Single phase PWM inverter design

- Design requirement:
  - Output voltage: U_o (V) ; Fundamental frequency f_1 *(Hz)*.
  - Output power: S (kVA)
- Example:

$$U_{om} = 220\sqrt{2} = 311(V); f_1 = 50Hz; S = 1.25\ kVA$$

- Design step:
- 1. DC voltage calculation: V_{Bus} (V).
  - Without over modulation, the modulation index $\mu \leq 1$, ex. choose $\mu_{max} = 0,9$.
  - Then: $V_{Bu} = U_{om}/0,9 = 311/0,9 = 346$ V.
  - Normally, the drop voltage on the output filter can be chosen about 10% of the output voltage, then the minimum of required DC voltage can be calculated as following: $V_{Bus\_min} = 1,1.346 = 380$ V.

Choose: V_{bus}=380V

Single phase PWM inverter design

- 2. Calculate the amplitude of the output current : I_{om} (A).
  - RMS value: $I_o = S_o/U_o$ = 1250/220 = 5,68 (A).
  - Amplitude value: $I_{om} = I_o.sqrt(2)$ = 5,68*1,4142 = 8 (A).
- 3. Choose switching frequency: f_s (Hz),
  - With low power inverter using IGBT, choose f_s = 20 kHz, period: $T_s = 0,5.10^{-4}$ (s).
- 4. IGBT and Diode calculation:
  - IGBT average current:
  - I_V = 2,29 A.
  - Diode average current:
  - I_D = 0,26 A.

$$I_V = \frac{1}{2\pi}\int_{\varphi}^{\pi} I_{om}\sin(\theta-\varphi)\,d\theta = \frac{1+\cos\varphi}{2\pi}I_{om}$$

$$I_D = \frac{1}{2\pi}\int_{0}^{\varphi} I_{om}\sin(\theta-\varphi)\,d\theta = \frac{1-\cos\varphi}{2\pi}I_{om}$$

IGBT and diode block voltage: V_{Bus}

Choose cos φ=0.8 is the minimum load power factor

Single phase PWM inverter design

- 5. Inductor design

The voltage across the inductor is given as:

$$V = L_i \times \frac{di}{dt}$$

For the full-bridge inverter with an AC output, write the equation as:

$$\Rightarrow (V_{Bus} - V_O) = L_i \times \frac{\Delta i_{pp}}{D \times T_s} \qquad \Rightarrow \Delta i_{pp} = \frac{D \times T_s \times (V_{Bus} - V_O)}{L_i}$$

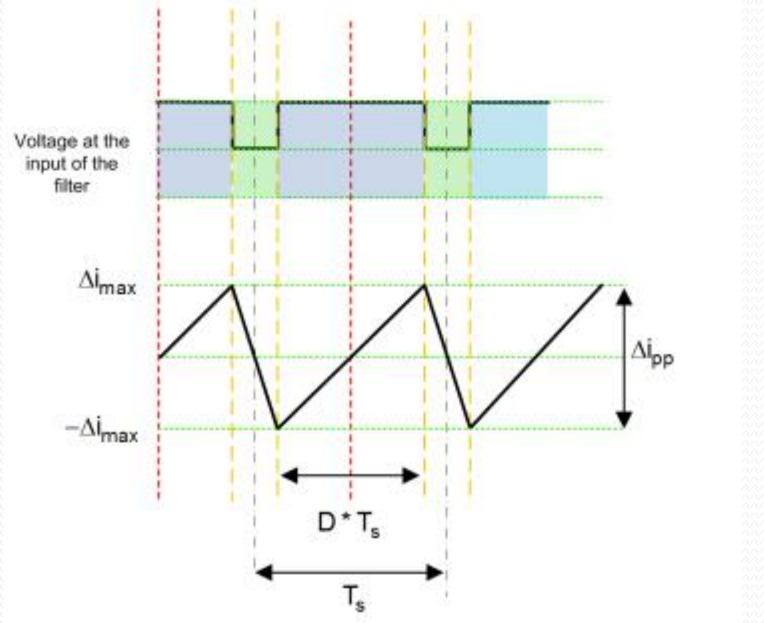
Where: D is duty circle

Replace duty circle D=μ.sin(ωt) and Vo= D.V_{Bus} , then:

$$\Delta i_{pp} = \frac{V_{Bus} \times T_s \times m_a \times \sin(\omega t) \times (1 - m_a \sin(\omega t))}{L_i}$$

