Electric Circuit Theory

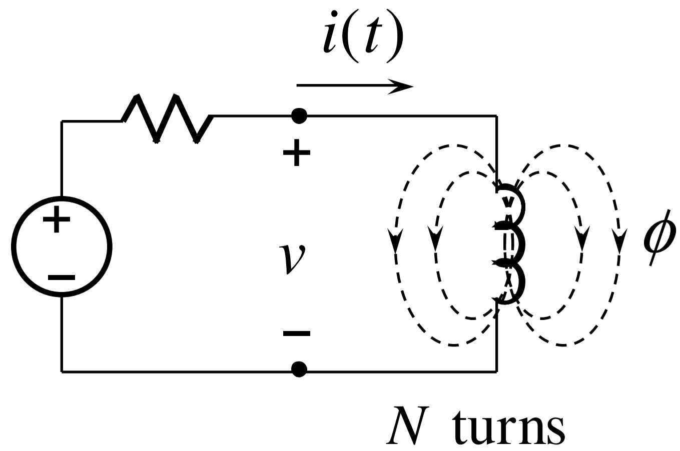
Magnetically Coupled Circuits

Contents

Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. Analysis of Magnetically Coupled Circuits
4. Energy in a Coupled Circuit
5. Transformers

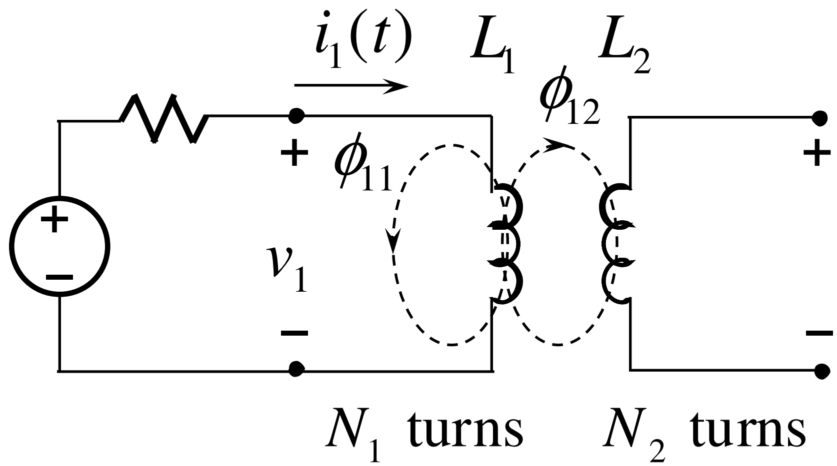
Mutual Inductance (1)



Faraday's law: $v = N\dfrac{d\phi}{dt} = N\dfrac{d\phi}{di}\dfrac{di}{dt} = L\dfrac{di}{dt}$

$$L = N\frac{d\phi}{di}$$

Mutual Inductance (2)



$$\boxed{v_2 = M_{21}\frac{di_1}{dt}}$$

$$\phi_1 = \phi_{11} + \phi_{12}$$

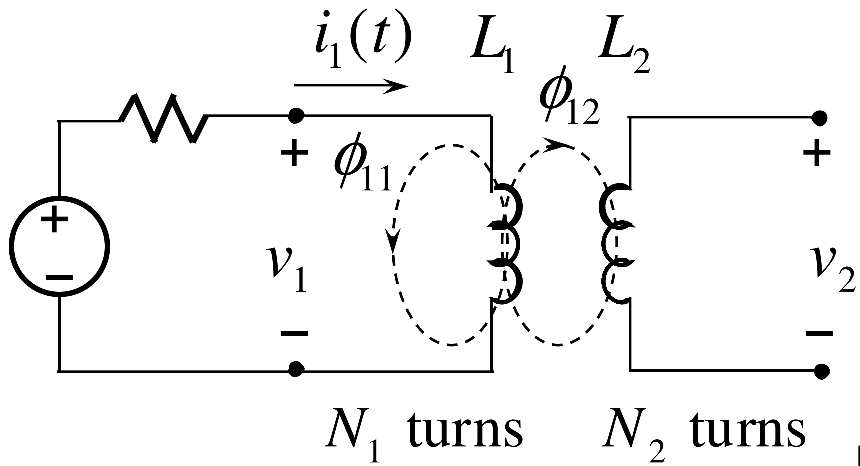
$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$= N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$= N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

Mutual Inductance (3)



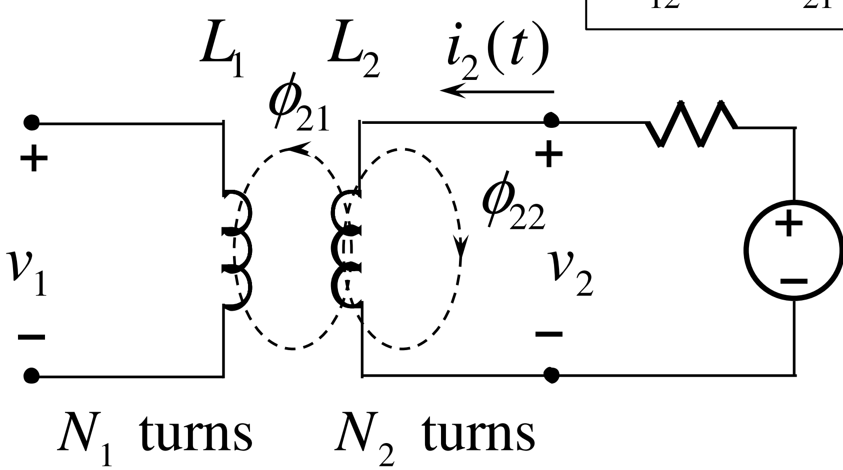
$$v_1 = L_1 \frac{di_1}{dt}$$

$$v_2 = M_{21} \frac{di_1}{dt}$$

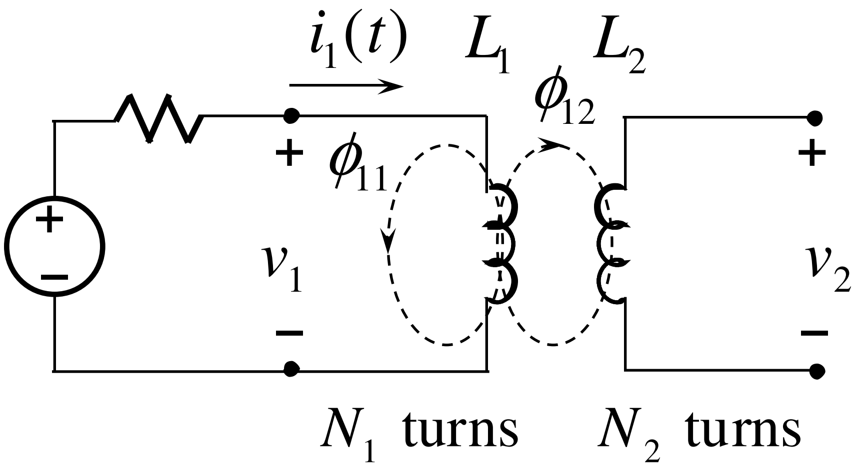
$$\boxed{M_{12} = M_{21} = M = k\sqrt{L_1 L_2}}$$

$$v_1 = M_{12} \frac{di_2}{dt}$$

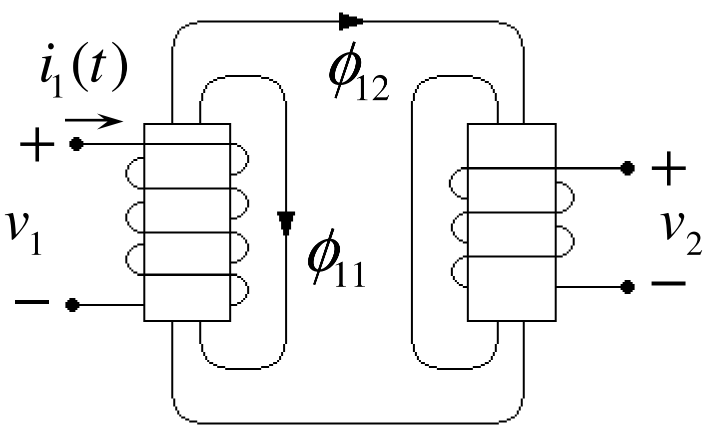
$$v_2 = L_2 \frac{di_2}{dt}$$



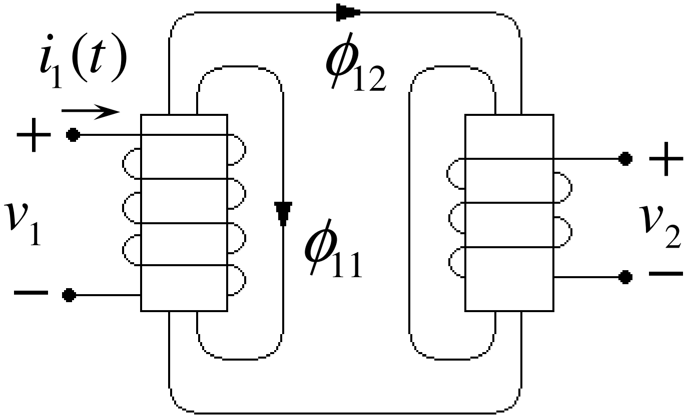
Mutual Inductance (4)



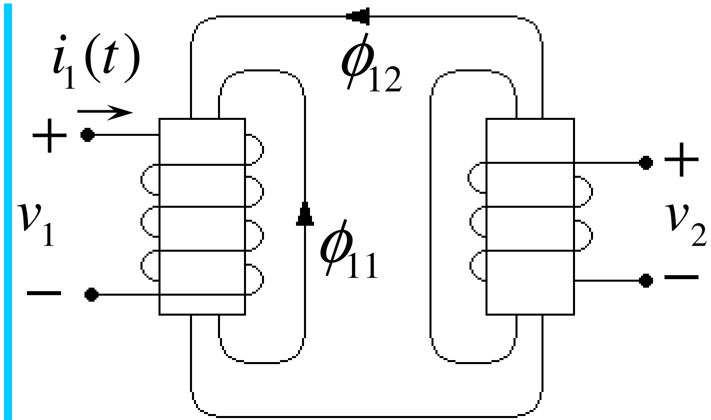
$$v_2 = M_{21}\frac{di_1}{dt}$$



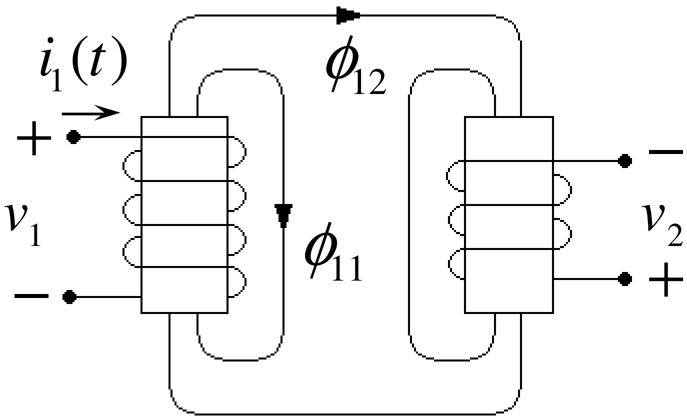
Mutual Inductance (5)



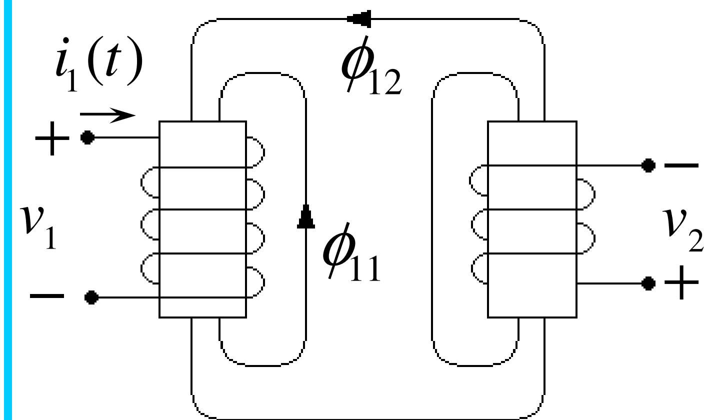
$$v_2 = M\frac{di_1}{dt}$$



$$v_2 = -M\frac{di_1}{dt}$$



$$v_2 = -M\frac{di_1}{dt}$$



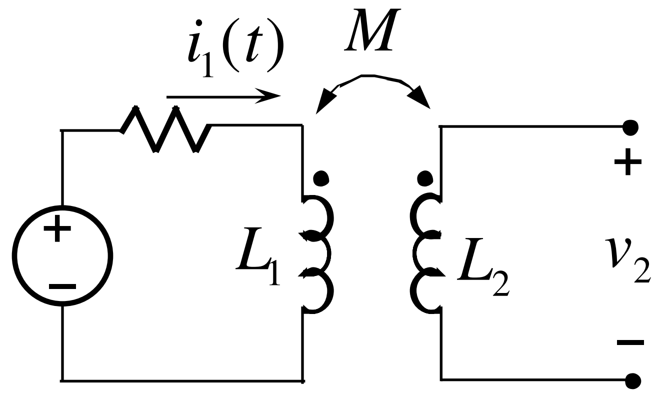
$$v_2 = M\frac{di_1}{dt}$$

Magnetically Coupled Circuits

1. Mutual Inductance
2. **Dot Convention**
3. Analysis of Magnetically Coupled Circuits
4. Energy in a Coupled Circuit
5. Transformers

Dot Convention (1)

- If a current **enters** a dotted terminal of one coil, it induces a **positive** voltage at the dotted terminal of the second coil

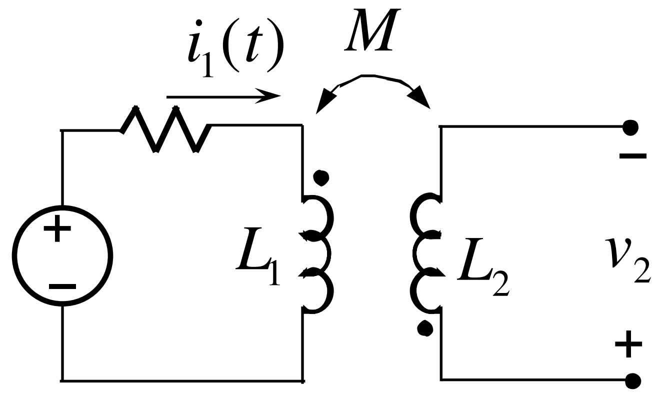


$$v_2 = M\frac{di_1}{dt}$$

- If a current **leaves** a dotted terminal of one coil, it induces a **negative** voltage at the dotted terminal of the second coil

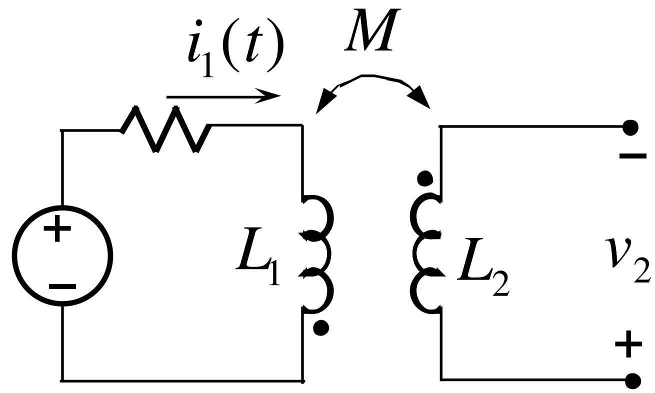
Dot Convention (2)

- If a current **enters** a dotted terminal of one coil, it induces a **positive** voltage at the dotted terminal of the second coil



- If a current **leaves** a dotted terminal of one coil, it induces a **negative** voltage at the dotted terminal of the second coil

Dot Convention (3)

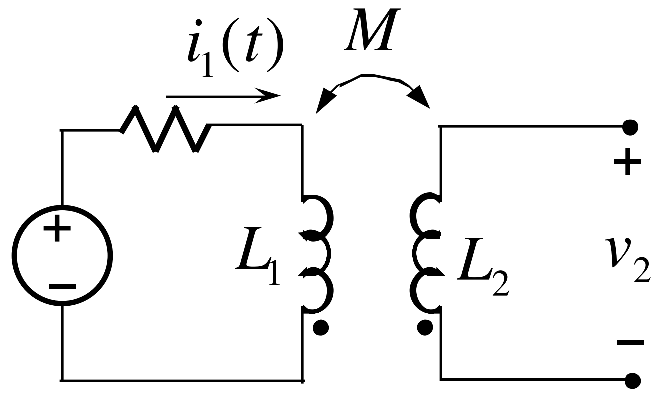


- If a current **enters** a dotted terminal of one coil, it induces a **positive** voltage at the dotted terminal of the second coil

- If a current **leaves** a dotted terminal of one coil, it induces a **negative** voltage at the dotted terminal of the second coil

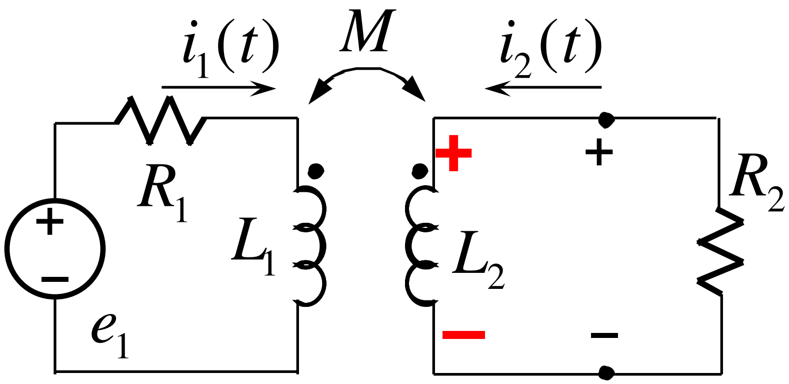
Dot Convention (4)

- If a current **enters** a dotted terminal of one coil, it induces a **positive** voltage at the dotted terminal of the second coil

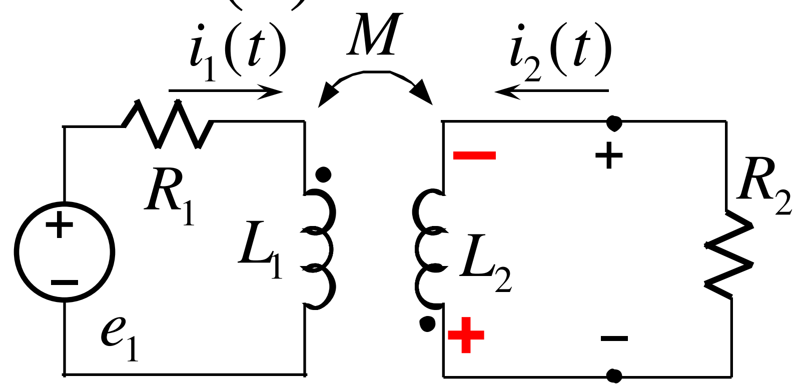


- If a current **leaves** a dotted terminal of one coil, it induces a **negative** voltage at the dotted terminal of the second coil

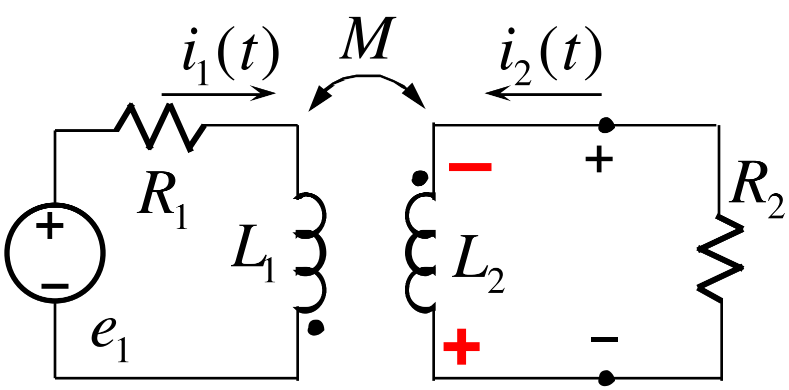
Dot Convention (5)



