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Engineering Electromagnetics

Current & Conductors

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Current & Conductors

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Current & Current Density (1)

- Current:

$$I = \frac{dQ}{dt}$$

- Unit A (ampère)
- Current is defined as the motion of positive charges

Current & Current Density (2)

- Current: rate of movement of charge crossing a given reference plane (of one coulomb per second)
- Current density: **J** (A/m^2)
- The increment of current ΔI crossing an incremental surface ΔS normal to the current density:

$$\Delta I = J_N \Delta S$$

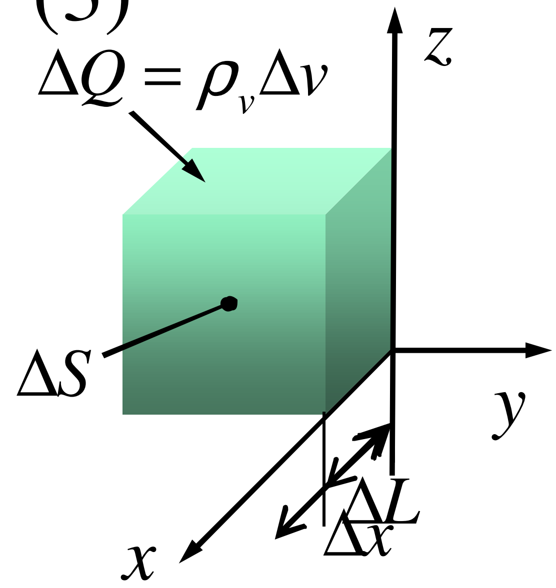
- If the current density is not perpendicular to the surface:

$$\Delta I = \mathbf{J}.\Delta\mathbf{S}$$

- Total current:

$$\boxed{I = \int_S \mathbf{J}.d\mathbf{S}}$$

Current & Current Density (3)



$$\Delta Q = \rho_v \Delta v = \rho_v \Delta S \Delta L$$

$$\left.\begin{aligned} \Delta Q &= \rho_v \Delta S \Delta x \\ \Delta I &= \frac{\Delta Q}{\Delta t} \end{aligned}\right\} \rightarrow \Delta I = \rho_v \Delta S \frac{\Delta x}{\Delta t}$$

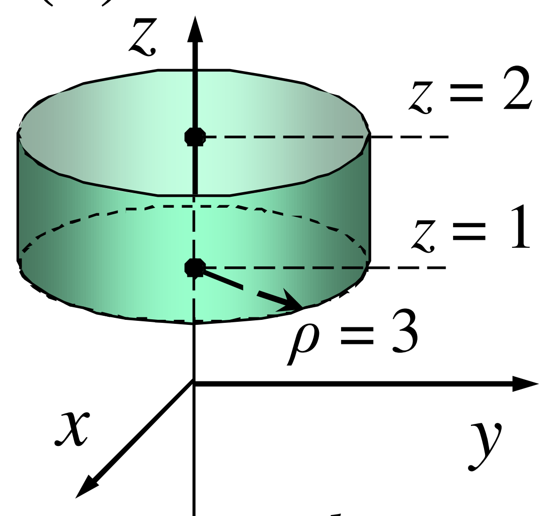
$$\left.\begin{aligned} &= \rho_v \Delta S v_x \\ \Delta I &= J_x \Delta S \end{aligned}\right\} \rightarrow J_x = \rho_v v_x$$

$$\rightarrow \boxed{\mathbf{J} = \rho_v \mathbf{v}}$$

Current & Current Density (4)

Ex. 1

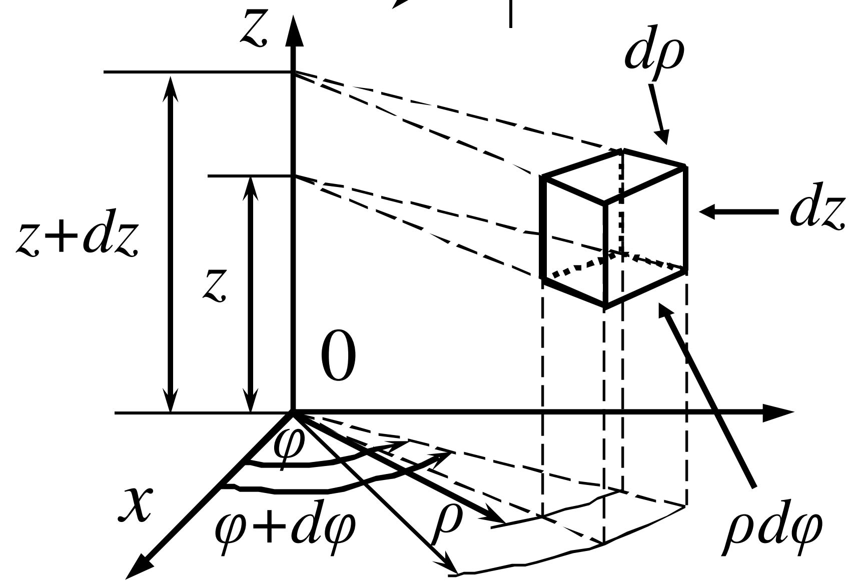
Given $\mathbf{J} = 2\rho z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi$ mA/m². Find the total current leaving the circular band.



$$I = \int_S \mathbf{J}.d\mathbf{S} = \int_S \mathbf{J}\big|_{\rho=3} .d\mathbf{S}$$

$$\mathbf{J}\big|_{\rho=3} = 2\times 3z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi$$

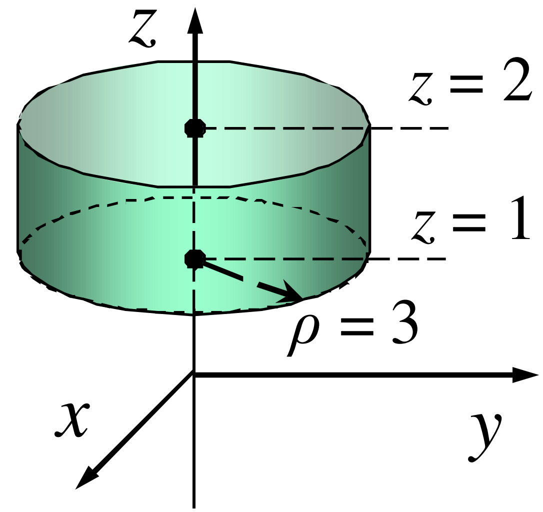
$$\left.\begin{aligned} &= 6z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi \\ d\mathbf{S} &= \rho d\varphi dz\mathbf{a}_\rho = 3d\varphi dz\mathbf{a}_\rho \end{aligned}\right\}$$



Current & Current Density (5)

Ex. 1

Given $\mathbf{J} = 2\rho z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi$ mA/m². Find the total current leaving the circular band.



$$I = \int_S \mathbf{J}.d\mathbf{S} = \int_S \mathbf{J}\big|_{\rho=3}.d\mathbf{S}$$

$$\mathbf{J}\big|_{\rho=3} = 2\times 3z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi$$

$$\left.\begin{aligned} &= 6z\mathbf{a}_\rho + 7z\sin^2\varphi\mathbf{a}_\varphi \\ d\mathbf{S} &= \rho d\varphi dz\mathbf{a}_\rho = 3d\varphi dz\mathbf{a}_\rho \end{aligned}\right\} \to \mathbf{J}\big|_{\rho=3}.d\mathbf{S} = 18zd\varphi dz$$

$$\to I = \int_{z=1}^{z=2}\int_{\varphi=0}^{\varphi=2\pi} 18zd\varphi dz = \int_{z=1}^{z=2} 2\pi\times 18zdz = 169\,\text{mA}$$

Current & Current Density (6)

The current leaving a closed surface: $I = \oint_S \mathbf{J}.d\mathbf{S}$

The total charge in the surface: Q_i

The law of conservation of charge

$$\rightarrow I = \oint_S \mathbf{J}.d\mathbf{S} = -\frac{dQ_i}{dt}$$

- in circuit analysis, $I = dQ/dt$ because this is an entering current
- in electromagnetism, $I = -\,dQ/dt$ because this is a leaving one

