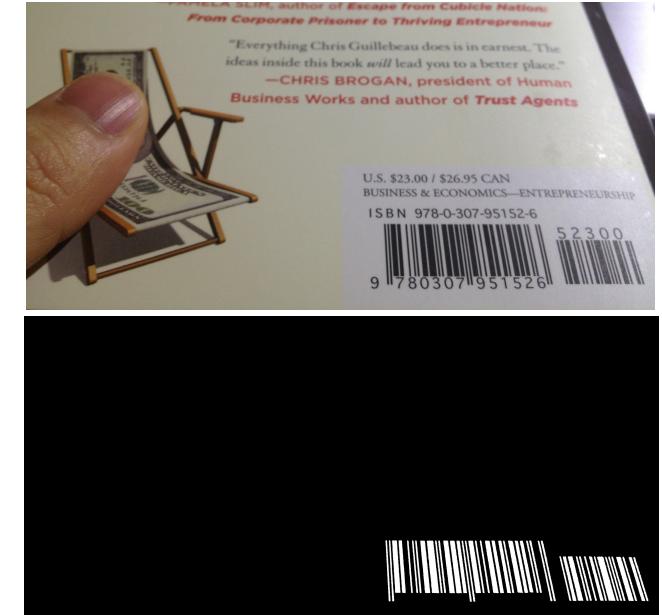
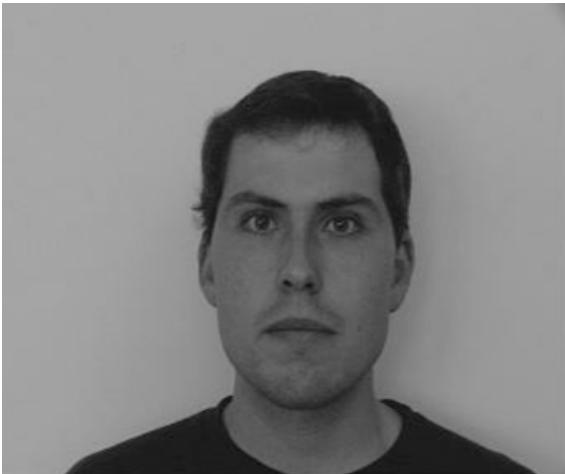


# Image Segmentation

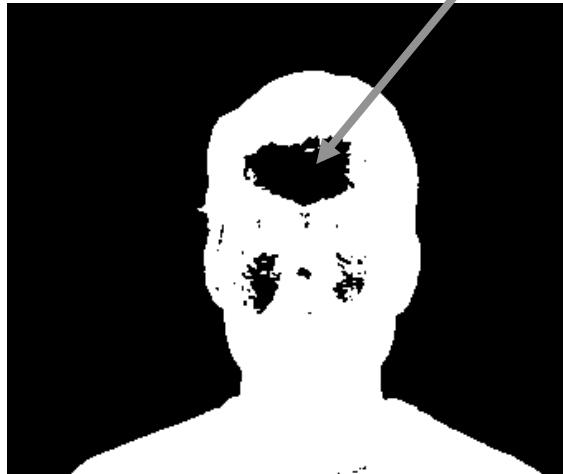
- Gray-level thresholding
- Supervised vs. unsupervised thresholding
- Binarization using Otsu's method
- Locally adaptive thresholding
- Maximally stable extremal regions
- Color-based segmentation
- Region labeling and counting
- Region moments



# Gray-level thresholding



Original image  
*Peter f [x,y]*



Thresholded  
*Peter m [x,y]*

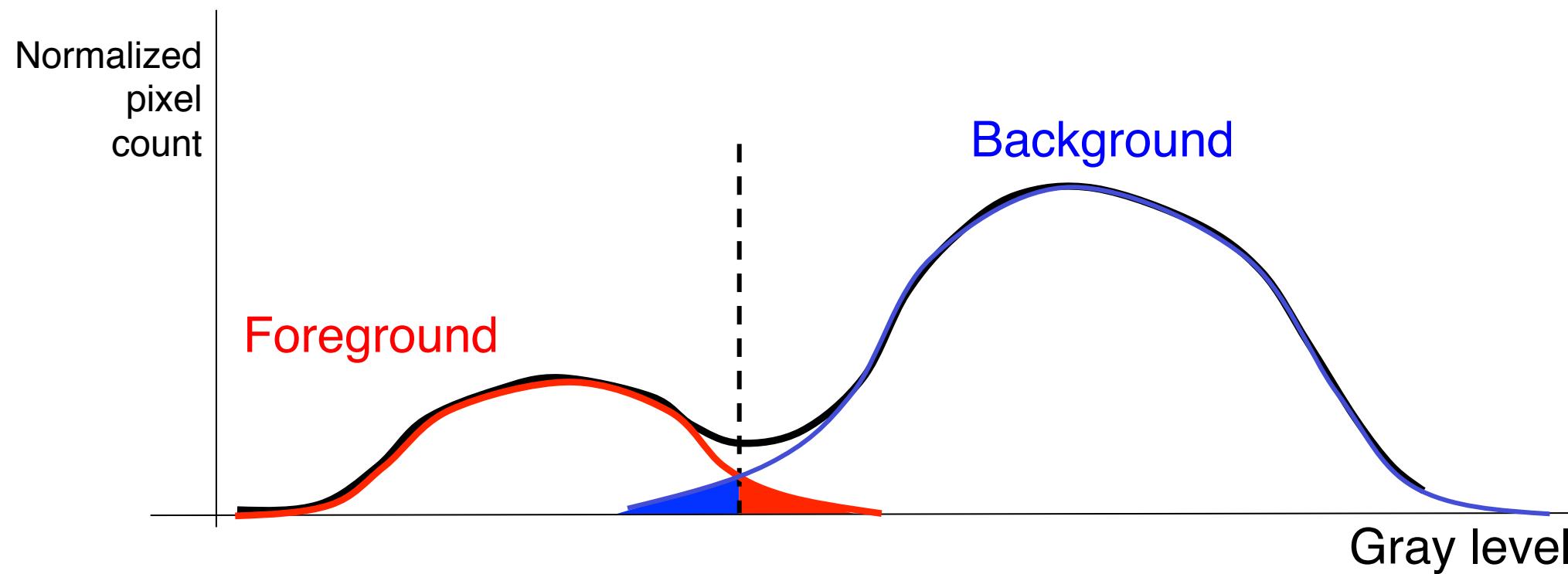


$$f[x,y] \cdot m[x,y]$$

How can holes be filled?



# How to choose the threshold?



# Unsupervised thresholding

- Idea: find threshold  $T$  that minimizes *within-class variance* of both foreground and background (same as k-means)

$$\sigma_{\text{within}}^2(T) = \frac{N_{\text{Fgrnd}}(T)}{N} \sigma_{\text{Fgrnd}}^2(T) + \frac{N_{\text{Bgrnd}}(T)}{N} \sigma_{\text{Bgrnd}}^2(T)$$

- Equivalently, maximize *between-class variance*

$$\begin{aligned}\sigma_{\text{between}}^2(T) &= \sigma^2 - \sigma_{\text{within}}^2(T) \\ &= \left( \frac{1}{N} \sum_{x,y} f^2[x,y] - \mu^2 \right) - \frac{N_{\text{Fgrd}}}{N} \left( \frac{1}{N_{\text{Fgrd}}} \sum_{x,y \in \text{Fgrnd}} f^2[x,y] - \mu_{\text{Fgrnd}}^2 \right) - \frac{N_{\text{Bgrnd}}}{N} \left( \frac{1}{N_{\text{Bgrnd}}} \sum_{x,y \in \text{Bgrnd}} f^2[x,y] - \mu_{\text{Bgrnd}}^2 \right) \\ &= -\mu^2 + \frac{N_{\text{Fgrnd}}}{N} \mu_{\text{Fgrnd}}^2 + \frac{N_{\text{Bgrnd}}}{N} \mu_{\text{Bgrnd}}^2 = \frac{N_{\text{Fgrnd}}}{N} (\mu_{\text{Fgrnd}} - \mu)^2 + \frac{N_{\text{Bgrnd}}}{N} (\mu_{\text{Bgrnd}} - \mu)^2 \\ &= \frac{N_{\text{Fgrnd}}(T) \cdot N_{\text{Bgrnd}}(T)}{N^2} (\mu_{\text{Fgrnd}}(T) - \mu_{\text{Bgrnd}}(T))^2\end{aligned}$$

[Otsu, 1979]