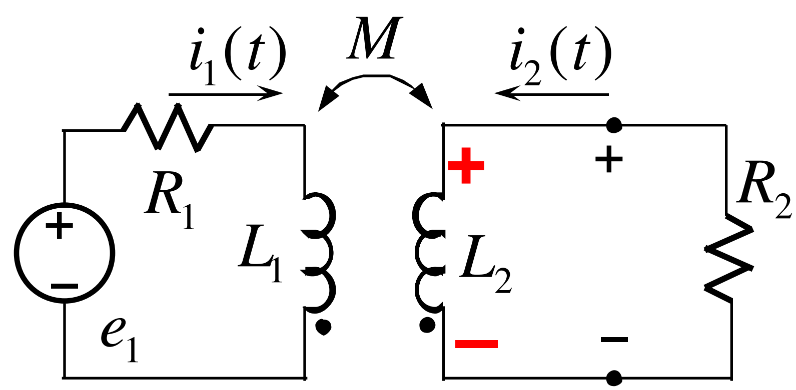
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. **Analysis of Magnetically Coupled Circuits**
   a) **MCC in Phasor Domain**
   b) **Branch Current Method**
   c) **Mesh Current Method**
   d) **Equivalent Subcircuits**
4. Energy in a Coupled Circuit
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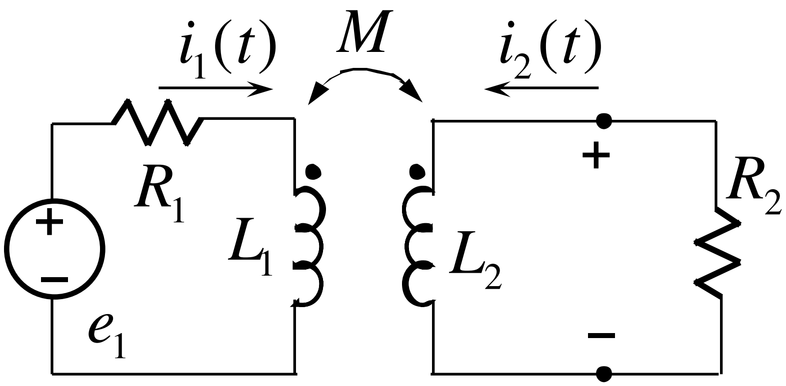
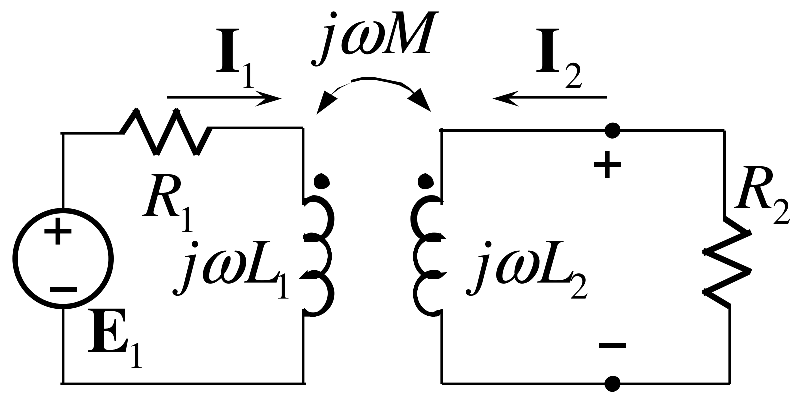
MCC in Phasor Domain (1)





$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \rightarrow \mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

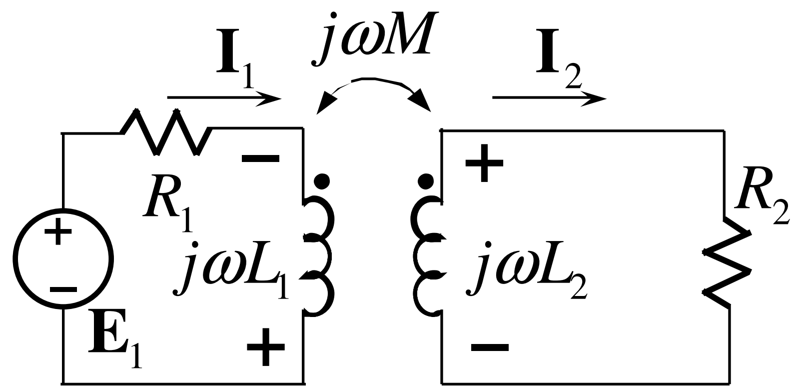
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \rightarrow \mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M \mathbf{I}_1$$

$$\boxed{\mathbf{Z}_M = j\omega M}$$

MCC in Phasor Domain (2)

Ex. 1

$\mathbf{E}_1 = 100\angle 0^\circ$ V; $\omega = 100$ rad/s;
$L_1 = 0.2\,\text{H}; L_2 = 0.3\,\text{H}; M = 0.1\,\text{H};$
$R_1 = 30\Omega; R_2 = 40\Omega;$ Find currents?



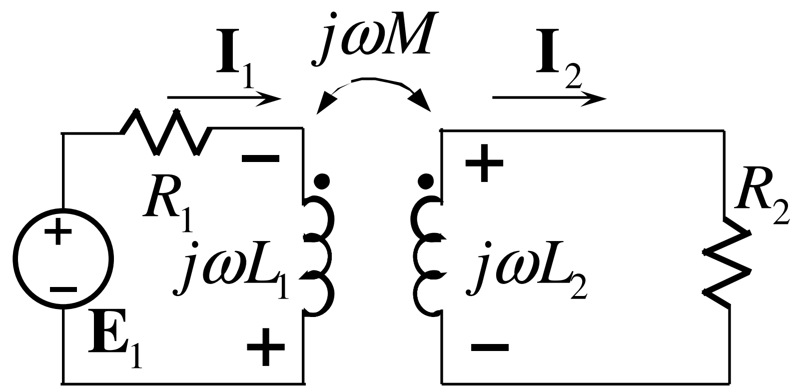
$$\left.\begin{aligned} \mathbf{V}_{M1} &= j\omega M\mathbf{I}_2 \\ \mathbf{V}_{1L} &= j\omega L_1\mathbf{I}_1 \\ \mathbf{V}_{R1} + \mathbf{V}_{1L} - \mathbf{V}_{1M} &= \mathbf{E}_1 \end{aligned}\right\} \rightarrow R_1\mathbf{I}_1 + j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 = \mathbf{E}_1$$

$$\left.\begin{aligned} \mathbf{V}_{M2} &= j\omega M\mathbf{I}_1 \\ \mathbf{V}_{2L} &= j\omega L_2\mathbf{I}_2 \\ \mathbf{V}_{R2} + \mathbf{V}_{2L} - \mathbf{V}_{2M} &= 0 \end{aligned}\right\} \rightarrow R_2\mathbf{I}_2 + j\omega L_2\mathbf{I}_2 - j\omega M\mathbf{I}_1 = 0$$

MCC in Phasor Domain (3)

Ex. 1

$\mathbf{E}_1 = 100\angle 0^\circ$ V; $\omega = 100$ rad/s;
$L_1 = 0.2\,\text{H}; L_2 = 0.3\,\text{H}; M = 0.1\,\text{H};$
$R_1 = 30\Omega; R_2 = 40\Omega;$ Find currents?



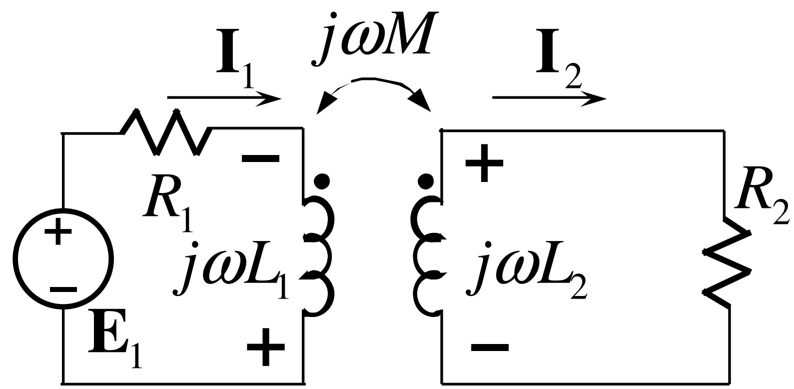
$$\left.\begin{aligned} R_1\mathbf{I}_1 + j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 &= \mathbf{E}_1 \\ R_2\mathbf{I}_2 + j\omega L_2\mathbf{I}_2 - j\omega M\mathbf{I}_1 &= 0 \end{aligned}\right\}$$

$$\rightarrow \begin{cases} 30\mathbf{I}_1 + j100\times 0.2\mathbf{I}_1 - j100\times 0.1\mathbf{I}_2 = 100\angle 0^\circ \\ 40\mathbf{I}_2 + j100\times 0.3\mathbf{I}_2 - j100\times 0.1\mathbf{I}_1 = 0 \end{cases} \rightarrow \begin{cases} \mathbf{I}_1 = 2.34 - j1.39\,\text{A} \\ \mathbf{I}_2 = 0.50 + j0.21\,\text{A} \end{cases}$$

MCC in Phasor Domain (4)

Ex. 1

$\mathbf{E}_1 = 100\angle 0^\circ$ V; $\omega = 100$ rad/s;
$L_1 = 0.2\,\text{H}; L_2 = 0.3\,\text{H}; M = 0.1\,\text{H};$
$R_1 = 30\Omega; R_2 = 40\Omega;$ Find currents?



$$\mathbf{V}_{M1} = j\omega M\mathbf{I}_2;\ \mathbf{V}_{M2} = j\omega M\mathbf{I}_1$$

$$\begin{cases} \mathbf{V}_{R1} + \mathbf{V}_{1L} - \mathbf{V}_{1M} = \mathbf{E}_1 \\ \mathbf{V}_{R2} + \mathbf{V}_{2L} - \mathbf{V}_{2M} = 0 \end{cases}$$

$$\rightarrow \begin{cases} R_1\mathbf{I}_1 + j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 = \mathbf{E}_1 \\ R_2\mathbf{I}_2 + j\omega L_2\mathbf{I}_2 - j\omega M\mathbf{I}_1 = 0 \end{cases}$$

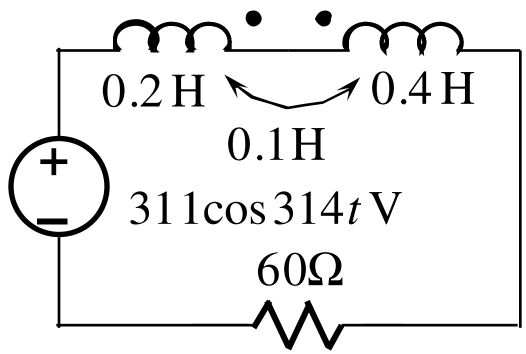
$$\rightarrow \begin{cases} \mathbf{I}_1 = 2.34 - j1.39\,\text{A} \\ \mathbf{I}_2 = 0.50 + j0.21\,\text{A} \end{cases}$$

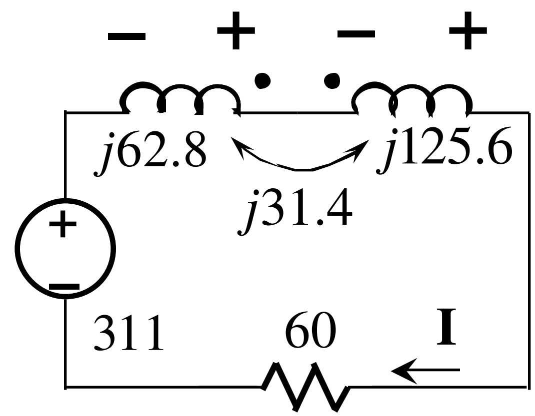
1. Write voltages of mutual inductance
2. Assign signs at dotted terminals (using dot convention)
3. Write KVL equations
4. Write the set of equations & solve for it

MCC in Phasor Domain (5)

Ex. 2

Find current?





$$j62.8\mathbf{I} - j31.4\mathbf{I} + j125.6\mathbf{I} - j31.4\mathbf{I} + 60\mathbf{I} = 311$$

$$\rightarrow \mathbf{I} = 2.23\angle -64.5^{\circ} \text{ A}$$

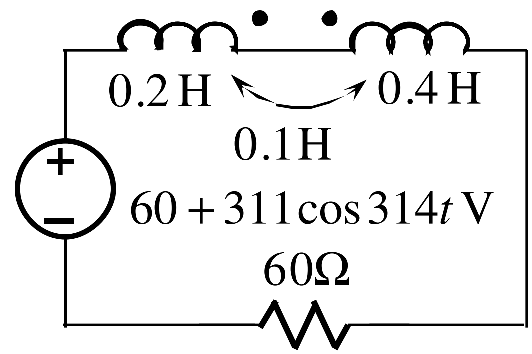
$$\rightarrow i = 2.23\cos(314t - 64.5^{\circ}) \text{ A}$$

1. Write voltages of mutual inductance
2. Assign signs at dotted terminals (using dot convention)
3. Write KVL equations
4. Write the set of equations & solve for it

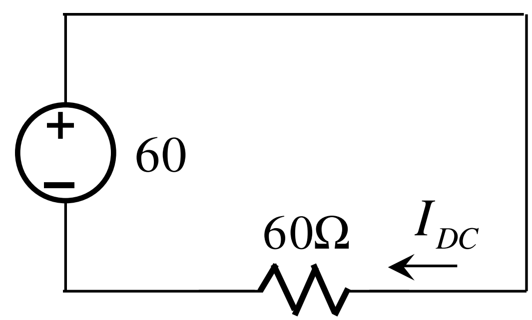
MCC in Phasor Domain (6)

Ex. 3

Find current?



$$I_{DC} = \frac{60}{60} = 1\text{ A}$$

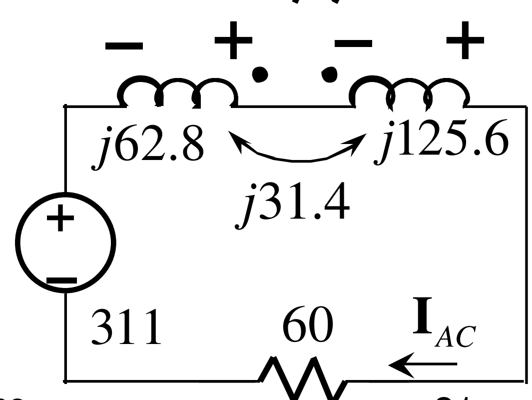


$$(j62.8 - j31.4 + j125.6 - j31.4 + 60)\mathbf{I}_{AC} = 311$$

$$\rightarrow \mathbf{I}_{AC} = 2.23\angle -64.5^\circ\text{ A}$$

$$\rightarrow i_{AC} = 2.23\cos(314t - 64.5^\circ)\text{ A}$$

$$\rightarrow i = I_{DC} + i_{AC} = \boxed{1 + 2.23\cos(314t - 64.5^\circ)\text{ A}}$$



Magnetically Coupled Circuits

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2. Dot Convention
3. **Analysis of Magnetically Coupled Circuits**
   a) MCC in Phasor Domain
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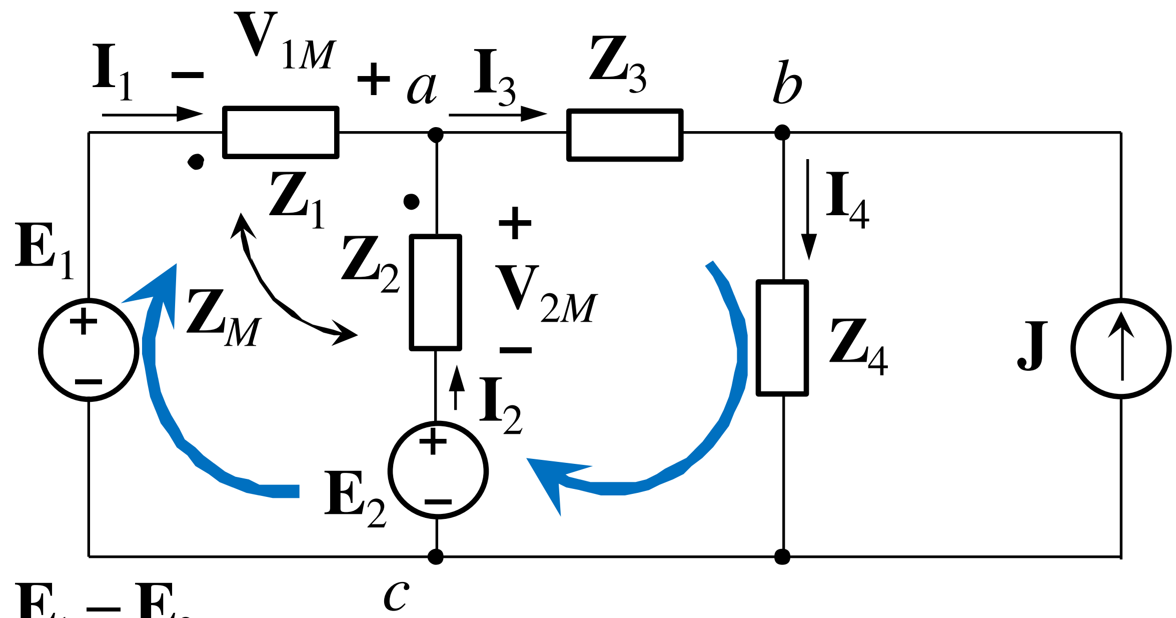
Branch Current Method (1)

Ex. 1



$$\mathbf{V}_{1M} = \mathbf{Z}_M \mathbf{I}_2$$

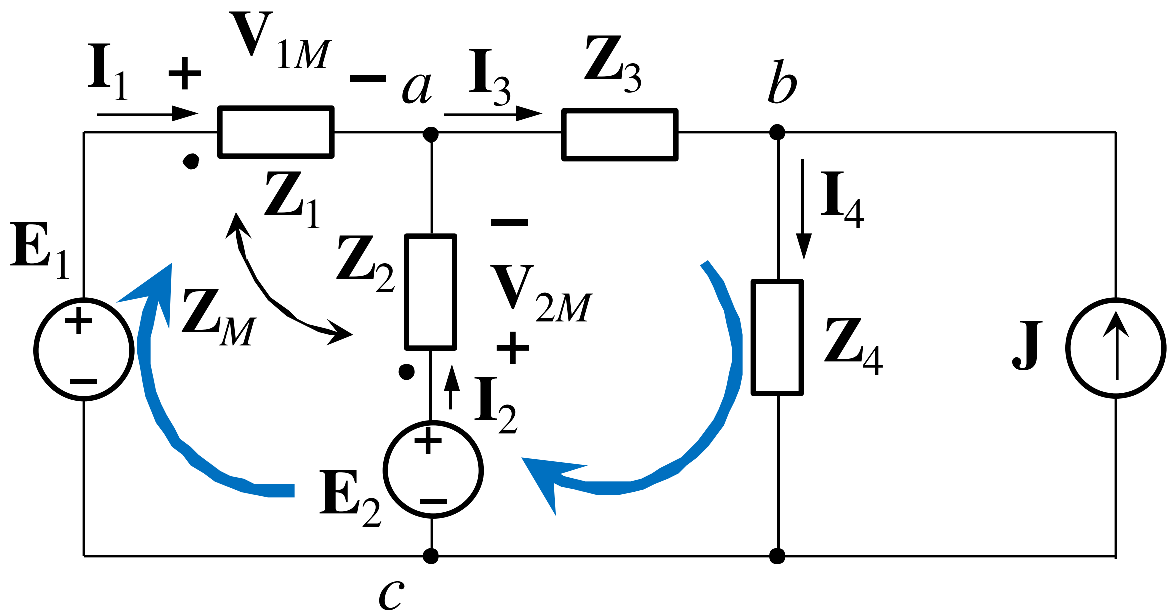
$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_1$$

