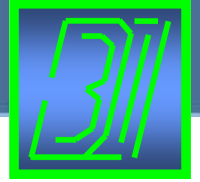




TRƯỜNG ĐẠI HỌC
BÁCH KHOA HÀ NỘI

Nguyễn Công Phương



Electric Circuit Theory

AC Power Analysis

Contents

- I. Basic Elements Of Electrical Circuits
- II. Basic Laws
- III. Electrical Circuit Analysis
- IV. Circuit Theorems
- V. Active Circuits
- VI. Capacitor And Inductor
- VII. First Order Circuits
- VIII. Second Order Circuits
- IX. Sinusoidal Steady State Analysis
- X. AC Power Analysis**
- XI. Three-phase Circuits
- XII. Magnetically Coupled Circuits
- XIII. Frequency Response
- XIV. The Laplace Transform
- XV. Two-port Networks





BOILERMAKER MOTOR COMPANY **TEFC**

LOW VOLTAGE LINE

HP 50 RPM 1765 FRAME 326T

VOLTS 230/460 PHASE 3 HERTZ 60

AMPS 122/61 TIME RATING CONT.

CAN BE USED ON 208 V SYSTEM UP TO 140 AMPS
MAY NOT MEET ALL NEMA PERFORMANCE LIMITS ON 208 V SYSTEM

INS F SERVICE FACTOR 1.15

HIGH VOLTAGE LINE

NEMA CODE G NEMA DESIGN B

MAX AMBIENT 40°C TEMP RISE 70°C

NEMA NOM. 92.4 EFF. 92.4 NEMA NOM. 91.0 EFF. 91.0 BOILERMAKER MIN. 91.0 EFF. 91.0

<http://electricalacademia.com/induction-motor/electric-motor-nameplate-details-explained-induction-motor-nameplate/>

SIEMENS

PE•21 PLUS™ PREMIUM EFFICIENCY

ORD.NO. 1LA02864SE41 I NO.

TYPE RGZ5SD FRAME 286T

H.P. 30.00 SERVICE FACTOR 1.15 3 PH

AMPS 34.9 VOLTS 460

R.P.M. 1765 HERTZ 60

DUTY CONT 40°C AMB. DATE CODE

CLASS INSUL F NEMA DESIGN B KVA CODE G NEMA NOM. EFF. 93.6

SH. END BRG. 50BC03JPP3 OPP. END BRG. 50BC03JPP3

MILL AND CHEMICAL DUTY QUALITY INDUCTION MOTOR

Siemens Energy & Automation, Inc, Little Rock, AR MADE IN U.S.A.

<http://poqynamexyoqep.oramanageability.com/understanding-induction-motor-nameplate-information-47374dan8099.html>

Dayton ELECTRIC BASEBOARD HEATER
CALEFACTOR ELÉCTRICO DE ZOCALO
PLINTE CHAUFFANTE ÉLECTRIQUE

MODEL NO. **3UG82D**

VOLTS 240/208 WATTS 500/376 AMPS 2.1/1.8

DATE CODE 1209

UL US
64E1 LISTED
ELECTRIC BASEBOARD HEATER
E256626

4104-2497-500

Made In USA/Hecho en EE.UU./Fab. aux Etats-Unis
Mfd. for/Fab. para/Fab. pour: Dayton Electric Mfg. Co., Niles, IL 60714 USA
For Repair Parts/Para Partes de Reparación/Pour pièces détachées,
Call/Llame/Appeler: 1-800-323-0620 or 001-800-527-2331 en Mexico

<https://www.cpsc.gov/Recalls/2010/marley-engineered-products-recalls-baseboard-heaters-sold-at-grainger-due-to-fire>

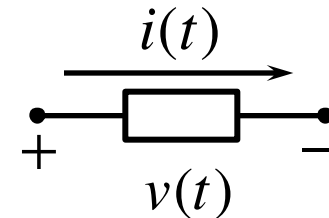
<https://sites.google.com/site/ncpdhbkhn/home>

AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals

Instantaneous Power (1)

$$\left. \begin{array}{l} p(t) = v(t)i(t) \\ v(t) = V_m \sin(\omega t + \phi_v) \\ i(t) = I_m \sin(\omega t + \phi_i) \end{array} \right\}$$



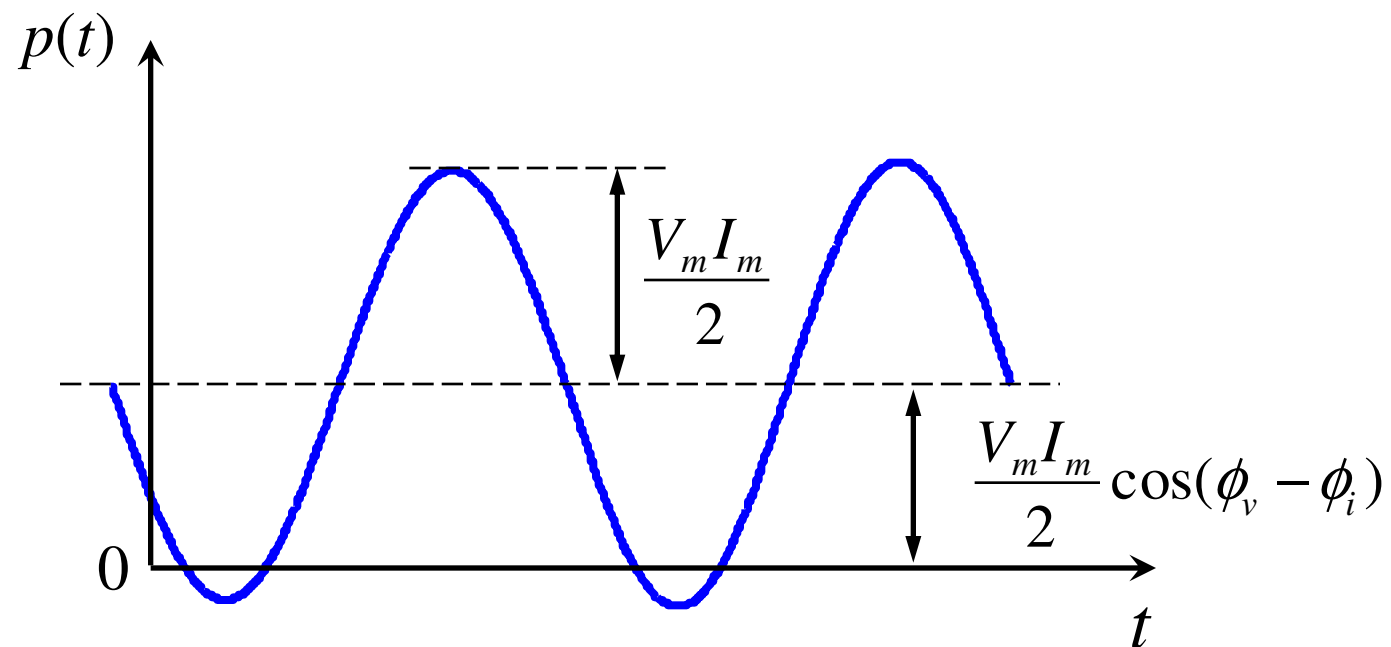
$$\rightarrow p(t) = V_m I_m \sin(\omega t + \phi_v) \sin(\omega t + \phi_i)$$

$$= \frac{V_m I_m}{2} [\cos(\phi_v - \phi_i) - \cos(2\omega t + \phi_v + \phi_i)]$$

$$= \frac{V_m I_m}{2} \cos(\phi_v - \phi_i) - \frac{V_m I_m}{2} \cos(2\omega t + \phi_v + \phi_i)$$

Instantaneous Power (2)

$$p(t) = \frac{V_m I_m}{2} \cos(\phi_v - \phi_i) - \frac{V_m I_m}{2} \cos(2\omega t + \phi_v + \phi_i)$$



Average Power (1)

$$P = \frac{1}{T} \int_0^T p(t) dt$$

$$p(t) = \frac{V_m I_m}{2} \cos(\phi_v - \phi_i) - \frac{V_m I_m}{2} \cos(2\omega t + \phi_v + \phi_i)$$

$$\rightarrow P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) \frac{1}{T} \int_0^T dt - \frac{1}{2} V_m I_m \frac{1}{T} \int_0^T \cos(2\omega t + \phi_v + \phi_i) dt$$

The average of a sinusoid over its period is zero

$$\rightarrow P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

Average Power (2)

$$\mathbf{V} = V_m \underline{\angle \phi_v}$$

$$\mathbf{I} = I_m \underline{\angle \phi_i} \rightarrow \mathbf{I}^* = I_m \underline{\angle -\phi_i}$$

$$\rightarrow \mathbf{VI}^* = V_m I_m \underline{\angle \phi_v - \phi_i}$$

$$\mathbf{VI}^* = V_m I_m \underline{\angle \phi_v - \phi_i} = V_m I_m \cos(\phi_v - \phi_i) + j V_m I_m \sin(\phi_v - \phi_i)$$
$$P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

$$\rightarrow P = \frac{1}{2} \text{Re}(\mathbf{VI}^*)$$

Average Power (3)

$$P = \frac{1}{2} \operatorname{Re}(\mathbf{V}\mathbf{I}^*) = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

$$\phi_v = \phi_i :$$

$$P = \frac{1}{2} V_m I_m \cos(0) = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$$

$$\phi_v - \phi_i = \pm 90^\circ :$$

$$P = \frac{1}{2} V_m I_m \cos(90^\circ) = 0$$

Average Power (4)

Ex.