Current & Current Density (7)

$$\left.\begin{array}{l} I = \oint_S \mathbf{J}.d\mathbf{S} = -\dfrac{dQ_i}{dt} \\ \oint_S \mathbf{J}.d\mathbf{S} = \int_V (\nabla.\mathbf{J})dv \quad \text{(div. theo.)} \end{array}\right\} \rightarrow \left.\begin{array}{l} \int_V (\nabla.\mathbf{J})dv = -\dfrac{dQ_i}{dt} \\ Q_i = \int_V \rho_v dv \end{array}\right\}$$

$$\rightarrow \int_V (\nabla.\mathbf{J})dv = -\frac{d}{dt}\int_V \rho_v dv = \int_V -\frac{\partial \rho_v}{\partial t} dv$$

$$\rightarrow (\nabla.\mathbf{J})\Delta v = -\frac{\partial \rho_v}{\partial t}\Delta v \rightarrow \boxed{\nabla.\mathbf{J} = -\frac{\partial \rho_v}{\partial t}}$$

Current & Current Density (9)

Ex. 2

Consider the current density $\mathbf{J} = \dfrac{e^{-t}}{r}\mathbf{a}_r$ A/m^2.

$$\left.\begin{aligned}
&-\frac{\partial \rho_v}{\partial t} = \nabla.\mathbf{J} = \nabla.\left(\frac{e^{-t}}{r}\mathbf{a}_r\right)\\
&\nabla.\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\varphi}{\partial\varphi}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\to -\frac{\partial \rho_v}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{e^{-t}}{r}\right) = \frac{e^{-t}}{r^2} \to \rho_v = -\int\frac{e^{-t}}{r^2}dt + K(r) = \frac{e^{-t}}{r^2} + K(r)\\
&\text{Suppose } \rho_v \to 0 \text{ as } t \to \infty, \text{ then } K(r) = 0
\end{aligned}\right\}$$

$$\to \rho_v = \frac{e^{-t}}{r^2} \ \mathrm{C/m^3} \qquad \to v_r = \frac{J_r}{\rho_v} = \left(\frac{e^{-t}}{r}\right)\Big/\left(\frac{e^{-t}}{r^2}\right) = r \ \ \mathrm{m/s}$$

Current & Conductors

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2. **Metallic Conductors**
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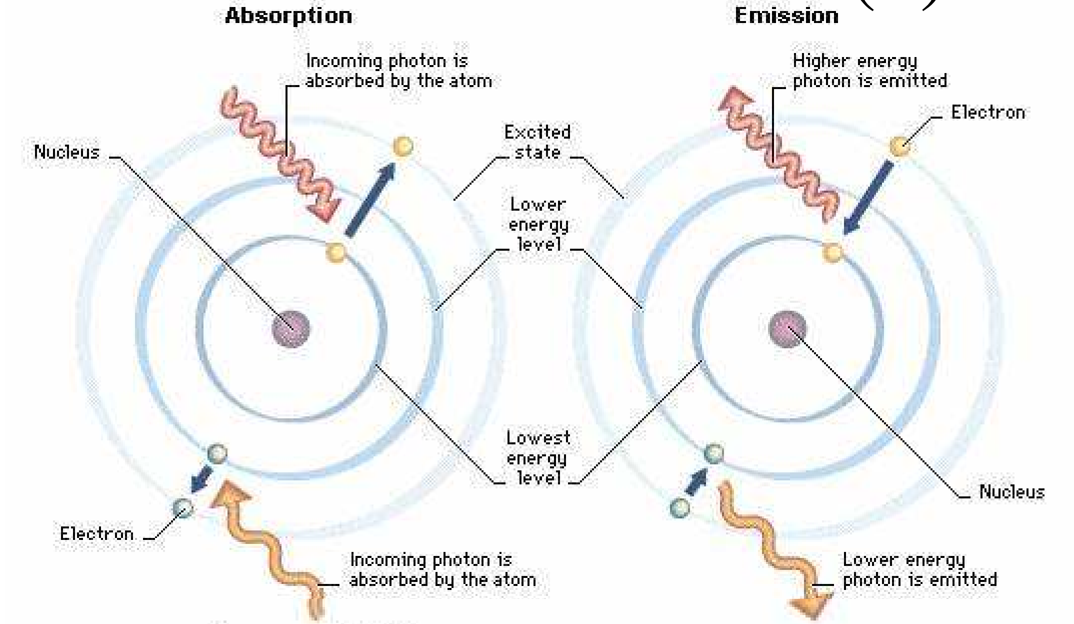
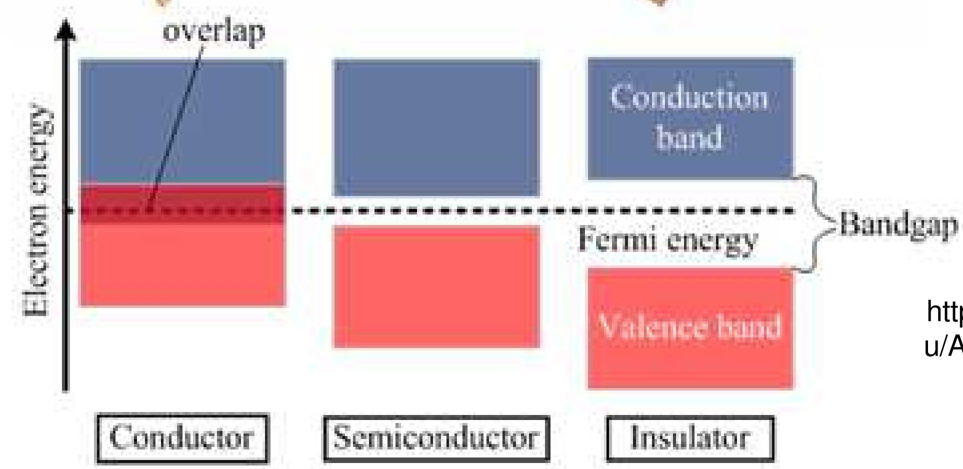
Metallic Conductors (1)



http://samuelsolema.blogspot.com/2013/03/hw10-quantum-theory.html



http://potenntial.eecs.utk.edu/About.php?topic=PowerSemiconductors

Metallic Conductors (2)

$$\mathbf{F} = -e\mathbf{E}$$

- In free space, the electron will accelerate
- In conductors, the electron will soon obtain a constant average velocity:

$$\mathbf{v}_d = -\mu_e\mathbf{E}$$

- μ_e: the mobility of an electron, m^2/Vs, positive
- Ex.: Al: 0.0012; Cu: 0.0032; Ag: 0.0056
- $\mathbf{J} = \rho_v\mathbf{v}$
- $\rightarrow \boxed{\mathbf{J} = -\rho_e\mu_e\mathbf{E}}$

Metallic Conductors (3)

$$\mathbf{J} = -\rho_e \mu_e \mathbf{E}$$

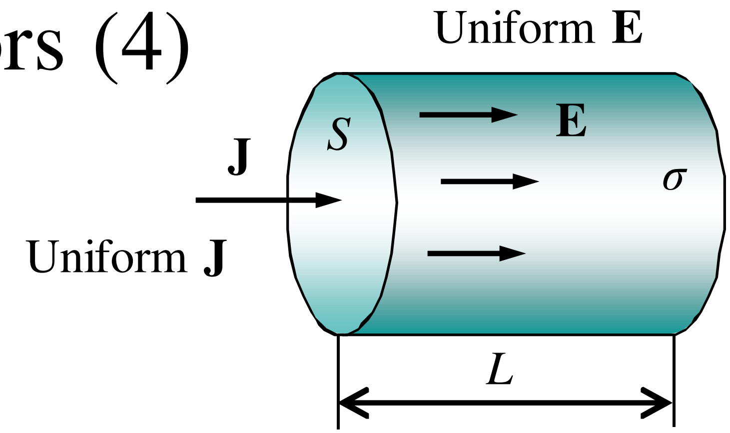
- ρ_e : free-electron charge density, negative
- **J** is in the same direction as **E**