$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_3 - \mathbf{I}_4 + \mathbf{J} = 0 \\ \mathbf{V}_{Z1} - \mathbf{V}_{1M} - \mathbf{V}_{Z2} + \mathbf{V}_{2M} = \mathbf{E}_1 - \mathbf{E}_2 \\ \quad \rightarrow \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 = \\ \quad = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{V}_{Z2} - \mathbf{V}_{2M} + \mathbf{V}_3 + \mathbf{V}_4 = \mathbf{E}_2 \\ \quad \rightarrow \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases}$$

1. Write voltages of mutual inductance
2. Assign signs at dotted terminals (using dot convention)
3. Write KVL equations
4. Write the set of equations & solve for it

Branch Current Method (2)

Ex. 2



$$\mathbf{V}_{1M} = \mathbf{Z}_M \mathbf{I}_2$$

$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_1$$

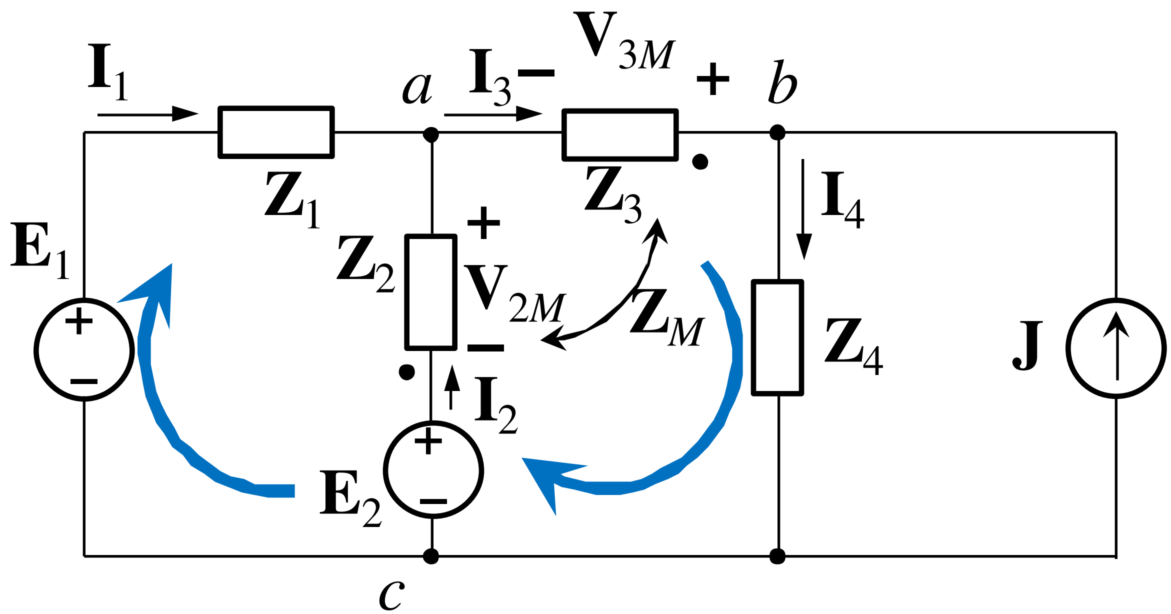
$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_3 - \mathbf{I}_4 + \mathbf{J} = 0 \\ \mathbf{V}_{Z1} + \mathbf{V}_{1M} - \mathbf{V}_{Z2} - \mathbf{V}_{2M} = \mathbf{E}_1 - \mathbf{E}_2 \rightarrow \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{V}_{Z2} + \mathbf{V}_{2M} + \mathbf{V}_3 + \mathbf{V}_4 = \mathbf{E}_2 \rightarrow \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases}$$

Branch Current Method (3)

Ex. 3

$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_3$$

$$\mathbf{V}_{3M} = \mathbf{Z}_M \mathbf{I}_2$$



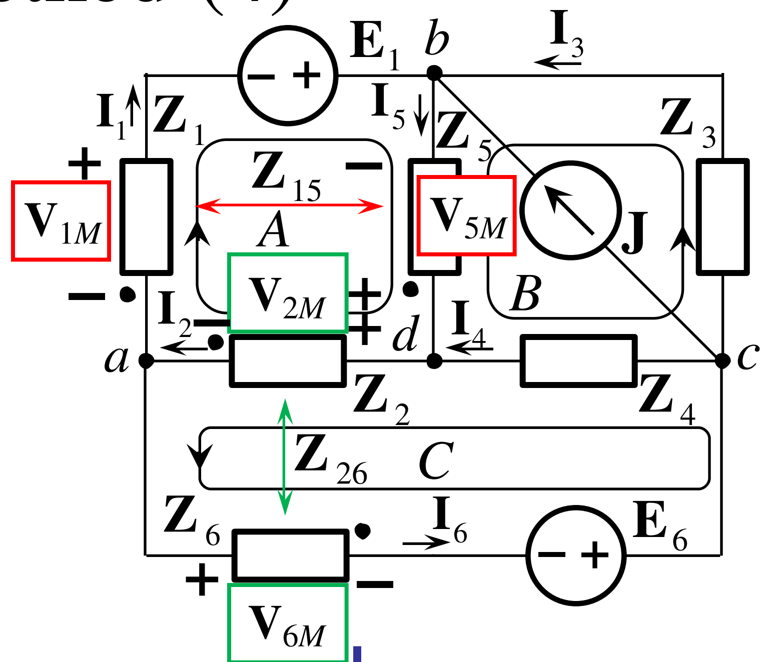
$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_3 - \mathbf{I}_4 + \mathbf{J} = 0 \\ \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_3 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_3 + \mathbf{Z}_3\mathbf{I}_3 - \mathbf{Z}_M\mathbf{I}_2 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases}$$

Branch Current Method (4)

Ex. 4

$$n_{KCL} = 4 - 1 = 3$$

$$n_{KVL} = 6 - 4 + 1 = 3$$



$$\begin{cases} a: & -\mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_6 = 0 \\ b: & \mathbf{I}_1 + \mathbf{I}_3 - \mathbf{I}_5 + \mathbf{J} = 0 \\ c: & -\mathbf{I}_3 - \mathbf{I}_4 + \mathbf{I}_6 - \mathbf{J} = 0 \\ A: & \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_{15}\mathbf{I}_5 + \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 + \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_{26}\mathbf{I}_6 = \mathbf{E}_1 \\ B: & \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 - \mathbf{Z}_4\mathbf{I}_4 = 0 \\ C: & \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_{26}\mathbf{I}_6 + \mathbf{Z}_6\mathbf{I}_6 + \mathbf{Z}_{26}\mathbf{I}_2 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_6 \end{cases}$$

$$\mathbf{V}_{1M} = \mathbf{Z}_{15}\mathbf{I}_5$$

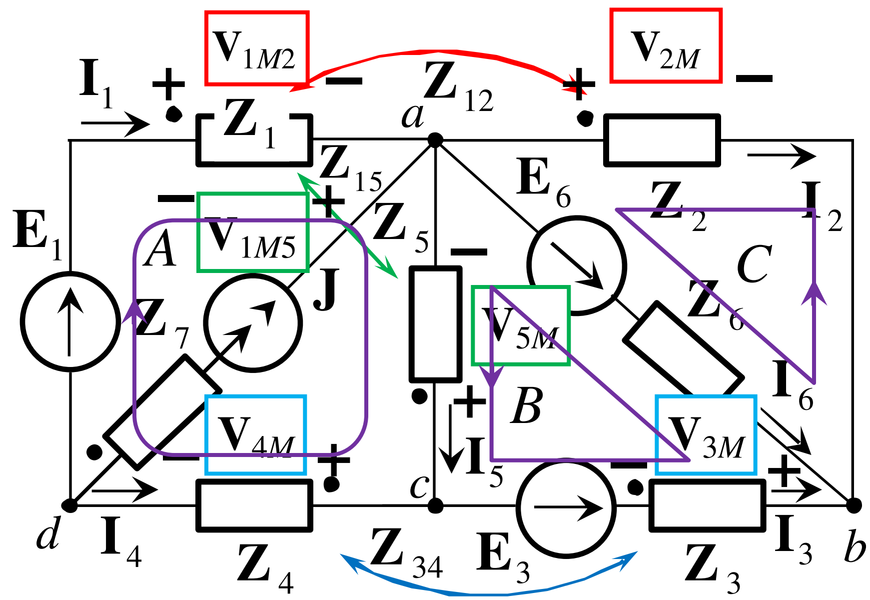
$$\mathbf{V}_{5M} = \mathbf{Z}_{15}\mathbf{I}_1$$

$$\mathbf{V}_{2M} = \mathbf{Z}_{26}\mathbf{I}_6$$

$$\mathbf{V}_{6M} = \mathbf{Z}_{26}\mathbf{I}_2$$

Branch Current Method (5)

Ex. 5



$$
\begin{cases}
b: \quad \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_6 = 0 \\
c: \quad \mathbf{I}_4 - \mathbf{I}_3 + \mathbf{I}_5 = 0 \\
d: \quad -\mathbf{I}_1 - \mathbf{I}_4 - \mathbf{J} = 0 \\
A: \quad \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2 - \mathbf{Z}_{15}\mathbf{I}_5 + \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 - \mathbf{Z}_4\mathbf{I}_4 + \mathbf{Z}_{34}\mathbf{I}_3 = \mathbf{E}_1 \\
B: \quad \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 - \mathbf{Z}_{34}\mathbf{I}_4 - \mathbf{Z}_6\mathbf{I}_6 = \mathbf{E}_3 - \mathbf{E}_6 \\
C: \quad \mathbf{Z}_6\mathbf{I}_6 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_{12}\mathbf{I}_1 = \mathbf{E}_6
\end{cases}
$$

$$\mathbf{V}_{1M2} = \mathbf{Z}_{12}\mathbf{I}_2; \ \mathbf{V}_{1M5} = \mathbf{Z}_{15}\mathbf{I}_5$$

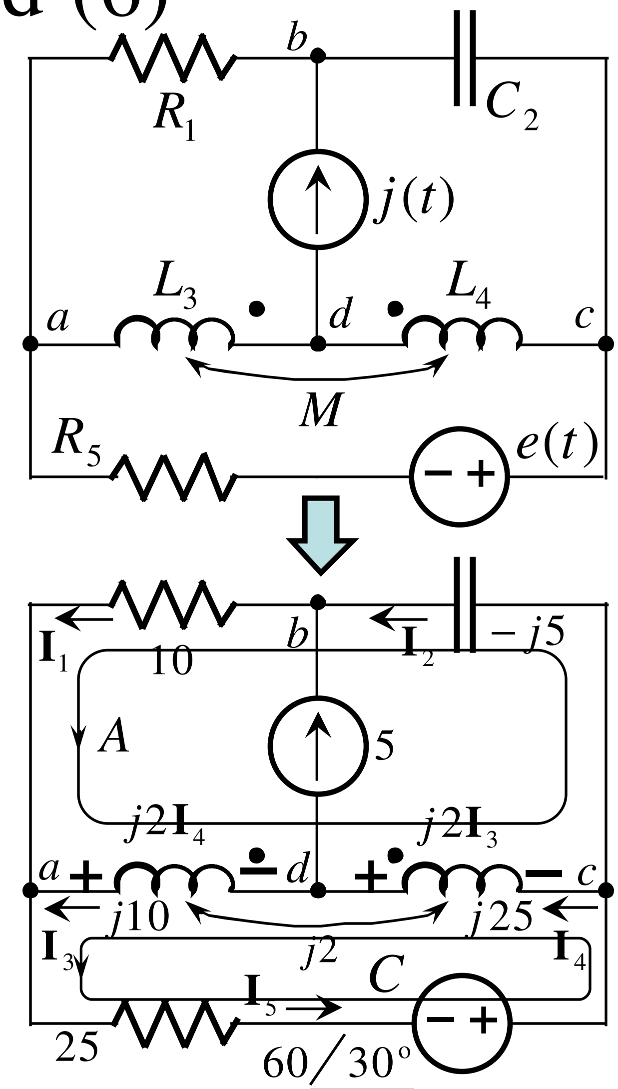
$$\mathbf{V}_{2M} = \mathbf{Z}_{12}\mathbf{I}_1; \quad \mathbf{V}_{5M} = \mathbf{Z}_{15}\mathbf{I}_1$$

$$\mathbf{V}_{3M} = \mathbf{Z}_{34}\mathbf{I}_4; \quad \mathbf{V}_{4M} = \mathbf{Z}_{34}\mathbf{I}_3$$

Branch Current Method (6)

Ex. 6

$R_1 = 10\ \Omega$, $R_5 = 25\ \Omega$, $L_3 = 0.2$ H, $L_4 = 0.5$ H, $C_2 = 4$ mF,
$M = 0.04$ H, $j(t) = 5\sin(50t)$ A, $e(t) = 60\sin(50t + 30^o)$ V.



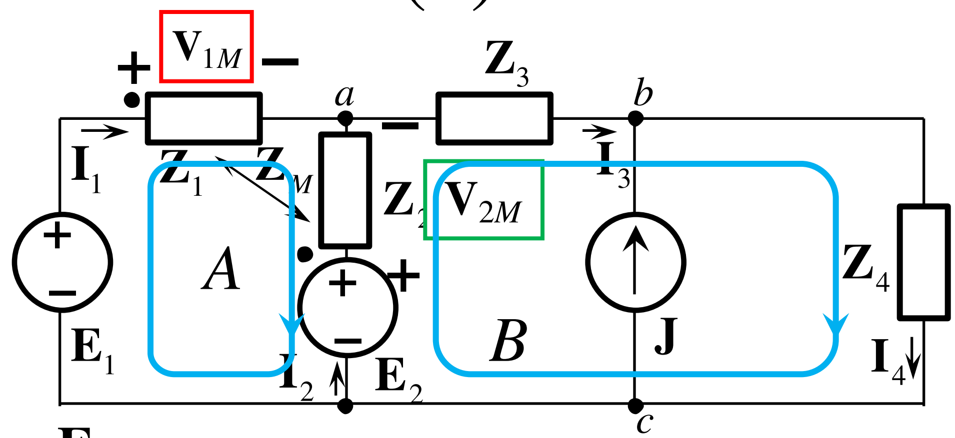
$$\begin{cases} a:\ \mathbf{I}_1 + \mathbf{I}_3 - \mathbf{I}_5 = 0 \\ b:\ -\mathbf{I}_1 + \mathbf{I}_2 + 5 = 0 \\ c:\ -\mathbf{I}_2 - \mathbf{I}_4 + \mathbf{I}_5 = 0 \\ A:\ 10\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_4 - j25\mathbf{I}_4 + j2\mathbf{I}_3 - j5\mathbf{I}_2 = 0 \\ C:\ 25\mathbf{I}_5 + j25\mathbf{I}_4 - j2\mathbf{I}_3 + j10\mathbf{I}_3 - 2j_2\mathbf{I}_4 = 60\angle 30^o \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = 3.71\angle 25.17^o \text{ A} \\ \mathbf{I}_2 = 2.28\angle 136.18^o \text{ A} \\ \mathbf{I}_3 = 3.23\angle -155.51^o \text{ A} \\ \mathbf{I}_4 = 2.46\angle -32.93^o \text{ A} \\ \mathbf{I}_5 = 0.48\angle 29.75^o \text{ A} \end{cases} \rightarrow \begin{cases} i_1 = 3.71\sin(50t + 25.17^o) \text{ A} \\ i_2 = 2.28\sin(50t + 136.18^o) \text{ A} \\ i_3 = 3.23\sin(50t - 155.51^o) \text{ A} \\ i_4 = 2.46\sin(50t - 32.93^o) \text{ A} \\ i_5 = 0,48\sin(50t + 29,75^o) \text{ A} \end{cases}$$

Branch Current Method (7)

Ex. 7

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\,\Omega;\ \mathbf{Z}_M = j2\,\Omega;$
$\mathbf{Z}_3 = -j20\,\Omega;\ \mathbf{Z}_4 = 25\,\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\underline{/30^\circ}\ \text{V};\ \mathbf{J} = 5\underline{/45^\circ}\ \text{A}.$



$$\boxed{\mathbf{V}_{1M} = \mathbf{Z}_M\mathbf{I}_2}$$

$$\boxed{\mathbf{V}_{2M} = \mathbf{Z}_M\mathbf{I}_1}$$

$$\begin{cases} a:\ \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ b:\ \mathbf{I}_3 + \mathbf{J} - \mathbf{I}_4 = 0 \\ A:\ \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2 \\ B:\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{I}_3 - \mathbf{I}_4 = -5\underline{/45^\circ} \\ (10 + j15 - j2)\mathbf{I}_1 + [j2 - (20 + j10)]\mathbf{I}_2 = 100 - 150\underline{/30^\circ} \\ j2\mathbf{I}_1 + (20 + j10)\mathbf{I}_2 - j20\mathbf{I}_3 + 25\mathbf{I}_4 = 150\underline{/30^\circ} \end{cases}$$

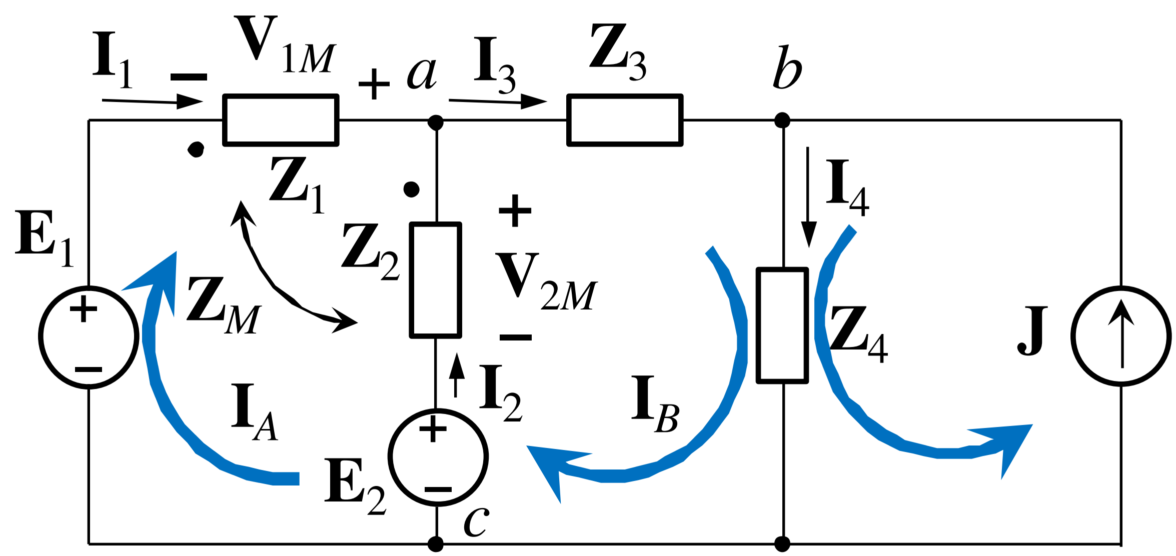
$$\rightarrow \begin{cases} \mathbf{I}_1 = -1{,}49 - j2{,}06\ \text{A}; \quad \mathbf{I}_2 = 2{,}40 + j0{,}79\ \text{A} \\ \mathbf{I}_3 = 0{,}91 - j1{,}28\ \text{A}; \quad \mathbf{I}_4 = 4{,}44 + j2{,}26\ \text{A} \end{cases}$$

Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. **Analysis of Magnetically Coupled Circuits**
   a) MCC in Phasor Domain
   b) Branch Current Method
   c) **Mesh Current Method**
   d) Equivalent Subcircuits
4. Energy in a Coupled Circuit
5. Transformers

Mesh Current Method (1)