$v(t) = 150\sin(314t - 30^\circ)$ V, $i(t) = 10\sin(314t + 45^\circ)$ A. Find P ?

$$P = \frac{1}{2} V_m I_m \cos(\varphi_u - \varphi_i) = \frac{1}{2} 150 \times 10 \cos(-30^\circ - 45^\circ) = \boxed{194.11 \text{ W}}$$

$$P = \frac{1}{2} \operatorname{Re}(\mathbf{V}\mathbf{I}^*)$$

$$\mathbf{V} = V_m \angle \varphi_u = 150 \angle -30^\circ$$

$$\mathbf{I} = I_m \angle \varphi_i = 10 \angle 45^\circ \rightarrow \mathbf{I}^* = 10 \angle -45^\circ$$

$$\mathbf{V}\mathbf{I}^* = (150 \angle -30^\circ)(10 \angle -45^\circ) = 1500 \angle -75^\circ = 388.23 - j1448.9 \text{ VA}$$

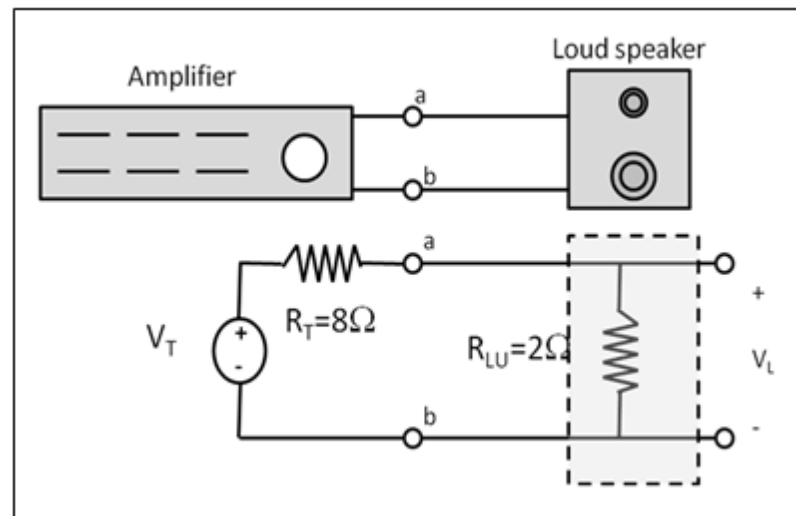
$$P = \frac{1}{2} \operatorname{Re}\{388.23 - j1448.9\} = \frac{1}{2} 388.23 = \boxed{191.11 \text{ W}}$$

AC Power Analysis

1. Instantaneous and Average Power
- 2. Maximum Average Power Transfer**
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals



Maximum Average Power Transfer (1)



<http://www.chegg.com/homework-help/questions-and-answers/use-maximum-power-transfer-theorem-determine-increase-power-delivered-loudspeaker-resultin-q6983635>

Maximum Average Power Transfer (2)

$$P_L = \frac{1}{2} I_{Lm}^2 R_L$$

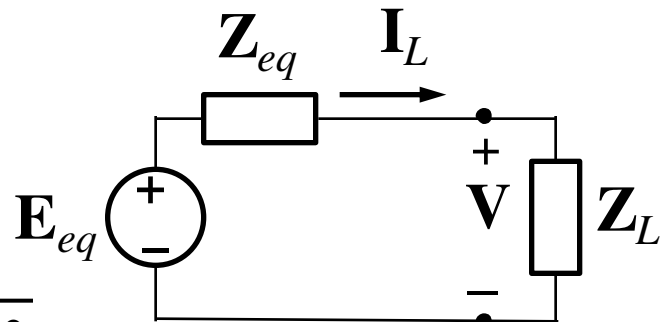
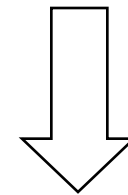
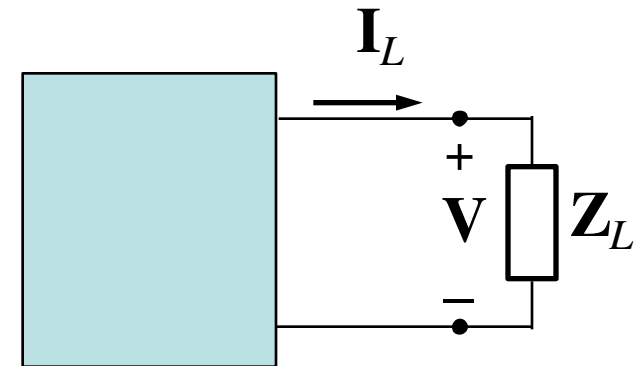
$$\mathbf{I}_L = \frac{\mathbf{E}_{eq}}{\mathbf{Z}_{eq} + \mathbf{Z}_L} \rightarrow I_{Lm} = \frac{|\mathbf{E}_{eq}|}{|\mathbf{Z}_{eq} + \mathbf{Z}_L|}$$

$$\left. \begin{aligned} \mathbf{Z}_{eq} &= R_{eq} + jX_{eq} \\ \mathbf{Z}_L &= R_L + jX_L \end{aligned} \right\} \rightarrow$$

$$\rightarrow \mathbf{Z}_{eq} + \mathbf{Z}_L = R_{eq} + jX_{eq} + R_L + jX_L$$

$$= (R_{eq} + R_L) + j(X_{eq} + X_L)$$

$$\rightarrow |\mathbf{Z}_{eq} + \mathbf{Z}_L| = \sqrt{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$



Maximum Average Power Transfer (3)

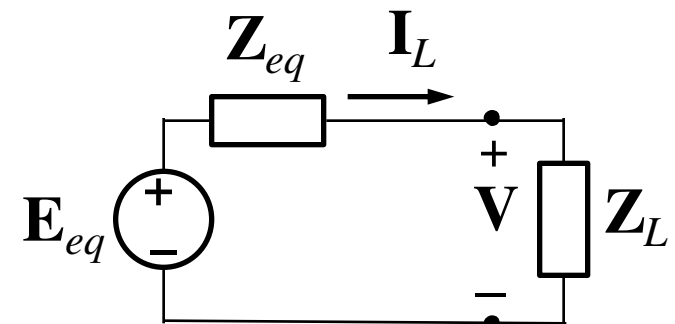
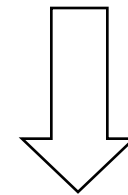
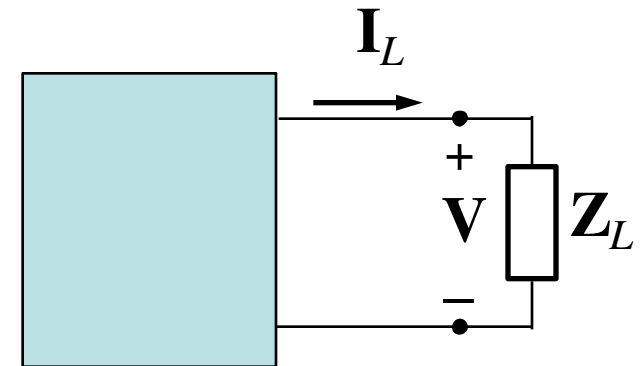
$$P_L = \frac{1}{2} I_{Lm}^2 R_L$$

$$I_{Lm} = \frac{|\mathbf{E}_{eq}|}{|\mathbf{Z}_{eq} + \mathbf{Z}_L|}$$

$$|\mathbf{Z}_{eq} + \mathbf{Z}_L| = \sqrt{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

$$\rightarrow P_L = \frac{1}{2} \times \frac{|\mathbf{E}_{eq}|^2 R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

$$P_L \text{ is maximum if: } \begin{cases} \frac{\partial P_L}{\partial R_L} = 0 \\ \frac{\partial P_L}{\partial X_L} = 0 \end{cases}$$