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

- σ : conductivity, S/m
- Ex.: Al: 3.82×10^7; Cu: 5.80×10^7; Ag: 6.17×10^7

$$\boxed{\sigma = -\rho_e \mu_e}$$

Metallic Conductors (4)



$$I = \int_S \mathbf{J}.d\mathbf{S} = JS \rightarrow J = \frac{I}{S}$$

$$V_{ab} = -\int_b^a \mathbf{E}.d\mathbf{L}$$

$$= -\mathbf{E}.\int_b^a d\mathbf{L}$$

$$= -\mathbf{E}.\mathbf{L}_{ba} = \mathbf{E}.\mathbf{L}_{ab}$$

$$\left.\begin{aligned} \rightarrow V &= EL \\ J &= \sigma E \end{aligned}\right\} \rightarrow J = \sigma \frac{V}{L}$$

$$\rightarrow \sigma \frac{V}{L} = \frac{I}{S} \rightarrow V = \frac{L}{\sigma S} I$$

$$\boxed{R = \frac{L}{\sigma S}}$$

$$\rightarrow \boxed{V = RI}$$

(Ohm's law)

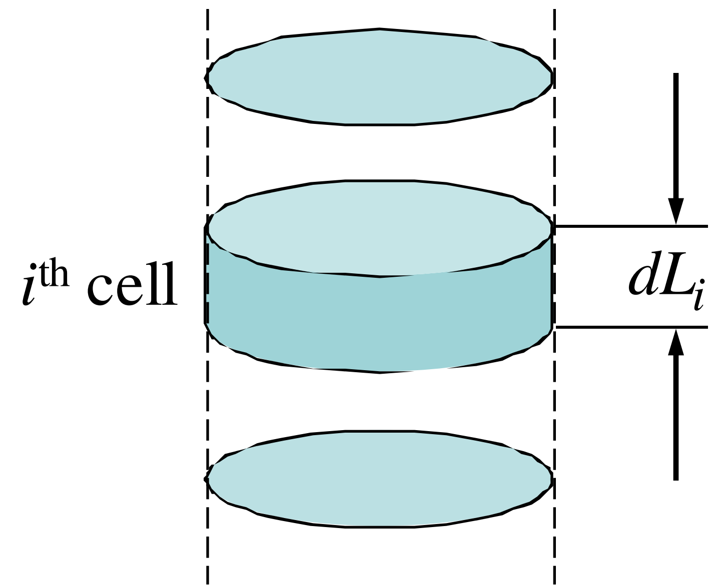
$$\boxed{R = \frac{V_{ab}}{I} = \frac{-\int_b^a \mathbf{E}.d\mathbf{L}}{\int_S \sigma \mathbf{E}.d\mathbf{S}}}$$

Metallic Conductors (5)

$$R_i = \frac{dL_i}{\sigma_i S_i}$$

$$\rightarrow R = \sum_{i=1}^{N} R_i = \sum_{i=1}^{N} \frac{dL_i}{\sigma_i S_i}$$

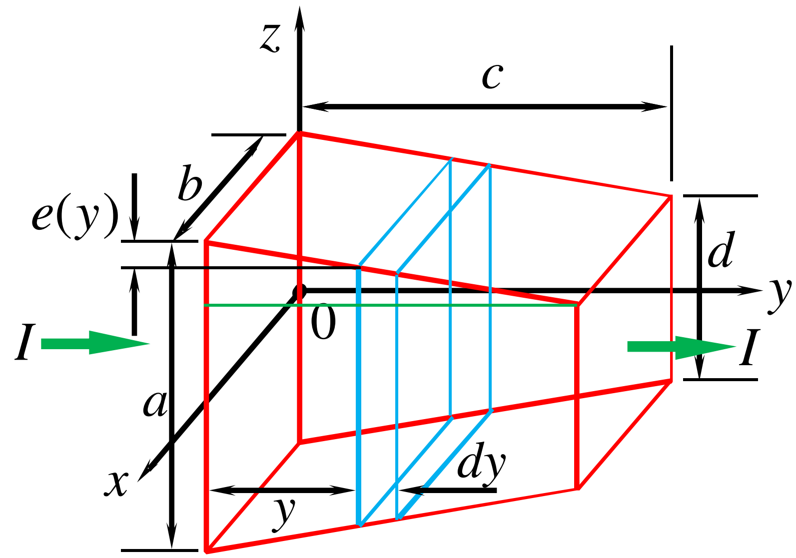
$$\rightarrow R = \int \frac{dL}{\sigma S}$$



Metallic Conductors (6)

Ex. 1

$$\left.\begin{aligned} dR &= \frac{dy}{\sigma S(y)} \\ S(y) &= b[a-2e(y)] \\ \frac{e(y)}{y} &= \frac{\frac{a-d}{2}}{c} \rightarrow e(y) = \frac{a-d}{2c}y \end{aligned}\right\}$$



$$\rightarrow dR = \frac{dy}{\sigma b\left[a - 2\frac{a-d}{2c}y\right]} = \frac{c}{\sigma b[ac-(a-d)y]}dy$$

$$\rightarrow R = \int_{y=0}^{c} \frac{c}{\sigma b[ac-(a-d)y]}dy$$

Ex. 2

Metallic Conductors (7)

A material with conductivity $\sigma = m/\rho + k$, where m & k are constants, fills the space between two concentric, cylindrical conductors of radii a & b. L is the length of each conductor. Find the resistance of the material?

$$\left.\begin{aligned} R_i &= \frac{dL_i}{\sigma_i S_i} \\ dL_i &= d\rho \\ \sigma_i &= \frac{m}{\rho} + k \\ S_i &= 2\pi\rho L \end{aligned}\right\} \rightarrow R = \int_a^b \frac{d\rho}{(k\rho + m)2\pi L} = \frac{1}{2\pi Lk} \ln\frac{kb+m}{ka+m}$$

Metallic Conductors (8)

Ex. 3

$$R = \frac{V}{I} = \frac{-\int_b^a \mathbf{E}.d\mathbf{L}}{\int_S \sigma \mathbf{E}.d\mathbf{S}}$$

$$\mathbf{E} = E(\rho)\mathbf{a}_\varphi$$

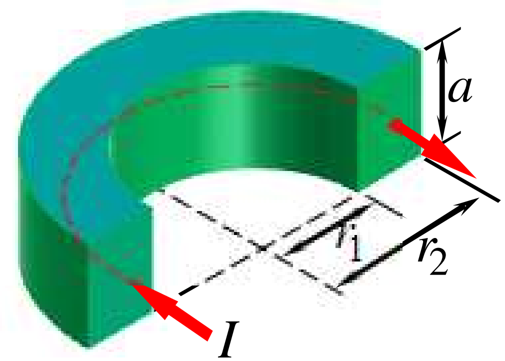
$$V = \int_{\varphi=0}^{\pi} \left[E(\rho)\mathbf{a}_\varphi \right].\left(\rho d\phi \mathbf{a}_\varphi \right)$$

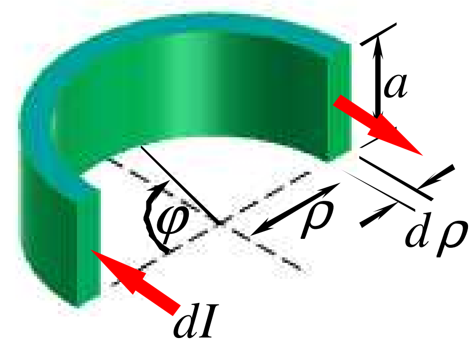
$$= \int_{\varphi=0}^{\pi} E(\rho)\rho d\phi = E(\rho)\rho \int_0^{\pi} d\phi = E(\rho)\rho\pi \rightarrow E(\rho) = \frac{V}{\pi\rho}$$

$$I = \int_S \sigma \mathbf{E}.d\mathbf{S} = \int_{\rho=r_1}^{r_2} \left[\sigma E(\rho)\mathbf{a}_\varphi \right].\left(ad\rho \mathbf{a}_\varphi \right) = \int_{\rho=r_1}^{r_2} \sigma E(\rho) a d\rho$$

$$= \int_{\rho=r_1}^{r_2} \sigma \frac{V}{\pi\rho} a d\rho = \frac{\sigma V a}{\pi} \ln \frac{r_2}{r_1}$$

$$\rightarrow R = \frac{V}{\dfrac{\sigma V a}{\pi} \ln \dfrac{r_2}{r_1}} = \boxed{\frac{\pi}{\sigma a \ln \dfrac{r_2}{r_1}}}$$

