambda} \left(j\dfrac{2\pi}{\lambda r} + \dfrac{1}{r^2} \right) \\[2ex]
E_{rs} = \dfrac{I_0 d\eta}{2\pi} \cos\theta e^{-j2\pi r/\lambda} \left(\dfrac{1}{r^2} + \dfrac{1}{j2\pi r^3} \right) \\[2ex]
E_{\theta s} = \dfrac{I_0 d\eta}{4\pi} \sin\theta e^{-j2\pi r/\lambda} \left(j\dfrac{2\pi}{\lambda r} + \dfrac{1}{r^2} + \dfrac{\lambda}{j2\pi r^3} \right)
\end{array}\right.$$

Basic Antenna Principles (5)

$$H_{\varphi s} = \frac{I_0 d}{4\pi} \sin\theta e^{-j2\pi r/\lambda}\left(j\frac{2\pi}{\lambda r} + \frac{1}{r^2} \right)$$

Ex: $I_0 d = 4\pi, \quad \theta = 90^{\mathrm{o}}, \quad t = 0, \quad f = 300\,\mathrm{MHz}, \quad v = 3.10^8\,\mathrm{m/s}, \quad \lambda = 1\mathrm{m}$

$$\to H_{\varphi s} = \left(j\frac{2\pi}{r} + \frac{1}{r^2} \right) e^{-j2\pi r}$$

$$\left.\begin{array}{l} \to H_{\varphi} = \sqrt{\left(\frac{2\pi}{r}\right)^2 + \frac{1}{r^4}} \cos\{[\mathrm{arctg}(2\pi r) - 2\pi r]\} \\ \cos(a-b) = \cos a \cos b + \sin a \sin b \\ \cos[\mathrm{arctg}(x)] = 1/\sqrt{1+x^2} \end{array}\right\}$$

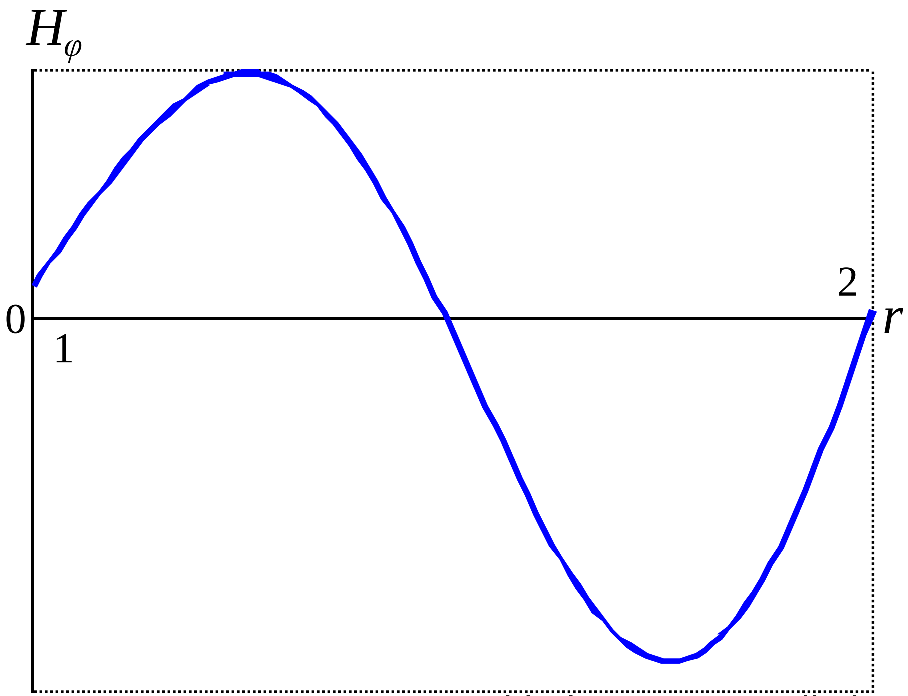
$$\to H_{\varphi} = \frac{1}{r^2}(\cos 2\pi r + 2\pi r \sin 2\pi r)$$

