EE3410E
POWER ELECTRONICS

Chap 1. (Cont.)
Fundamental definitions

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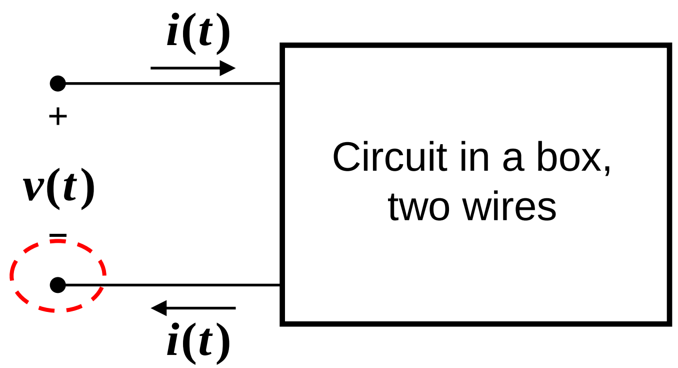
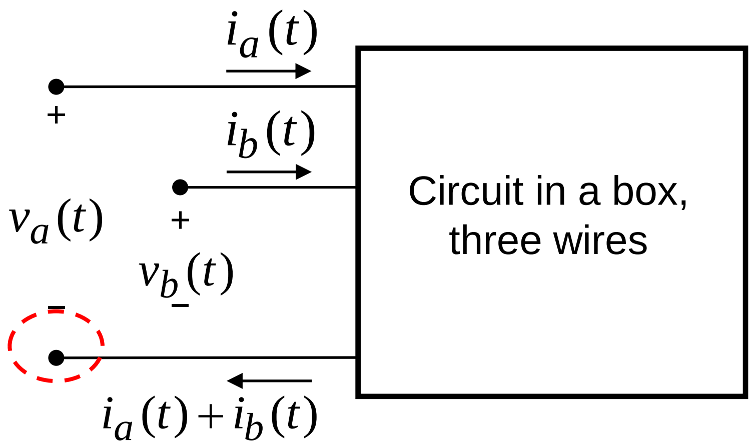
Instantaneous power p(t) flowing into the box

$$p(t) = v(t) \bullet i(t)$$



$$p(t) = v_a(t) \bullet i_a(t) + v_b(t) \bullet i_b(t)$$



Any wire can be the voltage reference

Works for any circuit, as long as all N wires are accounted for. There must be (N – 1) voltage measurements, and (N – 1) current measurements.

Average value of periodic instantaneous power p(t)

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

Two-wire sinusoidal case

$$v(t) = V\sin(\omega_o t + \delta), \qquad i(t) = I\sin(\omega_o t + \theta)$$

$$p(t) = v(t) \bullet i(t) = V\sin(\omega_o t + \delta) \bullet I\sin(\omega_o t + \theta)$$

zero average

$$p(t) = VI\left[\frac{\cos(\delta - \theta) - \cancel{\cos(2\omega_o t + \delta + \theta)}}{2}\right]$$

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o + T} p(t)dt = \frac{VI}{2}\cos(\delta - \theta) = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\cos(\delta - \theta)$$

$$\boxed{P_{avg} = V_{rms}I_{rms}\boxed{\cos(\delta - \theta)}}$$

Displacement power factor

Average power

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

The average value of the squared voltage

Compare to the average power expression

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

compare

Apply v(t) to a resistor

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T}\left[\frac{v^2(t)}{R}\right]dt = \frac{1}{RT}\int_{t_o}^{t_o+T} v^2(t)dt$$

$$P_{avg} = \frac{V_{rms}^2}{R}$$

rms is based on a power concept, describing the equivalent voltage that will produce a given average power to a resistor

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

For the sinusoidal case $v(t) = V\sin(\omega_o t + \delta)$,

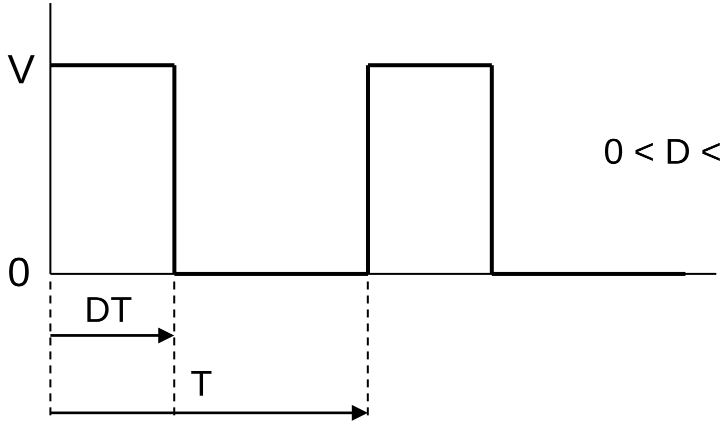
$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} V^2 \sin^2(\omega_o t + \delta)dt$$

$$V_{rms}^2 = \frac{V^2}{2T}\int_{t_o}^{t_o+T} [1 - \cos 2(\omega_o t + \delta)]dt = \frac{V^2}{2T}\left[t - \frac{\sin 2(\omega_o t + \delta)}{2\omega_o}\right]_{t_o}^{t_o+T}$$

$$V_{rms}^2 = \frac{V^2}{2}, \quad \boxed{V_{rms} = \frac{V}{\sqrt{2}}}$$

RMS of some common periodic waveforms

Duty cycle controller



0 < D < 1

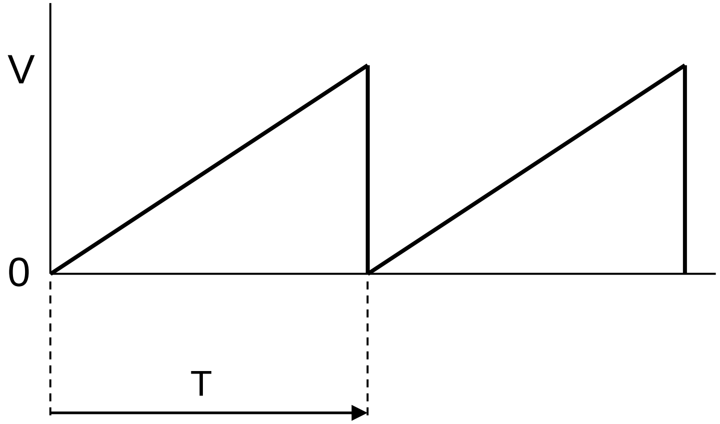
By inspection, this is the average value of the squared waveform

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{T}\int_0^{DT} V^2 dt = \frac{V^2}{T} \bullet DT = DV^2$$

$$V_{rms} = V\sqrt{D}$$

RMS of common periodic waveforms, cont.

Sawtooth

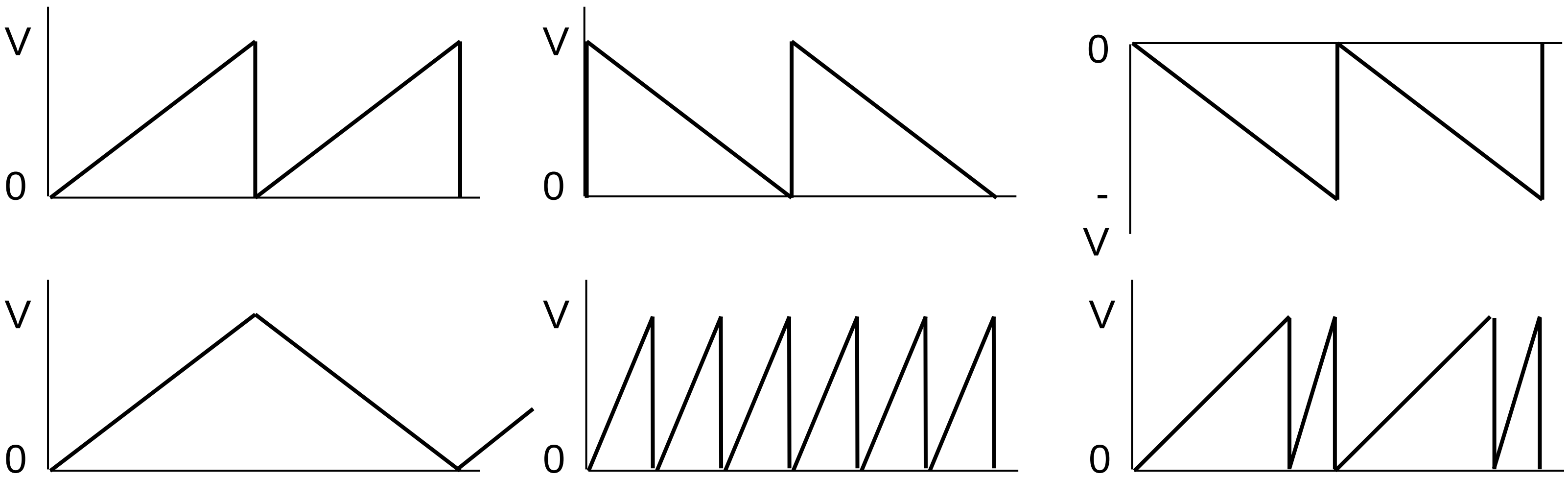


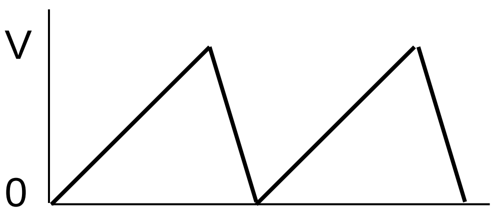
$$V_{rms}^2 = \frac{1}{T}\int_0^T \left[\frac{V}{T}t\right]^2 dt = \frac{V^2}{T^3}\int_0^T t^2 dt = \frac{V^2}{3T^3}t^3\Big|_0^T$$