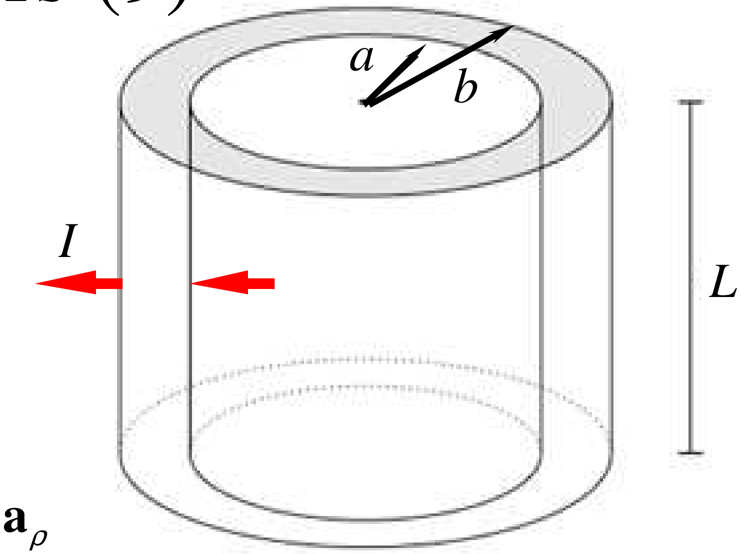




Metallic Conductors (9)

Ex. 4

(Method 1)



$$R = \frac{V}{I} = \frac{-\int_b^a \mathbf{E}.d\mathbf{L}}{\int_S \sigma \mathbf{E}.d\mathbf{S}}$$

$$\left.\begin{array}{l} \nabla.\mathbf{D} = \rho_v = 0 \rightarrow \nabla.\varepsilon\mathbf{E} = 0 \\ \nabla.\mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial\varphi} + \frac{\partial D_z}{\partial z} \\ \mathbf{E} = E_\rho \mathbf{a}_\rho \end{array}\right\} \rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\varepsilon E_\rho) = 0$$

$$\rightarrow E_\rho = \frac{C}{\rho} \rightarrow \mathbf{E} = \frac{C}{\rho}\mathbf{a}_\rho$$

$$V = \int_a^b \mathbf{E}.d\mathbf{L} = \int_a^b \frac{C}{\rho}\mathbf{a}_\rho.d\rho\mathbf{a}_\rho = C\ln\frac{b}{a} \rightarrow C = \frac{V}{\ln(b/a)} \rightarrow \mathbf{E} = \frac{V}{\rho\ln(b/a)}\mathbf{a}_\rho$$

$$I = \int_S \sigma\mathbf{E}.d\mathbf{S} = \int_{z=0}^{L}\int_{\varphi=0}^{2\pi} \sigma\frac{V}{\rho\ln(b/a)}\mathbf{a}_\rho.\rho d\varphi dz\mathbf{a}_\rho = \frac{\sigma V 2\pi L}{\ln(b/a)}$$

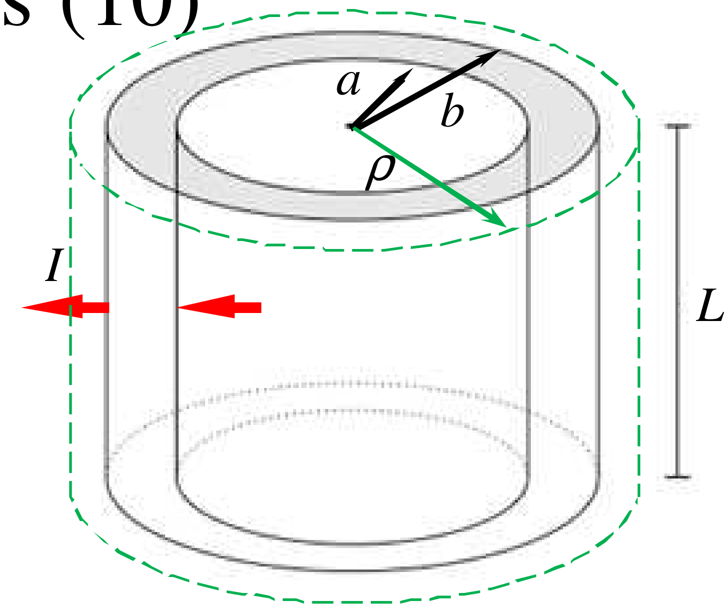
$$\rightarrow R = \frac{V}{I} = \frac{V}{\frac{\sigma V 2\pi L}{\ln(b/a)}} = \boxed{\frac{\ln(b/a)}{2\pi\sigma L}}$$

Metallic Conductors (10)

Ex. 4

(Method 2)

$$R = \frac{V}{I} = \frac{-\int_b^a \mathbf{E}.d\mathbf{L}}{\int_S \sigma \mathbf{E}.d\mathbf{S}}$$



$$\left.\begin{array}{l} Q = \oint_S \mathbf{D}.d\mathbf{S} \\ \mathbf{E} = E_\rho \mathbf{a}_\rho \end{array}\right\} \rightarrow Q = \oint_S \varepsilon E_\rho \mathbf{a}_\rho .d\mathbf{S}$$

$$= \varepsilon E_\rho (2\pi\rho L)$$

$$\rightarrow \mathbf{E} = \frac{Q}{2\pi\varepsilon\rho L}\mathbf{a}_\rho$$

$$\left.\begin{array}{l} V = \int_a^b \mathbf{E}.d\mathbf{L} = \int_a^b \frac{Q}{2\pi\varepsilon\rho L}\mathbf{a}_\rho .d\rho\mathbf{a}_\rho = \frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a} \\ I = \int_S \sigma \mathbf{E}.d\mathbf{S} = \int\limits_{z=0}^{L}\int\limits_{\varphi=0}^{2\pi} \sigma \frac{Q}{2\pi\varepsilon\rho L}\mathbf{a}_\rho .\rho d\varphi dz \mathbf{a}_\rho = \frac{\sigma Q}{\varepsilon} \end{array}\right\} \rightarrow R = \frac{V}{I} = \frac{\frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a}}{\frac{\sigma Q}{\varepsilon}} = \boxed{\frac{\ln(b/a)}{2\pi\sigma L}}$$

Current & Conductors

1. Current & Current Density
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3. **Conductor Properties & Boundary Conditions**
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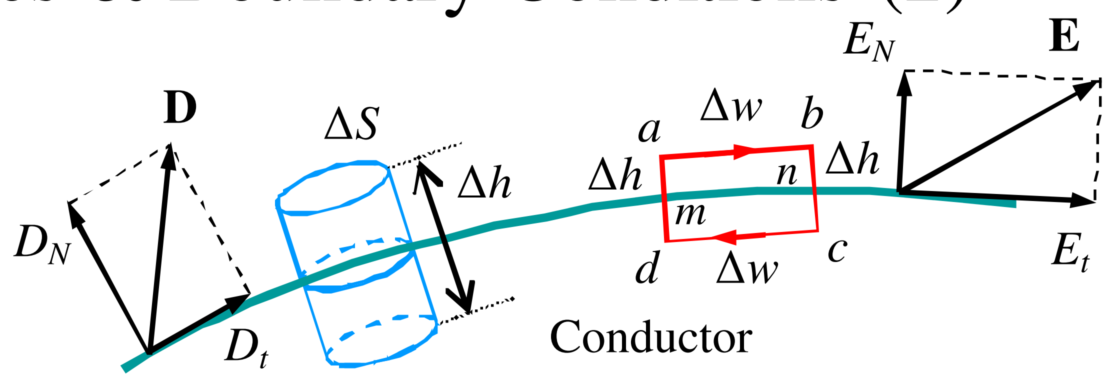
Conductor Properties & Boundary Conditions (1)

