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Electric Circuit Theory

Sinusoidal Steady-State Analysis

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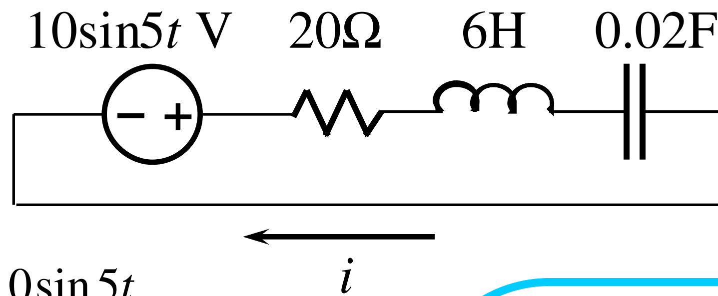
Sinusoidal Steady-State Analysis (1)

Ex. 1



$$20i + 6\frac{di}{dt} + \frac{1}{0.02}\int i\,dt = 10\sin 5t$$

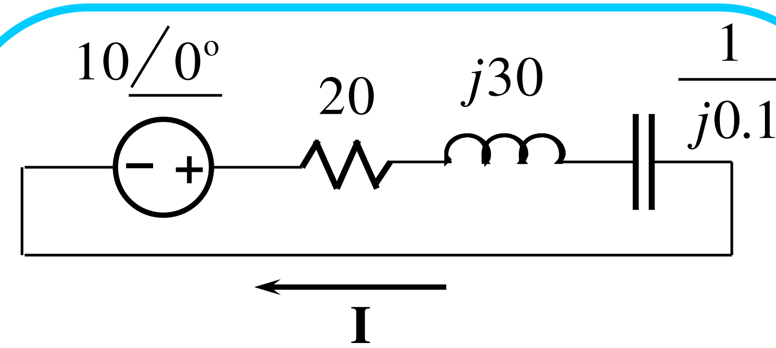
$$\left.\begin{aligned}\rightarrow 20\frac{di}{dt} + 6\frac{d^2 i}{dt^2} + \frac{i}{0.02} &= 50\cos 5t \\ i &= I_m \sin(5t+\phi)\end{aligned}\right\}$$

$$\rightarrow 100 I_m \cos(5t+\phi) - 150 I_m \sin(5t+\phi) + \\ + 50 I_m \sin(5t+\phi) = 50\cos 5t$$

$$\rightarrow 2\sqrt{2} I_m \sin(5t+\phi+135^{\circ}) = \sin(5t+90^{\circ})$$

$$\rightarrow \begin{cases} 2\sqrt{2} I_m = 1 \\ \phi + 135^{\circ} = 90^{\circ} \end{cases} \rightarrow \begin{cases} I_m = 0.35 \\ \phi = -45^{\circ} \end{cases}$$

$$\rightarrow \boxed{i = 0.35\sin(5t - 45^{\circ})\,\text{A}}$$

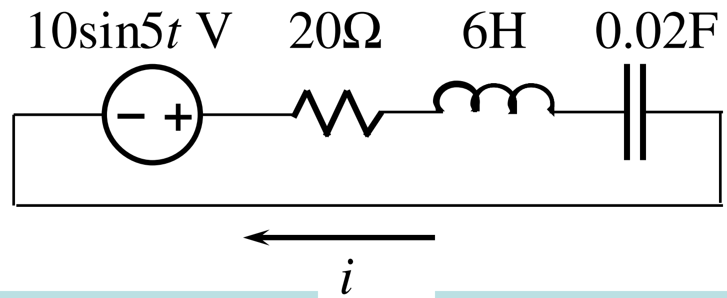


$$\mathbf{I} = \frac{10\angle 0^{\circ}}{20 + j30 + \frac{1}{j0.1}} = 0.35\angle -45^{\circ}\ \text{A}$$

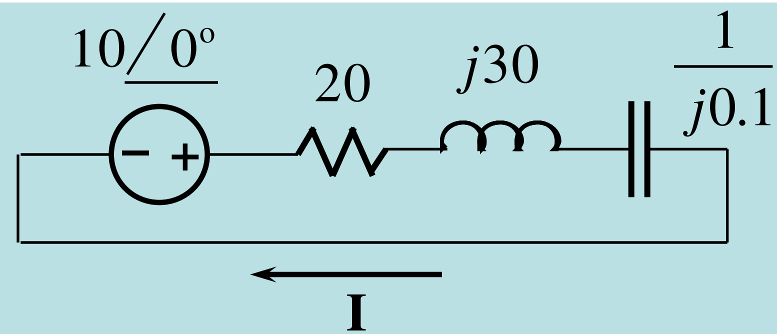
$$\rightarrow \boxed{i = 0.35\sin(5t - 45^{\circ})\,\text{A}}$$

Sinusoidal Steady-State Analysis (2)

Ex. 1



1. Transform to phasor domain
2. Solve the problem using dc circuit analysis
3. Transform the resulting phasor to the time-domain.



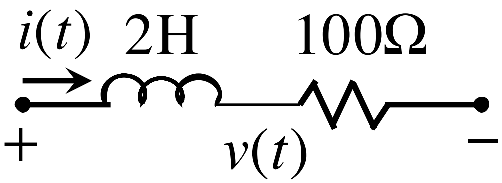
$$\mathbf{I} = \frac{10\angle 0^\circ}{20 + j30 + \dfrac{1}{j0.1}} = 0.35\angle -45^\circ \text{ A}$$

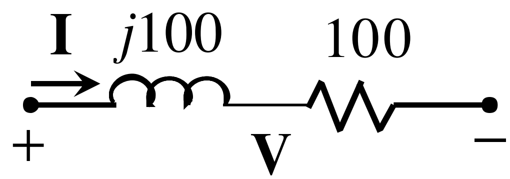
$$\rightarrow i = 0.35\sin(5t - 45^\circ)\text{ A}$$

Sinusoidal Steady-State Analysis (3)

Ex. 2

Given $i(t) = 2\sin(50t)$. Find $v(t)$?





$\mathbf{I} = 2\text{A}$

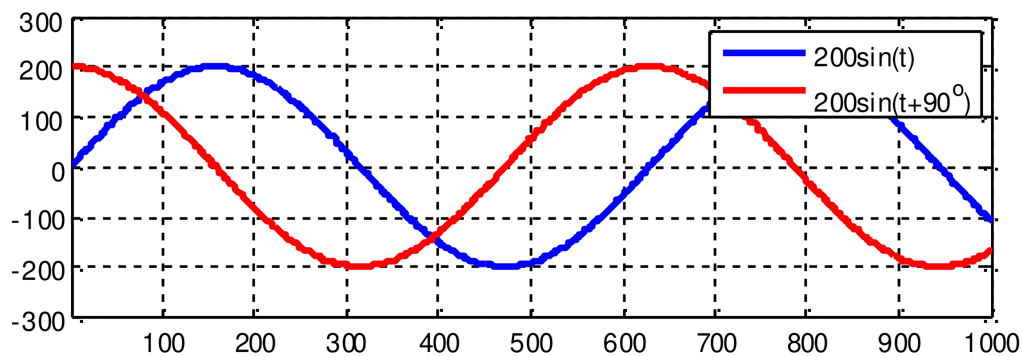
$\mathbf{V}_R = R\mathbf{I} = 100 \times 2 = 200 \text{ V}$

$\mathbf{V}_L = \mathbf{Z}_L \mathbf{I} = j100 \times 2 = j200 \text{ V}$

$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L = 200 + j200$

$= 200\sqrt{2}\,\underline{/\,45^\circ} \text{ V}$

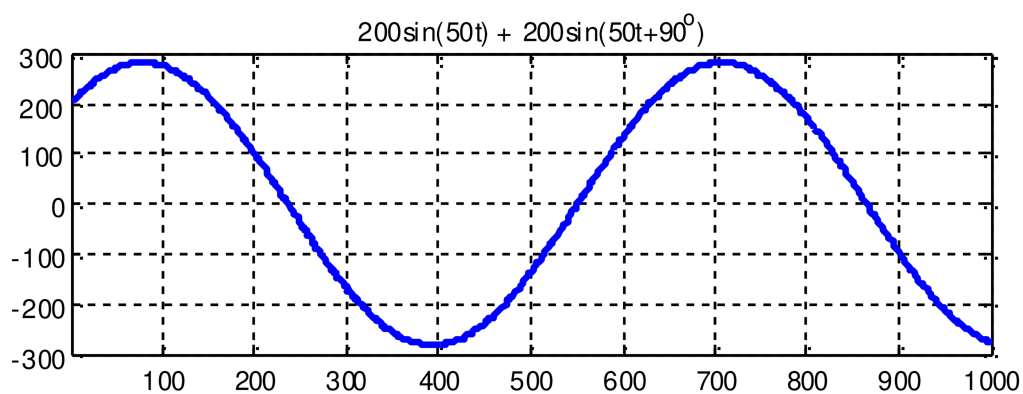
$\rightarrow v(t) = 200\sqrt{2}\sin(50t + 45^\circ) \text{ V}$



$V_R = 100 \times 2 = 200\text{V}$

$V_L = 100 \times 2 = 200\text{V}$

$V = 200 + 200 = 400\text{V}$



Sinusoidal Steady-State Analysis

1. Sinusoidal Steady-State Analysis
2. **Ohm's Law**
3. Kirchhoff's Laws
4. Impedance Combinations
5. Branch Current Method
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Ohm's Law (1)

$$\left.\begin{aligned}
\mathbf{V}_R &= R\mathbf{I} && \rightarrow \frac{\mathbf{V}_R}{\mathbf{I}} = R \\
\mathbf{V}_L &= j\omega L\mathbf{I} && \rightarrow \frac{\mathbf{V}_L}{\mathbf{I}} = j\omega L \\
\mathbf{V}_C &= \frac{\mathbf{I}}{j\omega C} && \rightarrow \frac{\mathbf{V}_C}{\mathbf{I}} = \frac{1}{j\omega C}
\end{aligned}\right\} \rightarrow \boxed{\frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}} \rightarrow \mathbf{V} = \mathbf{Z}\mathbf{I}$$

Z: impedance (Ω)

Admittance (S): $\mathbf{Y} = \dfrac{1}{\mathbf{Z}}$

Ohm's Law (2)

$$\boxed{\frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}}$$

$$\frac{\mathbf{V}_R}{\mathbf{I}} = R \quad \rightarrow \mathbf{Z}_R = R \qquad \mathbf{Y}_R = \frac{1}{R}$$

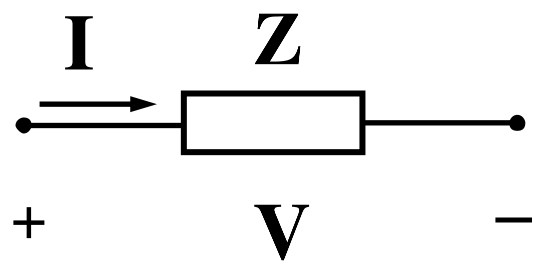
$$\frac{\mathbf{V}_L}{\mathbf{I}} = j\omega L \quad \rightarrow \mathbf{Z}_L = j\omega L \qquad \mathbf{Y}_L = \frac{1}{j\omega L} = \frac{-j}{\omega L}$$

$$\frac{\mathbf{V}_C}{\mathbf{I}} = \frac{1}{j\omega C} \rightarrow \mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} \qquad \mathbf{Y}_C = j\omega C$$

Ohm's Law (3)

	$\mathbf{Z}_L = j\omega L$	$\mathbf{Z}_C = \dfrac{-j}{\omega C}$
$\omega = 0$	$\mathbf{Z}_L = 0$ Short circuit	$\mathbf{Z}_C \to \infty$ Open circuit
$\omega \to \infty$	$\mathbf{Z}_L \to \infty$ Open circuit	$\mathbf{Z}_C = 0$ Short circuit

Ohm's Law (4)

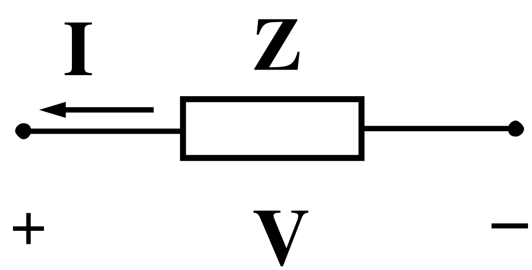


$$\mathbf{V} = \mathbf{ZI}$$

$$\mathbf{Z} = R + jX$$

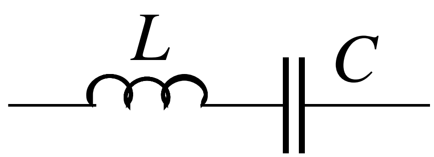
R: resistance

X: reactance

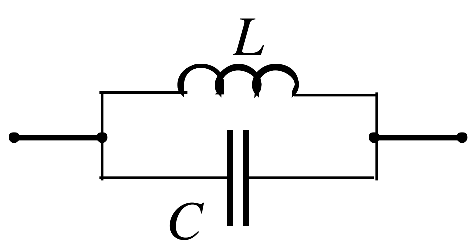


$$\mathbf{V} = -\mathbf{ZI}$$

Ohm's Law (5)



$$\left.\begin{array}{l} \mathbf{Z} = j\omega L + \dfrac{1}{j\omega C} \\ \text{If} \quad j\omega L + \dfrac{1}{j\omega C} = 0 \rightarrow \dfrac{-\omega^2 LC + 1}{j\omega C} = 0 \rightarrow \omega = \dfrac{1}{\sqrt{LC}} \end{array}\right\} \rightarrow \mathbf{Z} = 0$$



$$\left.\begin{array}{l} \mathbf{Z} = \dfrac{j\omega L \dfrac{1}{j\omega C}}{j\omega L + \dfrac{1}{j\omega C}} = \dfrac{L/C}{j\omega L + \dfrac{1}{j\omega C}} \\ \text{If} \quad j\omega L + \dfrac{1}{j\omega C} = 0 \rightarrow \dfrac{-\omega^2 LC + 1}{j\omega C} = 0 \rightarrow \omega = \dfrac{1}{\sqrt{LC}} \end{array}\right\} \rightarrow \mathbf{Z} = \infty$$

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Kirchhoff's Law (1)

$$v_1 + v_2 + ... + v_n = 0$$

$$\rightarrow V_{m1}\sin(\omega t + \phi_1) + V_{m2}\sin(\omega t + \phi_2) + ... + V_{mn}\sin(\omega t + \phi_n) = 0$$

$$\rightarrow \boxed{\mathbf{V}_1 + \mathbf{V}_2 + ... + \mathbf{V}_n = 0}$$

Kirchhoff's Law (2)

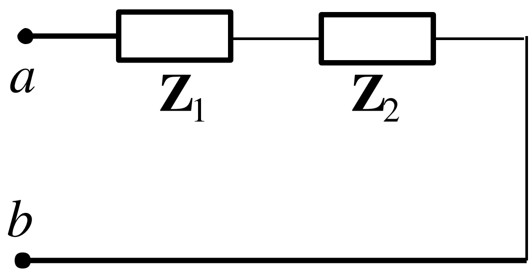
$$i_1 + i_2 + \ldots + i_n = 0$$

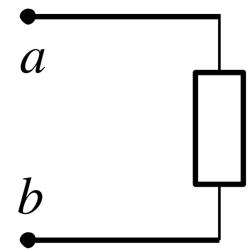
$$\rightarrow I_{m1}\sin(\omega t + \phi_1) + I_{m2}\sin(\omega t + \phi_2) + \ldots + I_{mn}\sin(\omega t + \phi_n) = 0$$

$$\rightarrow \boxed{\mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_n = 0}$$

Sinusoidal Steady-State Analysis

Impedance Combinations (1)



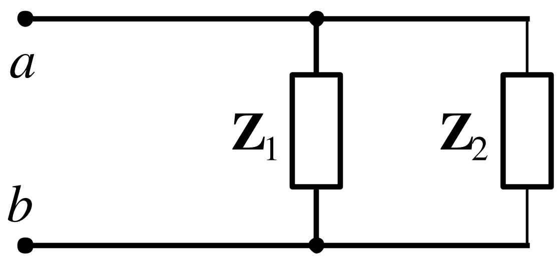


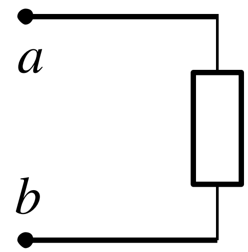
$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\boxed{\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_{ab}}$$

$$\boxed{\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \ldots + \mathbf{Z}_n}$$

Impedance Combinations (2)





$$\mathbf{Z}_{eq} = \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

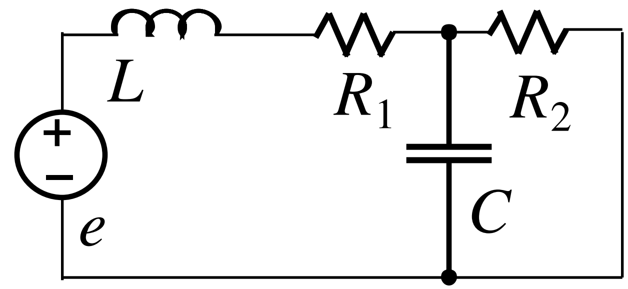
$$\boxed{\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{I}_{ab}}$$

$$\boxed{\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \ldots + \frac{1}{\mathbf{Z}_n}}$$

Impedance Combinations (3)

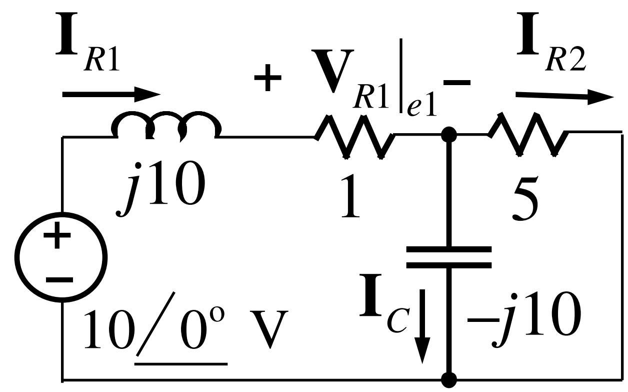
Ex. 1

$e = 10\sin 10t$ V; $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; find i_C, i_{R2}, i_{R1}, & v_{R1}?



$$\mathbf{Z}_{R2,C} = \frac{5(-j10)}{5-j10} = 4 - j2\,\Omega$$

$$\mathbf{Z}_t = j10 + 1 + \mathbf{Z}_{R2,C} = j10 + 1 + 4 - j2 = 5 + j8\,\Omega$$



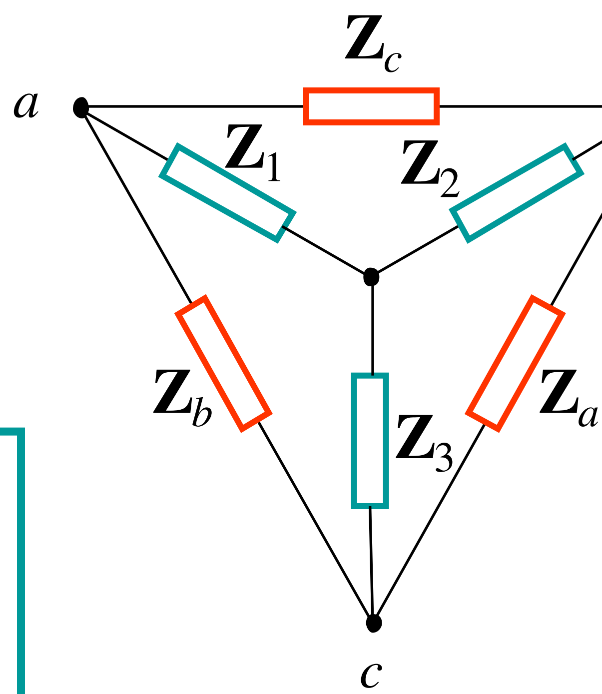
$$\mathbf{I}_{R1} = \frac{\mathbf{E}}{\mathbf{Z}_t} = \frac{10}{5+j8} = 1.06\angle -58^\circ \text{ A}$$

$$\mathbf{V}_{R1} = R_1\mathbf{I}_{R1} = 1\times 1.06\angle -58^\circ = 1.06\angle -58^\circ \text{ V}$$

$$\mathbf{I}_{R2} = \frac{\mathbf{I}_{R1}(-j10)}{5-j10} = \frac{(1.06\angle -58^\circ)(-j10)}{5-j10} = 0.95\angle -84.6^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}_{R1} - \mathbf{I}_{R2} = 1.06\angle -58^\circ - 0.95\angle -84.6^\circ = 0.47\angle 5.4^\circ \text{ A}$$

Impedance Combinations (4)



$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

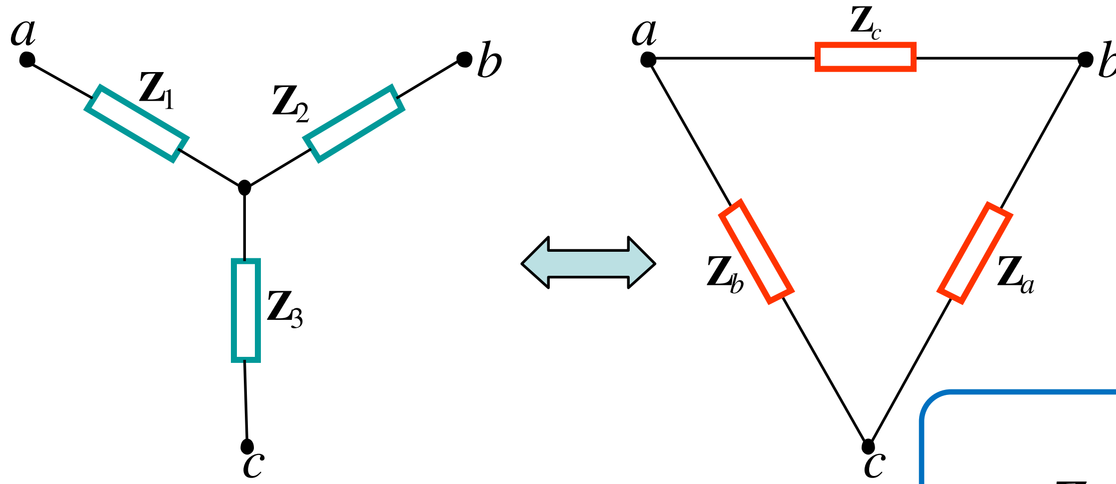
$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

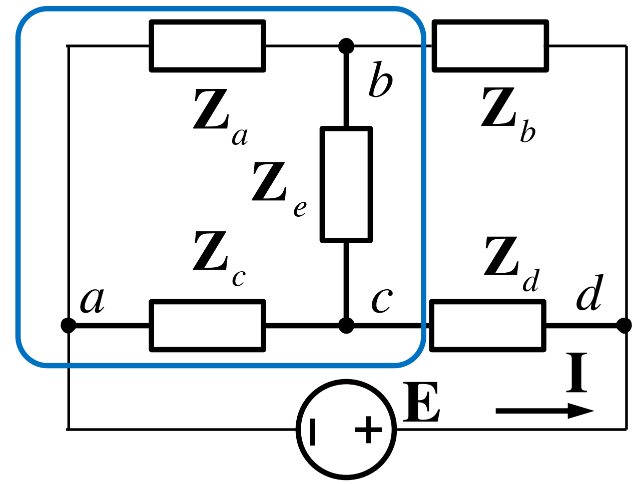
$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

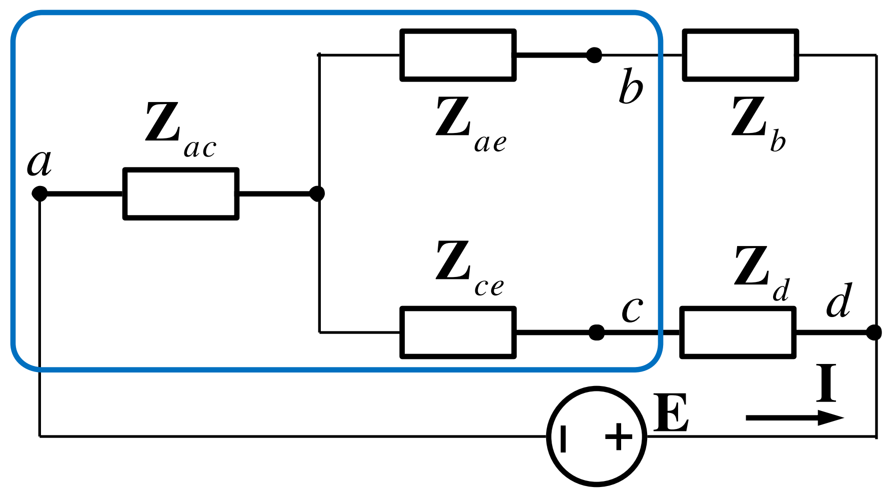
Impedance Combination (5)

Ex. 2

$e = 100\sin 20t$ V; $\mathbf{Z}_a = 20\ \Omega$; $\mathbf{Z}_b = 5 - j10\ \Omega$;
$\mathbf{Z}_c = j25\ \Omega$; $\mathbf{Z}_d = 15 + j15\ \Omega$; $\mathbf{Z}_e = -j8\ \Omega$. find i?