Maximum current ripple can be calculated as:

$$\Delta i_{pp}\Big|_{max} = \frac{V_{Bus} \times T_s}{4 \times L_i}$$ When: $$\sin(\omega t) = \frac{1}{2\, m_a}$$



Current ripple calculation

Single phase PWM inverter design

- 5. Inductor design

Normally, the current ripple ≤ 30% is tolerable by the inductor core, then the inductor can be calculated as following:

$$L_i = \frac{V_{Bus}}{4 \times F_{sw} \times \Delta i_{pp}\big|_{max}}$$

Example design: choose current ripple= 20%

$$L_i = \frac{380}{4.20000.0.2.5.68} = 4{,}18\ (mH)$$

Check drop voltage condition (less than 10%)

Drop voltage = $\omega_s L_i I_o$=2.3,14.50.4,18.10^{-3}.5,68= 7,45V ≈ 3.4%

Single phase PWM inverter design

- 6. Capacitance selection
  - The output inductor and capacitor form a low pass filter that filters out the switching frequency. To get good switching frequency attenuation the cut off frequency is kept at $f_{sw}/10$ or lower.
  - Example design, choose $\omega_{CL} = 0,1\omega_s \Rightarrow \omega_{CL} = 12,5664.10^3$ (rad/s)

Then:

$$C = \frac{1}{L}\frac{1}{\omega_{CL}^2} = \frac{1}{4.18.10^{-3}}\frac{1}{(12,5664.10^3)^2} = 1.51(\mu F)$$

5. Space vector modulation
5.1 Clark transformation (Alpha-beta transformation)

- Three phase voltage/current system $X = (X_A, X_B, X_C)$,

if : $X_a + X_b + X_c = 0$

By Clark transformation, it equal to a space vector:

$$\bar{u} = \frac{2}{3}(u_A + a u_B + a^2 u_C)$$

Where: $a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

- Express on Alpha-beta axis:

$\bar{u}$

$$\begin{cases} u_\alpha = \frac{1}{3}(2u_A - u_B - u_C) \\ u_\beta = \frac{1}{\sqrt{3}}(u_B - u_C) \end{cases}$$

- Express by transformation matrix:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}[u_A u_B u_C]^T$$

$$= T_1 . [u_A u_B u_C]^T$$

- If:

$$\begin{cases} u_A = U^m \cos(\omega t) \\ u_B = U^m \cos\left(\omega t - \frac{2\pi}{3}\right) \\ u_C = U^m \cos\left(\omega t + \frac{2\pi}{3}\right) \end{cases}$$

- Then $\bar{u}$ is rotating vector:

$$\bar{u} = U^m e^{j(\omega t)}$$

5. Space vector modulation

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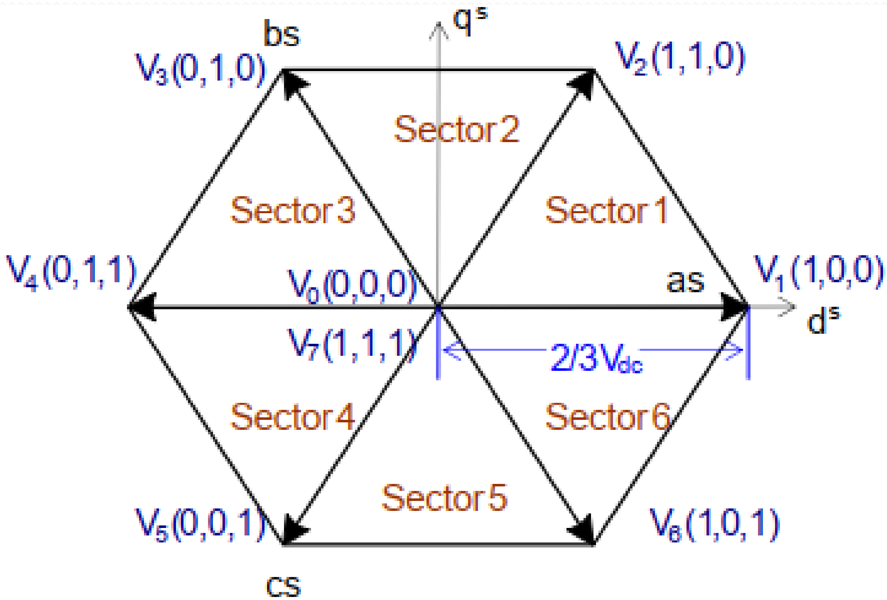
$$\bar{u} = U^m e^{j(\omega t)}$$