$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

Using the power concept, it is easy to reason that the following waveforms would all produce the same average power to a resistor, and thus their rms values are identical and equal to the previous example

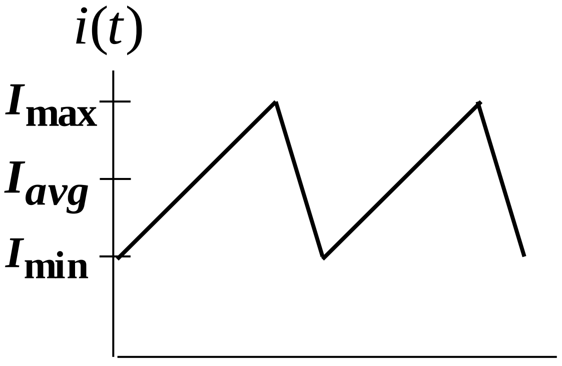


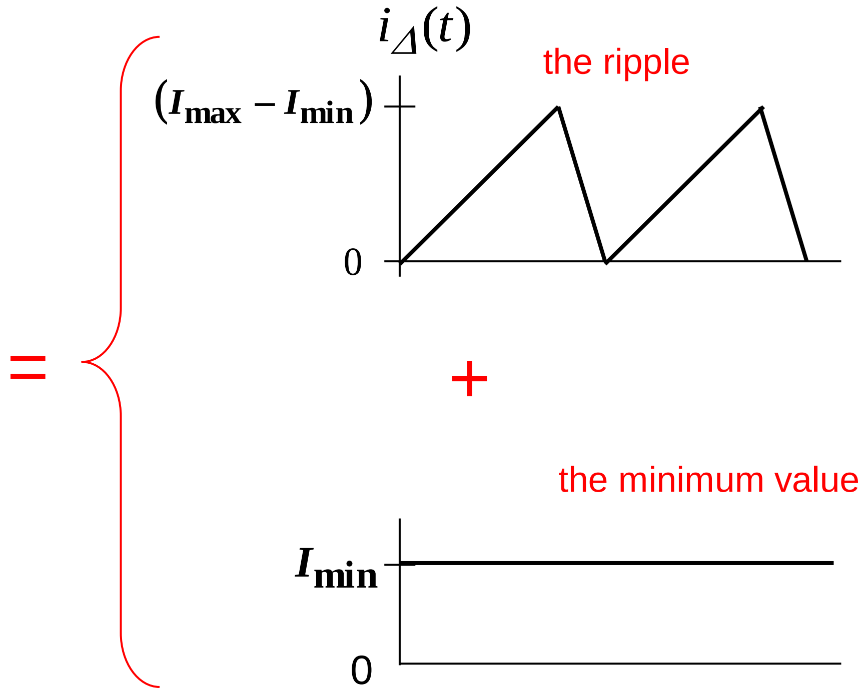


$$V_{rms} = \frac{V}{\sqrt{3}}$$

RMS of common periodic waveforms, cont.

Now, consider a useful example, based upon a waveform that is often seen in DC-DC converter currents. Decompose the waveform into its ripple, plus its minimum value.





$$I_{avg} = \frac{(I_{max} + I_{min})}{2}$$

RMS of common periodic waveforms, cont.

$$I_{rms}^2 = Avg\left\{\left(i_\Delta(t) + I_{\min}\right)^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_\Delta^2(t) + 2i_\Delta(t) \bullet I_{\min} + I_{\min}^2\right\}$$

$$I_{rms}^2 = Avg\left\{i_\Delta^2(t)\right\} + 2I_{\min} \bullet Avg\left\{i_\Delta(t)\right\} + I_{\min}^2$$

$$I_{rms}^2 = \frac{\left(I_{\max} - I_{\min}\right)^2}{3} + 2I_{\min} \bullet \frac{\left(I_{\max} - I_{\min}\right)}{2} + I_{\min}^2$$

Define $I_{PP} = I_{\max} - I_{\min}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{\min} I_{PP} + I_{\min}^2$$

RMS of common periodic waveforms, cont.

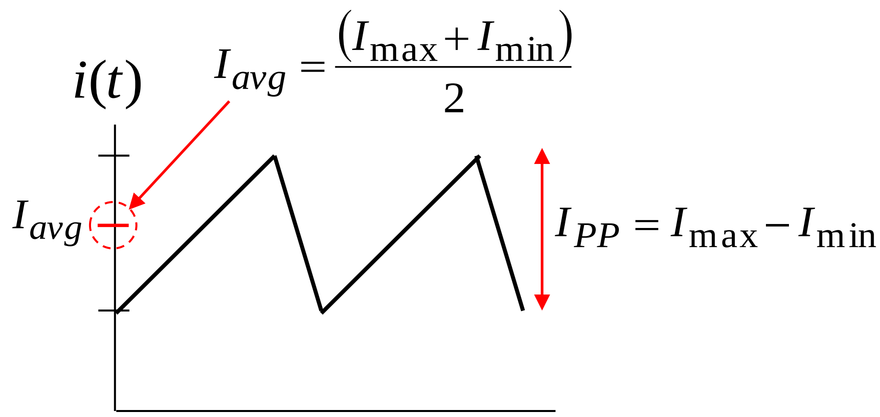
Recognize that $$I_{\min} = I_{avg} - \frac{I_{PP}}{2}$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + \left(I_{avg} - \frac{I_{PP}}{2}\right) I_{PP} + \left(I_{avg} - \frac{I_{PP}}{2}\right)^2$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{avg} I_{PP} - \frac{I_{PP}^2}{2} + I_{avg}^2 - I_{avg} I_{PP} + \frac{I_{PP}^2}{4}$$

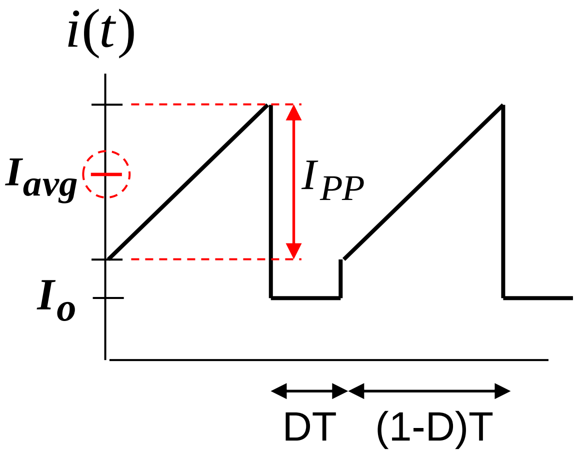
$$I_{rms}^2 = \frac{I_{PP}^2}{3} - \frac{I_{PP}^2}{4} + I_{avg}^2$$

$$\boxed{I_{rms}^2 = I_{avg}^2 + \frac{I_{PP}^2}{12}}$$



RMS of segmented waveforms

Consider a modification of the previous example. A constant value exists during D of the cycle, and a sawtooth exists during (1-D) of the cycle.



In this example, I_{avg} is defined as the average value of the sawtooth portion

$$I_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} i^2(t)dt = \frac{1}{T}\left[\int_{t_o}^{t_o+DT} i^2(t)dt + \int_{t_o+DT}^{t_o+T} i^2(t)dt\right]$$

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet \frac{1}{DT}\int_{t_o}^{t_o+DT} i^2(t)dt + (1-D)T \bullet \frac{1}{(1-D)T}\int_{t_o+DT}^{t_o+T} i^2(t)dt\right]$$

RMS of segmented waveforms, cont.