# Unsupervised thresholding (cont.)

- Algorithm: Search for threshold  $T$  to maximize

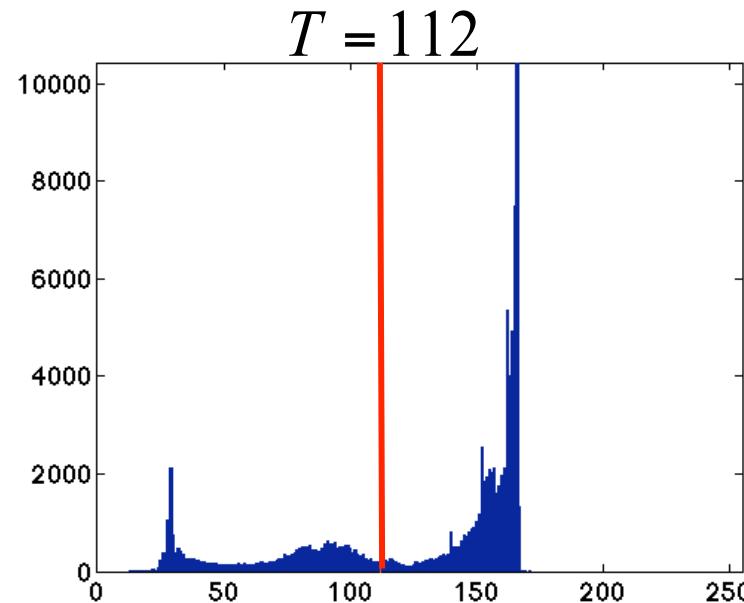
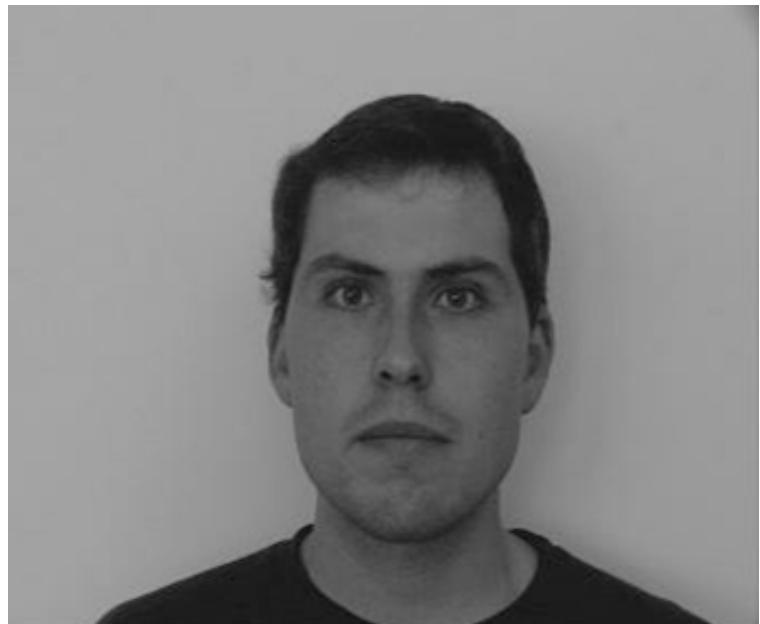
$$\sigma_{between}^2(T) = \frac{N_{Fgrnd}(T) \cdot N_{Bgrnd}(T)}{N^2} (\mu_{Fgrnd}(T) - \mu_{Bgrnd}(T))^2$$

- Useful recursion for sweeping  $T$  across histogram:

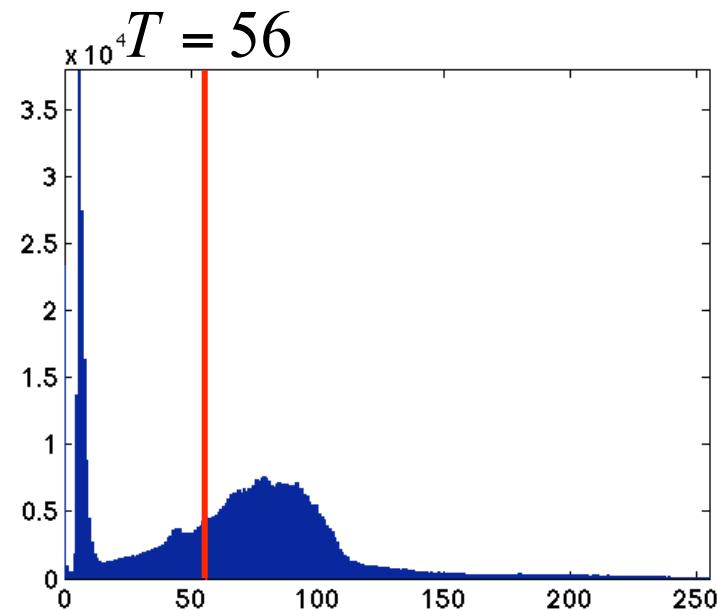
$$\begin{aligned} N_{Fgrnd}(T+1) &= N_{Fgrnd}(T) + n_T \\ N_{Bgrnd}(T+1) &= N_{Bgrnd}(T) - n_T \\ \mu_{Fgrnd}(T+1) &= \frac{\mu_{Fgrnd}(T)N_{Fgrnd}(T) + n_T T}{N_{Fgrnd}(T+1)} \\ \mu_{Bgrnd}(T+1) &= \frac{\mu_{Bgrnd}(T)N_{Bgrnd}(T) - n_T T}{N_{Fgrnd}(T+1)} \end{aligned}$$

[Otsu, 1979]

# Unsupervised thresholding (cont.)



# Unsupervised thresholding (cont.)



# Unsupervised thresholding (cont.)

The Stanford Daily Tuesday, September 18, 2012 ♦ 13

**FOOTBALL**

## The winding road ahead

**By SAM FISHER**  
FOOTBALL EDITOR



SIMON WARBY/The Stanford Daily

Andrew Luck may be gone, but with Saturday's win over USC, the Stanford Cardinal put itself in position to achieve beyond the path paved by number 12. You heard right, though: the team that Luck left to do this, 2012 Stanford team showed that it's capable of playing at a national championship level.

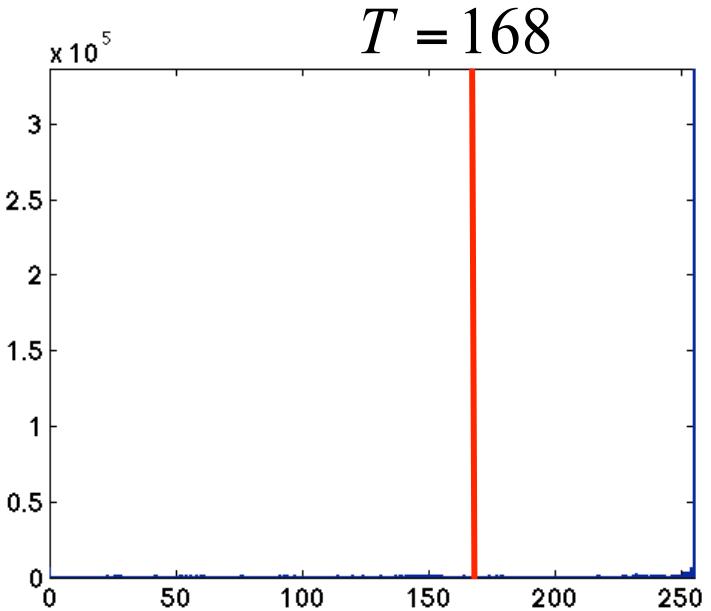
Though Stanford survived one of its toughest tests in the gauntlet that is the BCS National Championship elimination, the road to 2013 is not walk in the park. The toughest challenges remain on the road ahead: games at Notre Dame, Oregon and UCLA, all of whom are currently ranked in the top 20. The next two games at Wisconsin and then home against Arizona, are no pushovers either. And as Stanford has faced top-ranked opponents in years past, any team on the Cardinal's schedule has the potential for a major upset.

From Stanford's current vantage point, there are three paths the rest of the season could take. Door Number One leads to The Pac-12 title, a berth in the BCS National Championship Game, and the big plays up the middle from the defensive line and the running backs. The rest of the D-line played great in support, but Mauro went above and beyond the call of duty to help end the Trojans at the end. His 213 total yards of offense, including a pair of TDs had fans on both sides forgetting about Andrew Luck.

**Josh Mauro:** The back-up defensive end saw most of his action at nose tackle in the second half, where he completely took over the ball game, dominating USC's backup center, Cyrus Hobbs, all half to provide the key pressure up the middle from the defensive line. The rest of the D-line played great in support, but Mauro went above and beyond the call of duty to help end the Trojans at the end. His 213 total yards of offense, including a pair of TDs had fans on both sides forgetting about Andrew Luck.

**Stephen Taylor:** It all starts and ends with Stanford's workhorse. Taylor was everything you could ask for and more against USC. He provided the big plays with a game-tying touchdown on the ground and another three passes in the air and the consistent ground-and-pound to wear down the Trojans at the end. His 213 total yards of offense, including a pair of TDs had fans on both sides forgetting about USC for the Pac-12 title.