Ex. 1



$$\mathbf{V}_{1M} = \mathbf{Z}_M \mathbf{I}_2$$

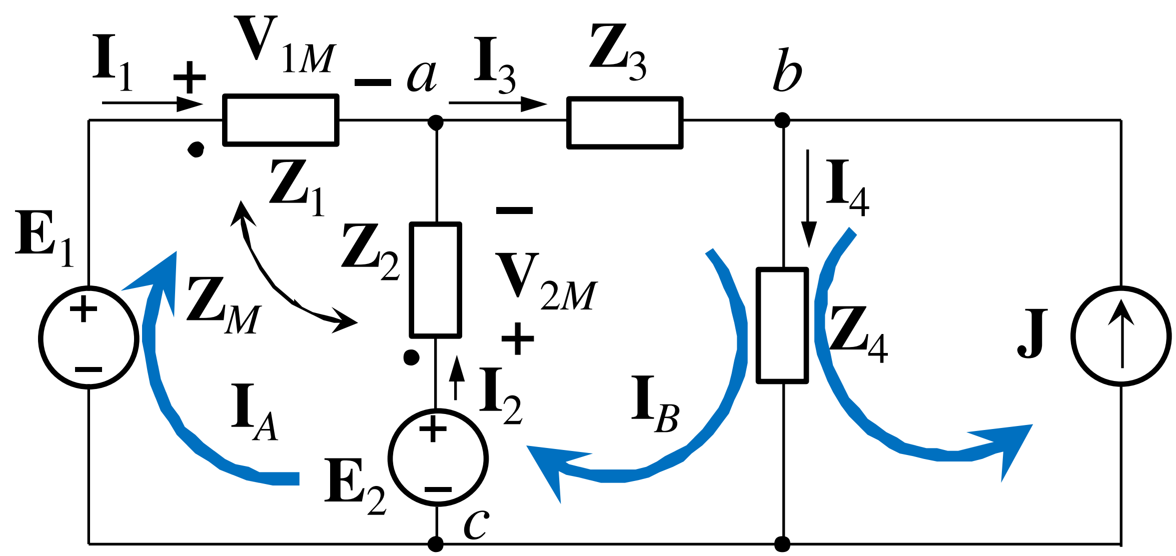
$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_1$$

$$\left.\begin{array}{l} \begin{cases} \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases} \\ \mathbf{I}_1 = \mathbf{I}_A,\ \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A,\ \mathbf{I}_3 = \mathbf{I}_B,\ \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J} \end{array}\right\}$$

$$\rightarrow \begin{cases} \mathbf{Z}_1\mathbf{I}_A - \mathbf{Z}_M(\mathbf{I}_B - \mathbf{I}_A) - \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_M\mathbf{I}_A = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) - \mathbf{Z}_M\mathbf{I}_A + \mathbf{Z}_3\mathbf{I}_B + \mathbf{Z}_4(\mathbf{I}_B + \mathbf{J}) = \mathbf{E}_2 \end{cases}$$

Mesh Current Method (2)

Ex. 2



$$\mathbf{V}_{1M} = \mathbf{Z}_M \mathbf{I}_2$$

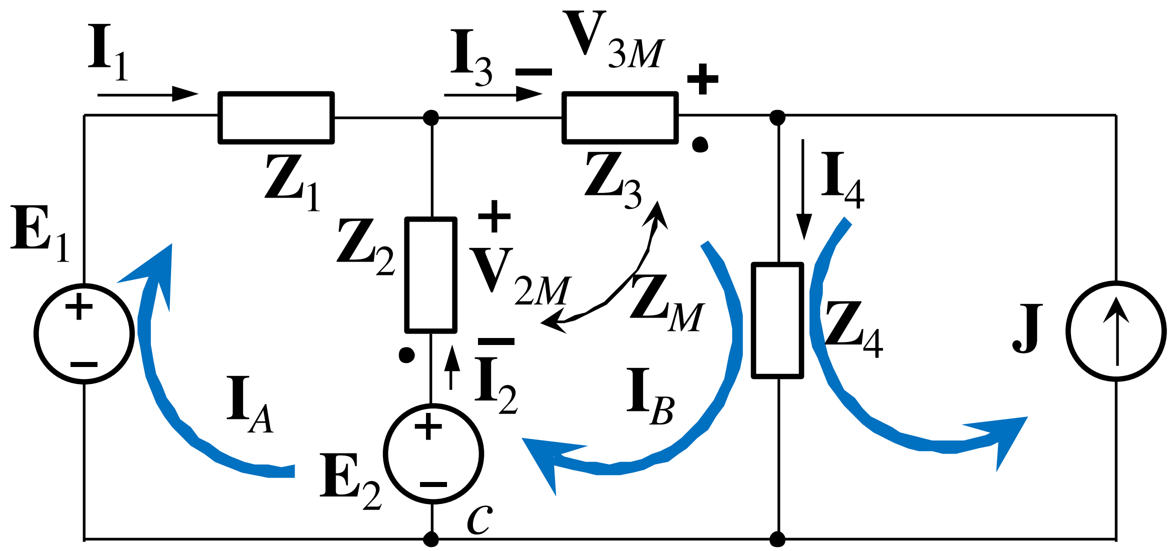
$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_1$$

$$\left.\begin{array}{l} \begin{cases} \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases} \\ \mathbf{I}_1 = \mathbf{I}_A,\ \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A,\ \mathbf{I}_3 = \mathbf{I}_B,\ \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J} \end{array}\right\}$$

$$\rightarrow \begin{cases} \mathbf{Z}_1\mathbf{I}_A + \mathbf{Z}_M(\mathbf{I}_B - \mathbf{I}_A) - \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_M\mathbf{I}_A = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_M\mathbf{I}_A + \mathbf{Z}_3\mathbf{I}_B + \mathbf{Z}_4(\mathbf{I}_B + \mathbf{J}) = \mathbf{E}_2 \end{cases}$$

Mesh Current Method (3)

Ex. 3



$$\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_3$$

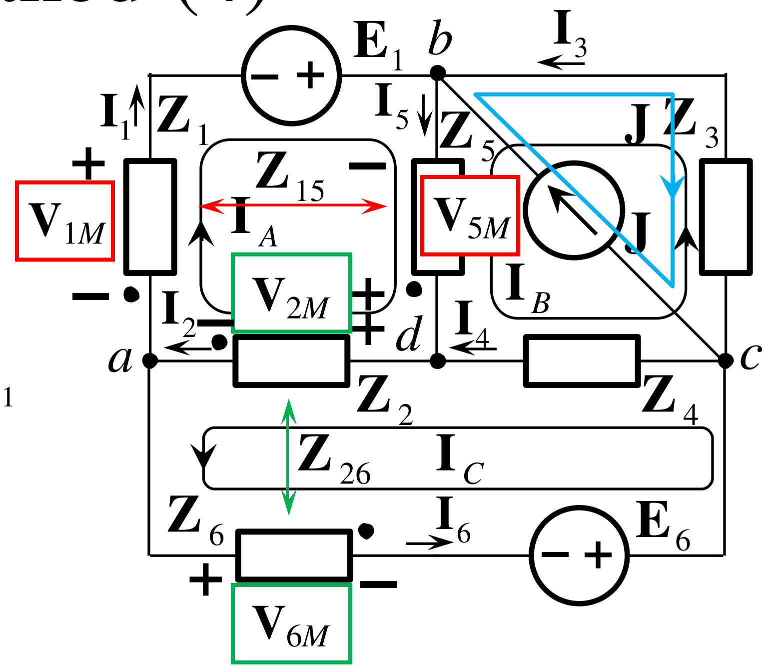
$$\mathbf{V}_{3M} = \mathbf{Z}_M \mathbf{I}_2$$

$$\left.\begin{array}{l} \begin{cases} \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_3 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_3 + \mathbf{Z}_3\mathbf{I}_3 - \mathbf{Z}_M\mathbf{I}_2 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \end{cases} \\ \mathbf{I}_1 = \mathbf{I}_A,\ \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A,\ \mathbf{I}_3 = \mathbf{I}_B,\ \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J} \end{array}\right\}$$

$$\rightarrow \begin{cases} \mathbf{Z}_1\mathbf{I}_A - \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_M\mathbf{I}_B = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2(\mathbf{I}_B - \mathbf{I}_A) - \mathbf{Z}_M\mathbf{I}_B + \mathbf{Z}_3\mathbf{I}_B - \mathbf{Z}_M(\mathbf{I}_B - \mathbf{I}_A) + \mathbf{Z}_4(\mathbf{I}_B + \mathbf{J}) = \mathbf{E}_2 \end{cases}$$

Mesh Current Method (4)

Ex. 4

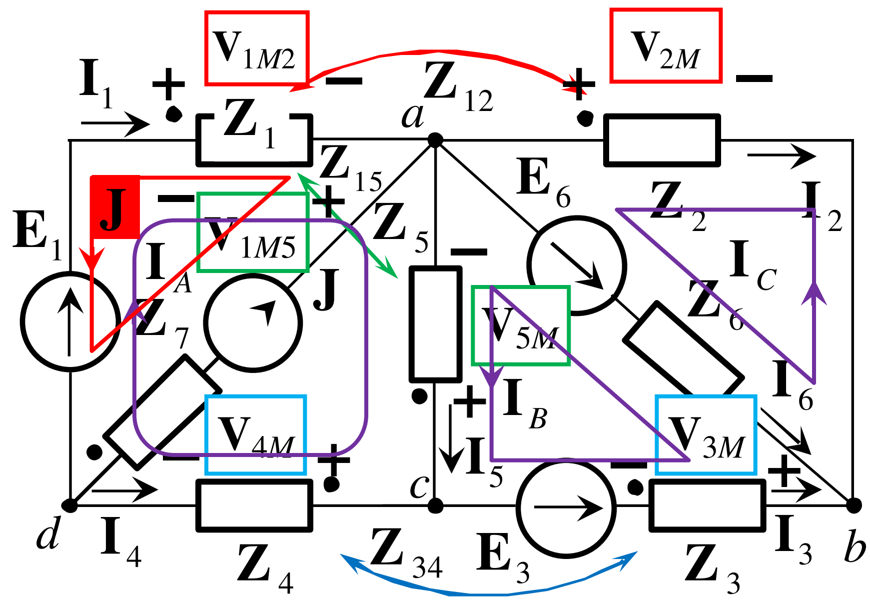


$$\begin{cases} A: \ \mathbf{Z}_1\mathbf{I}_1 - \mathbf{Z}_{15}\mathbf{I}_5 + \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 + \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_{26}\mathbf{I}_6 = \mathbf{E}_1 \\ B: \ \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 - \mathbf{Z}_4\mathbf{I}_4 = 0 \\ C: \ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_{26}\mathbf{I}_6 + \mathbf{Z}_6\mathbf{I}_6 + \mathbf{Z}_{26}\mathbf{I}_2 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_6 \\ \mathbf{I}_1 = \mathbf{I}_A, \ \mathbf{I}_2 = \mathbf{I}_A + \mathbf{I}_C, \ \ \mathbf{I}_3 = \mathbf{I}_B - \mathbf{J} \\ \mathbf{I}_4 = \mathbf{I}_C - \mathbf{I}_B, \ \mathbf{I}_5 = \mathbf{I}_A + \mathbf{I}_B, \ \mathbf{I}_6 = \mathbf{I}_C \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{Z}_1\mathbf{I}_A - \mathbf{Z}_{15}(\mathbf{I}_A + \mathbf{I}_B) + \mathbf{Z}_5(\mathbf{I}_A + \mathbf{I}_B) - \mathbf{Z}_{15}\mathbf{I}_A + \mathbf{Z}_2(\mathbf{I}_A + \mathbf{I}_C) + \mathbf{Z}_{26}\mathbf{I}_C = \mathbf{E}_1 \\ \mathbf{Z}_3(\mathbf{I}_B - \mathbf{J}) + \mathbf{Z}_5(\mathbf{I}_A + \mathbf{I}_B) - \mathbf{Z}_{15}\mathbf{I}_A - \mathbf{Z}_4(\mathbf{I}_C - \mathbf{I}_B) = 0 \\ \mathbf{Z}_2(\mathbf{I}_A + \mathbf{I}_C) + \mathbf{Z}_{26}\mathbf{I}_C + \mathbf{Z}_6\mathbf{I}_C + \mathbf{Z}_{26}(\mathbf{I}_A + \mathbf{I}_C) + \mathbf{Z}_4(\mathbf{I}_C - \mathbf{I}_B) = \mathbf{E}_6 \end{cases}$$

Mesh Current Method (5)

Ex. 5



$$\begin{cases} A: \quad \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2 - \mathbf{Z}_{15}\mathbf{I}_5 + \mathbf{Z}_5\mathbf{I}_5 \\ \qquad -\mathbf{Z}_{15}\mathbf{I}_1 - \mathbf{Z}_4\mathbf{I}_4 + \mathbf{Z}_{34}\mathbf{I}_3 = \mathbf{E}_1 \\ B: \quad \mathbf{Z}_5\mathbf{I}_5 - \mathbf{Z}_{15}\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 - \mathbf{Z}_{34}\mathbf{I}_4 \\ \qquad -\mathbf{Z}_6\mathbf{I}_6 = \mathbf{E}_3 - \mathbf{E}_6 \\ C: \quad \mathbf{Z}_6\mathbf{I}_6 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_{12}\mathbf{I}_1 = \mathbf{E}_6 \\ \mathbf{I}_1 = \mathbf{I}_A - \mathbf{J}, \ \mathbf{I}_2 = -\mathbf{I}_C, \ \mathbf{I}_3 = \mathbf{I}_B \\ \mathbf{I}_4 = -\mathbf{I}_A, \ \mathbf{I}_5 = \mathbf{I}_A + \mathbf{I}_B, \ \mathbf{I}_6 = \mathbf{I}_C - \mathbf{I}_B \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{Z}_1(\mathbf{I}_A - \mathbf{J}) + \mathbf{Z}_{12}(-\mathbf{I}_C) - \mathbf{Z}_{15}(\mathbf{I}_A + \mathbf{I}_B) + \mathbf{Z}_5(\mathbf{I}_A + \mathbf{I}_B) - \mathbf{Z}_{15}(\mathbf{I}_A - \mathbf{J}) - \mathbf{Z}_4(-\mathbf{I}_A) + \mathbf{Z}_{34}\mathbf{I}_B = \mathbf{E}_1 \\ \mathbf{Z}_5(\mathbf{I}_A + \mathbf{I}_B) - \mathbf{Z}_{15}(\mathbf{I}_A - \mathbf{J}) + \mathbf{Z}_3\mathbf{I}_B - \mathbf{Z}_{34}(-\mathbf{I}_A) - \mathbf{Z}_6(\mathbf{I}_C - \mathbf{I}_B) = \mathbf{E}_3 - \mathbf{E}_6 \\ \mathbf{Z}_6(\mathbf{I}_C - \mathbf{I}_B) - \mathbf{Z}_2(-\mathbf{I}_C) - \mathbf{Z}_{12}(\mathbf{I}_A - \mathbf{J}) = \mathbf{E}_6 \end{cases}$$

Mesh Current Method (6)

Ex. 6

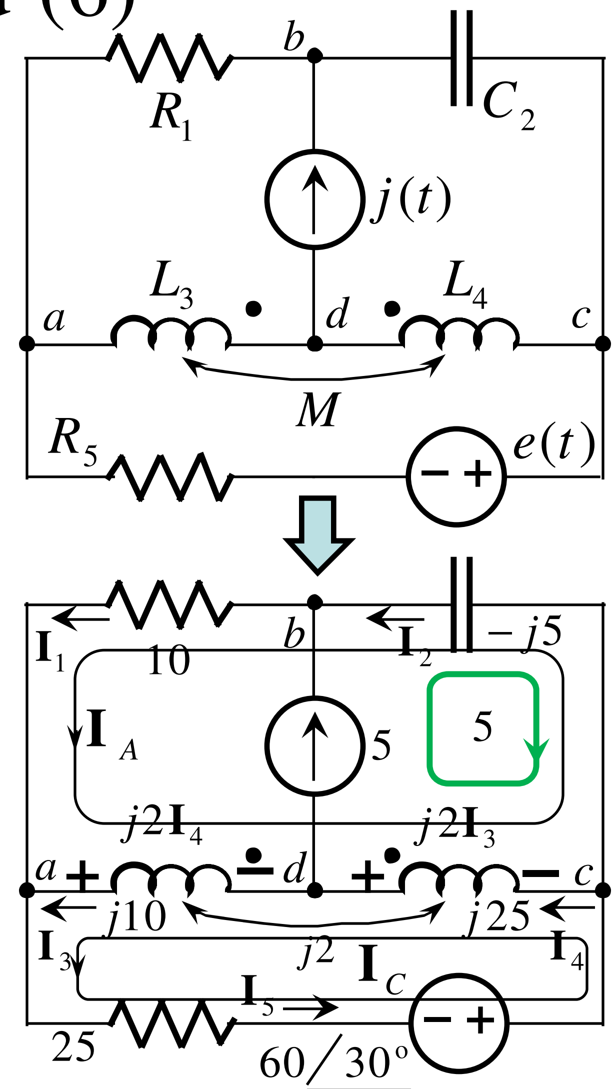
$R_1 = 10\ \Omega$, $R_5 = 25\ \Omega$, $L_3 = 0.2$ H, $L_4 = 0.5$ H, $C_2 = 4$ mF,
$M = 0.04$ H, $j(t) = 5\sin(50t)$ A, $e(t) = 60\sin(50t + 30^o)$ V.

$$\begin{cases}\begin{cases} A:\ 10\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_4 - j25\mathbf{I}_4 + j2\mathbf{I}_3 - j5\mathbf{I}_2 = 0 \\ C:\ 25\mathbf{I}_5 + j25\mathbf{I}_4 - j2\mathbf{I}_3 + j10\mathbf{I}_3 - 2j_2\mathbf{I}_4 = 60\underline{/30^o} \end{cases} \\ \mathbf{I}_1 = \mathbf{I}_A,\ \mathbf{I}_2 = \mathbf{I}_A - 5,\ \mathbf{I}_3 = \mathbf{I}_C - \mathbf{I}_A,\ \mathbf{I}_4 = \mathbf{I}_C - \mathbf{I}_A + 5,\ \mathbf{I}_5 = \mathbf{I}_C \end{cases}$$

$$\rightarrow \begin{cases} (10 + j26)\mathbf{I}_A - j31\mathbf{I}_C = j90 \\ -j31\mathbf{I}_A + (25 + j31)\mathbf{I}_C = 51.96 - j85 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_A = 3.36 + j1.58\ \text{A} \\ \mathbf{I}_C = 0.42 + j0.24\ \text{A} \end{cases}$$

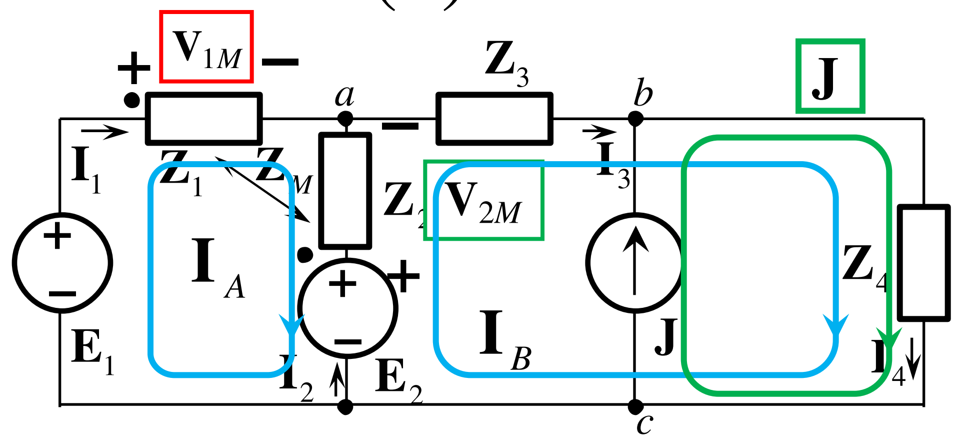
$$\rightarrow \begin{cases} \mathbf{I}_1 = 3.71\underline{/25.17^o}\ \text{A} \\ \mathbf{I}_2 = 2.28\underline{/136.18^o}\ \text{A} \\ \mathbf{I}_3 = 3.23\underline{/-155.51^o}\ \text{A} \\ \mathbf{I}_4 = 2.46\underline{/-32.93^o}\ \text{A} \\ \mathbf{I}_5 = 0.48\underline{/29.75^o}\ \text{A} \end{cases} \rightarrow \begin{cases} i_1 = 3.71\sin(50t + 25.17^o)\ \text{A} \\ i_2 = 2.28\sin(50t + 136.18^o)\ \text{A} \\ i_3 = 3.23\sin(50t - 155.51^o)\ \text{A} \\ i_4 = 2.46\sin(50t - 32.93^o)\ \text{A} \\ i_5 = 0,48\sin(50t + 29,75^o)\ \text{A} \end{cases}$$