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet \frac{1}{DT} \int_{t_o}^{t_o+DT} i^2(t)dt + (1-D)T \bullet \frac{1}{(1-D)T} \int_{t_o+DT}^{t_o+T} i^2(t)dt \right]$$

$$I_{rms}^2 = \frac{1}{T}\left[DT \bullet Avg\left\{i^2(t)\right\}_{over\ DT} + (1-D)T \bullet Avg\left\{i^2(t)\right\}_{over\ (1\text{-}D)T} \right]$$

$$I_{rms}^2 = D \bullet Avg\left\{i^2(t)\right\}_{over\ DT} + (1-D) \bullet Avg\left\{i^2(t)\right\}_{over\ (1\text{-}D)T}$$

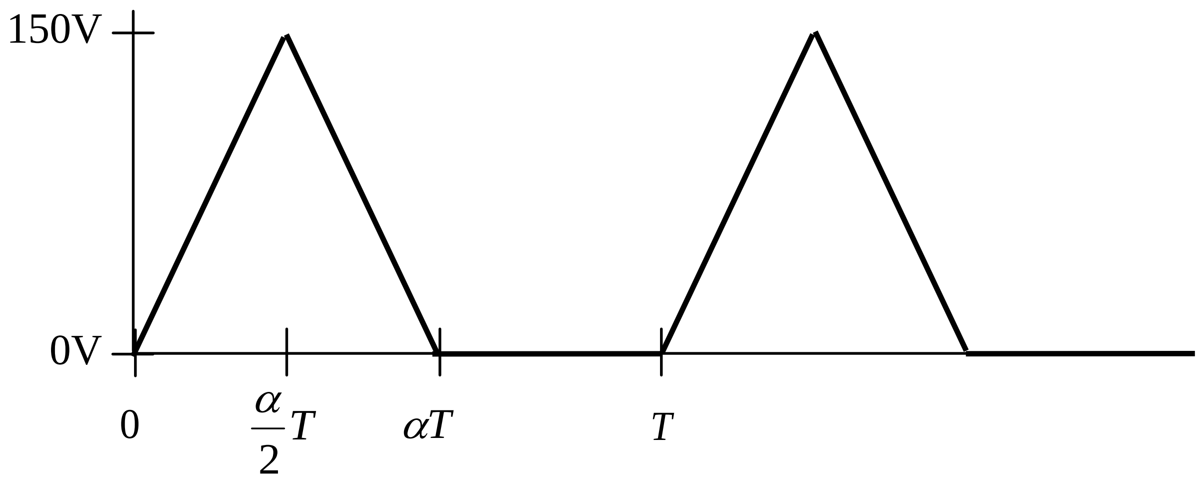
$$I_{rms}^2 = D \bullet I_o^2 + (1-D) \bullet \left[I_{avg}^2 + \frac{I_{PP}^2}{12} \right] \Longleftrightarrow$$

a weighted average

So, the squared rms value of a segmented waveform can be computed by finding the squared rms values of each segment, weighting each by its fraction of T, and adding

Practice Problem

The periodic waveform shown is applied to a 100Ω resistor. What value of α yields 50W average power to the resistor?



Fourier series for any physically realizable periodic waveform with period T

$$i(t) = I_{avg} + \sum_{k=1}^{\infty} I_k \sin(k\omega_o t + \theta_k) = I_{avg} + \sum_{k=1}^{\infty} I_k \cos(k\omega_o t + \theta_k - 90^o)$$

$$T = \frac{2\pi}{\omega_o} = \frac{2\pi}{2\pi f_o} = \frac{1}{f_o}$$

$$I_k = \sqrt{a_k^2 + b_k^2}$$

$$I_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} i(t)dt$$

$$\sin(\theta_k) = \frac{a_k}{\sqrt{a_k^2 + b_k^2}}$$

$$a_k = \frac{2}{T}\int_0^T i(t)\cos(k\omega_o t)dt$$

$$\cos(\theta_k) = \frac{b_k}{\sqrt{a_k^2 + b_k^2}}$$

$$b_k = \frac{2}{T}\int_0^T i(t)\sin(k\omega_o t)dt$$

$$\tan(\theta_k) = \frac{\sin(\theta_k)}{\cos(\theta_k)} = \frac{a_k}{b_k}$$

When using arctan, be careful to get the correct quadrant

Two interesting properties

Half-wave symmetry,

$$i(t \pm \frac{T}{2}) = -i(t)$$

then no even harmonics

(remove the average value from i(t) before making the above test)

Time shift,

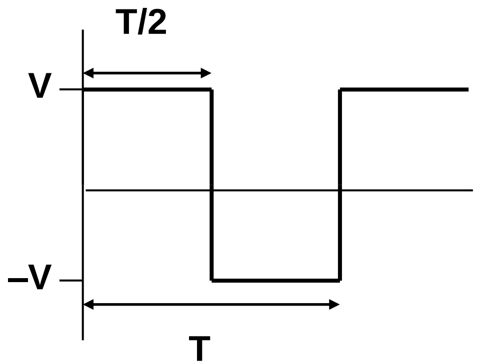
$$i(t-\Delta T) = \sum_{k=1}^{\infty} I_k \sin(k\omega_o(t-\Delta T)+\theta_k)$$

$$= \sum_{k=1}^{\infty} I_k \sin(k\omega_o t + \theta_k - k\theta_o)$$

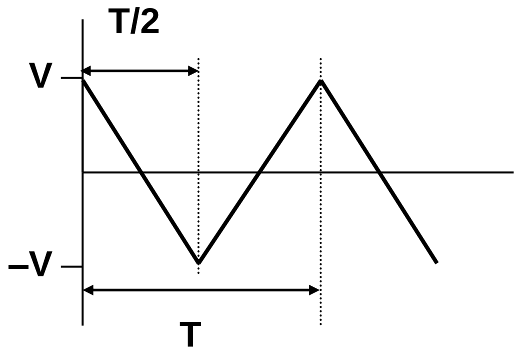
where the fundamental angle shift is

$$\theta_o = \omega_o T.$$

Thus, harmonic k is shifted by k times the fundamental angle shift



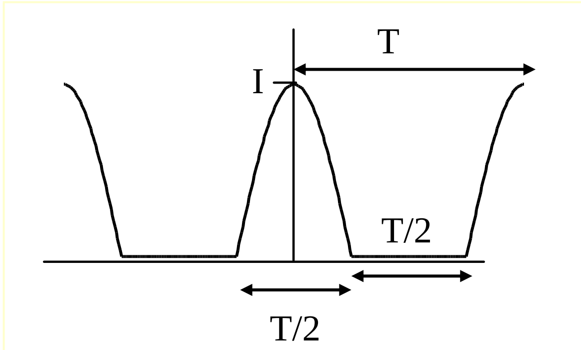
$$v(t)=\frac{4V}{\pi}\sum_{k=1,\,k\,odd}^{\infty}\frac{1}{k}\sin(k\omega_0 t)=\frac{4V}{\pi}\left[\sin(1\omega_0 t)+\frac{1}{3}\sin(3\omega_0 t)+\frac{1}{5}\sin(5\omega_0 t)+\cdots\right]$$



$$v(t) = \frac{8V}{\pi^2} \sum_{k=1,\, k\ odd}^{\infty} \frac{1}{k^2} \cos(k\omega_0 t)$$

$$= \frac{8V}{\pi^2}\left[\cos(1\omega_0 t) + \frac{1}{9}\cos(3\omega_0 t) + \frac{1}{25}\cos(5\omega_0 t) + \cdots\right]$$

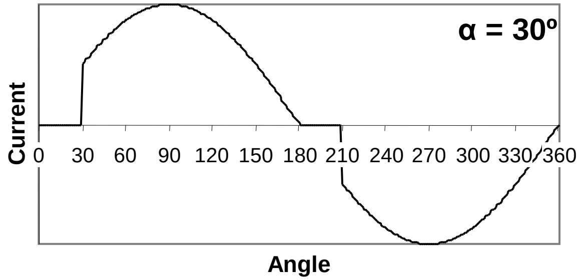
Half-wave rectified cosine wave

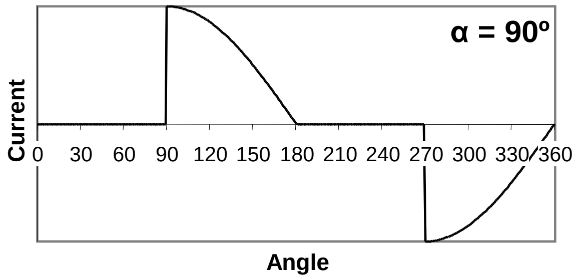


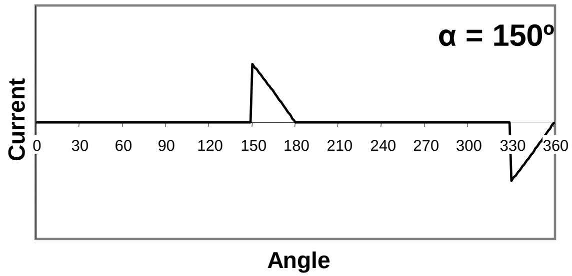
$$i(t)=\frac{I}{\pi}+\frac{I}{2}\cos(\omega_o t)+\frac{2I}{\pi}\sum_{k=2,4,6,\cdots}^{\infty}(-1)^{k/2+1}\frac{1}{k^2-1}\cos(k\omega_o t)$$

$$=\frac{I}{\pi}+\frac{I}{2}\cos(\omega_o t)+\frac{2I}{\pi}\left[\frac{1}{3}\cos(2\omega_o t)-\frac{1}{15}\cos(4\omega_o t)+\frac{1}{35}\cos(6\omega_o t)-\cdots\right]$$

Triac light dimmer waveshapes (bulb voltage and current waveforms are identical)







$$a_1 = \frac{-V_p}{\pi}\sin^2\alpha, \quad b_1 = V_p\left[1 - \frac{\alpha}{\pi} + \frac{1}{2\pi}\sin 2\alpha\right]$$

$$a_k = \frac{V_p}{\pi}\left[\frac{1}{1-k}\big(\cos(1-k)\alpha - \cos(1-k)\pi\big) + \frac{1}{1+k}\big(\cos(1+k)\alpha - \cos(1+k)\pi\big)\right], k = 3,5,7,\ldots$$

$$b_k = \frac{V_p}{\pi}\left[\frac{1}{1-k}\big(\sin(1-k)\pi - \sin(1-k)\alpha\big) + \frac{1}{1+k}\big(\sin(1+k)\alpha - \sin(1+k)\pi\big)\right], k = 3,5,7,\ldots$$