Please see AWARDS, page 15

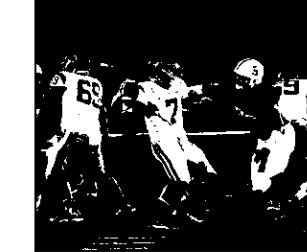


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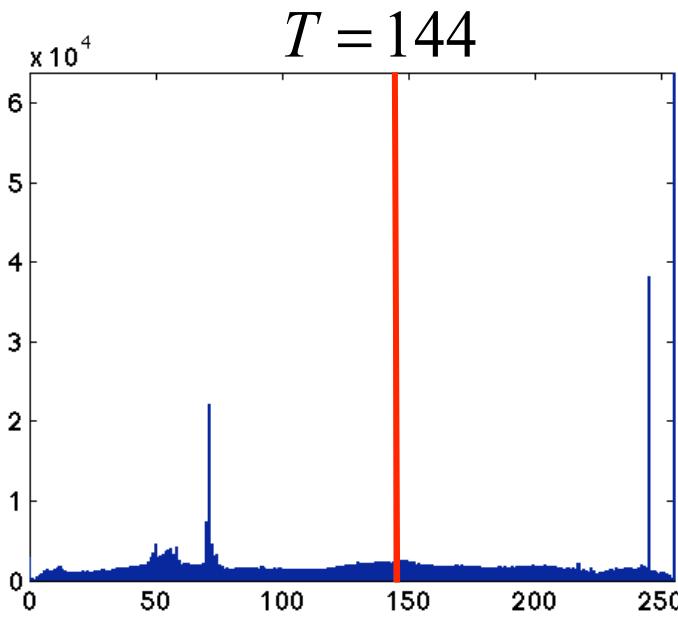
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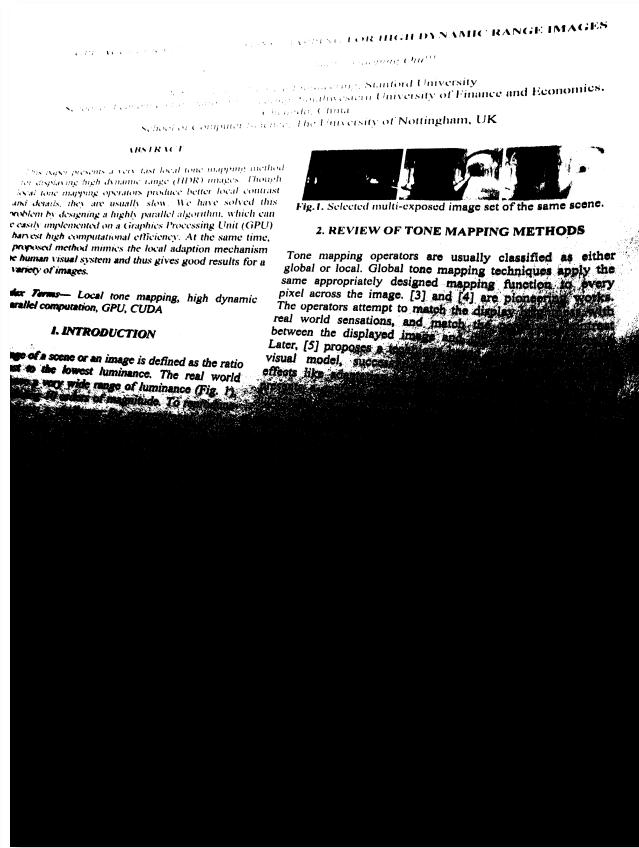
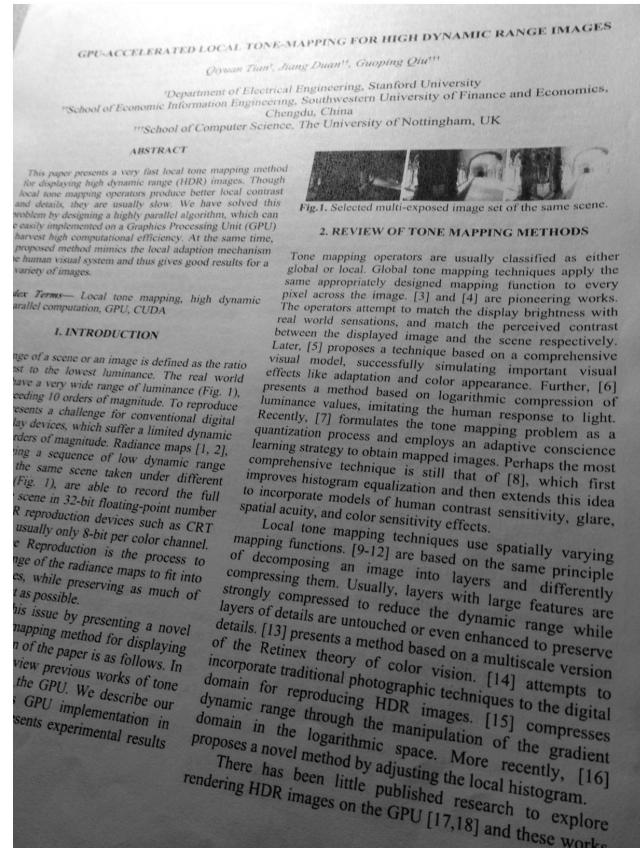


# Unsupervised thresholding (cont.)



# Sometimes, a global threshold does not work

Original image

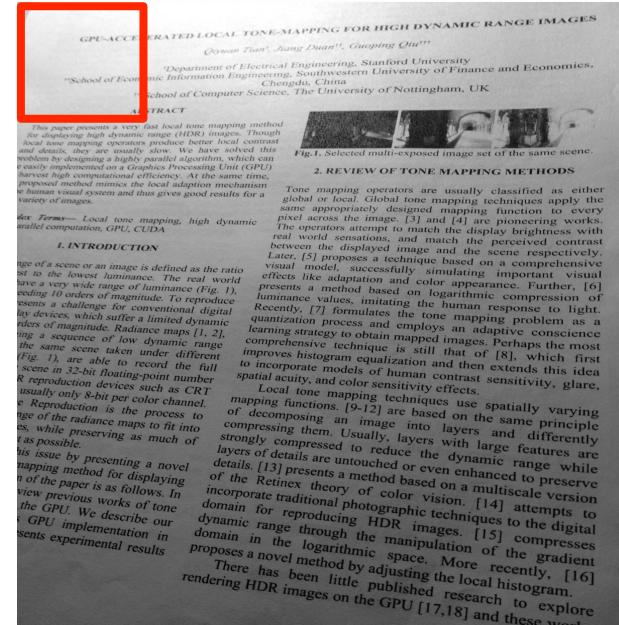


## Thresholded with Otsu's Method



# Locally adaptive thresholding

- Slide a window over the image
- For each window position, decide whether to perform thresholding
  - Thresholding should not be performed in uniform areas
  - Use variance or other suitable criterion
- Non-uniform areas: apply Otsu's method (based on local histogram)
- Uniform areas: classify the entire area as foreground or background based on mean value



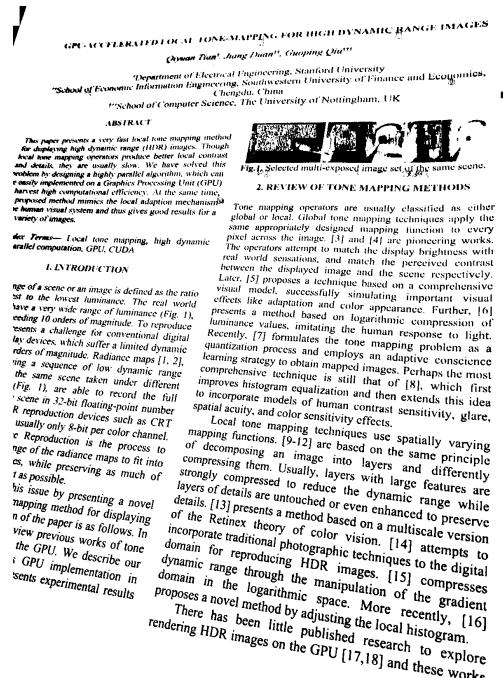
# Locally adaptive thresholding (example)



Non-uniform areas



Local threshold values



Locally thresholded result

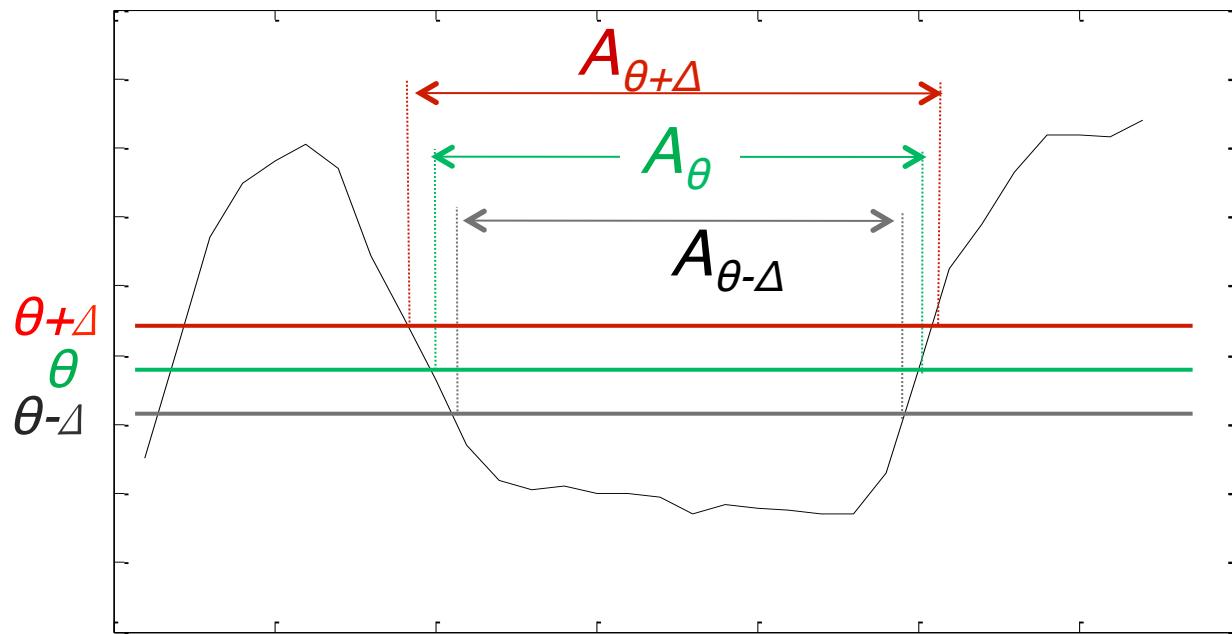


# Maximally stable extremal regions

- Extremal region: any connected region in an image with all pixel values above (or below) a threshold
  - Observations:
    - Nested extremal regions result when the threshold is successively raised (or lowered).
    - The nested extremal regions form a “component tree.”
  - Key idea: choose thresholds  $\theta$  such that the resulting bright (or dark) extremal regions are nearly constant when these thresholds are perturbed by  $+/-\Delta$
- “***maximally stable***” ***extremal regions (MSER)***

[Matas, Chum, Urba, Pajdla, 2002]

# MSERs: illustration



$$\text{Local minimum of } \left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right| \rightarrow \text{MSER}$$

[Matas, Chum, Urba, Pajdla, 2002]

# Level sets of an image

1	1	1	1	1	1	1	1	1	1	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	5	5	5	5	1	5	4	4	8
1	6	6	4	2	2	5	5	5	6	1	5	4	4	4
1	6	6	4	2	2	6	6	6	6	1	5	5	5	5
1	4	4	4	2	2	6	6	6	6	1	5	5	5	5
1	1	1	1	1	2	6	1	1	1	1	2	2	2	2
1	8	8	5	1	2	6	1	7	7	1	2	2	2	2
1	8	8	5	1	1	1	1	7	7	1	1	1	1	2
1	8	8	5	5	5	3	3	7	7	1	1	1	1	2
1	8	8	5	5	3	3	3	7	7	7	1	1	1	1
1	8	8	5	5	3	3	3	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$$f[x, y]$$

