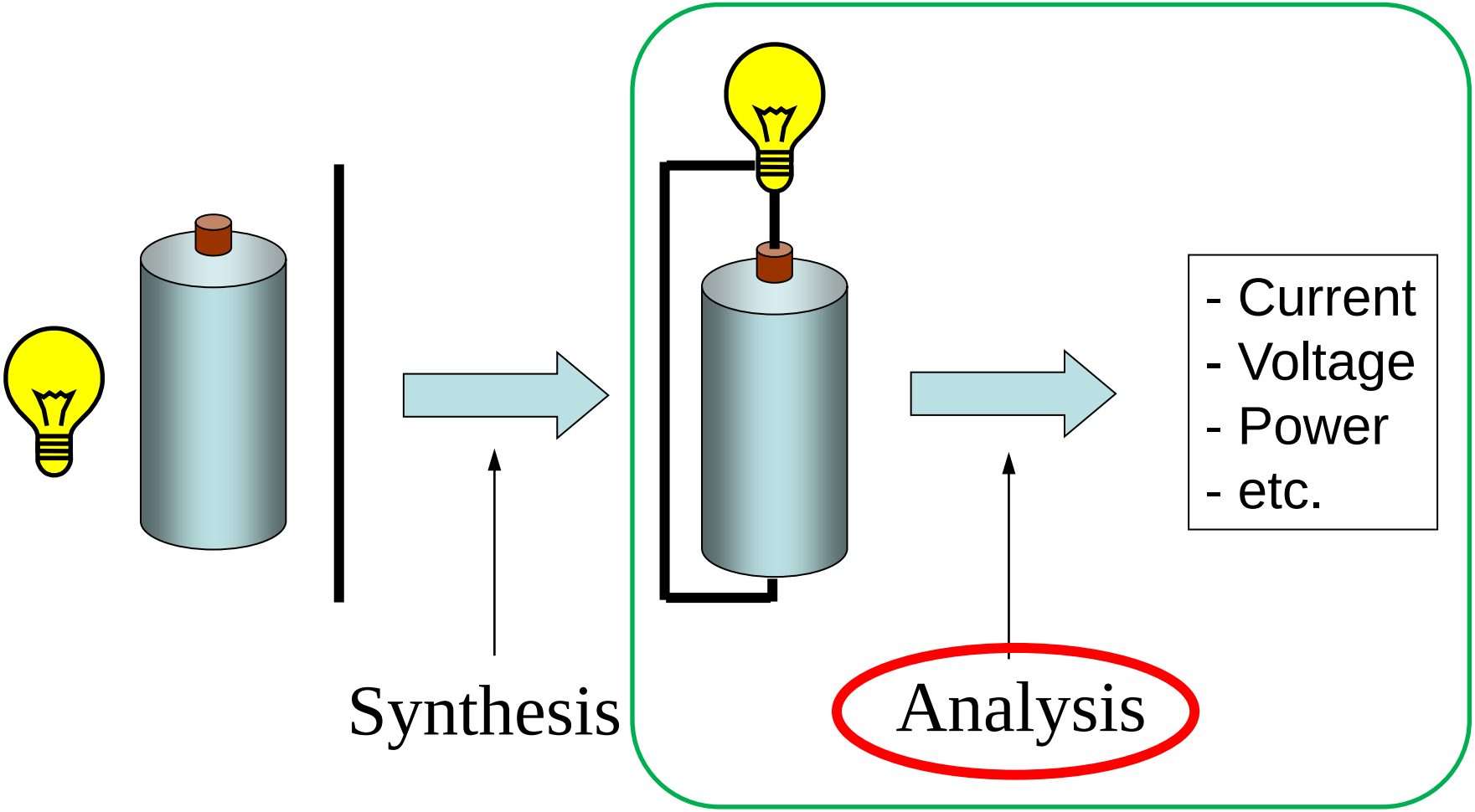
Nguyễn Công Phương

Electric Circuit Theory

Sinusoid & Phasors

Introduction



Contents

Bibliography

1. C. K. Alexander, M. N. O. Sadiku. *Fundamentals of Electric Circuits*. McGraw-Hill, 2001
2. J. Bird. *Electrical Circuit Theory and Technology*. Newnes, 2003
3. A. M. Davis. *Linear Circuit Analysis*. PWS, 1998
4. Nguyễn Bình Thành, Nguyễn Trần Quân, Phạm Khắc Chương. *Cơ sở kỹ thuật điện.* Đại học & trung học chuyên nghiệp, 1971
5. J. W. Nilsson, S. A. Riedel. *Electric Circuits*. Addison-Wesley, 1996
6. J. O'Malley. *Theory and Problems of Basic Circuit Analysis*. McGraw-Hill, 1992
7. Nguyễn Công Phương. *Lỗi thường gặp khi làm bài tập Lý thuyết mạch*. Khoa học & Kỹ thuật, 2021.
8. Nguyễn Công Phương & Nguyễn Tuấn Ninh. *Giải bài tập Lý thuyết mạch bằng Python*. Khoa học & Kỹ thuật, 2022.
9. https://sites.google.com/site/ncpdhbkhn/

Sinusoid & Phasors

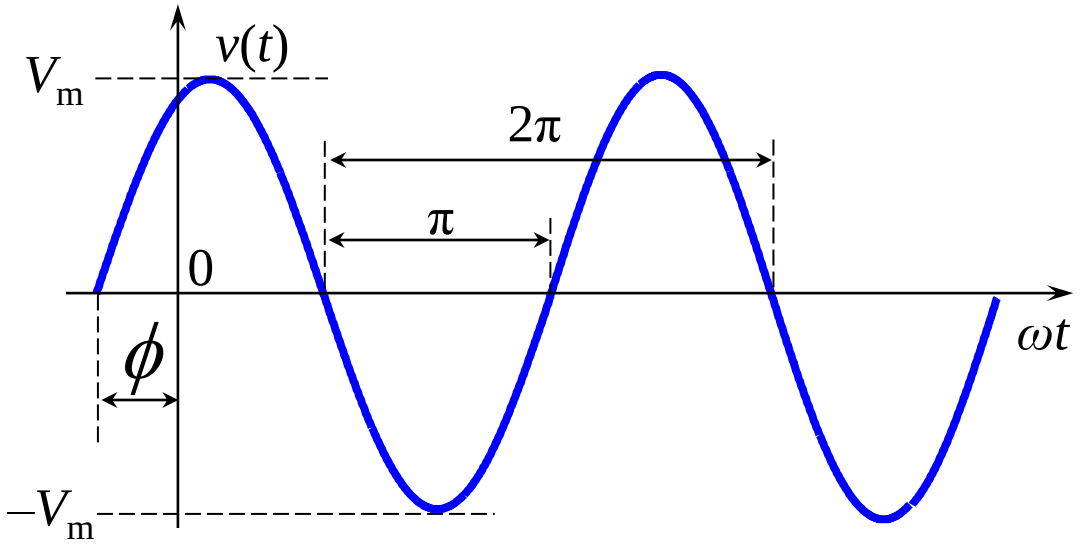
1. Sinusoids
2. Complex Numbers
3. Phasors
4. Phasor Relationships for Circuit Elements

Sinusoids (1)

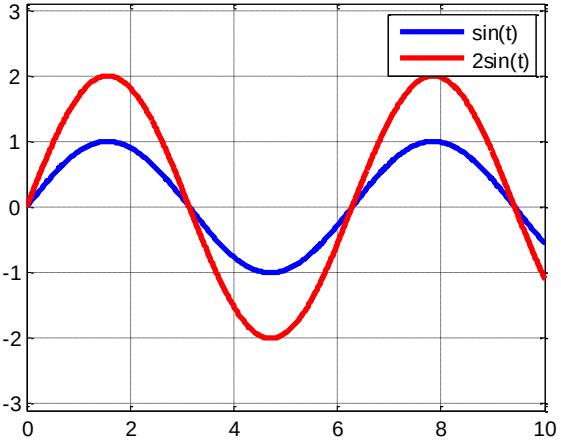
$$v(t) = V_m \sin(\omega t + \phi)$$

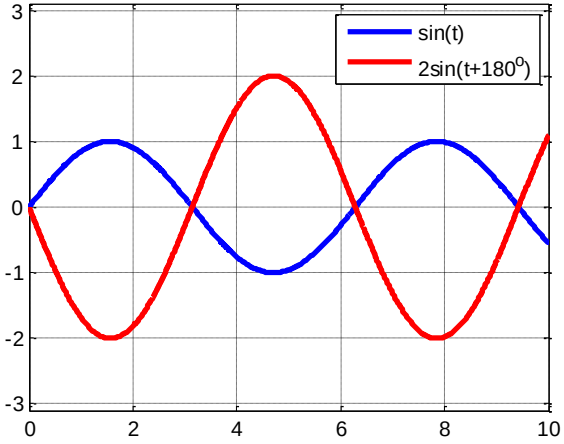
$T = \dfrac{2\pi}{\omega}$ (period, in s)