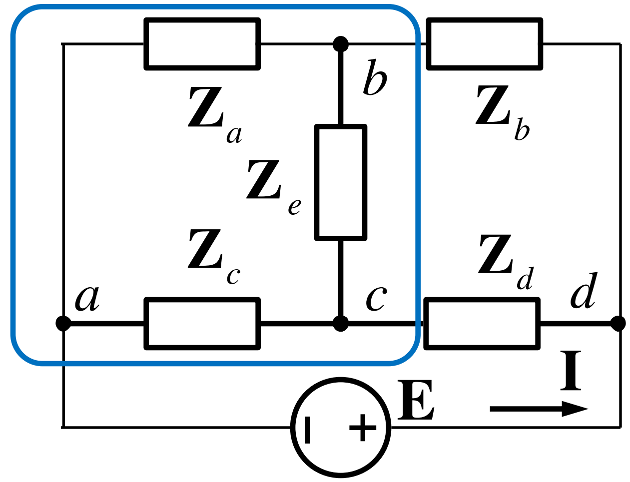




Impedance Combination (6)

Ex. 2

$e = 100\sin 20t$ V; $\mathbf{Z}_a = 20\ \Omega$; $\mathbf{Z}_b = 5 - j10\ \Omega$;
$\mathbf{Z}_c = j25\ \Omega$; $\mathbf{Z}_d = 15 + j15\ \Omega$; $\mathbf{Z}_e = -j8\ \Omega$. find i?

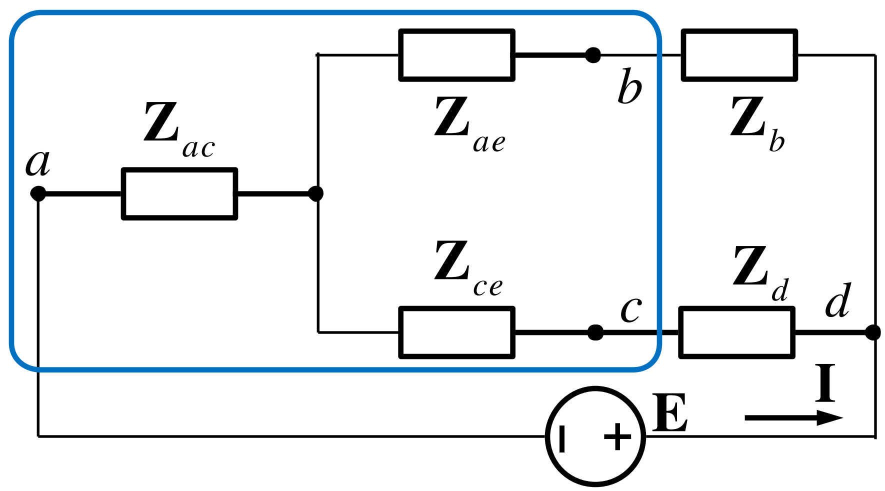


$$\mathbf{Z}_{ac} = \frac{\mathbf{Z}_a\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_c + \mathbf{Z}_e} = \frac{20(j25)}{20 + j25 - j8} = 12.33 + j14.51\ \Omega$$

$$\mathbf{Z}_{ae} = \frac{\mathbf{Z}_a\mathbf{Z}_e}{\mathbf{Z}_a + \mathbf{Z}_c + \mathbf{Z}_e} = \frac{20(-j8)}{20 + j25 - j8} = -3.95 - j4.64\ \Omega$$

$$\mathbf{Z}_{ce} = \frac{\mathbf{Z}_c\mathbf{Z}_e}{\mathbf{Z}_a + \mathbf{Z}_c + \mathbf{Z}_e} = \frac{j25(-j8)}{20 + j25 - j8}$$

$$= 5.81 - j4.93\ \Omega$$



$$\mathbf{Z}_t = \mathbf{Z}_{ac} + [(\mathbf{Z}_{ae} + \mathbf{Z}_b) // (\mathbf{Z}_{ce} + \mathbf{Z}_d)]$$

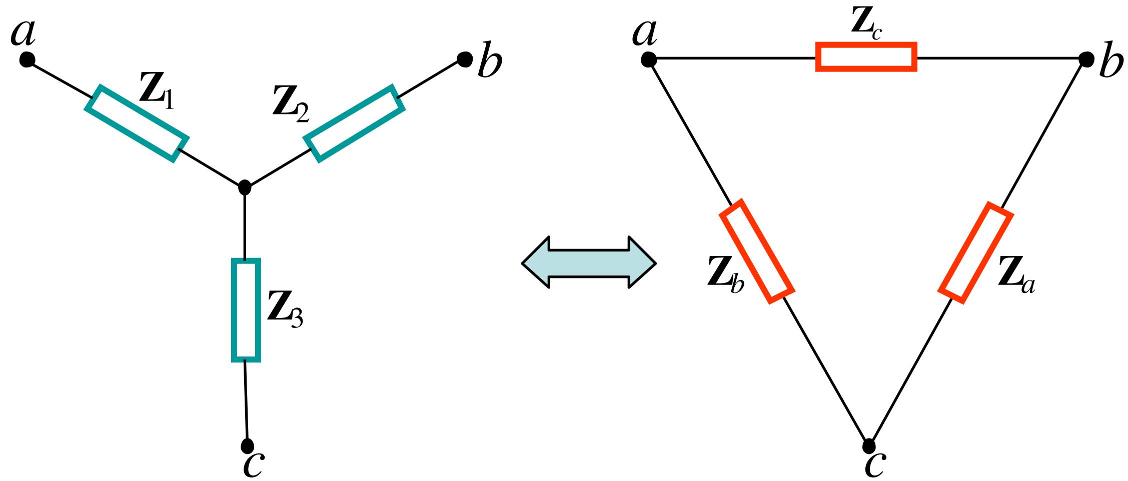
$$= 22.46 + j3.18\ \Omega$$

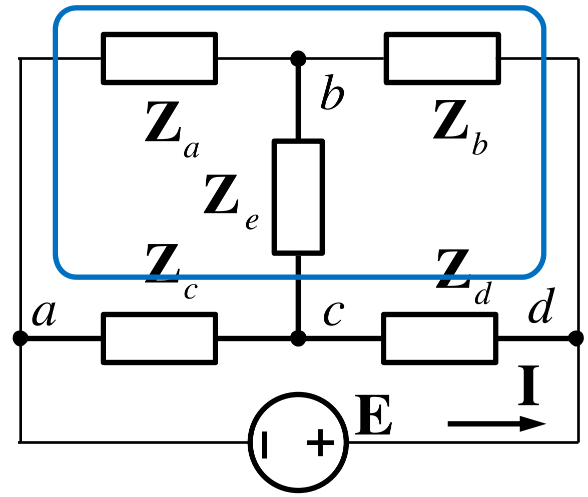
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_t} = \frac{100}{22.46 + j3.18} = \boxed{4.37 - j0.62\ \text{A}}$$

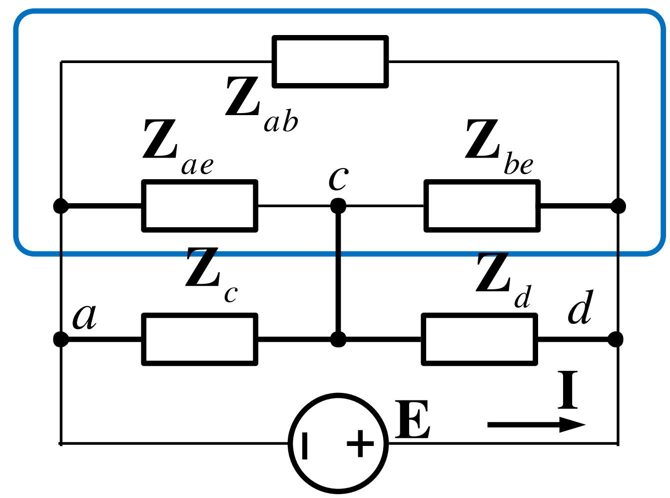
Impedance Combination (7)

Ex. 2

$e = 100\sin 20t$ V; $\mathbf{Z}_a = 20\ \Omega$; $\mathbf{Z}_b = 5 - j10\ \Omega$;
$\mathbf{Z}_c = j25\ \Omega$; $\mathbf{Z}_d = 15 + j15\ \Omega$; $\mathbf{Z}_e = -j8\ \Omega$. find i?



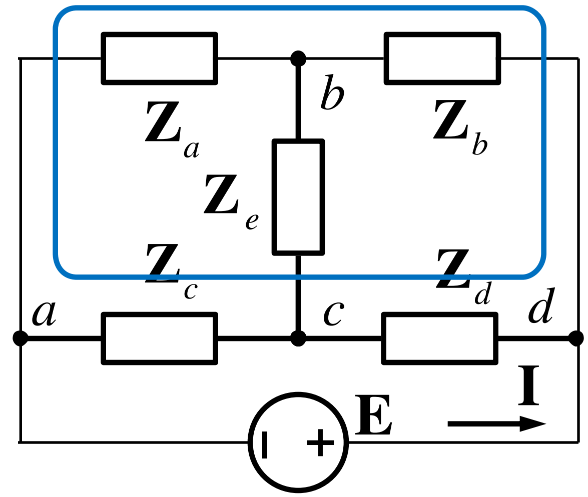




Impedance Combination (8)

Ex. 2

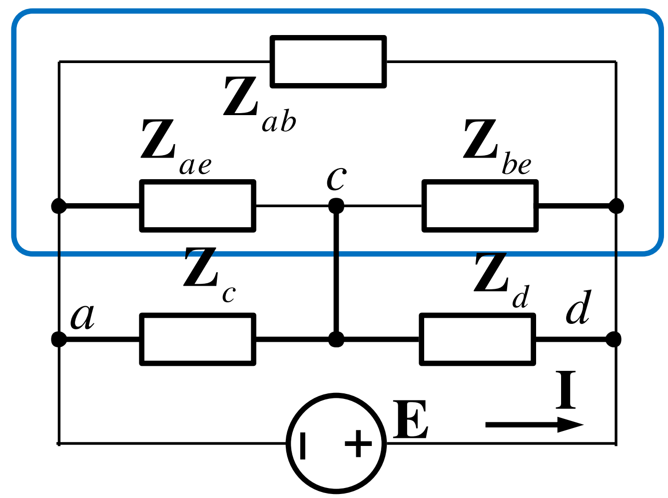
$e = 100\sin 20t$ V; $\mathbf{Z}_a = 20\ \Omega$; $\mathbf{Z}_b = 5 - j10\ \Omega$; $\mathbf{Z}_c = j25\ \Omega$; $\mathbf{Z}_d = 15 + j15\ \Omega$; $\mathbf{Z}_e = -j8\ \Omega$. find i?



$$\mathbf{Z}_{ab} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_e + \mathbf{Z}_e\mathbf{Z}_a}{\mathbf{Z}_e} = 50 + j2.50\,\Omega$$

$$\mathbf{Z}_{ae} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_e + \mathbf{Z}_e\mathbf{Z}_a}{\mathbf{Z}_b} = 32.80 - j14.40\ \Omega$$

$$\mathbf{Z}_{be} = \frac{\mathbf{Z}_a\mathbf{Z}_b + \mathbf{Z}_b\mathbf{Z}_e + \mathbf{Z}_e\mathbf{Z}_a}{\mathbf{Z}_a} = 1.00 - j20\ \Omega$$



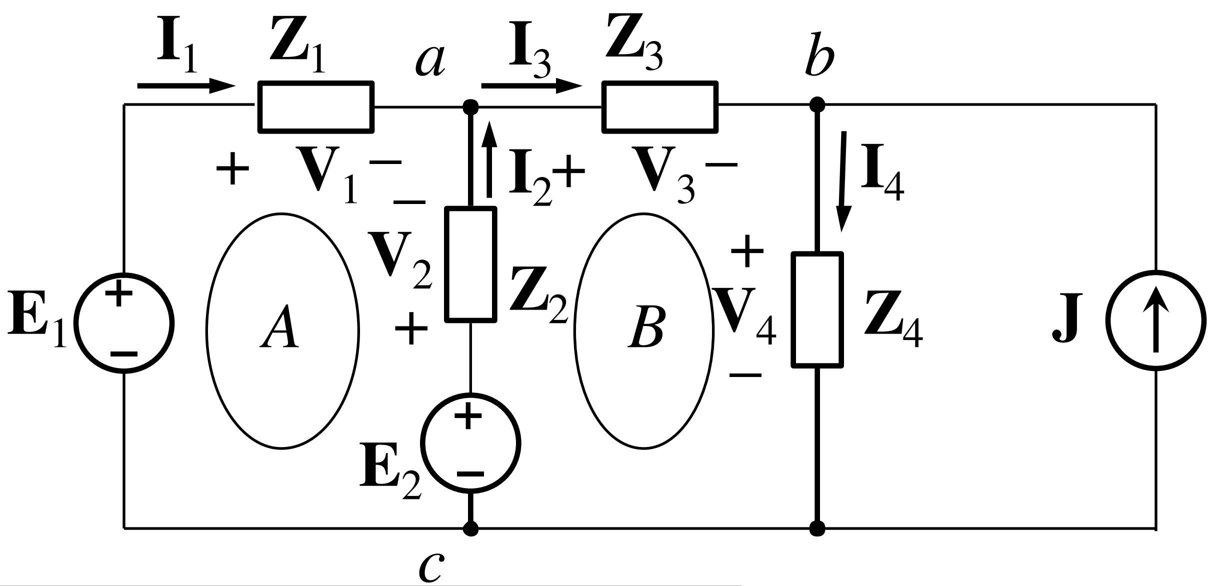
$$\mathbf{Z}_t = \mathbf{Z}_{ab} // [(\mathbf{Z}_{ae} // \mathbf{Z}_c) + (\mathbf{Z}_{be} + \mathbf{Z}_d)]$$
$$= 22.46 + j3.18\,\Omega$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_t} = \frac{100}{22.46 + j3.18} = \boxed{4.37 - j0.62\ \text{A}}$$

Sinusoidal Steady-State Analysis

Branch Current Method (1)

Ex. 1



1. Find:

   $n_{KCL} = n - 1$, and

   $n_{KVL} = b - n + 1$

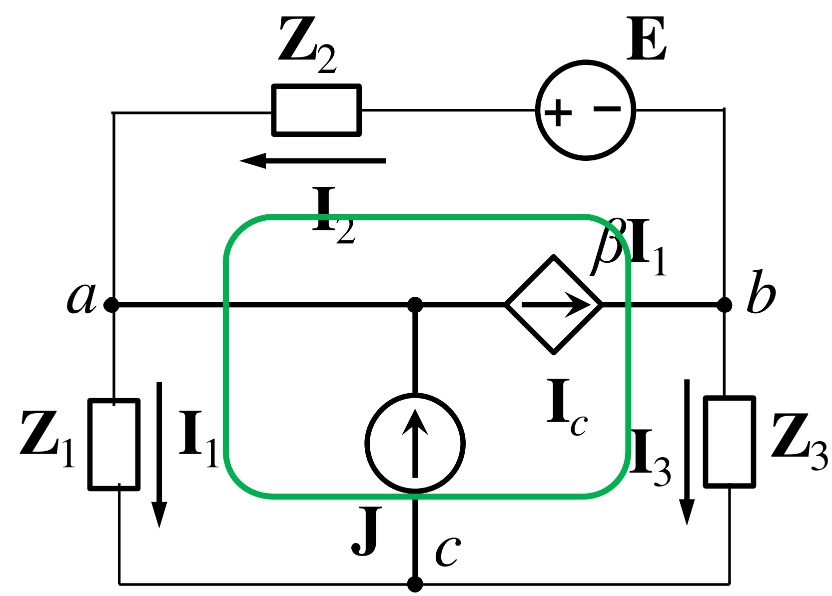
2. Apply KCL at n_{KCL} nodes
3. Apply KVL at n_{KVL} loops
4. Solve simultaneous equations

$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_3 - \mathbf{I}_4 = -\mathbf{J} \\ \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases}$$

Branch Current Method (2)

Ex. 2

$$\left.\begin{aligned}&\begin{cases} b: \mathbf{I}_c - \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ c: \mathbf{I}_1 + \mathbf{I}_3 - \mathbf{J} = 0 \\ A: \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_2\mathbf{I}_2 - \mathbf{E} = 0 \end{cases} \\ &\mathbf{I}_c = \beta\mathbf{I}_1 \end{aligned}\right\}$$



$$\rightarrow \begin{cases} \beta\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_1 + \mathbf{I}_3 - \mathbf{J} = 0 \\ \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_2\mathbf{I}_2 - \mathbf{E} = 0 \end{cases}$$

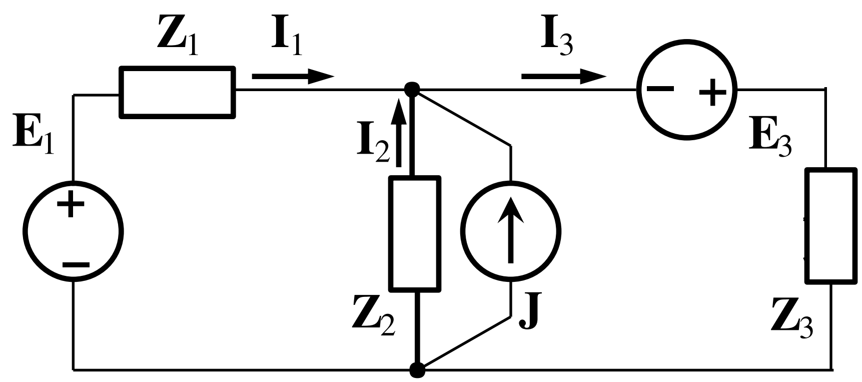
Branch Current Method (3)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\underline{/15^\circ}\ \text{V};\ \mathbf{J} = 2\underline{/-30^\circ}\ \text{A};$

Find currents?



$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 + \mathbf{J} = 0 \\ \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 \\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_3\mathbf{I}_3 = \mathbf{E}_3 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = -2\underline{/-30^\circ} \\ 10\mathbf{I}_1 - j20\mathbf{I}_2 = 30 \\ j20\mathbf{I}_2 + (5 - j10)\mathbf{I}_3 = 45\underline{/15^\circ} \end{cases}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta};\quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta};\quad \mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

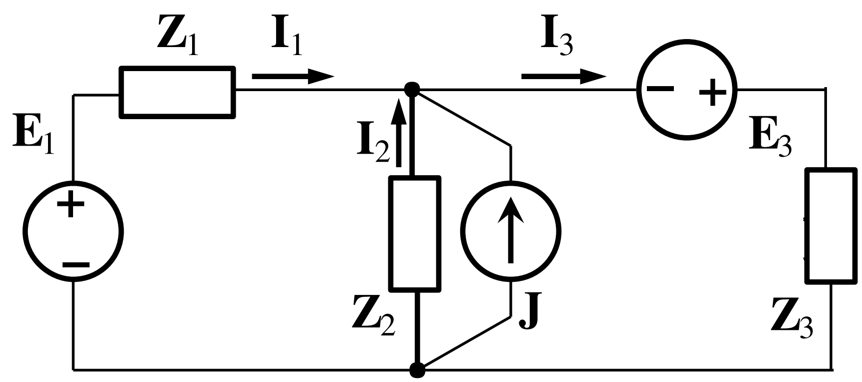
Branch Current Method (4)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\angle 15^\circ\ \text{V};\ \mathbf{J} = 2\angle -30^\circ\ \text{A};$

Find currents?



$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = -2\angle -30^\circ \\ 10\mathbf{I}_1 - j20\mathbf{I}_2 = 30 \\ j20\mathbf{I}_2 + (5 - j10)\mathbf{I}_3 = 45\angle 15^\circ \end{cases}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta};\quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta};\quad \mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 10 & -j20 & 0 \\ 0 & j20 & 5 - j10 \end{vmatrix} = 1\begin{vmatrix} -j20 & 0 \\ j20 & 5 - j10 \end{vmatrix} - 10\begin{vmatrix} 1 & -1 \\ j20 & 5 - j10 \end{vmatrix} + 0\begin{vmatrix} 1 & -1 \\ -j20 & 0 \end{vmatrix}$$

$$= -250 - j200$$

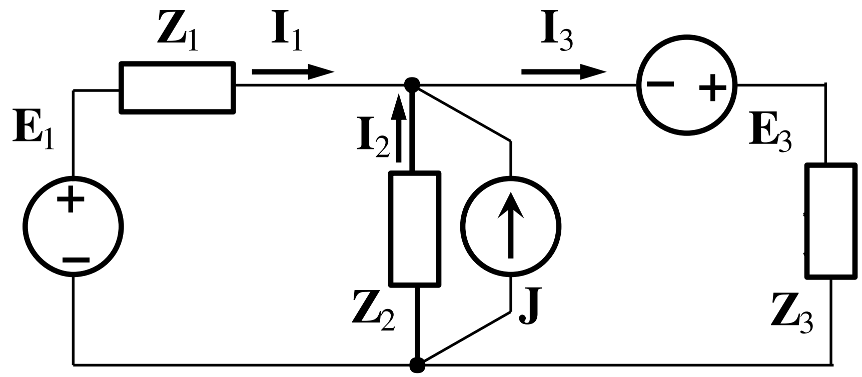
Branch Current Method (5)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\underline{/15^\circ}\ \text{V};\ \mathbf{J} = 2\underline{/-30^\circ}\ \text{A};$

Find currents?



$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = -2\underline{/-30^\circ} \\ 10\mathbf{I}_1 - j20\mathbf{I}_2 = 30 \\ j20\mathbf{I}_2 + (5 - j10)\mathbf{I}_3 = 45\underline{/15^\circ} \end{cases}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta};\quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta};\quad \mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} -2\underline{/-30^\circ} & 1 & -1 \\ 30 & -j20 & 0 \\ 45\underline{/15^\circ} & j20 & 5 - j10 \end{vmatrix}}{-250 - j200} = 1.04 + j3.95 = 4.09\underline{/75.2^\circ}\ \text{A}$$

$$\rightarrow i_1 = 4.09\sin(\omega t + 75.2^\circ)\ \text{A}$$

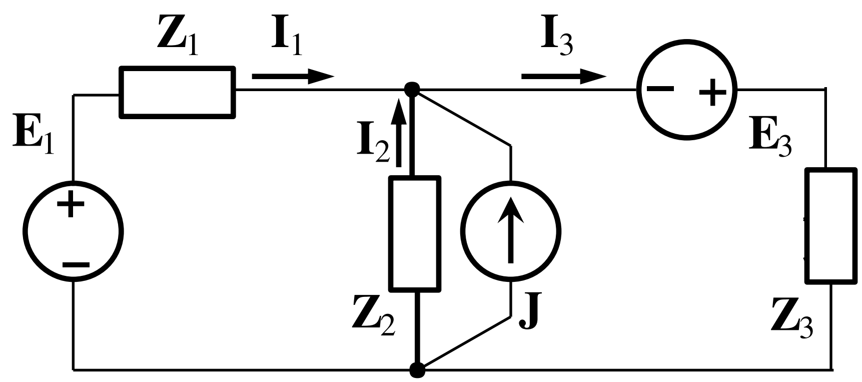
Branch Current Method (6)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\angle 15^\circ\ \text{V};\ \mathbf{J} = 2\angle -30^\circ\ \text{A};$

Find currents?



