Nguyễn Công Phương

Engineering Electromagnetics

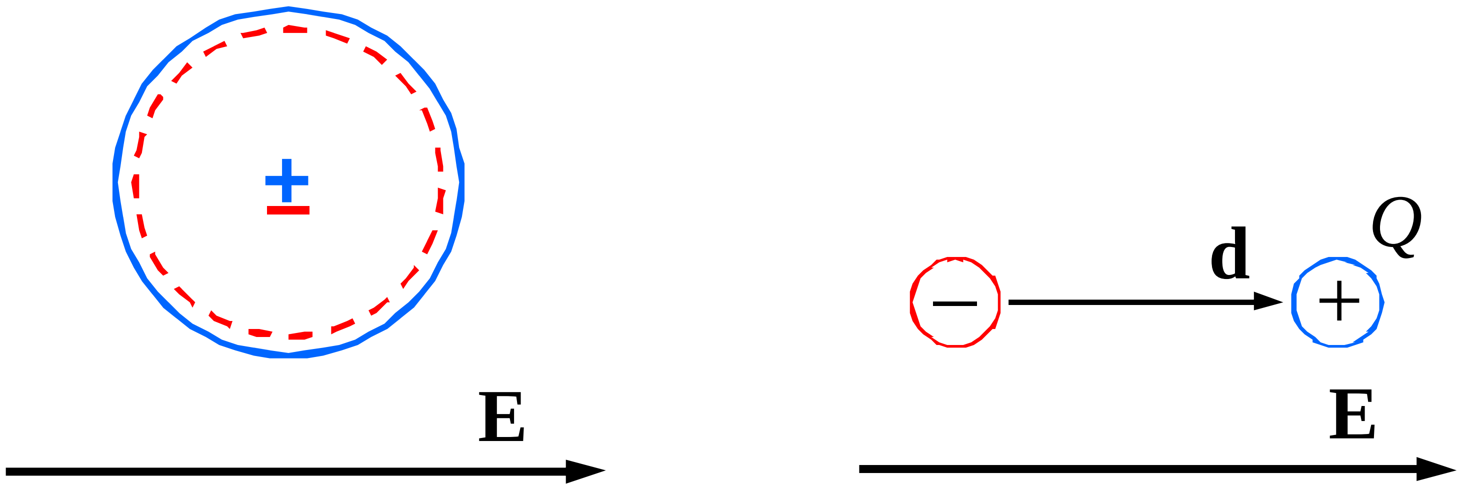
Dielectrics & Capacitance

Contents

Dielectrics & Capacitance

1. Dielectric Materials
2. Boundary Conditions for Perfect Dielectric Materials
3. Capacitance
4. Using Field Sketches to Estimate Capacitance
5. Current Density & Flux Density

Dielectric Materials (1)



- Dipole moment: $\mathbf{p} = Q\mathbf{d}$
- Q: the positive one of the 2 bound charges
- $\mathbf{d}$: the vector from the negative to the positive charge

Dielectric Materials (2)

- Dipole moment: $\mathbf{p} = Q\mathbf{d}$
- If there are n dipoles per unit volume, then the total dipole moment in Δv:

$$\mathbf{p}_{total} = \sum_{i=1}^{n\Delta v} \mathbf{p}_i$$

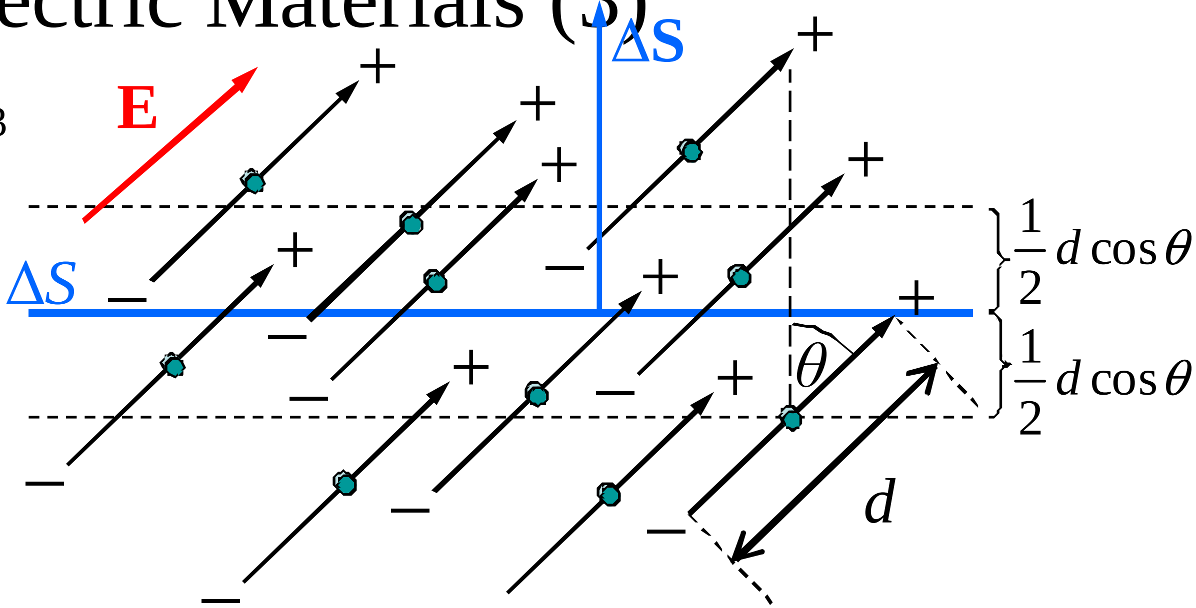
- The polarization:

$$\mathbf{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} \mathbf{p}_i$$

- Unit: C/m^2

Dielectric Materials (3)

Density: n molecules/m^3



$$\left.\begin{array}{l}\Delta v = d\cos\theta\Delta S \\ \Delta Q_b = nQ\Delta v\end{array}\right\}$$

$$\rightarrow \Delta Q_b = nQd\cos\theta\Delta S$$

$$\left.\begin{array}{r} = nQ\mathbf{d}.\Delta\mathbf{S} \\ \mathbf{p} = Q\mathbf{d} \;\rightarrow \mathbf{P} = nQ\mathbf{d}\end{array}\right\}\rightarrow \Delta Q_b = \mathbf{P}.\Delta\mathbf{S}$$

$$\left.\begin{array}{l}\rightarrow Q_b = -\oint_S \mathbf{P}.d\mathbf{S} \\ \text{Gauss's law:}\quad Q_T = \oint_S \varepsilon_0\mathbf{E}.d\mathbf{S} \\ Q_T = Q_b + Q \quad \rightarrow Q = Q_T - Q_b\end{array}\right\}\rightarrow Q = \oint_S \left(\varepsilon_0\mathbf{E} + \mathbf{P}\right).d\mathbf{S}$$

(Q: the total free charge)

Dielectric Materials (4)

$$\left.\begin{aligned} Q &= \oint_S (\varepsilon_0 \mathbf{E} + \mathbf{P}).d\mathbf{S} \\ \text{Gauss's law:}\quad Q &= \oint_S \mathbf{D}.d\mathbf{S} \end{aligned}\right\} \rightarrow \boxed{\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}}$$

$$\left.\begin{aligned} \text{Divergence theorem:}\quad \oint_S \mathbf{D}.d\mathbf{S} &= \int_v \nabla.\mathbf{D}dv \\ Q &= \int_V \rho_v dv \end{aligned}\right\} \rightarrow \boxed{\nabla.\mathbf{D} = \rho_v}$$

Dielectric Materials (5)

- $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$
- In an isotropic material, **E** & **P** are always parallel, regardless of the orientation of the field
- $\mathbf{P} = \chi_e\varepsilon_0\mathbf{E}$
- χ_e : the electric susceptibility
- $\rightarrow \mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\mathbf{E} + \chi_e\varepsilon_0\mathbf{E} = (\chi_e + 1)\varepsilon_0\mathbf{E}$
- $\varepsilon_r = \chi_e + 1$: the relative permitivity
- $\rightarrow \boxed{\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} = \varepsilon\mathbf{E}}$
- $\varepsilon = \varepsilon_0\varepsilon_r$: the permitivity

Dielectric Materials (6)

Material	ε_r	Material	ε_r	Material	ε_r
Quartz	3.8–5	Paper	3.0	Silica	3.8
GaAs*	13	Bakelite	5.0	Quartz	3.8
Nylon	3.1	Glass	6.0 (4–7)	Snow	3.8
Paraffin	3.2	Mica	6.0	Soil (dry)	2.8
Perspex	2.6	Water (distilled)	81	Wood (dry)	1.5–4
Polystyrene foam	1.05	Polyethylene	2.2	Silicon	11.8
Teflon	2.0	Polyvinyl chloride	6.1	Ethyl alcohol	25
$BaTiO_3$**	10,000	Germanium	16	Amber	2.7
Air	1.0006	Glycerin	50	Plexiglas	3.4
Rubber	3.0	Nylon	3.5	Aluminum oxide	8.8