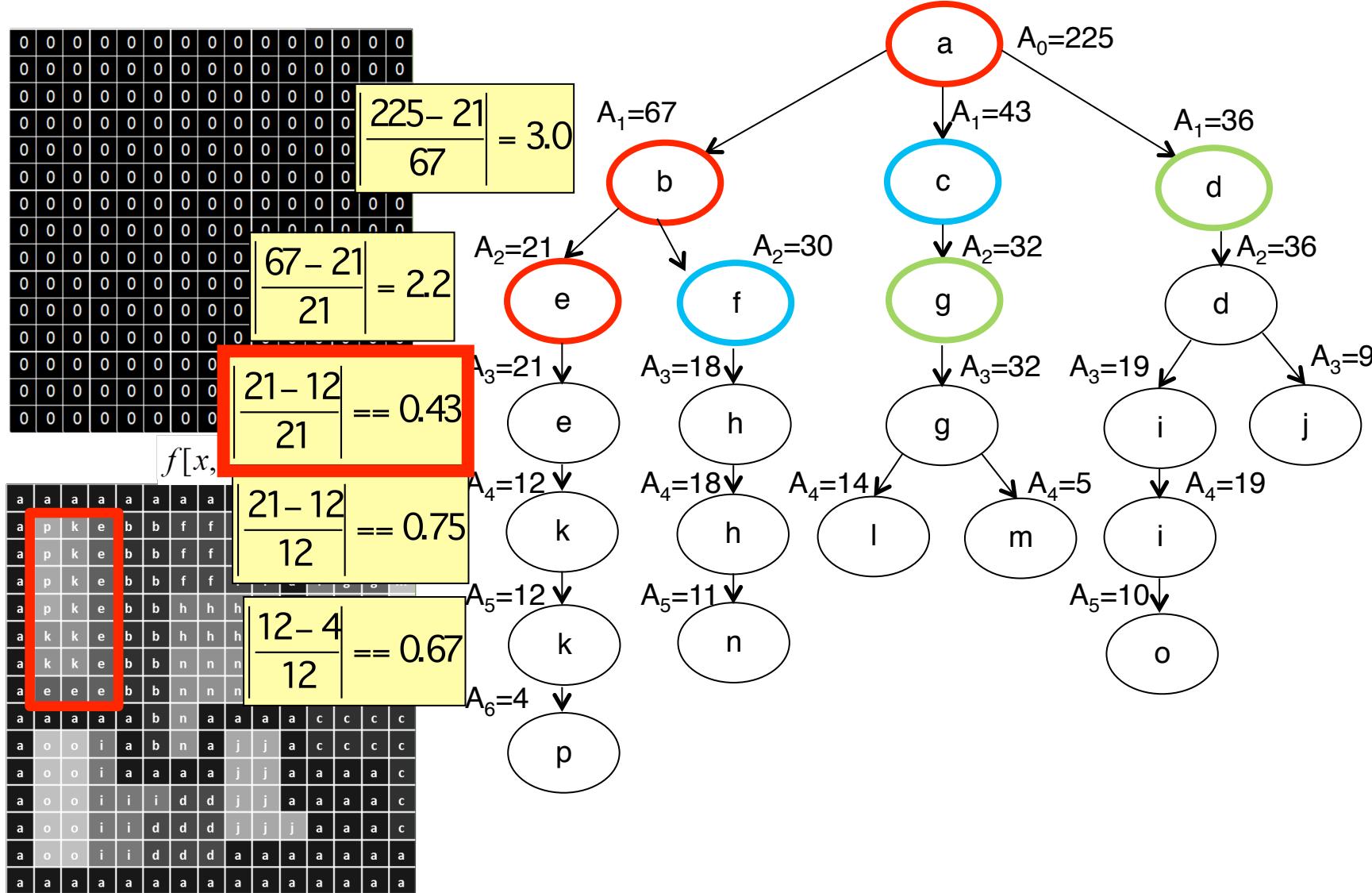
Image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$f[x, y] > 8$$