Vp is the peak value of the underlying AC waveform

RMS in terms of Fourier Coefficients

$$(V_{rms})^2 = V_{avg}^2 + \sum_{k=1}^{\infty} \frac{V_k^2}{2}$$

which means that $V_{rms} \geq \left| V_{avg} \right|$

and that

$$V_{rms} \geq \frac{\left| V_k \right|}{\sqrt{2}}$$ for any k

Bounds on RMS

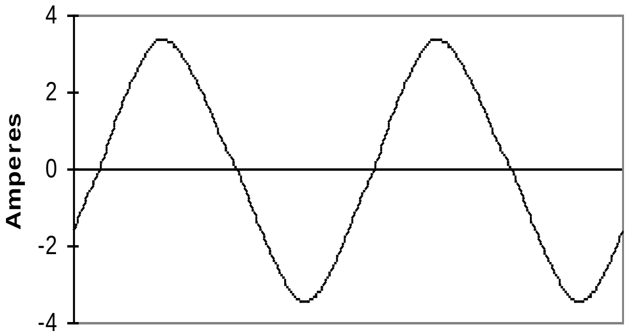
From the power concept, it is obvious that the rms voltage or current can never be greater than the maximum absolute value of the corresponding v(t) or i(t)

From the Fourier concept, it is obvious that the rms voltage or current can never be less than the absolute value of the average of the corresponding v(t) or i(t)

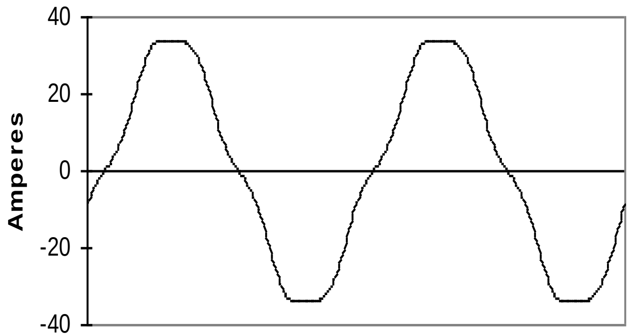
Total harmonic distortion – THD (for voltage or current)

$$\left(THD_V\right)^2 = \frac{\sum_{k=2}^{\infty} V_k^2}{V_1^2}$$

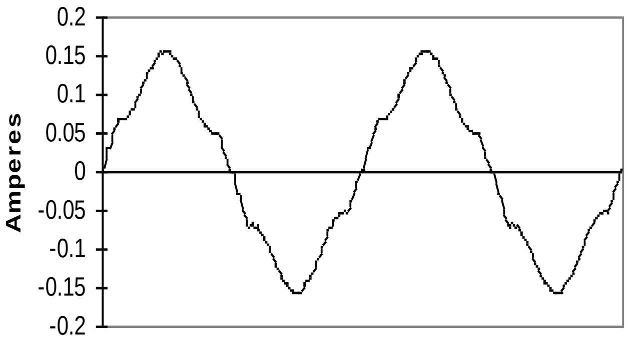
Some measured current waveforms



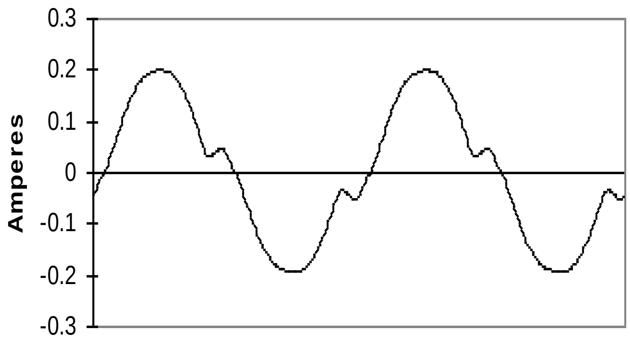
Refrigerator
THDi = 6.3%



240V residential air conditioner
THDi = 10.5%

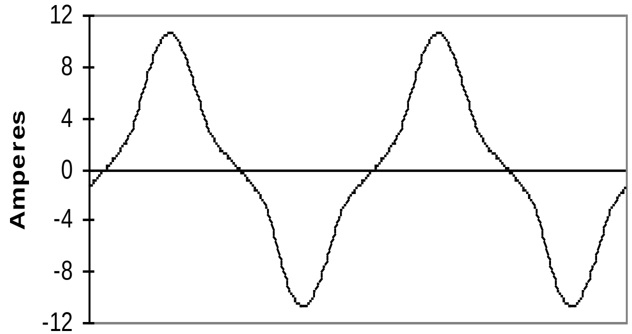


277V fluorescent light (electronic ballast)
THDi = 11.6%

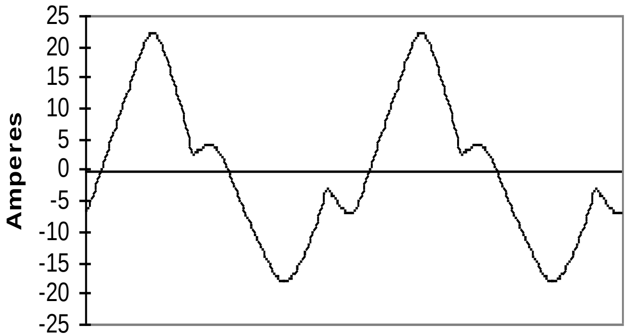


277V fluorescent light (magnetic ballast)
THDi = 18.5%

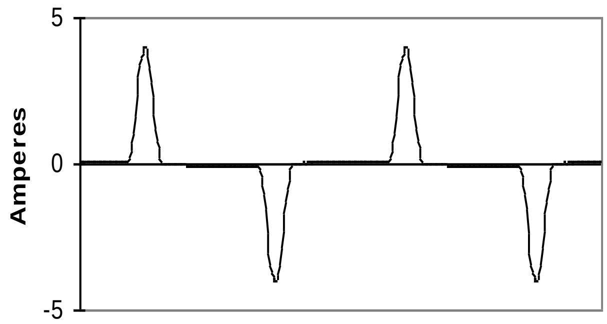
Some measured current waveforms, cont.