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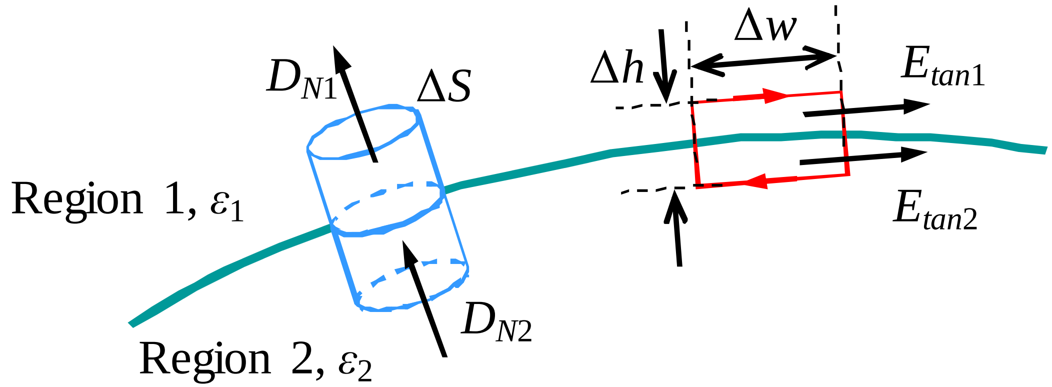
Dielectrics & Capacitance

1. Dielectric Materials
2. **Boundary Conditions for Perfect Dielectric Materials**
3. Capacitance
4. Using Field Sketches to Estimate Capacitance
5. Current Density & Flux Density

Boundary Conditions for Perfect Dielectric Materials (1)



$$\oint \mathbf{E}.d\mathbf{L} = 0$$

$$\rightarrow E_{tan1}\Delta w - E_{tan2}\Delta w = 0$$

$$\rightarrow \boxed{E_{tan1} = E_{tan2}}$$

$$\rightarrow \frac{D_{tan1}}{\varepsilon_1} = E_{tan1} = E_{tan2} = \frac{D_{tan2}}{\varepsilon_2} \rightarrow \boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2}}$$

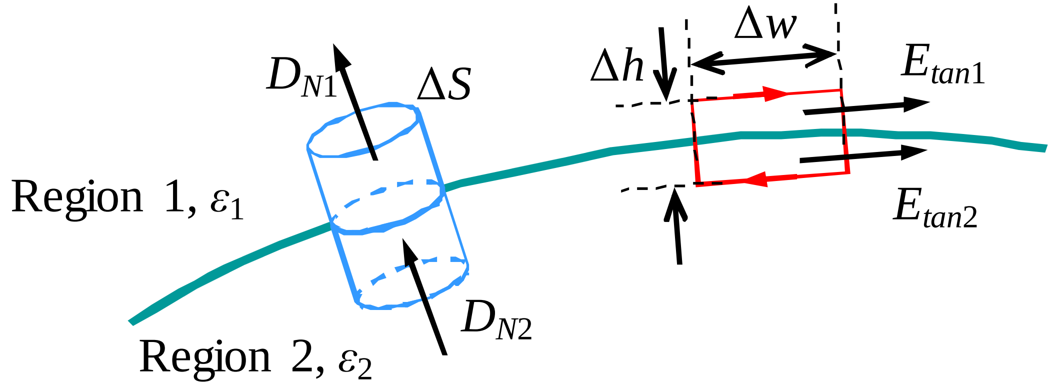
$$\left.\begin{array}{l} \Delta Q = \rho_S \Delta S \\ \Delta Q = D_{N1}\Delta S - D_{N2}\Delta S \end{array}\right\} \rightarrow \boxed{D_{N1} - D_{N2} = \rho_S}$$

No free charge on the interface $\rightarrow \rho_S = 0$

$$\left.\begin{array}{l} D_{N1} - D_{N2} = \rho_S \\ \rho_S = 0 \end{array}\right\} \rightarrow \boxed{D_{N1} = D_{N2}}$$

$$\rightarrow \varepsilon_1 E_{N1} = \varepsilon_2 E_{N2} \rightarrow \boxed{\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1}}$$

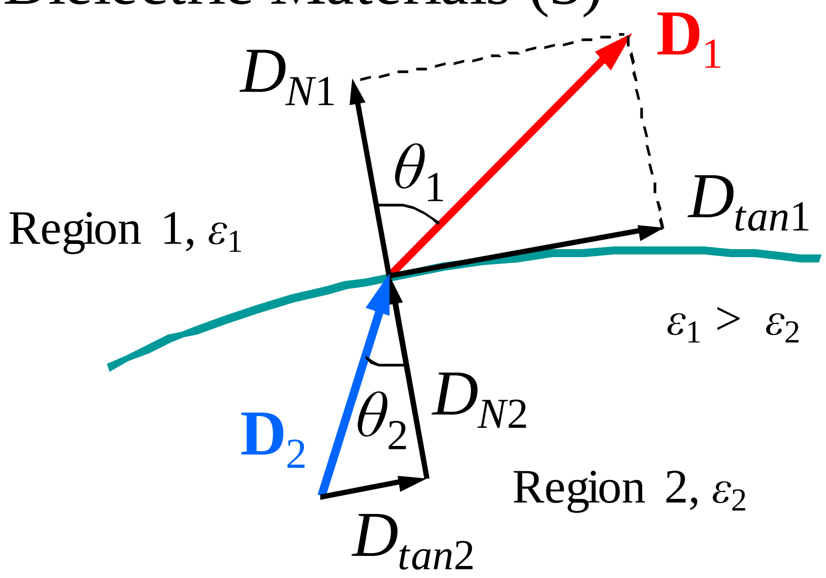
Boundary Conditions for Perfect Dielectric Materials (2)



$$\left.\begin{aligned} & E_{tan1} = E_{tan2} \\ & \frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} \\ & D_{N1} = D_{N2} \\ & \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} \end{aligned}\right\} \rightarrow$$

If we know the field on one side (e.g **E**$_1$ or **D**$_1$) of a boundary, we cand find quickly the field on the other side (**E**$_2$ & **D**$_2$)

Boundary Conditions for Perfect Dielectric Materials (3)



$$\left.\begin{aligned} D_{N1} &= D_{N2} \\ D_{N1} &= D_1 \cos\theta_1 \\ D_{N2} &= D_2 \cos\theta_2 \end{aligned}\right\} \rightarrow D_1 \cos\theta_1 = D_2 \cos\theta_2$$

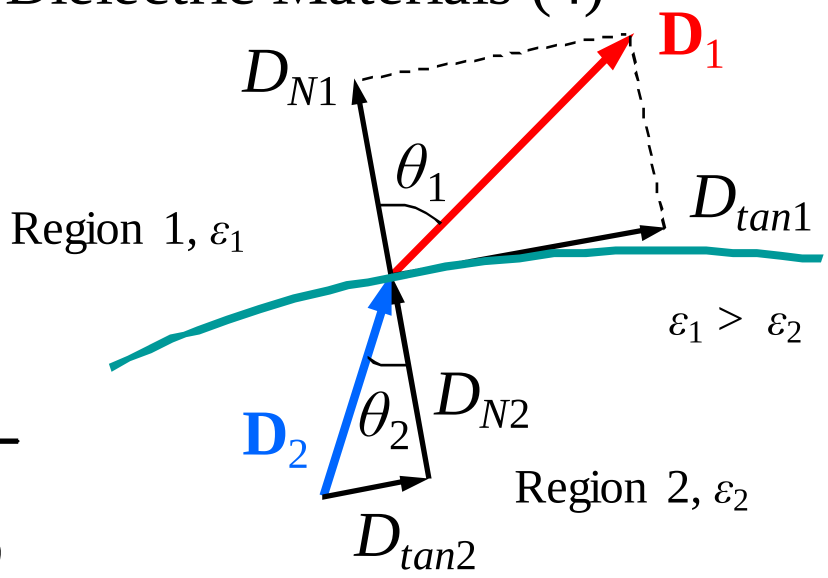
$$\left.\begin{aligned} \frac{D_{tan1}}{D_{tan2}} &= \frac{\varepsilon_1}{\varepsilon_2} \\ D_{tan1} &= D_1 \sin\theta_1 \\ D_{tan2} &= D_2 \sin\theta_2 \end{aligned}\right\} \rightarrow \varepsilon_2 D_1 \sin\theta_1 = \varepsilon_1 D_2 \sin\theta_2$$

$$\left.\begin{aligned} &D_1 \cos\theta_1 = D_2 \cos\theta_2 \\ &\varepsilon_2 D_1 \sin\theta_1 = \varepsilon_1 D_2 \sin\theta_2 \end{aligned}\right\} \rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \rightarrow \theta_2$$

$$\left.\begin{aligned} &\theta_2 \\ &D_1 \cos\theta_1 = D_2 \cos\theta_2 \end{aligned}\right\} \rightarrow D_2$$

Boundary Conditions for Perfect Dielectric Materials (4)

$$\theta_2 = \operatorname{atan}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right)$$



$$D_2 = D_1\sqrt{\cos^2\theta_1 + \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 \sin^2\theta_1}$$

$$E_2 = E_1\sqrt{\sin^2\theta_1 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \cos^2\theta_1}$$

Boundary Conditions for Perfect Dielectric Materials (6)

Ex.