$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = -2\angle -30^\circ \\ 10\mathbf{I}_1 - j20\mathbf{I}_2 = 30 \\ j20\mathbf{I}_2 + (5 - j10)\mathbf{I}_3 = 45\angle 15^\circ \end{cases}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta};\quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta};\quad \mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 1 & -2\angle -30^\circ & -1 \\ 10 & 30 & 0 \\ 0 & 45\angle 15^\circ & 5 - j10 \end{vmatrix}}{-250 - j200} = 1.98 + j0.98 = 2.20\angle 26.4^\circ\ \text{A}$$

$$\rightarrow i_2 = 2.20\sin(\omega t + 26.4^\circ)\ \text{A}$$

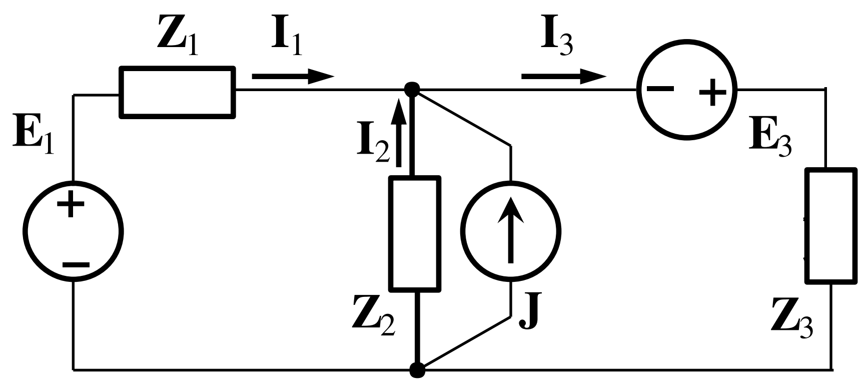
Branch Current Method (7)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\angle 15^\circ\ \text{V};\ \mathbf{J} = 2\angle -30^\circ\ \text{A};$

Find currents?



$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = -2\angle -30^\circ \\ 10\mathbf{I}_1 - j20\mathbf{I}_2 = 30 \\ j20\mathbf{I}_2 + (5 - j10)\mathbf{I}_3 = 45\angle 15^\circ \end{cases}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta};\quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta};\quad \mathbf{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 1 & 1 & -2\angle -30^\circ \\ 10 & -j20 & 30 \\ 0 & j20 & 45\angle 15^\circ \end{vmatrix}}{-250 - j200} = 4.75 + j3.93 = 6.16\angle 39.6^\circ\ \text{A}$$

$$\rightarrow i_3 = 6.16\sin(\omega t + 39.6^\circ)\ \text{A}$$

Sinusoidal Steady-State Analysis

1. Sinusoidal Steady-State Analysis
2. Ohm's Law
3. Kirchhoff's Laws
4. Impedance Combinations
5. Branch Current Method
6. **Node Voltage Method**
7. Mesh Current Method
8. Superposition Theorem
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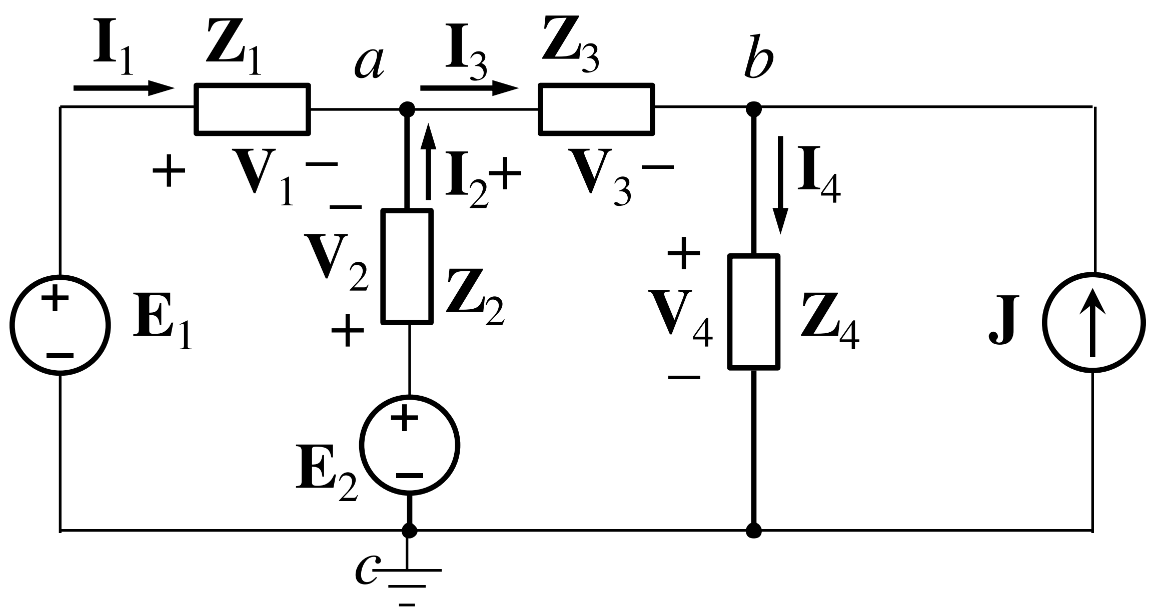
11. Op Amp AC Circuits

Node Voltage Method (1)

Ex. 1

1. the reference node
2. the sum of the reciprocals of all impedances connected to each node
3. the negative sum of the reciprocals of the impedances of all branches joining each pair of node
4. current source(s) for each node
5. node voltage equations
6. node voltages
7. branch currents

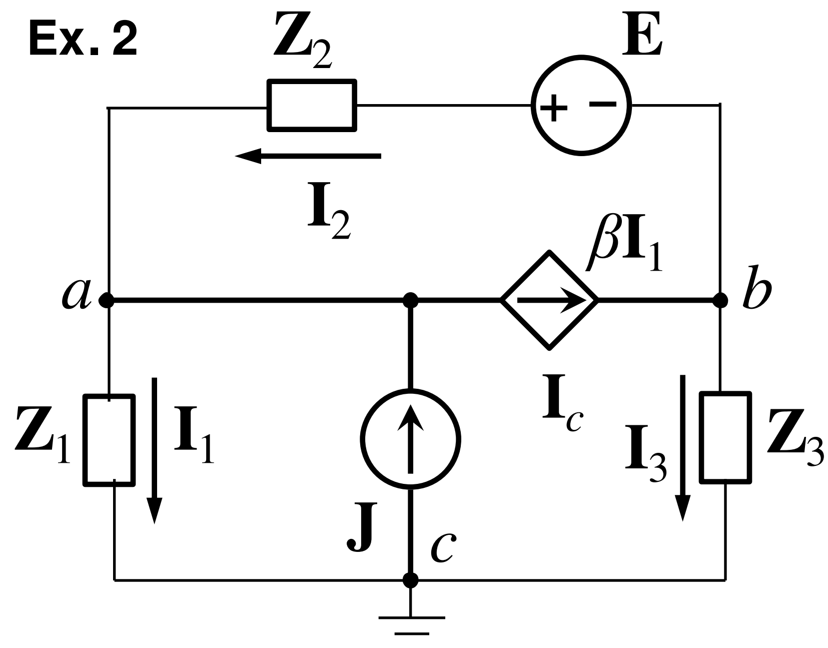


$$\begin{cases} \left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}+\dfrac{1}{Z_3}\right)V_a - \left(\dfrac{1}{Z_3}\right)V_b = \dfrac{E_1}{Z_1}+\dfrac{E_2}{Z_2} \\ -\left(\dfrac{1}{Z_3}\right)V_a + \left(\dfrac{1}{Z_3}+\dfrac{1}{Z_4}\right)V_b = J \end{cases}$$

Node Voltage Method (2)

$$\left.\begin{cases} \left(\dfrac{1}{\mathbf{Z}_1}+\dfrac{1}{\mathbf{Z}_2}\right)\mathbf{V}_a - \dfrac{1}{\mathbf{Z}_2}\mathbf{V}_b = \mathbf{J} - \mathbf{I}_c + \dfrac{\mathbf{E}}{\mathbf{Z}_2} \\ -\dfrac{1}{\mathbf{Z}_2}\mathbf{V}_a + \left(\dfrac{1}{\mathbf{Z}_2}+\dfrac{1}{\mathbf{Z}_3}\right)\mathbf{V}_b = \mathbf{I}_c - \dfrac{\mathbf{E}}{\mathbf{Z}_2} \end{cases} \\ \mathbf{I}_c = \beta\mathbf{I}_1 = \beta\dfrac{\mathbf{V}_a}{\mathbf{Z}_1} \right\}$$

Ex. 2



$$\rightarrow \begin{cases} \left(\dfrac{1}{\mathbf{Z}_1}+\dfrac{1}{\mathbf{Z}_2}+\beta\dfrac{1}{\mathbf{Z}_1}\right)\mathbf{V}_a - \dfrac{1}{\mathbf{Z}_2}\mathbf{V}_b = \mathbf{J} + \dfrac{\mathbf{E}}{\mathbf{Z}_2} \\ -\left(\dfrac{1}{\mathbf{Z}_2}+\beta\dfrac{1}{\mathbf{Z}_1}\right)\mathbf{V}_a + \left(\dfrac{1}{\mathbf{Z}_2}+\dfrac{1}{\mathbf{Z}_3}\right)\mathbf{V}_b = -\dfrac{\mathbf{E}}{\mathbf{Z}_2} \end{cases}$$

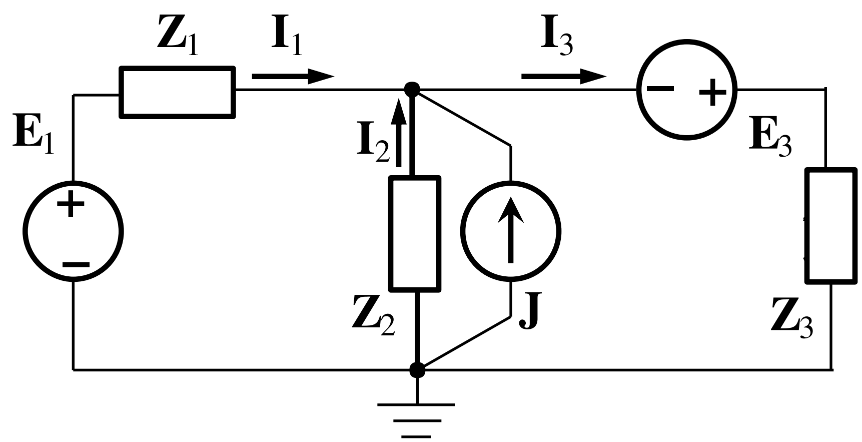
Node Voltage Method (3)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\underline{/15^\circ}\ \text{V};\ \mathbf{J} = 2\underline{/-30^\circ}\ \text{A};$

Find currents?



$$\left(\frac{1}{10} + \frac{1}{j20} + \frac{1}{5 - j10}\right)\mathbf{V}_a = \frac{30}{10} + 2\underline{/-30^\circ} - \frac{45\underline{/15^\circ}}{5 - j10}$$

$$\rightarrow \mathbf{V}_a = 19.57 - j39.50\ \text{V}$$

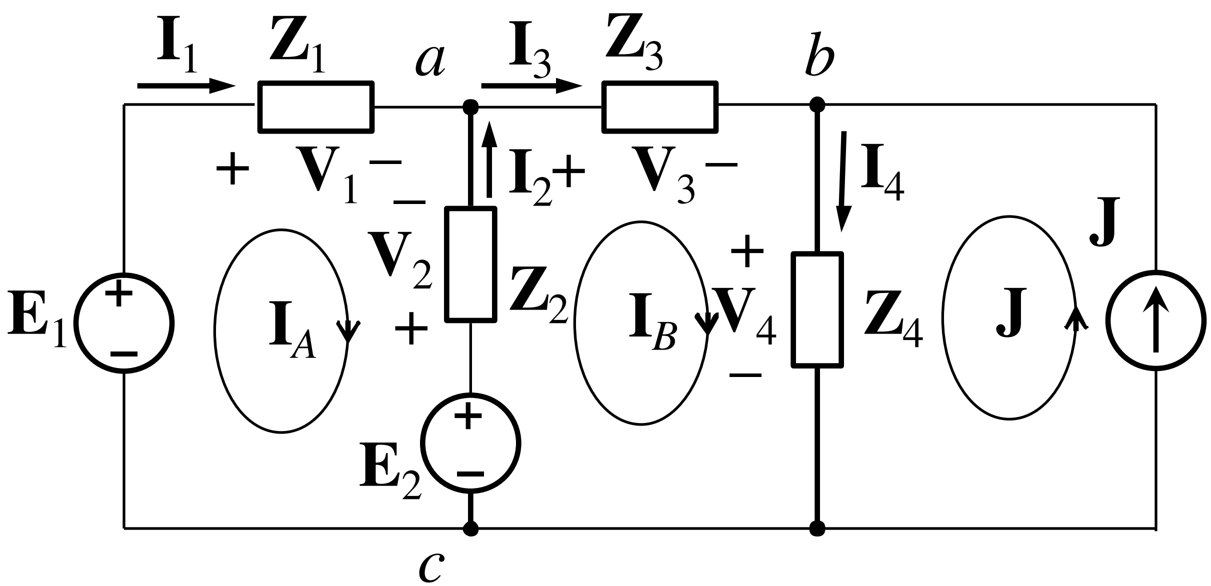
$$\rightarrow \begin{cases} \mathbf{I}_1 = \dfrac{30 - (19.57 - j39.50)}{10} = 1.04 + j3.95 = 4.09\underline{/75.2^\circ}\ \text{A} \\ \mathbf{I}_2 = \dfrac{-(19.57 - j39.50)}{j20} = 1.98 + j0.98 = 2.20\underline{/26.4^\circ}\ \text{A} \\ \mathbf{I}_3 = \dfrac{45\underline{/15^\circ} + (19.57 - j39.50)}{5 - j10} = 4.75 + j3.93 = 6.16\underline{/39.6^\circ}\ \text{A} \end{cases}$$

$$\rightarrow \begin{cases} i_1 = 4.09\sin(\omega t + 75.2^\circ)\ \text{A} \\ i_2 = 2.20\sin(\omega t + 26.4^\circ)\ \text{A} \\ i_3 = 6.16\sin(\omega t + 39.6^\circ)\ \text{A} \end{cases}$$

Sinusoidal Steady-State Analysis

Mesh Current Method (1)

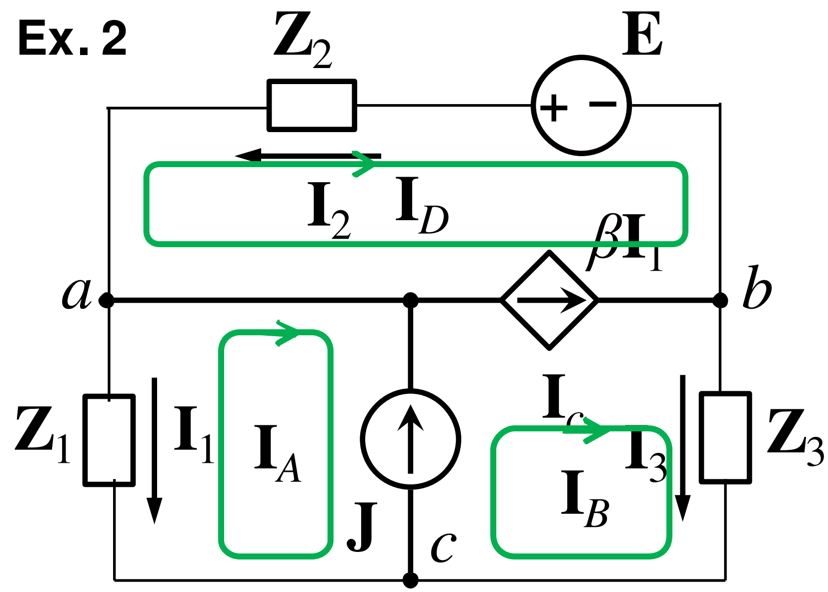
Ex. 1



$$\left.\begin{array}{l}\begin{cases}\mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 = \mathbf{E}_1 - \mathbf{E}_2\\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2\end{cases}\\ \mathbf{I}_1 = \mathbf{I}_A, \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A, \mathbf{I}_3 = \mathbf{I}_B, \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J}\end{array}\right\} \rightarrow \begin{cases}\mathbf{Z}_1\mathbf{I}_A - \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) = \mathbf{E}_1 - \mathbf{E}_2\\ \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_3\mathbf{I}_B + \mathbf{Z}_4(\mathbf{I}_B + \mathbf{J}) = \mathbf{E}_2\end{cases}$$

Mesh Current Method (2)

Ex. 2



$$\left.\begin{aligned} \mathbf{Z}_1\mathbf{I}_A + \mathbf{Z}_2\mathbf{I}_D + \mathbf{Z}_3\mathbf{I}_B + \mathbf{E} &= 0 \\ \mathbf{I}_B - \mathbf{I}_A &= \mathbf{J} \\ \mathbf{I}_B - \mathbf{I}_D &= \mathbf{I}_c \\ \mathbf{I}_c &= \beta\mathbf{I}_1 \end{aligned}\right\}$$

$$\left.\begin{aligned} \rightarrow (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{I}_A &= -\mathbf{E} - (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{J} + \mathbf{Z}_2\beta\mathbf{I}_1 \\ \mathbf{I}_A &= -\mathbf{I}_1 \end{aligned}\right\}$$

$$\rightarrow (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \beta\mathbf{Z}_2)\mathbf{I}_A = \mathbf{E} + (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{J}$$

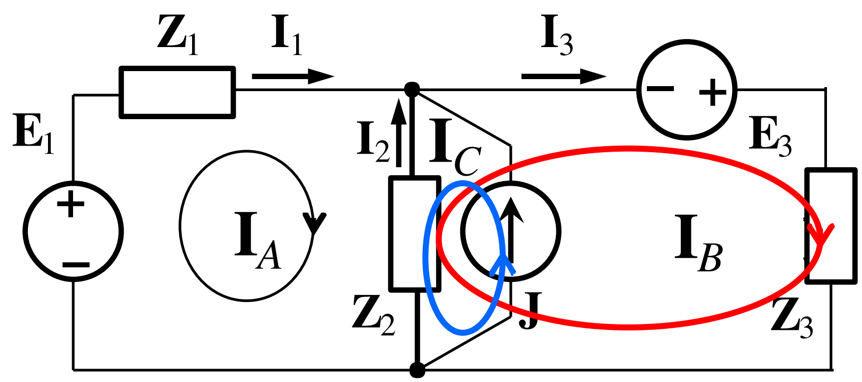
Mesh Current Method (3)

Ex. 3

$\mathbf{Z}_1 = 10\Omega;\ \mathbf{Z}_2 = j20\Omega;\ \mathbf{Z}_3 = 5 - j10\Omega;$

$\mathbf{E}_1 = 30\,\text{V};\ \mathbf{E}_3 = 45\angle 15^\circ\ \text{V};\ \mathbf{J} = 2\angle -30^\circ\ \text{A};$

Find currents?



$$\begin{cases} 10\mathbf{I}_A + j20(\mathbf{I}_A - \mathbf{I}_B + 2\angle -30^\circ) = 30 \\ j20(\mathbf{I}_B - \mathbf{I}_A - 2\angle -30^\circ) + (5 - j10)\mathbf{I}_B = 45\angle 15^\circ \end{cases}$$

$$\rightarrow \begin{cases} (10 + j20)\mathbf{I}_A - j20\mathbf{I}_B = 30 - j20 \times 2\angle -30^\circ \\ -j20\mathbf{I}_A + (5 + j20)\mathbf{I}_B = j20 \times 2\angle -30^\circ + 45\angle 15^\circ \end{cases} \rightarrow \begin{cases} \mathbf{I}_A = 1.04 + j3.95\ \text{A} \\ \mathbf{I}_B = 4.75 + j3.93\ \text{A} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = \mathbf{I}_A = 1.04 + j3.95 = 4.09\angle 75.2^\circ\ \text{A} \\ \mathbf{I}_2 = -\mathbf{I}_A + \mathbf{I}_B - \mathbf{J} = 2.20\angle 26.4^\circ\ \text{A} \\ \mathbf{I}_3 = \mathbf{I}_B = 4.75 + j3.93 = 6.16\angle 39.6^\circ\ \text{A} \end{cases} \rightarrow \begin{cases} i_1 = 4.09\sin(\omega t + 75.2^\circ)\ \text{A} \\ i_2 = 2.20\sin(\omega t + 26.4^\circ)\ \text{A} \\ i_3 = 6.16\sin(\omega t + 39.6^\circ)\ \text{A} \end{cases}$$

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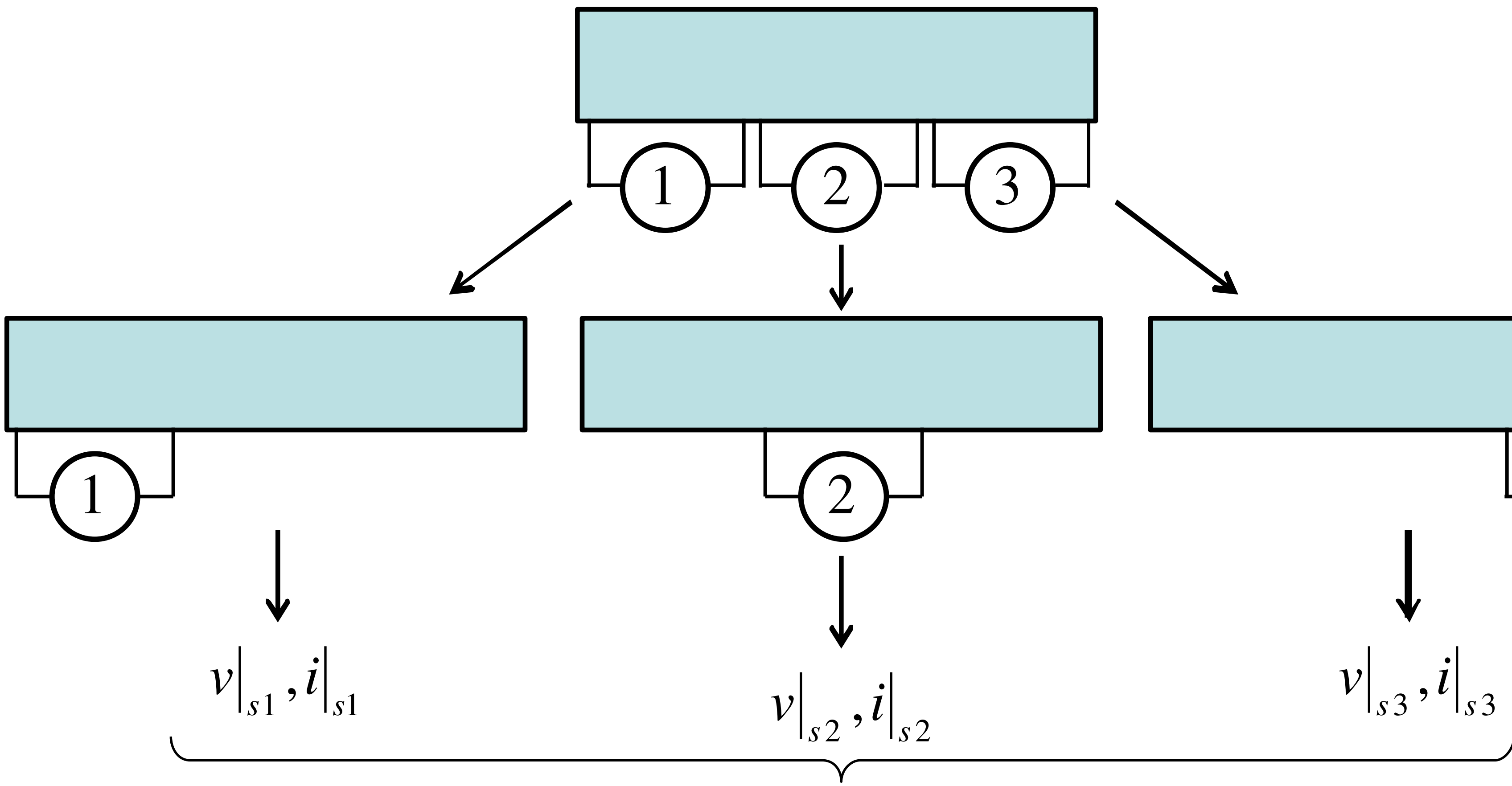
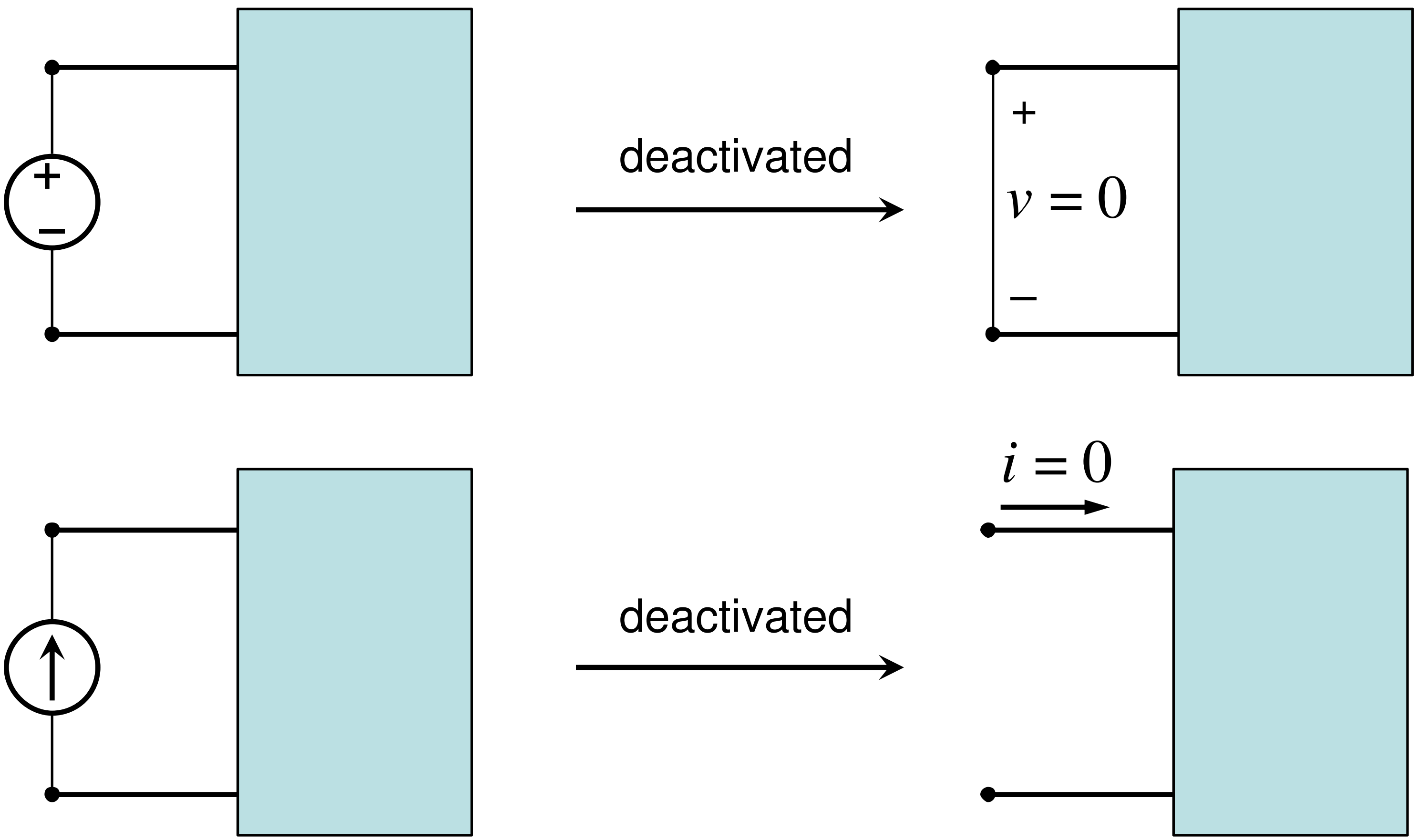
Superposition Theorem (1)