Level Set

# Component tree of an image



Local minima of sequence

$$\left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right|$$

$\theta = \Delta, \Delta + 1, \dots \rightarrow \text{MSERS}$

# MSER: examples



Dark MSERs,  $\Delta=15$



Original image



Bright MSERs,  $\Delta=15$

# MSER: examples



Dark MSERs,  $\Delta=15$

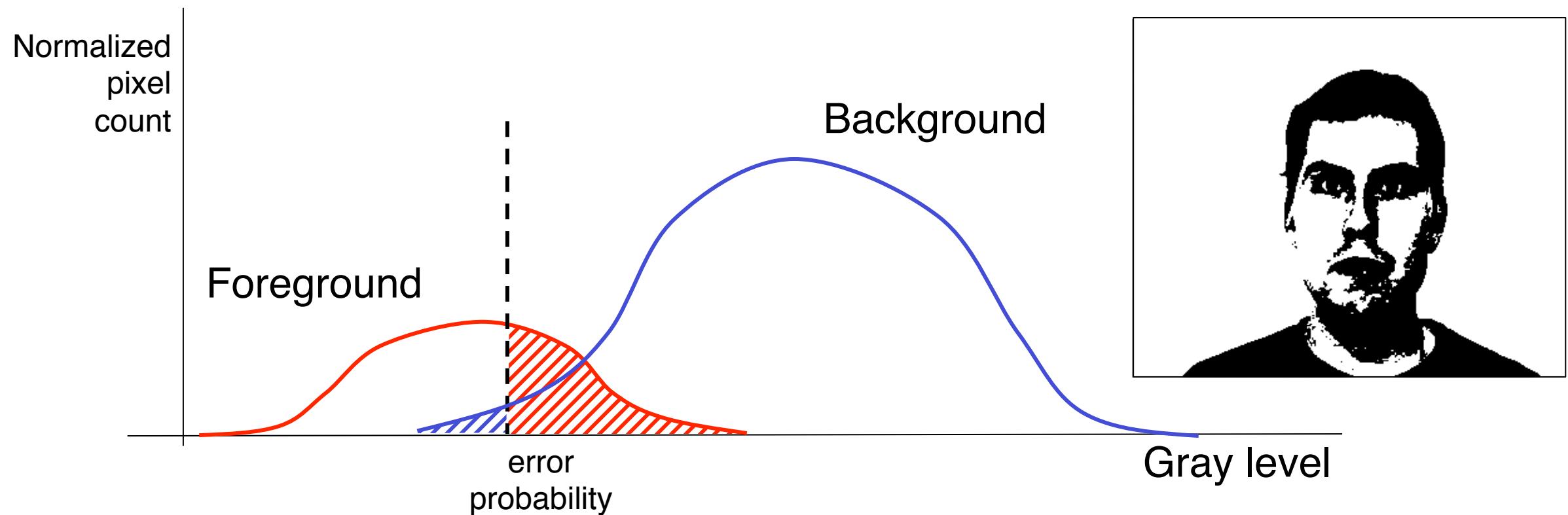


Original image

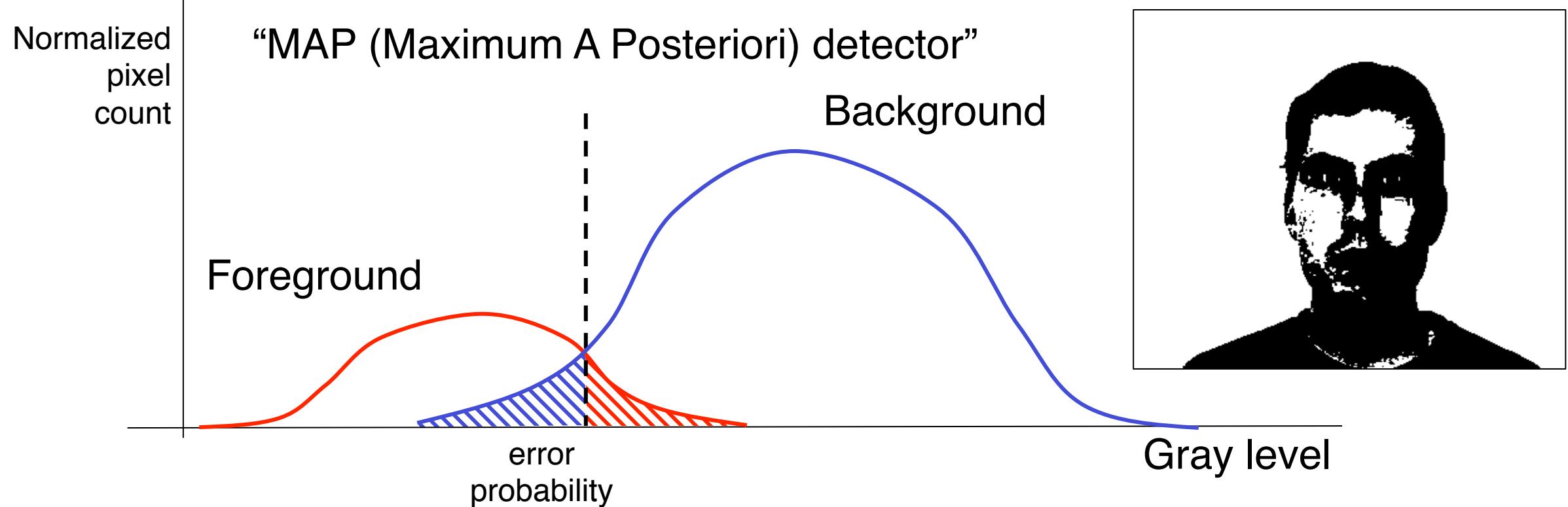


Bright MSERs,  $\Delta=15$

# Supervised thresholding



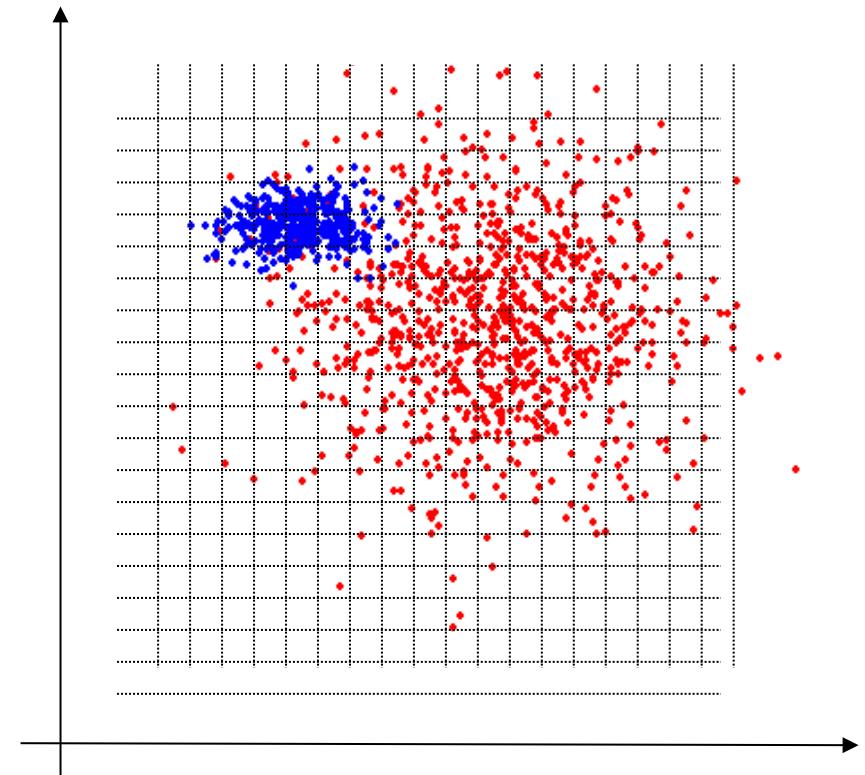
# Supervised thresholding



If errors  $\text{BG} \rightarrow \text{FG}$  and  $\text{FG} \rightarrow \text{BG}$  are associated with different costs:  
“Bayes minimum risk detector” is optimal.

# Multidimensional MAP detector

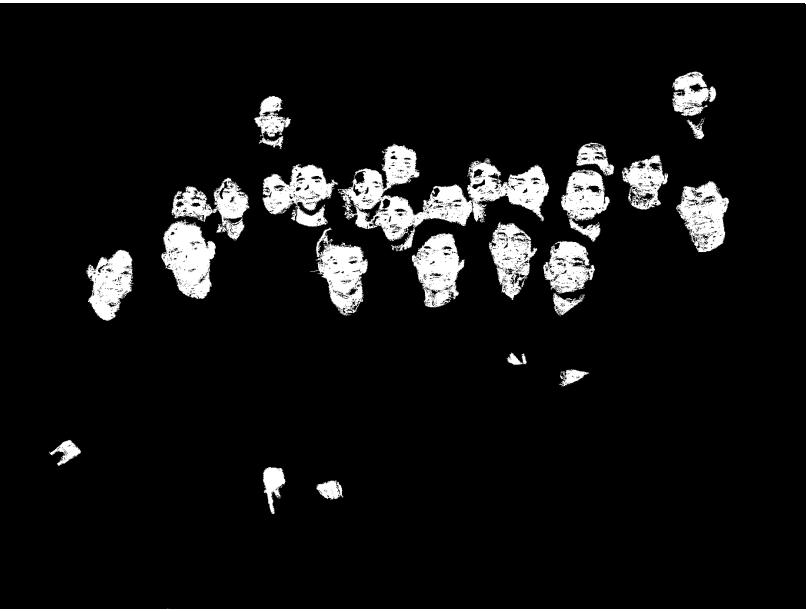
- Training
  - Provide labelled set of training data
  - Subdivide n-dimensional space into small bins
  - Count frequency of occurrence for each bin and class in training set, label bin with most probable class
  - (Propagate class labels to empty bins)
- For test data: identify bin, look up the most probable class