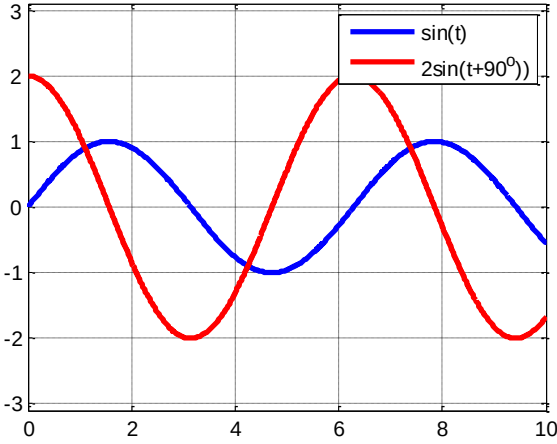
$f = \dfrac{1}{T}$ (frequency, in Hz)



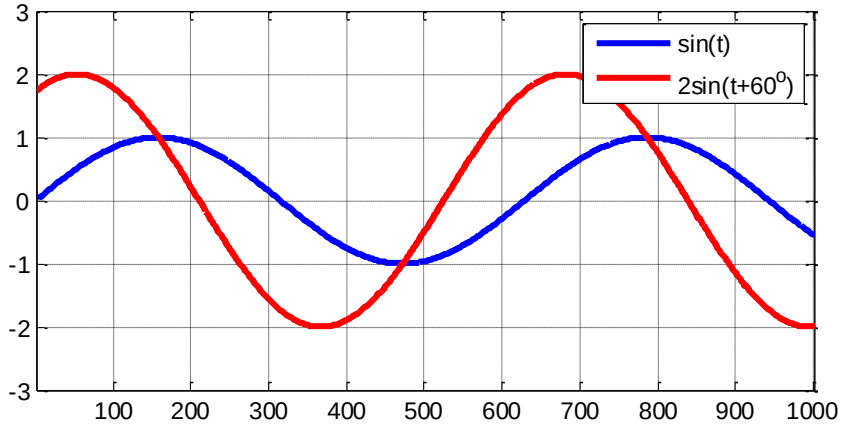
Sinusoids (2)

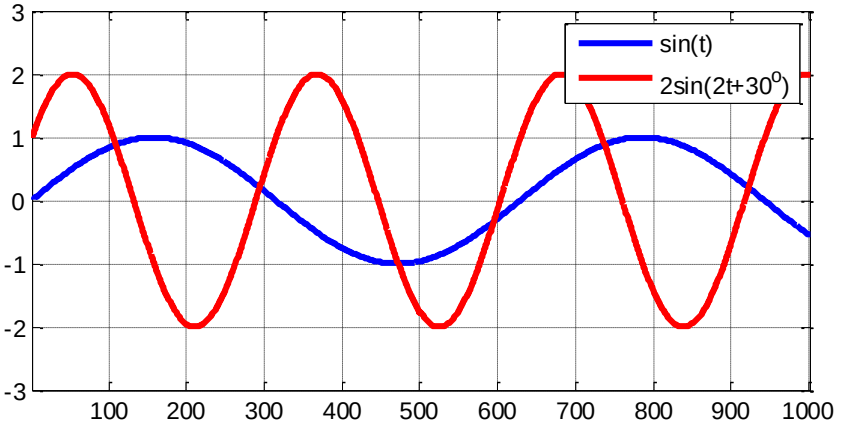


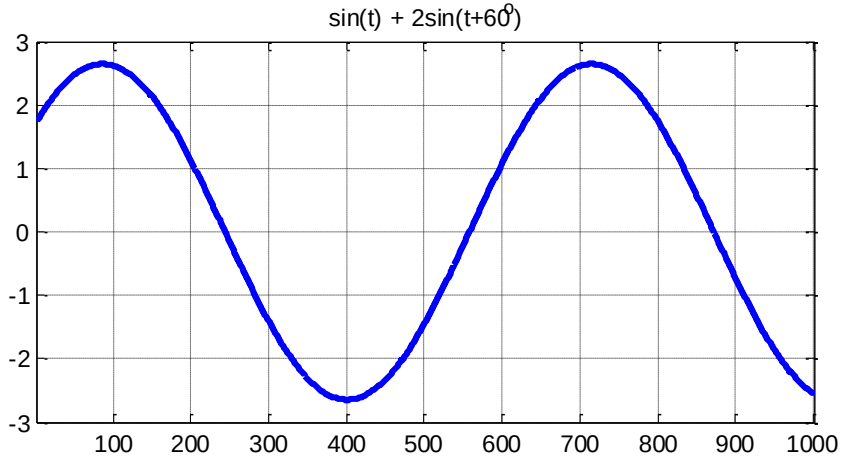


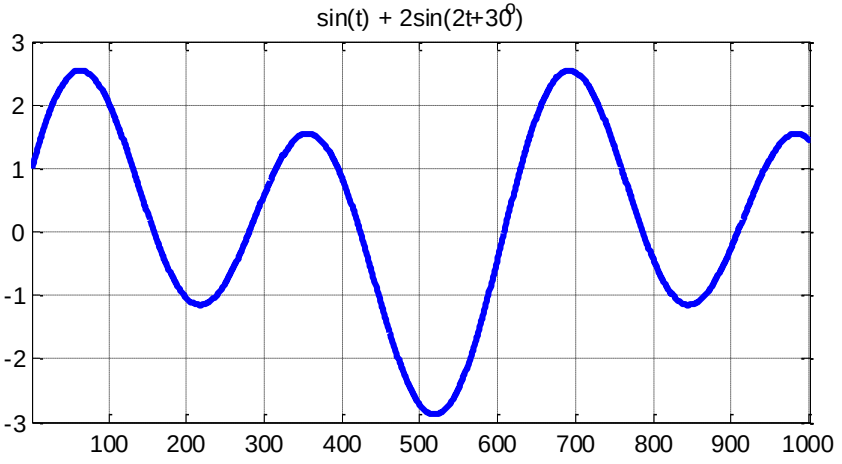


Sinusoids (3)


sin(t)
2sin(t+60^o)

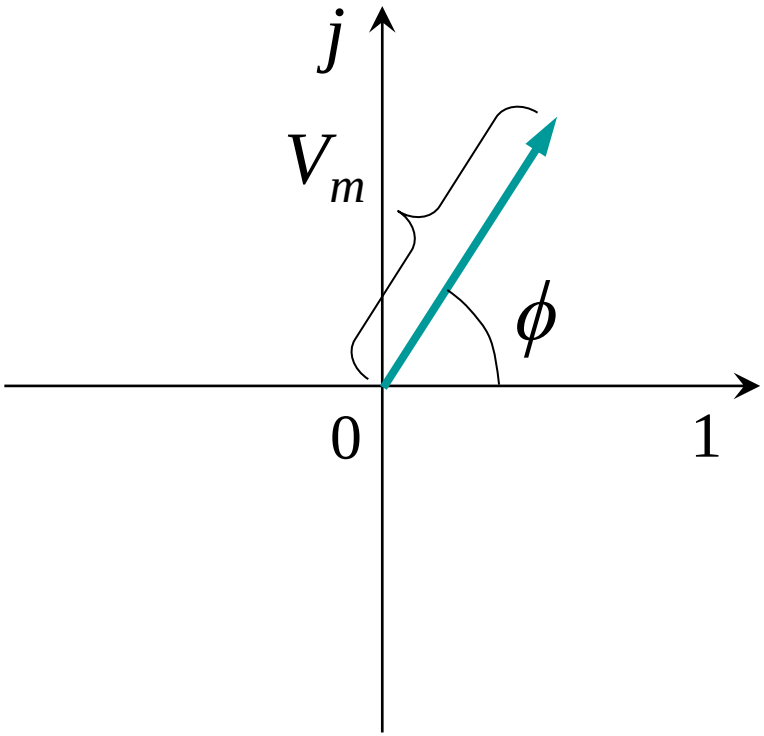

sin(t)
2sin(2t+30^o)

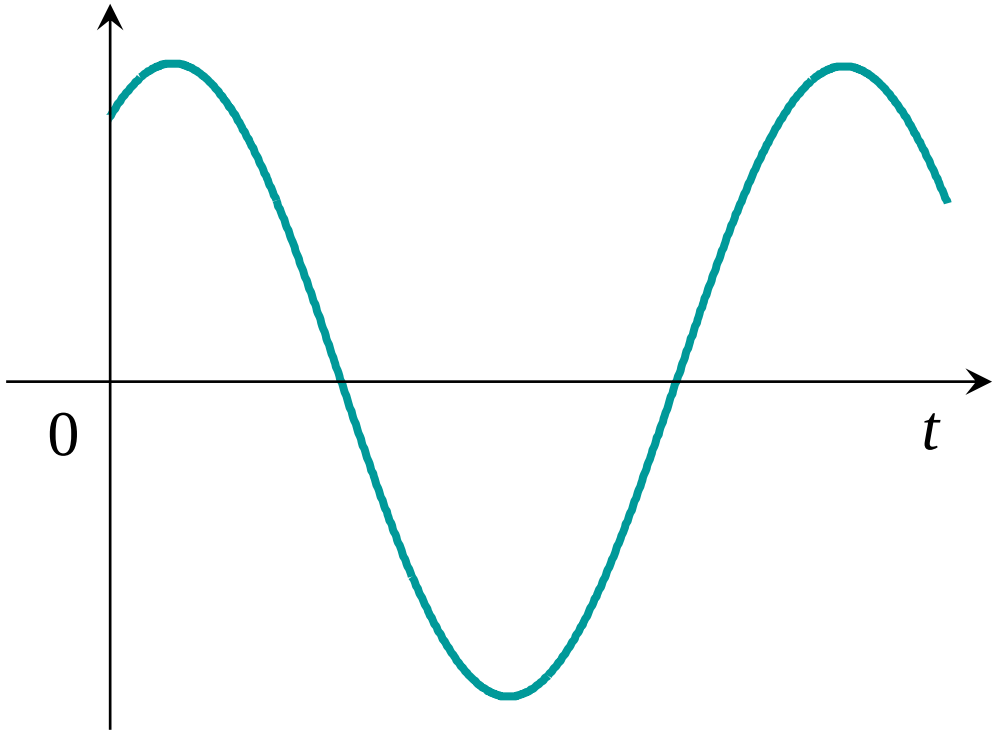

sin(t) + 2sin(t+60^o)


sin(t) + 2sin(2t+30^o)