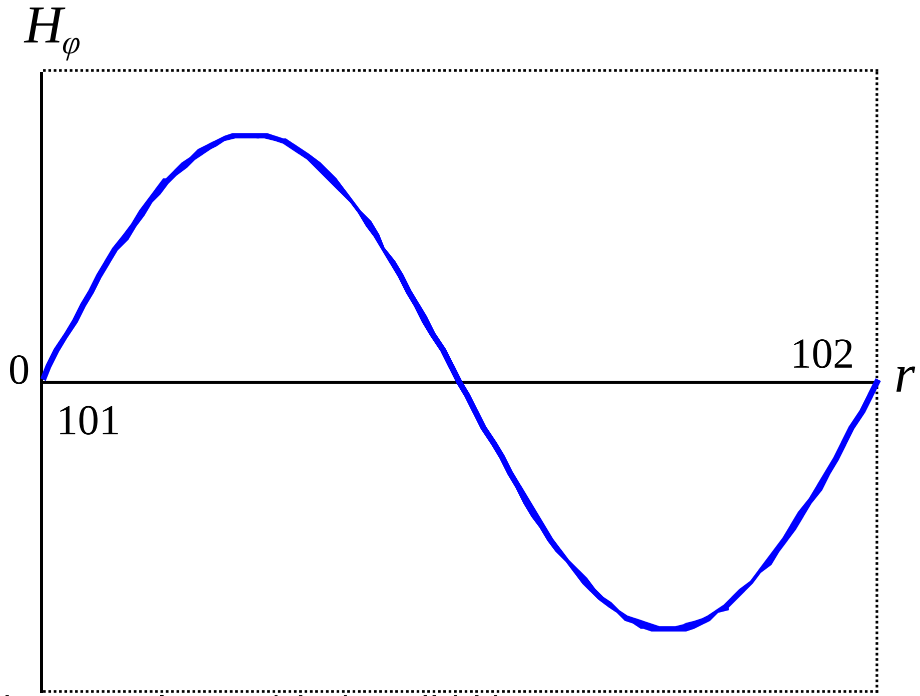
Basic Antenna Principles (6)

$$H_{\varphi s} = \frac{I_0 d}{4\pi} \sin\theta e^{-j2\pi r/\lambda} \left(j\frac{2\pi}{\lambda r} + \frac{1}{r^2} \right)$$

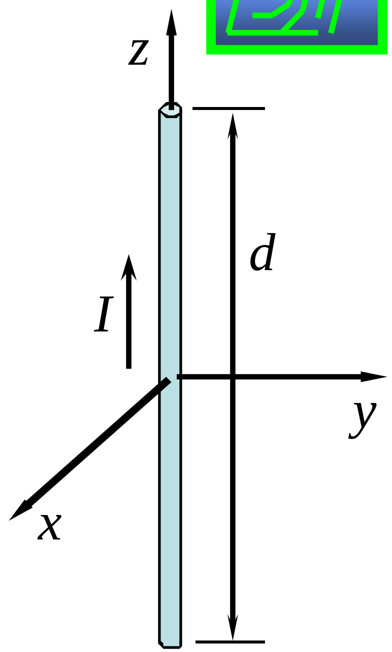
Ex. $I_0 d = 4\pi, \quad \theta = 90^\circ, \quad t = 0, \quad f = 300\,\text{MHz}, \quad v = 3.10^8\,\text{m/s}, \quad \lambda = 1\,\text{m}$

$$\rightarrow H_\varphi = \frac{1}{r^2}(\cos 2\pi r + 2\pi r \sin 2\pi r)$$



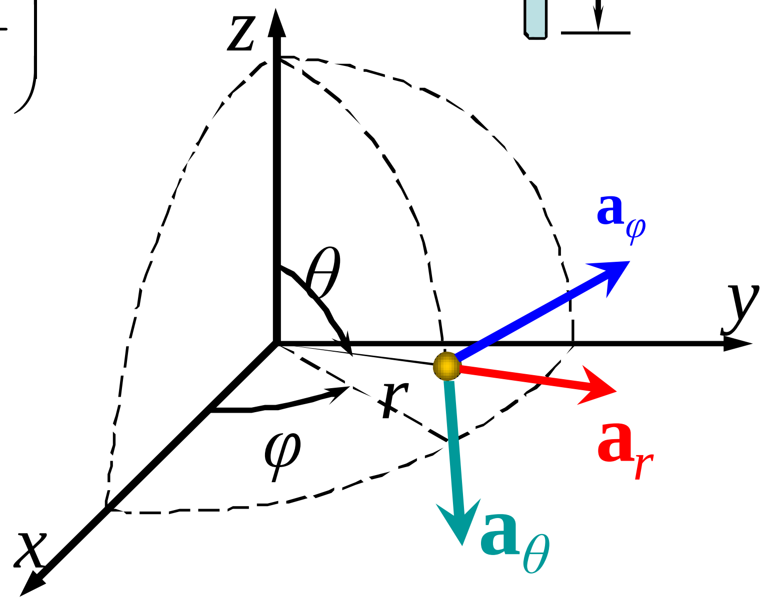


Basic Antenna Principles (7)