- Given some electrons in the interior of a conductor
- They will begin to accelerate away from each other, until they reach the surface of the conductor
- *Characteristic* 1: the charge density inside a conductor is zero, the exterior surface has a surface charge density
- Within a conductor: no charge → no current → no electric field intensity (Ohm)
- *Characteristic* 2: the electric field intensity within the conductor is zero

Conductor Properties & Boundary Conditions (2)



$$\boxed{\oint \mathbf{E}.d\mathbf{L} = 0}$$

$$\rightarrow \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$$

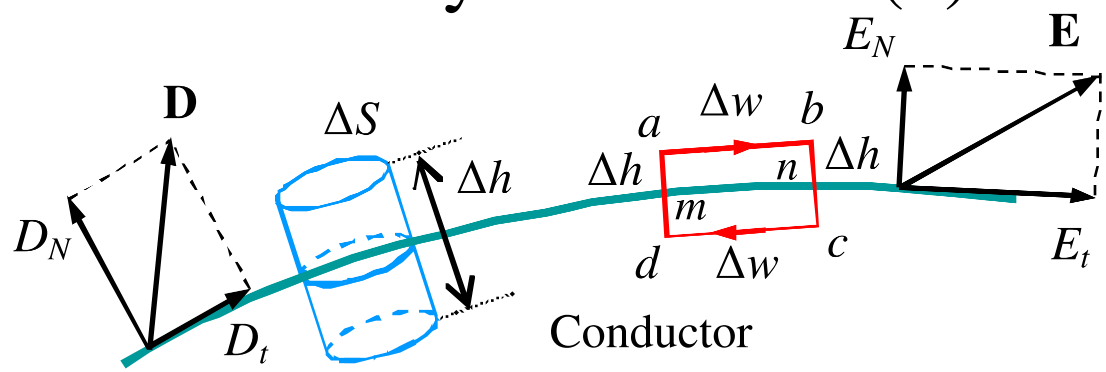
$$\left.\begin{matrix} \int_a^b \mathbf{E}_{ab}.d\mathbf{L}_{ab} \\ \mathbf{E}_{ab} = \mathbf{E}_N + \mathbf{E}_t \end{matrix}\right\} \rightarrow \left.\begin{matrix} \int_a^b \mathbf{E}_{ab}.d\mathbf{L}_{ab} = \int_a^b (\mathbf{E}_N + \mathbf{E}_t).d\mathbf{L}_{ab} = \int_a^b E_t dL_{ab} \\ E_t \approx \text{const} \end{matrix}\right\}$$

$$\rightarrow \int_a^b \mathbf{E}_{ab}.d\mathbf{L}_{ab} = E_t \int_a^b dL_{ab} = E_t \Delta w$$

$$\left.\begin{matrix} \int_c^d \mathbf{E}_{cd}.d\mathbf{L}_{cd} \\ \mathbf{E}_{\text{within conductor}} = 0 \end{matrix}\right\} \rightarrow \mathbf{E}_{cd} = 0 \rightarrow \int_c^d \mathbf{E}_{cd}.d\mathbf{L}_{cd} = 0$$

Conductor Properties & Boundary Conditions (3)

$$\boxed{\oint \mathbf{E}.d\mathbf{L} = 0}$$



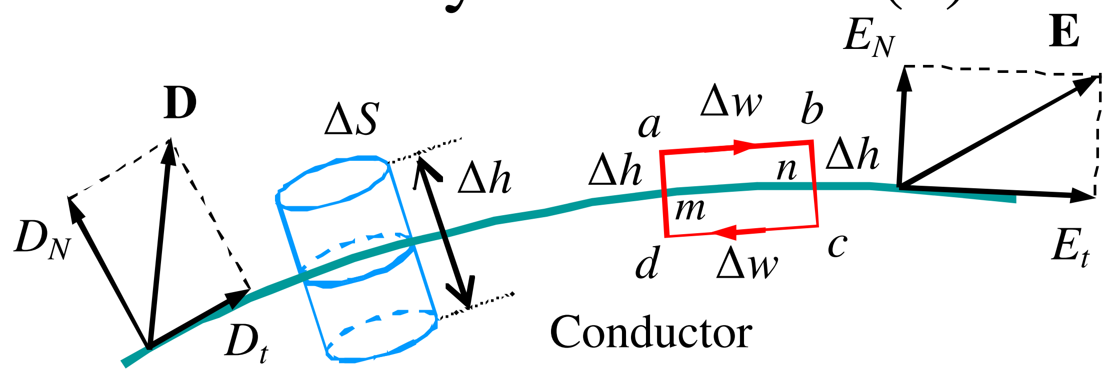
$$\rightarrow \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$$

$$\left.\begin{array}{l} \int_b^c \mathbf{E}_{bc}.d\mathbf{L}_{bc} = \int_b^n \mathbf{E}_{bn}.d\mathbf{L}_{bn} + \int_n^c \mathbf{E}_{nc}.d\mathbf{L}_{nc} \\ \mathbf{E}_{\text{within conductor}} = 0 \;\rightarrow \mathbf{E}_{nc} = 0 \\ \mathbf{E}_{bn} = \mathbf{E}_{N,b} + \mathbf{E}_{tt} \end{array}\right\} \rightarrow \left.\begin{array}{r} \int_b^c \mathbf{E}_{bc}.d\mathbf{L}_{bc} = -\int_b^n E_{N,b} dL_{bn} \\ E_{N,b} \approx \text{const} \end{array}\right\}$$

$$\rightarrow \int_b^c \mathbf{E}_{bc}.d\mathbf{L}_{bc} = -E_{N,b}\int_b^n dL_{bn} = -\frac{E_{N,b}\Delta h}{2}$$

$$\int_d^a \mathbf{E}_{da}.d\mathbf{L}_{da} = \frac{E_{N,a}\Delta h}{2}$$

Conductor Properties & Boundary Conditions (4)



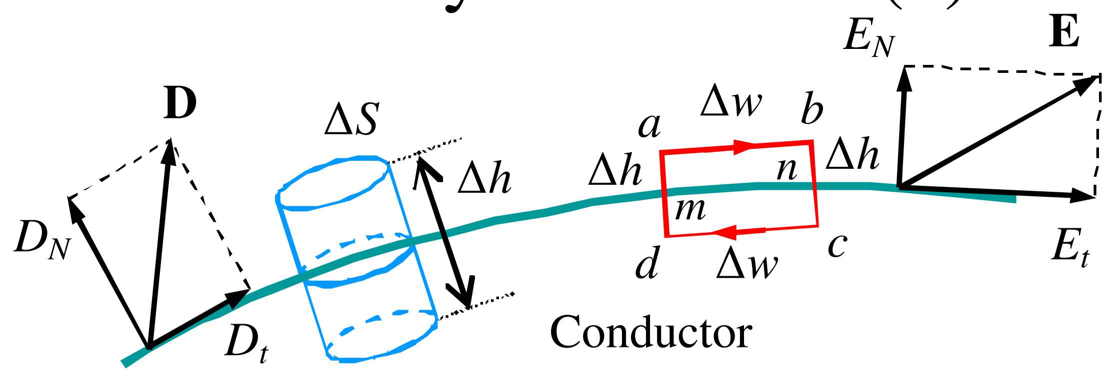
$$\boxed{\oint \mathbf{E}.d\mathbf{L} = 0}$$

$$\left.\begin{array}{l} \rightarrow \int_a^b + \int_b^c + \int_c^d + \int_d^a = 0 \\ \int_a^b = E_{tt}\Delta w \\ \int_b^c = -\dfrac{E_{N,b}\Delta h}{2} \\ \int_c^d = 0 \\ \int_d^a = \dfrac{E_{N,a}\Delta h}{2} \end{array}\right\} \rightarrow \left.\begin{array}{r} E_t\Delta w - \dfrac{E_{N,b}\Delta h}{2} + 0 + \dfrac{E_{N,a}\Delta h}{2} = 0 \\ \Delta h \rightarrow 0 \end{array}\right\}$$

$$\rightarrow E_t\Delta w = 0 \;\rightarrow E_t = 0$$

$$\rightarrow D_t = \varepsilon_0 E_t = 0 \;\rightarrow \boxed{D_t = E_t = 0}$$

Conductor Properties & Boundary Conditions (5)