$$v = v|_{s1} + v|_{s2} + v|_{s3}; \qquad i = i|_{s1} + i|_{s2} + i|_{s3}$$

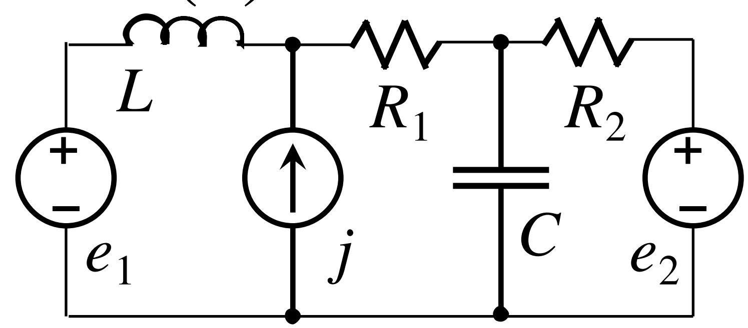
Superposition Theorem (2)



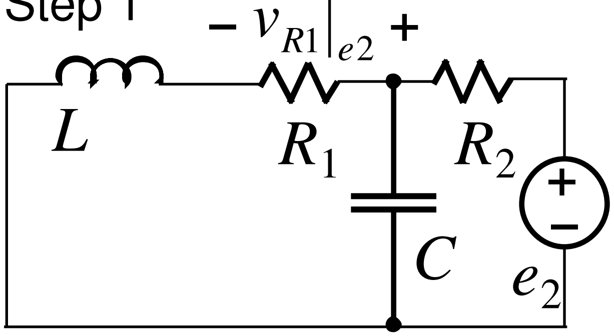
Superposition Theorem (3)

Ex. 1

$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^o)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1$ Ω; $R_2 = 5$ Ω; $C = 0.01$ F; $v_{R1} = ?$

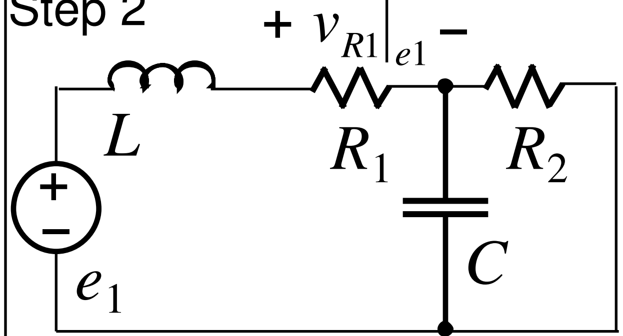


Step 1



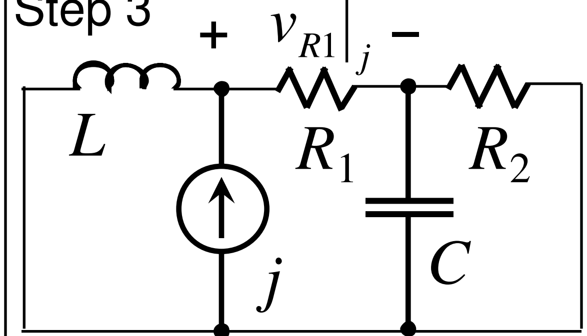
$v_{R1}|_{e2} = ?$

Step 2



$v_{R1}|_{e1} = ?$

Step 3



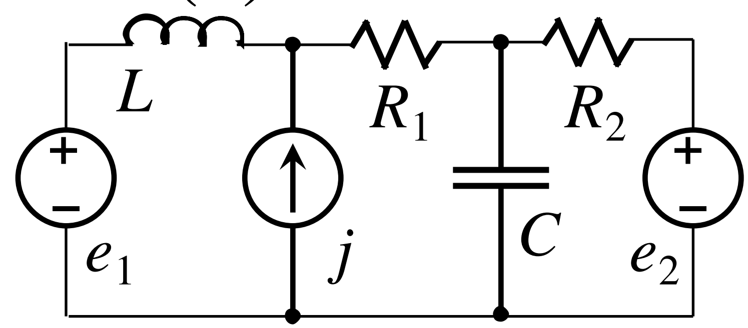
$v_{R1}|_{j} = ?$

Step 4: $v_{R1} = -v_{R1}|_{e2} + v_{R1}|_{e1} + v_{R1}|_{j}$

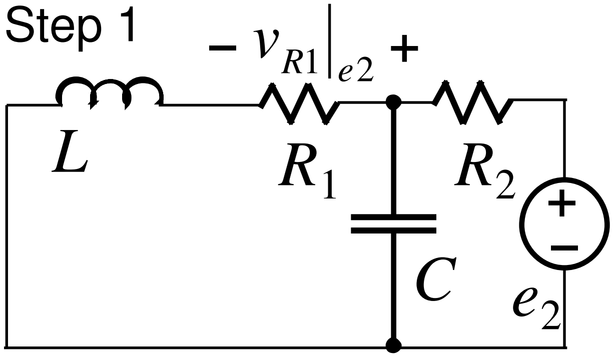
Superposition Theorem (4)

Ex. 1

$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^o)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; $v_{R1} = ?$



Step 1



$$v_{R1}\big|_{e2} = ?$$

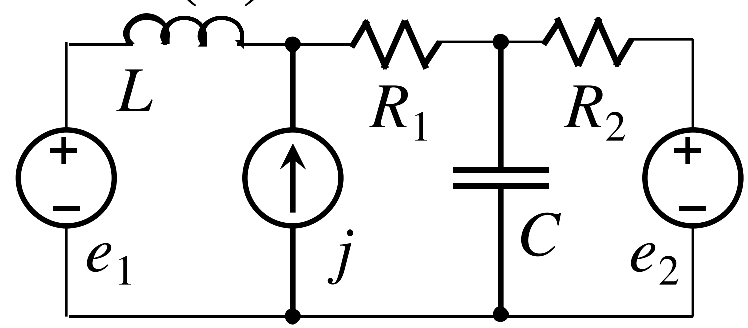
$$i\big|_{e2} = \frac{6}{1+5} = 1\text{A}$$

$$v_{R1}\big|_{e2} = 1 \times 1 = \boxed{1\text{V}}$$

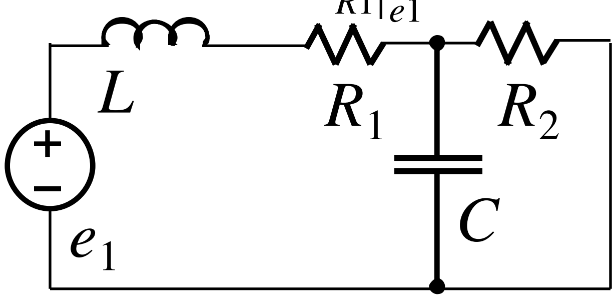
Superposition Theorem (5)

Ex. 1

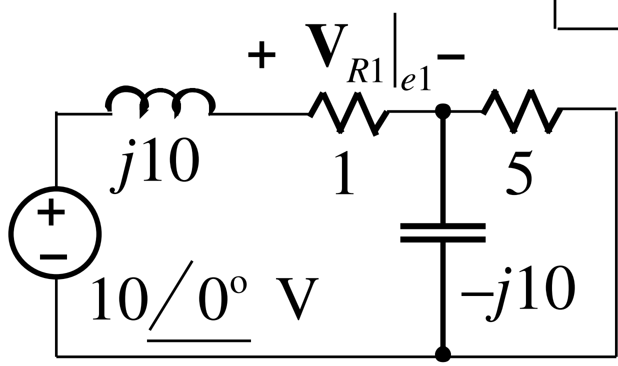
$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^\circ)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; $v_{R1} = ?$



Step 2



$v_{R1}\big|_{e1} = ?$



$$\mathbf{Z} = j10 + 1 + \frac{5(-j10)}{5 - j10}$$

$$= 5 + j8 = 9.43\angle 58^\circ\ \Omega$$

$$\mathbf{I}_{R1}\big|_{e1} = \frac{\mathbf{E}_1}{\mathbf{Z}} = \frac{10\angle 0}{9.43\angle 58^\circ} = 1.06\angle -58^\circ\ \text{A}$$

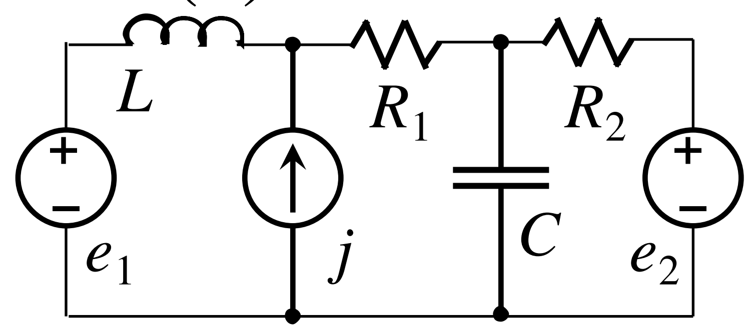
$$\mathbf{V}_{R1}\big|_{e1} = R_1 \mathbf{I}_{R1}\big|_{e1} = 1 \times 1.06\angle -58^\circ = 1.06\angle -58^\circ\ \text{V}$$

$$\rightarrow v_{R2}\big|_{e1} = \boxed{1.06\sin(10t - 58^\circ)\ \text{V}}$$

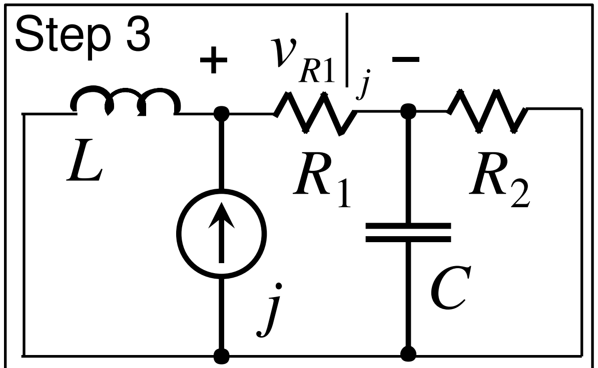
Superposition Theorem (6)

Ex. 1

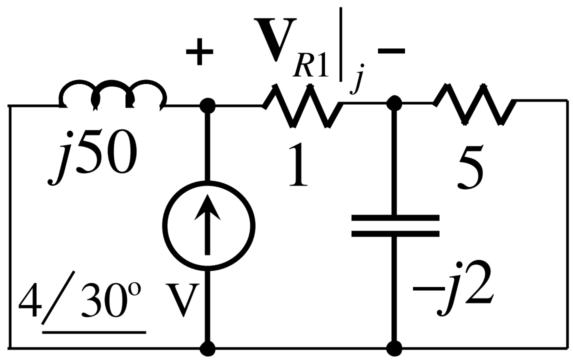
$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^{\circ})$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; $v_{R1} = ?$

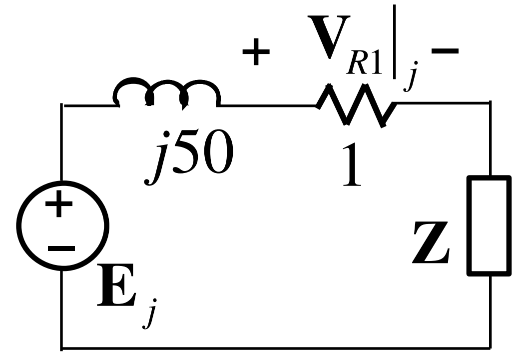


Step 3



$v_{R1}\big|_j = ?$





$$\mathbf{E}_j = (j50)(4\angle 30^{\circ}) = 200\angle 120^{\circ}\ \text{V}$$

$$\mathbf{Z} = \frac{5(-j2)}{5 - j2} = 0.69 - j1.72\ \Omega$$

$$\mathbf{I}\big|_j = \frac{200\angle 120^{\circ}}{j50 + 1 + 0.69 - j1.72} = 4.14\angle 32^{\circ}\ \text{A}$$

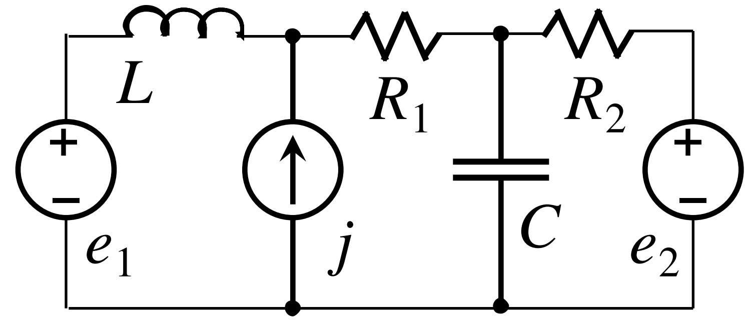
$$\mathbf{V}_{R1}\big|_j = 1 \times 4.14\angle 32^{\circ} = 4.14\angle 32^{\circ}\ \text{V}$$

$$\rightarrow v_{R1}\big|_j = \boxed{4.14\sin(50t + 32^{\circ})\ \text{V}}$$

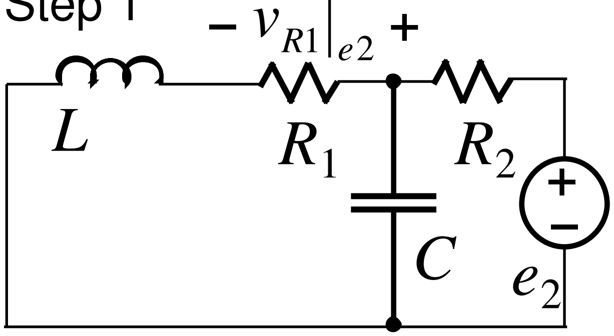
Superposition Theorem (7)

Ex. 1

$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^o)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1\ \Omega$; $R_2 = 5\ \Omega$; $C = 0.01$ F; $v_{R1} = ?$

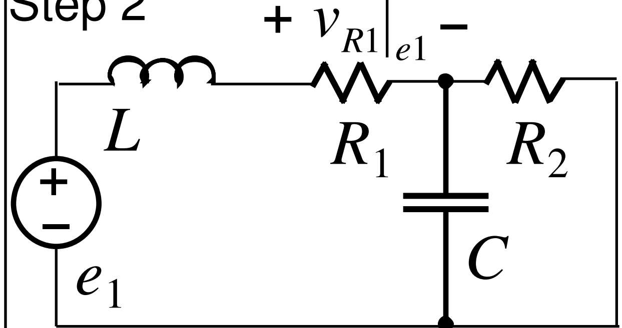


Step 1



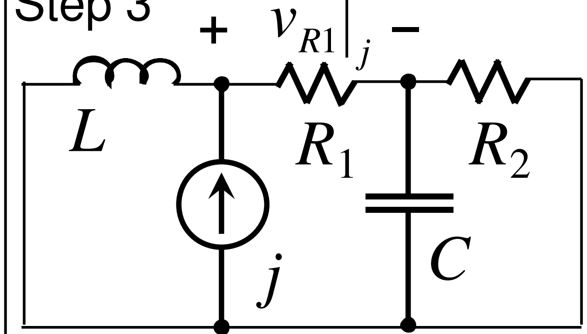
$v_{R1}\big|_{e2} = 1\,\text{V}$

Step 2



$v_{R1}\big|_{e1} = 1.06\sin(10t - 58^o)\,\text{V}$

Step 3



$v_{R1}\big|_{j} = 4.14\sin(50t + 32^o)\,\text{V}$

$$v_{R1} = -v_{R1}\big|_{e2} + v_{R1}\big|_{e1} + v_{R1}\big|_{j} = \boxed{-1 + 1.06\sin(10t - 58^o) + 4.14\sin(50t + 32^o)\,\text{V}}$$

Superposition Theorem (8)

$$v_{R1} = -v_{R1}\big|_{e2} + v_{R1}\big|_{e1} + v_{R1}\big|_{j} = \boxed{-1 + 1.06\sin(10t - 58^{o}) + 4.14\sin(50t + 32^{o})\ \text{V}}$$

$$v_{R1}\big|_{e2} = 1\ \text{V}$$

$$\mathbf{V}_{R1}\big|_{e1} = 1.06\angle -58^{o}\ \text{V}$$

$$\mathbf{V}_{R1}\big|_{j} = 4.14\angle 32^{o}\ \text{V}$$

$$\mathbf{V}_{R1} = -1 + 1.06\angle -58^{o} + 4.14\angle 32^{o}$$

$$= -1 + (0.56 - j0.90) + (3.51 + j2.19)$$

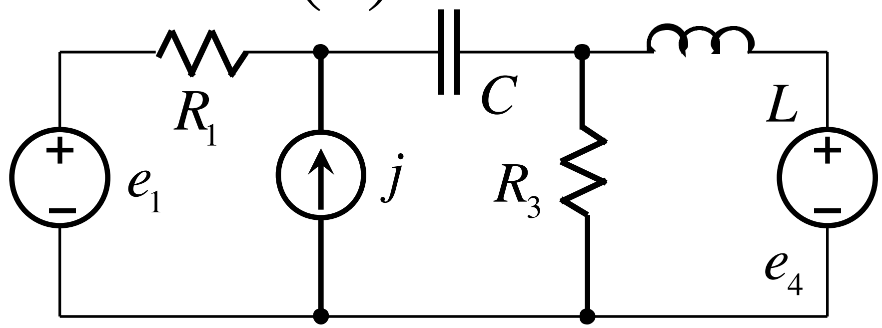
$$= 3.07 + j1.29 = 3.33\angle 22.8^{o}\ \text{V}$$

$$\rightarrow v_{R1} = 3.33\sin(? + 22.8^{o})\ \text{V}$$

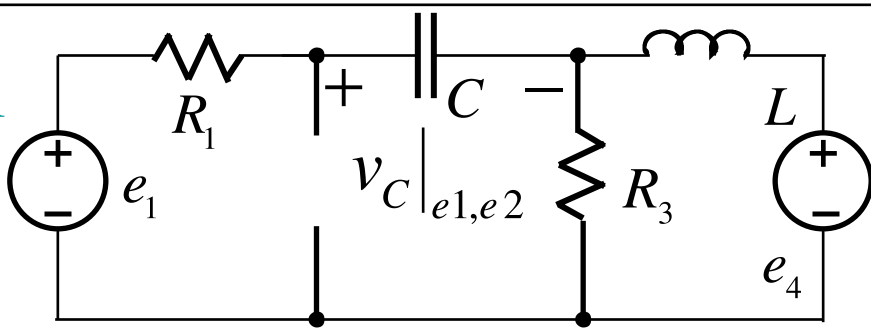
Superposition Theorem (9)

Ex. 2

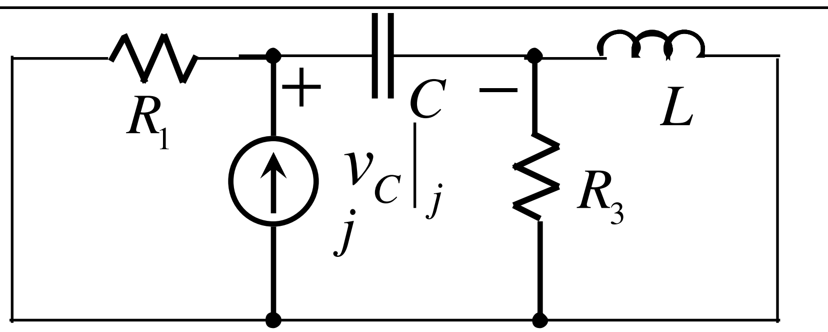
$e_1 = 45\text{V (DC)}$; $e_4 = 60\text{V (DC)}$; $j = 10\sin(100t)$ A;
$R_1 = 5\Omega$; $R_3 = 10\Omega$; $C = 2\text{mF}$; $L = 0.1\text{H}$; $v_C = ?$