Sinusoids (4)

$$v(t) = V_m \sin(\omega t + \phi)$$



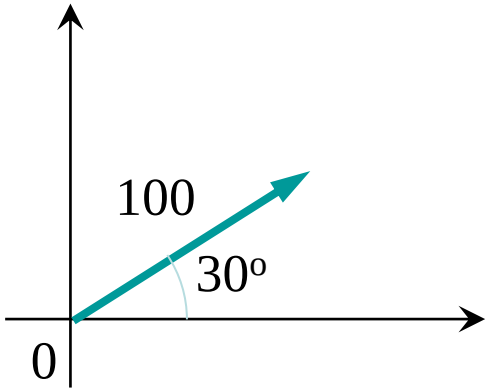


Phasor diagram

Sinusoids (5)

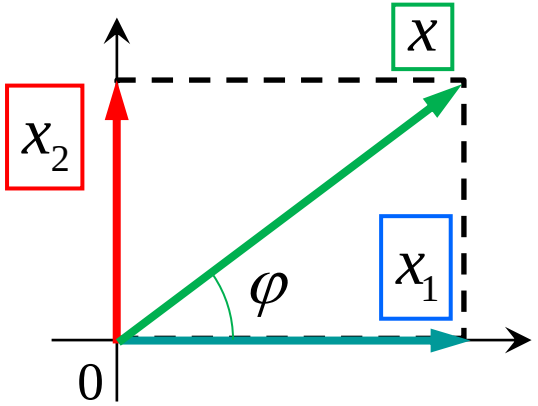
Ex. 1

$x(t) = 100\sin(20t + 30^o)$.



Ex. 2

$x_1(t) = 100\sin(20t)$, $x_2(t) = 80\sin(20t + 90^o)$,
Find $x = x_1(t) + x_2(t)$?



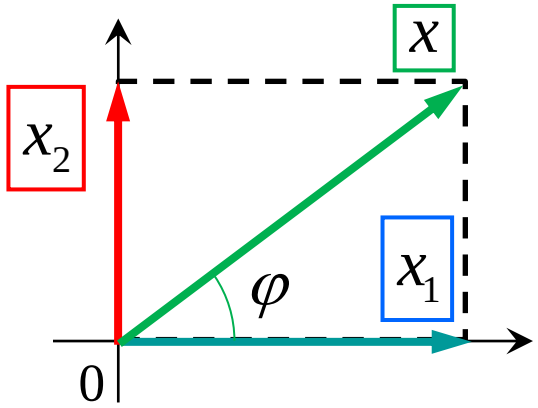
$$X_m = \sqrt{X_{1m}^2 + X_{2m}^2} = \sqrt{100^2 + 80^2} = 128.06$$

$$\varphi = \arctan\frac{X_{2m}}{X_{1m}} = \arctan\frac{80}{100} = 38.66^o$$

$$x(t) = 128.06\sin(20t + 38.66^o)$$

Sinusoids (6)

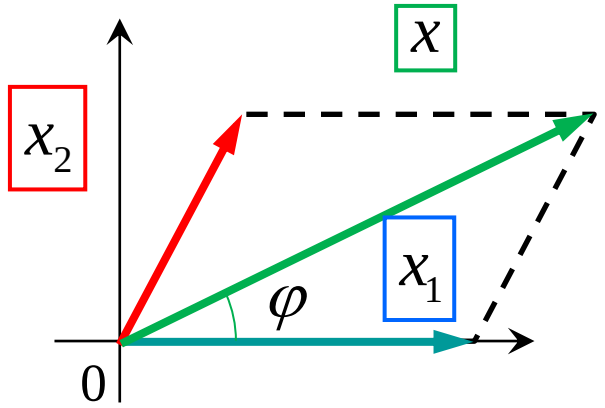
$x_1(t) = 100\sin(20t)$,
$x_2(t) = 80\sin(20t + 90^o)$,
Find $x = x_1(t) + x_2(t)$?



$$X_m = \sqrt{100^2 + 80^2} = 128.06$$

$$\varphi = \arctan\frac{80}{100} = 38.66^o$$

$x_1(t) = 100\sin(20t)$,
$x_2(t) = 80\sin(20t + 60^o)$,
Find $x = x_1(t) + x_2(t)$?



$$X_m = ? \quad \varphi = ?$$

Complex numbers

Sinusoid & Phasors

1. Sinusoids
2. **Complex Numbers**
3. Phasors
4. Phasor Relationships for Circuit Elements

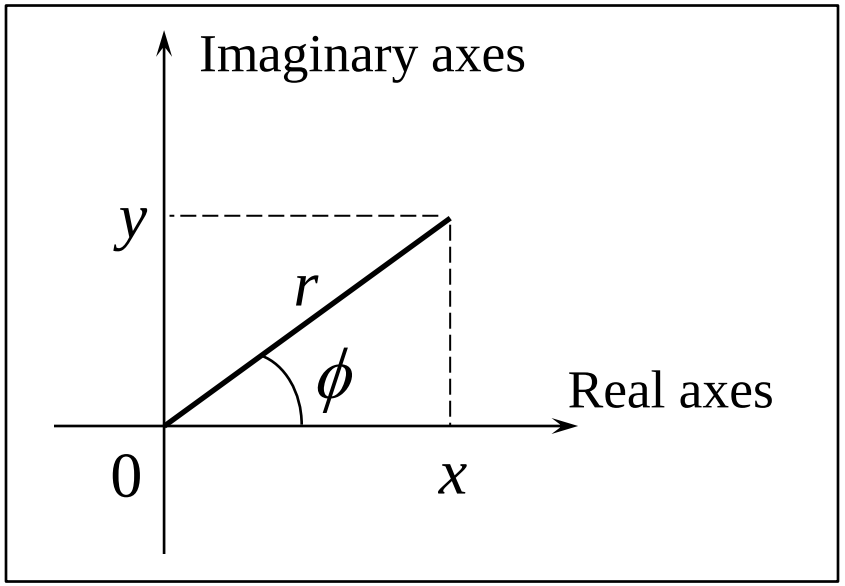
Complex Numbers (1)

$$z = x + jy \quad = r\angle\phi \quad = re^{j\phi}; \qquad j = \sqrt{-1}$$

$$r = \sqrt{x^2 + y^2}; \quad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\phi; \quad y = r\sin\phi$$

$$x = \mathrm{Re}(z); \quad y = \mathrm{Im}(z)$$



$$z = x + jy = r\angle\phi = re^{j\phi} = r(\cos\phi + j\sin\phi)$$

Complex Numbers (2)

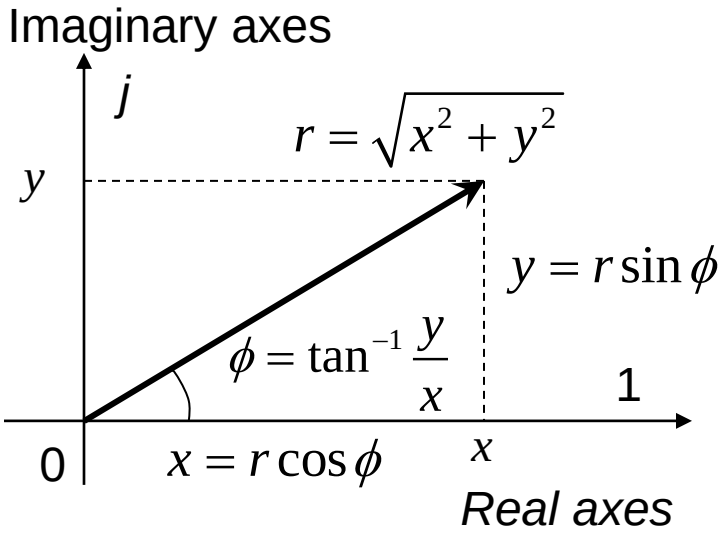
Ex. 1

$3 + j4 \rightarrow r\angle\phi$?

$$r = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = 5$$

$$\varphi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{4}{3} = 53.1^{o}$$

$3 + j4 \rightarrow 5\angle 53.1^{o}$



Ex. 2

$10\angle 60^{o} \rightarrow x + jy$?

$$x = 10\cos 60^{o} = 5$$

$$y = 10\sin 60^{o} = 8.66$$

$10\angle 60^{o} \rightarrow 5 + j8.66$

Complex Numbers (3)

$$z = x + jy; \quad z_1 = x_1 + jy_1 = r_1 \angle \phi_1; \quad z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

$$\frac{1}{z} = \frac{1}{r} \angle -\phi$$

$$\sqrt{z} = \sqrt{r} \angle \phi / 2$$

$$z^* = x - jy = r \angle -\phi = re^{-j\phi}$$

Complex Numbers (4)