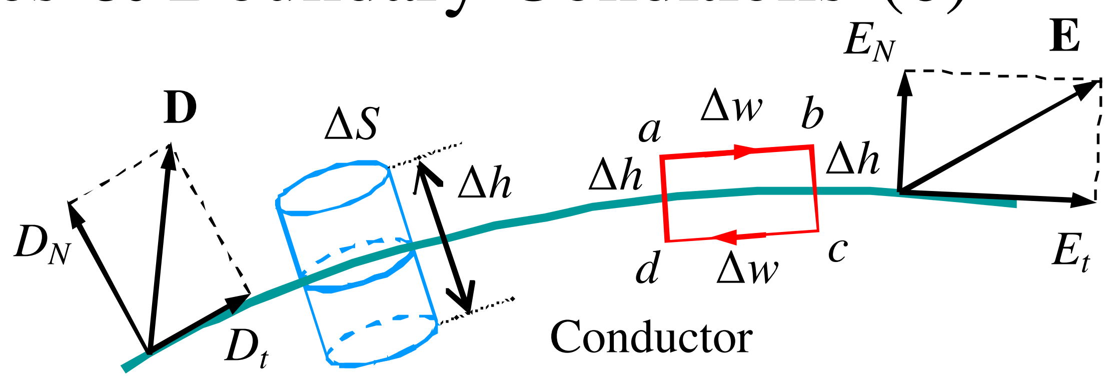
$$E_{tt} = 0$$

$$\boxed{\oint_S \mathbf{D}.d\mathbf{S} = Q}$$

$$\left.\begin{aligned}
&\rightarrow \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{sides}} = \rho_S \Delta S \\
&\int_{\text{top}} = \int_{\text{top}} \mathbf{D}_N . d\mathbf{S}_{\text{top}} = \int_{\text{top}} D_N dS_{\text{top}} = D_N \Delta S \\
&\int_{\text{bottom}} = \int_{\text{bottom}} 0.d\mathbf{S}_{\text{bottom}} = 0 \\
&\int_{\text{sides}} = \int_{\text{s, top}} \mathbf{D}_N .d\mathbf{S}_{\text{s, top}} + \int_{\text{s, bottom}} 0.d\mathbf{S}_{\text{s, bottom}} = 0
\end{aligned}\right\}$$

$$\rightarrow D_N \Delta S = \rho_S \Delta S \rightarrow \boxed{D_N = \rho_S = \varepsilon_0 E_N}$$

Conductor Properties & Boundary Conditions (6)



$$\boxed{D_t = E_t = 0}$$

$$\boxed{D_N = \varepsilon_0 E_N = \rho_S}$$

$$\boxed{V_{xy} = -\int_y^x \mathbf{E}.d\mathbf{L} = 0}$$

Characteristics of conductors in static field:

1. The static EFI inside a conductor is zero
2. The static EFI at the surface of a conductor is everywhere directed normal to that surface
3. The conductor surface is an equipotential surface

Conductor Properties & Boundary Conditions (7)

Ex.

Given $V = x^2 - 10yz$ V & $P(2, 1, 2)$ lies on a conductor – free space boundary. Find V, $\mathbf{E}$, $\mathbf{D}$, ρ_S at P, & the equation of the conductor surface.

$$V_P = 2^2 - 10\times1\times2 = -16 \text{ V} \qquad \to -16 = x^2 - 10yz$$

$$\mathbf{E} = -\nabla V = -\nabla(x^2 - 10yz) = -2x\mathbf{a}_x + 10z\mathbf{a}_y + 10y\mathbf{a}_z \text{ V/m}$$

$$\to \mathbf{E}_P = \left(-2x\mathbf{a}_x + 10z\mathbf{a}_y + 10y\mathbf{a}_z\right)\Big|_{x=2,\ y=1,\ z=2}$$

$$= -40\mathbf{a}_x + 20\mathbf{a}_y + 10\mathbf{a}_z \text{ V/m}$$

$$\mathbf{D}_P = \varepsilon_0\mathbf{E}_P = 8.854\times10^{-12}(-40\mathbf{a}_x + 20\mathbf{a}_y + 10\mathbf{a}_z) \text{ C/m}^2$$

$$\left.\begin{array}{l}\rho_{S,P} = D_N \\ D_{N,P} = |\mathbf{D}_P| = 8.854\times10^{-12}\sqrt{40^2 + 20^2 + 10^2} = 406 \text{ pC/m}^2\end{array}\right\}$$

$$\to \rho_{S,P} = 406 \text{ pC/m}^2$$

Current & Conductors

1. Current & Current Density
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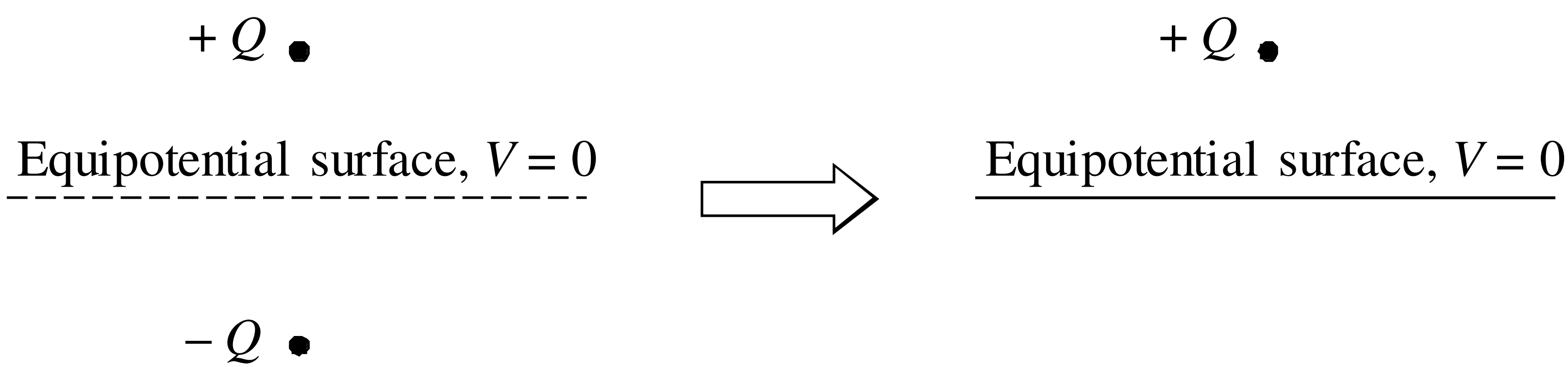
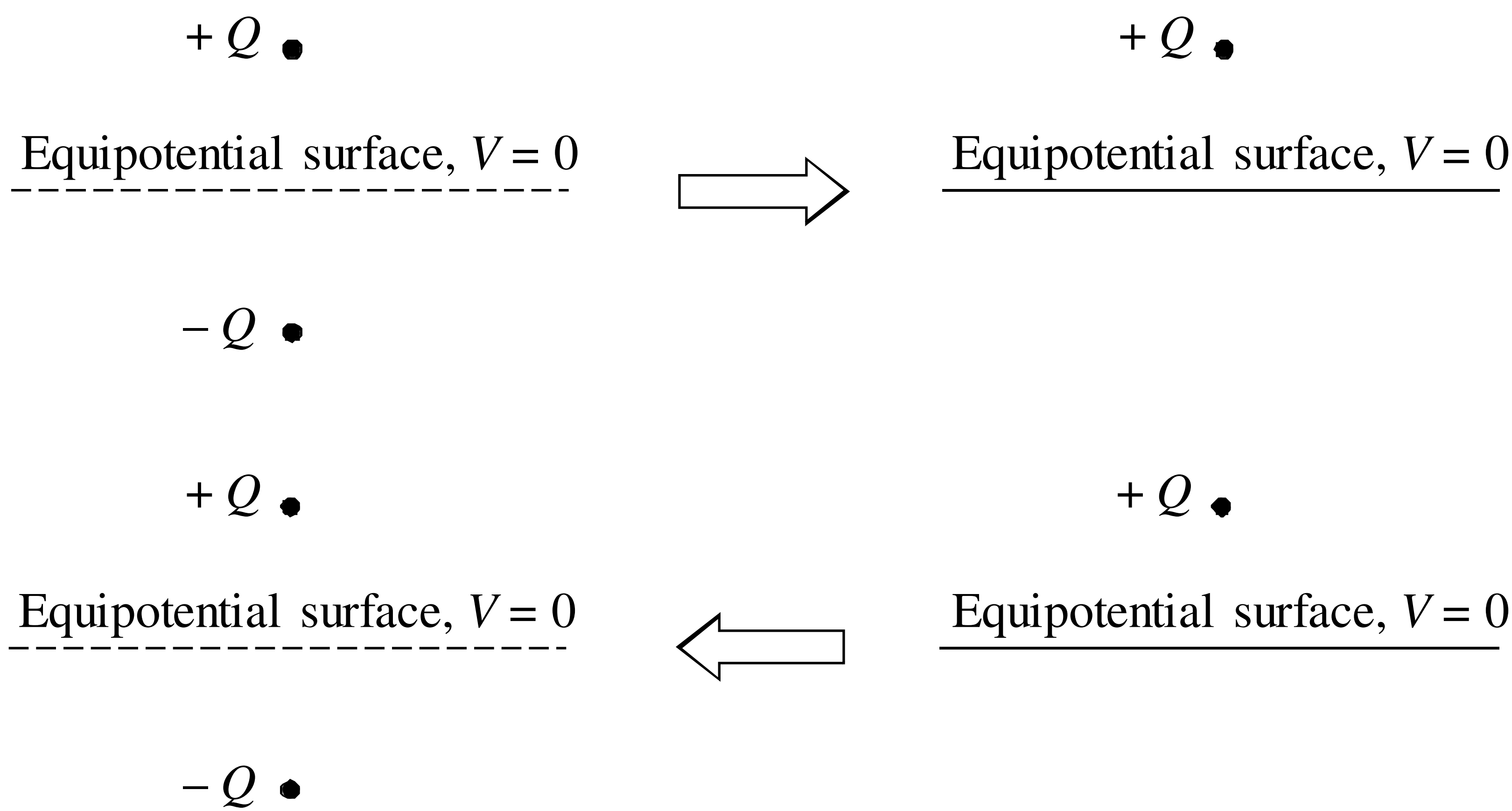
4. **The Method of Images**
5. Semiconductors
6. Applications