Maximum Average Power Transfer (4)

$$P_L = \frac{1}{2} \times \frac{|\mathbf{E}_{eq}|^2 R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}$$

$$\begin{cases} \frac{\partial P_L}{\partial X_L} = 0 \rightarrow \frac{\partial P_L}{\partial X_L} = |\mathbf{E}_{eq}|^2 \frac{R_L (X_{eq} + X_L)}{[(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2]^2} = 0 \\ \frac{\partial P_L}{\partial R_L} = 0 \rightarrow \frac{\partial P_L}{\partial R_L} = |\mathbf{E}_{eq}|^2 \frac{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2 - 2R_L(R_{eq} + R_L)}{2[(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2]^2} = 0 \end{cases}$$

$$\rightarrow \begin{cases} X_L = -X_{eq} \\ R_L = \sqrt{R_{eq}^2 + (X_{eq} + X_L)^2} \end{cases} \quad \rightarrow \begin{cases} X_L = -X_{eq} \\ R_L = R_{eq} \end{cases}$$

$$\rightarrow \boxed{\mathbf{Z}_L = \mathbf{Z}_{eq}^*}$$

For maximum average power transfer, the load impedance must be equal to the complex conjugate of the equivalent impedance

Maximum Average Power Transfer (5)

$$P_L = \frac{1}{2} \times \frac{|\mathbf{E}_{eq}|^2 R_L}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2} \left\{ \begin{array}{l} X_L = -X_{eq} \\ R_L = R_{eq} \end{array} \right\} \rightarrow P_{L\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$$

Maximum Average Power Transfer (6)

For maximum average power transfer, the load impedance must be equal to the complex conjugate of the equivalent impedance

$$\mathbf{Z}_L = \mathbf{Z}_{eq}^*$$

$$\left. \begin{array}{l} \text{If } Z_L = R_L ? \rightarrow X_L = 0 \\ \frac{\partial P_L}{\partial R_L} = 0 \rightarrow R_L = \sqrt{R_{eq}^2 + (X_{eq} + X_L)^2} \end{array} \right\} \rightarrow$$

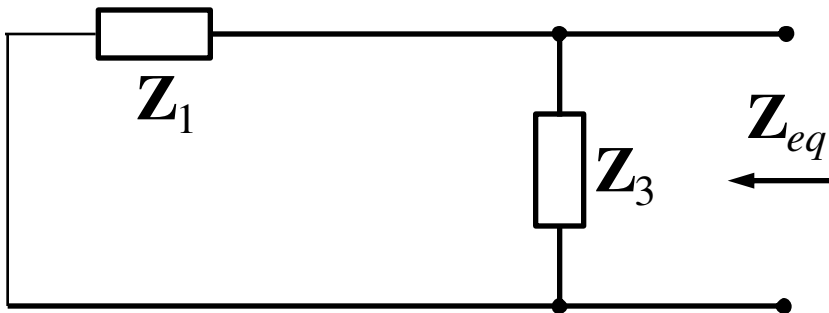
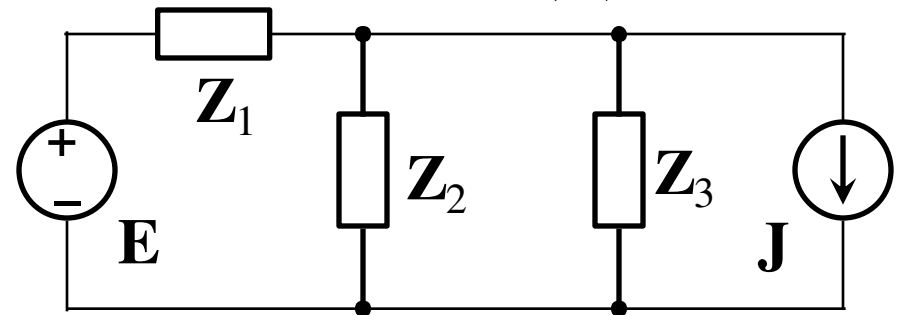
$$\rightarrow R_L = \sqrt{R_{eq}^2 + X_{eq}^2} = |\mathbf{Z}_{eq}|$$

Ex. 1 Maximum Average Power Transfer (7)

$$\mathbf{E} = 20 \angle -45^\circ \text{ V}; \mathbf{J} = 5 \angle 60^\circ \text{ A}$$

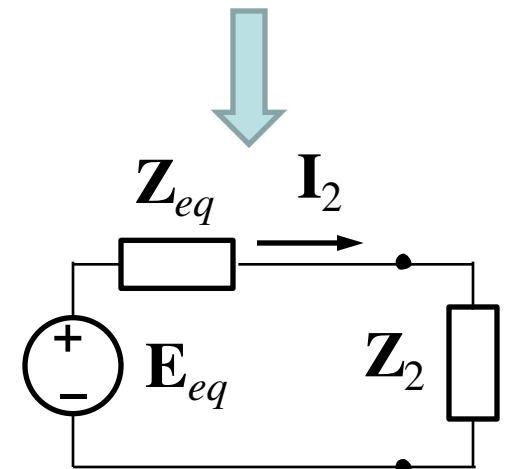
$$\mathbf{Z}_1 = 12 \Omega; \mathbf{Z}_3 = -j16 \Omega$$

Determine the load impedance \mathbf{Z}_2 that maximize the average power. What is the maximum average power?



$$\mathbf{Z}_{eq} = \frac{\mathbf{Z}_1 \mathbf{Z}_3}{\mathbf{Z}_1 + \mathbf{Z}_3} = \frac{12(-j16)}{12 - j16} = 7.68 - j5.76 \Omega$$

$$\rightarrow \mathbf{Z}_2 = \boxed{7.68 + j5.76 \Omega}$$



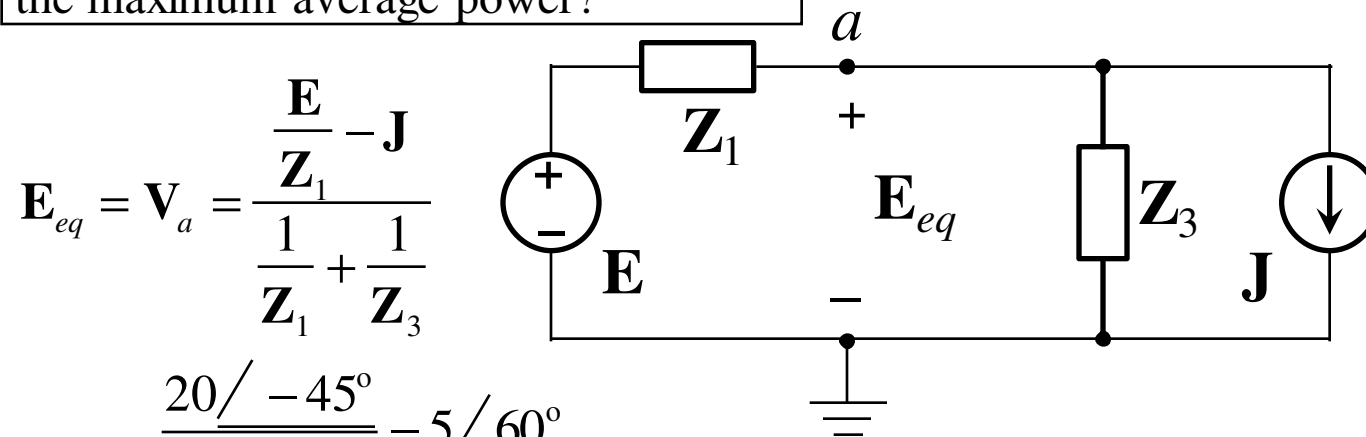
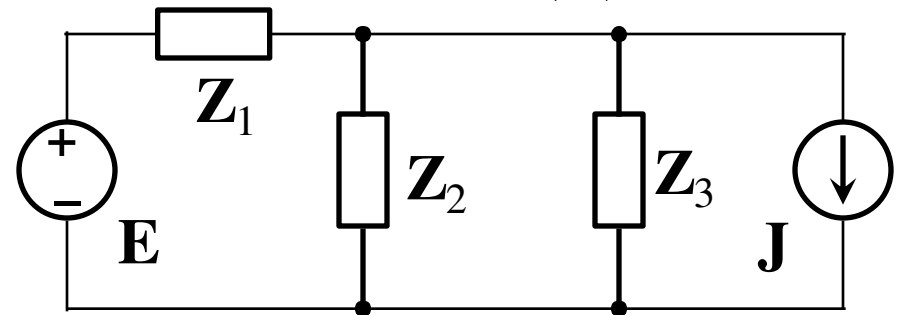
$$\mathbf{Z}_2 = \mathbf{Z}_{eq}^*$$

Ex. 1 Maximum Average Power Transfer (8)

$$\mathbf{E} = 20 \angle -45^\circ \text{ V}; \mathbf{J} = 5 \angle 60^\circ \text{ A}$$

$$\mathbf{Z}_1 = 12 \Omega; \mathbf{Z}_3 = -j16 \Omega$$

Determine the load impedance \mathbf{Z}_2 that maximize the average power. What is the maximum average power?