Ex. 3

$$3+j4+5-j6=(3+5)+j(4-6)=\boxed{8-j2}$$

$$3+j4-(5-j6)=(3-5)+j[4-(-6)]=\boxed{-2+j10}$$

$$3+j4-5\angle 30^{o}=3+j4-[(5\cos 30^{o})+j(5\sin 30^{o})]=3+j4-(4.33+j2.50)$$

$$=\boxed{-1.33+j1.50}$$

$$(3+j4)(5-j6)=3\times 5+j4\times 5-3\times j6-j4\times j6$$

$$=15+j20-j18-(-1)24=\boxed{39+j2}$$

$$=\left(5\angle 53.1^{o}\right)\left(7.81\angle -50.2^{o}\right)=(5\times 7.81)\angle 53.1^{o}-50.2^{o}$$

$$=\boxed{39.1\angle 2.9^{o}}$$

Complex Numbers (5)

Ex. 4

$$\frac{3+j4}{5-j6}=\frac{3+j4}{5-j6}\times\frac{5+j6}{5+j6}=\frac{3\times5+j4\times5+3\times j6+j4\times j6}{5^2-(j6)^2}=\frac{15+j20+j18-24}{25-(-1)36}$$

$$=\frac{-9+j38}{61}=\frac{-9}{61}+\frac{j38}{61}=\boxed{-0.15+j0.62}$$

$$=\frac{5\angle 53.1^o}{7.81\angle -50.2^o}=\frac{5}{7.81}\angle 53.1^o-(-50.2^o)=\boxed{0.64\angle 103.3^o}$$

Complex Numbers (6)

Ex. 5

$$\sqrt{\frac{3+j4+5\angle 30^o}{(4+j5)(6-j7)^*}} = \sqrt{\frac{7.33+j6.50}{(4+j5)(6-j7)^*}} = \sqrt{\frac{7.33+j6.50}{59.00\angle 100.7^o}} = \sqrt{\frac{9.80\angle 41.6^o}{59.00\angle 100.7^o}}$$

$$= \sqrt{0.17\angle -51.1^o}$$

$$= \boxed{0.41\angle -25.6^o}$$

$$5\angle 30^o = (5\cos 30^o) + j(5\sin 30^o) = 4.33 + j2.50$$

$$3+j4+5\angle 30^o = (3+j4) + (4.33+j2.50) = 7.33+j6.50$$

$$(4+j5)(6-j7)^* = (4+j5)(6+j7)$$

$$4+j5 = \sqrt{4^2+5^2}\angle \tan^{-1}(5/4) = 6.40\angle 51.3^o$$

$$6+j7 = \sqrt{6^2+7^2}\angle \tan^{-1}(7/6) = 9.22\angle 49.4^o$$

$$(4+j5)(6+j7) = \left(6.40\angle 51.3^o\right)\left(9.22\angle 49.4^o\right) = 59.00\angle 100.7^o$$

$$7.33+j6.50 = \sqrt{7.33^2+6.50^2}\angle \tan^{-1}(6.50/7.33) = 9.80\angle 41.6^o$$

Complex Numbers (7)

$$10\angle 0^{o} \leftrightarrow 10$$

$$10\angle 90^{o} \leftrightarrow j10$$

$$10\angle -90^{o} \leftrightarrow -j10$$

$$10\angle 180^{o} = 10\angle -180^{o} \leftrightarrow -10$$

$$\dot{A} = A\angle \varphi,\ \dot{B} = A\angle \varphi + 90^{o} \leftrightarrow \dot{B} = (j)(\dot{A})$$

$$\dot{A} = A\angle \varphi,\ \dot{B} = A\angle \varphi - 90^{o} \leftrightarrow \dot{B} = (-j)(\dot{A})$$

$$\dot{A} = A\angle \varphi,\ \dot{B} = A\angle \varphi \pm 180^{o} \leftrightarrow \dot{B} = -\dot{A}$$

$$\frac{M}{j} = -jM$$

Sinusoid & Phasors

1. Sinusoids
2. Complex Numbers
3. **Phasors**
4. Phasor Relationships for Circuit Elements

Phasors (1)

$$x(t) = X_m \sin(\omega t + \varphi) = X\sqrt{2}\sin(\omega t + \varphi) \leftrightarrow \mathbf{X} = X\angle\varphi$$

$$x(t) = X_m \sin(\omega t + \varphi) = X\sqrt{2}\sin(\omega t + \varphi) \leftrightarrow \mathbf{X} = X_m\angle\varphi$$

$$x(t) = X_m \cos(\omega t + \varphi) = X\sqrt{2}\cos(\omega t + \varphi) \leftrightarrow \mathbf{X} = X\angle\varphi$$

$$x(t) = X_m \cos(\omega t + \varphi) = X\sqrt{2}\cos(\omega t + \varphi) \leftrightarrow \mathbf{X} = X_m\angle\varphi$$

$$\dot{X} \quad \mathbf{X} \quad \bar{X} \quad \vec{X}$$

Phasors (2)

$2\sin t + 4\sin(t+30^o)$

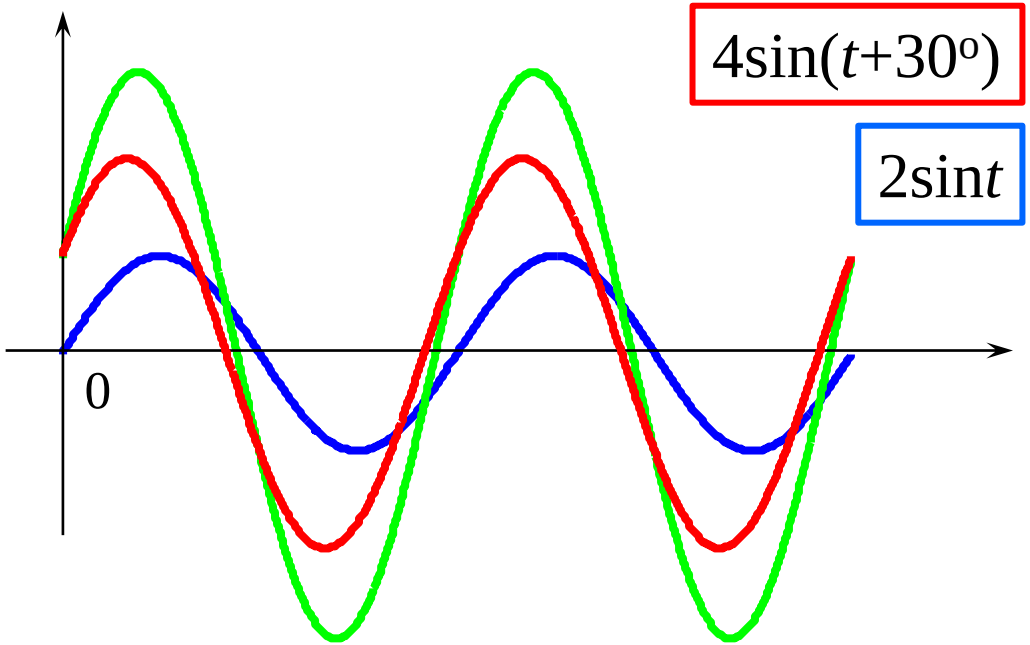
$4\sin(t+30^o)$

$2\sin t$

Ex. 1

$$x(t) = 2\sin t \leftrightarrow \mathbf{X} = 2\angle 0^o$$

$$y(t) = 4\sin(t+30^o) \leftrightarrow \mathbf{Y} = 4\angle 30^o$$



$$x(t)+y(t) \leftrightarrow \mathbf{X}+\mathbf{Y}$$

$$\mathbf{X} = 2\angle 0^o = 2$$

$$\mathbf{Y} = 4\angle 30^o = 3.46 + j2$$

$$\rightarrow \mathbf{X}+\mathbf{Y} = 2+(3.46+j2) = 5.46+j2 = 5.82\angle 20^o$$

$$\rightarrow x(t)+y(t) = 5.82\sin(t+20^o)$$

$$\boxed{x(t) = X_m \sin(\omega t+\phi) \leftrightarrow \mathbf{X} = X_m\angle\phi}$$

Phasors (3)

Ex. 2

$$4\sin(20t+40^o) \leftrightarrow 4\angle 40^o$$

$$6\sin(314t-120^o) \leftrightarrow 6\angle -120^o$$

$$-5\cos(100t+20^o) = -5\sin(100t+110^o) \leftrightarrow -5\angle 110^o$$

$$12\angle 30^o \leftrightarrow 12\sin(\omega t+30^o)$$

$$-24\angle 60^o \leftrightarrow -24\sin(\omega t+60^o)$$

$$3+j4 \rightarrow 5\angle 53.1^o \leftrightarrow 5\sin(\omega t+53.1^o)$$

Phasors (4)