Mesh Current Method (7)

Ex. 7

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\,\Omega;\ \mathbf{Z}_M = j2\,\Omega;$
$\mathbf{Z}_3 = -j20\,\Omega;\ \mathbf{Z}_4 = 25\,\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\angle 30^\circ\ \text{V};\ \mathbf{J} = 5\angle 45^\circ\ \text{A}.$



$$\boxed{\mathbf{V}_{1M} = \mathbf{Z}_M\mathbf{I}_2} \qquad \boxed{\mathbf{V}_{2M} = \mathbf{Z}_M\mathbf{I}_1}$$

$$\begin{cases} A:\ \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 \\ \qquad = \mathbf{E}_1 - \mathbf{E}_2 \\ B:\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{Z}_4\mathbf{I}_4 = \mathbf{E}_2 \\ \mathbf{I}_1 = \mathbf{I}_A;\ \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A;\ \mathbf{I}_3 = \mathbf{I}_B;\ \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J} \end{cases}$$

$$\rightarrow \begin{cases} (\mathbf{Z}_1 + \mathbf{Z}_2 - 2\mathbf{Z}_M)\mathbf{I}_A + (\mathbf{Z}_M - \mathbf{Z}_2)\mathbf{I}_B = \mathbf{E}_1 - \mathbf{E}_2 \\ (\mathbf{Z}_M - \mathbf{Z}_2)\mathbf{I}_A + (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4)\mathbf{I}_B = \mathbf{E}_2 - \mathbf{Z}_4\mathbf{J} \end{cases} \rightarrow \begin{cases} \mathbf{I}_A = -1.49 - j2.06\ \text{A} \\ \mathbf{I}_B = 0.91 - j1.28\ \text{A} \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_1 = \mathbf{I}_A = -1,49 - j2,06\ \text{A} \\ \mathbf{I}_2 = \mathbf{I}_B - \mathbf{I}_A = 2,40 + j0,79\ \text{A} \\ \mathbf{I}_3 = \mathbf{I}_B = 0,91 - j1,28\ \text{A} \\ \mathbf{I}_4 = \mathbf{I}_B + \mathbf{J} = 4,44 + j2,26\ \text{A} \end{cases}$$

Analysis of Magnetically Coupled Circuits

1. Write voltages of mutual inductance
2. Assign signs at dotted terminals (using dot convention)
3. Write KVL equations (branch current method or mesh current method)
4. Write the set of equations & solve for it

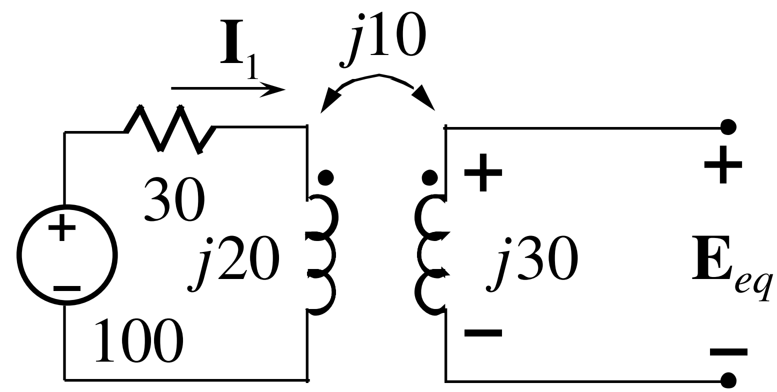
Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. **Analysis of Magnetically Coupled Circuits**
   a) MCC in Phasor Domain
   b) Branch Current Method
   c) Mesh Current Method
   d) **Equivalent Subcircuits**
4. Energy in a Coupled Circuit
5. Transformers

Equivalent Subcircuits (1)

Ex. 1

Find the Thevenin equivalent subcircuit?



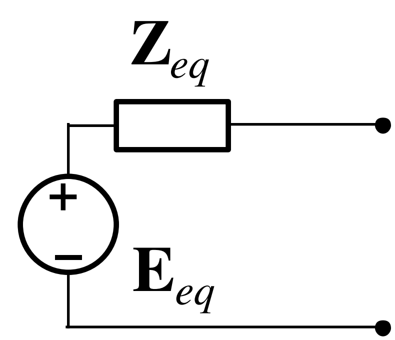
$$\mathbf{V}_{M2} = j10\mathbf{I}_1$$

$$\mathbf{E}_{eq} = \mathbf{V}_{M2} = j10\mathbf{I}_1$$

$$(30 + j20)\mathbf{I}_1 = 100$$

$$\to \mathbf{I}_1 = 2.31 - j1.54\ \text{A}$$

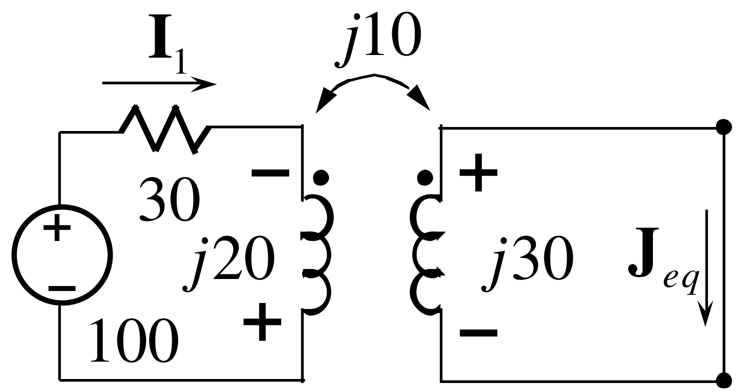
$$\to \boxed{\mathbf{E}_{eq} = 15.38 + j23.08\ \text{V}}$$



Equivalent Subcircuits (2)

Ex. 1

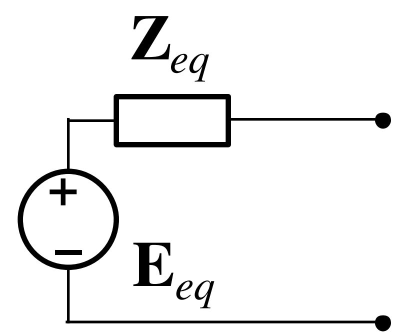
Find the Thevenin equivalent subcircuit?



$$\mathbf{Z}_{eq} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$

$$\begin{cases} (30 + j20)\mathbf{I}_1 - j10\mathbf{J}_{eq} = 100 \\ -j10\mathbf{I}_1 + j30\mathbf{J}_{eq} = 0 \end{cases}$$

$$\rightarrow \mathbf{J}_{eq} = 0.85 - j0.47\text{ A}$$



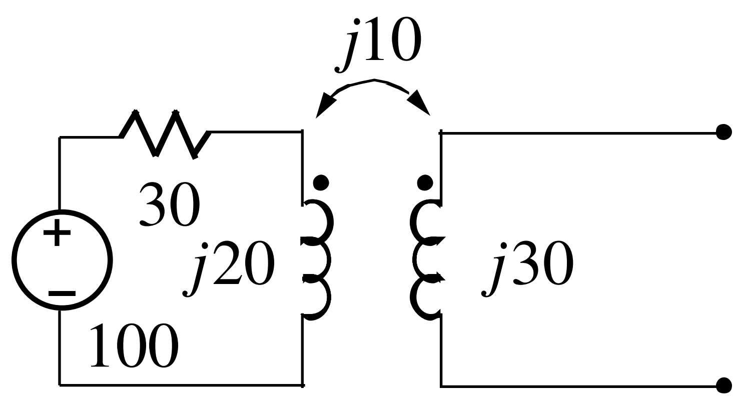
$$\rightarrow \mathbf{Z}_{eq} = \frac{15.38 + j23.08}{0.85 - j0.47} = \boxed{2.31 + j28.46\Omega}$$

$$\boxed{\mathbf{E}_{eq} = 15.38 + j23.08\text{ V}; \mathbf{Z}_{eq} = 2.31 + j28.46\Omega}$$

Equivalent Subcircuits (3)

Ex. 1

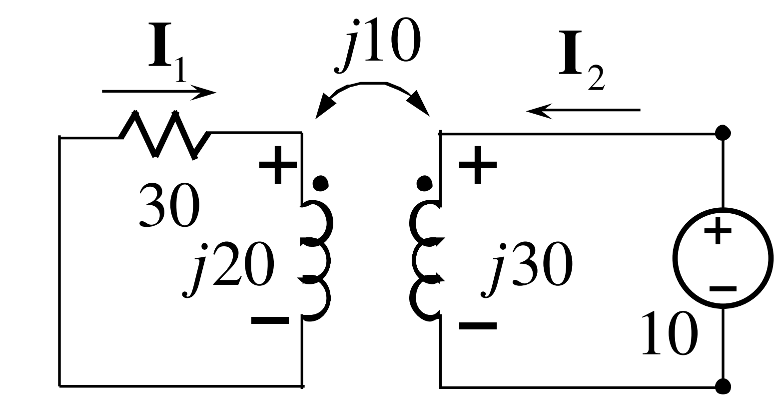
Find the Thevenin equivalent subcircuit?



$$\mathbf{Z}_{eq} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}} = 2.31 + j28.46\Omega$$

Method 1

Method 2



$$\mathbf{Z}_{eq} = \frac{10}{\mathbf{I}_2}$$

$$\begin{cases} (30 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 0 \\ j10\mathbf{I}_1 + j30\mathbf{I}_2 = 10 \end{cases}$$

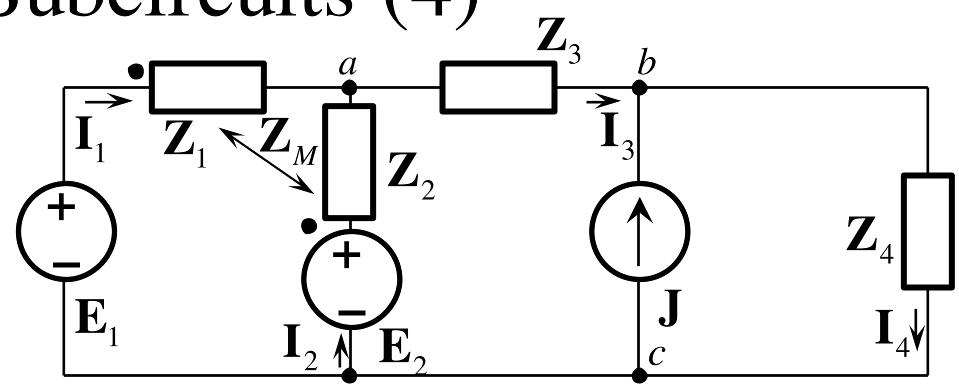
$$\rightarrow \mathbf{I}_2 = 0.28 - j0.35\,\text{A}$$

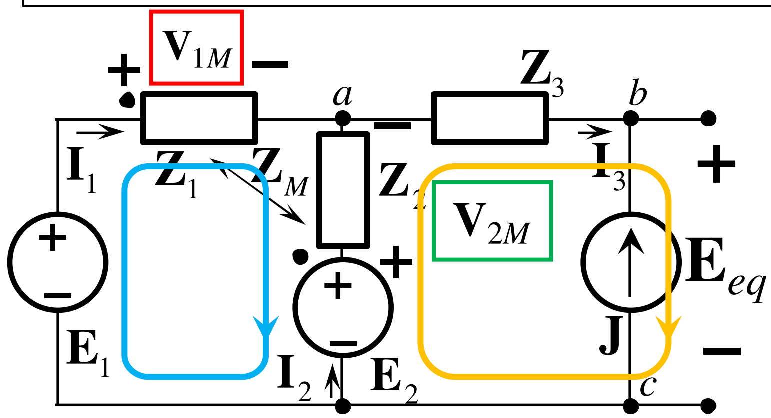
$$\rightarrow \mathbf{Z}_{eq} = \frac{10}{0.28 - j0.35} = \boxed{2.31 + j28.46\,\Omega}$$

Equivalent Subcircuits (4)

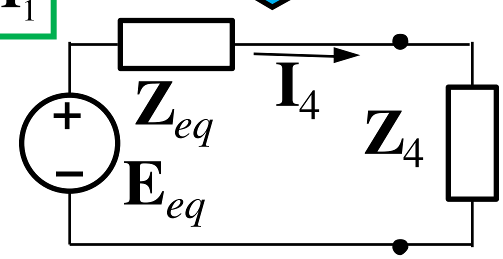
Ex. 2

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\,\Omega;\ \mathbf{Z}_M = j2\,\Omega;$
$\mathbf{Z}_3 = -j20\,\Omega;\ \mathbf{Z}_4 = 25\,\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\angle 30^\circ\ \text{V};\ \mathbf{J} = 5\angle 45^\circ\ \text{A}.\ \text{Find}\ \mathbf{I}_4\,?$





$\boxed{\mathbf{V}_{1M} = \mathbf{Z}_M \mathbf{I}_2}$ $\boxed{\mathbf{V}_{2M} = \mathbf{Z}_M \mathbf{I}_1}$



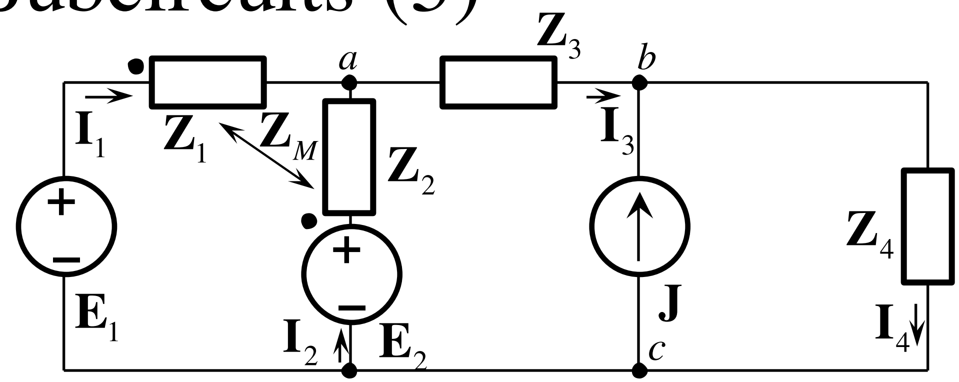
$$\begin{cases} a: \mathbf{I}_1 + \mathbf{I}_2 = \mathbf{I}_3 = -\mathbf{J} \\ \boxed{\mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2} \end{cases}$$

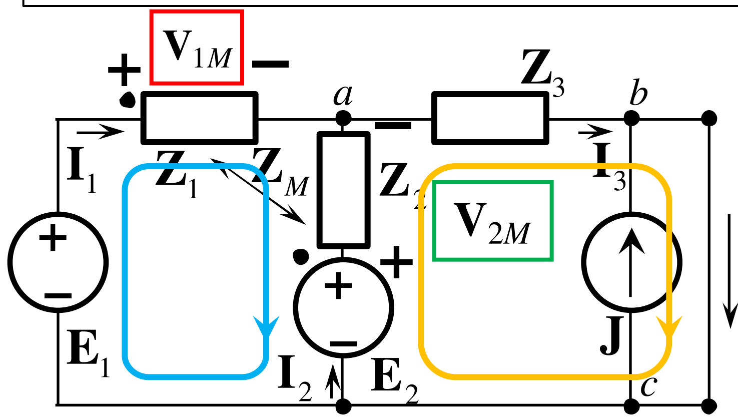
$$\left.\begin{array}{l} \rightarrow \mathbf{I}_1 = -4.34 - j2.76\,\text{A};\ \mathbf{I}_2 = 0.81 - j0.78\,\text{A} \\ \boxed{\mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 + \mathbf{E}_{eq} = \mathbf{E}_2} \end{array}\right\} \rightarrow \mathbf{E}_{eq} = 171.19 + j20.42\,\text{V}$$

Equivalent Subcircuits (5)

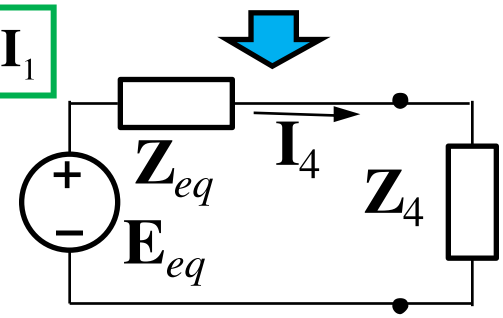
Ex. 2

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\,\Omega;\ \mathbf{Z}_M = j2\,\Omega;$
$\mathbf{Z}_3 = -j20\,\Omega;\ \mathbf{Z}_4 = 25\,\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\underline{/30^\circ}\ \text{V};\ \mathbf{J} = 5\underline{/45^\circ}\ \text{A}.$ Find $\mathbf{I}_4$?





$\mathbf{V}_{1M} = \mathbf{Z}_M\mathbf{I}_2$ $\quad$ $\mathbf{V}_{2M} = \mathbf{Z}_M\mathbf{I}_1$



$$\mathbf{J}_{eq} = \mathbf{I}_3 + \mathbf{J}$$

$$\begin{cases} \mathbf{I}_1 + \mathbf{I}_2 - \mathbf{I}_3 = 0 \\ \mathbf{Z}_1\mathbf{I}_1 + \mathbf{Z}_M\mathbf{I}_2 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_M\mathbf{I}_1 = \mathbf{E}_1 - \mathbf{E}_2 \\ \mathbf{Z}_2\mathbf{I}_2 + \mathbf{Z}_M\mathbf{I}_1 + \mathbf{Z}_3\mathbf{I}_3 = \mathbf{E}_2 \end{cases}$$

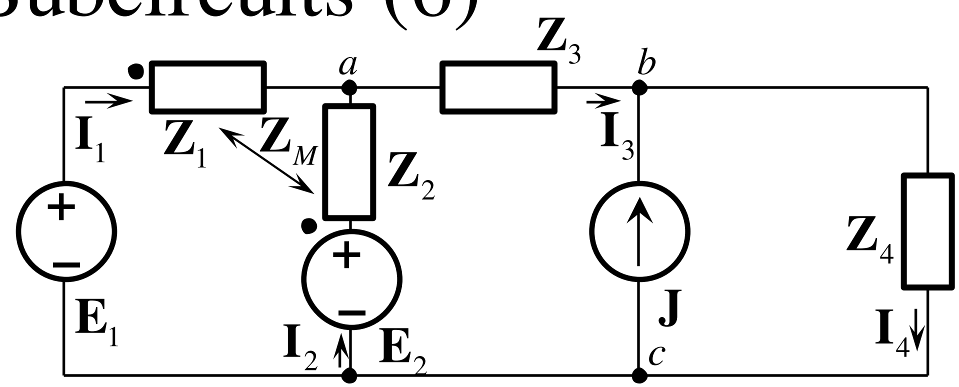
$$\rightarrow \mathbf{I}_3 = 1.71 + j7.56\ \text{A}$$

$$\rightarrow \mathbf{J}_{eq} = 5.25 + j11.09\ \text{A}$$

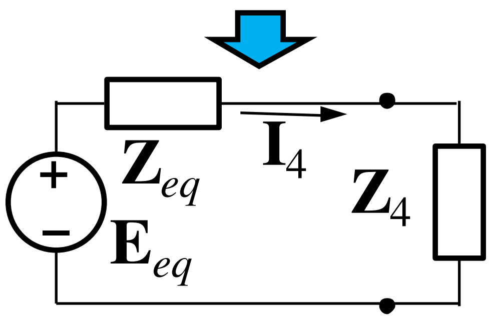
Equivalent Subcircuits (6)

Ex. 2

$\mathbf{Z}_1 = 10 + j15\,\Omega;\ \mathbf{Z}_2 = 20 + j10\Omega;\ \mathbf{Z}_M = j2\Omega;$
$\mathbf{Z}_3 = -j20\Omega;\ \mathbf{Z}_4 = 25\Omega;\ \mathbf{E}_1 = 100\,\text{V};$
$\mathbf{E}_2 = 150\angle 30^\circ\ \text{V};\ \mathbf{J} = 5\angle 45^\circ\ \text{A}$. Find $\mathbf{I}_4$?