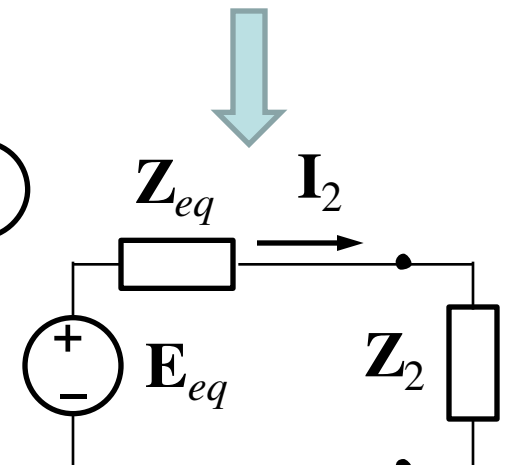


$$\mathbf{E}_{eq} = \mathbf{V}_a = \frac{\frac{\mathbf{E}}{\mathbf{Z}_1} - \mathbf{J}}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3}}$$

$$= \frac{\frac{20 \angle -45^\circ}{12} - 5 \angle 60^\circ}{\frac{1}{12} + \frac{1}{-j16}} = 54.38 \angle -140.4^\circ \text{ V}$$

$$\rightarrow |\mathbf{E}_{eq}| = 54.38 \text{ V} \rightarrow P_{2\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}} = \frac{54.38^2}{8 \times 7.68} = 48.13 \text{ W}$$

<https://sites.google.com/site/ncpdhbkhn/home>



$$P_{2\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$$

Maximum Average Power Transfer (9)

1. Find the Thevenin equivalent

a. \mathbf{Z}_{eq}

b. \mathbf{E}_{eq}

2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

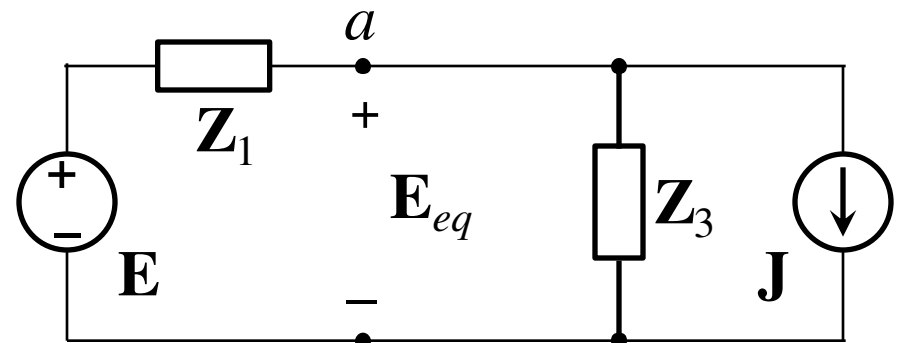
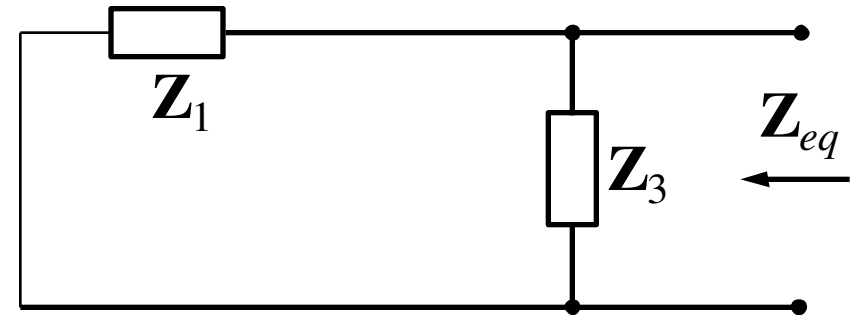
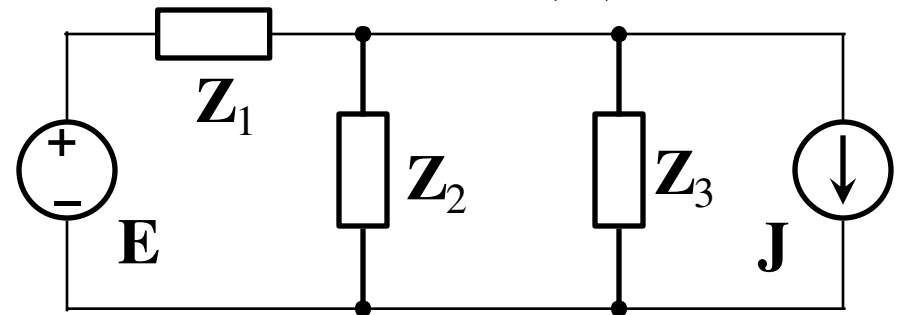
3. $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

1a. $\mathbf{Z}_{eq} = 7.68 - j5.76\Omega$

1b. $\mathbf{E}_{eq} = 54.38 \angle -140.4^\circ \text{ V}$

2. $\mathbf{Z}_L = 7.68 + j5.76\Omega$

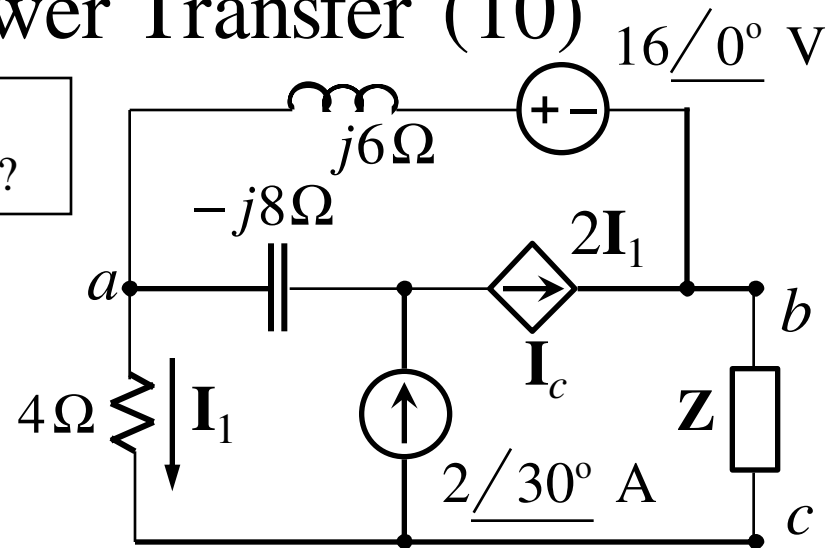
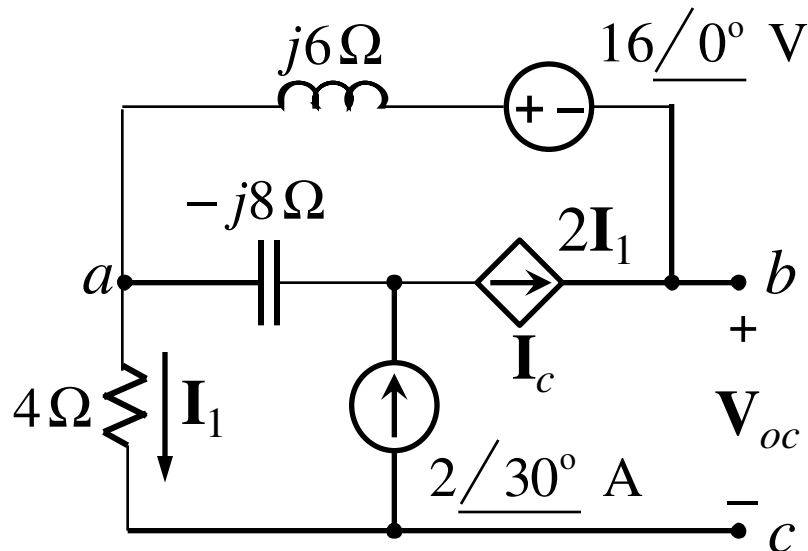
3. $P_{2\max} = 48.13 \text{ W}$



Ex. 2 Maximum Average Power Transfer (10)

Determine the load impedance \mathbf{Z} that maximize the average power. What is the maximum average power?