The Method of Images (1)



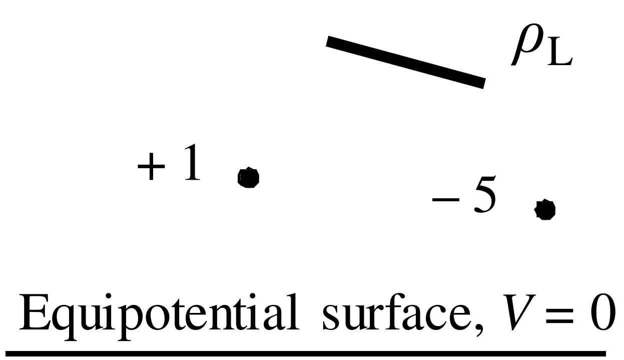
- Dipole: the plane between the 2 charges is zero potential
- That plane can be represented by a vanishingly thin conducting plane, infinite in extent
- $\rightarrow$ the dipole can be substituted for a system of a charge and a conducting plane, & then the fields above the conducting plane obtain equivalence

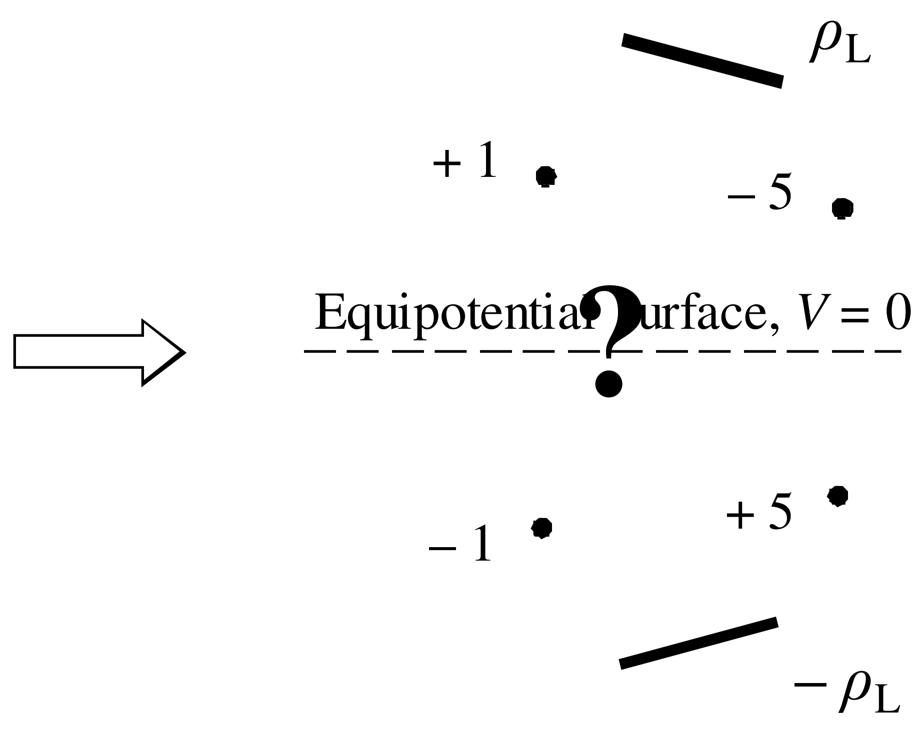
The Method of Images (2)



The Method of Images (3)

Ex. 1



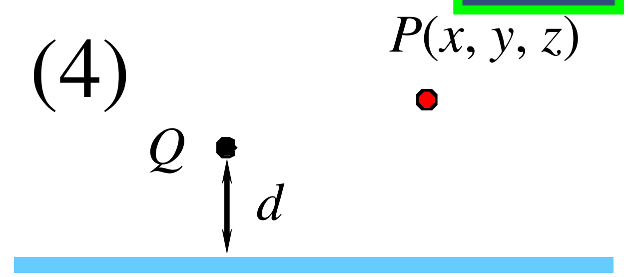


1. Coulomb' s law
2. Gauss' law
3. Laplace's equation

   & $\mathbf{E} = -\nabla V$

Ex. 2 The Method of Images (4)

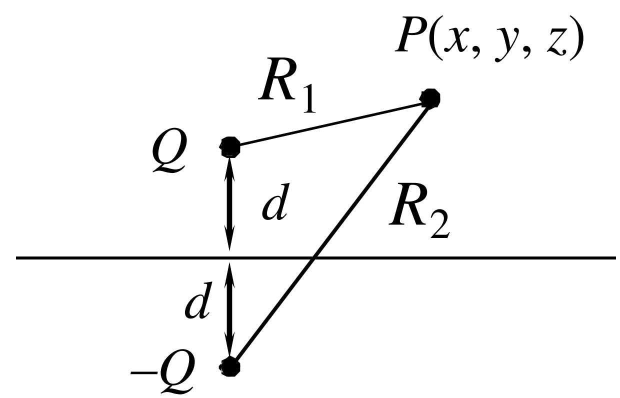
Given Q at $(0, 0, d)$. Find the potential & EFI at P ?