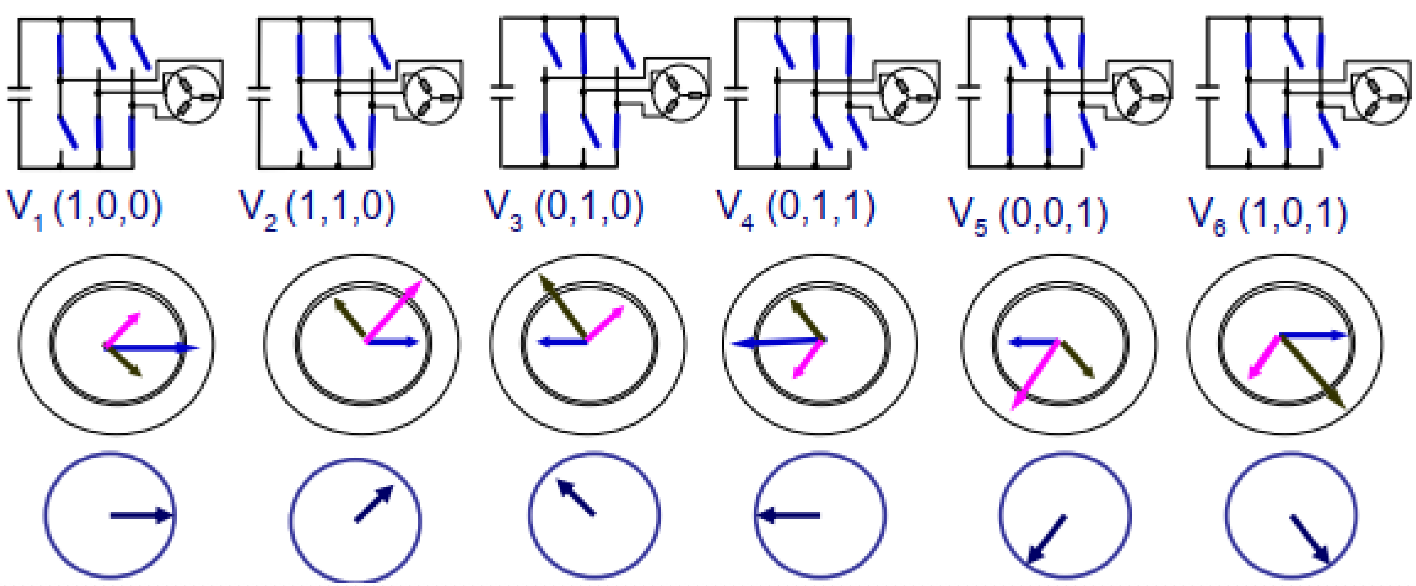
5. Space vector modulation
5.2 Switching states and vectors

No	Van dẫn	u_A	u_B	u_C	ū
U0	V2, V4, V6	0	0	0	0
U1	V6, V1, V2	2/3U_{DC}	-1/3U_{DC}	-1/3U_{DC}	$\frac{2}{3}U_{DC}e^{-j0}$
U2	V1, V2, V3	1/3U_{DC}	1/3U_{DC}	-2/3U_{DC}	$\frac{2}{3}U_{DC}e^{j\frac{\pi}{3}}$
U3	V2, V3, V4	-1/3U_{DC}	2/3U_{DC}	-1/3U_{DC}	$\frac{2}{3}U_{DC}e^{j\frac{2\pi}{3}}$
U4	V3, V4, V5	-2/3U_{DC}	1/3U_{DC}	1/3U_{DC}	$\frac{2}{3}U_{DC}e^{-j\pi}$
U5	V4, V5, V6	-1/3U_{DC}	-1/3U_{DC}	2/3U_{DC}	$\frac{2}{3}U_{DC}e^{-j\frac{2\pi}{3}}$
U6	V5, V6, V1	1/3U_{DC}	-2/3U_{DC}	1/3U_{DC}	$\frac{2}{3}U_{DC}e^{-j\frac{\pi}{3}}$
U7	V1, V3, V5	0	0	0	0