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open-circuit}}{\mathbf{I}_{short-circuit}}$$



1. Find the Thevenin equivalent

a. \mathbf{Z}_{eq}

b. \mathbf{E}_{eq}

2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

3. $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (11)

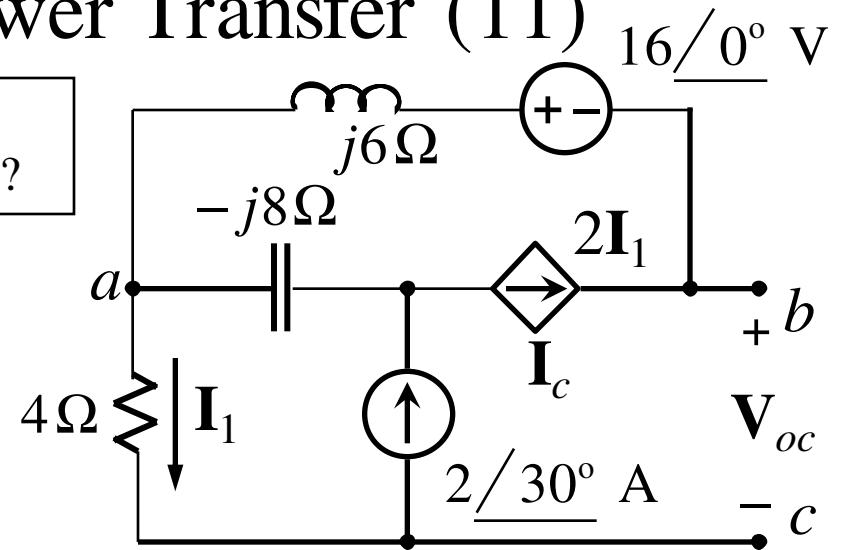
Determine the load impedance \mathbf{Z} that maximize the average power. What is the maximum average power?

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open-circuit}}{\mathbf{I}_{short-circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$

$$\left. \begin{aligned} (\mathbf{V}_c - \mathbf{V}_b) - 16 + j6\mathbf{I}_2 + 4\mathbf{I}_1 &= 0 \\ \mathbf{V}_{oc} &= \mathbf{V}_b - \mathbf{V}_c \end{aligned} \right\}$$

$$\left. \begin{aligned} \rightarrow \mathbf{V}_{oc} &= -16 + j6\mathbf{I}_2 + 4\mathbf{I}_1 \\ \mathbf{I}_1 &= 2 \angle 30^\circ \\ \mathbf{I}_2 = \mathbf{I}_c &= 2\mathbf{I}_1 = 2 \times 2 \angle 30^\circ \end{aligned} \right\}$$

$$\begin{aligned} \rightarrow \mathbf{V}_{oc} &= -16 + j6 \times 2 \times 2 \angle 30^\circ + 4 \times 2 \angle 30^\circ \\ &= \boxed{-21.07 + j24.78 \text{ V}} \end{aligned}$$



1. Find the Thevenin equivalent

a. \mathbf{Z}_{eq}

b. \mathbf{E}_{eq}

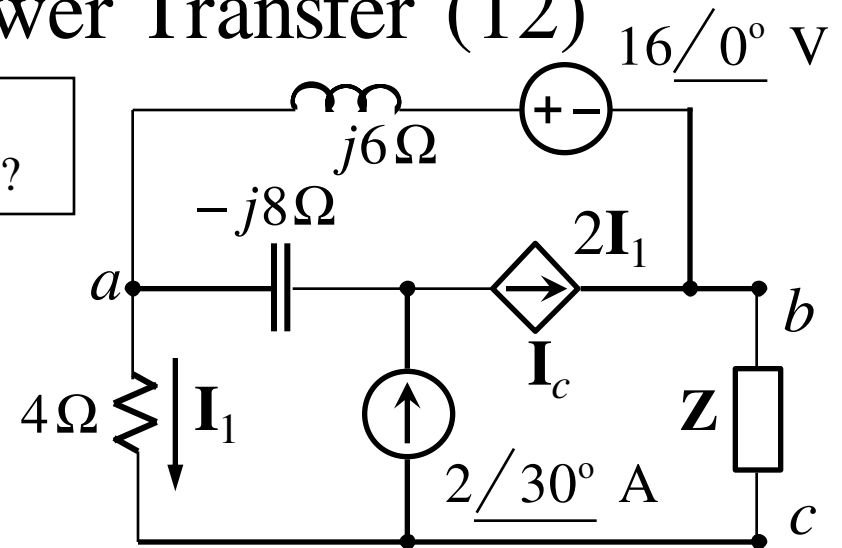
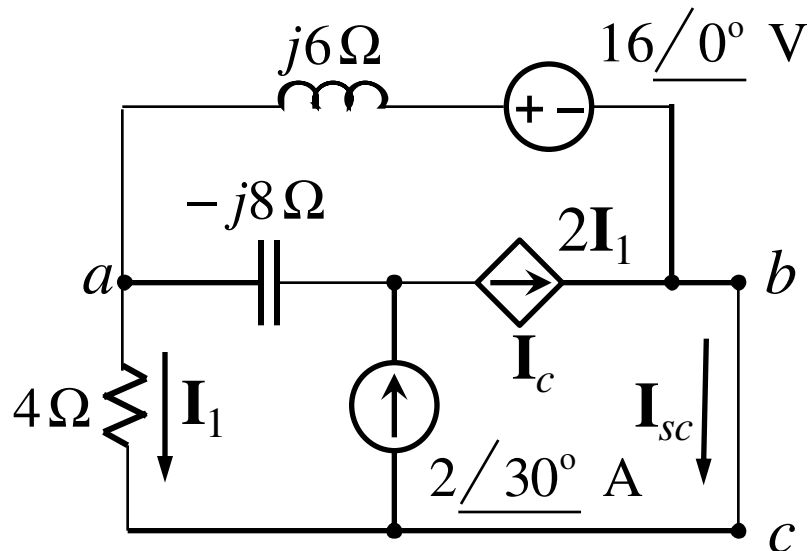
2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

3. $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (12)

Determine the load impedance \mathbf{Z} that maximize the average power. What is the maximum average power?

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open-circuit}}{\mathbf{I}_{short-circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$



1. Find the Thevenin equivalent

a. \mathbf{Z}_{eq}

b. \mathbf{E}_{eq}

2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

$$3. P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$$

Ex. 2 Maximum Average Power Transfer (13)

Determine the load impedance \mathbf{Z} that maximize the average power. What is the maximum average power?

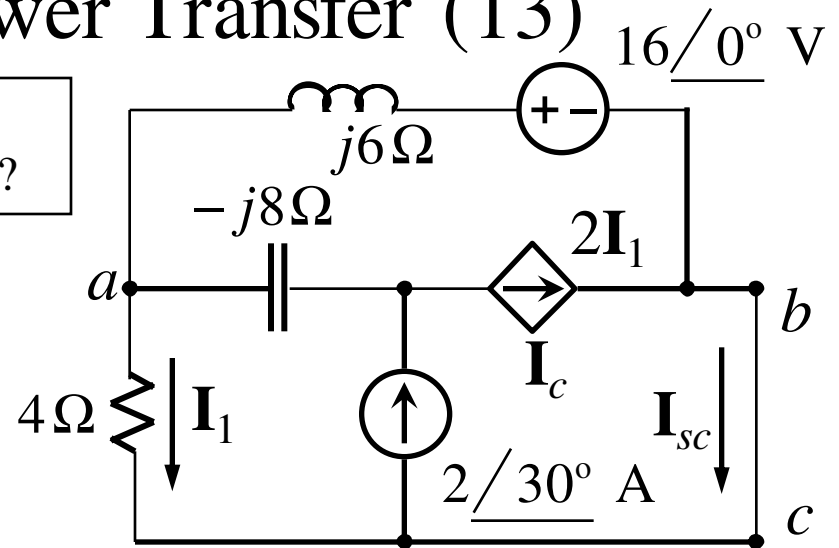
$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_{open-circuit}}{\mathbf{I}_{short-circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}}$$

$$\mathbf{I}_1 - 2\angle 30^\circ + \mathbf{I}_{sc} = 0 \rightarrow \mathbf{I}_{sc} = 2\angle 30^\circ - \mathbf{I}_1$$

$$\left. \begin{aligned} j6\mathbf{I}_2 + 4\mathbf{I}_1 &= 16\angle 0^\circ \\ \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^\circ - \mathbf{I}_c &= 0 \\ \rightarrow \mathbf{I}_2 - \mathbf{I}_1 + 2\angle 30^\circ - 2\mathbf{I}_1 &= 0 \\ \rightarrow 3\mathbf{I}_1 - \mathbf{I}_2 &= 2\angle 30^\circ \end{aligned} \right\}$$

$$\rightarrow \mathbf{I}_1 = 0.67 - j0.41 \text{ A}$$

$$\rightarrow \mathbf{I}_{sc} = 2\angle 30^\circ - (0.67 - j0.41) = 1.06 + j1.41 \text{ A}$$