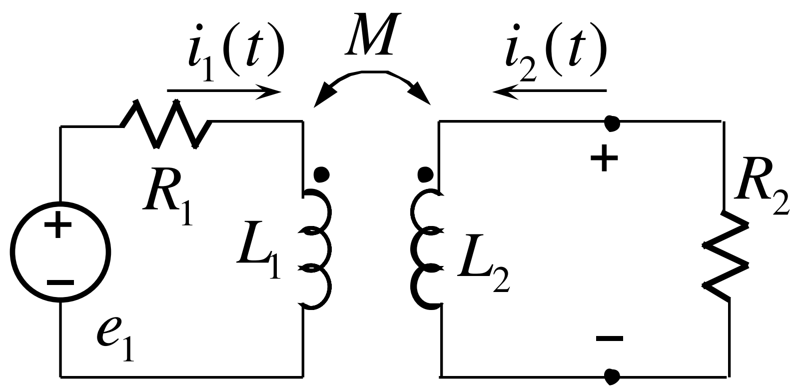
$$\left.\begin{aligned}\mathbf{E}_{eq} &= 171.19 + j20.42\,\text{V}\\ \mathbf{J}_{eq} &= 5.25 + j11.09\,\text{A}\\ \mathbf{Z}_{eq} &= \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}\end{aligned}\right\} \rightarrow \mathbf{Z}_{eq} = 7.47 - j11.90\ \Omega$$



$$\mathbf{I}_4 = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_4} = \boxed{4.44 + j2.26\,\text{A}}$$

Magnetically Coupled Circuits

Energy in a Coupled Circuit



$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

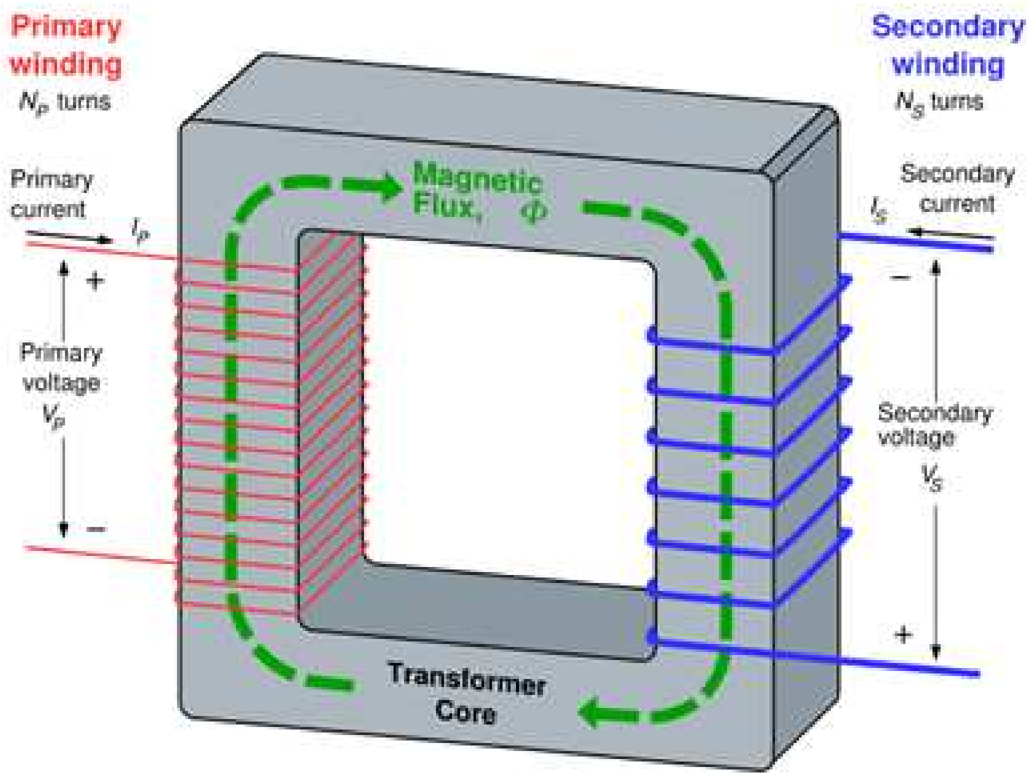
$$M = k\sqrt{L_1 L_2}$$

$$0 \le k \le 1$$

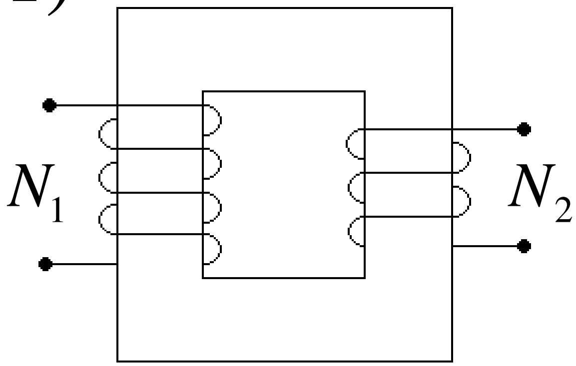
Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. Analysis of Magnetically Coupled Circuits
4. Energy in a Coupled Circuit
5. **Transformers**
    a) **Linear Transformers**
    b) **Ideal Transformers**
    c) **Ideal Autotransformers**
    d) **Three – Phase Transformers**

Linear Transformers (1)

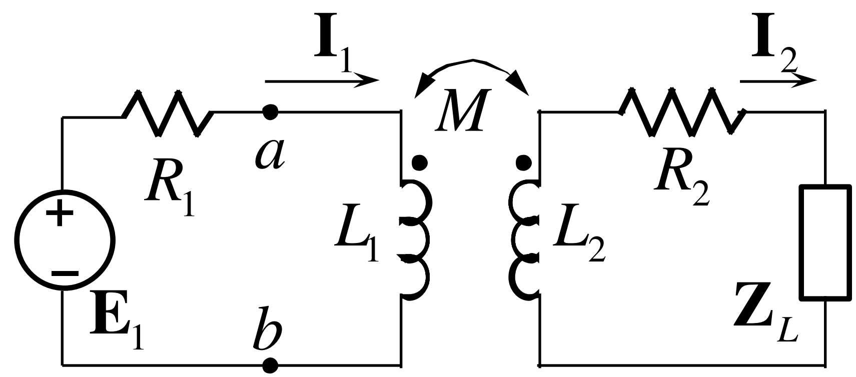


http://www.splung.com/content/sid/3/page/applications_of_induction



https://www.alibaba.com/product-detail/10KVA-Single-Phase-Transformer-to-VDE_60699387302.html

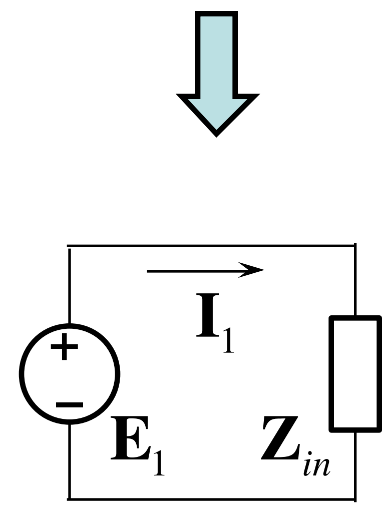
Linear Transformers (2)



$$\left\{\begin{array}{l}(R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 = \mathbf{E}_1 \\ -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_1 + \mathbf{Z}_L)\mathbf{I}_2 = 0\end{array}\right.$$

$$\mathbf{Z}_{in} = \frac{\mathbf{E}_1}{\mathbf{I}_1}$$

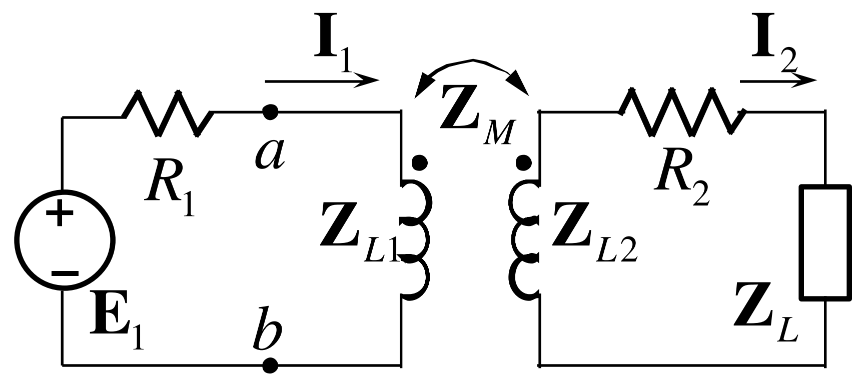
$$\rightarrow \boxed{\mathbf{Z}_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}}$$



Linear Transformers (3)

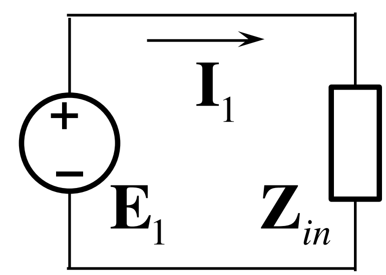
Ex.

$\mathbf{E}_1 = 100\angle 30^o$ V; $R_1 = 60\,\Omega$; $R_2 = 40\,\Omega$;
$\mathbf{Z}_L = 80 + j10\,\Omega$; $\mathbf{Z}_{L1} = j20\,\Omega$; $\mathbf{Z}_{L2} = j40\,\Omega$;
$\mathbf{Z}_M = j5\,\Omega$. Find $\mathbf{I}_1$?



$$\mathbf{Z}_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

$$= 60 + j20 + \frac{5^2}{40 + j40 + 80 + j10} = 60.18 + j19.93\,\Omega$$

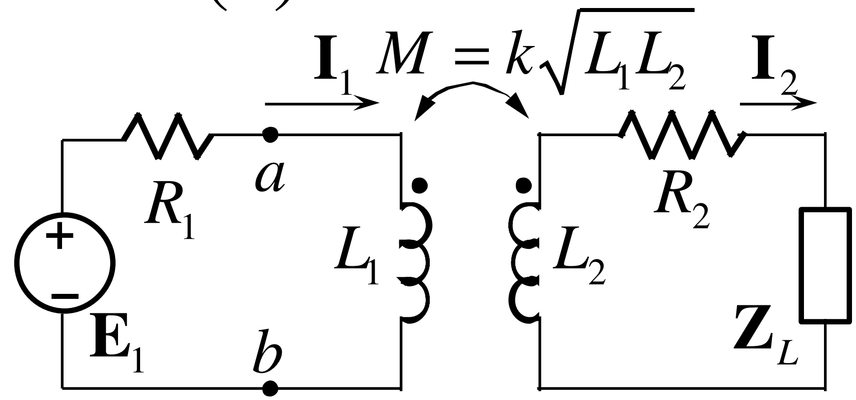


$$\mathbf{I}_1 = \frac{\mathbf{E}_1}{\mathbf{Z}_{in}} = \frac{100\angle 30^o}{60.18 + j19.93} = 1.54 + j0.32\,\Omega$$

Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. Analysis of Magnetically Coupled Circuits
4. Energy in a Coupled Circuit
5. **Transformers**
   a) Linear Transformers
   b) **Ideal Transformers**
   c) Ideal Autotransformers
   d) Three – Phase Transformers

Ideal Transformers (1)



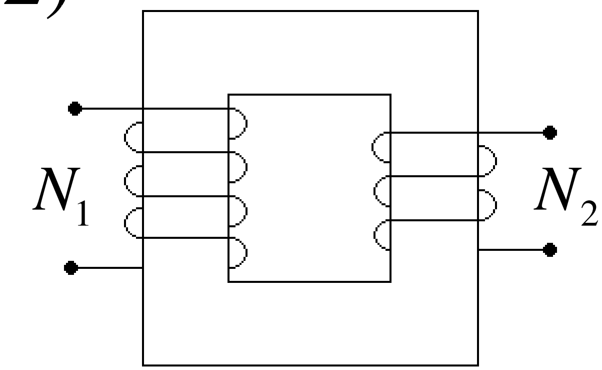
A transformer is said to be ideal if:

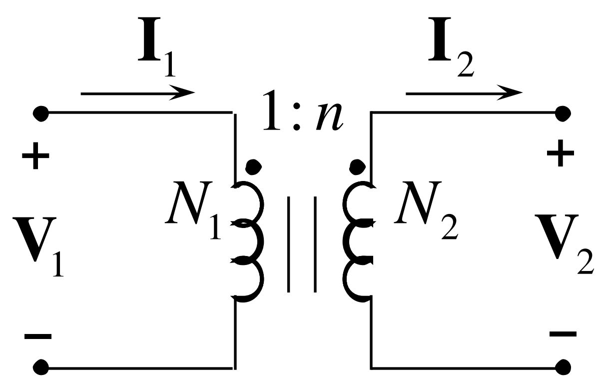
1. Coils have very large reactances (L_1, L_2, $M \to \infty$).
2. Coupling coefficient is equal to unity ($k = 1$).
3. Primary & secondary coils are lossless ($R_1 = R_2 = 0$).

Ideal Transformers (2)

$$\left.\begin{array}{l}\begin{cases}\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 \\ \mathbf{V}_2 = -j\omega L_1\mathbf{I}_2 + j\omega M\mathbf{I}_1\end{cases} \\ \text{If } k = 1 \rightarrow M = \sqrt{L_1 L_2}\end{array}\right\} \rightarrow \mathbf{V}_2 = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

$$\rightarrow \frac{\mathbf{V}_2}{\mathbf{V}_1} = \sqrt{\frac{L_2}{L_1}} = n$$





$$\left.\begin{array}{l} v_1 = N_1\dfrac{d\phi}{dt} \\ v_2 = N_2\dfrac{d\phi}{dt}\end{array}\right\} \rightarrow \frac{v_2}{v_1} = \frac{N_2}{N_1} = n \rightarrow \boxed{\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n}$$

$$p_1 = p_2 \rightarrow v_1 i_1 = v_2 i_2 \rightarrow \frac{i_2}{i_1} = \frac{v_1}{v_2} \rightarrow \boxed{\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}}$$

Ex. 1

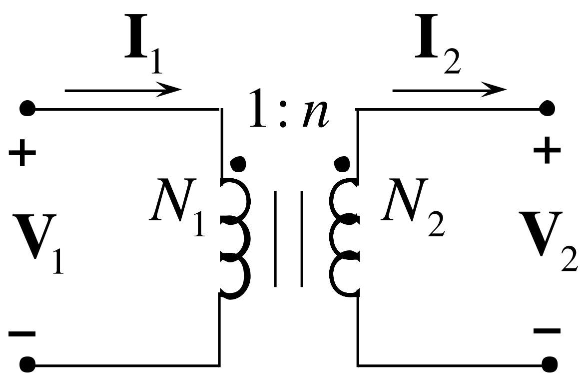
Ideal Transformers (3)

Given an ideal step-down transformer rated at 22/0.4 kV, 1000 turns on the primary side. Find:

a) The turn ratio?

b) The number of turns on the secondary side?

Ideal Transformers (4)

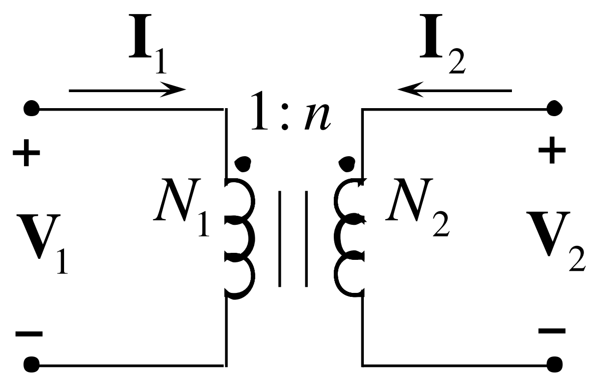


$$\begin{cases} \mathbf{V}_1 = j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 \\ \mathbf{V}_2 = -j\omega L_2\mathbf{I}_2 + j\omega M\mathbf{I}_1 \end{cases}$$

$$M = \sqrt{L_1L_2}$$

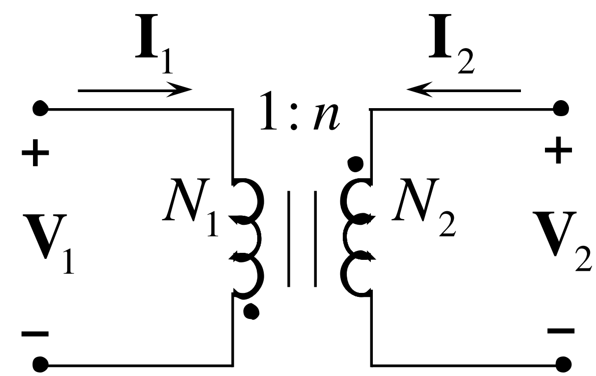
$$p_1 = p_2$$

$$\rightarrow \begin{cases} \dfrac{\mathbf{V}_2}{\mathbf{V}_1} = \dfrac{N_2}{N_1} = n \\ \dfrac{\mathbf{I}_2}{\mathbf{I}_1} = \dfrac{N_1}{N_2} = \dfrac{1}{n} \end{cases}$$

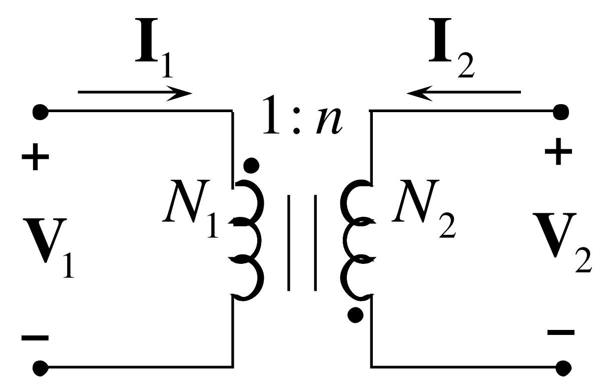


$$\begin{cases} \mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2 \\ \mathbf{V}_2 = j\omega L_1\mathbf{I}_2 + j\omega M\mathbf{I}_1 \end{cases}$$

$$\rightarrow \begin{cases} \dfrac{\mathbf{V}_2}{\mathbf{V}_1} = \dfrac{N_2}{N_1} = n \\ \dfrac{\mathbf{I}_2}{\mathbf{I}_1} = -\dfrac{N_1}{N_2} = -\dfrac{1}{n} \end{cases}$$