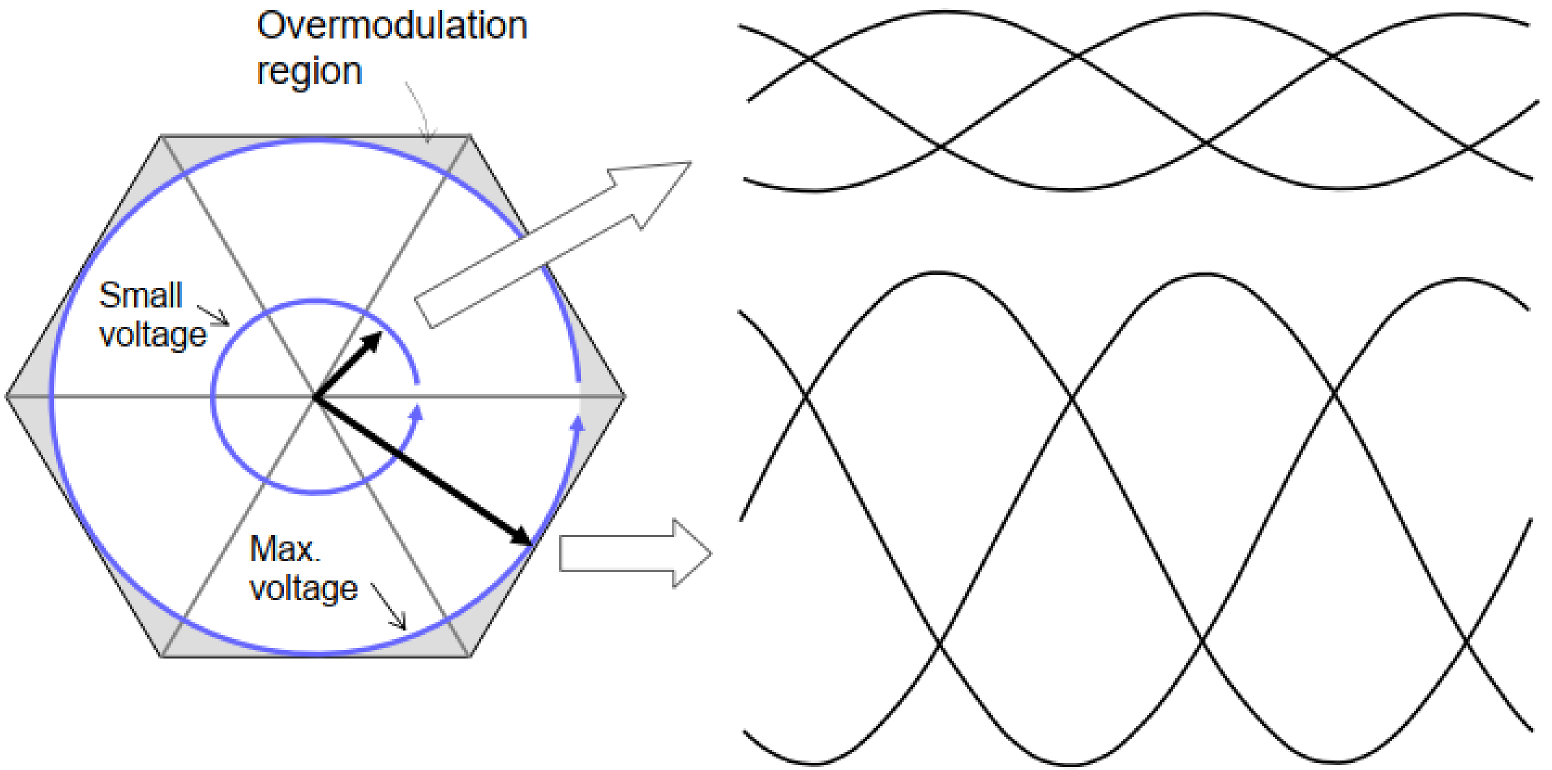
5. Space vector modulation
5.2 Switching states and space vectors