Step 1



Step 2



Step 3

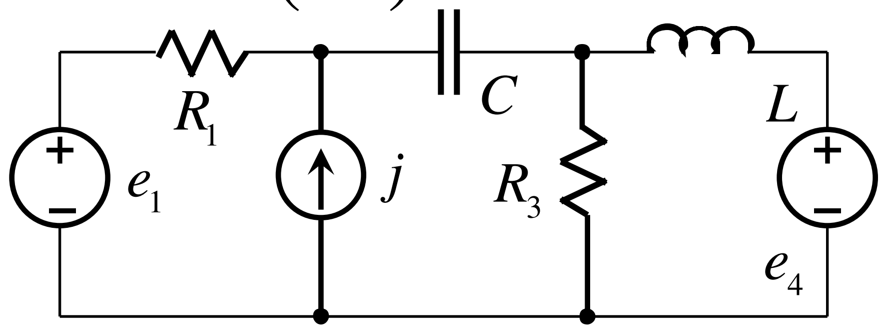
$$v_C = v_C\big|_{e1,e2} + v_C\big|_j$$

https://sites.google.com/site/ncpdhbkhn/home

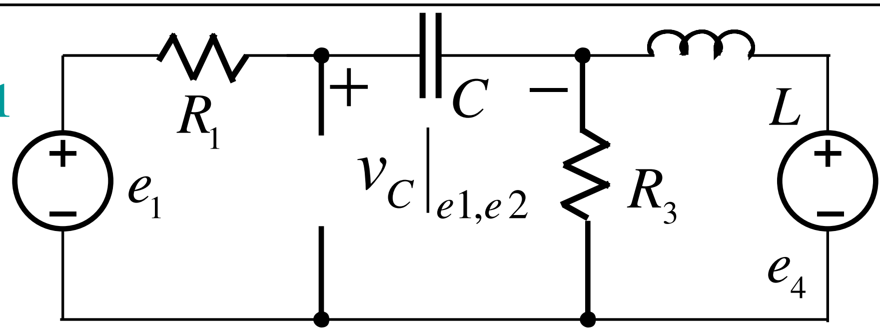
Superposition Theorem (10)

Ex. 2

$e_1 = 45\text{V (DC)}$; $e_4 = 60\text{V (DC)}$; $j = 10\sin(100t)$ A;
$R_1 = 5\Omega$; $R_3 = 10\Omega$; $C = 2\text{mF}$; $L = 0.1\text{H}$; $v_C = ?$



Step 1

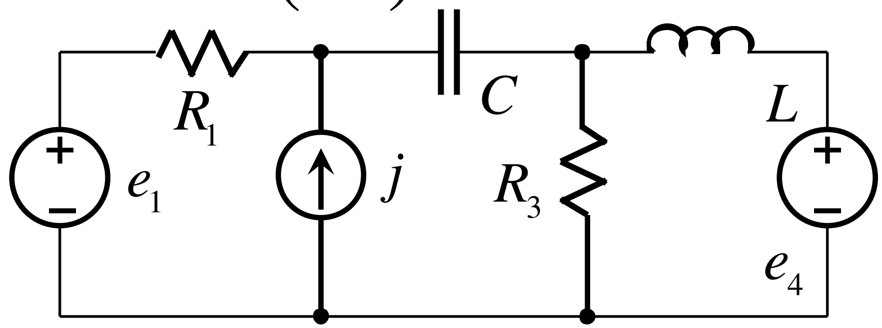


$$v_C\big|_{e1,e2} = e_1 - e_4 = 45 - 60 = -15\text{ V}$$

Superposition Theorem (11)

Ex. 2

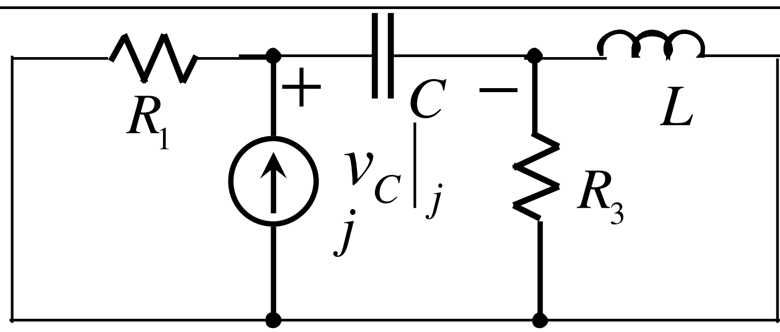
$e_1 = 45$V (DC); $e_4 = 60$V (DC); $j = 10\sin(100t)$ A;
$R_1 = 5\Omega$; $R_3 = 10\Omega$; $C = 2$mF; $L = 0.1$H; $v_C = ?$



$$\mathbf{V}_C\Big|_j = \frac{R_1\mathbf{J}}{R_1 + \dfrac{1}{j\omega C} + \dfrac{R_3(j\omega L)}{R_3 + j\omega L}} \times \frac{1}{j\omega C} = \frac{5\times 10}{5 + \dfrac{1}{j100\times 0.002} + \dfrac{10(j100\times 0.1)}{10 + j100\times 0.1}} \times \frac{1}{j100\times 0.002}$$

$$= -j25.00 = 25.00\underline{/-90^{\circ}} \text{ V}$$

Step 2

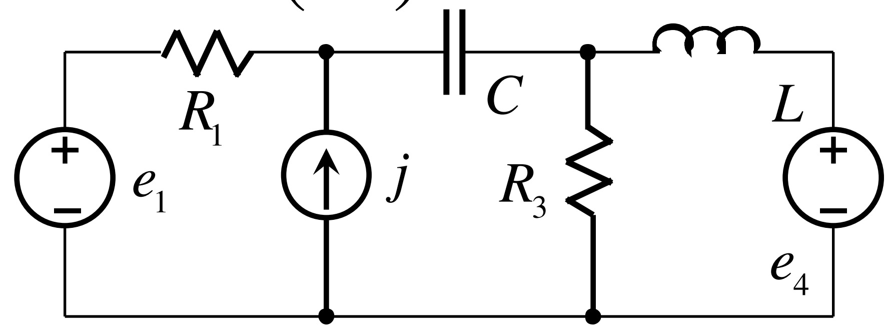


$$\rightarrow v_C\Big|_j = 25\sin(100t - 90^{\circ}) \text{ V}$$

Superposition Theorem (12)

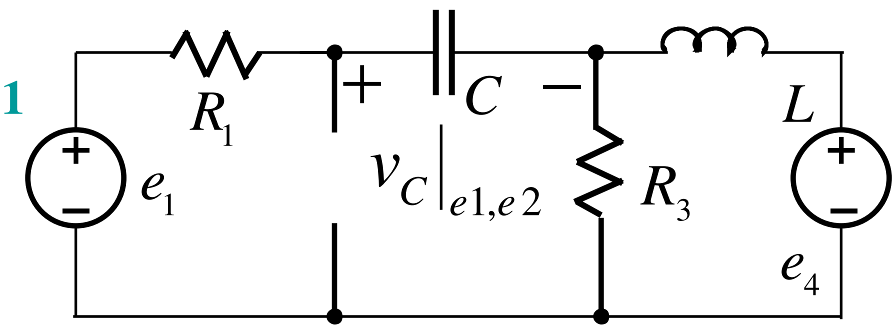
Ex. 2

$e_1 = 45\text{V (DC)}$; $e_4 = 60\text{V (DC)}$; $j = 10\sin(100t)$ A;
$R_1 = 5\Omega$; $R_3 = 10\Omega$; $C = 2\text{mF}$; $L = 0.1\text{H}$; $v_C = ?$

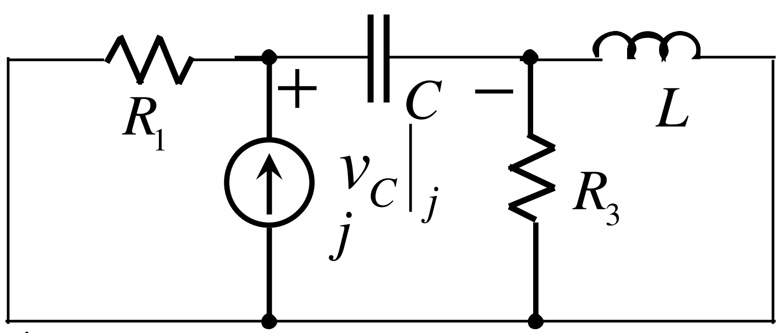


$$v_C\big|_{e1,e2} = -15\,\text{V}$$

Step 1



Step 2



$$v_C\big|_j = 25\sin(100t - 90^\circ)\ \text{V}$$

Step 3

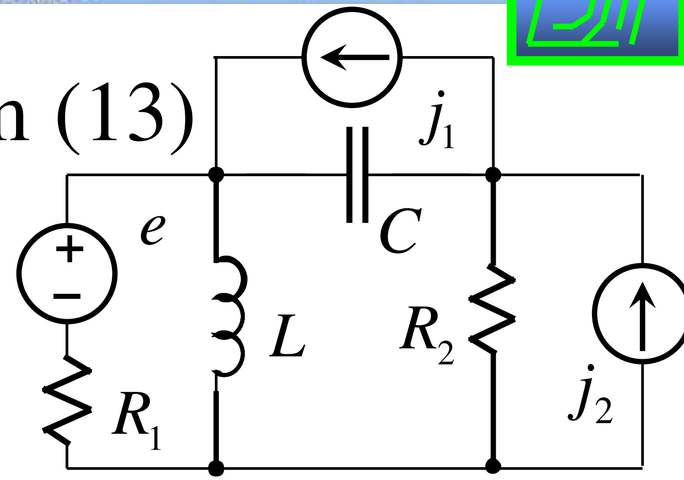
$$\rightarrow v_C = v_C\big|_{e1,e2} + v_C\big|_j$$

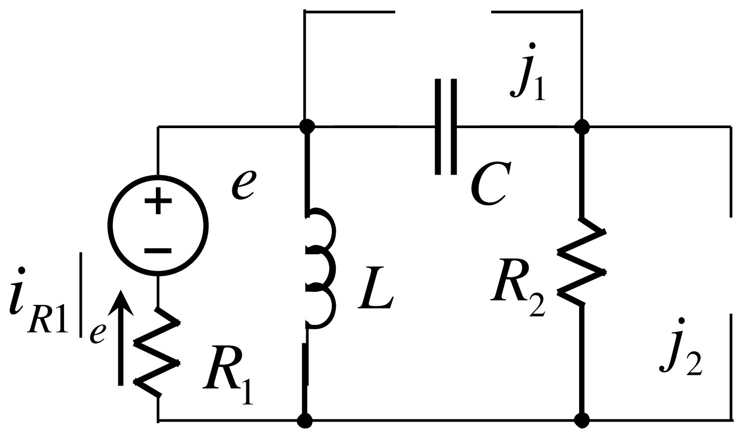
$$= \boxed{-15 + 25\sin(100t - 90^\circ)\ \text{V}}$$

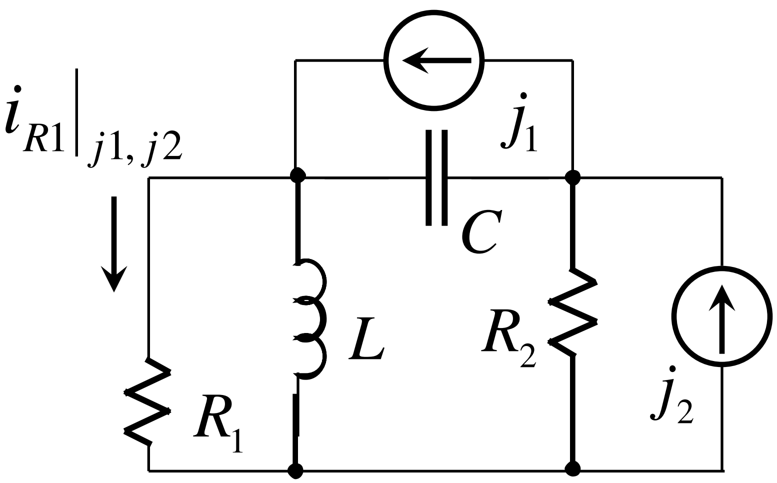
Superposition Theorem (13)

Ex. 3

$e = 45\text{V (DC)}; j_1 = 6\sin(100t + 15^o)\text{ A}; j_2 = 10\sin(100t)\text{ A};$
$R_1 = 5\Omega; R_2 = 10\Omega; C = 2\text{mF}; L = 0.1\text{H}; i_{R1} = ?$







$$i_{R1}\big|_e = \frac{e}{R_1} = \frac{45}{5} = 9\text{ A}$$

$$\mathbf{I}_{R1}\big|_{j1,j2} = \frac{\dfrac{1}{j\omega C}\mathbf{J}_1 + R_2\mathbf{J}_2}{R_2 + \dfrac{1}{j\omega C} + \dfrac{R_1(j\omega L)}{R_1 + j\omega L}} \times \frac{j\omega L}{R_1 + j\omega L}$$

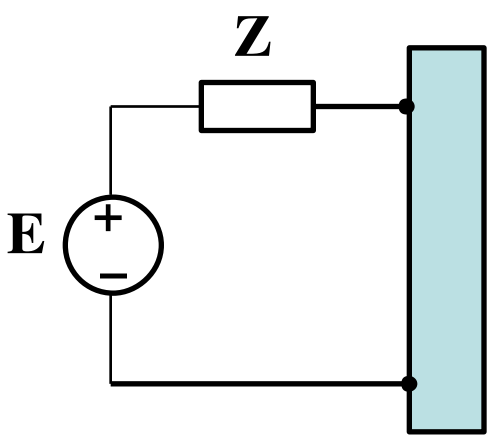
$$= 6.39 + j2.79 = 6.97\underline{/23.6^o}\text{ A}$$

$$\rightarrow i_{R1} = \boxed{-9 + 6.97\sin(100t + 23.6^o)\text{ A}}$$

Sinusoidal Steady-State Analysis

1. Sinusoidal Steady-State Analysis
2. Ohm's Law
3. Kirchhoff's Laws
4. Impedance Combinations
5. Branch Current Method
6. Node Voltage Method
7. Mesh Current Method
8. Superposition Theorem
9. **Source Transformation**
10. Thévenin & Norton Equivalent Circuits
11. Op Amp AC Circuits

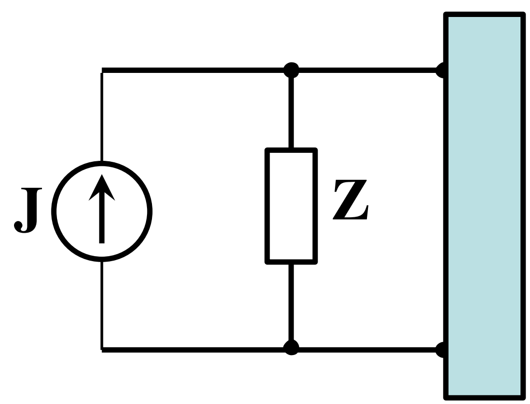
Source Transformation (1)



$$J = \frac{E}{Z}$$

$$\Longleftrightarrow$$

$$E = ZJ$$



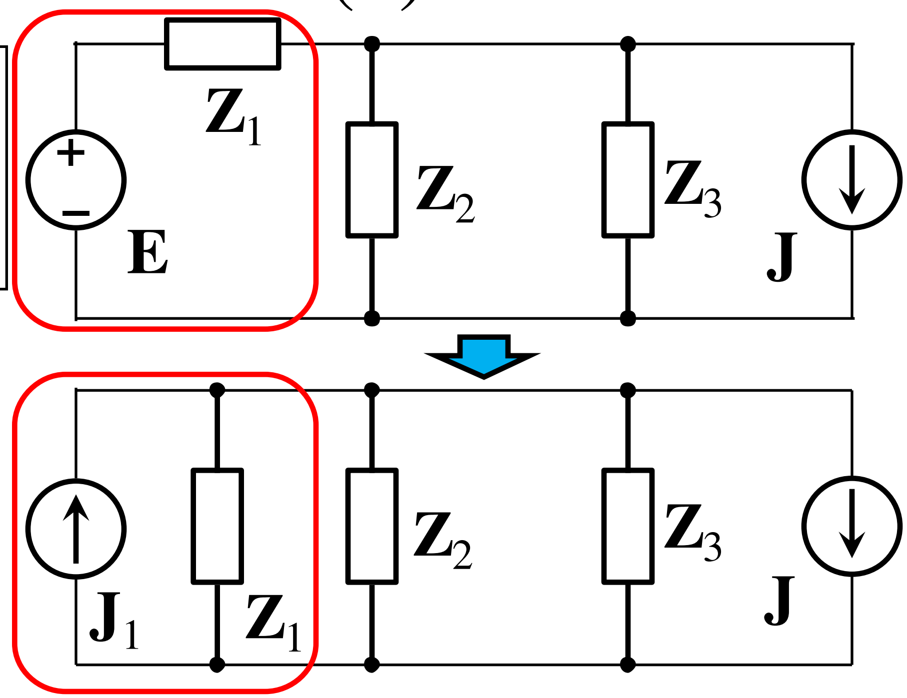
$$V = E + ZI \quad \rightleftarrows \quad I = -\frac{E}{Z} + \frac{V}{Z}$$

Source Transformation (2)

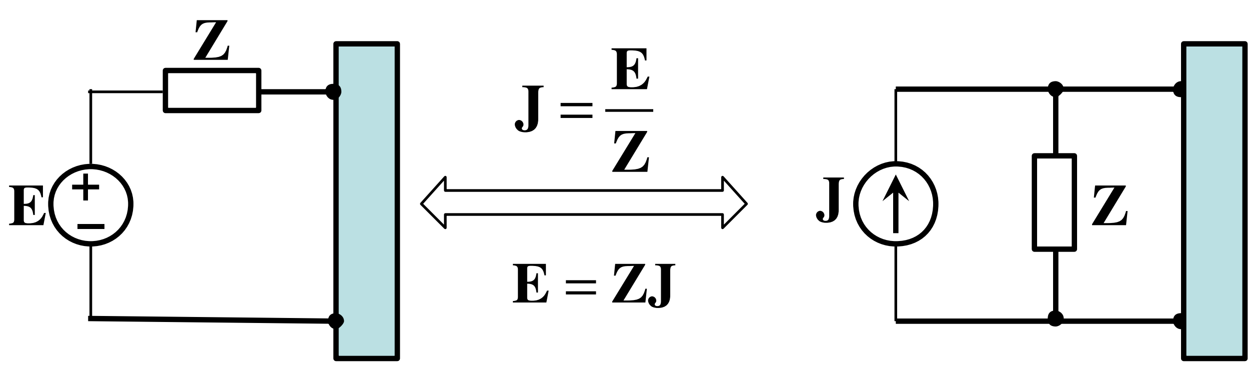
Ex. 1

$\mathbf{E} = 20\underline{/-45^\circ}$ V; $\mathbf{J} = 5\underline{/60^\circ}$ A;

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_2 = j10\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$;
Find the current of $\mathbf{Z}_2$?



$$\mathbf{J}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\underline{/-45^\circ}}{12} = 1.67\underline{/-45^\circ}\text{ A}$$



$$\mathbf{J} = \frac{\mathbf{E}}{\mathbf{Z}}$$

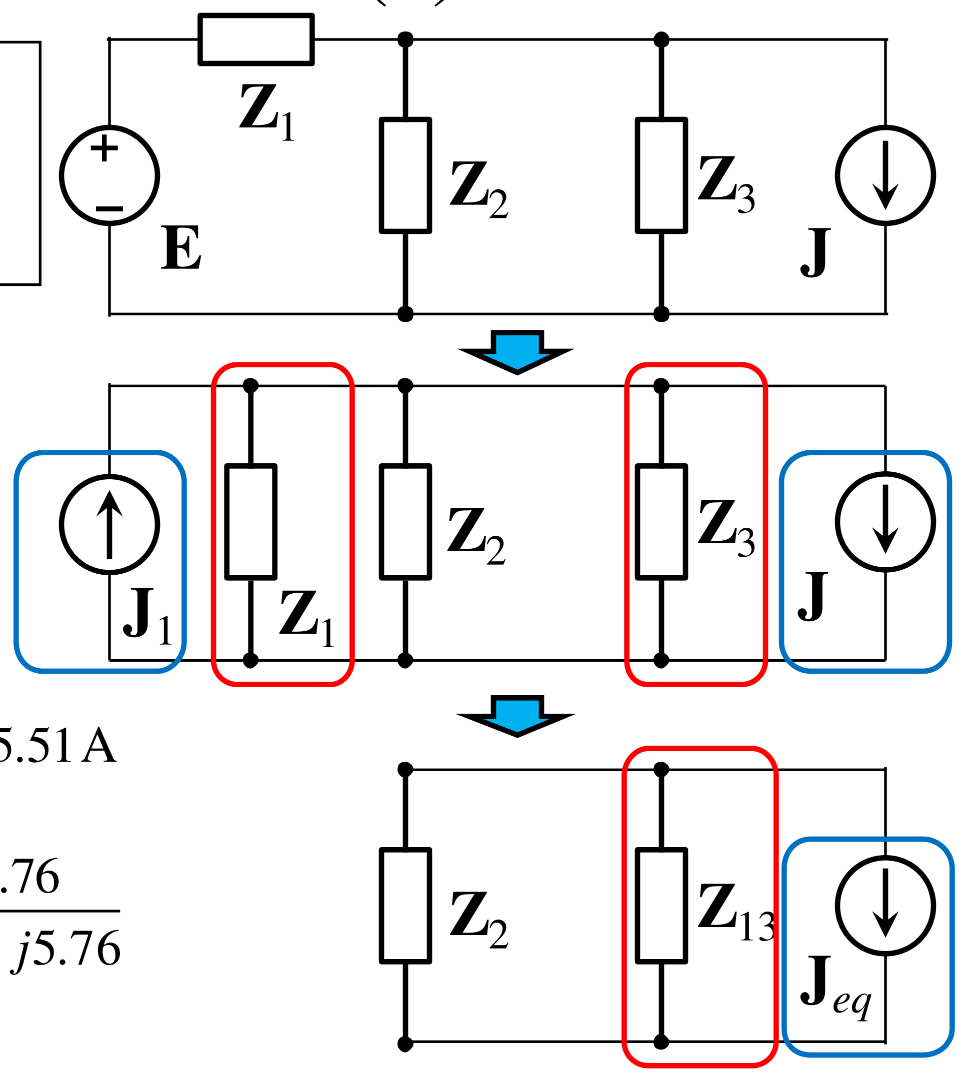
$$\mathbf{E} = \mathbf{Z}\mathbf{J}$$

Source Transformation (3)

Ex. 1

$\mathbf{E} = 20\angle -45^\circ$ V; $\mathbf{J} = 5\angle 60^\circ$ A;

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_2 = j10\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$;
Find the current of $\mathbf{Z}_2$?



$$\mathbf{J}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\angle -45^\circ}{12} = 1.67\angle -45^\circ \text{ A}$$

$$\mathbf{Z}_{13} = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_3} = \frac{12(-j16)}{12 - j16} = 7.68 - j5.76\,\Omega$$

$$\mathbf{J}_{eq} = \mathbf{J} - \mathbf{J}_1 = 5\angle 60^\circ - 1.67\angle -45^\circ = 1.32 + j5.51\text{ A}$$

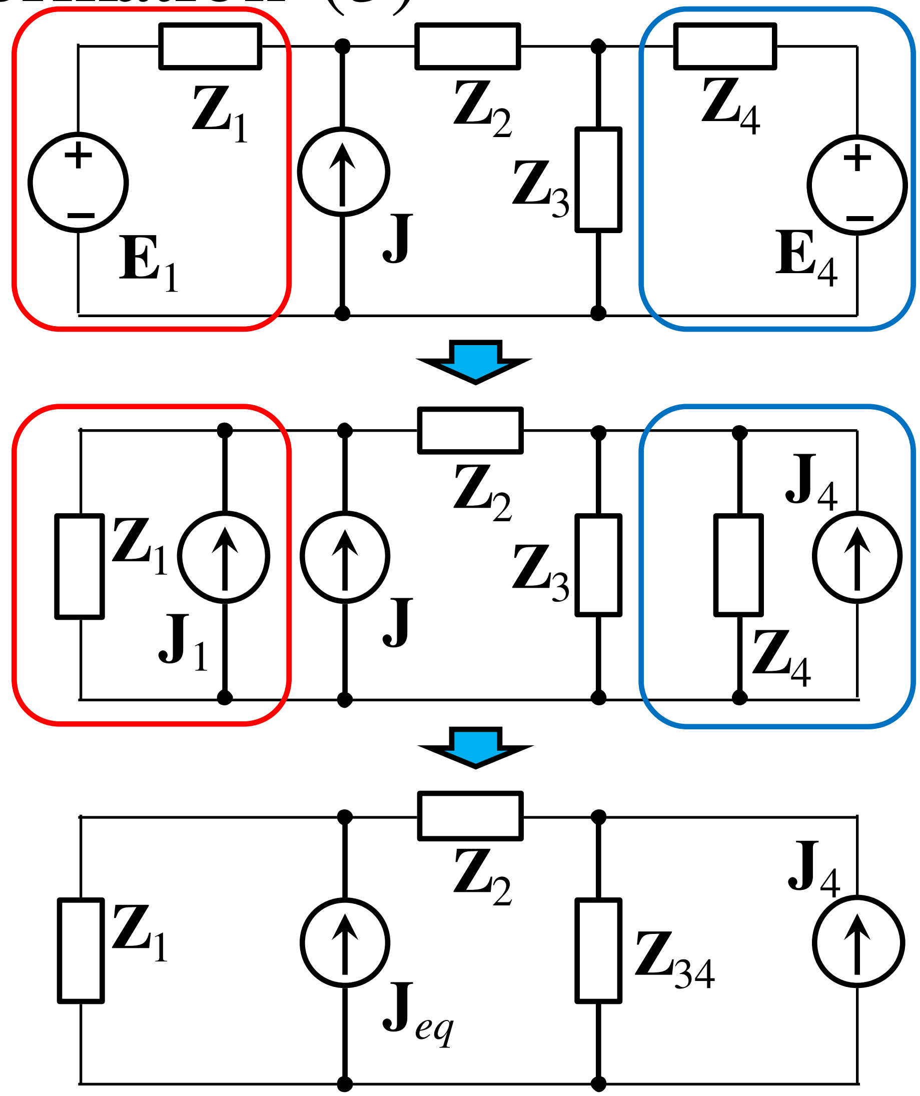
$$\mathbf{I}_2 = \mathbf{J}_{eq}\frac{\mathbf{Z}_{13}}{\mathbf{Z}_2 + \mathbf{Z}_{13}} = (1.32 + j5.51)\frac{7.68 - j5.76}{j10 + 7.68 - j5.76}$$

$$= \boxed{6.09 + j1.16 \text{ A}}$$

Source Transformation (3)