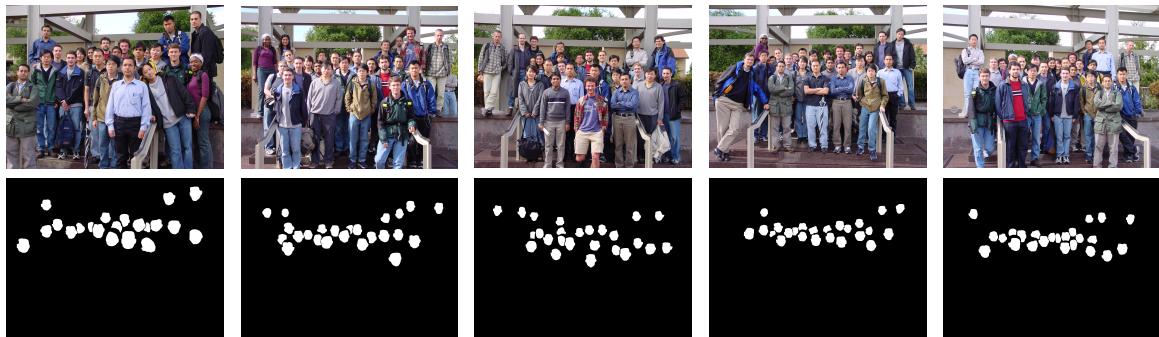
# MAP detector in RGB-space



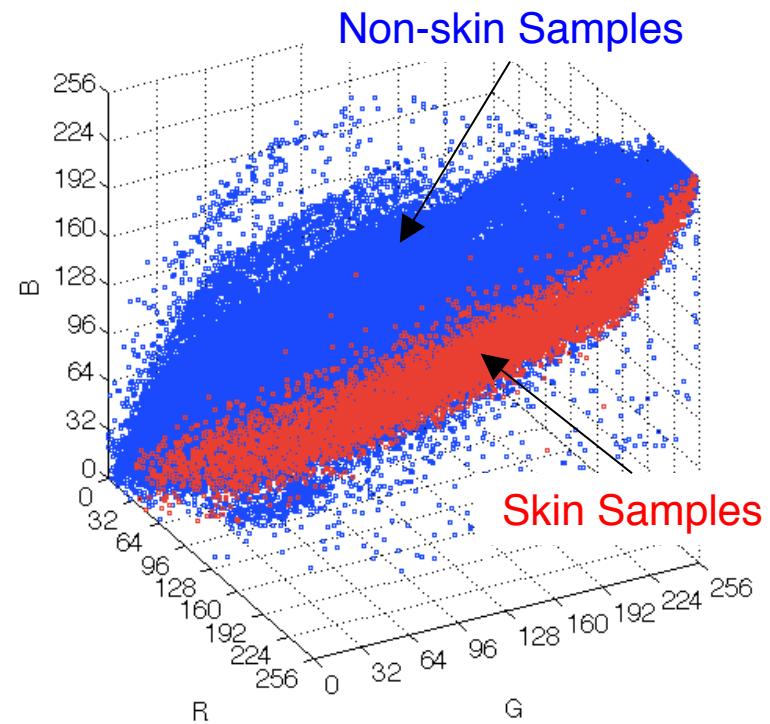
Original image



Skin color detector



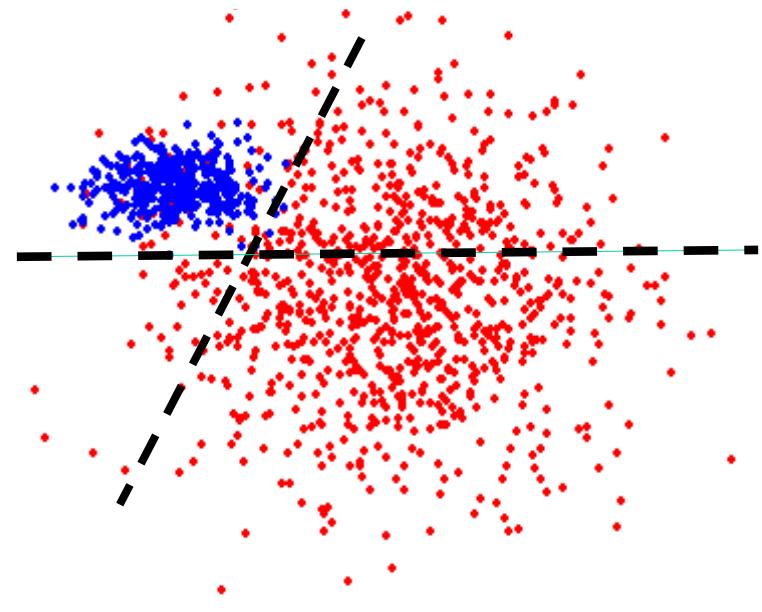
Five training images



# Linear discriminant function

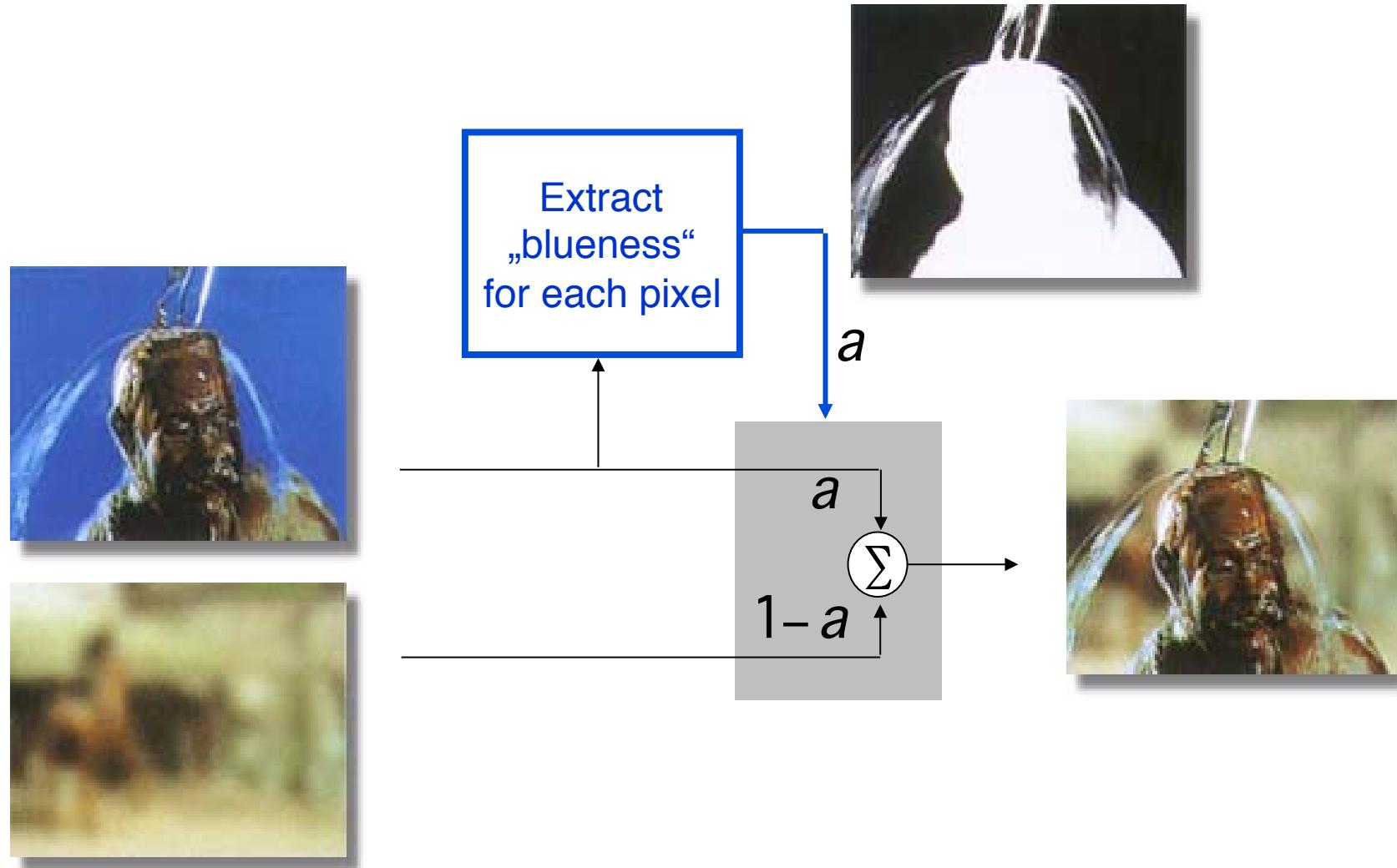
- To segment image with  $n$  components  $f_i, i=1,2,\dots,n$  into two classes, perform test

$$\sum_i w_i f_i + w_0 \geq 0 ?$$

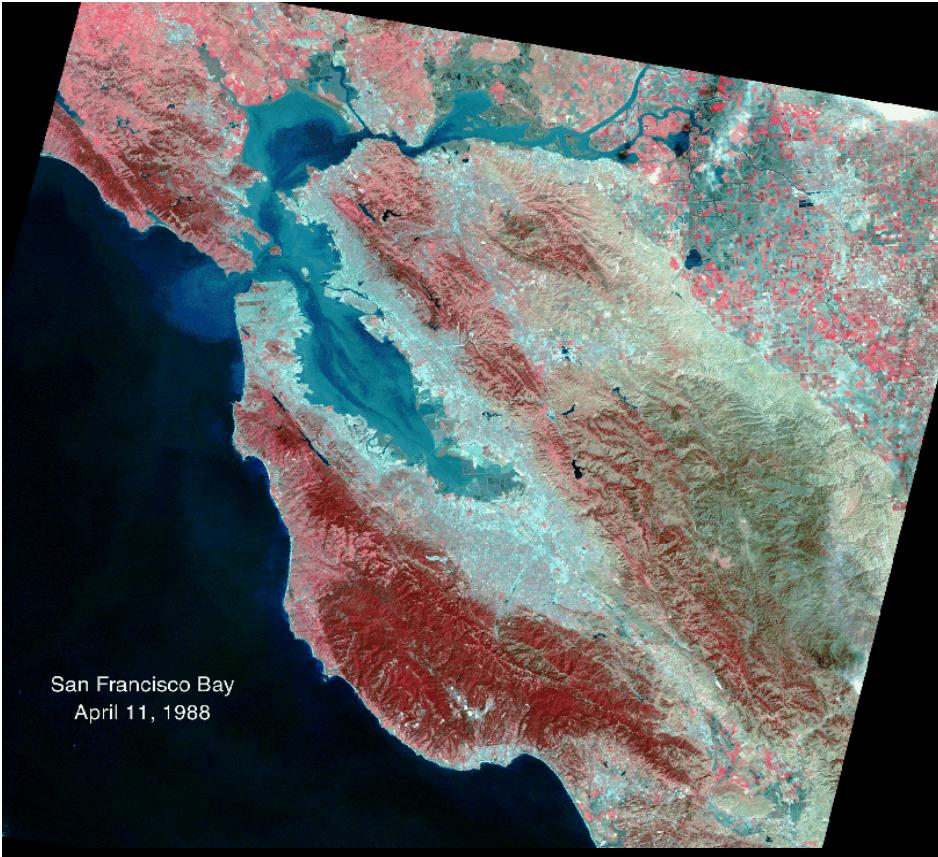


- Categories are separated by hyperplane in  $n$ -space
- Numerous techniques to determine weights  
 $w_i, i=0,1,2,\dots,n$ , see, e.g., [\[Duda, Hart, Stork, 2001\]](#)
- Can be extended to the intersection of several linear discriminant functions
- Can be extended to multiple classes

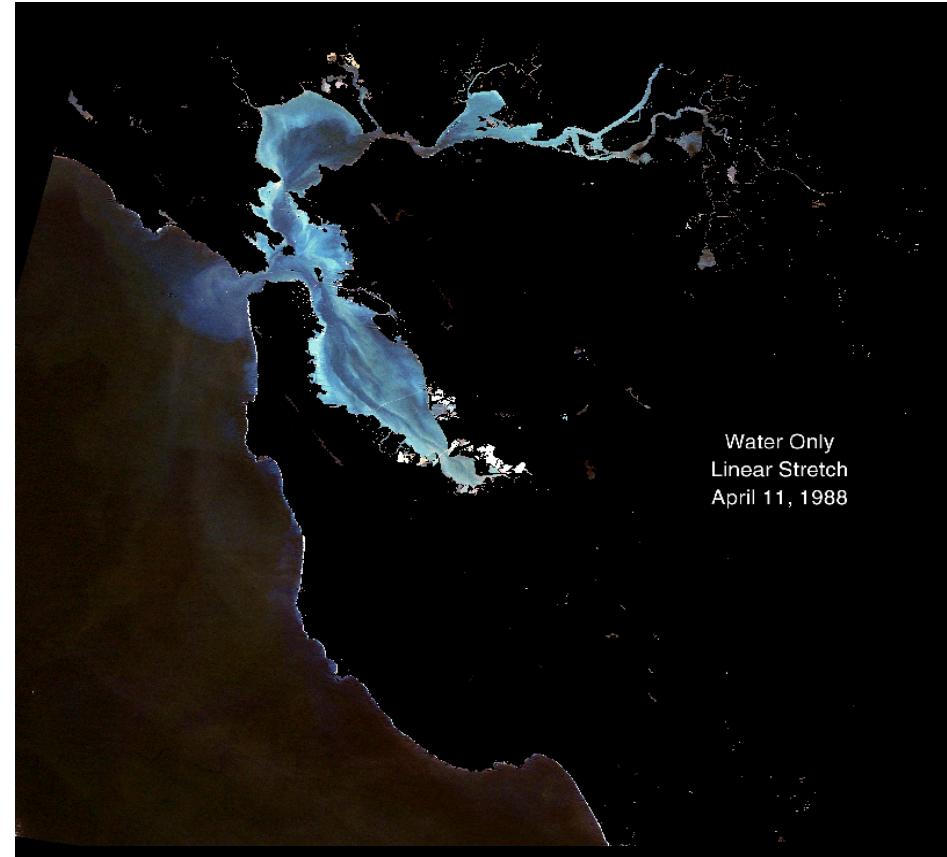
# Chroma keying



# Landsat image processing



Original Landsat image false color picture out  
of bands 4,5,6

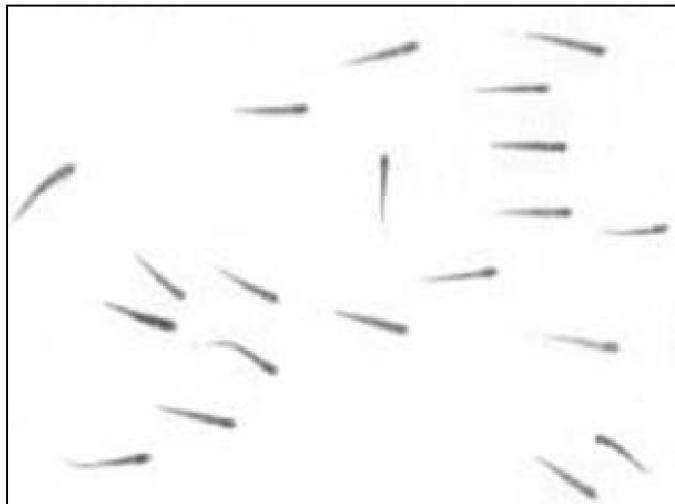


Water area segmented and enhanced to show  
sediments

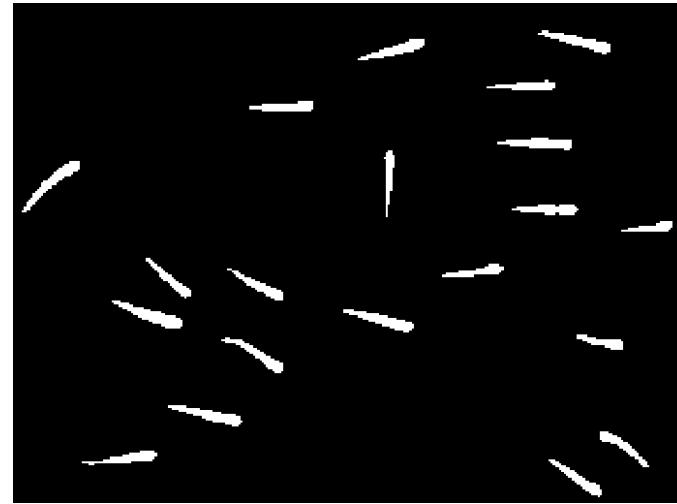
Source: US Geological Survey USGS, <http://sfbay.wr.usgs.gov/>

# Region labeling and counting

- How many fish in this picture?