Given the region $z < 0$ with $\varepsilon_{r1} = 3.2$ & $\mathbf{D}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ nC/m². The region $z > 0$ possesses $\varepsilon_{r2} = 2$. Find D_{N1}, $\mathbf{D}_{tan1}$, D_{tan1}, θ_1, $\mathbf{D}_{N2}$, $\mathbf{D}_{tan2}$, $\mathbf{D}_2$, θ_2 ?

$$D_{N1} = D_{1z} = 70 \text{ nC/m}^2$$

$$\mathbf{D}_{tan1} = -30\mathbf{a}_x + 50\mathbf{a}_y \text{ nC/m}^2$$

$$D_{tan1} = |\mathbf{D}_{tan1}| = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ nC/m}^2$$

$$D_1 = |\mathbf{D}_1| = \sqrt{(-30)^2 + 50^2 + 70^2} = 91.1 \text{ nC/m}^2$$

$$\theta_1 = \text{atan}\frac{D_{tan1}}{D_{N1}} = \text{atan}\frac{58.3}{70} = 39.8^{\circ}$$

Boundary Conditions for Perfect Dielectric Materials (7)

Ex.

Given the region $z < 0$ with $\varepsilon_{r1} = 3.2$ & $\mathbf{D}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ nC/m². The region $z > 0$ possesses $\varepsilon_{r2} = 2$. Find D_{N1}, $\mathbf{D}_{tan1}$, D_{tan1}, θ_1, $\mathbf{D}_{N2}$, $\mathbf{D}_{tan2}$, $\mathbf{D}_2$, θ_2 ?

$$D_{N2} = D_{N1} = 70 \text{ nC/m}^2 \rightarrow \mathbf{D}_{N2} = 70\mathbf{a}_z \text{ nC/m}^2$$

$$\frac{D_{tan1}}{D_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} \rightarrow \frac{\mathbf{D}_{tan1}}{\mathbf{D}_{tan2}} = \frac{\varepsilon_1}{\varepsilon_2} \rightarrow \mathbf{D}_{tan2} = \frac{\varepsilon_2}{\varepsilon_1}\mathbf{D}_{tan1} = \frac{2}{3.2}(-30\mathbf{a}_x + 50\mathbf{a}_y)$$

$$= -18.75\mathbf{a}_x + 31.25\mathbf{a}_y \text{ nC/m}^2$$

$$\mathbf{D}_2 = \mathbf{D}_{tan2} + \mathbf{D}_{N2} = -18.75\mathbf{a}_x + 31.25\mathbf{a}_y + 70\mathbf{a}_z \text{ nC/m}^2$$

$$\theta_2 = \operatorname{atan}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right) = \operatorname{atan}\left(\frac{2}{3.2}\tan 39.8^\circ\right) = 27.5^\circ$$

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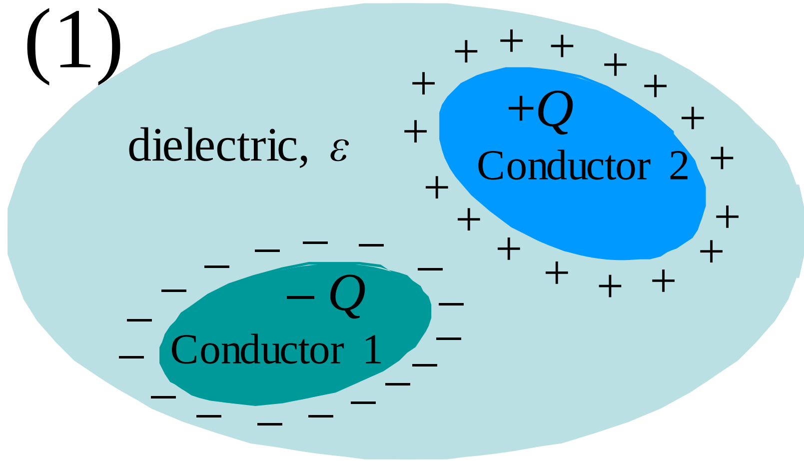
Capacitance (1)

$$\left.\begin{array}{l} \text{Capacitance: } \boxed{C = \dfrac{Q}{V_0}} \\ Q = \oint_S \varepsilon \mathbf{E}.d\mathbf{S} \\ V_0 = -\int_{-}^{+} \mathbf{E}.d\mathbf{L} \end{array}\right\} \rightarrow C = \frac{\oint_S \varepsilon \mathbf{E}.d\mathbf{S}}{-\int_{-}^{+} \mathbf{E}.d\mathbf{L}}$$

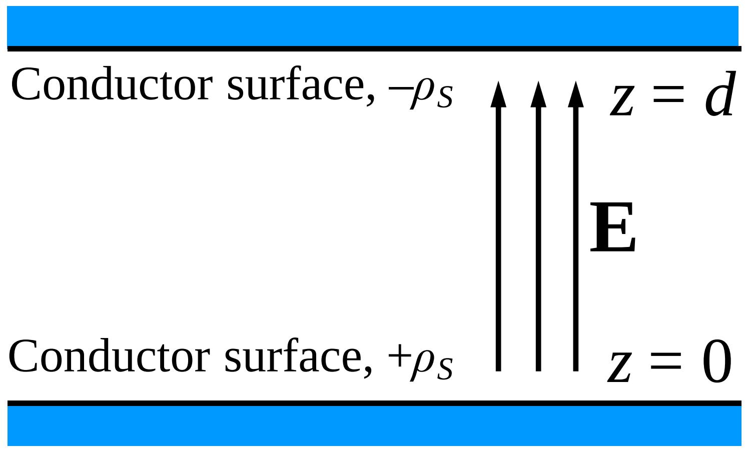


- V_0: work to carry a unit positive charge from the surface 1 to the surface 2
- C depends on the physical dimensions (of the system of conductors) & on the permittivity
- Unit: F (farad), C/V, practically μF, nF, pF

Capacitance (2)

$$\mathbf{E} = \frac{\rho_S}{\varepsilon}\mathbf{a}_z$$

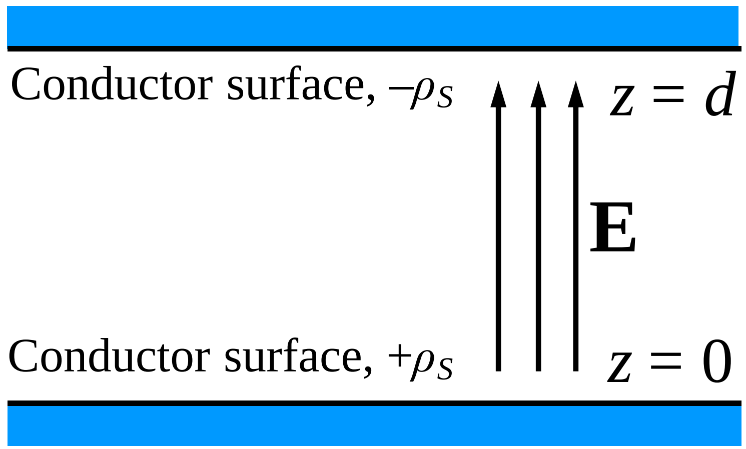
$$\mathbf{D} = \rho_S \mathbf{a}_z$$



$$\left.\begin{aligned} V_0 &= -\int_{top}^{bottom} E.dL = -\int_d^0 \frac{\rho_S}{\varepsilon} dz = \frac{\rho_S}{\varepsilon} d \\ Q &= \rho_S S \\ C &= \frac{Q}{V_0} \end{aligned}\right\} \rightarrow \boxed{C = \frac{\varepsilon S}{d}}$$

Capacitance (3)

$$\left.\begin{aligned} W_E &= \frac{1}{2}\int_V \varepsilon E^2 dv \\ E &= \frac{\rho_S}{\varepsilon} \end{aligned}\right\}$$

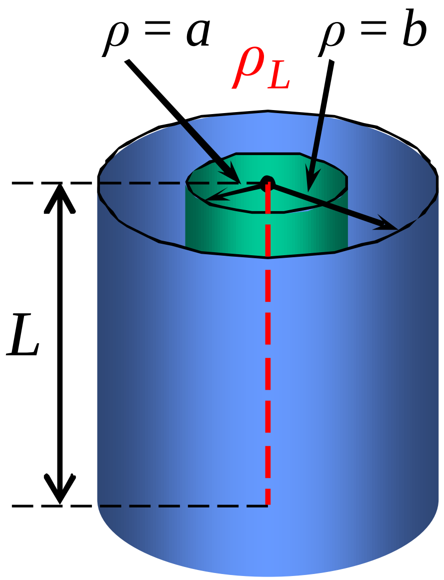


$$\rightarrow W_E = \frac{1}{2}\int_0^S \int_0^d \frac{\varepsilon \rho_S^2}{\varepsilon^2} dz dS$$

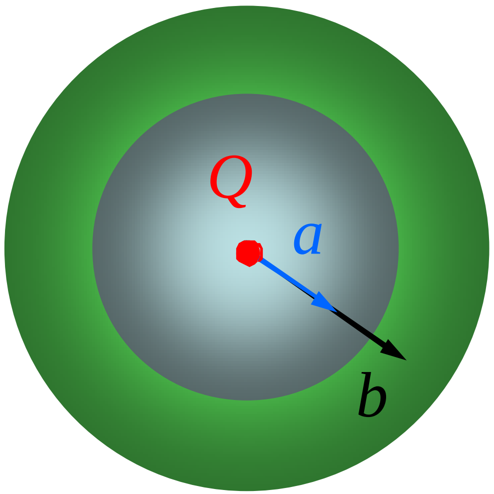
$$\left.\begin{aligned} &= \frac{1}{2}\frac{\rho_S^2}{\varepsilon} Sd = \frac{1}{2}\frac{\varepsilon S}{d}\frac{\rho_S^2 d^2}{\varepsilon^2} \\ C &= \frac{\varepsilon S}{d} \\ V_0 &= \frac{\rho_S}{\varepsilon} d \end{aligned}\right\} \rightarrow \boxed{W_E = \frac{1}{2} C V_0^2 = \frac{1}{2} Q V_0 = \frac{1}{2}\frac{Q^2}{C}}$$