Ex. 3

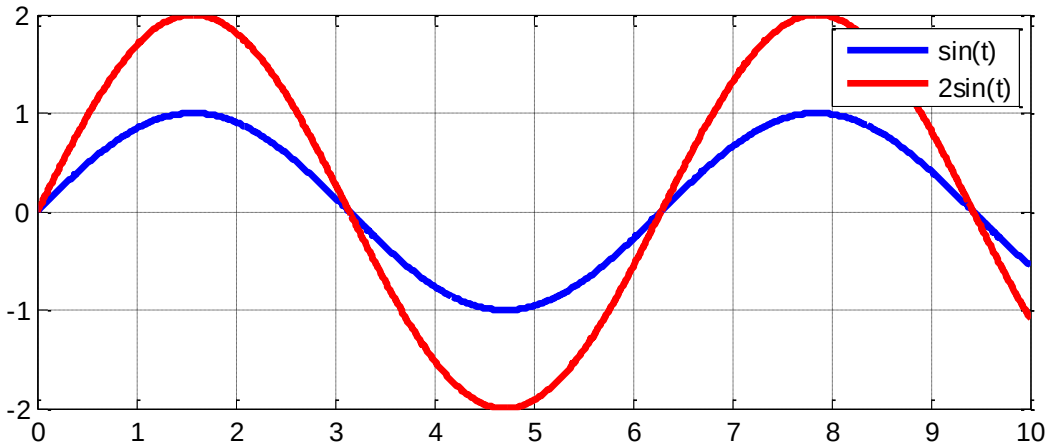
$$\boxed{\sin(t) + 2\sin(t)}$$

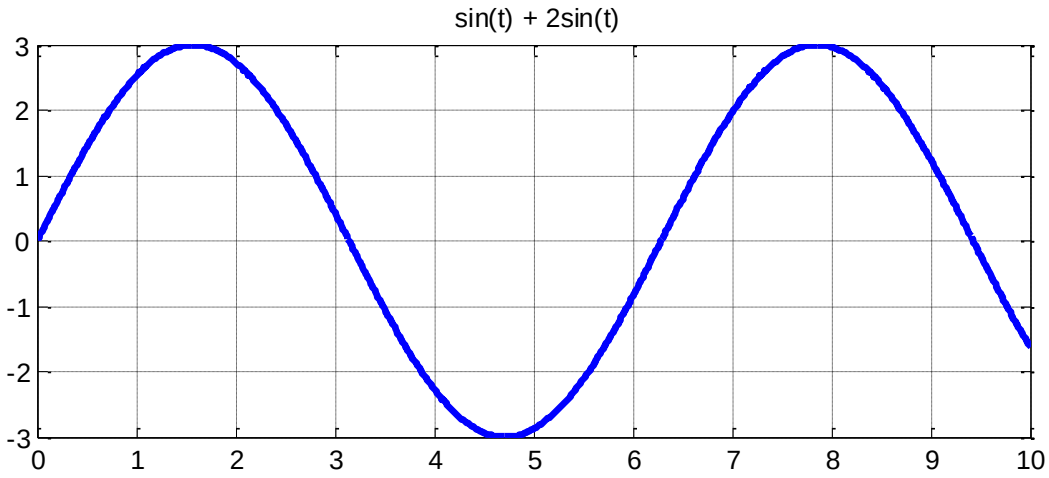
$$\sin(t) \leftrightarrow 1\angle 0^{o} = 1$$

$$2\sin(t) \leftrightarrow 2\angle 0^{o} = 2$$

$$1 + 2 = 3$$

$$3 \leftrightarrow \boxed{3\sin(t)}$$





Phasors (5)

Ex. 4

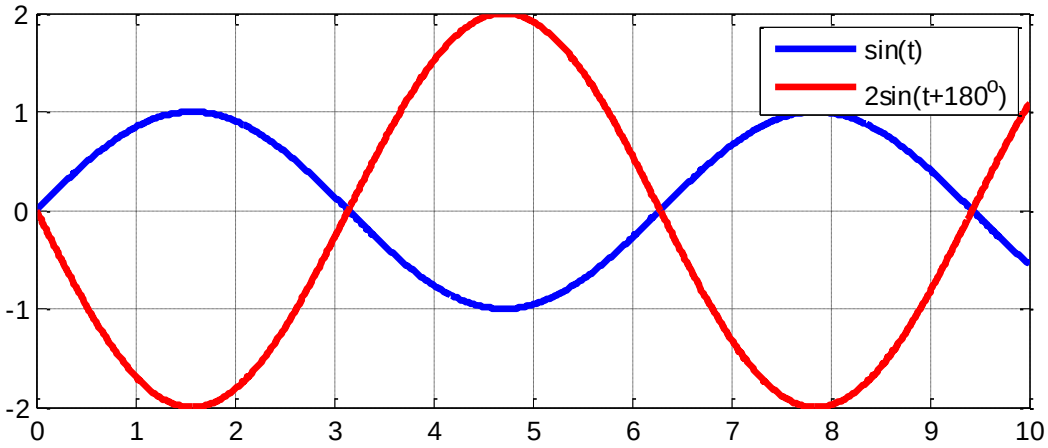
$$\sin(t) + 2\sin(t + 180^o)$$

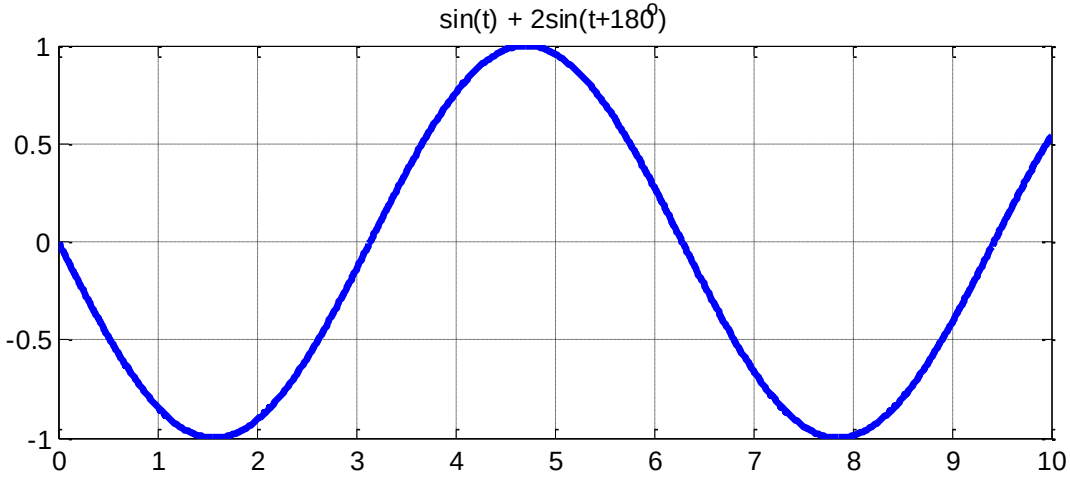
$$\sin(t) \leftrightarrow 1\angle 0^o = 1$$

$$2\sin(t + 180^o) \leftrightarrow 2\angle 180^o = -2$$

$$1 - 2 = -1$$

$$-1 = 1\angle 180^o \leftrightarrow \sin(t + 180^o)$$





Phasor (6)

Ex. 5

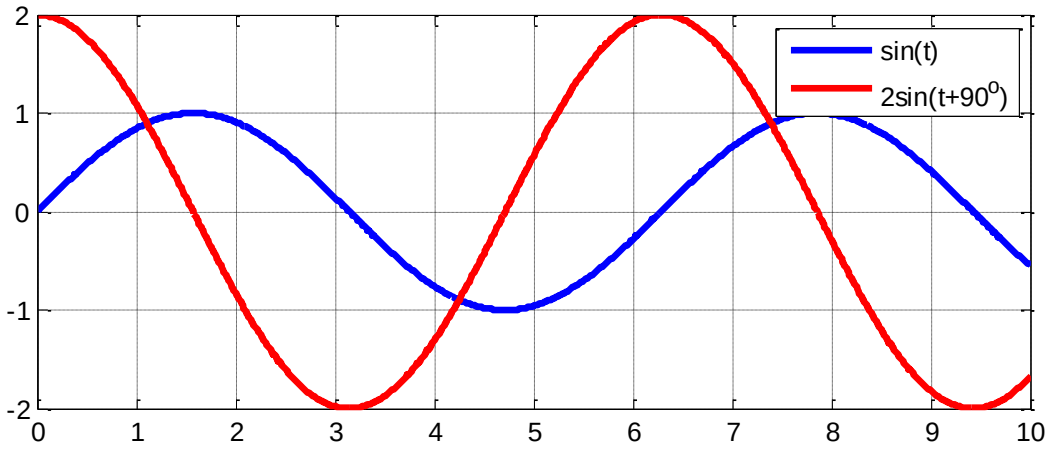
$$\boxed{\sin(t) + 2\sin(t + 90^o)}$$

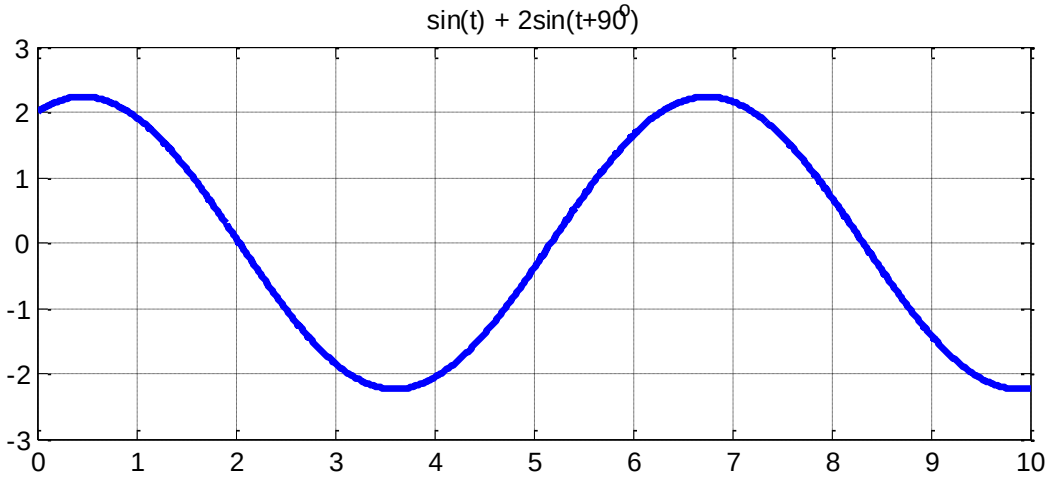
$$\sin(t) \leftrightarrow 1\angle 0^o = 1$$