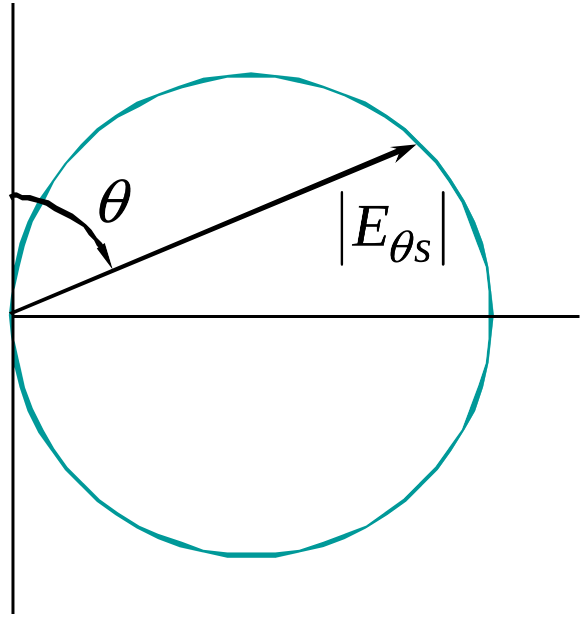
$$\begin{cases} H_{\varphi s} = \dfrac{I_0 d}{4\pi} \sin\theta e^{-j2\pi r/\lambda}\left(j\dfrac{2\pi}{\lambda r} + \dfrac{1}{r^2}\right) \\ E_{rs} = \dfrac{I_0 d\eta}{2\pi} \cos\theta e^{-j2\pi r/\lambda}\left(\dfrac{1}{r^2} + \dfrac{1}{j2\pi r^3}\right) \\ E_{\theta s} = \dfrac{I_0 d\eta}{4\pi} \sin\theta e^{-j2\pi r/\lambda}\left(j\dfrac{2\pi}{\lambda r} + \dfrac{1}{r^2} + \dfrac{\lambda}{j2\pi r^3}\right) \end{cases}$$

$$\rightarrow \begin{cases} H_{\varphi s} = j\dfrac{I_0 d}{2\lambda r} \sin\theta e^{-j2\pi r/\lambda} \\ E_{rs} = 0 \\ E_{\theta s} = j\dfrac{I_0 d\eta}{2\lambda r} \sin\theta e^{-j2\pi r/\lambda} \end{cases} \qquad \rightarrow E_{\theta s} = \eta H_{\varphi s}$$



Basic Antenna Principles (8)

$$\begin{cases} H_{\varphi s} = j\dfrac{I_0 d}{2\lambda r}\sin\theta e^{-j2\pi r/\lambda} \\ E_{\theta s} = j\dfrac{I_0 d\eta}{2\lambda r}\sin\theta e^{-j2\pi r/\lambda} \end{cases}$$



Basic Antenna Principles (9)

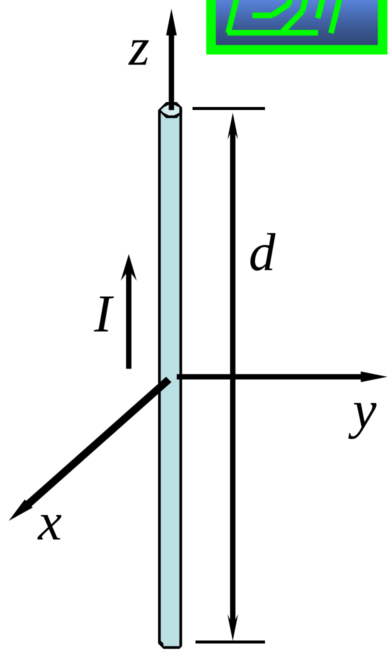
$$\left.\begin{aligned} H_{\varphi s} &= j\frac{I_0 d}{2\lambda r}\sin\theta e^{-j2\pi r/\lambda} \\ E_{\theta s} &= j\frac{I_0 d\eta}{2\lambda r}\sin\theta e^{-j2\pi r/\lambda} \\ E_{\theta s} &= \eta H_{\varphi s} \end{aligned}\right\} \rightarrow \begin{cases} E_\theta = \eta H_\varphi \\ H_\varphi = -\dfrac{I_0 d}{2\lambda r}\sin\theta\sin\left(\omega t - \dfrac{2\pi r}{\lambda}\right) \end{cases}$$