Equipotential surface, $V = 0$

$$V_{+Q} = \frac{Q}{4\pi\varepsilon_0 R_1} = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2 + y^2 + (z-d)^2}}$$

$$V_{-Q} = \frac{-Q}{4\pi\varepsilon_0 R_2} = \frac{-Q}{4\pi\varepsilon_0\sqrt{x^2 + y^2 + (z+d)^2}}$$



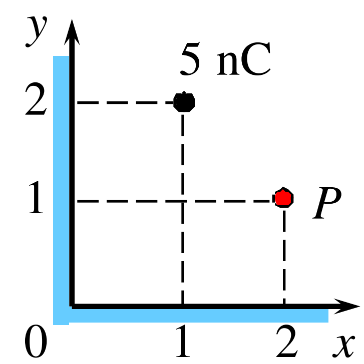
$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z+d)^2}}\right]$$

$$\mathbf{E} = -\nabla V = -\frac{Q}{4\pi\varepsilon_0}\left[\left(\frac{x}{R_2^3} - \frac{x}{R_1^3}\right)\mathbf{a}_x + \left(\frac{y}{R_2^3} - \frac{y}{R_1^3}\right)\mathbf{a}_y + \left(\frac{z+d}{R_2^3} - \frac{z-d}{R_1^3}\right)\mathbf{a}_z\right]$$

Ex. 3

The Method of Images (5)

Find the potential at P ?

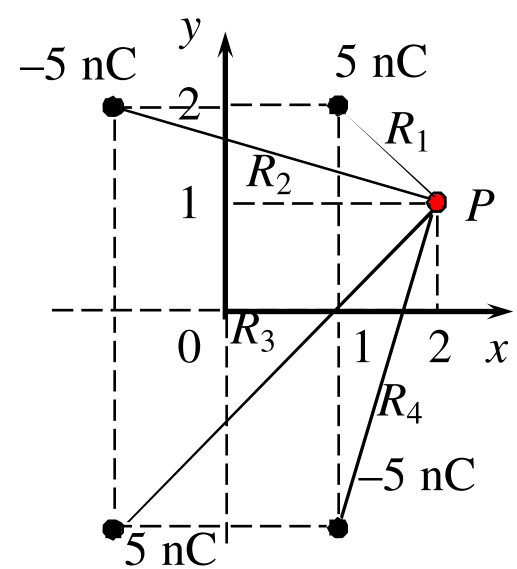


$$R_1 = \sqrt{1^2 + 1^2} = 1.41$$

$$R_2 = \sqrt{3^2 + 1^2} = 3.16$$

$$R_3 = \sqrt{3^2 + 3^2} = 4.24$$

$$R_4 = \sqrt{1^2 + 3^2} = 3.16$$

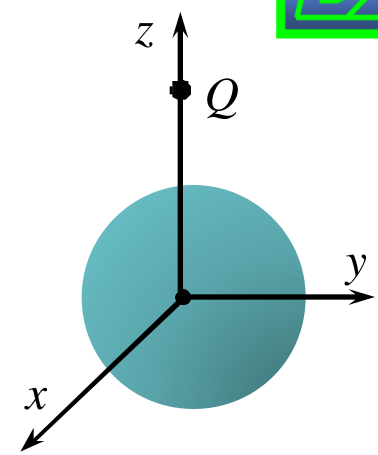


$$V_P = \frac{5\times10^{-9}}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}\right) = 14.03\ \text{V}$$

Ex. 4

The Method of Images (6)

A point charge Q at a distance d from a center of a grounded conducting sphere of radius a. Find the image charge?



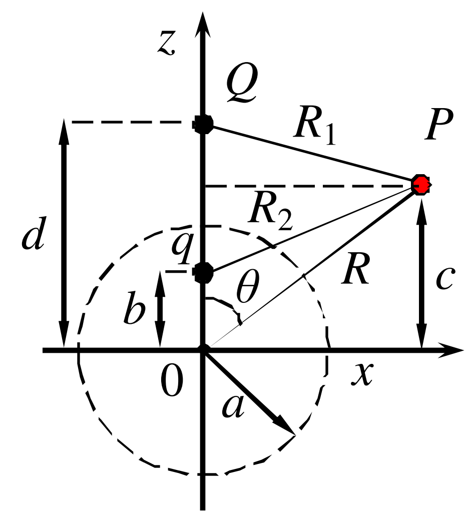
Problem: find q & b

$$R_1 = \sqrt{(d - R\cos\theta)^2 + (R\sin\theta)^2} = \sqrt{R^2 + d^2 - 2Rd\cos\theta}$$

$$R_2 = \sqrt{(R\cos\theta - b)^2 + (R\sin\theta)^2} = \sqrt{R^2 + b^2 - 2Rb\cos\theta}$$

$$V_P = \frac{Q}{4\pi\varepsilon R_1} - \frac{q}{4\pi\varepsilon R_2} = \frac{Q}{4\pi\varepsilon R_1} - \frac{mQ}{4\pi\varepsilon R_2} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{m}{R_2}\right)$$

$$R = a \rightarrow V_P = 0 \qquad \rightarrow \frac{1}{R_1} - \frac{m}{R_2} = 0 \qquad \rightarrow \begin{cases} m = \dfrac{a}{d} \rightarrow q = -\dfrac{a}{d}Q \\ b = \dfrac{a^2}{d} \end{cases}$$



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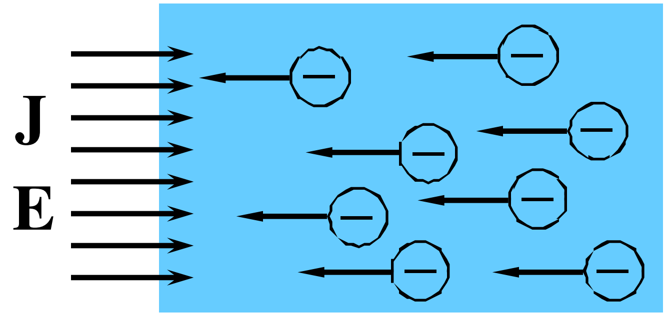
Semiconductors

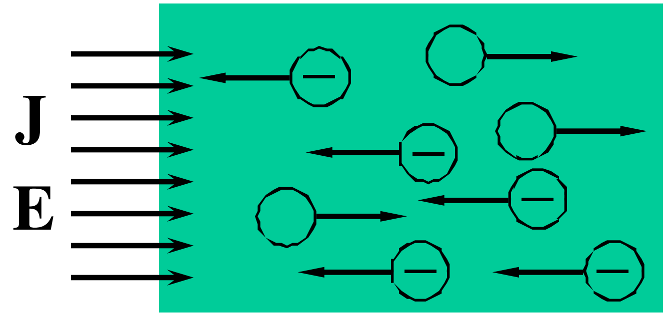


- Germani, silicon
- Conductivity of conductors:

$$\sigma = -\rho_e \mu_e$$

- Conductivity of semiconductors:

$$\sigma = -\rho_e \mu_e + \rho_h \mu_h$$



- *h*: hole
- At 300K:
  - $\mu_{e,\ \text{Germani}}$: 0.36 m^2/Vs; $\mu_{h,\ \text{Germani}}$: 0.17 m^2/Vs
  - $\mu_{e,\ \text{Silicon}}$: 0.12 m^2/Vs; $\mu_{h,\ \text{Silicon}}$: 0.025 m^2/Vs

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Applications – Faraday's Cage

https://lifeonthebluehighways.com/2013/04/20/faradays-cage/

$$Q \longrightarrow \mathbf{F} = \frac{Q_1 Q_2}{4\pi\varepsilon R^2}\mathbf{a}_R \longrightarrow \mathbf{E} = \frac{Q}{4\pi\varepsilon R^2}\mathbf{a}_R \longrightarrow \mathbf{D} = \varepsilon\mathbf{E}$$

$$\mathbf{F} \longrightarrow W = -Q\int \mathbf{E}.d\mathbf{L} \longrightarrow V = -\int \mathbf{E}.d\mathbf{L} \longrightarrow C = \frac{Q}{V}$$

$$Q \longrightarrow I = \frac{dQ}{dt} \longrightarrow R = \frac{V}{I}$$