Capacitance (4)

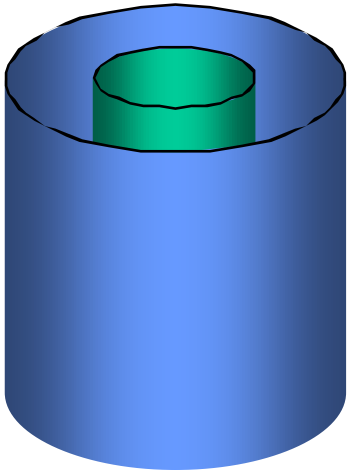
$$\left.\begin{aligned} V_{ab} &= \frac{\rho_L}{2\pi\varepsilon}\ln\frac{b}{a} \\ Q &= \rho_L L \\ C &= \frac{Q}{V_{ab}} \end{aligned}\right\} \rightarrow \boxed{C = \frac{2\pi\varepsilon L}{\ln\dfrac{b}{a}}}$$



$$\left.\begin{aligned} V_{ab} &= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right) \\ C &= \frac{Q}{V_{ab}} \end{aligned}\right\} \rightarrow \boxed{C = \frac{4\pi\varepsilon}{\dfrac{1}{a}-\dfrac{1}{b}}}$$



Capacitance (5)

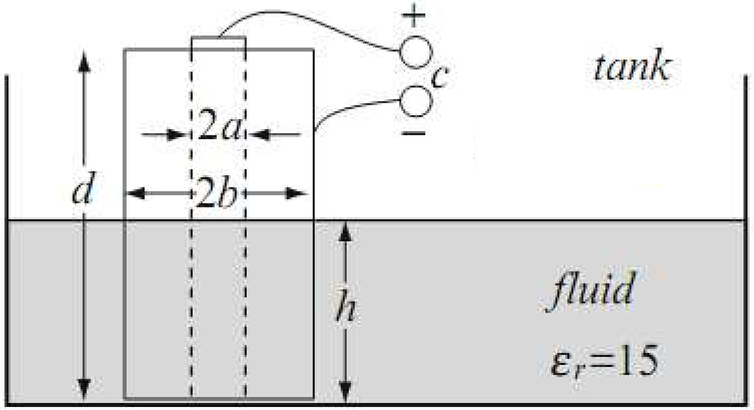


$$C_{immerse} = \frac{2\pi\varepsilon h}{\ln(b/a)}$$

$$C_{above} = \frac{2\pi\varepsilon_0 (d-h)}{\ln(b/a)}$$

$$C_{total} = C_{immerse} + C_{above}$$
$$= \frac{2\pi\varepsilon h}{\ln(b/a)} + \frac{2\pi\varepsilon_0 (d-h)}{\ln(b/a)}$$

$$\rightarrow C(h) = \frac{2\pi\varepsilon_0}{\ln(b/a)}(h\varepsilon_r + d - b)$$



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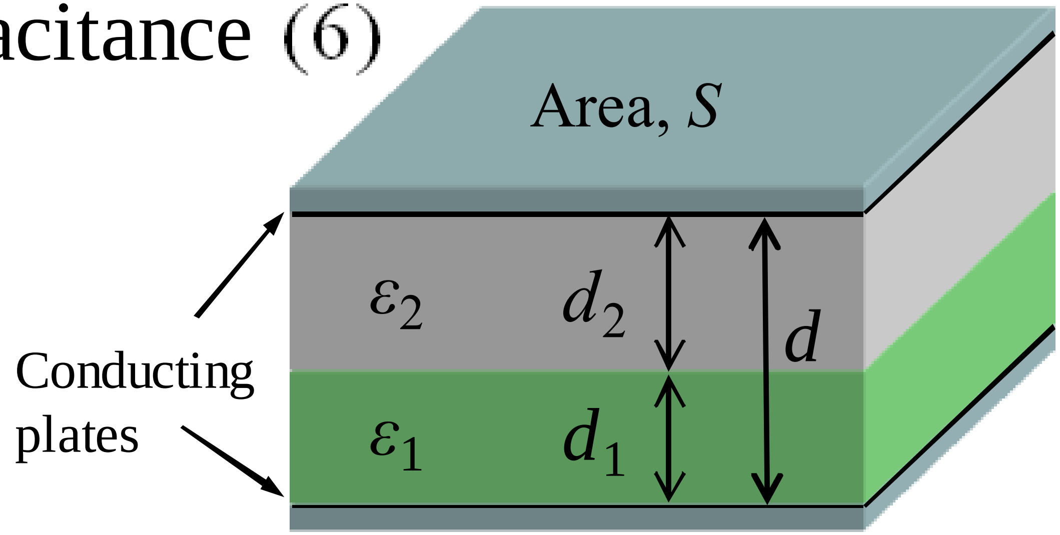
Capacitance (6)

$$\left.\begin{array}{l} V_0 = E_1 d_1 + E_2 d_2 \\ D_{N1} = D_{N2} \rightarrow \varepsilon_1 E_1 = \varepsilon_2 E_2 \end{array}\right\}$$

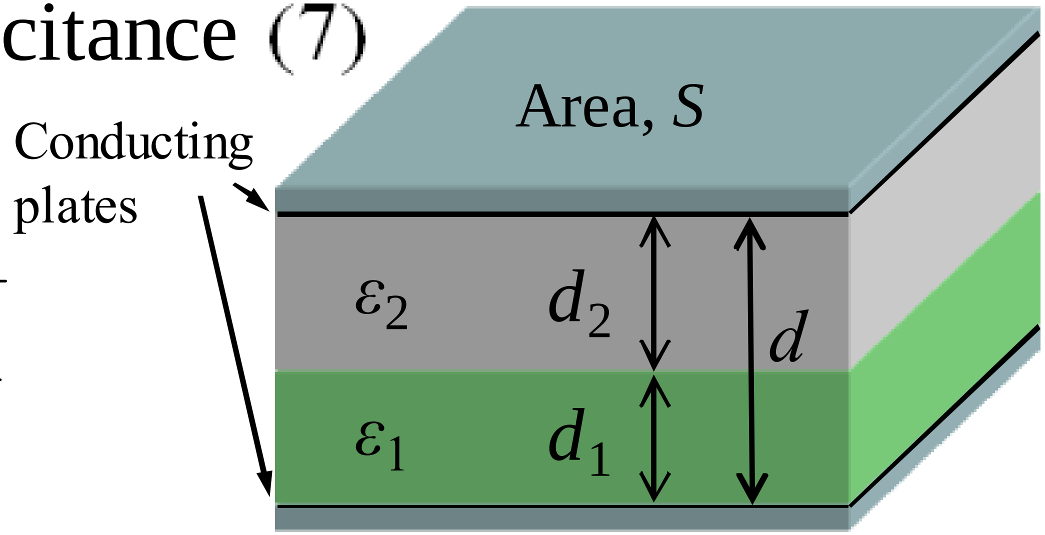
$$\rightarrow E_1 = \frac{V_0}{d_1 + d_2 \dfrac{\varepsilon_1}{\varepsilon_2}}$$



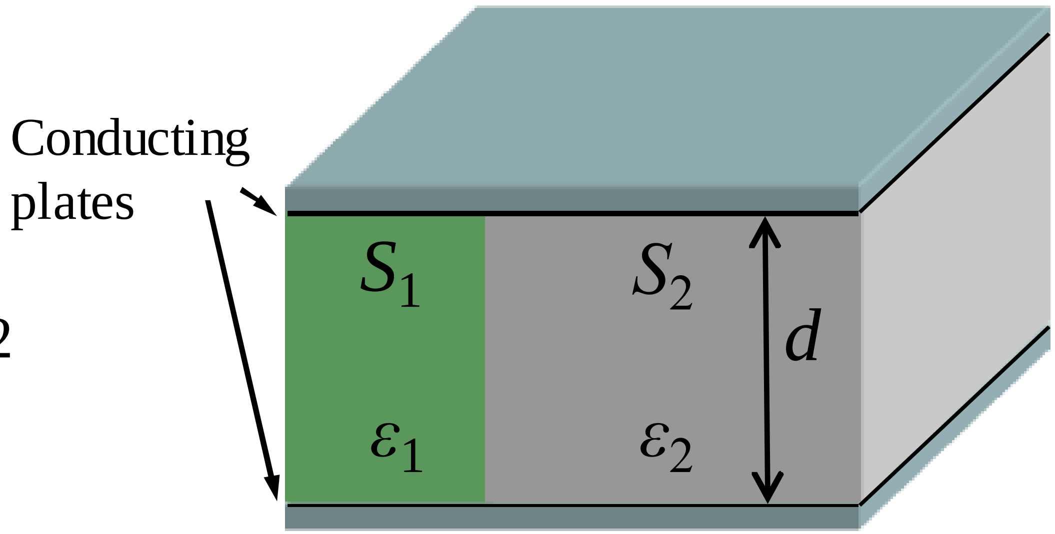
$$\left.\begin{array}{r} \rightarrow \rho_{S1} = D_1 = \varepsilon_1 E_1 = \dfrac{V_0}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}} \\ Q = \rho_S S = \rho_{S1} S \\ C = \dfrac{Q}{V_0} \end{array}\right\} \rightarrow \boxed{C = \frac{1}{\dfrac{d_1}{\varepsilon_1 S} + \dfrac{d_2}{\varepsilon_2 S}}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$$