$$2\sin(t + 90^o) \leftrightarrow 2\angle 90^o = j2$$

$$1 + j2 = \sqrt{5}\angle 63.4^o$$

$$\sqrt{5}\angle 63.4^o \leftrightarrow \boxed{\sqrt{5}\sin(t + 63.4^o)}$$





Phasor (7)

Ex. 6

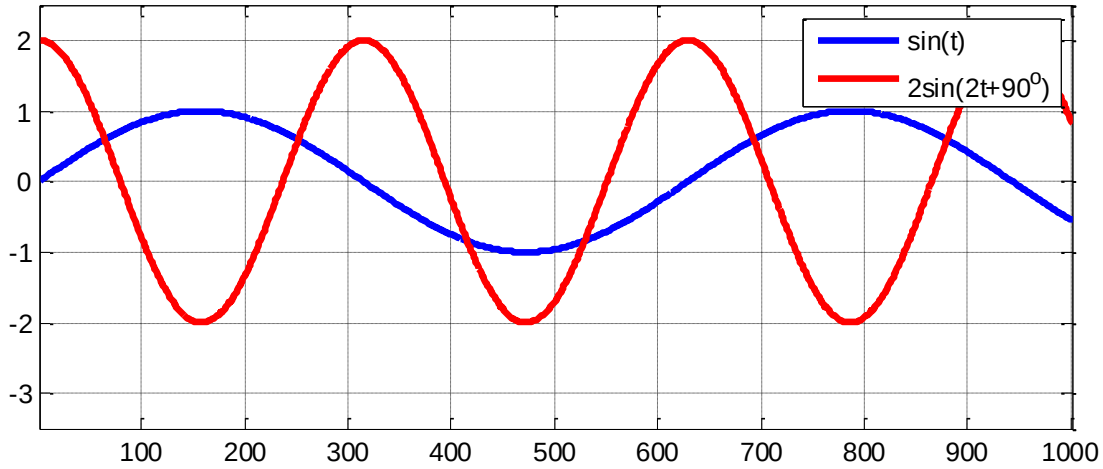
$$\sin(t) + 2\sin(2t + 90^o)$$

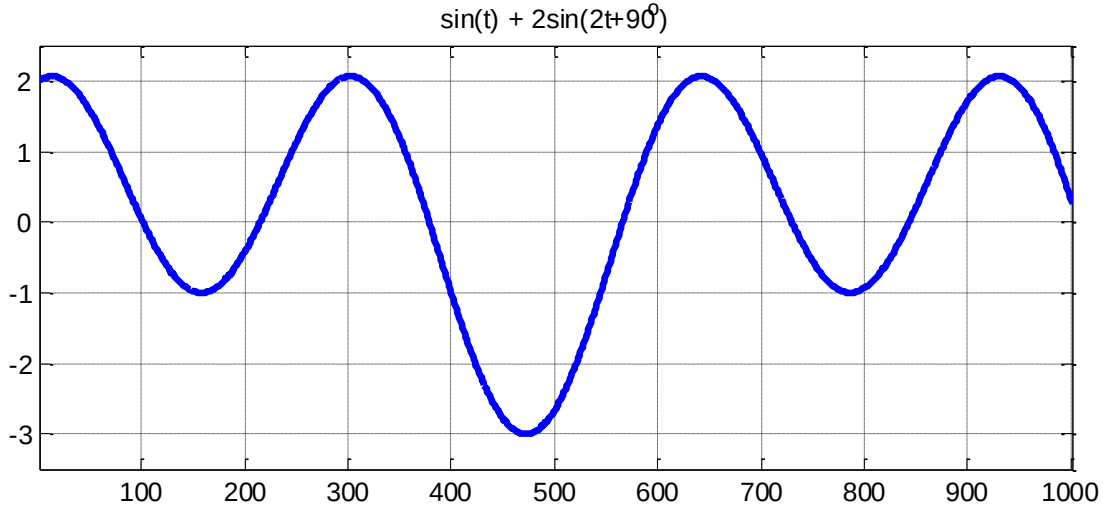
$$\sin(t) \leftrightarrow 1\angle 0^o = 1$$

$$2\sin(2t + 90^o) \leftrightarrow 2\angle 90^o = j2$$

$$1 + j2 = \sqrt{5}\angle 63.4^o$$

$$\sqrt{5}\angle 63.4^o \leftrightarrow \sqrt{5}\sin(\omega t + 63.4^o)$$





Phasors (8)

Ex. 7

$x_1(t) = 100\sin(20t)$,
$x_2(t) = 80\sin(20t + 60^o)$,
Find $x(t) = x_1(t) + x_2(t)$?

$$x_1(t) \leftrightarrow 100\angle 0^o = 100$$

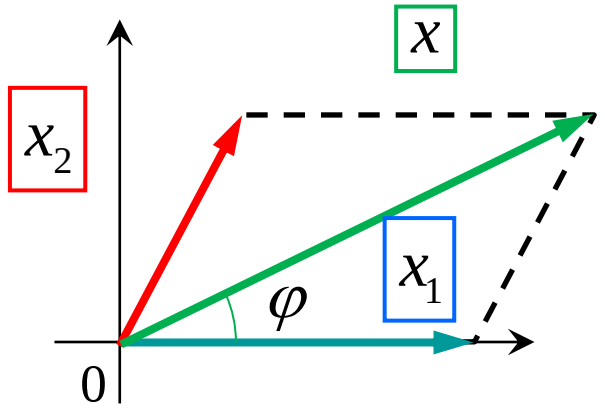
$$x_2(t) \leftrightarrow 80\angle 60^o$$

$$x_1(t) + x_2(t) \leftrightarrow 100 + 80\angle 60^o$$

$$= 156.21\angle 26.33^o$$

$$\rightarrow \begin{cases} X_m = 156.21 \\ \varphi = 26.33^o \end{cases}$$

$$\rightarrow \boxed{x(t) = 156.21\sin(20t + 26.33^o)}$$



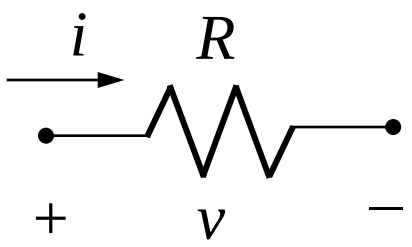
$X_m = ?\ \varphi = ?$

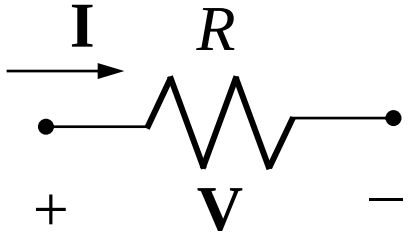
Complex numbers

Sinusoid & Phasors

1. Sinusoids
2. Complex Numbers
3. Phasors
4. **Phasor Relationships for Circuit Elements**

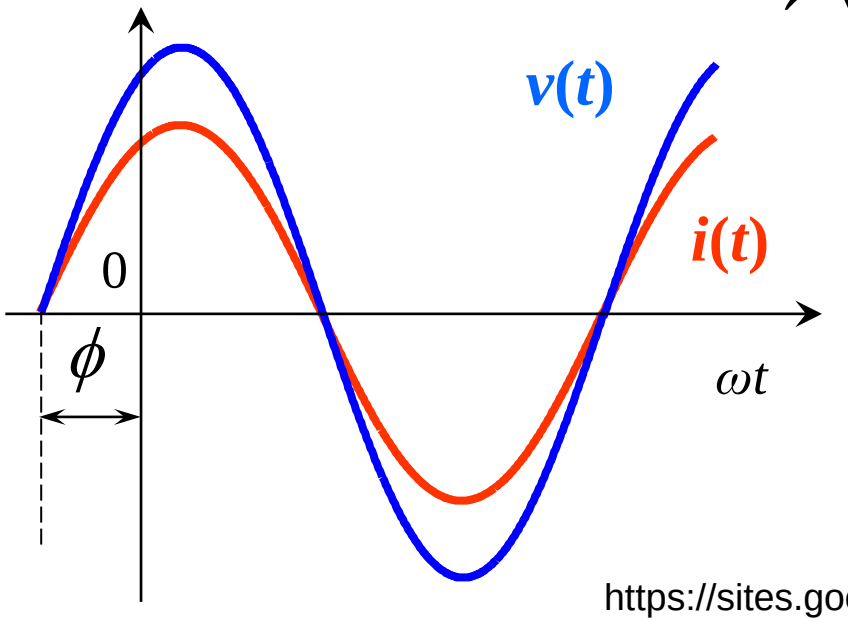
Phasor Relationships for Circuit Elements (1)