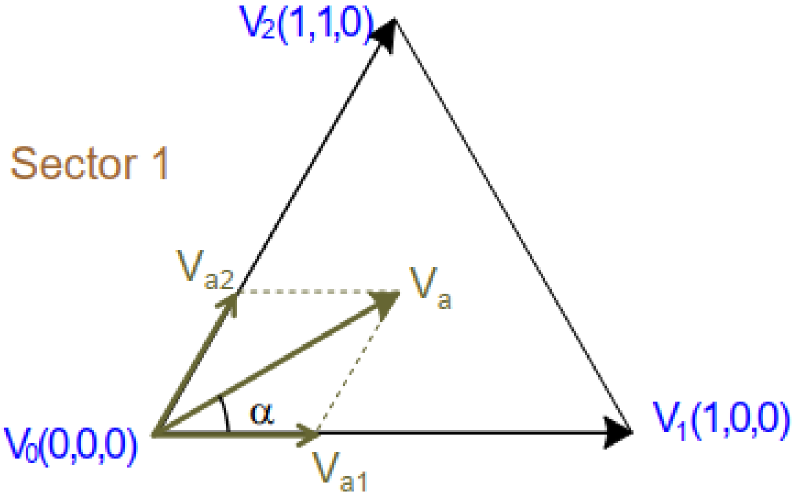
$$
\begin{aligned}
V_1 &: (S_a, S_b, S_c) = (1, 0, 0);\\
V_2 &: (S_a, S_b, S_c) = (1, 1, 0);\\
V_3 &: (S_a, S_b, S_c) = (0, 1, 0);\\
V_4 &: (S_a, S_b, S_c) = (0, 1, 1);\\
V_5 &: (S_a, S_b, S_c) = (0, 0, 1);\\
V_6 &: (S_a, S_b, S_c) = (1, 0, 1);\\
V_7 &: (S_a, S_b, S_c) = (1, 1, 1);\\
V_0 &: (S_a, S_b, S_c) = (0, 0, 0).
\end{aligned}
$$



5. Space vector modulation
5.3 Modulation



5. Space vector modulation
5.3 Modulation



$$\frac{V_{a1}}{(2/3)V_{dc}} = \frac{T_1}{T_s},$$
$$\frac{V_{a2}}{(2/3)V_{dc}} = \frac{T_2}{T_s}.$$

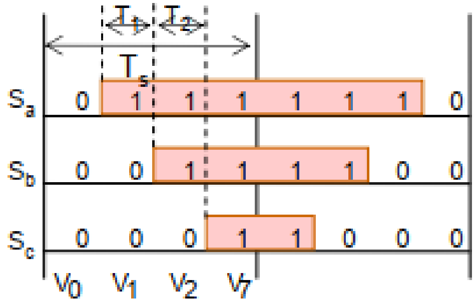
$$\begin{bmatrix} v_d^s \\ v_q^s \end{bmatrix} = V_a \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} = \frac{T_1}{T_s}\frac{2}{3}V_{dc}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{T_2}{T_s}\frac{2}{3}V_{dc}\begin{bmatrix} \cos\frac{\pi}{3} \\ \sin\frac{\pi}{3} \end{bmatrix}$$
$$= \frac{2}{3}\frac{V_{dc}}{T_s}\begin{bmatrix} 1 & \cos\frac{\pi}{3} \\ 0 & \sin\frac{\pi}{3} \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}.$$

$$\Rightarrow \boxed{\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \frac{\sqrt{3}T_s}{V_{dc}}\begin{bmatrix} \sin\frac{\pi}{3} & -\cos\frac{\pi}{3} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_d^s \\ v_q^s \end{bmatrix}.}$$

$T_0 = T_s - T_1 - T_2$

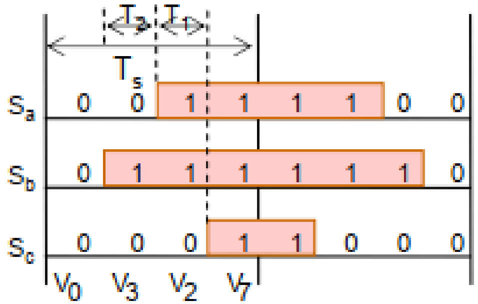
5. Space vector modulation
5.3 Modulation

Sector 1



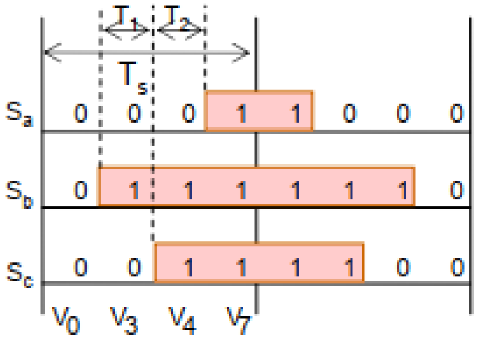
$$
\begin{aligned}
S_a &= T_1 + T_2 + (T_s - T_1 - T_2)/2 \\
S_b &= T_2 + (T_s - T_1 - T_2)/2 \\
S_c &= (T_s - T_1 - T_2)/2
\end{aligned}
$$