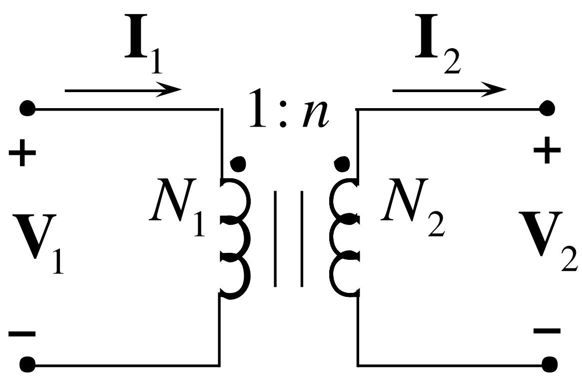


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$

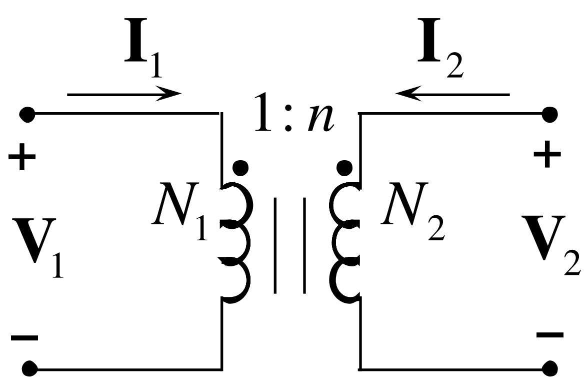


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

Ideal Transformers (5)

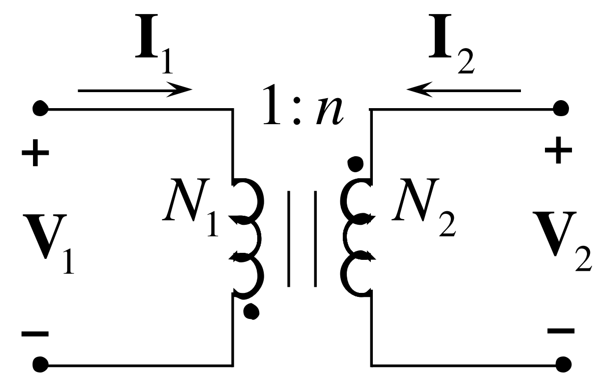


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1}{n}$$

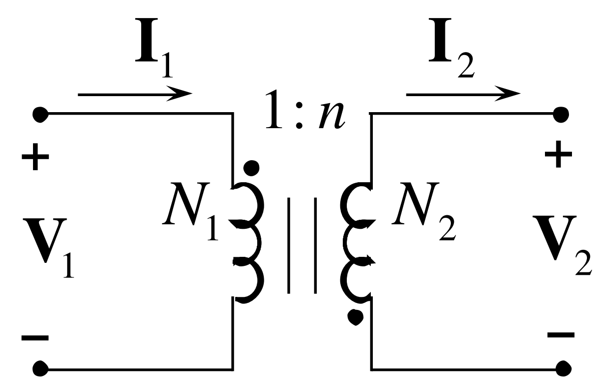


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{1}{n}$$

- If v_1 & v_2 are both positive or both negative at the dotted terminals, use $+n$. Otherwise, use $-n$
- If i_1 & i_2 both enter into or both leave the dotted terminals, use $-n$. Otherwise, use $+n$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1}{n}$$

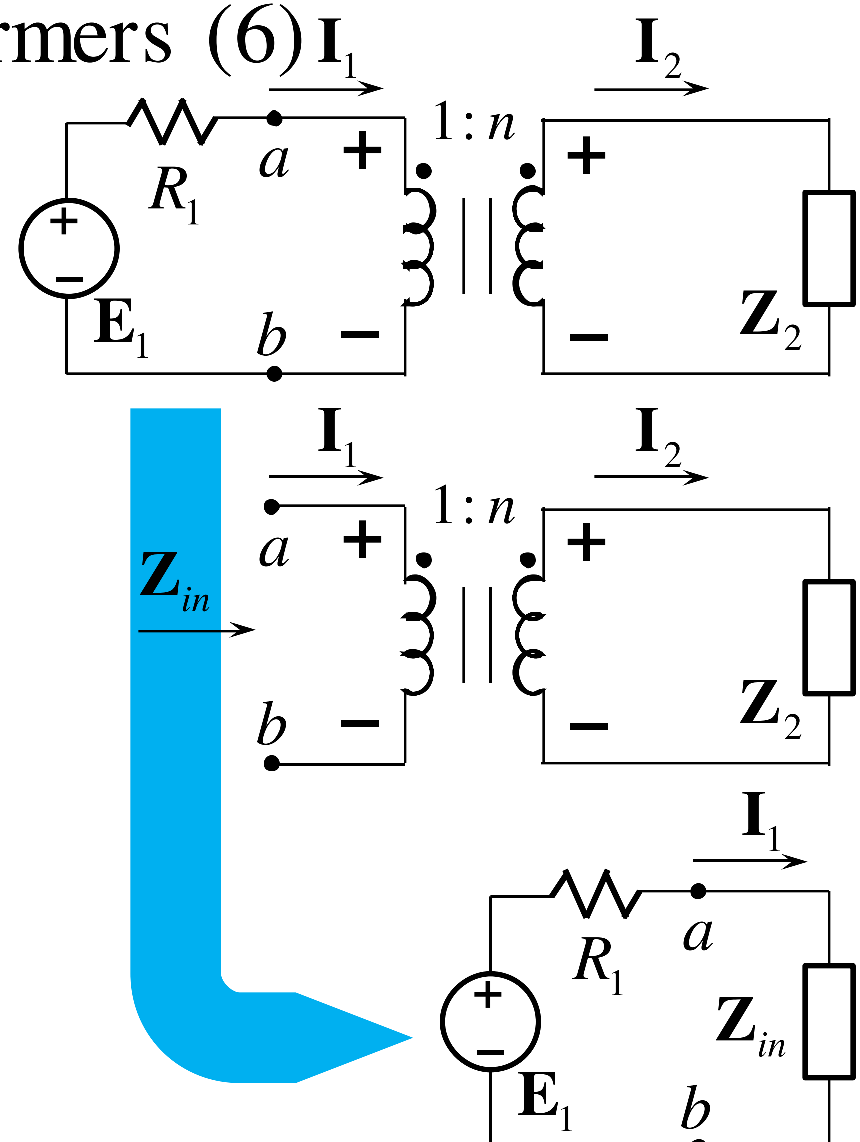


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{1}{n}$$

Ideal Transformers (6)

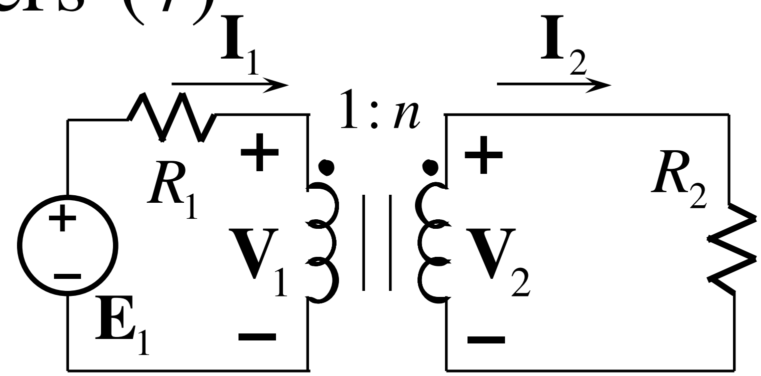
$$\left.\begin{aligned} \mathbf{Z}_{in} &= \frac{\mathbf{V}_{ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1}{\mathbf{I}_1} \\ \mathbf{V}_1 &= \mathbf{V}_2 / n \\ \mathbf{I}_1 &= n\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{Z}_2\mathbf{I}_2 \end{aligned}\right\}$$

$$\rightarrow \mathbf{Z}_{in} = \frac{\mathbf{Z}_2}{n^2}$$



Ideal Transformers (7)

Ex. 2



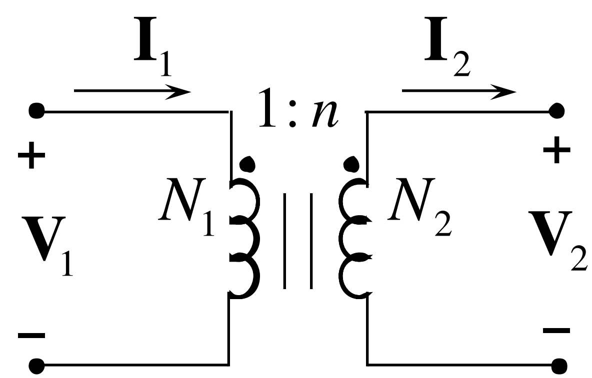
Given an ideal transformer, find currents if $\mathbf{E}_1 = 100\underline{/0^o}$ V; $n = 5$; $R_1 = 6\Omega$; $R_2 = 100\Omega$?

Method 1

$$\left.\begin{aligned} R_1\mathbf{I}_1 + \mathbf{V}_1 &= \mathbf{E}_1 \\ -\mathbf{V}_2 + R_2\mathbf{I}_2 &= 0 \end{aligned}\right\}$$

$$\rightarrow \begin{cases} 6\mathbf{I}_1 + \mathbf{V}_1 = 100\underline{/0^o} \\ -5\mathbf{V}_1 + 100 \times \dfrac{\mathbf{I}_1}{5} = 0 \end{cases}$$

$$\rightarrow \mathbf{I}_1 = \boxed{10\,\text{A}} \quad \rightarrow \mathbf{I}_2 = \frac{\mathbf{I}_1}{5} = \boxed{2\,\text{A}}$$

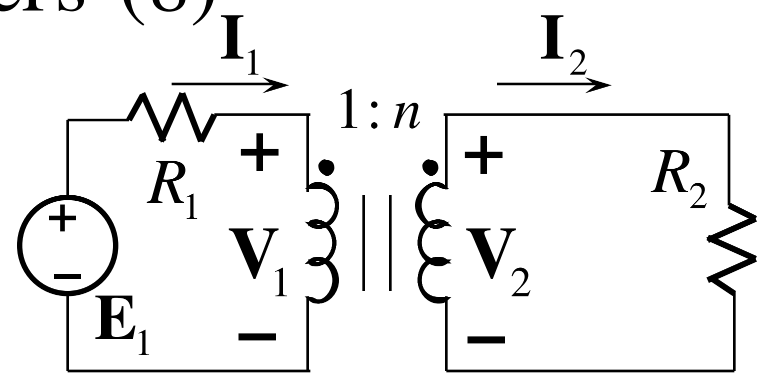


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1}{n}$$

Ideal Transformers (8)

Ex. 2

Given an ideal transformer, find currents if $\mathbf{E}_1 = 100\angle 0^\circ$ V; $n = 5$; $R_1 = 6\Omega$; $R_2 = 100\Omega$?

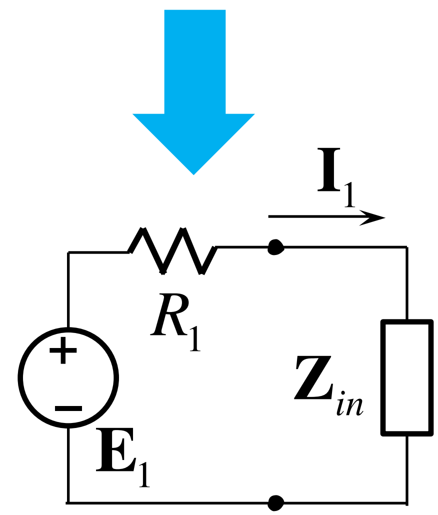


Method 2

$$\mathbf{Z}_{in} = \frac{R_2}{n^2} = \frac{100}{25} = 4\Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{E}_1}{R_1 + Z_{in}} = \frac{100}{6+4} = \boxed{10\,\text{A}}$$

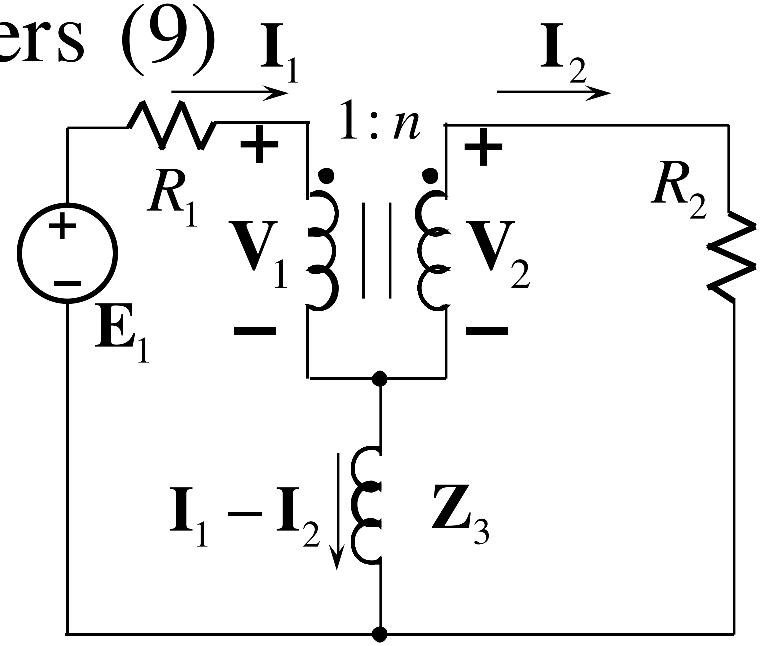
$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = n = 5 \rightarrow \mathbf{I}_2 = \frac{\mathbf{I}_1}{5} = \boxed{2\,\text{A}}$$



Ideal Transformers (9)

Ex. 3

Given an ideal transformer, find currents if $\mathbf{E}_1 = 100\underline{/0^o}$ V; $n = 5$; $R_1 = 6\Omega$; $R_2 = 100\Omega$; $\mathbf{Z}_3 = j20\Omega$?

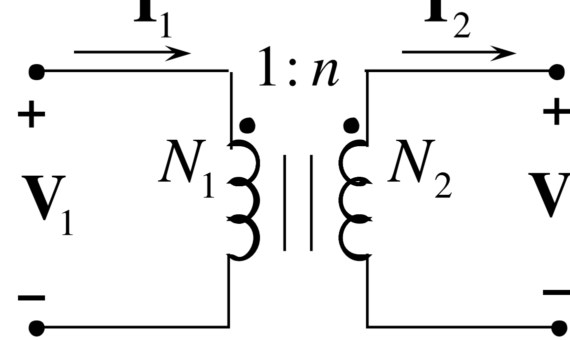


$$\left.\begin{aligned} &R_1\mathbf{I}_1 + \mathbf{V}_1 + \mathbf{Z}_3(\mathbf{I}_1 - \mathbf{I}_2) = \mathbf{E}_1 \\ &-\mathbf{V}_2 + R_2\mathbf{I}_2 - \mathbf{Z}_3(\mathbf{I}_1 - \mathbf{I}_2) = 0 \end{aligned}\right\}$$

$$\rightarrow \begin{cases} 6\mathbf{I}_1 + \mathbf{V}_1 + j20\left(\mathbf{I}_1 - \dfrac{\mathbf{I}_1}{5}\right) = 100\underline{/0^o} \\ -5\mathbf{V}_1 + 100 \times \dfrac{\mathbf{I}_1}{5} - j20\left(\mathbf{I}_1 - \dfrac{\mathbf{I}_1}{5}\right) = 0 \end{cases}$$

$$\rightarrow \mathbf{I}_1 = \boxed{3.79 - j4.85\,\text{A}}$$

$$\rightarrow \mathbf{I}_2 = \frac{\mathbf{I}_1}{5} = \boxed{1.90 - j2.43\,\text{A}}$$

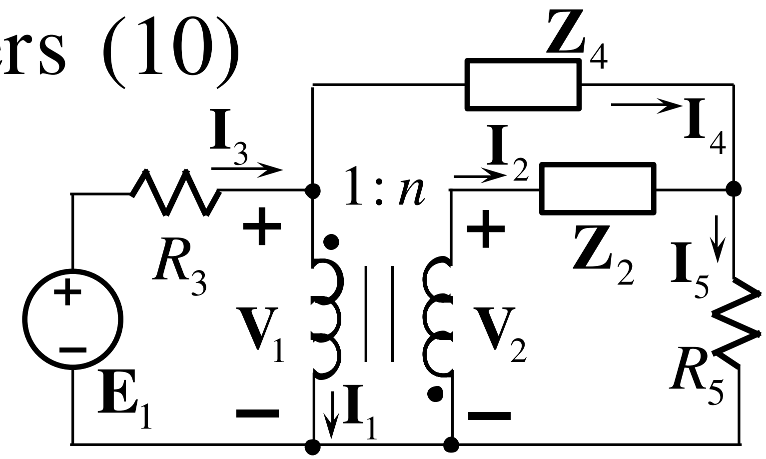


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = n; \; \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1}{n}$$

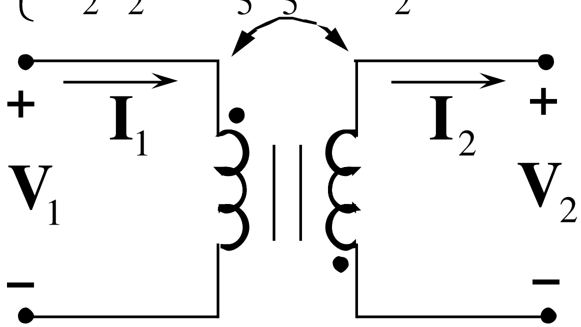
Ideal Transformers (10)

Ex. 4

$$\mathbf{E}_1 = 100\angle 0^\circ \text{ V}; n = 2; R_3 = 6\Omega; R_5 = 100\Omega;$$
$$\mathbf{Z}_2 = j20\Omega; \mathbf{Z}_4 = 30 - j40\Omega.$$



$$\left.\begin{cases} \mathbf{I}_3 - \mathbf{I}_1 - \mathbf{I}_4 = 0 \\ \mathbf{I}_2 + \mathbf{I}_4 - \mathbf{I}_5 = 0 \\ R_3\mathbf{I}_3 + \mathbf{V}_1 = \mathbf{E}_1 \\ \mathbf{Z}_4\mathbf{I}_4 - \mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 - \mathbf{V}_1 = 0 \\ \mathbf{Z}_2\mathbf{I}_2 + R_5\mathbf{I}_5 - \mathbf{V}_2 = 0 \end{cases}\right\} \rightarrow \begin{cases} \mathbf{I}_3 + 2\mathbf{I}_2 - \mathbf{I}_4 = 0 \\ \mathbf{I}_2 + \mathbf{I}_4 - \mathbf{I}_5 = 0 \\ R_3\mathbf{I}_3 + \mathbf{V}_1 = \mathbf{E}_1 \\ \mathbf{Z}_4\mathbf{I}_4 - \mathbf{Z}_2\mathbf{I}_2 - 2\mathbf{V}_1 - \mathbf{V}_1 = 0 \\ \mathbf{Z}_2\mathbf{I}_2 + R_5\mathbf{I}_5 + 2\mathbf{V}_1 = 0 \end{cases}$$

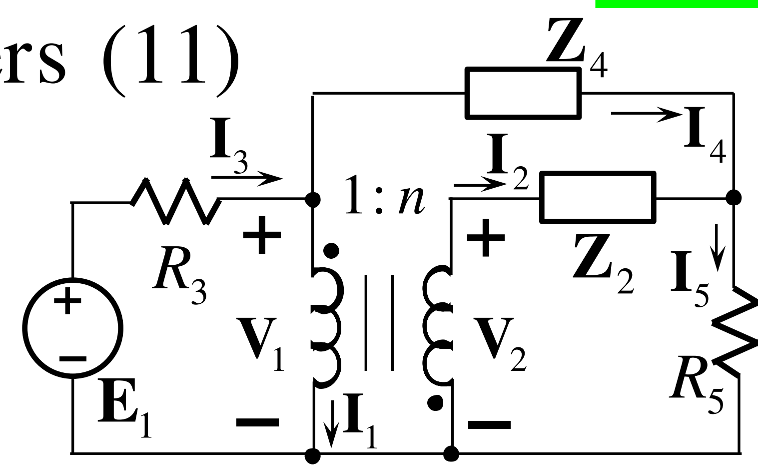


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -n; \quad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{1}{n}$$

Ideal Transformers (11)