Capacitance (7)

$$C = \frac{1}{\dfrac{d_1}{\varepsilon_1 S} + \dfrac{d_2}{\varepsilon_2 S}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$$



$$C = \frac{\varepsilon_1 S_1 + \varepsilon_2 S_2}{d} = C_1 + C_2$$



Capacitance (8)

$$Q = \oint_S \mathbf{D}(r).d\mathbf{S} = D(r).4\pi r^2$$

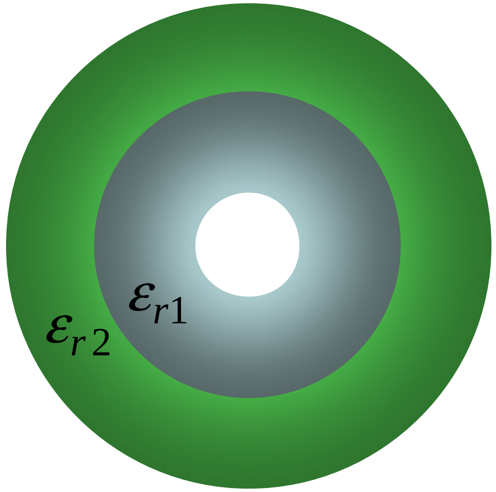
$$\rightarrow D(r) = \frac{Q}{4\pi r^2}$$

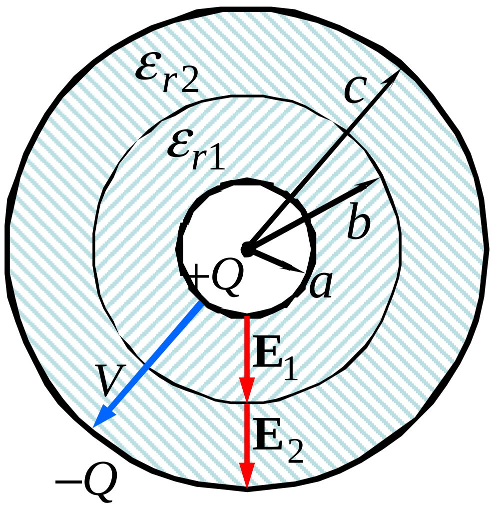
$$V = \int_{r=a}^{b} \mathbf{E}_1.d\mathbf{L} + \int_{r=b}^{c} \mathbf{E}_2.d\mathbf{L}$$

$$= \int_a^b \frac{Q}{4\pi\varepsilon_1 r^2} dr + \int_b^c \frac{Q}{4\pi\varepsilon_2 r^2} dr$$

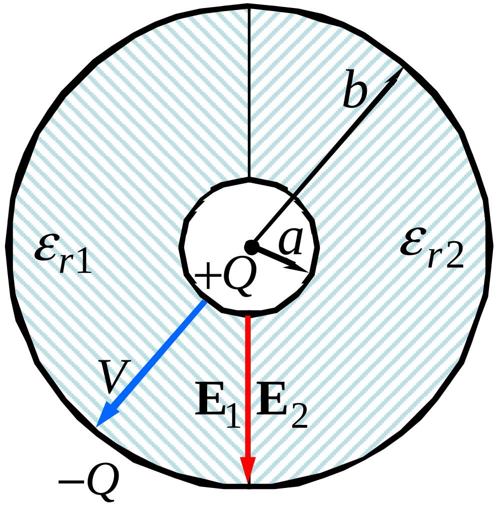
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}a} - \frac{1}{\varepsilon_{r1}b} + \frac{1}{\varepsilon_{r2}b} - \frac{1}{\varepsilon_{r2}c}\right)$$

$$\rightarrow C = \frac{Q}{V} = \boxed{\frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}a} - \frac{1}{\varepsilon_{r1}b} + \frac{1}{\varepsilon_{r2}b} - \frac{1}{\varepsilon_{r2}c}}}$$





Capacitance (9)



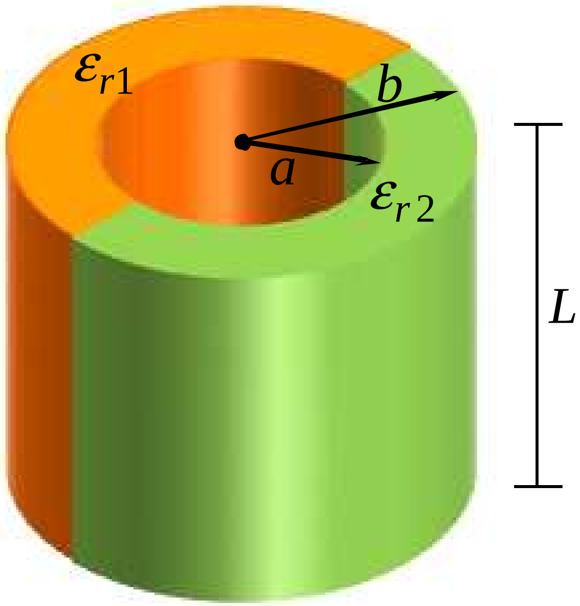
$$Q = \oint_S \mathbf{D}(r).d\mathbf{S}$$
$$= \int_{S_1} \mathbf{D}_1(r).d\mathbf{S} + \int_{S_2} \mathbf{D}_2(r).d\mathbf{S}$$
$$= \int_{S_1} \varepsilon_{r1}\varepsilon_0\mathbf{E}_1(r).d\mathbf{S} + \int_{S_2} \varepsilon_{r2}\varepsilon_0\mathbf{E}_2(r).d\mathbf{S}$$
$$= \int_{S_1} \varepsilon_{r1}\varepsilon_0\mathbf{E}(r).d\mathbf{S} + \int_{S_2} \varepsilon_{r2}\varepsilon_0\mathbf{E}(r).d\mathbf{S}$$
$$= \varepsilon_0 E(r)\left(\varepsilon_{r1}\frac{4\pi r^2}{2} + \varepsilon_{r2}\frac{4\pi r^2}{2}\right) \to E(r) = \frac{Q}{2\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})\pi r^2}$$
$$V = \int_{r=a}^{b} \mathbf{E}.d\mathbf{L} = \int_a^b \frac{Q}{2\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})\pi r^2}dr = \frac{Q}{2\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$
$$\to C = \frac{Q}{V} = \boxed{\frac{2\pi\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})ab}{b-a}}$$

Capacitance (10)

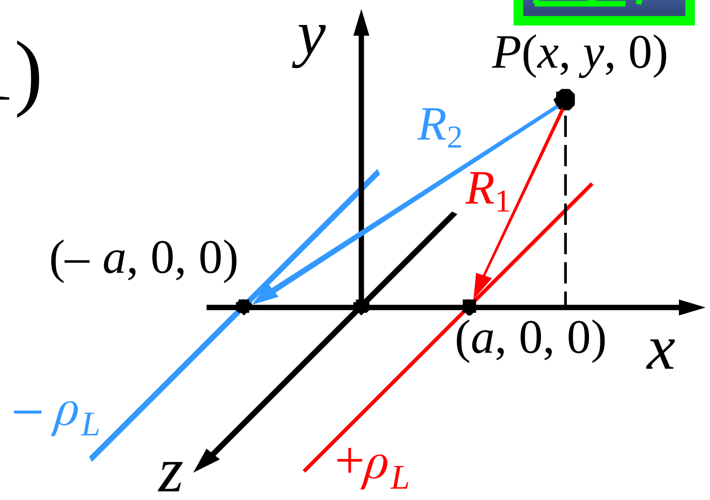
$$C = \frac{2\pi\varepsilon L}{\ln(b/a)}$$

$$= C_1 + C_2 = C = \frac{\pi\varepsilon_{r1}\varepsilon_0 L}{\ln(b/a)} + \frac{\pi\varepsilon_{r2}\varepsilon_0 L}{\ln(b/a)}$$

$$= \frac{2\pi\varepsilon_{r,tb}\varepsilon_0 L}{\ln(b/a)}, \quad \varepsilon_{r,tb} = \frac{\varepsilon_{r1} + \varepsilon_{r2}}{2}$$



Capacitance (11)