Ex. 2

$\mathbf{E}_1 = 100\angle 30^\circ\ \text{V}; \mathbf{E}_4 = 80\angle -45^\circ\ \text{V}; \mathbf{J} = 5\ \text{A};$
$\mathbf{Z}_1 = 10\Omega; \mathbf{Z}_2 = 5\Omega; \mathbf{Z}_3 = j20\Omega; \mathbf{Z}_4 = -j25\Omega;$
Find the current of $\mathbf{Z}_2$?



$$\mathbf{J}_1 = \frac{\mathbf{E}_1}{\mathbf{Z}_1} = \frac{100\angle 30^\circ}{10} = 10\angle 30^\circ\ \text{A}$$

$$\mathbf{J}_4 = \frac{\mathbf{E}_4}{\mathbf{Z}_4} = \frac{80\angle -45^\circ}{-j25} = 2.26 + j2.26\ \text{A}$$

$$\mathbf{J}_{eq} = \mathbf{J}_1 + \mathbf{J} = 10\angle 30^\circ + 5 = 13.66 + j5.00\ \text{A}$$

$$\mathbf{Z}_{34} = \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} = \frac{j20(-j25)}{j20 - j25} = j100\,\Omega$$

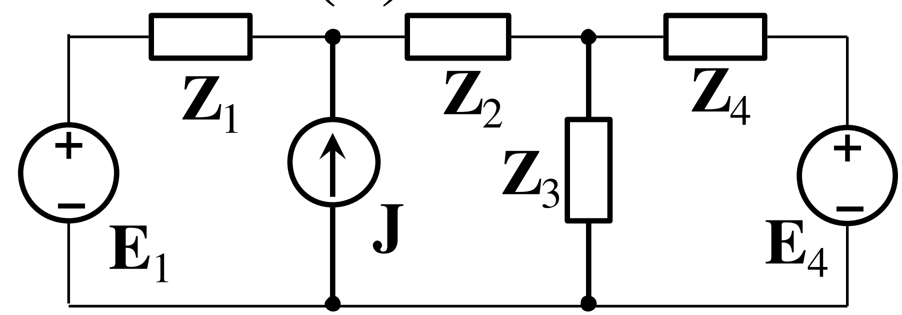
Source Transformation (4)

Ex. 2

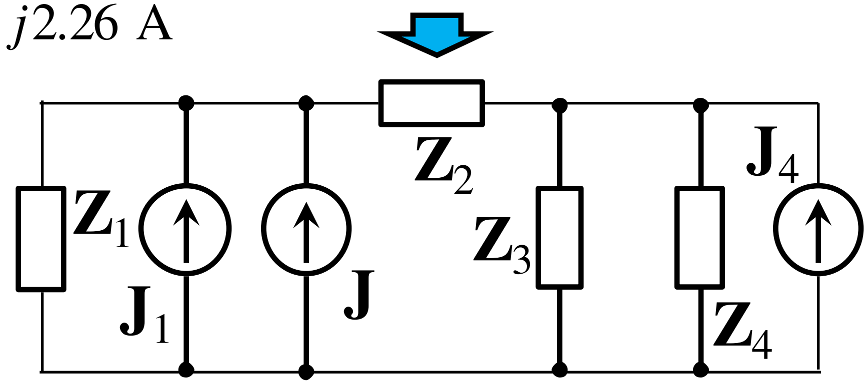
$\mathbf{E}_1 = 100\angle 30^\circ \text{ V}; \mathbf{E}_4 = 80\angle -45^\circ \text{ V}; \mathbf{J} = 5 \text{ A};$

$\mathbf{Z}_1 = 10\Omega; \mathbf{Z}_2 = 5\Omega; \mathbf{Z}_3 = j20\Omega; \mathbf{Z}_4 = -j25\Omega;$

Find the current of $\mathbf{Z}_2$?



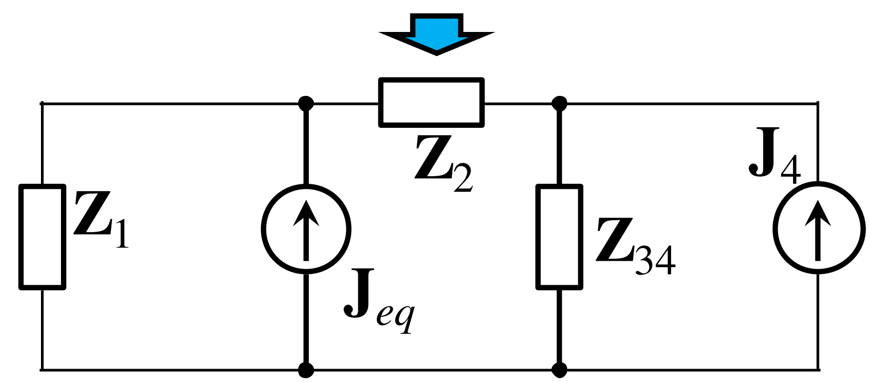
$$\mathbf{J}_{eq} = 13.66 + j5.00 \text{ A}; \;\; \mathbf{Z}_{34} = j100\,\Omega; \; \mathbf{J}_4 = 2.26 + j2.26 \text{ A}$$

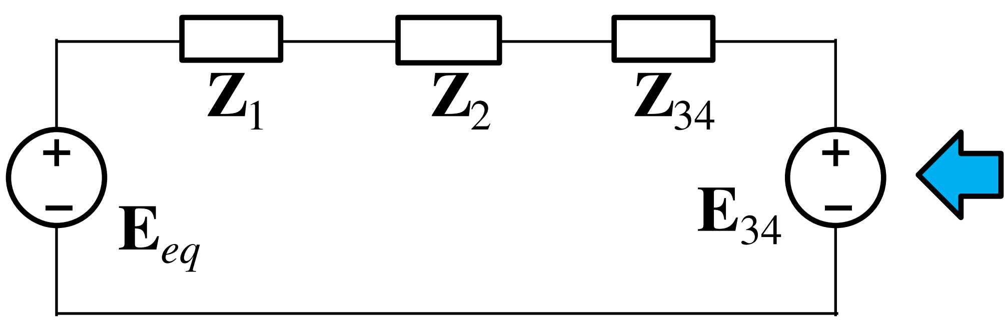


$$\mathbf{E}_{eq} = \mathbf{Z}_1\mathbf{J}_{eq} = 10(13.66 + j5.00) = 136.6 + j50 \text{ V}$$

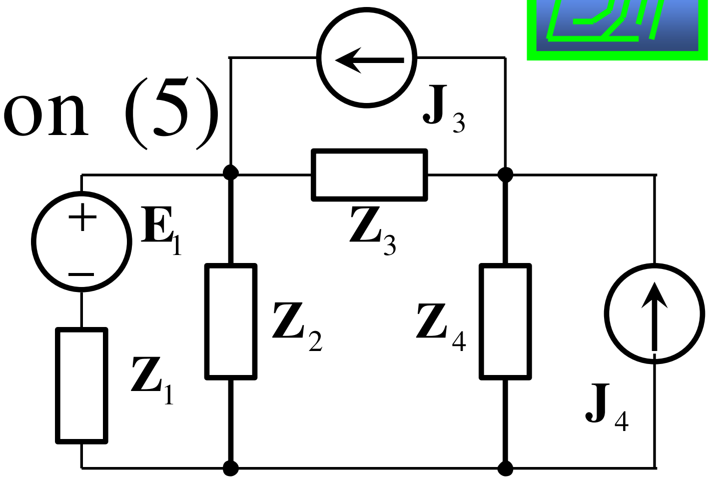
$$\mathbf{E}_{34} = \mathbf{Z}_{34}\mathbf{J}_4 = -226 + j226 \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_{eq} - \mathbf{E}_{34}}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_{34}} = \boxed{-1.19 - j3.81 \text{ A}}$$





Source Transformation (5)



Ex. 3

$\mathbf{E}_1 = 100\angle 30^\circ$ V; $\mathbf{J}_3 = 5$ A; $\mathbf{J}_4 = 8\angle -45^\circ$ A;

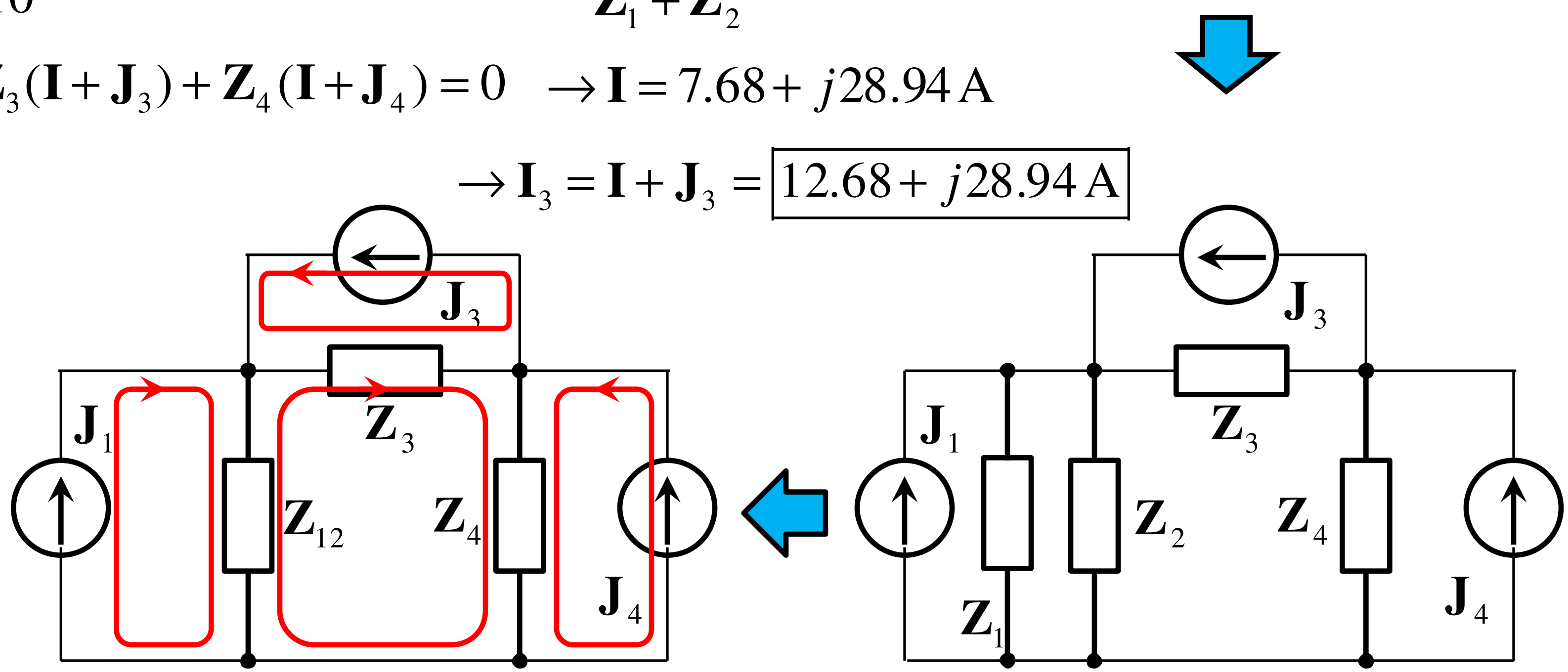
$\mathbf{Z}_1 = 10\Omega; \mathbf{Z}_2 = 5\Omega; \mathbf{Z}_3 = j20\Omega; \mathbf{Z}_4 = -j25\Omega;$

Find the current of $\mathbf{Z}_3$?

$$\mathbf{J}_1 = \frac{\mathbf{E}_1}{\mathbf{Z}_1} = \frac{100\angle 30^\circ}{10} = 10\angle 30^\circ \text{ A} \qquad \mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = 3.33\Omega$$

$$\mathbf{Z}_{12}(\mathbf{I} - \mathbf{J}_1) + \mathbf{Z}_3(\mathbf{I} + \mathbf{J}_3) + \mathbf{Z}_4(\mathbf{I} + \mathbf{J}_4) = 0 \quad \rightarrow \mathbf{I} = 7.68 + j28.94 \text{ A}$$

$$\rightarrow \mathbf{I}_3 = \mathbf{I} + \mathbf{J}_3 = \boxed{12.68 + j28.94 \text{ A}}$$



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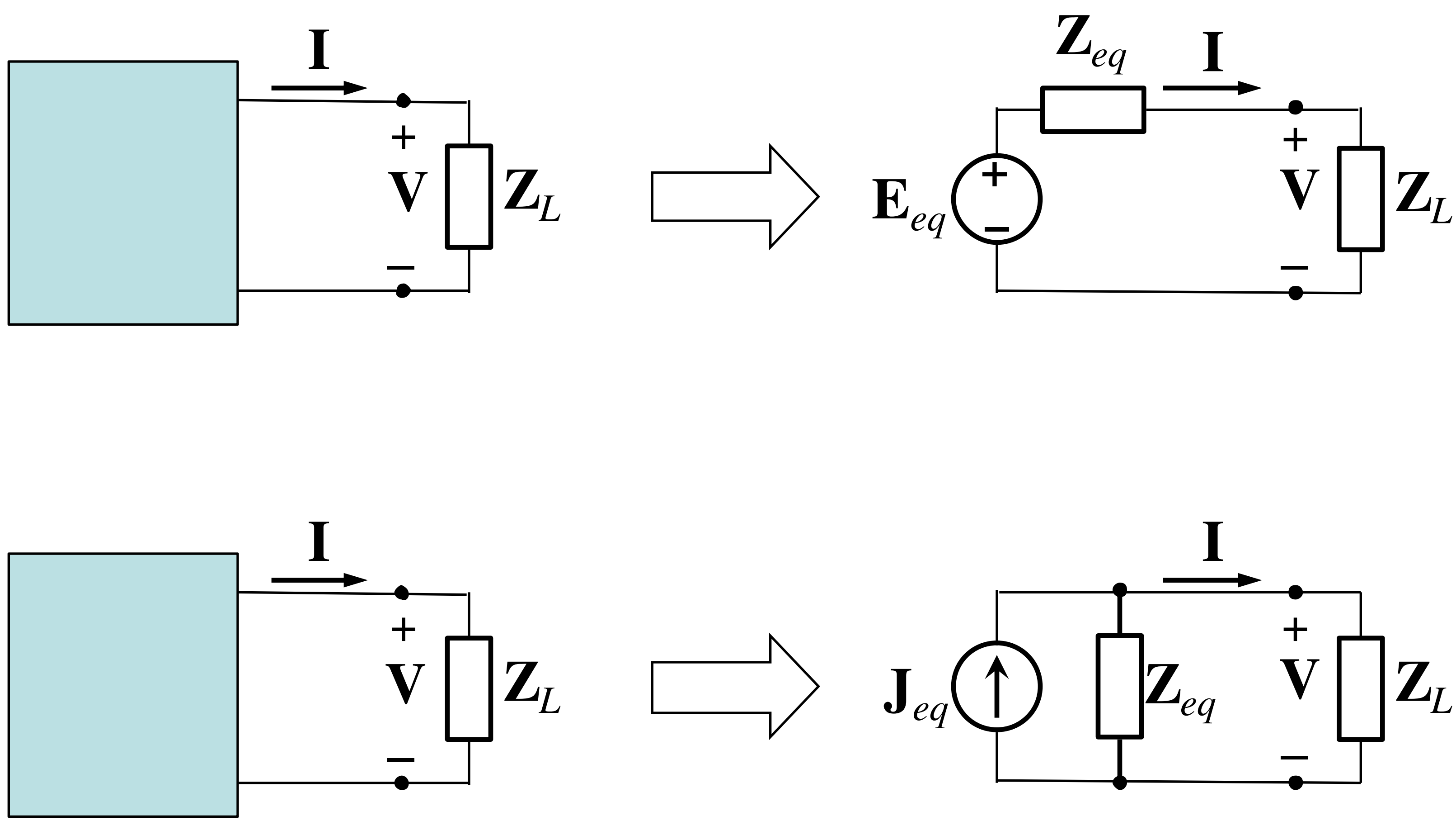
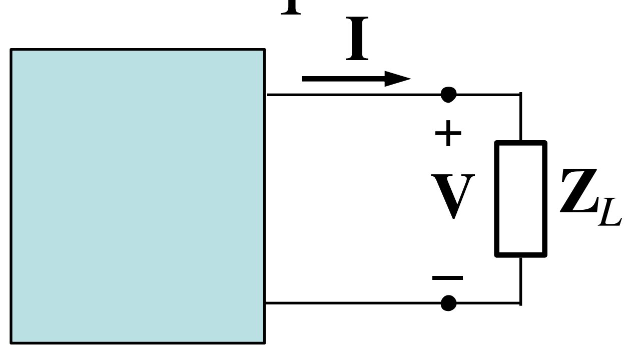
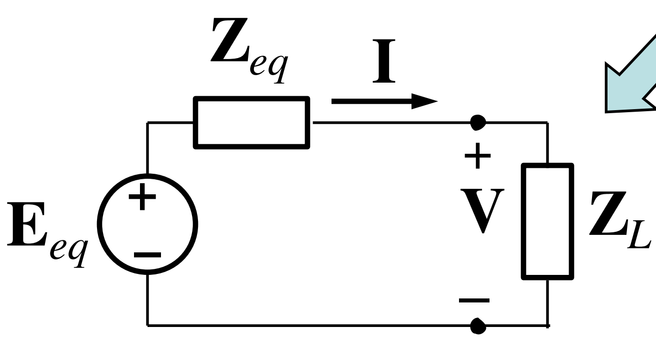
Thévenin & Norton Equivalent Circuits (1)

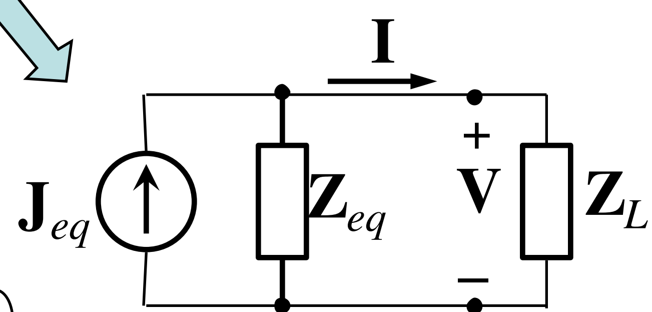


Thévenin & Norton Equivalent Circuits (2)





$$E_{eq} = Z_{eq} J_{eq}$$

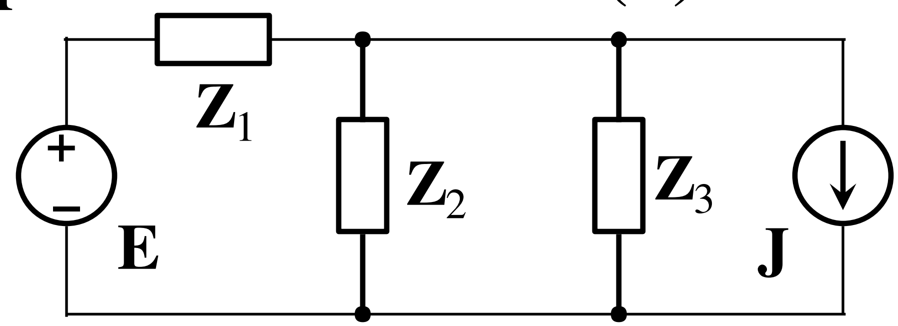


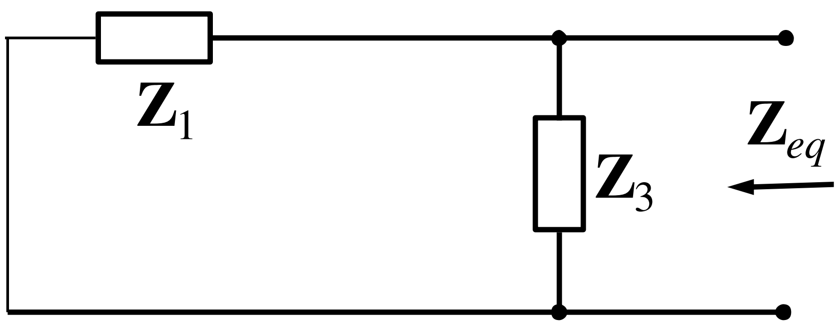
$$\left.\begin{aligned} Z_{eq} &= \frac{E_{eq}}{J_{eq}} \\ E_{eq} &= V_{open\text{-}circuit} \\ J_{eq} &= I_{short\text{-}circuit} \end{aligned}\right\} \rightarrow \boxed{Z_{eq} = \frac{V_{open\text{-}circuit}}{I_{short\text{-}circuit}}}$$

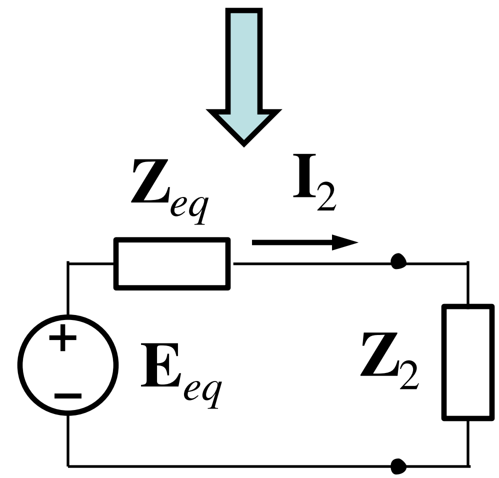
Ex. 1 Thévenin & Norton Equivalent Circuits (3)

$\mathbf{E} = 20\angle -45^{\circ}$ V; $\mathbf{J} = 5\angle 60^{\circ}$ A;

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_2 = j10\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$;
Find the current of $\mathbf{Z}_2$?