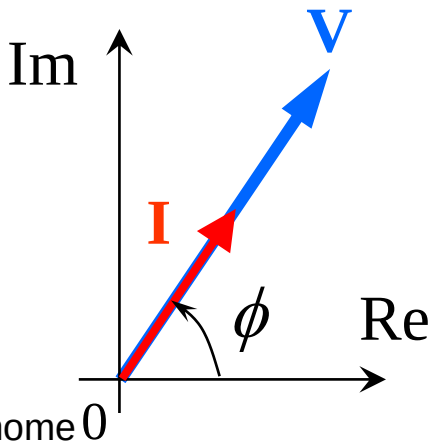




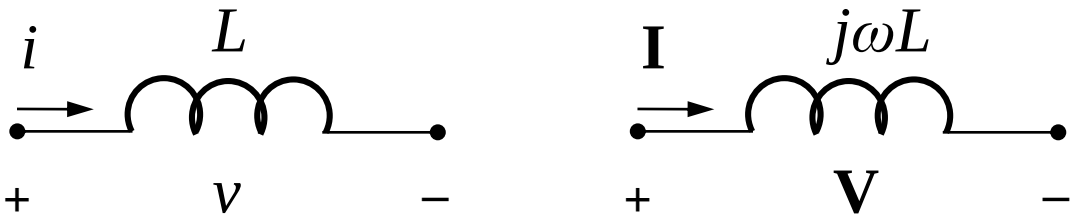
$$\left.\begin{aligned} i &= I_m \sin(\omega t + \phi) \\ v &= Ri \end{aligned}\right\} \rightarrow v = RI_m \sin(\omega t + \phi)$$

$$\left.\begin{aligned} \rightarrow \mathbf{V} &= RI_m \angle \phi \\ \mathbf{I} &= I_m \angle \phi \end{aligned}\right\} \rightarrow \boxed{\mathbf{V} = R\mathbf{I}}$$

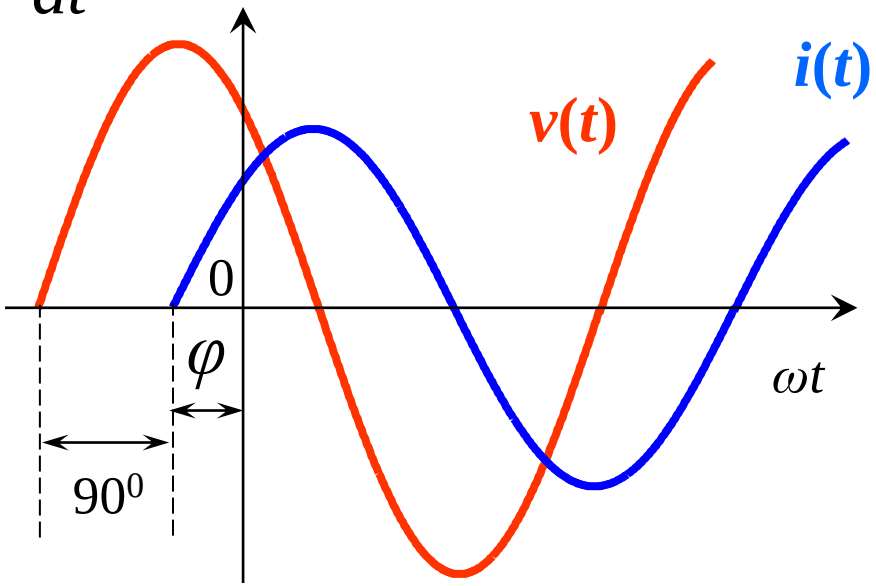


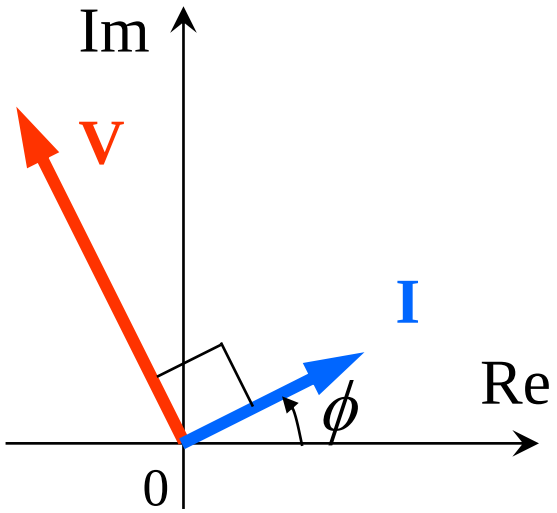


Phasor Relationships for Circuit Elements (2)

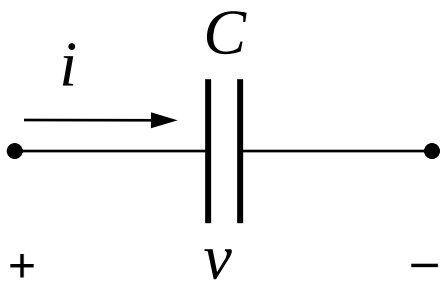


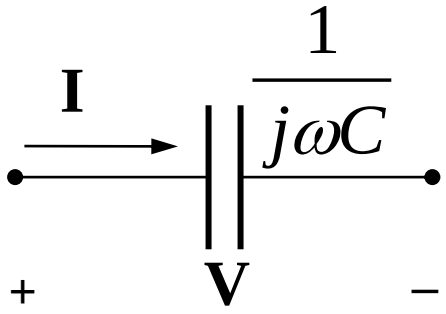
$$\left.\begin{aligned} i &= I_m \sin(\omega t + \phi) \\ v &= L\frac{di}{dt} \end{aligned}\right\} \rightarrow v = \omega L I_m \sin(\omega t + \phi + 90^o) \rightarrow \boxed{\mathbf{V} = j\omega L\mathbf{I}}$$





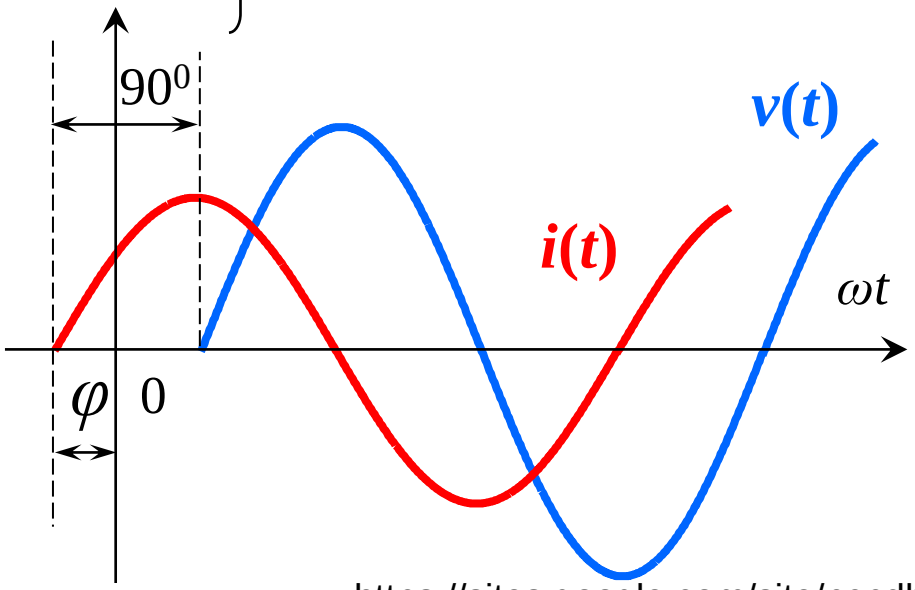
Phasor Relationships for Circuit Elements (3)

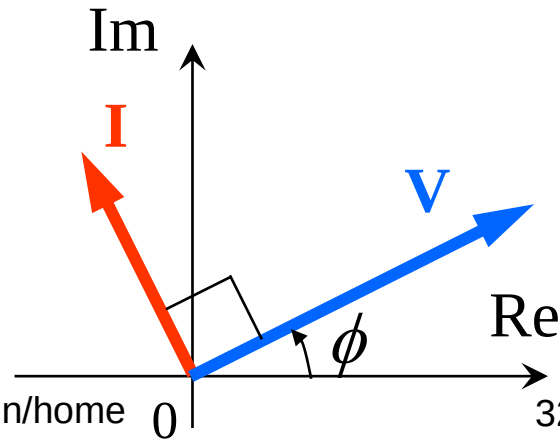




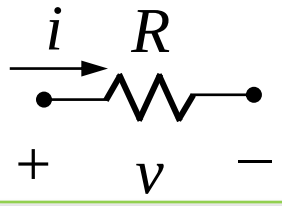
$$\left.\begin{aligned} v &= V_m \sin(\omega t + \phi) \\ i &= C\frac{dv}{dt} \end{aligned}\right\} \rightarrow i = \omega C V_m \sin(\omega t + \phi + 90^o) \rightarrow \mathbf{I} = j\omega C\mathbf{V}$$

$$\rightarrow \boxed{\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}}$$

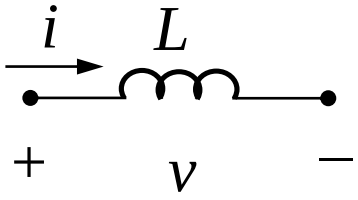




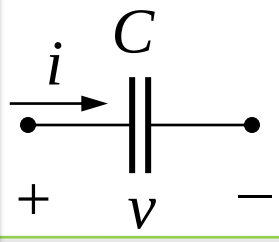
Phasor Relationships for Circuit Elements (4)