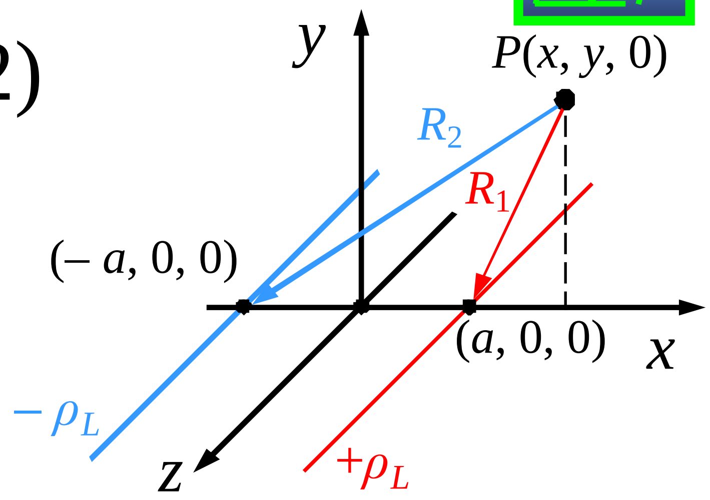
$$\left.\begin{aligned} V_1 &= \frac{\rho_L}{2\pi\varepsilon}\ln\frac{R_{01}}{R_1} \\ V_2 &= \frac{-\rho_L}{2\pi\varepsilon}\ln\frac{R_{02}}{R_2} \end{aligned}\right\}$$

$$\to V = V_1 + V_2 = \frac{\rho_L}{2\pi\varepsilon}\left(\ln\frac{R_{01}}{R_1} - \ln\frac{R_{02}}{R_2}\right)$$

$$\left.\begin{aligned} &= \frac{\rho_L}{2\pi\varepsilon}\ln\frac{R_{01}R_2}{R_{02}R_1} \\ R_{01} &= R_{02} \\ R_1 &= \sqrt{(x-a)^2 + y^2} \\ R_2 &= \sqrt{(x+a)^2 + y^2} \end{aligned}\right\} \to V = \frac{\rho_L}{2\pi\varepsilon}\ln\sqrt{\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon}\ln\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$$

Capacitance (12)



$$V = \frac{\rho_L}{4\pi\varepsilon}\ln\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

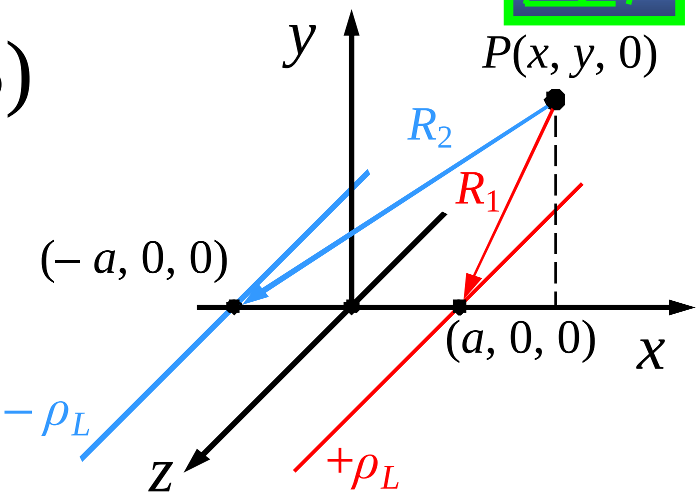
Choosing an equipotential surface V_1, we define:

$$K_1 = e^{4\pi\varepsilon V_1/\rho_L}$$

$$\to K_1 = \frac{(x+a)^2+y^2}{(x-a)^2+y^2} \to x^2 - 2ax\frac{K_1+1}{K_1-1} + y^2 + a^2 = 0$$

$$\to \left(x - a\frac{K_1+1}{K_1-1}\right)^2 + y^2 = \left(\frac{2a\sqrt{K_1}}{K_1-1}\right)^2$$

Capacitance (13)



$$K_1 = e^{4\pi\varepsilon V_1/\rho_L}$$

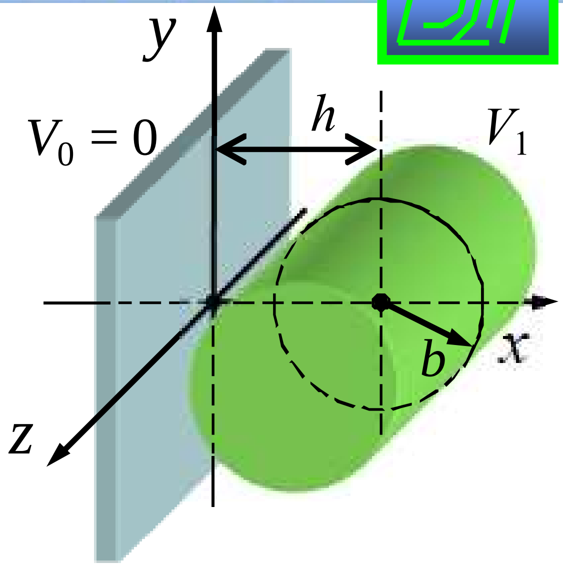
$$\rightarrow \left(x - a\frac{K_1+1}{K_1-1} \right)^2 + y^2 = \left(\frac{2a\sqrt{K_1}}{K_1-1} \right)^2$$

- The $V = V_1$ surface is independent of $z \rightarrow$ it is a cylinder
- It intersects the xy plane in a circle of radius:

$$b = \frac{2a\sqrt{K_1}}{K_1-1}$$

& this circle is centered at ($x = h$, $y = 0$) where $h = a\dfrac{K_1+1}{K_1-1}$

Capacitance (14)



The V_1 surface intersects the xy plane in a circle of radius

$$\left.\begin{array}{l} b = \dfrac{2a\sqrt{K_1}}{K_1 - 1} \\ \text{\& centered at } (x = h, y = 0) \text{ where } h = a\dfrac{K_1 + 1}{K_1 - 1} \end{array}\right\}$$

$$\to \left.\begin{array}{l} \left\{\begin{array}{l} a = \sqrt{h^2 - b^2} \\ \sqrt{K_1} = \dfrac{h + \sqrt{h^2 - b^2}}{b} \end{array}\right. \\ K_1 = e^{4\pi\varepsilon V_1/\rho_L} \end{array}\right\} \to \rho_L = \frac{4\pi\varepsilon V_1}{\ln K_1}$$

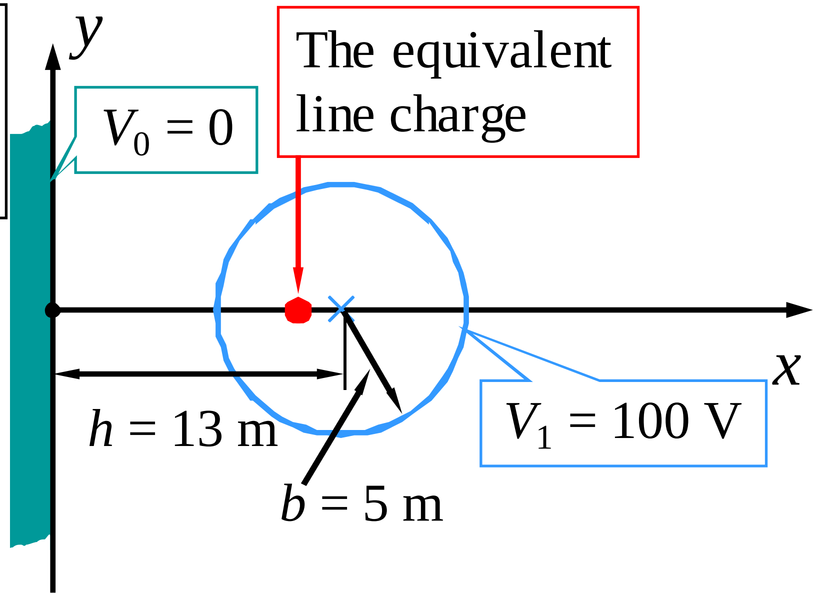
If h, b & V_1 are given then a, ρ_L & K_1 can be found

$$\to C_{plane,\ cylinder} = \frac{\rho_L L}{V_1} = \frac{4\pi\varepsilon L}{\ln K_1} = \frac{2\pi\varepsilon L}{\ln[(h + \sqrt{h^2 - b^2})/b]} = \boxed{\frac{2\pi\varepsilon L}{\cosh^{-1}(h/b)}}$$

Capacitance (15)

Ex.

Given the system, find the location & the magnitude of the equivalent line charge, & the location of the 50V equipotential surface.