1. Find the Thevenin equivalent

a. \mathbf{Z}_{eq}

b. \mathbf{E}_{eq}

2. $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

3. $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

Ex. 2 Maximum Average Power Transfer (14)

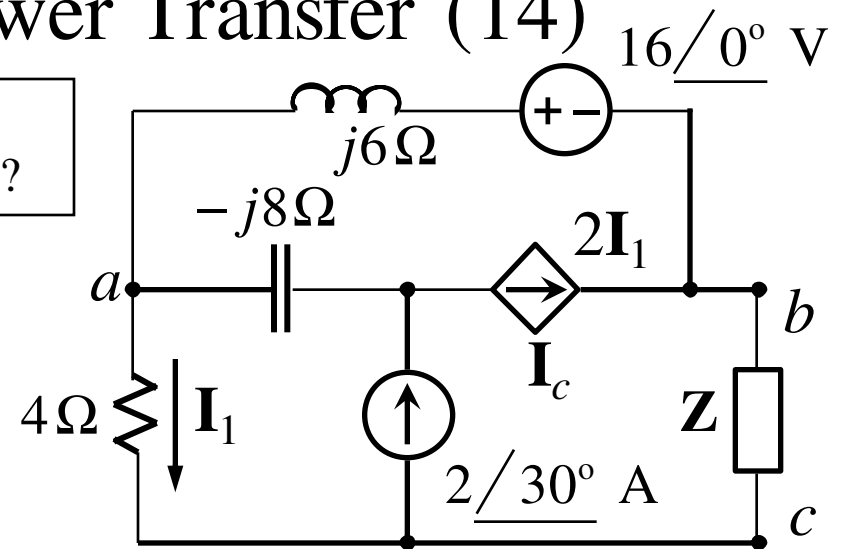
Determine the load impedance \mathbf{Z} that maximize the average power. What is the maximum average power?

$$\left. \begin{aligned} \mathbf{Z}_{eq} &= \frac{\mathbf{V}_{open-circuit}}{\mathbf{I}_{short-circuit}} = \frac{\mathbf{E}_{eq}}{\mathbf{J}_{eq}} \\ \mathbf{V}_{oc} &= -21.07 + j24.78 \text{ V} \\ \mathbf{I}_{sc} &= 1.06 + j1.41 \text{ A} \end{aligned} \right\}$$

$$\rightarrow \mathbf{Z}_{eq} = \frac{-21.07 + j24.78}{1.06 + j1.41} = [4.00 + j18.00 \Omega]$$

$$\rightarrow \boxed{\mathbf{Z} = 4.00 - j18.00 \Omega}$$

$$P_{\max} = \frac{21.07^2 + 24.78^2}{8 \times 4} = \boxed{33.06 \text{ W}}$$



1. Find the Thevenin equivalent

a. ✓ \mathbf{Z}_{eq}

b. ✓ \mathbf{E}_{eq}

2. ✓ $\mathbf{Z}_L = \mathbf{Z}_{eq}^*$

3. ✓ $P_{\max} = \frac{|\mathbf{E}_{eq}|^2}{8R_{eq}}$

AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
- 3. RMS Value**
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals



RMS Value (1)

$$P = \frac{1}{T} \int_0^T i^2 R dt = \frac{R}{T} \int_0^T i^2 dt$$

$$P = I^2 R$$

$$\left. \begin{array}{l} P = \frac{1}{T} \int_0^T i^2 R dt \\ P = I^2 R \end{array} \right\} \rightarrow I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

$$V_{eff} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

I is the effective/RMS value of $i(t)$

$$X_{eff} = X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2 dt}$$

RMS Value (2)

$$\left. \begin{aligned} I_{rms} &= \sqrt{\frac{1}{T} \int_0^T i^2 dt} \\ i(t) &= I_m \sin \omega t \end{aligned} \right\} \rightarrow I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = \sqrt{\frac{1}{T} \int_0^T [I_m \sin \omega t]^2 dt}$$
$$= \sqrt{\frac{1}{T} \int_0^T I_m^2 \frac{1 - \cos 2\omega t}{2} dt}$$
$$= \sqrt{\frac{I_m^2}{2T} \int_0^T dt} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

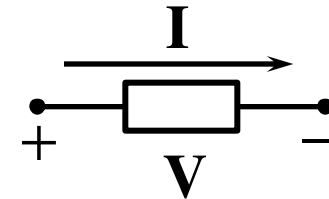
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

RMS Value (3)

$$P = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i)$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$



$$P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)$$

$$P = \frac{1}{2} \text{Re}(\mathbf{V} \mathbf{I}^*)$$

RMS Value (4)

Ex.

$v(t) = 150\sin(314t - 30^\circ)$ V, $i(t) = 10\sin(314t + 45^\circ)$ A. Find V_{rms} & I_{rms} ?

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{150}{\sqrt{2}} = 106.07 \text{ V}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

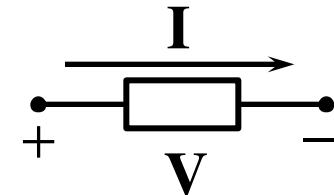


AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
- 4. Apparent Power and Power Factor**
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals



Apparent Power (1)



$$S = V_{rms} I_{rms} \quad (\text{in volt- ampere, VA})$$

JEFFERSON ELECTRIC
BY PIONEER POWER SOLUTIONS

Cat. No. **423-7865-M01**
Dry Type Isolation Transformer
Transformateur D'Isolation À Sec

kVA 150.0 High Volt/Prim: 600 Delta Low Volt/Sec 208ZZ/120

Phase 3
Hz 60
%IZ 4.0
Temp 130 °C
Class 220 °C
Wgt/Poids 2150 lbs
Class ANN
Wdg. Mtl. AL
Connect Cu-Al

423 Series

Outdoor Type 3R
Enclosure/Boltier

High Volt/Prim: on H1, H2, H3

Connect	1	2	3	4	5
Volts	630	615	600	585	570

Low Volt/Sec on X1, X2, X3

Line - Line	Line - Neutral
208 V	120 V

ENERGY PERFORMANCE VERIFIED
RENDERONT ENERGETIQUE VERIFIE

MEETS CAN/CSA C412.2-08
MEETS TP-1 1996 EFFICIENCY

Datecode: R1210 Job#: 1AP2 SN: 0001

<https://www.amazon.com/Ventilated-Transformer-Enclosure-Nameplate-Details/dp/B07G3DNTXN>

Apparent Power (2)

Ex.

$v(t) = 150\sin(314t - 30^\circ)$ V, $i(t) = 10\sin(314t + 45^\circ)$ A. Find S ?

$$S = V_{rms} I_{rms} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = \frac{150}{\sqrt{2}} \times \frac{10}{\sqrt{2}} = 750 \text{ VA}$$

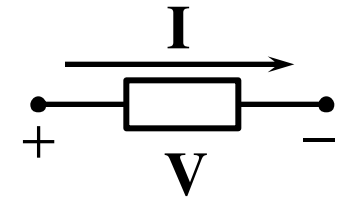


Power Factor (1)

$$P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)$$

$$S = V_{rms} I_{rms}$$

$$pf = \frac{P}{S} = \cos(\phi_v - \phi_i)$$



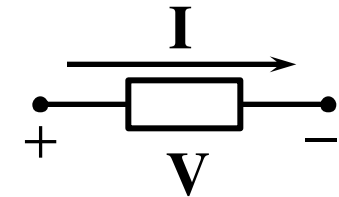
- $\phi_v - \phi_i = 0 \rightarrow pf = 1 \rightarrow P = S = V_{rms} I_{rms}$
- $\phi_v - \phi_i = \pm 90^\circ \rightarrow pf = 0 \rightarrow P = 0$



Power Factor (2)

Ex.