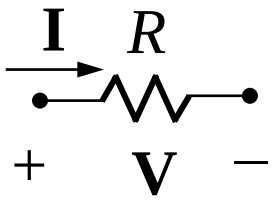
$$v = Ri$$



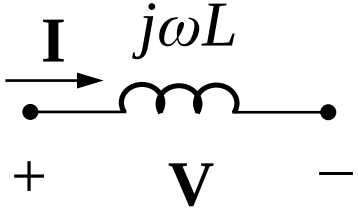
$$v = L\frac{di}{dt}$$



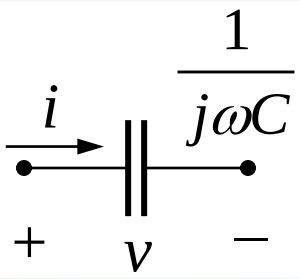
$$v = \frac{1}{C}\int idt$$



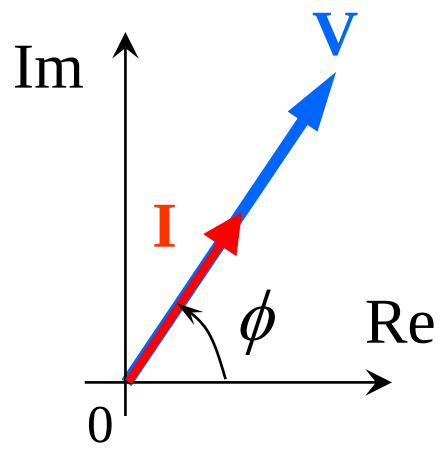
$$\mathbf{V} = R\mathbf{I}$$

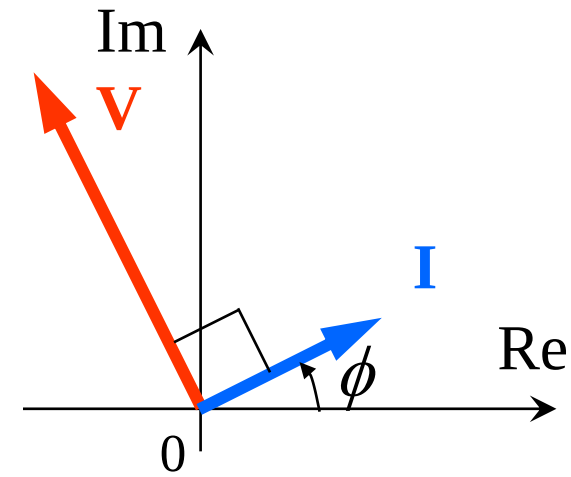


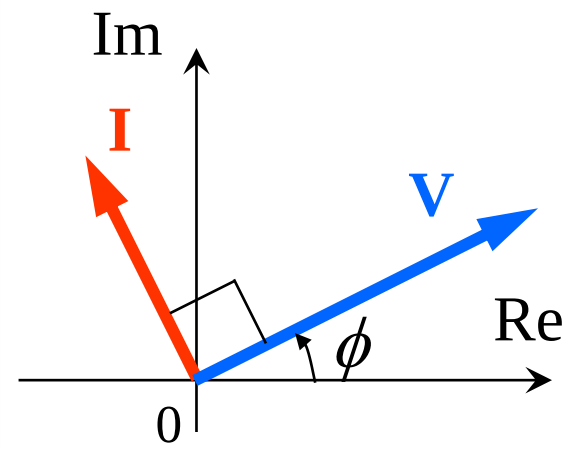
$$\mathbf{V} = j\omega L\mathbf{I}$$



$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

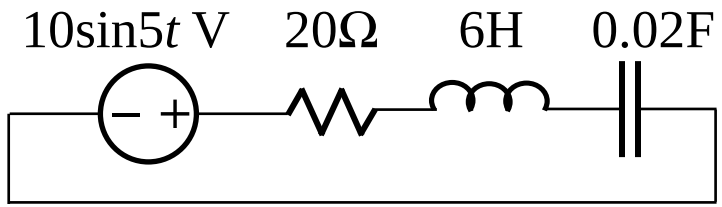


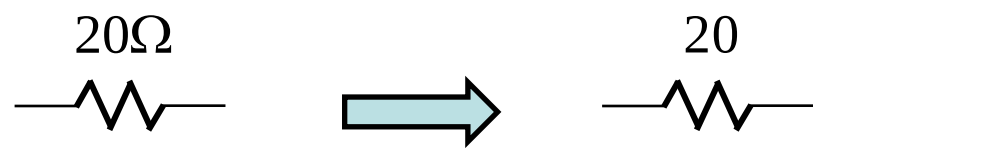


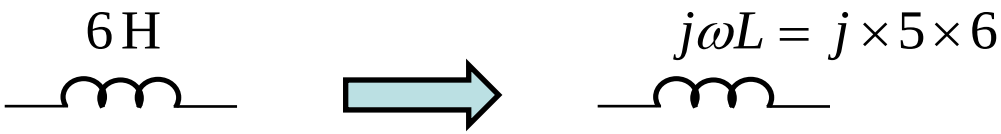


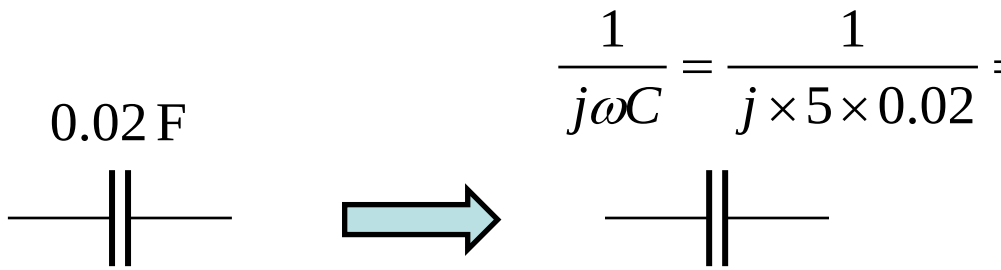
Phasor Relationships for Circuit Elements (5)

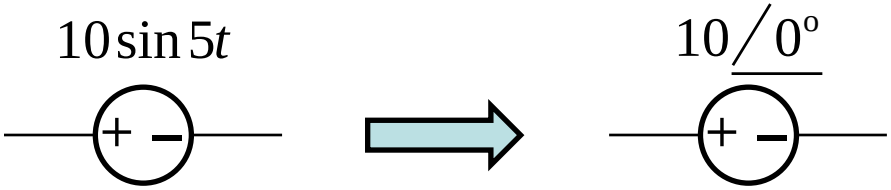
Ex. 1

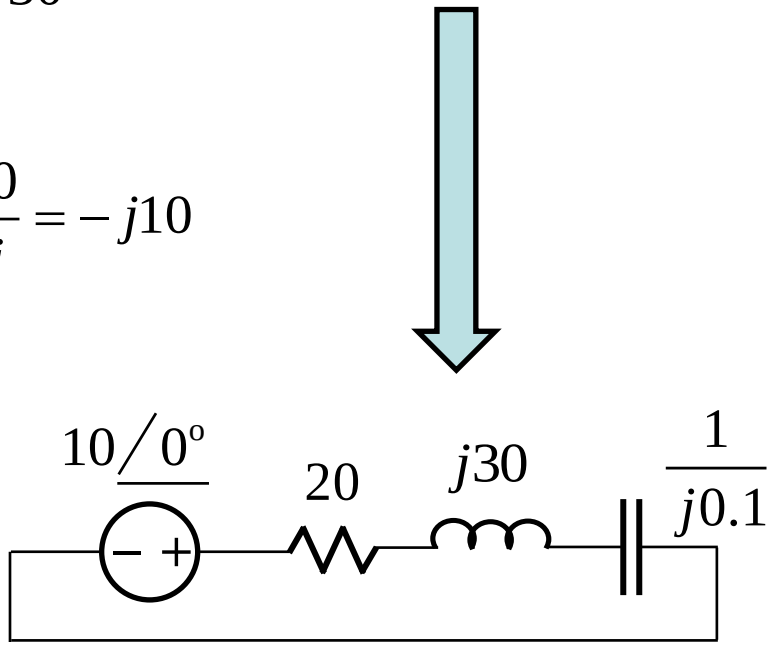












Phasor Relationships for Circuit Elements (6)

Ex. 2

$e_1 = 10\sin 10t$ V; $j = 4\sin(10t + 30^o)$ A;
$e_2 = 6\sin(10t + 60^o)$ V; $L = 1$ H; $R_1 = 1\ \Omega$;
$R_2 = 5\ \Omega$; $C = 0.01$ F.