Vacuum cleaner
THDi = 25.9%



Microwave oven
THDi = 31.9%

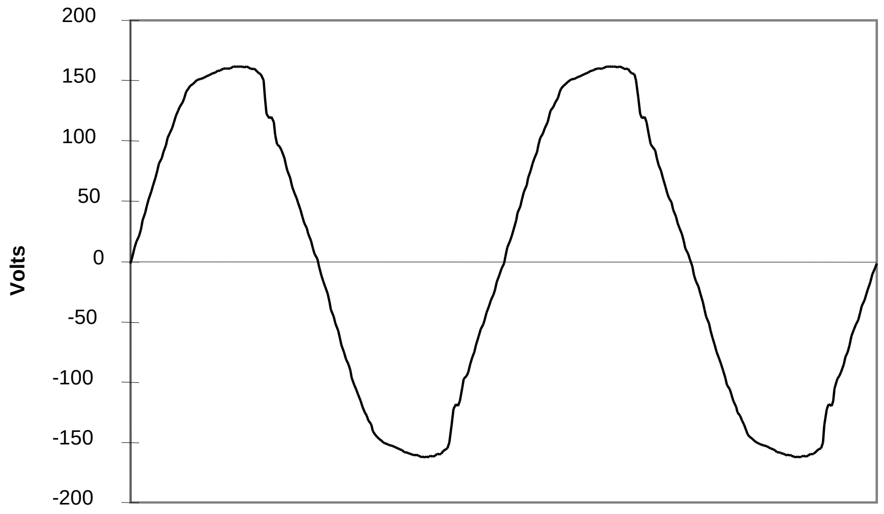


PC
THDi = 134%

Resulting voltage waveform at the service panel for a room filled with PCs

THDV = 5.1%

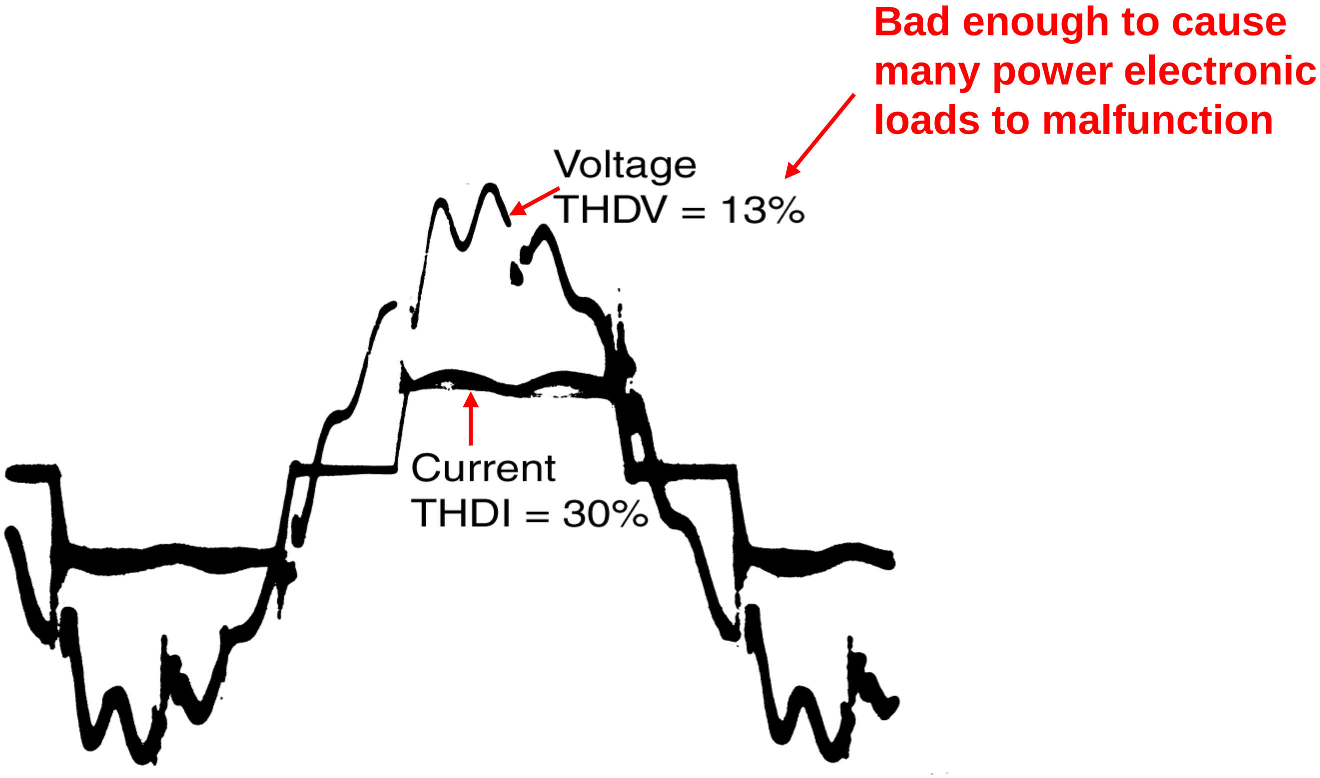
(2.2% of 3rd, 3.9% of 5th, 1.4% of 7th)



THDV = 5% considered to be the upper limit before problems are noticed

THDV = 10% considered to be terrible

Some measured current waveforms, cont.

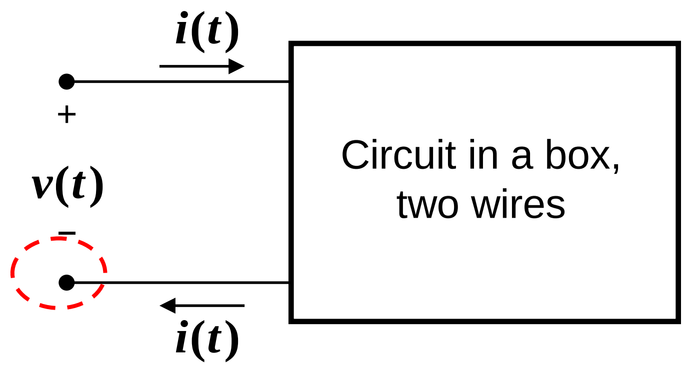


5000HP, three-phase, motor drive

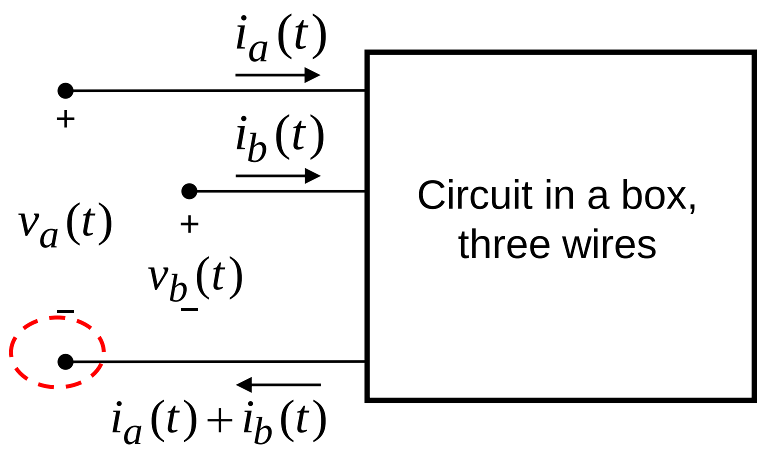
(locomotive-size)

Now, back to instantaneous power p(t)

$$p(t) = v(t) \bullet i(t)$$



$$p(t) = v_a(t) \bullet i_a(t) + v_b(t) \bullet i_b(t)$$



Any wire can be the voltage reference

Average power in terms of Fourier coefficients

$$v(t) = V_{avg} + \sum_{k=1}^{\infty} V_k \sin(k\omega_o t + \delta_k)$$

$$i(t) = I_{avg} + \sum_{k=1}^{\infty} I_k \sin(k\omega_o t + \theta_k)$$

$$p(t) = \left[V_{avg} + \sum_{k=1}^{\infty} V_k \sin(k\omega_o t + \delta_k) \right] \bullet \left[I_{avg} + \sum_{k=1}^{\infty} I_k \sin(k\omega_o t + \theta_k) \right]$$ Messy!

$$P_{avg} = \frac{1}{T} \int_{t_o}^{t_o+T} p(t)dt$$

Average power in terms of Fourier coefficients, cont.

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

$$P_{avg} = V_{avg} \bullet I_{avg} + \sum_{k=1}^{\infty} \frac{V_k}{\sqrt{2}} \bullet \frac{I_k}{\sqrt{2}} \cos(\delta_k - \theta_k)$$

$\frac{V_k}{\sqrt{2}} = V_{k,rms}$, $\frac{I_k}{\sqrt{2}} = I_{k,rms}$

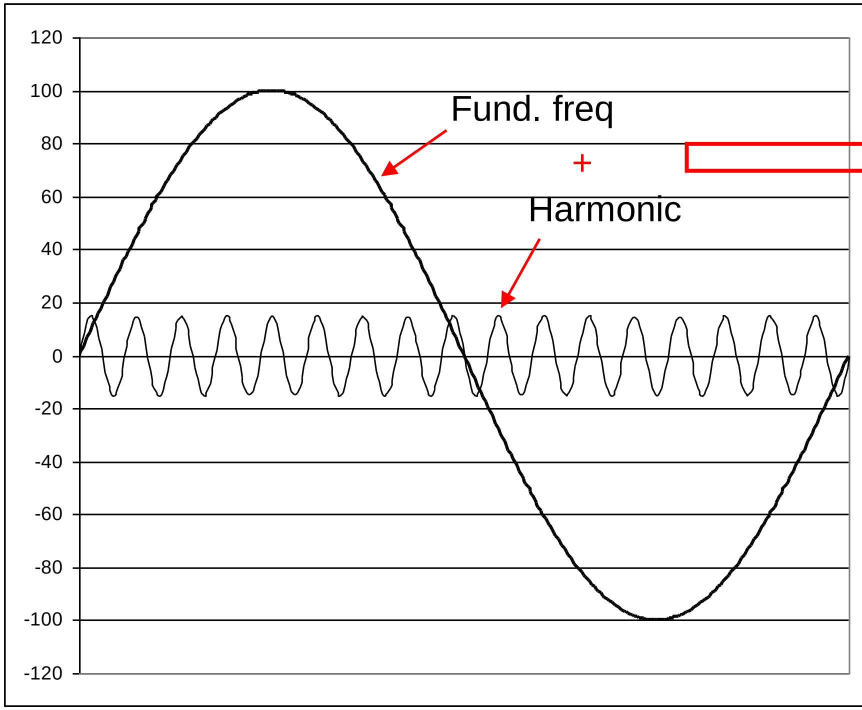
Cross products disappear because the product of unlike harmonics are themselves harmonics whose averages are zero over T!