Original *Fish* image



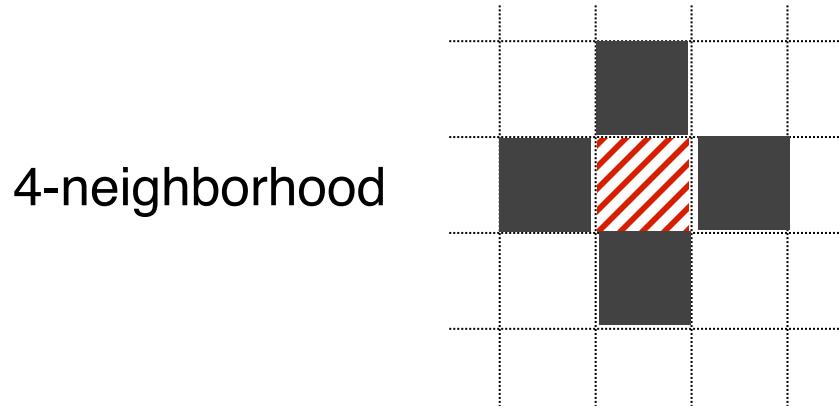
after thresholding

- Which pixels belong to the same object (region labeling)?
- How large is each object (region counting)?

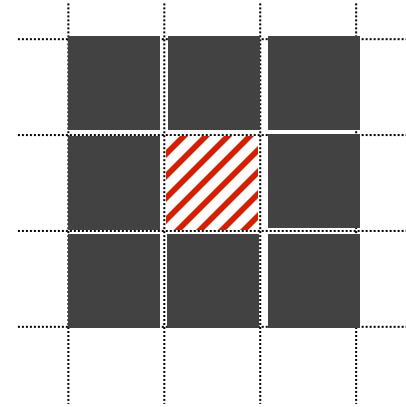


# 4-connected and 8-connected neighborhoods

- Definition: a ***region*** is a set of pixels, where each pixel can be reached from any other pixel in the region by a finite number of steps, with each step starting at a pixel and ending in the neighborhood of the pixel.



4-neighborhood

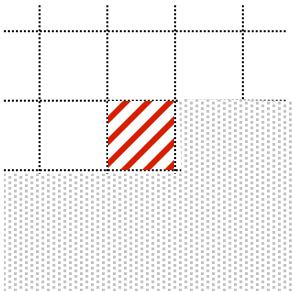


8-neighborhood

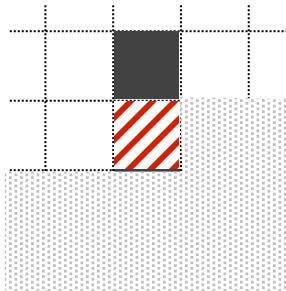
- Typically, either definition leads to the same regions, except when a region is only connected across diagonally adjacent pixels.

# Region labeling algorithm (4-neighborhood)

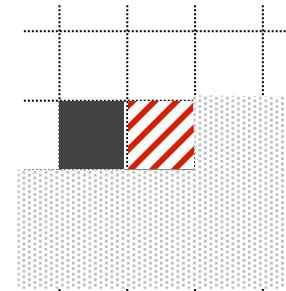
- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
- If  $f[x,y]=0$ , do nothing.
- If  $f[x,y]=1$ , distinguish 4 cases



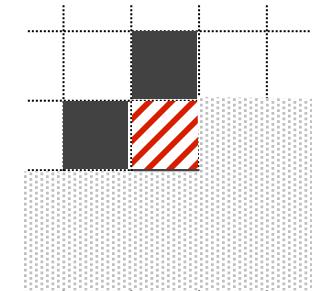
Generate new region label



Copy label from above



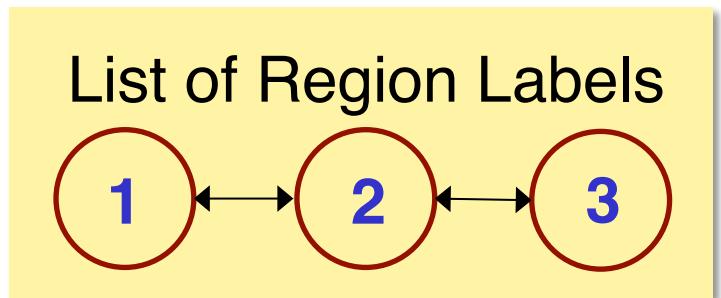
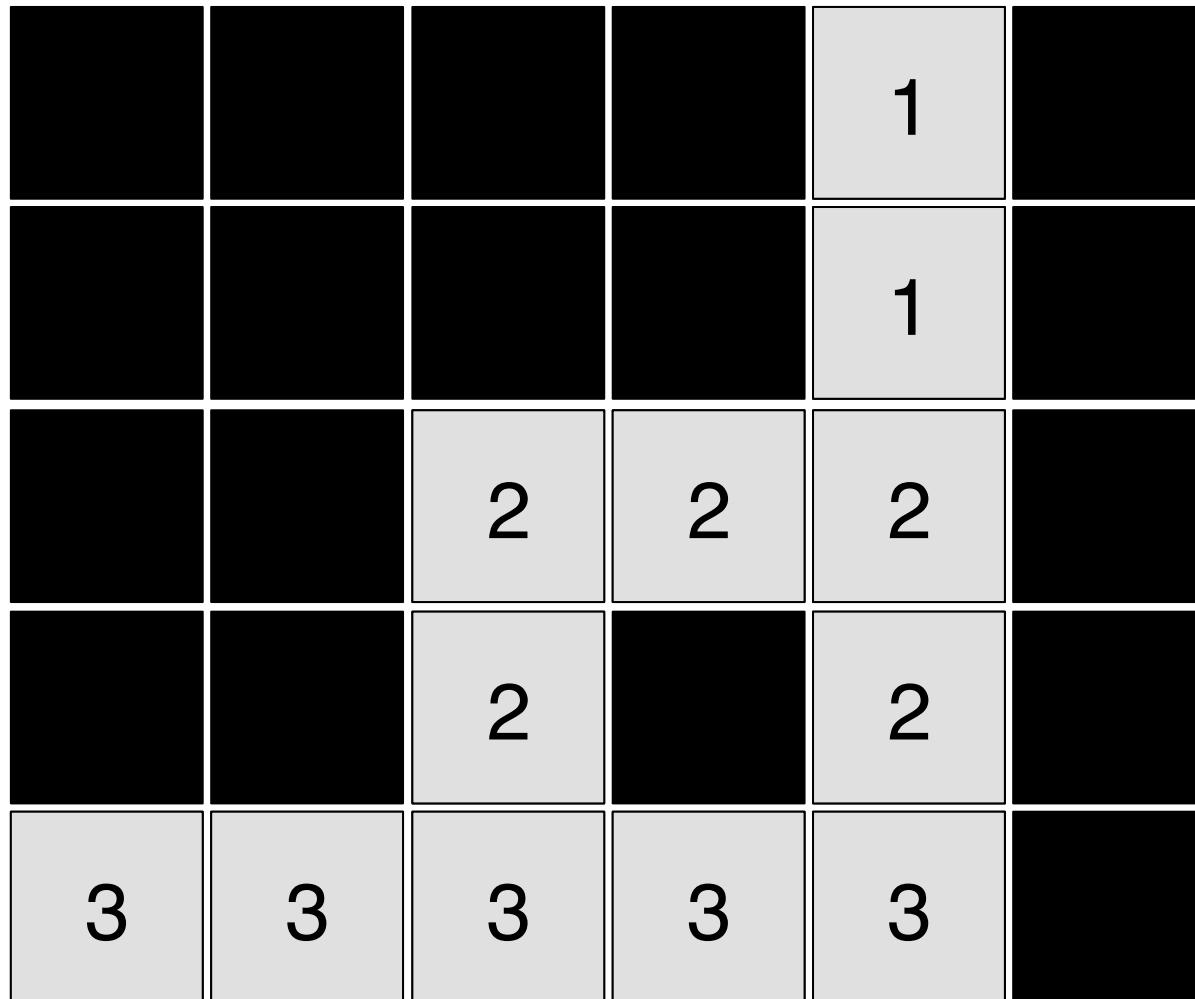
Copy label from the left



Copy label from the left. If labels above and to the left are different, store equivalence.