Sector 2



$$
\begin{aligned}
S_a &= T_1 + (T_s - T_1 - T_2)/2 \\
S_b &= T_2 + T_1 + (T_s - T_1 - T_2)/2 \\
S_c &= (T_s - T_1 - T_2)/2
\end{aligned}
$$

Sector 3

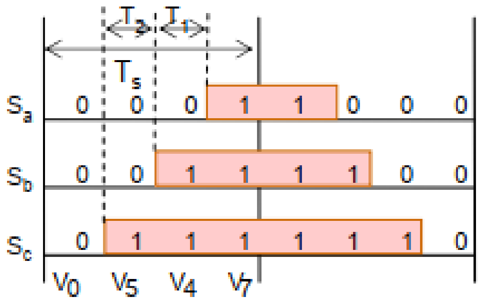


$$
\begin{aligned}
S_a &= (T_s - T_1 - T_2)/2 \\
S_b &= T_1 + T_2 + (T_s - T_1 - T_2)/2 \\
S_c &= T_2 + (T_s - T_1 - T_2)/2
\end{aligned}
$$

5. Space vector modulation

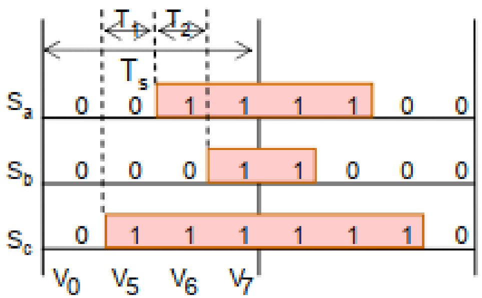
5.3 Modulation

Sector 4



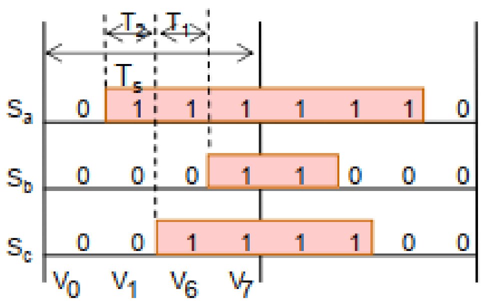
$$
\begin{aligned}
S_a &= (T_s - T_1 - T_2)/2 \\
S_b &= T_1 \;+\; (T_s - T_1 - T_2)/2 \\
S_c &= T_2 + T_1 \;+\; (T_s - T_1 - T_2)/2
\end{aligned}
$$