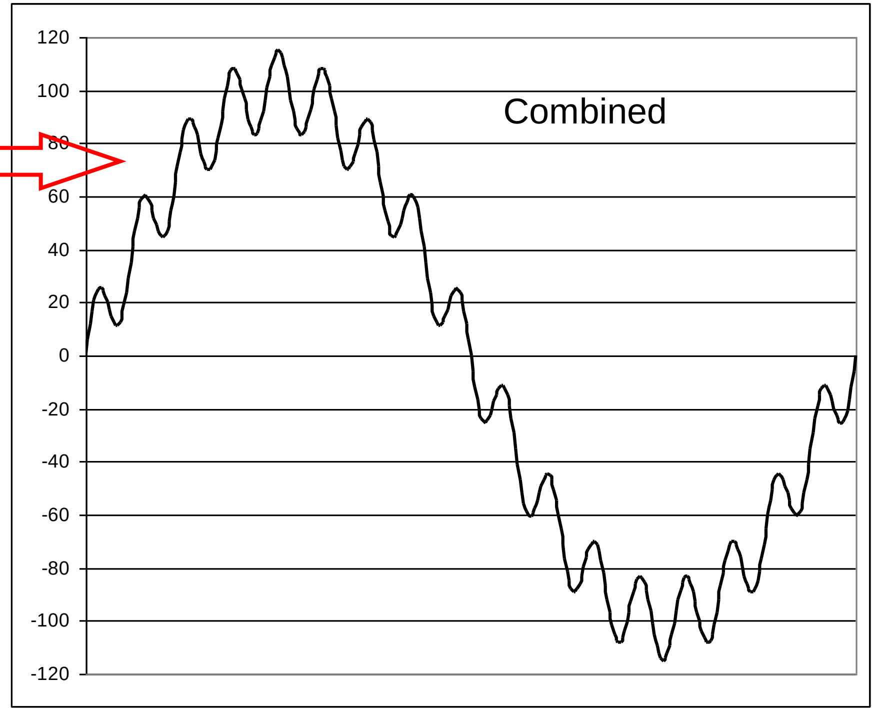
Not wanted in an AC system

$$P_{avg} = \underbrace{P_{dc}}_{\text{Due to the DC}} + \underbrace{P_1}_{\text{Due to the 1st harmonic}} + \underbrace{P_2}_{\text{Due to the 2nd harmonic}} + \underbrace{P_3}_{\text{Due to the 3rd harmonic}} + \cdots$$

Harmonic power – usually small wrt. P_1

Consider a special case where one single harmonic is superimposed on a fundamental frequency sine wave



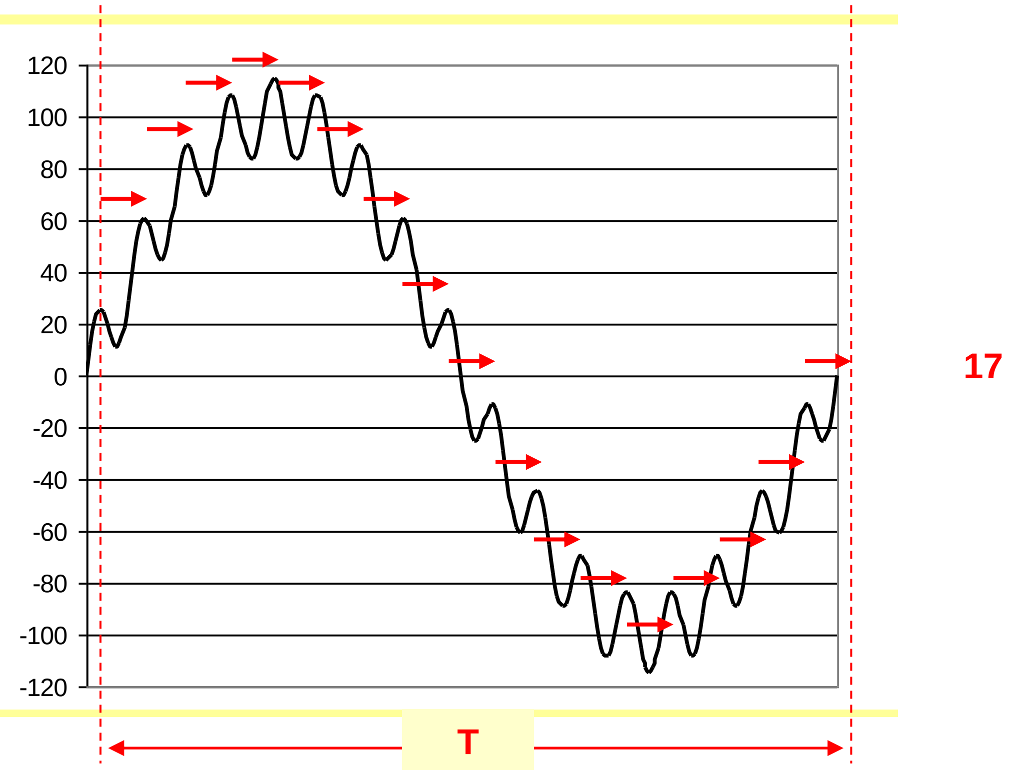


Using the combined waveform,

- Determine the order of the harmonic
- Estimate the magnitude of the harmonic
- From the above, estimate the RMS value of the waveform,
- and the THD of the waveform

Single harmonic case, cont.
Determine the order of the harmonic

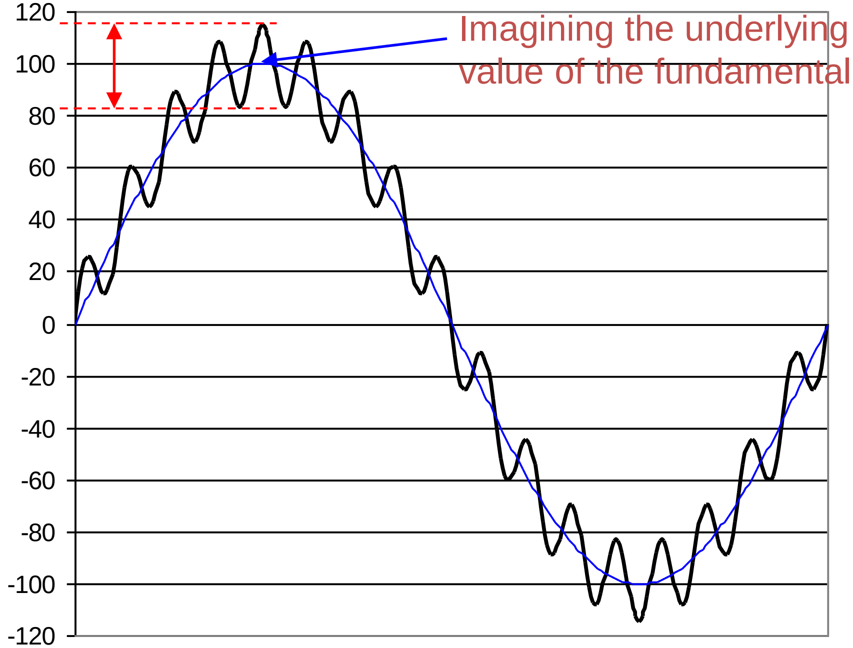
- **Count the number of cycles of the harmonic, or the number of peaks of the harmonic**



Single harmonic case, cont.
Estimate the magnitude of the harmonic

- **Estimate the peak-to-peak value of the harmonic where the fundamental is approximately constant**

Viewed near the peak of the underlying fundamental (where the fundamental is reasonably constant), the peak-to-peak value of the harmonic appears to be about 30



Imagining the underlying fundamental, the peak value of the fundamental appears to be about 100

Thus, the peak value of the harmonic is about 15

Single harmonic case, cont.
Estimate the RMS value of the waveform

$$(V_{rms})^2 = V_{avg}^2 + \sum_{k=1}^{\infty} \frac{V_k^2}{2} = 0^2 + \frac{V_1^2}{2} + \frac{V_{17}^2}{2}$$

$$= \frac{100^2}{2} + \frac{15^2}{2} = \frac{10225}{2} = 5113V^2$$

$$V_{rms} = 71.5V$$

Note – without the harmonic, the rms value would have been 70.7V (almost as large!)

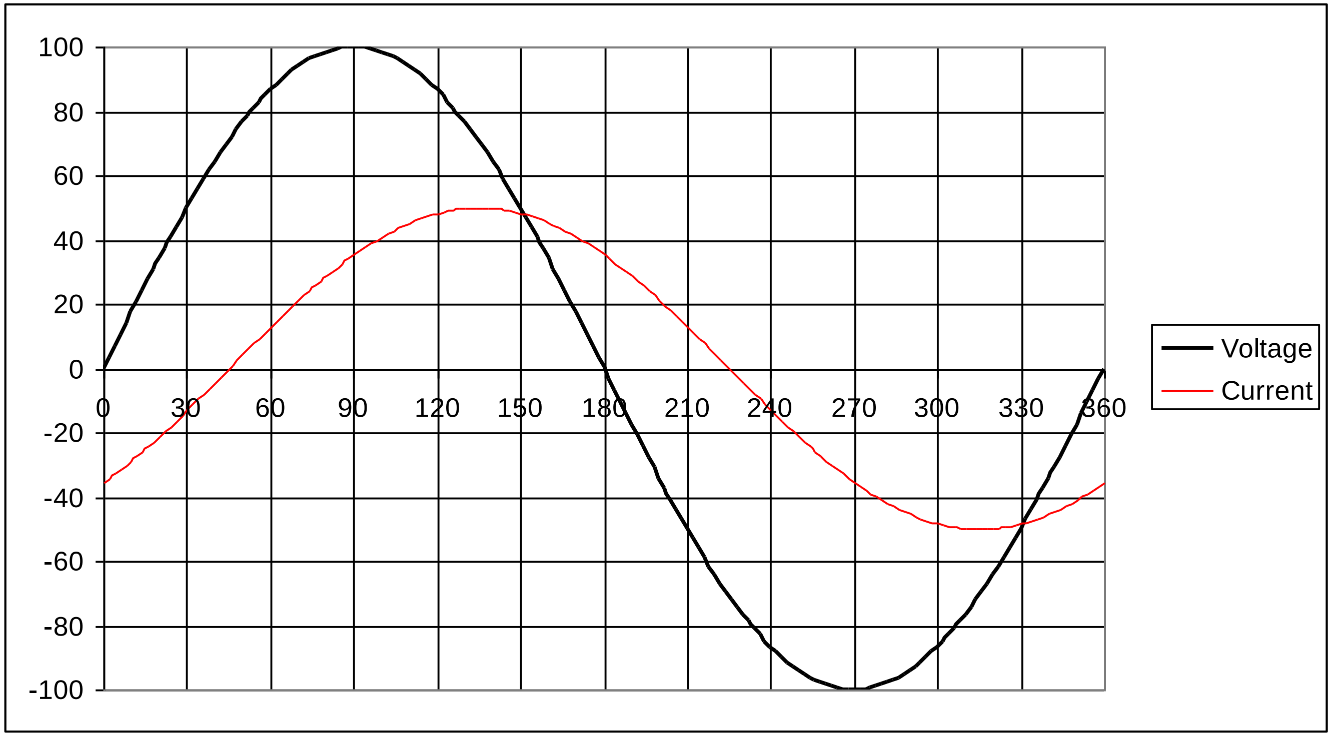
Single harmonic case, cont.
Estimate the THD of the waveform