$$a = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12 \text{ m}$$

$$\sqrt{K_1} = \frac{h + \sqrt{h^2 - b^2}}{b} = \frac{13 + 12}{5} = 5$$

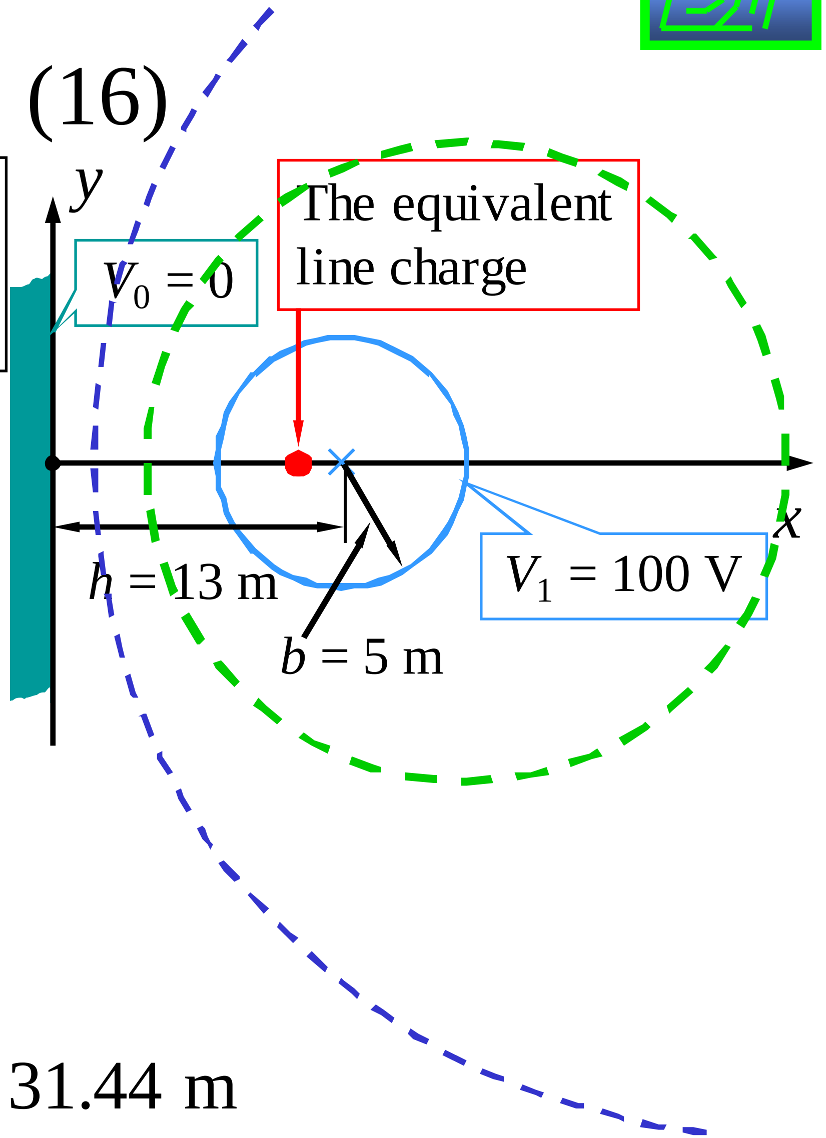
$$\left.\begin{aligned} &\to K_1 = 25 \\ &\rho_L = \frac{4\pi\varepsilon V_1}{\ln K_1} \end{aligned}\right\} \to \rho_L = \frac{4\pi \times 8.854 \times 10^{-12} \times 100}{\ln 25} = 3.46 \text{ nC/m}$$

$$C_{plane,\,cylinder} = \frac{2\pi\varepsilon}{\cosh^{-1}(h/b)} = \frac{2\pi \times 8.854 \times 10^{-12}}{\cosh^{-1}(13/5)} = 34.6 \text{ pF/m}$$

Capacitance (16)

Ex.

Given the system, find the location & the magnitude of the equivalent line charge, & the location of the 50V equipotential surface.



$$K_2 = e^{4\pi\varepsilon V_2/\rho_L}$$

$$= e^{4\pi\times 8.854\times 10^{-12}\times 50/3.46\times 10^{-9}} = 5.00$$

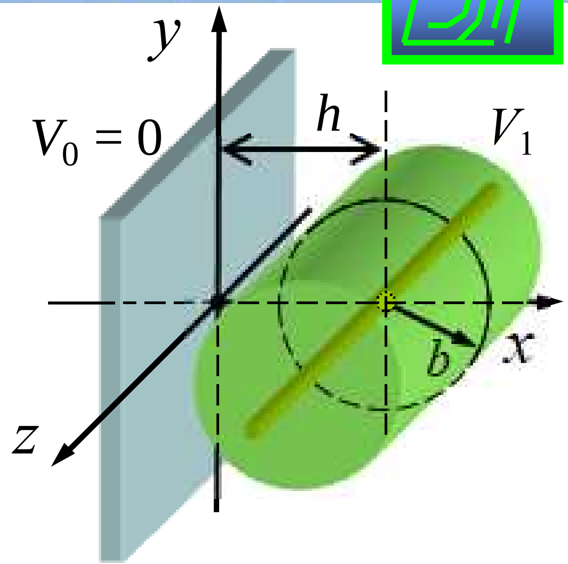
$$\to b_2 = \frac{2a\sqrt{K_2}}{K_2 - 1} = \frac{2\times 12\sqrt{5}}{5-1} = 13.42 \text{ m}$$

$$h_2 = a\frac{K_2+1}{K_2-1} = 12\frac{5+1}{5-1} = 18 \text{ m}$$

$$V_3 = 25 \text{ V} \to b_3 = 29.06 \text{ m},\ h_3 = 31.44 \text{ m}$$

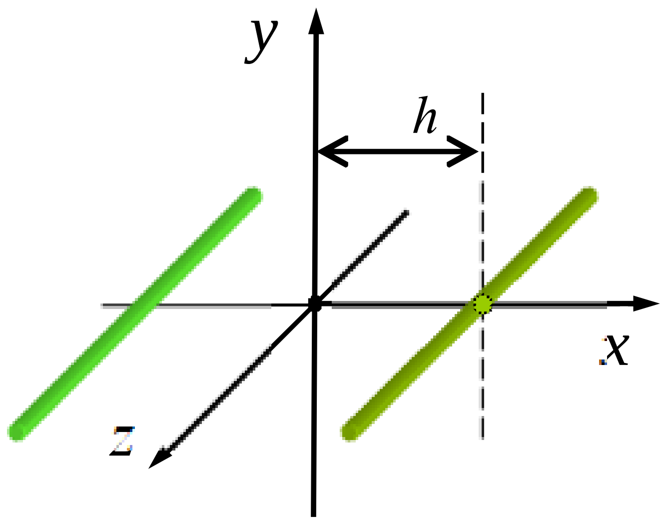
Capacitance (17)

$$\left.\begin{array}{r} C_{plane,\,cylinder} = \dfrac{2\pi\varepsilon L}{\ln[(h+\sqrt{h^2-b^2})/b]} \\ b \ll h \end{array}\right\}$$



$$\rightarrow C_{plane,\,cylinder} = C_{plane,\,wire} = \frac{2\pi\varepsilon L}{\ln\dfrac{2h}{b}}$$

$$\rightarrow \boxed{C_{wire,\,wire} = \frac{\pi\varepsilon L}{\ln\dfrac{2h}{b}}}$$



Dielectrics & Capacitance

1. Dielectric Materials
2. Boundary Conditions for Perfect Dielectric Materials
3. Capacitance
4. **Using Field Sketches to Estimate Capacitance**
5. Current Density & Flux Density

Using Field Sketches to Estimate Capacitance (1)

- A conductor boundary is an equipotential surface
- The electric field intensity **E** & the electric flux **D** are both perpendicular to the equipotential surfaces
- **E** & **D** are perpendicular to the conductor boundaries & posses zero tangential values
- The lines of electric flux, or streamlines, begin & terminate on charge & therefore, in a charge-free, homogeneous dielectric, begin & terminate only on the conductor boundaries

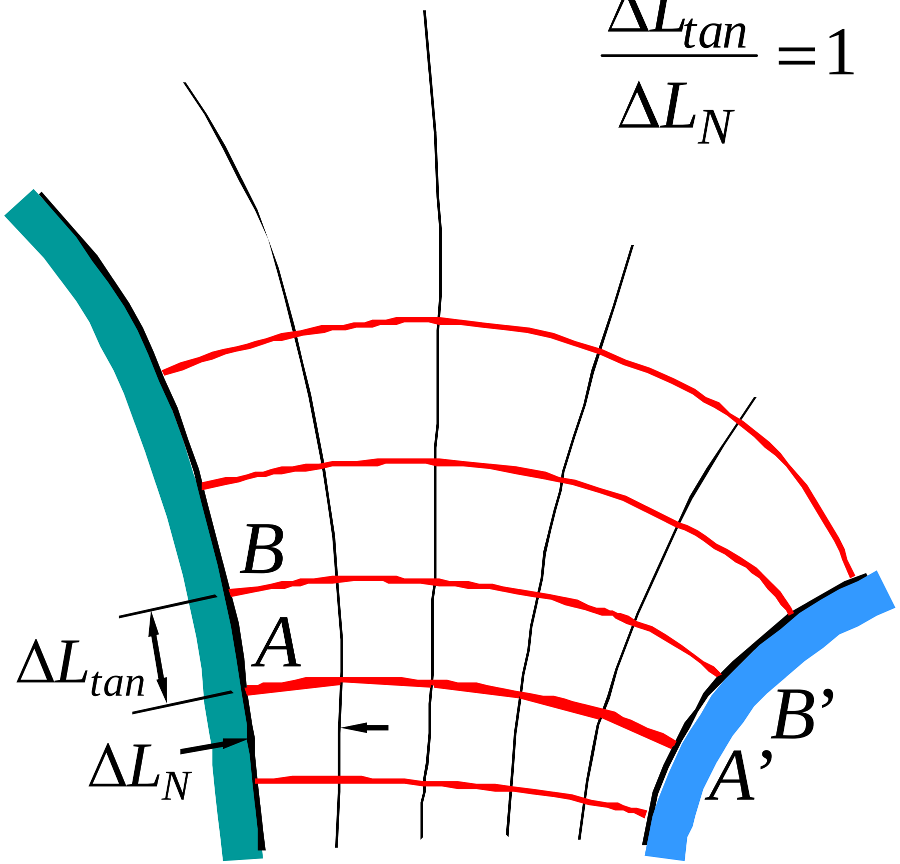
Using Field Sketches to Estimate Capacitance (2)