Ex. 4

$$\mathbf{E}_1 = 100\underline{/0^o}\ \text{V}; n = 2; R_3 = 6\Omega; R_5 = 100\Omega;$$
$$\mathbf{Z}_2 = j20\Omega; \mathbf{Z}_4 = 30 - j40\Omega.$$



$$\begin{cases} \mathbf{I}_3 + 2\mathbf{I}_2 - \mathbf{I}_4 = 0 \\ \mathbf{I}_2 + \mathbf{I}_4 - \mathbf{I}_5 = 0 \\ R_3\mathbf{I}_3 + \mathbf{V}_1 = \mathbf{E}_1 \\ \mathbf{Z}_4\mathbf{I}_4 - \mathbf{Z}_2\mathbf{I}_2 - 2\mathbf{V}_1 - \mathbf{V}_1 = 0 \\ \mathbf{Z}_2\mathbf{I}_2 + R_5\mathbf{I}_5 + 2\mathbf{V}_1 = 0 \end{cases}$$

$$\rightarrow \begin{cases} \mathbf{I}_3 + 2\mathbf{I}_2 - \mathbf{I}_4 = 0 \\ \mathbf{I}_2 + \mathbf{I}_4 - \mathbf{I}_5 = 0 \\ 6\mathbf{I}_3 + \mathbf{V}_1 = 100 \\ (30 - j40)\mathbf{I}_4 - j20\mathbf{I}_2 - 3\mathbf{V}_1 = 0 \\ j20\mathbf{I}_2 + 100\mathbf{I}_5 + 2\mathbf{V}_1 = 0 \end{cases} \rightarrow \begin{cases} \mathbf{I}_2 = -3.63 - j0.012\,\text{A} \\ \mathbf{I}_3 = 10.09 + j0.87\,\text{A} \\ \mathbf{I}_4 = 2.83 + j0.84\,\text{A} \\ \mathbf{I}_5 = -0.79 + j0.83\,\text{A} \end{cases}$$

$$\mathbf{I}_1 = -2\mathbf{I}_2 = 7.25 + j0.025\,\text{A}$$

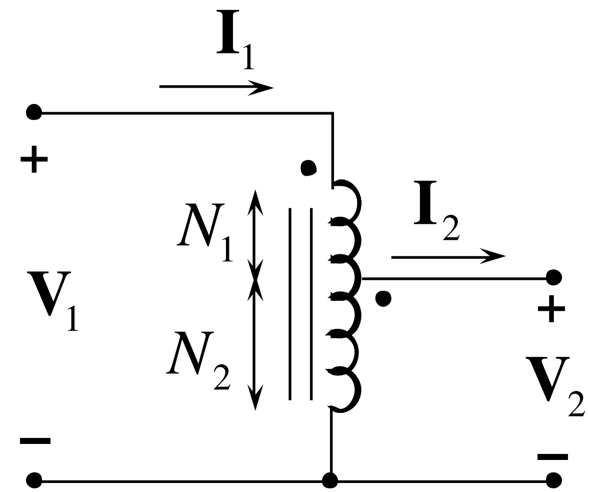
Magnetically Coupled Circuits

1. Mutual Inductance
2. Dot Convention
3. Analysis of Magnetically Coupled Circuits
4. Energy in a Coupled Circuit
5. **Transformers**
    a) Linear Transformers
    b) Ideal Transformers
    c) **Ideal Autotransformers**
    d) Three – Phase Transformers

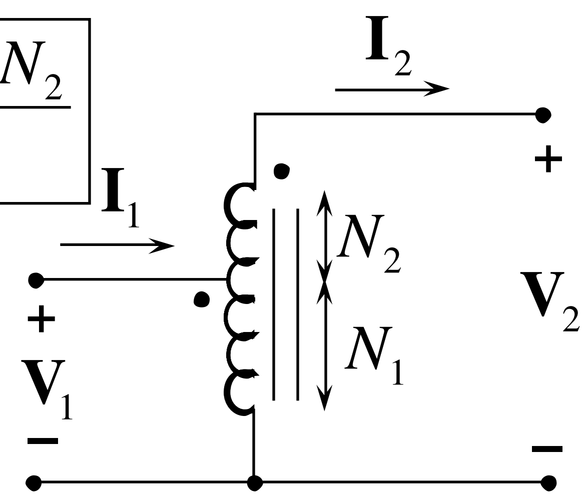
Ideal Autotransformers (1)

$$\left.\begin{aligned} v_1 &= (N_1 + N_2)\frac{d\phi}{dt} \\ v_2 &= N_2\frac{d\phi}{dt} \end{aligned}\right\} \rightarrow \boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2}}$$

$$p_1 = p_2 \rightarrow \boxed{\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}}$$



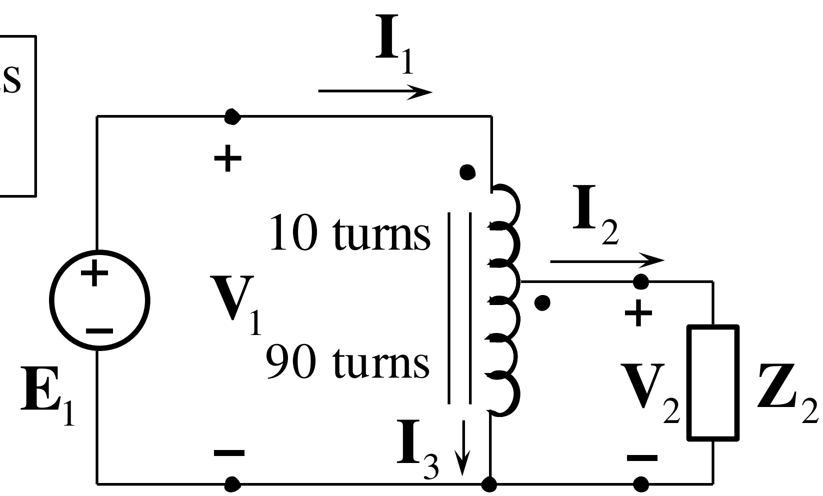
$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}\ ;\ \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1}}$$



https://proactivemarketplace.org/new-technipower-variac-variable-autotransformer-model-w20.html

Ex.

Ideal Autotransformers (2)



> Given an ideal autotransformer, find currents if $\mathbf{E}_1 = 100\underline{/0^\circ}$ V; $Z_2 = 5 + j10\Omega$?

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = \frac{10+90}{90} = 1.11$$

$$\rightarrow \mathbf{V}_2 = \frac{\mathbf{V}_1}{1.11} = \frac{100\underline{/0^\circ}}{1.11} = 90\underline{/0^\circ} \text{ V}$$

$$\rightarrow \mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_2} = \frac{90\underline{/0^\circ}}{5 + j10} = \boxed{3.60 - j7.20 \text{ A}}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2} = \frac{90}{10+90} = 0.9 \rightarrow \mathbf{I}_1 = 0.9\mathbf{I}_2 = \boxed{3.24 - j6.48 \text{ A}}$$

$$\mathbf{I}_3 = \mathbf{I}_1 - \mathbf{I}_2 = \boxed{-0.36 + j0.72 \text{ A}}$$

Magnetically Coupled Circuits

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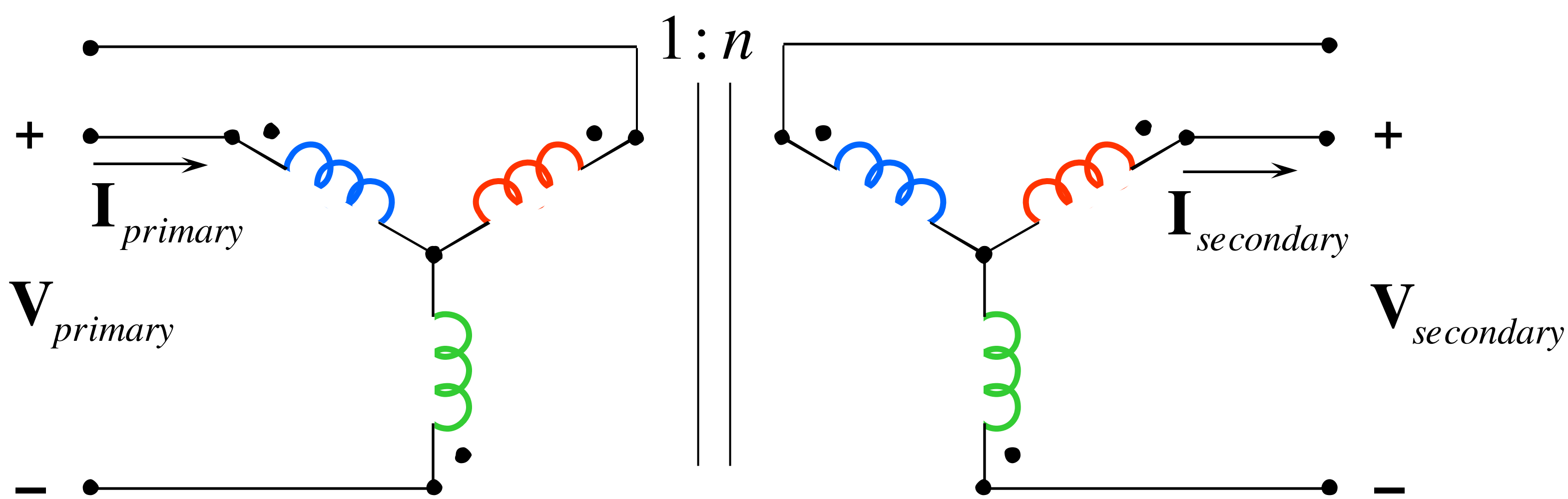
   c) Ideal Autotransformers
   d) **Three – Phase Transformers**

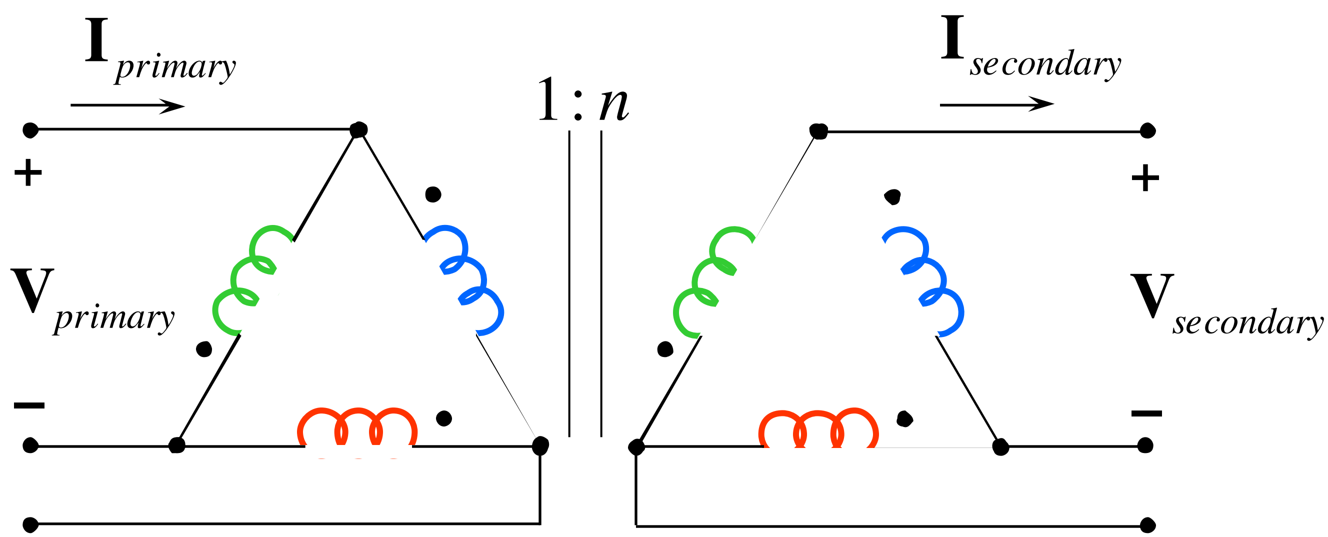
Three – Phase Transformers (1), Y–Y



$$\mathbf{V}_{secondary} = n\mathbf{V}_{primary}$$

$$\mathbf{I}_{secondary} = \frac{\mathbf{I}_{primary}}{n}$$

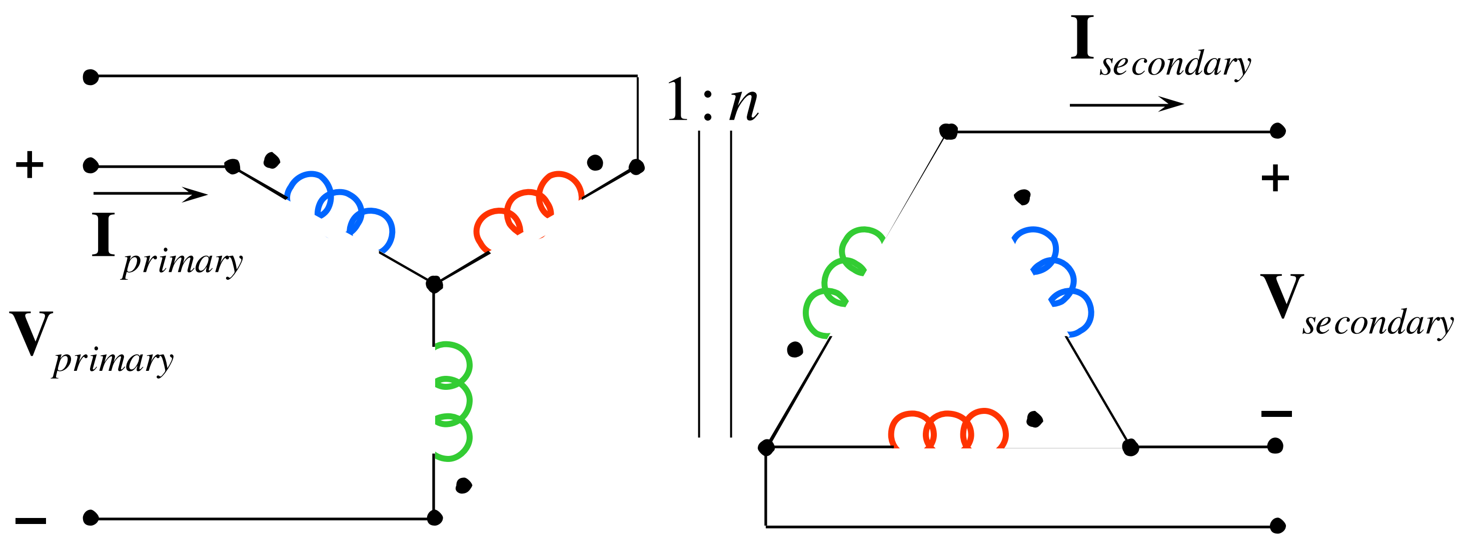
Three – Phase Transformers (2), Δ–Δ



$$\mathbf{V}_{secondary} = n\mathbf{V}_{primary}$$

$$\mathbf{I}_{secondary} = \frac{\mathbf{I}_{primary}}{n}$$

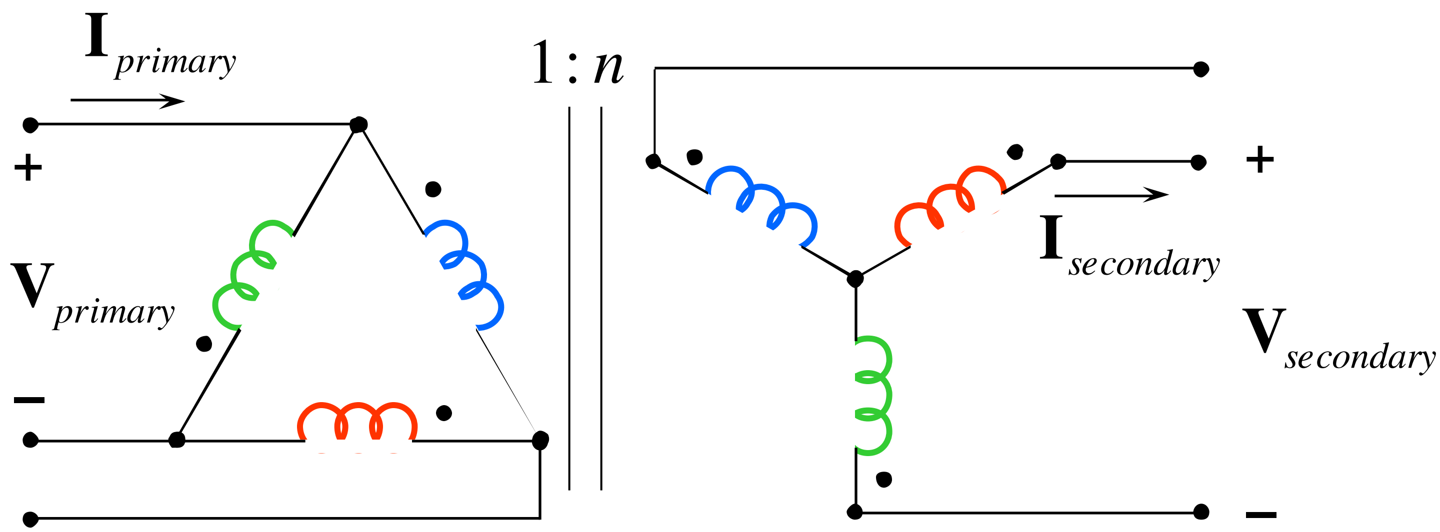
Three – Phase Transformers (3), Y–Δ



$$\mathbf{V}_{secondary} = \frac{n\mathbf{V}_{primary}}{\sqrt{3}}$$

$$\mathbf{I}_{secondary} = \frac{\sqrt{3}\mathbf{I}_{primary}}{n}$$

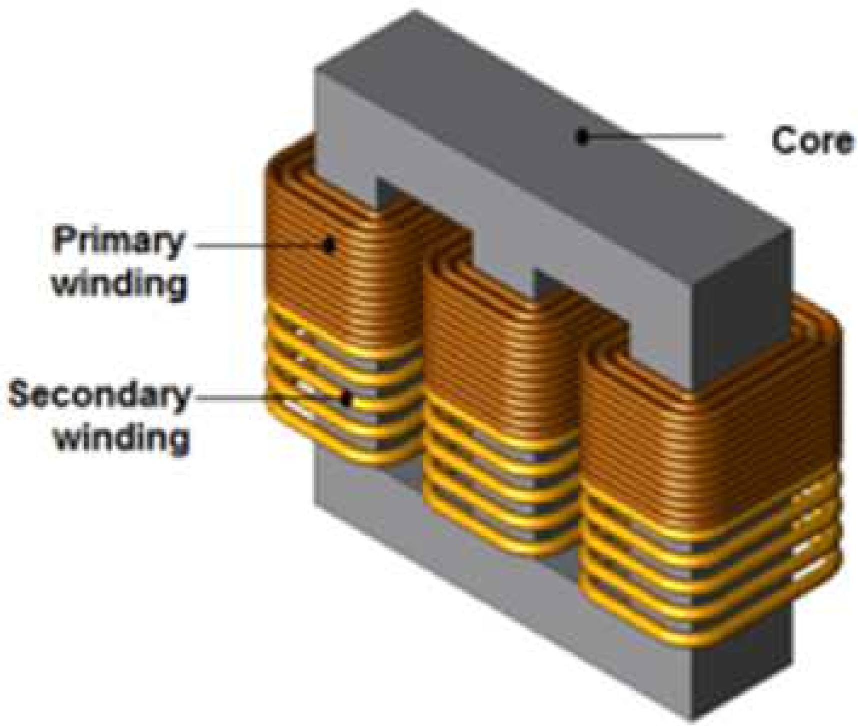
Three – Phase Transformers (4), Δ–Y



$$\mathbf{V}_{secondary} = n\sqrt{3}\mathbf{V}_{primary}$$

$$\mathbf{I}_{secondary} = \frac{\mathbf{I}_{primary}}{n\sqrt{3}}$$

Three – Phase Transformers (5)



https://www.jmag-international.com/catalog/132_threephasetransformer_loss/