$$THD^2 = \frac{\sum_{k=2}^{\infty} \frac{V_k^2}{2}}{\frac{V_1^2}{2}} = \frac{\frac{V_{17}^2}{2}}{\frac{V_1^2}{2}} = \frac{V_{17}^2}{V_1^2}$$

$$THD = \frac{V_{17}}{V_1} = \frac{15}{100} = 0.15$$

Given single-phase v(t) and i(t) waveforms for a load

- Determine their magnitudes and phase angles
- Determine the average power
- Determine the impedance of the load
- Using a series RL or RC equivalent, determine the R and L or C



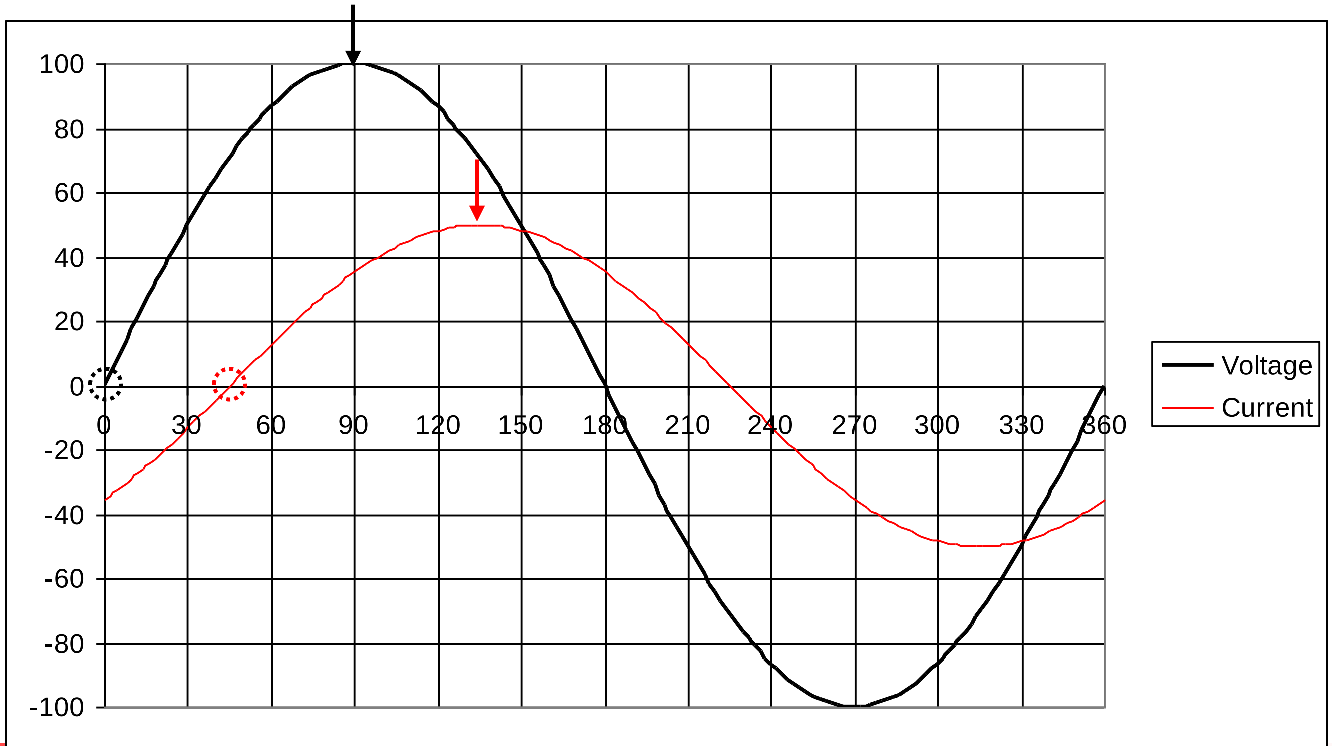
Determine voltage and current magnitudes and phase angles

Voltage sinewave has peak = 100V, phase angle = 0º

Current sinewave has peak = 50A, phase angle = -45º

Using a sine reference,

$$\tilde{V} = 100\angle 0° \, V, \;\; \tilde{I} = 50\angle -45° \, A$$



The average power is

$$P_{avg} = V_{avg} \bullet I_{avg} + \sum_{k=1}^{\infty} \frac{V_k}{\sqrt{2}} \bullet \frac{I_k}{\sqrt{2}} \cos(\delta_k - \theta_k)$$

$$P_{avg} = 0 \bullet 0 + \frac{V_1}{\sqrt{2}} \bullet \frac{I_1}{\sqrt{2}} \cos(\delta_1 - \theta_1)$$

$$P_{avg} = \frac{100}{\sqrt{2}} \bullet \frac{50}{\sqrt{2}} \cos\left(0 - (-45)\right)$$

$$P_{avg} = 1767W$$

The equivalent series impedance is inductive because the current lags the voltage

$$Z_{eq} = \frac{\tilde{V}}{\tilde{I}} = \frac{100\angle 0°}{50\angle -45°} = 2\angle 45°\Omega = R_{eq} + j\omega L_{eq}$$

$$R_{eq} = 2\cos(45°) = 1.414\Omega$$

$$\omega L_{eq} = 2\sin(45°) = 1.414\Omega$$

where *ω* is the radian frequency (2πf)

If the current leads the voltage, then the impedance angle is negative, and there is an equivalent capacitance

C's and L's operating in periodic steady-state

Examine the current passing through a capacitor that is operating in periodic steady state. The governing equation is

$$i(t) = C\frac{dv(t)}{dt} \quad \text{which leads to} \quad v(t) = v(t_o) + \frac{1}{C}\int_{t_o}^{t_o+t} i(t)dt$$

Since the capacitor is in periodic steady state, then the voltage at time t_o is the same as the voltage one period T later, so

$$v(t_o+T) = v(t_o), \quad \text{or} \quad v(t_o+T) - v(t_o) = 0 = \frac{1}{C}\int_{t_o}^{t_o+T} i(t)dt$$

The conclusion is that $\int_{t_o}^{t_o+T} i(t)dt = 0$ **which means that**

the average current through a capacitor operating in periodic steady state is zero

Now, an inductor

Examine the voltage across an inductor that is operating in periodic steady state. The governing equation is

$$v(t) = L\frac{di(t)}{dt} \quad \text{which leads to} \quad i(t) = i(t_o) + \frac{1}{L}\int_{t_o}^{t_o+t} v(t)dt$$

Since the inductor is in periodic steady state, then the voltage at time t_o is the same as the voltage one period T later, so

$$i(t_o+T) = i(t_o), \quad \text{or} \quad i(t_o+T) - i(t_o) = 0 = \frac{1}{L}\int_{t_o}^{t_o+T} v(t)dt$$

The conclusion is that $\int_{t_o}^{t_o+T} v(t)dt = 0$ **which means that**

the average voltage across an inductor operating in periodic steady state is zero