Sector 5



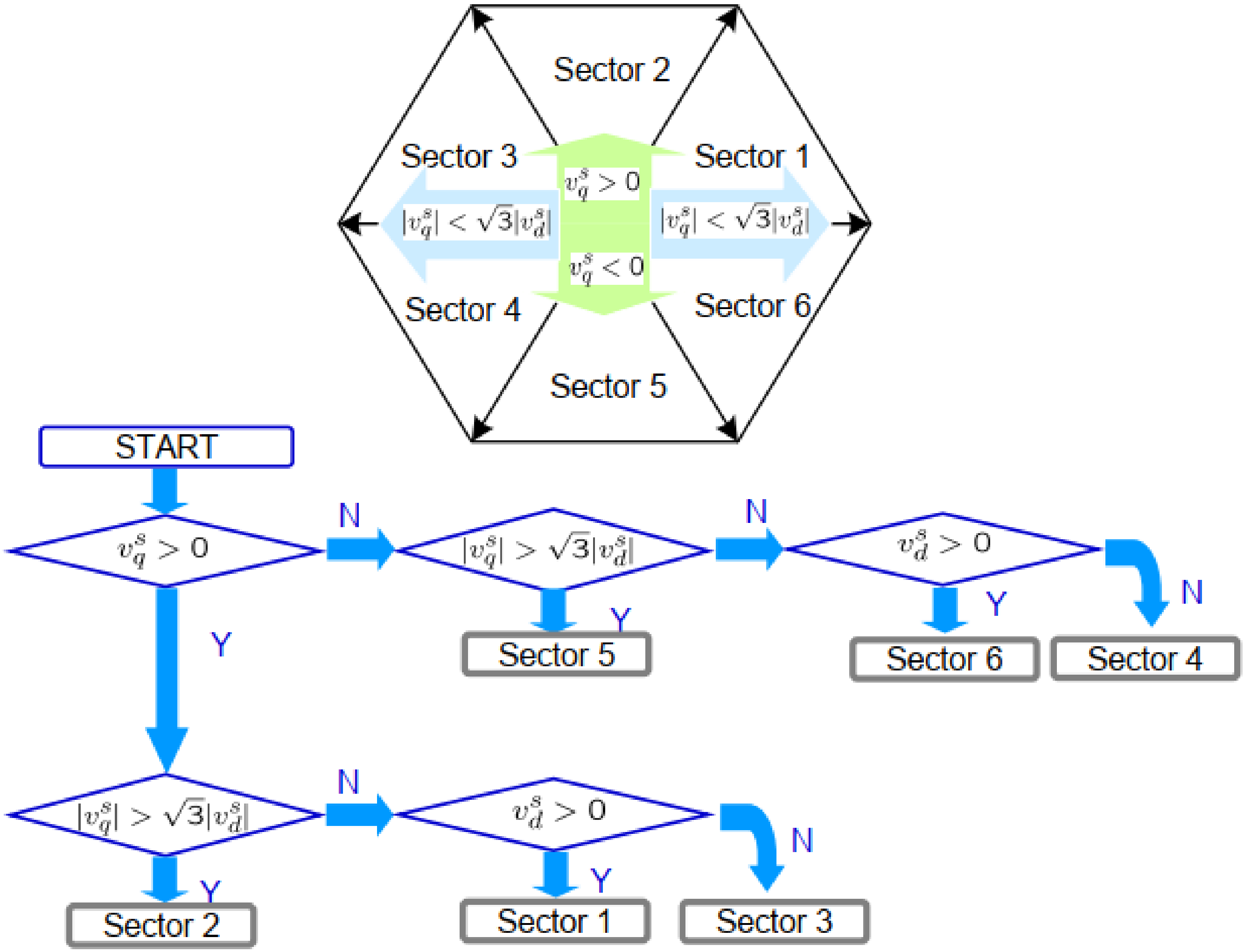
$$
\begin{aligned}
S_a &= T_2 \;+\; (T_s - T_1 - T_2)/2 \\
S_b &= (T_s - T_1 - T_2)/2 \\
S_c &= T_1 + T_2 \;+\; (T_s - T_1 - T_2)/2
\end{aligned}
$$

Sector 6



$$
\begin{aligned}
S_a &= T_2 + T_1 \;+\; (T_s - T_1 - T_2)/2 \\
S_b &= (T_s - T_1 - T_2)/2 \\
S_c &= T_1 \;+\; (T_s - T_1 - T_2)/2
\end{aligned}
$$

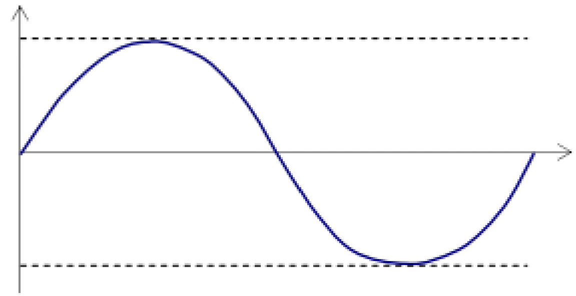
5. Space vector modulation
5.4 Sector finding method

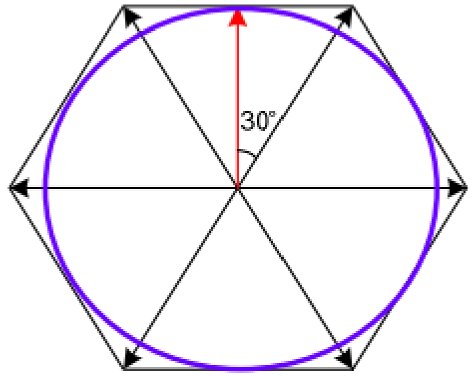


5. Space vector modulation
5.5 SPWM versus SVPWM

- SPWM

$$V_{o,\max} = \frac{1}{2}V_{dc}$$





- SVPWM
  - Voltage Usage : 15.5% increase
  - No of switching decreases.

$$V_{o,\max} = \frac{\sqrt{3}}{2}\frac{2}{3}V_{dc}$$