E & **D** are both perpendicular to the equipotential surfaces

$$\left.\begin{aligned} E &= \frac{1}{\varepsilon}\frac{\Delta\psi}{\Delta L_{tan}} \\ E &= \frac{\Delta V}{\Delta L_N} \end{aligned}\right\} \rightarrow \frac{1}{\varepsilon}\frac{\Delta\psi}{\Delta L_{tan}} = \frac{\Delta V}{\Delta L_N}$$

$$\rightarrow \boxed{\frac{\Delta L_{tan}}{\Delta L_N} = \text{const} = \frac{1}{\varepsilon}\frac{\Delta\psi}{\Delta V}}$$

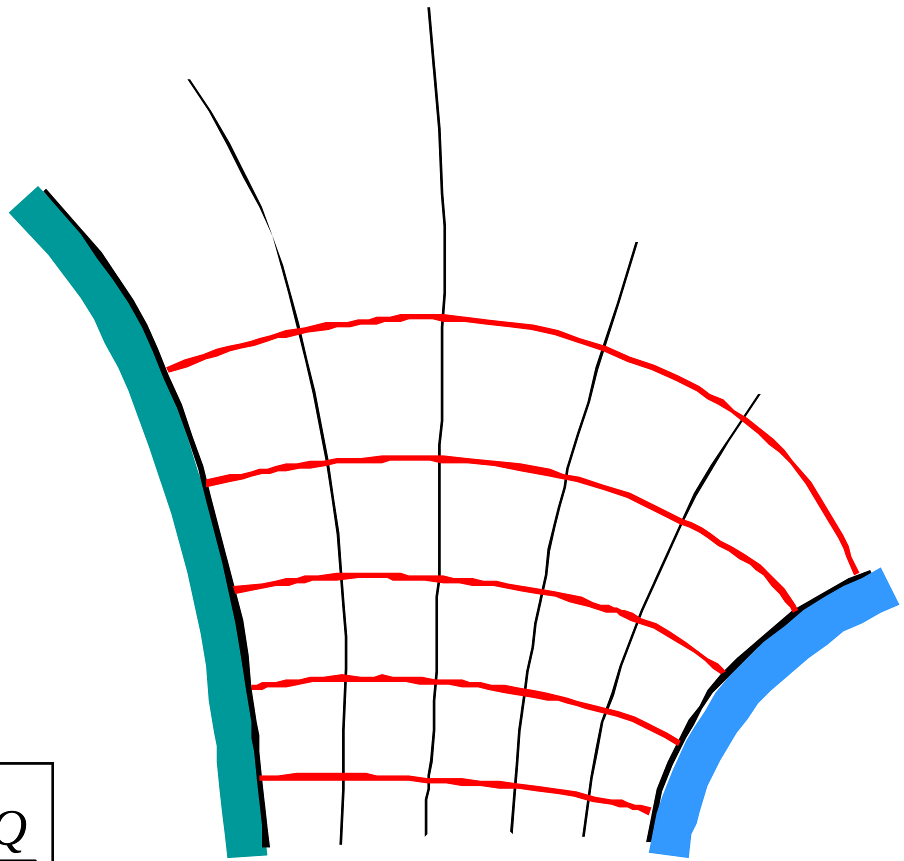
$$\frac{\Delta L_{tan}}{\Delta L_N} = 1$$



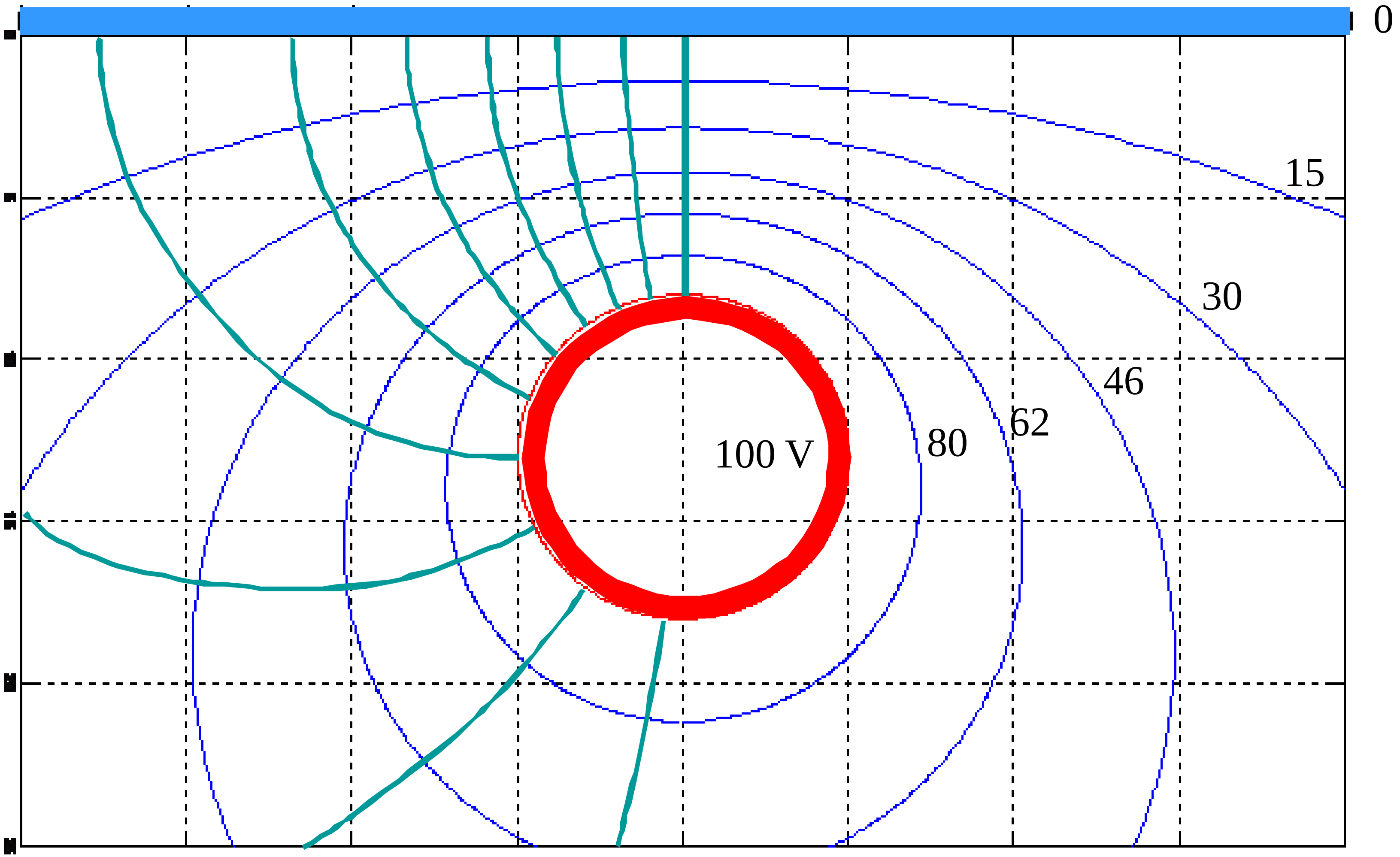
Using Field Sketches to Estimate Capacitance (3)

$$\left.\begin{aligned} C &= \frac{Q}{V_0} \\ Q &= N_Q \Delta Q = N_Q \Delta \psi \\ V_0 &= N_V \Delta V \end{aligned}\right\} \rightarrow C = \frac{N_Q \Delta \psi}{N_V \Delta V}$$

$$\left.\begin{aligned} &\rightarrow C = \frac{N_Q \Delta \psi}{N_V \Delta V} \\ &\frac{\Delta L_{tan}}{\Delta L_N} = \text{const} = \frac{1}{\varepsilon}\frac{\Delta \psi}{\Delta V} = 1 \end{aligned}\right\} \rightarrow C = \frac{N_Q}{N_V}\varepsilon\frac{\Delta L_{tan}}{\Delta L_N} = \boxed{\varepsilon\frac{N_Q}{N_V}}$$



Using Field Sketches to Estimate Capacitance (4)



Dielectrics & Capacitance

1. Dielectric Materials
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5. **Current Density & Flux Density**

Current Density & Flux Density

$\mathbf{J} = \sigma \mathbf{E}_\sigma$	$\mathbf{D} = \varepsilon \mathbf{E}_\varepsilon$
$E_\sigma = -\nabla V_\sigma$	$E_\varepsilon = -\nabla V_\varepsilon$

$$\left.\begin{aligned}
I &= \oint_S \mathbf{J}.d\mathbf{S} = \sigma \oint_S \mathbf{E}_\sigma.d\mathbf{S} \\
V_{\sigma 0} &= -\int \mathbf{E}_\sigma.d\mathbf{L} \\
Q &= \varepsilon \oint_S \mathbf{E}_\varepsilon.d\mathbf{S} \\
V_{\varepsilon 0} &= -\int \mathbf{E}_\varepsilon.d\mathbf{L}
\end{aligned}\right\} \rightarrow \left\{\begin{aligned}
R &= \frac{V_{\sigma 0}}{I} = \frac{-\int \mathbf{E}_\sigma.d\mathbf{L}}{\sigma \oint_S \mathbf{E}_\sigma.d\mathbf{S}} \\
C &= \frac{Q}{V_{\varepsilon 0}} = \frac{\varepsilon \oint_S \mathbf{E}_\varepsilon.d\mathbf{S}}{-\int \mathbf{E}_\varepsilon.d\mathbf{L}}
\end{aligned}\right.$$

$$\rightarrow \boxed{RC = \frac{\varepsilon}{\sigma}}$$