$v(t) = 150\sin(314t - 30^\circ)$ V, $i(t) = 10\sin(314t + 45^\circ)$ A. Find pf ?



$$pf = \cos(-30^\circ - 45^\circ) = 0.2588$$

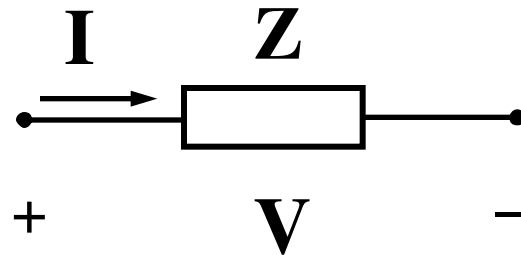


AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
- 5. Complex Power**
6. Conservation of AC Power
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals



Complex Power (1)



$$\mathbf{S} = \frac{1}{2} \mathbf{V} \mathbf{I}^*$$

$$\mathbf{V} = V_m \angle \phi_v$$

$$\mathbf{I} = I_m \angle \phi_i \rightarrow \mathbf{I}^* = I_m \angle -\phi_i$$

$$\rightarrow \mathbf{S} = \frac{1}{2} V_m I_m \angle \phi_v - \phi_i = \frac{1}{2} V_m I_m \cos(\phi_v - \phi_i) + j \frac{1}{2} V_m I_m \sin(\phi_v - \phi_i)$$

Complex Power (2)

$$\left. \begin{aligned} \mathbf{S} &= \frac{1}{2} \mathbf{V} \mathbf{I}^* \\ \mathbf{V} &= \mathbf{Z} \mathbf{I} \end{aligned} \right\}$$

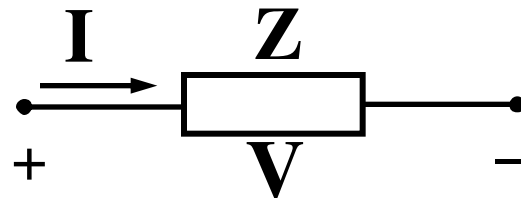
$$\rightarrow \left\{ \begin{aligned} \mathbf{S} &= \frac{1}{2} \mathbf{V} \left(\frac{\mathbf{V}}{\mathbf{Z}} \right)^* = \frac{1}{2} \frac{\mathbf{V} \mathbf{V}^*}{\mathbf{Z}^*} = \frac{1}{2} \frac{(V_m / \phi_v)(V_m / -\phi_v)}{\mathbf{Z}^*} = \boxed{\frac{V_{rms}^2}{\mathbf{Z}^*}} \\ \mathbf{S} &= \frac{1}{2} \mathbf{Z} \mathbf{I} \mathbf{I}^* = \frac{1}{2} \mathbf{Z} (I_m / \phi_i)(I_m / -\phi_i) = \frac{1}{2} \mathbf{Z} I_m^2 = \boxed{\mathbf{Z} I_{rms}^2} \end{aligned} \right\}$$

$\mathbf{Z} = R + jX$

$$\rightarrow \mathbf{S} = (R + jX) I_{rms}^2 = R I_{rms}^2 + jX I_{rms}^2 \rightarrow \left\{ \begin{aligned} P &= \text{Re}(\mathbf{S}) = R I_{rms}^2 \\ Q &= \text{Im}(\mathbf{S}) = X I_{rms}^2 \end{aligned} \right.$$

Reactive power (in volt-ampere reactive, VAR)

Complex Power (3)



$$\mathbf{S} = \frac{1}{2} \mathbf{V} \mathbf{I}^* = P + jQ = \frac{1}{2} V_m I_m \angle \phi_v - \phi_i = V_{rms} I_{rms} \angle \phi_v - \phi_i = \mathbf{Z} I_{rms}^2 = \frac{V_{rms}^2}{\mathbf{Z}^*}$$

$$S = |\mathbf{S}| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$$

$$P = \text{Re}(\mathbf{S}) = V_{rms} I_{rms} \cos(\phi_v - \phi_i) = S \cos(\phi_v - \phi_i) = R I_{rms}^2$$

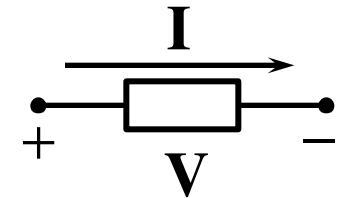
$$Q = \text{Im}(\mathbf{S}) = S \sin(\phi_v - \phi_i) = X I_{rms}^2$$

$$pf = \frac{P}{S} = \cos(\phi_v - \phi_i)$$

Complex Power (4)

Ex.

$$v(t) = 150\sin(314t - 30^\circ) \text{ V}, i(t) = 10\sin(314t + 45^\circ) \text{ A.}$$



$$\mathbf{V} = 150 \angle -30^\circ \text{ V}, \mathbf{I} = 10 \angle 45^\circ \text{ A}$$

$$\mathbf{S} = \frac{1}{2} \mathbf{V} \mathbf{I}^* = \frac{1}{2} (150 \angle -30^\circ) (10 \angle -45^\circ) = 750 \angle -75^\circ \text{ VA}$$

$$S = |\mathbf{S}| = 750 \text{ VA}$$

$$P = S \cos(\varphi_u - \varphi_i) = 750 \cos(-75^\circ) = 194.11 \text{ W}$$

$$Q = S \sin(\varphi_u - \varphi_i) = 750 \sin(-75^\circ) = -724.44 \text{ VAR}$$

$$pf = \cos(\varphi_u - \varphi_i) = \cos(-75^\circ) = 0.26$$

AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
- 6. Conservation of AC Power**
7. Power Factor Improvement
8. Average Power and RMS Value of Periodic Signals



Conservation of AC Power (1)

$$\sum_{i=1}^M \mathbf{S}_{source,i} = \sum_{i=1}^N \mathbf{S}_{load,i}$$

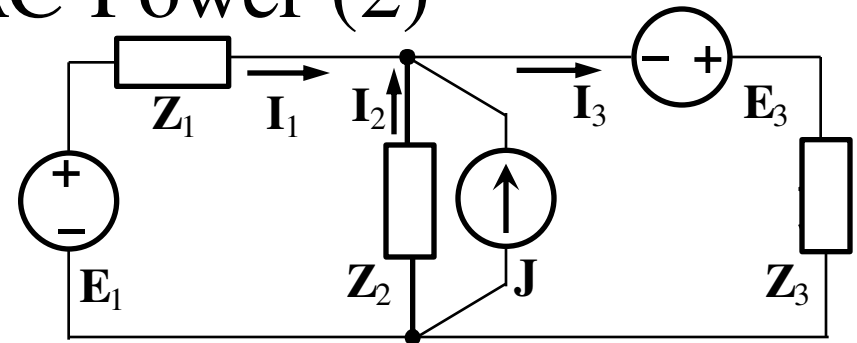
$$\rightarrow \begin{cases} \sum_{i=1}^M P_{source,i} = \sum_{i=1}^N P_{load,i} \\ \sum_{i=1}^M Q_{source,i} = \sum_{i=1}^N Q_{load,i} \end{cases}$$

$$\sum_{i=1}^M \mathbf{S}_{source,i} \neq \sum_{i=1}^N \mathbf{S}_{load,i}$$

Ex.

Conservation of AC Power (2)

$Z_1 = 10\Omega$; $Z_2 = j20\Omega$; $Z_3 = 5 - j10\Omega$;
 $E_1 = 30\text{ V}$; $E_3 = 45\angle 15^\circ\text{ V}$; $J = 2\angle -30^\circ\text{ A}$;
 Find currents?



$$I_1 = 4.09\angle 75.2^\circ\text{ A}, I_2 = 2.20\angle 26.4^\circ\text{ A}, I_3 = 6.16\angle 39.6^\circ\text{ A}$$

$$S_{Z1} = Z_1 I_{1rms}^2 = 10(4.09)^2 / 2 = 83.64\text{ VA}$$

$$S_{Z2} = Z_2 I_{2rms}^2 = j20(2.20)^2 / 2 = j48.40\text{ VA}$$

$$S_{Z3} = Z_3 I_{3rms}^2 = (5 - j10)(6.16^2) / 2 = 94.86 - j189.73\text{ VA}$$

$$S_{E1} = \frac{1}{2} E_1 I_1^* = \frac{1}{2} 30 \cdot 4.09\angle -75.2^\circ = 61.35\angle -75.2^\circ\text{ VA}$$