$$S_r = E_\theta H_\varphi = \left(\frac{I_0 d}{2\lambda r}\right)^2 \eta \sin^2\theta \sin^2\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$$S = \int_{\varphi=0}^{\varphi=2\pi}\int_{\theta=0}^{\theta=\pi} S_r r_0^2 \sin\theta d\theta d\varphi = \left(\frac{I_0 d}{2\lambda r}\right)^2 \eta \frac{2\pi}{3}\sin^2\left(\omega t - \frac{2\pi r_0}{\lambda}\right)$$

$$\rightarrow S_{\text{avr}} = \left(\frac{I_0 d}{2\lambda r}\right)^2 \eta\frac{\pi}{3} = 40\pi^2\left(\frac{I_0 d}{2\lambda r}\right)^2$$

Basic Antenna Principles (10)

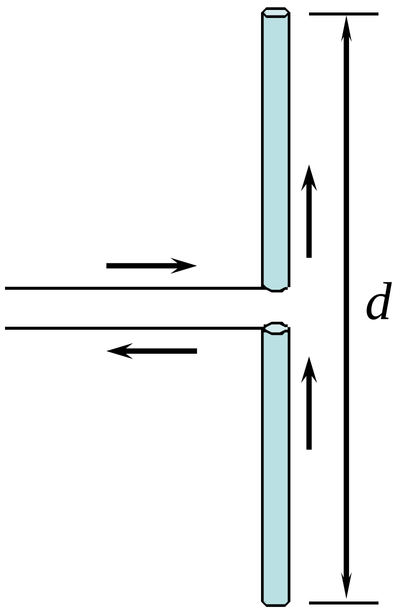


$$S_{\text{avr}} = 40\pi^2 \left(\frac{I_0 d}{2\lambda r} \right)^2$$

$$P_{\text{avr}} = \frac{1}{2} I_0^2 R_{\text{radiation}}$$

$$R_{\text{radiation}} = \frac{2P_{\text{avr}}}{I_0^2} = 80\pi^2 \left(\frac{d}{\lambda} \right)^2$$

Basic Antenna Principles (11)



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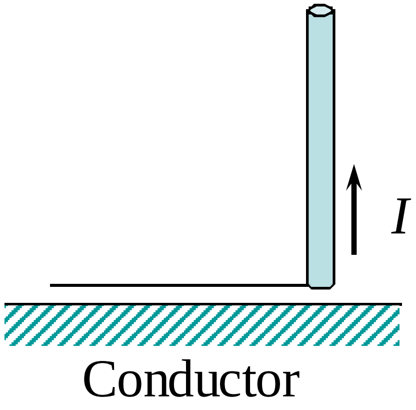

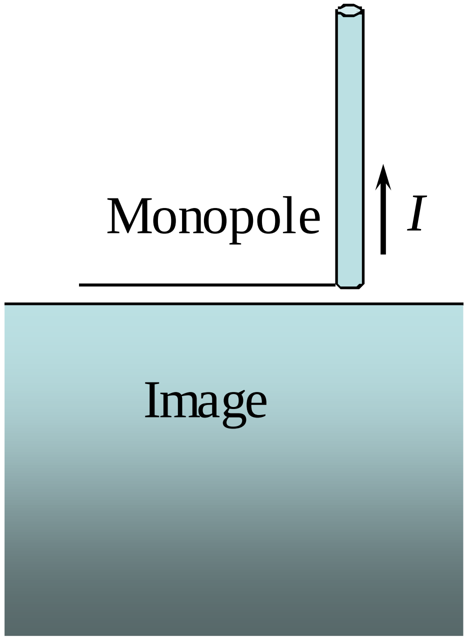

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