KVL and KCL in periodic steady-state

Since KVL and KCL apply at any instance, then they must also be valid in averages. Consider KVL,

$$\sum_{Around\ loop} v(t) = 0, \quad v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t) = 0$$

$$\frac{1}{T}\int_{t_o}^{t_o+T} v_1(t)dt + \frac{1}{T}\int_{t_o}^{t_o+T} v_2(t)dt + \frac{1}{T}\int_{t_o}^{t_o+T} v_3(t)dt + \cdots + \frac{1}{T}\int_{t_o}^{t_o+T} v_N(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T} (0)dt = 0$$

$$V_{1avg} + V_{2avg} + V_{3avg} + \cdots + V_{Navg} = 0$$

KVL applies in the average sense

The same reasoning applies to KCL

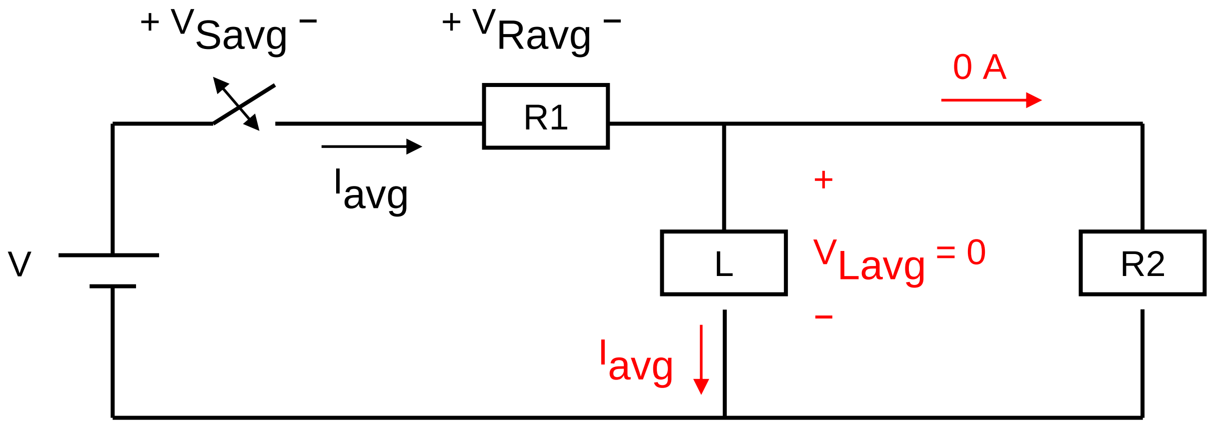
$$\sum_{Out\ of\ node} i(t) = 0, \quad i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t) = 0$$

$$I_{1avg} + I_{2avg} + I_{3avg} + \cdots + I_{Navg} = 0$$

KCL applies in the average sense

KVL and KCL in the average sense

Consider the circuit shown that has a constant duty cycle switch

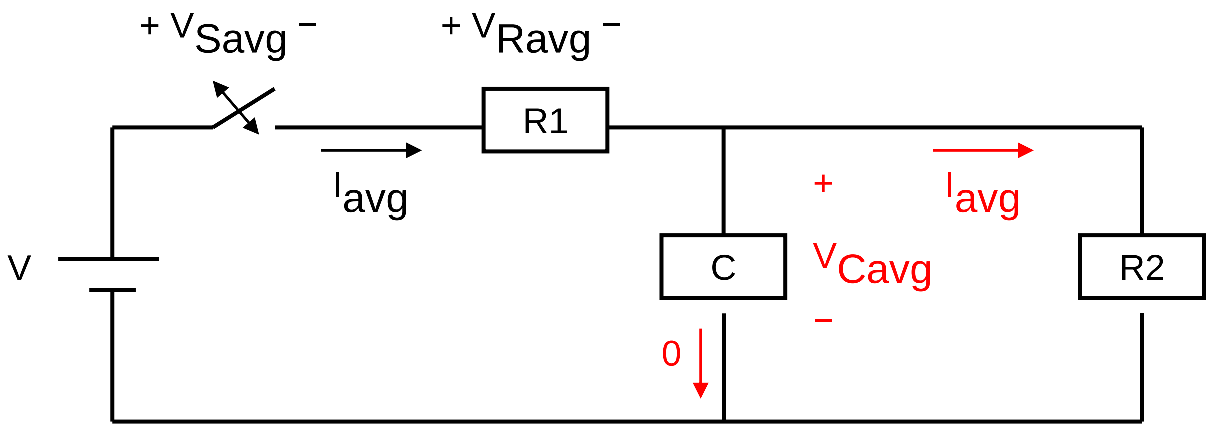


A DC multimeter (i.e., averaging) would show

and would show $V = V_{Savg} + V_{Ravg}$

KVL and KCL in the average sense, cont.

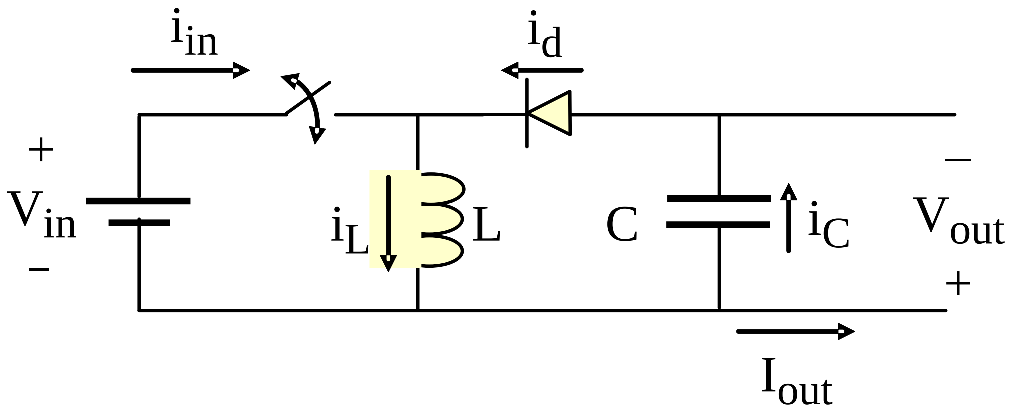
Consider the circuit shown that has a constant duty cycle switch



A DC multimeter (i.e., averaging) would show

and would show $V = V_{Savg} + V_{Ravg} + V_{Cavg}$

Practice Problem

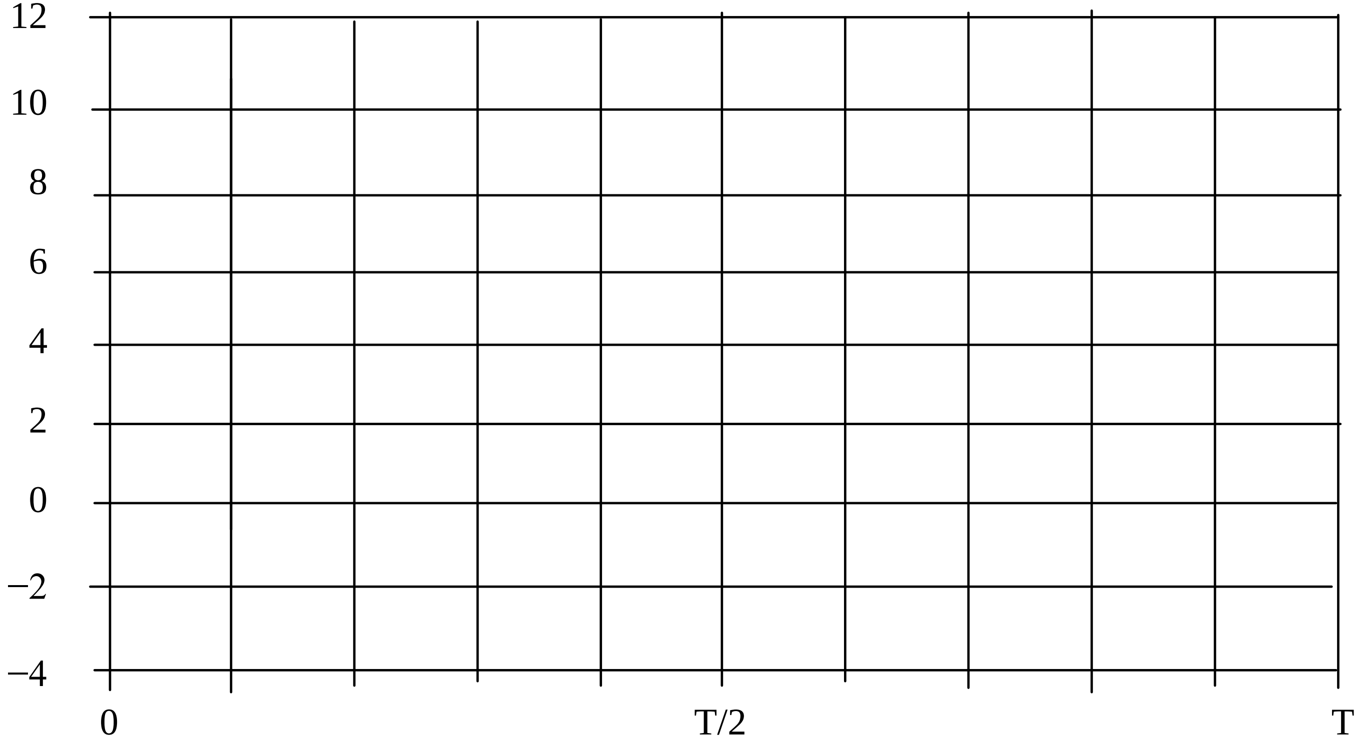


4a. Assuming continuous conduction in L, and ripple free V_{out} and I_{out} , draw the "switch closed" and switch open" circuits and use them to develop the $\frac{V_{out}}{V_{in}}$ equation.

4b. Consider the case where the converter is operating at 50kHz, V_{in} = 40V, V_{out} = 120V, P = 240W. Components L = 100μH, C = 1500μF. Carefully sketch the inductor and capacitor currents on the graph provided.

4c. Use the graphs to determine the inductor's rms current, and the capacitor's peak-to-peak current.

4d. Use the graphs to determine the capacitor's peak-to-peak ripple voltage.


12
10
8
6
4
2
0
−2
−4
0
T/2
T