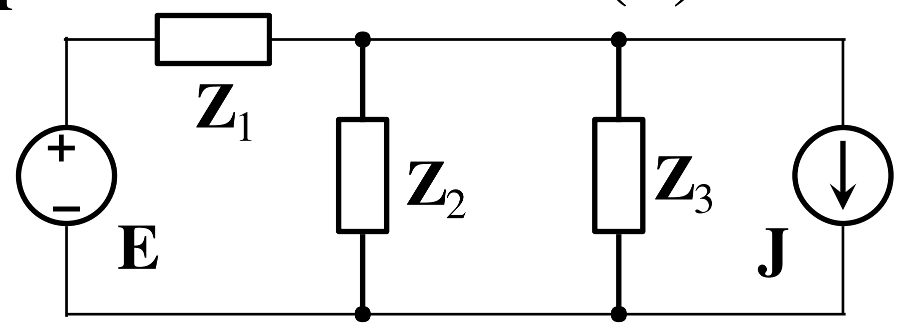


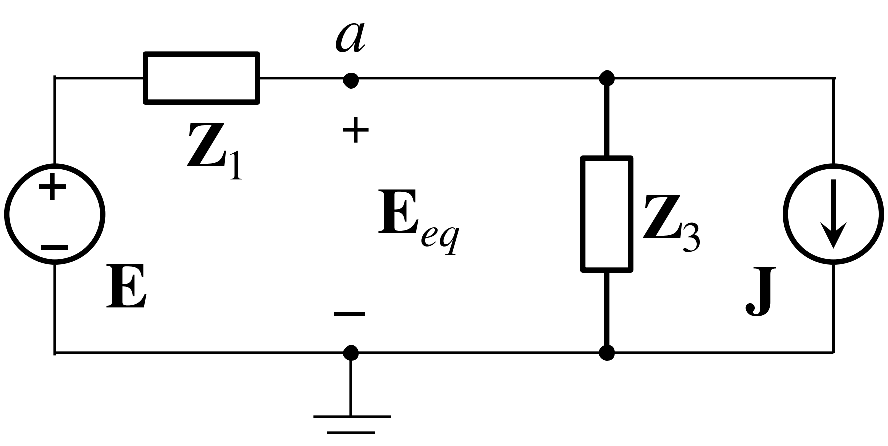
$$\mathbf{Z}_{eq} = \frac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_3} = \frac{12(-j16)}{12 - j16} = \boxed{7.68 - j5.76\,\Omega}$$

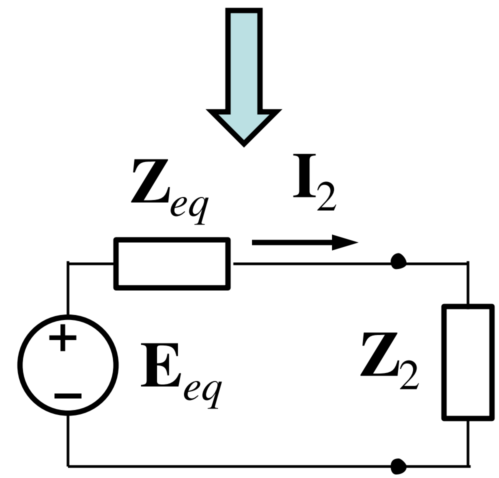
Ex. 1 Thévenin & Norton Equivalent Circuits (4)

$\mathbf{E} = 20\angle -45^\circ$ V; $\mathbf{J} = 5\angle 60^\circ$ A;

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_2 = j10\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$;
Find the current of $\mathbf{Z}_2$?





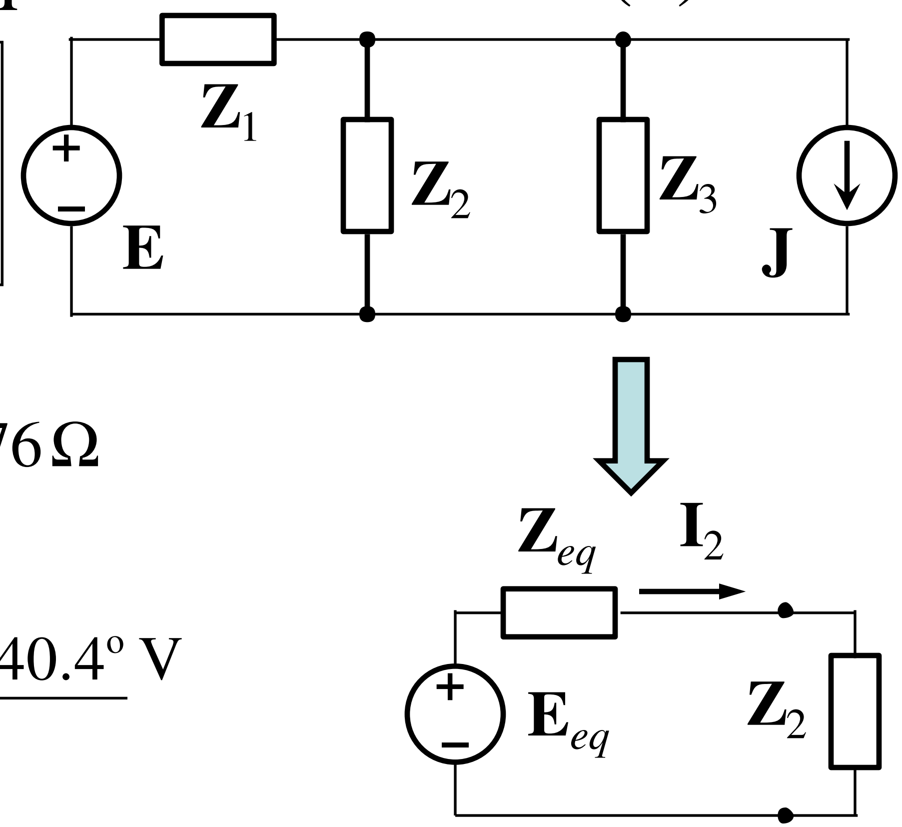


$$\mathbf{E}_{eq} = \mathbf{V}_a = \frac{\dfrac{\mathbf{E}}{\mathbf{Z}_1} - \mathbf{J}}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_3}} = \frac{\dfrac{20\angle -45^\circ}{12} - 5\angle 60^\circ}{\dfrac{1}{12} + \dfrac{1}{-j16}} = \boxed{54.38\angle -140.4^\circ \text{ V}}$$

Ex. 1 Thévenin & Norton Equivalent Circuits (5)

$\mathbf{E} = 20\angle -45^{\circ}$ V; $\mathbf{J} = 5\angle 60^{\circ}$ A;

$\mathbf{Z}_1 = 12\,\Omega$; $\mathbf{Z}_2 = j10\,\Omega$; $\mathbf{Z}_3 = -j16\,\Omega$;
Find the current of $\mathbf{Z}_2$?



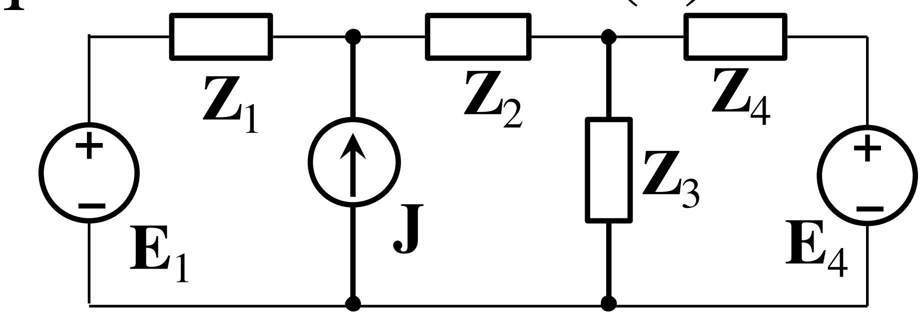
$$\mathbf{Z}_{eq} = 7.68 - j5.76\,\Omega$$

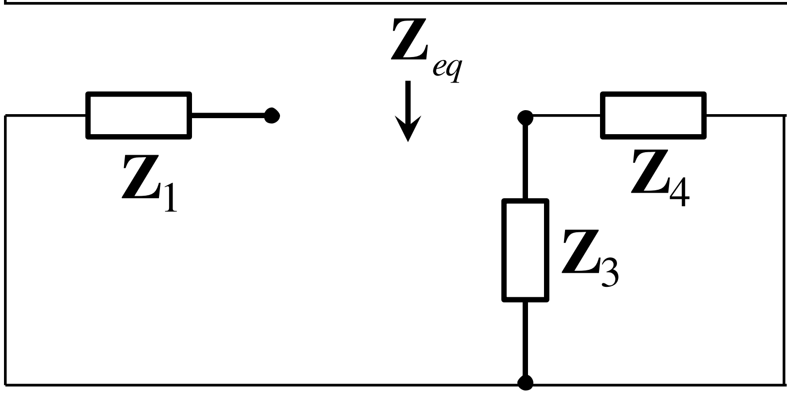
$$\mathbf{E}_{eq} = 54.38\angle -140.4^{\circ}\text{ V}$$

$$\rightarrow \mathbf{I}_2 = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_2} = \frac{54.38\angle -140.4^{\circ}}{7.68 - j5.76 + j10} = \boxed{6.20\angle -169.3^{\circ}\text{ A}}$$

Ex. 2 Thévenin & Norton Equivalent Circuits (6)

$\mathbf{E}_1 = 100\underline{/30^\circ}$ V; $\mathbf{E}_4 = 80\underline{/-45^\circ}$ V; $\mathbf{J} = 5$ A;
$\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = 5\Omega$; $\mathbf{Z}_3 = j20\Omega$; $\mathbf{Z}_4 = -j25\Omega$;
Find the current of $\mathbf{Z}_2$?

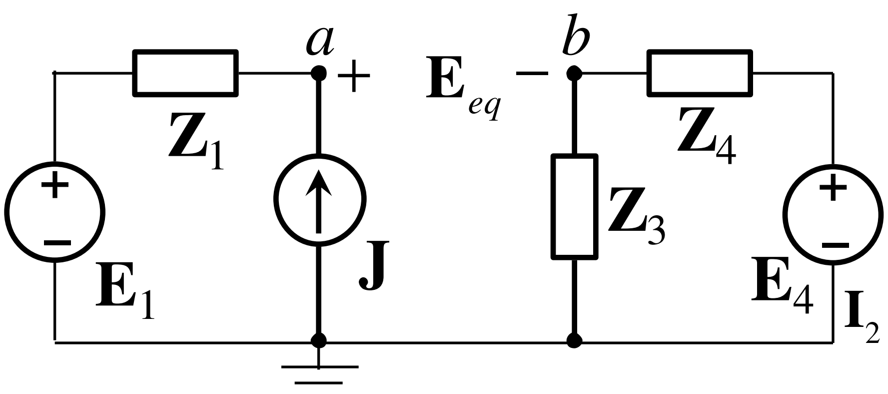




$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} = 10 + \frac{j20(-j25)}{j20 - j25} = 10 + j100\,\Omega$$

$$\mathbf{E}_{eq} = \mathbf{v}_a - \mathbf{v}_b$$

$$-\mathbf{Z}_1\mathbf{J} + \mathbf{v}_a = \mathbf{E}_1 \rightarrow \mathbf{v}_a = \mathbf{E}_1 + \mathbf{Z}_1\mathbf{J} = 137 + j50\text{ V}$$



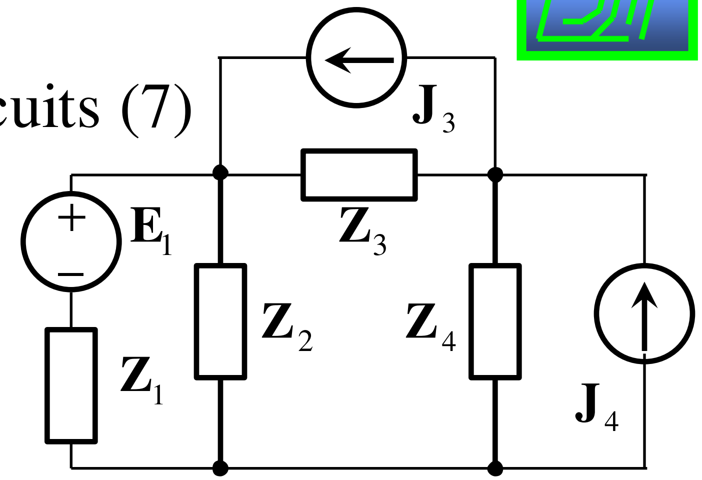
$$\mathbf{v}_b = \mathbf{Z}_3\mathbf{I}_3 = \mathbf{Z}_3\frac{\mathbf{E}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} = -226 + j226\text{V}$$

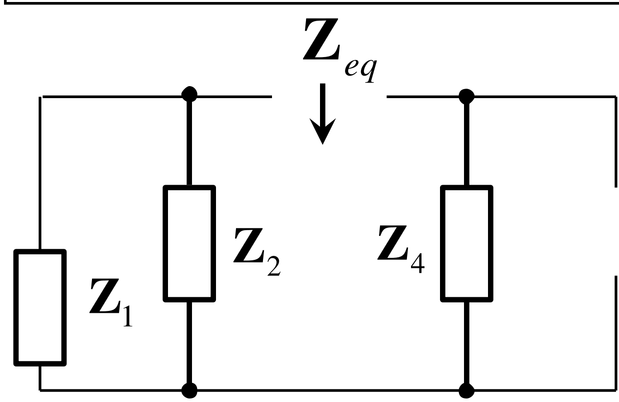
$$\mathbf{E}_{eq} = \mathbf{v}_a - \mathbf{v}_b = 363 - j176\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_2} = \frac{363 - j176}{10 + j100 + 5} = \boxed{-1.19 - j3.81\text{A}}$$

Ex. 3 Thévenin & Norton Equivalent Circuits (7)

$\mathbf{E}_1 = 100\angle 30^\circ$ V; $\mathbf{J}_3 = 5$ A; $\mathbf{J}_4 = 8\angle -45^\circ$ A;
$\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = 5\Omega$; $\mathbf{Z}_3 = j20\Omega$; $\mathbf{Z}_4 = -j25\Omega$;
Find the current of $\mathbf{Z}_3$?



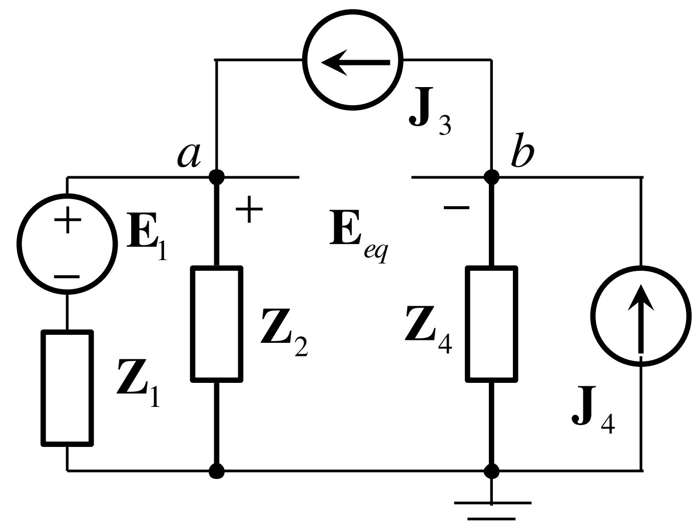


$$\mathbf{Z}_{eq} = \mathbf{Z}_4 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = -j25 + \frac{10\times 5}{10+5} = 3.33 - j25\,\Omega$$

$$\mathbf{E}_{eq} = \mathbf{v}_a - \mathbf{v}_b$$

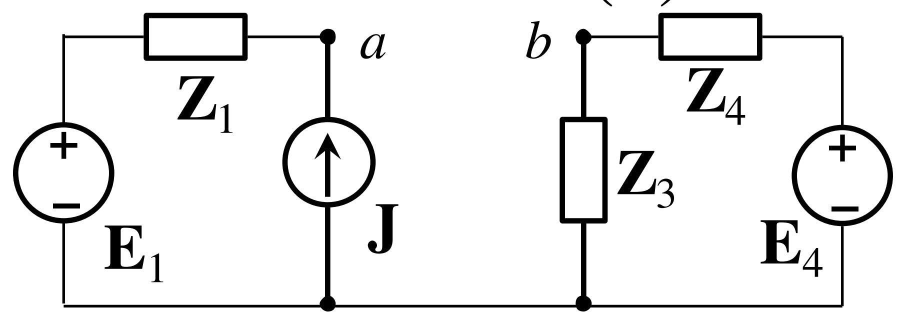
$$\begin{cases} \left(\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2}\right)\mathbf{v}_a = \dfrac{\mathbf{E}_1}{\mathbf{Z}_1} + \mathbf{J}_3 \\ \dfrac{1}{\mathbf{Z}_4}\mathbf{v}_b = -\mathbf{J}_3 + \mathbf{J}_4 \end{cases} \rightarrow \begin{cases} \mathbf{v}_a = 45{,}53 + j16{,}67\text{ V} \\ \mathbf{v}_b = -141{,}4 - j16{,}4\text{ V} \end{cases}$$

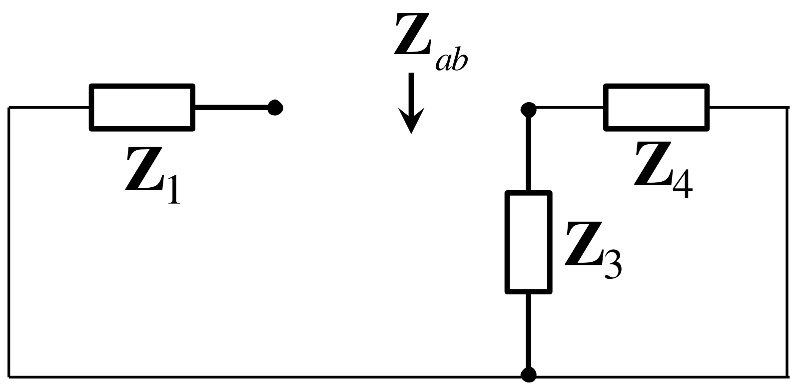
$$\mathbf{I}_3 = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_3} = \boxed{12.68 + j28.94\text{ A}}$$



Ex. 4 Thévenin & Norton Equivalent Circuits (8)

$\mathbf{E}_1 = 100\underline{/30^\circ}$ V; $\mathbf{E}_4 = 80\underline{/-45^\circ}$ V; $\mathbf{J} = 5$ A;
$\mathbf{Z}_1 = 10\Omega$; $\mathbf{Z}_2 = 5\Omega$; $\mathbf{Z}_3 = j20\Omega$; $\mathbf{Z}_4 = -j25\Omega$;
Find $\mathbf{Z}_{ab}$?



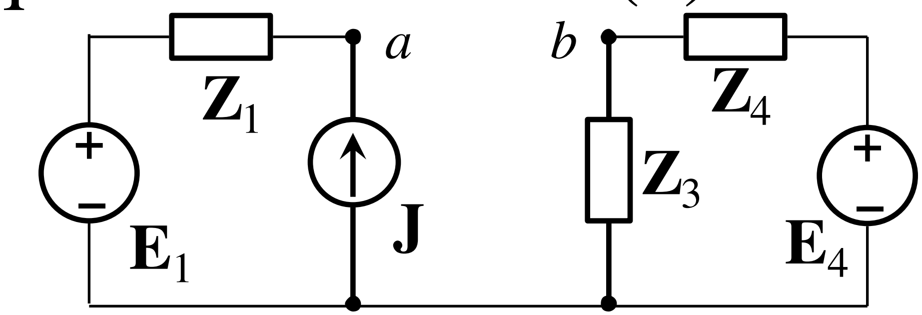


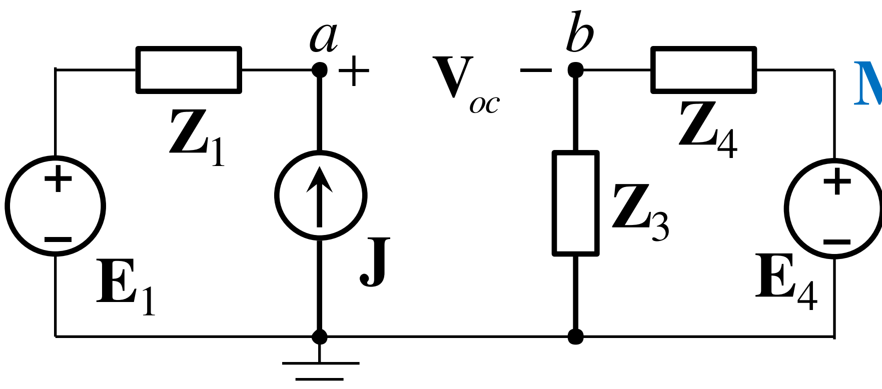
Method 1

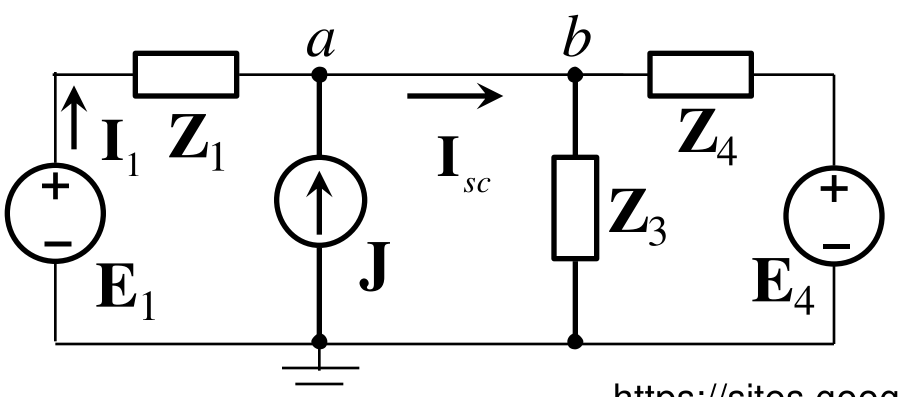
$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} = 10 + \frac{j20(-j25)}{j20 - j25} = \boxed{10 + j100\,\Omega}$$

Ex. 4 Thévenin & Norton Equivalent Circuits (9)

$\mathbf{E}_1 = 100\underline{/30^o}\text{ V}; \mathbf{E}_4 = 80\underline{/-45^o}\text{ V}; \mathbf{J} = 5\text{ A};$
$\mathbf{Z}_1 = 10\Omega; \mathbf{Z}_2 = 5\Omega; \mathbf{Z}_3 = j20\Omega; \mathbf{Z}_4 = -j25\Omega;$
Find $\mathbf{Z}_{ab}$?







Method 2

$$\mathbf{Z}_{ab} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}}$$

$$\mathbf{V}_{oc} = 363 - j176\text{ V}$$

$$\mathbf{I}_{sc} = \mathbf{I}_1 + \mathbf{J} = -1,39 - j3,77\text{ A}$$

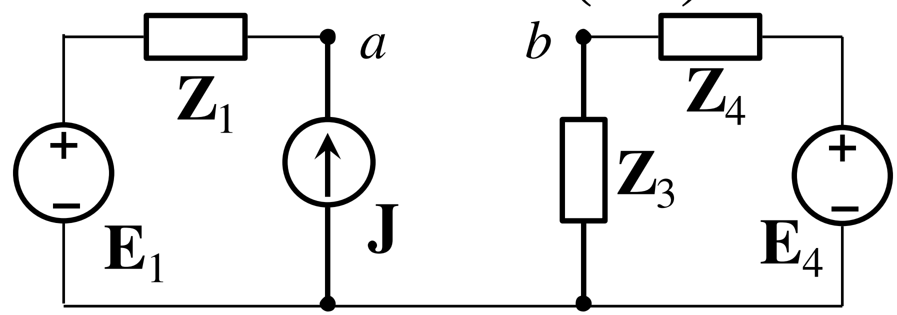
$$\mathbf{v}_a = \frac{\mathbf{E}_1/\mathbf{Z}_1 + \mathbf{J} + \mathbf{E}_4/\mathbf{Z}_4}{1/\mathbf{Z}_1 + 1/\mathbf{Z}_3 + 1/\mathbf{Z}_4} = 150 + j88\text{ V}$$

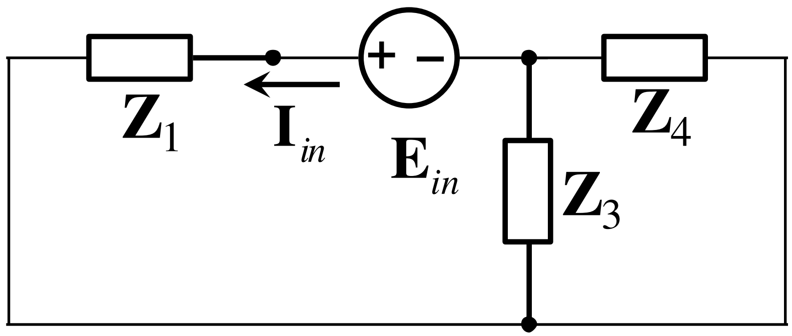
$$\mathbf{I}_1 = (\mathbf{E}_1 - \mathbf{v}_a)/\mathbf{Z}_1 = -6.39 - j3.77\text{ A}$$

$$\rightarrow \boxed{\mathbf{Z}_{ab} = 10 + j100\,\Omega}$$

Ex. 4 Thévenin & Norton Equivalent Circuits (10)

$\mathbf{E}_1 = 100\underline{/30^o}$ V; $\mathbf{E}_4 = 80\underline{/-45^o}$ V; $\mathbf{J} = 5$ A;
$\mathbf{Z}_1 = 10\Omega; \mathbf{Z}_2 = 5\Omega; \mathbf{Z}_3 = j20\Omega; \mathbf{Z}_4 = -j25\Omega;$
Find $\mathbf{Z}_{ab}$?