$$S_{E3} = \frac{1}{2} E_3 I_3^* = 138.60\angle -24.6^\circ\text{ VA}$$

$$S_J = \frac{1}{2} U_J J^* = (-Z_2 I_2) J^* = 36.65 - j24.35\text{ VA}$$

$$\sum S_{load} = 178.50 - j141.33\text{ VA}$$

$$\sum S_{source} = 178.34 - j141.36\text{ VA}$$

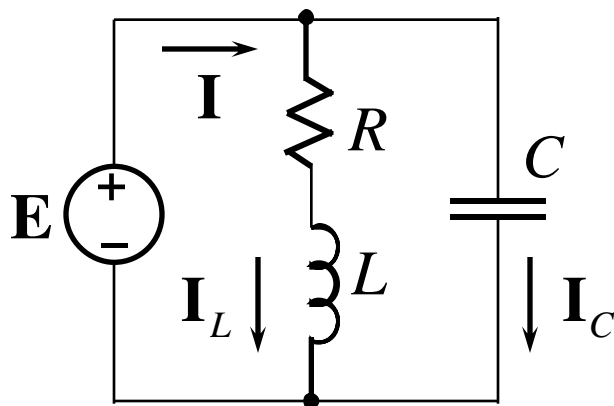
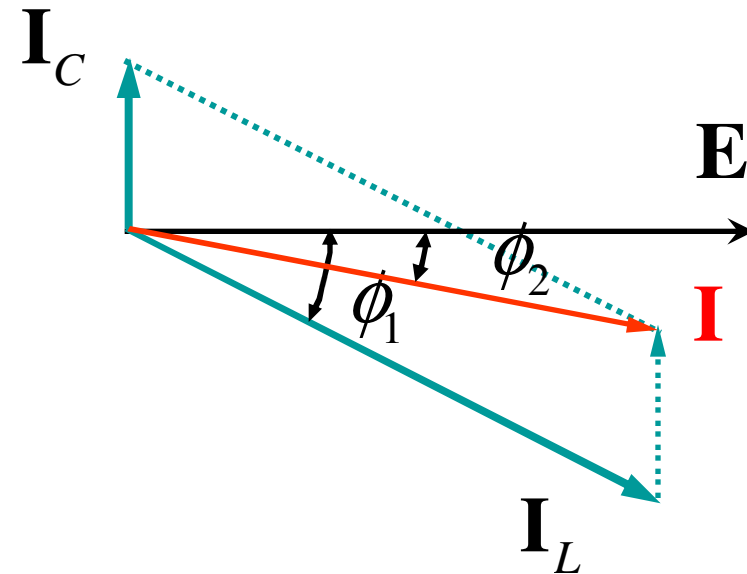
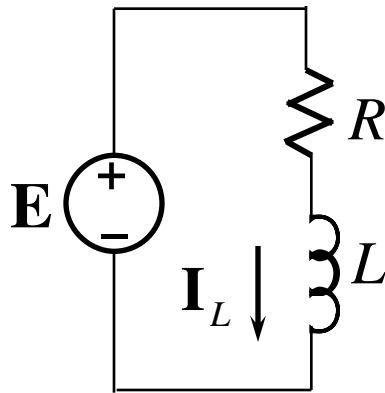
AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
- 7. Power Factor Improvement**
8. Average Power and RMS Value of Periodic Signals

Power Factor Improvement (1)

$$pf = \cos \phi$$

$$pf_2 > pf_1 \rightarrow \phi_2 < \phi_1$$



$$\phi_2 < \phi_1 \rightarrow C = ? \quad (P = \text{const})$$

Power Factor Improvement (2)

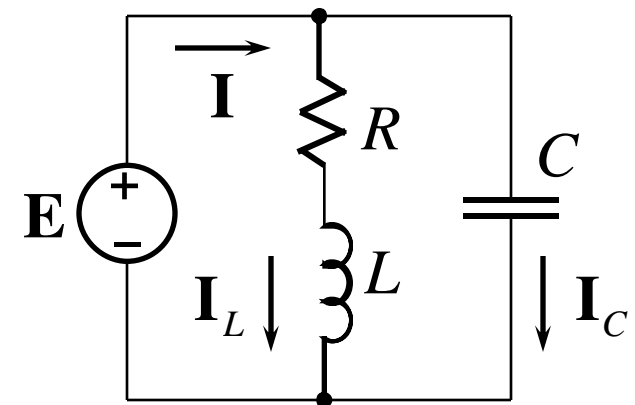
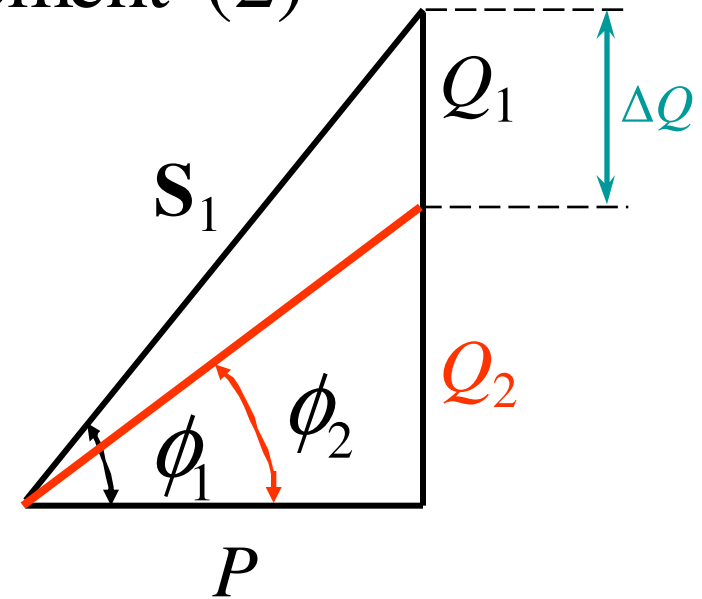
$$Q_1 = P \tan \phi_1, \quad Q_2 = P \tan \phi_2$$

$$\Delta Q = Q_1 - Q_2$$

$$\Delta Q = \frac{E_{rms}^2}{X_C} = \omega C E_{rms}^2 \rightarrow C = \frac{\Delta Q}{\omega E_{rms}^2}$$

$$C = \frac{Q_1 - Q_2}{\omega E_{rms}^2} = \frac{P \tan \phi_1 - P \tan \phi_2}{\omega E_{rms}^2}$$

$$= \boxed{P \frac{\tan \phi_1 - \tan \phi_2}{\omega E_{rms}^2}}$$



Ex Power Factor Improvement (3)

A load is connected to a 220 V (rms), 50 Hz power line. This load absorbs a power of 1000kW. Its power factor is 0.8. Find the capacitor required to raise the pf to 0.9?

$$C = P \frac{\tan \phi_1 - \tan \phi_2}{\omega E_{rms}^2}$$

$$pf_1 = 0.8 \rightarrow \cos \phi_1 = 0.8 \rightarrow \phi_1 = 36.9^\circ \rightarrow \tan \phi_1 = 0.75$$

$$pf_2 = 0.9 \rightarrow \cos \phi_2 = 0.9 \rightarrow \phi_2 = 25.8^\circ \rightarrow \tan \phi_2 = 0.48$$

$$\rightarrow C = 1000 \times 10^3 \frac{0.75 - 0.48}{314 \times (220)^2} = \boxed{0.0178 \text{ F}}$$

AC Power Analysis

1. Instantaneous and Average Power
2. Maximum Average Power Transfer
3. RMS Value
4. Apparent Power and Power Factor
5. Complex Power
6. Conservation of AC Power
7. Power Factor Improvement
- 8. Average Power and RMS Value of Periodic Signals**



Average Power and RMS Value of Periodic Signals (1)

$$v(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \sin(n\omega_0 t - \theta_n)$$

$$i(t) = I_{dc} + \sum_{m=1}^{\infty} I_m \sin(m\omega_0 t - \phi_m)$$

$$P = V_{dc} I_{dc} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

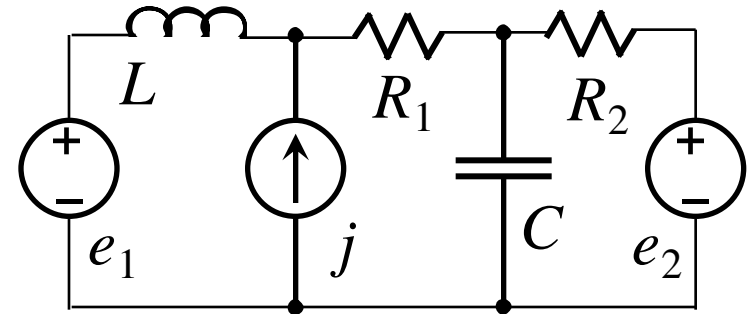
$$V_{rms} = \sqrt{V_{dc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} V_n^2}$$

$$I_{rms} = \sqrt{I_{dc}^2 + \frac{1}{2} \sum_{m=1}^{\infty} I_m^2}$$

Average Power and RMS Value of Periodic Signals (2)

Ex.

$e_1 = 10\sin 10t$ V; $j = 4\sin(50t + 30^\circ)$ V; $e_2 = 6$ V (DC); $L = 1$ H; $R_1 = 1 \Omega$; $R_2 = 5 \Omega$; $C = 0.01$ F; find the RMS value of the voltage across R_1 ?



$$v_{R1} = -1 + 1.06\sin(10t - 58^\circ) + 4.14\sin(50t + 32^\circ) \text{ V}$$

$$V_{rms} = \sqrt{V_{dc}^2 + \frac{1}{2} \sum_{n=1}^{\infty} V_n^2}$$

$$= \sqrt{(-1)_{dc}^2 + \frac{1}{2} (1.06^2 + 4.14^2)} = \boxed{3.18 \text{ V}}$$