Method 3

$$\mathbf{Z}_{ab} = \frac{\mathbf{E}_{in}}{\mathbf{I}_{in}}$$

$$\mathbf{E}_{in} = 100\text{V}$$

$$\mathbf{I}_{in} = \frac{100}{\mathbf{Z}_1 + \dfrac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}} = \frac{100}{10 + \dfrac{j20(-j25)}{j20 - j25}} = 0.099 - j0.099\,\text{A}$$

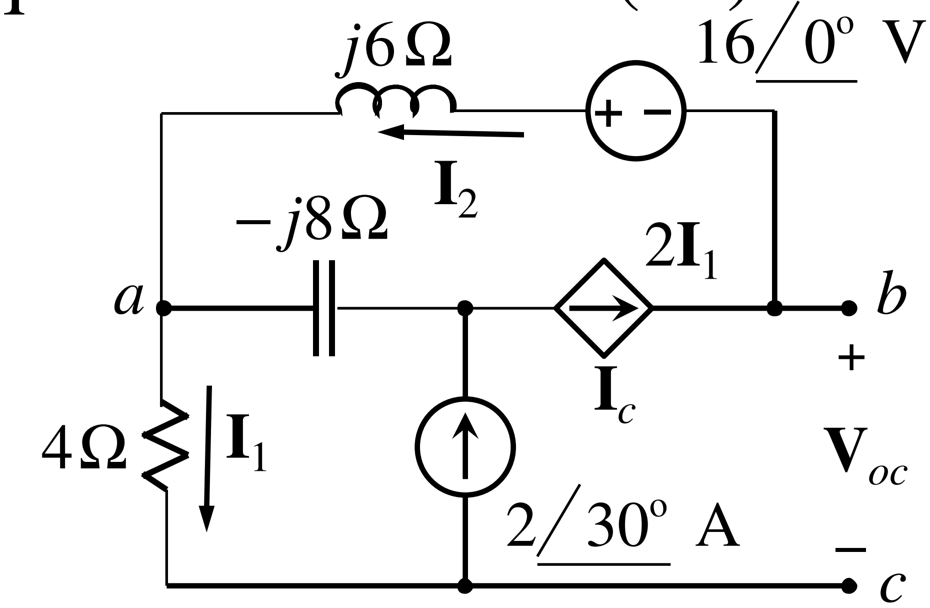
$$\rightarrow \mathbf{Z}_{ab} = \frac{100}{0.099 - j0.099} = \boxed{10 + j100\,\Omega}$$

Thévenin & Norton Equivalent Circuits (11)

Ex. 5

Find $\mathbf{Z}_{eq}$?

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}}$$



$$\left.\begin{array}{r}(\mathbf{V}_c - \mathbf{V}_b) - 16 + j6\mathbf{I}_2 + 4\mathbf{I}_1 = 0 \\ \mathbf{V}_{oc} = \mathbf{V}_b - \mathbf{V}_c\end{array}\right\}$$

$$\left.\begin{array}{l}\rightarrow \mathbf{V}_{oc} = -16 + j6\mathbf{I}_2 + 4\mathbf{I}_1 \\ \mathbf{I}_1 = 2\angle 30^\circ \\ \mathbf{I}_2 = \mathbf{I}_c = 2\mathbf{I}_1 = 2\times 2\angle 30^\circ\end{array}\right\}$$

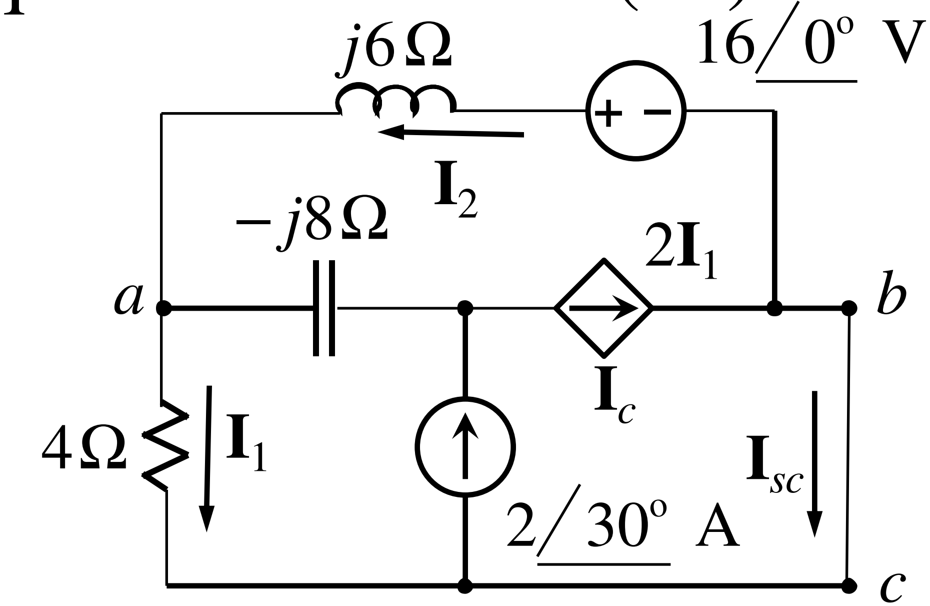
$$\rightarrow \mathbf{V}_{oc} = -16 + j6\times 2\times 2\angle 30^\circ + 4\times 2\angle 30^\circ = \boxed{-21.07 + j24.78 \text{ V}}$$

Thévenin & Norton Equivalent Circuits (12)

Ex. 5

Find $\mathbf{Z}_{eq}$?

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}}$$



$$\mathbf{I}_1 - 2\angle 30^\circ + \mathbf{I}_{sc} = 0$$

$$\to \mathbf{I}_{sc} = 2\angle 30^\circ - \mathbf{I}_1$$

$$\left.\begin{aligned} j6\mathbf{I}_2 + 4\mathbf{I}_1 &= 16\angle 0^\circ \\ \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^\circ - \mathbf{I}_c &= 0 \\ \to \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^\circ - 2\mathbf{I}_1 &= 0 \\ \to 3\mathbf{I}_1 - \mathbf{I}_2 &= 2\angle 30^\circ \end{aligned}\right\} \to \mathbf{I}_1 = 0.67 - j0.41\text{ A}$$

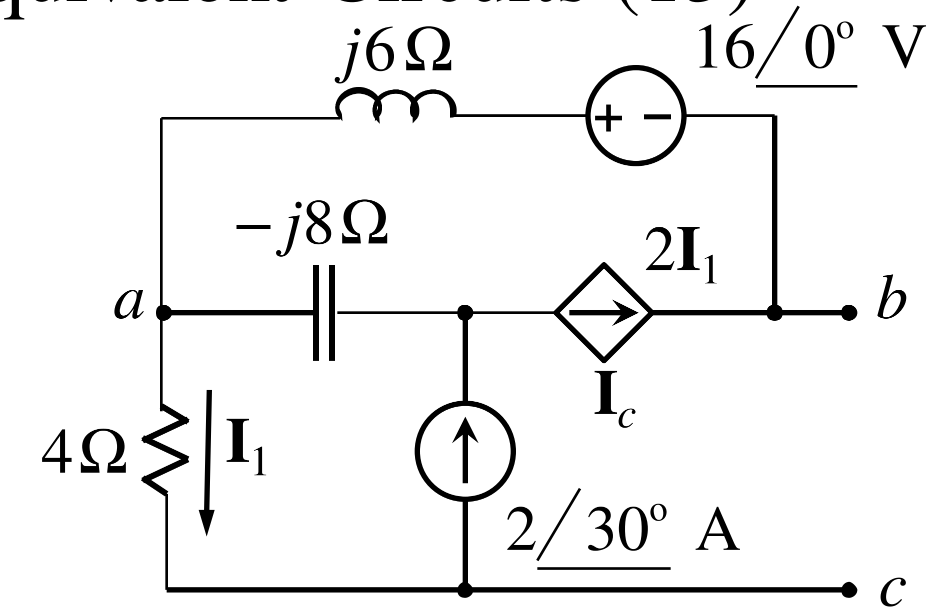
$$\to \mathbf{I}_{sc} = 2\angle 30^\circ - (0.67 - j0.41)$$

$$= \boxed{1.06 + j1.41\text{ A}}$$

Thévenin & Norton Equivalent Circuits (13)

Ex. 5

Find $\mathbf{Z}_{eq}$? **Method 1**



$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open\text{-}circuit}}{\mathbf{I}_{short\text{-}circuit}}$$

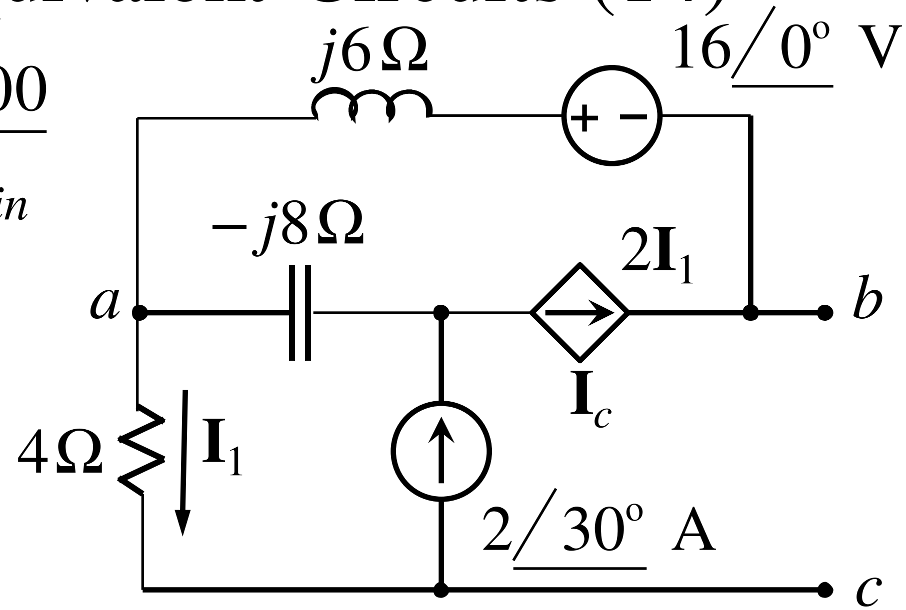
$$\mathbf{V}_{oc} = -21.07 + j24.78 \text{ V}$$

$$\mathbf{I}_{sc} = 1.06 + j1.41 \text{ A}$$

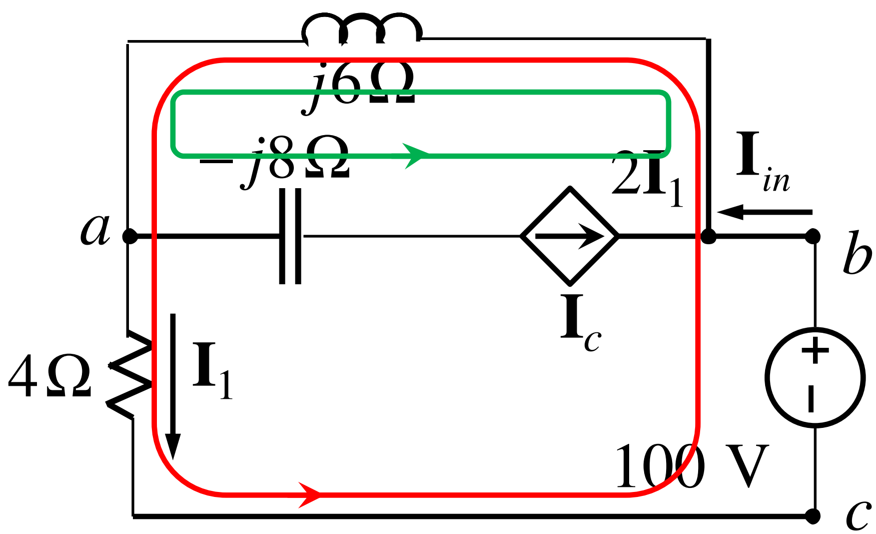
$$\rightarrow \mathbf{Z}_{eq} = \frac{-21.07 + j24.78}{1.06 + j1.41} = \boxed{4.00 + j18.00\ \Omega}$$

Thévenin & Norton Equivalent Circuits (14)

Ex. 5



Find $\mathbf{Z}_{eq}$? **Method 2** $\mathbf{Z}_{eq} = \dfrac{100}{\mathbf{I}_{in}}$

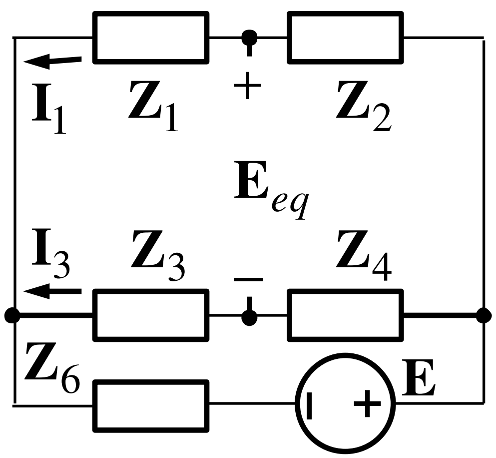


$$\left.\begin{aligned} j6(\mathbf{I}_r + \mathbf{I}_g) + 4\mathbf{I}_r &= 100 \\ \mathbf{I}_g = 2\mathbf{I}_1 &= 2\mathbf{I}_r \end{aligned}\right\} \rightarrow \mathbf{I}_r = 1.18 - j5.29\text{ A} = \mathbf{I}_{in}$$

$$\rightarrow \mathbf{Z}_{eq} = \frac{100}{1.18 - j5.29} = \boxed{4.00 + j18.00\ \Omega}$$

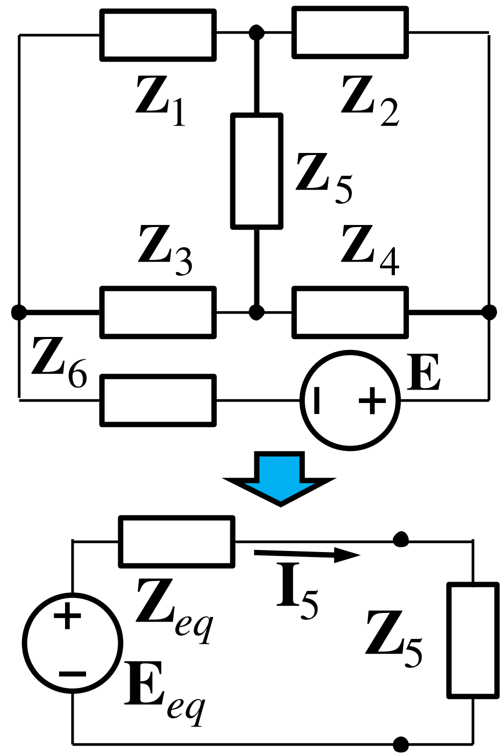
Ex. 6 Thévenin & Norton Equivalent Circuits (15)

$\mathbf{E} = 100\text{V}$; $\mathbf{Z}_1 = j10$; $\mathbf{Z}_2 = 5 - j10$; $\mathbf{Z}_3 = 10 + j5$; $\mathbf{Z}_4 = 20$; $\mathbf{Z}_5 = -j20$; $\mathbf{Z}_6 = 5 + j5$; find the current flowing through $\mathbf{Z}_5$?



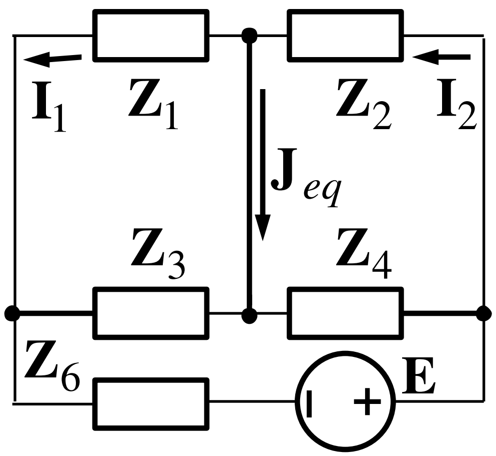
$$\left.\begin{aligned}
&\mathbf{E}_{eq} + \mathbf{Z}_3\mathbf{I}_3 - \mathbf{Z}_1\mathbf{I}_1 = 0 \\
&\mathbf{I}_6 = \frac{\mathbf{E}}{\dfrac{(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4} + \mathbf{Z}_6} \\
&\quad = 8.27 - j4.53\,\text{A} \\
&\mathbf{I}_1 = \frac{(\mathbf{Z}_3+\mathbf{Z}_4)\mathbf{I}_6}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4} = 7.20 - j3.73\,\text{A} \\
&\mathbf{I}_3 = \mathbf{I}_6 - \mathbf{I}_1 = 1.07 - j0.80\,\text{A}
\end{aligned}\right\}$$

$$\rightarrow \mathbf{E}_{eq} = -6.67 + j13.33\,\text{V}$$

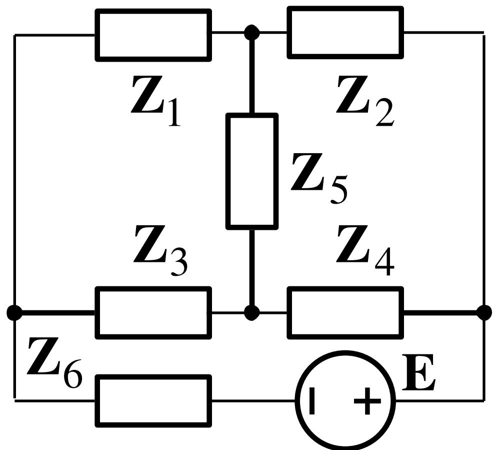


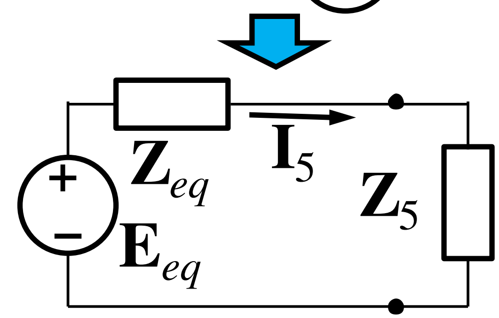
Ex. 6 Thévenin & Norton Equivalent Circuits (16)

$\mathbf{E} = 100\text{V}$; $\mathbf{Z}_1 = j10$; $\mathbf{Z}_2 = 5 - j10$; $\mathbf{Z}_3 = 10 + j5$; $\mathbf{Z}_4 = 20$; $\mathbf{Z}_5 = -j20$; $\mathbf{Z}_6 = 5 + j5$; find the current flowing through $\mathbf{Z}_5$?



$$\left.\begin{aligned}
&\mathbf{I}_2 - \mathbf{I}_1 - \mathbf{J}_{eq} = 0 \\
&\mathbf{I}_6 = \frac{\mathbf{E}}{\dfrac{\mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_3} + \dfrac{\mathbf{Z}_2\mathbf{Z}_4}{\mathbf{Z}_2 + \mathbf{Z}_4} + \mathbf{Z}_6} \\
&= 6.27 - j2.14\text{ A} \\
&\mathbf{I}_1 = \frac{\mathbf{Z}_3\mathbf{I}_6}{\mathbf{Z}_1 + \mathbf{Z}_3} = 2.72 - j3.08\text{ A} \\
&\mathbf{I}_2 = \frac{\mathbf{Z}_4\mathbf{I}_6}{\mathbf{Z}_2 + \mathbf{Z}_4} = 4.92 + j0.26\text{ A}
\end{aligned}\right\}$$





$$\mathbf{E}_{eq} = -6.67 + j13.33\text{ V}$$

$$\rightarrow \mathbf{J}_{eq} = 2.20 + j3.34\text{ A} \rightarrow \mathbf{I}_5 = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_5 + \dfrac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}} = \boxed{-0.83 - j0.31\text{ A}}$$

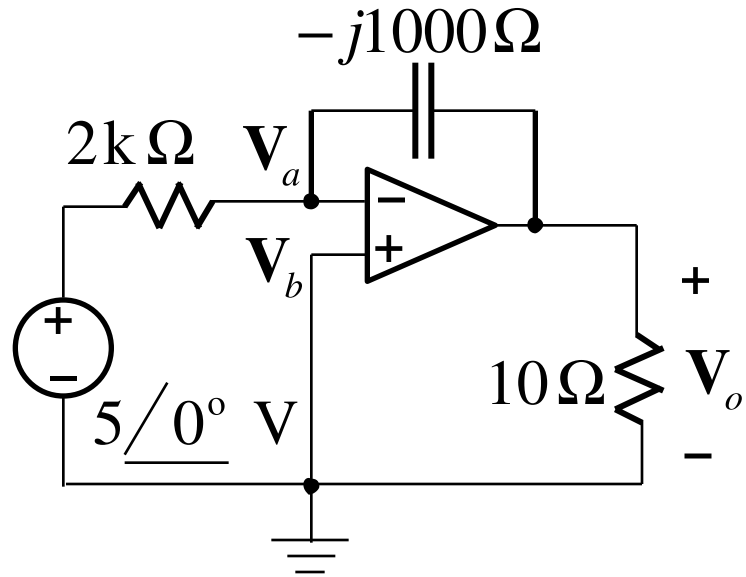
Sinusoidal Steady-State Analysis

1. Sinusoidal Steady-State Analysis
2. Ohm's Law
3. Kirchhoff's Laws
4. Impedance Combinations
5. Branch Current Method
6. Node Voltage Method
7. Mesh Current Method
8. Superposition Theorem
9. Source Transformation
10. Thévenin & Norton Equivalent Circuits
11. **Op Amp AC Circuits**

Op Amp AC Circuits (1)

Ex. 1

Find $\mathbf{V}_o$?

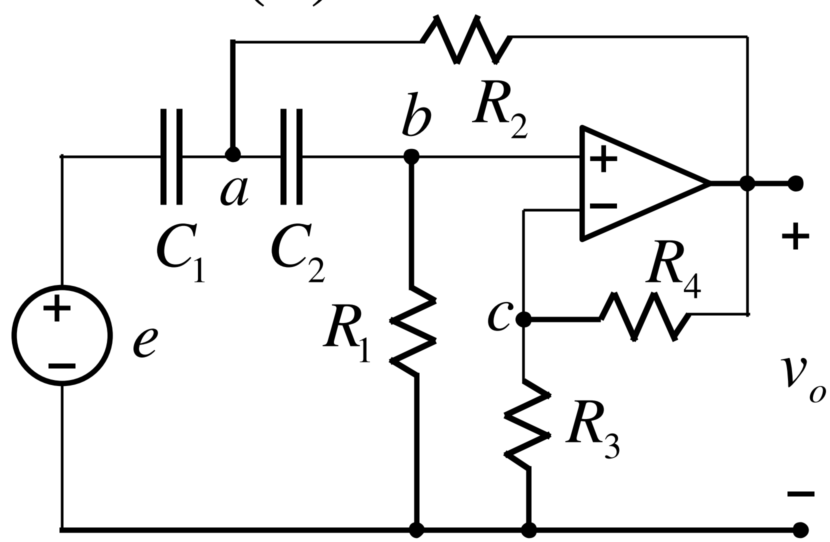


$$\frac{5\underline{/0^o}}{2000} + \frac{\mathbf{V}_o}{-j1000} = 0$$

$$\rightarrow \mathbf{V}_o = j1000\frac{5\underline{/0^o}}{2000} = j2.5\underline{/0^o} = \boxed{2.5\underline{/90^o}\ \mathrm{V}}$$

Op Amp AC Circuits (2)

Ex. 2



Find v_o?

$$a: \quad \frac{\mathbf{E} - \mathbf{V}_a}{\mathbf{Z}_{C1}} = \frac{\mathbf{V}_a - \mathbf{V}_o}{R_2} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_{C2}}$$

$$b: \quad \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_{C2}} = \frac{\mathbf{V}_b}{R_1}$$

$$c: \quad \mathbf{V}_c = \frac{R_3}{R_3 + R_4}\mathbf{V}_o = \mathbf{V}_b$$

$$\rightarrow \mathbf{V}_o = f(\mathbf{E}, R_1, R_2, R_3, R_4, \mathbf{Z}_{C1}, \mathbf{Z}_{C2})$$