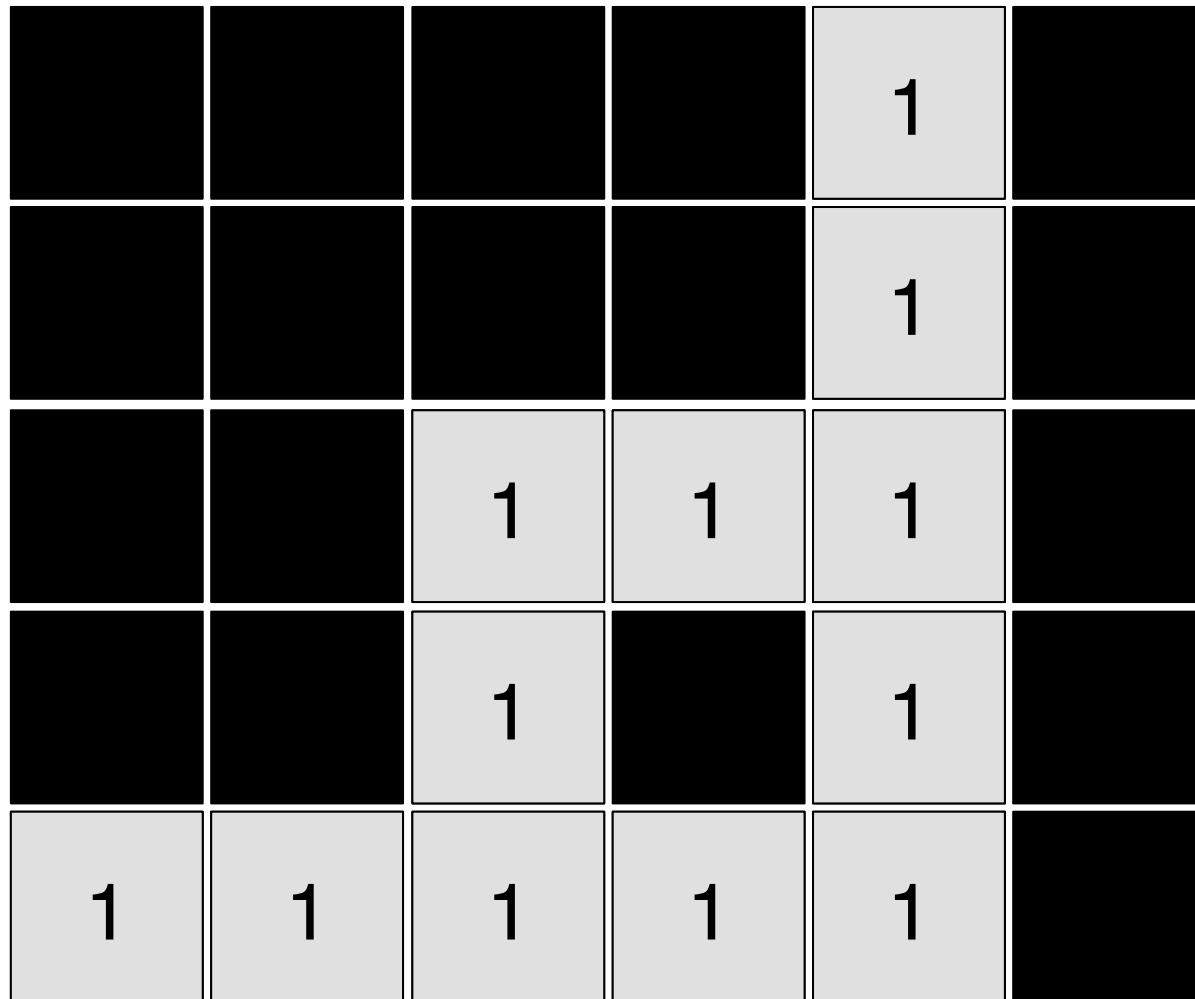
- Second pass through image to replace equivalent label by the same label.

# Region labeling example (4-neighborhood)



*All three labels are equivalent, so merge into single label.*

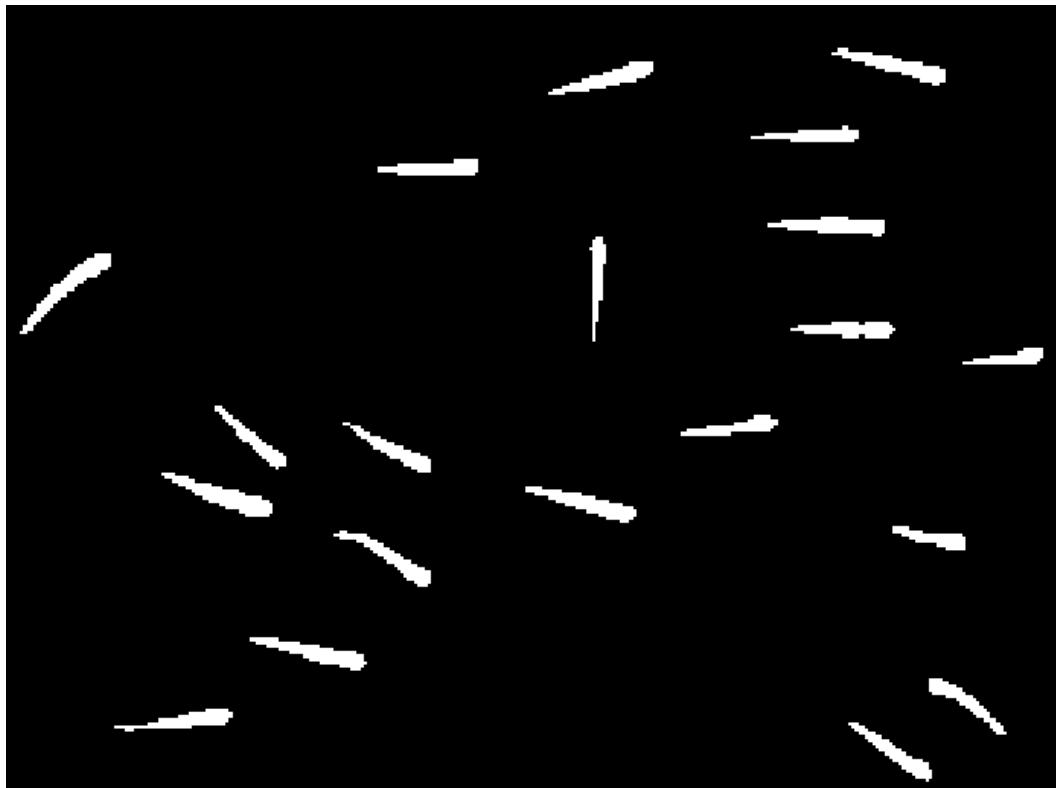
# Region labeling example (4-neighborhood)



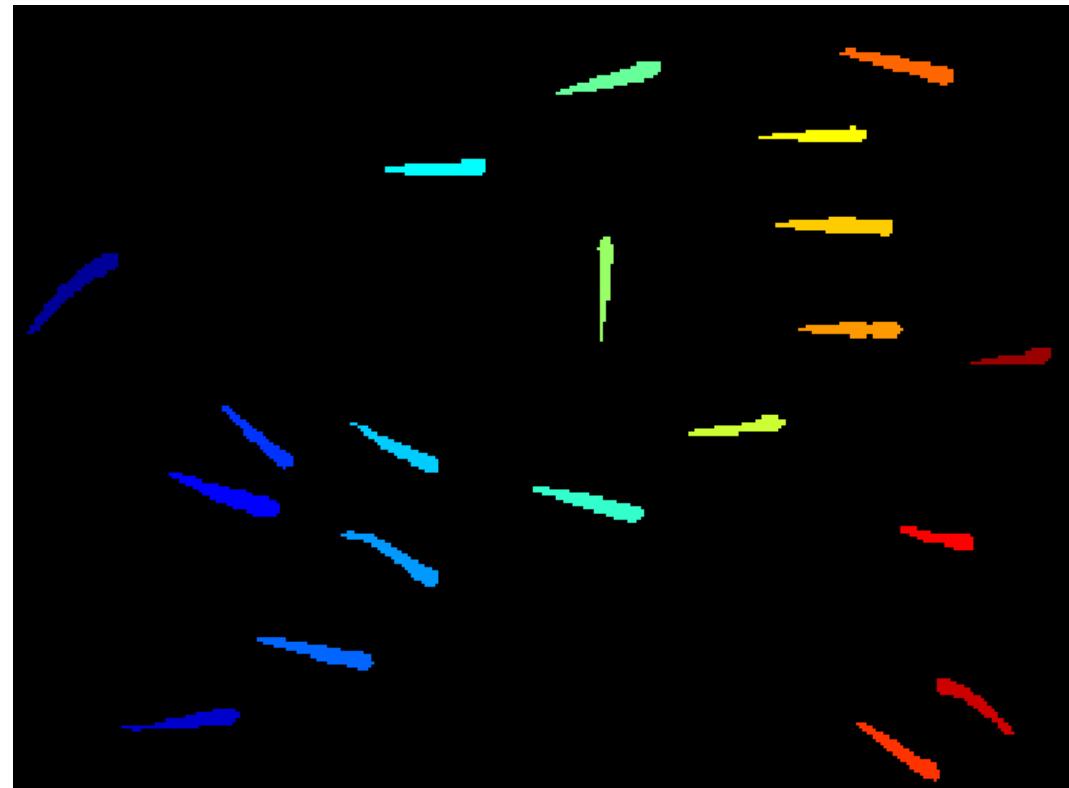
List of Region Labels

1

# Example: region labeling



Thresholded image



20 labeled regions



# Region counting algorithm

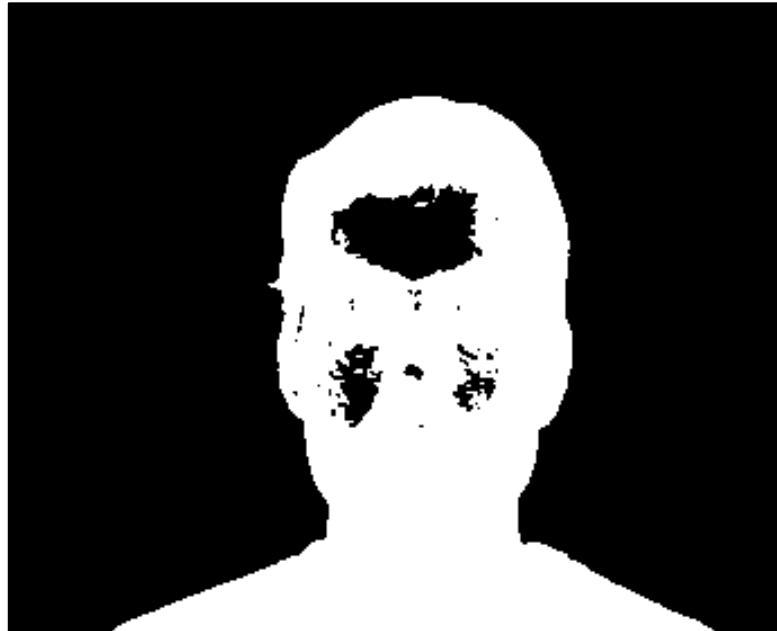
- Measures the size of each region
- Initialize  $counter[label]=0$  for all  $label$
- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
  - If  $f[x,y]=0$ , do nothing.
  - If  $f[x,y]=1$ , increment  $counter[label[x,y]]$

# Small region removal

- Loop through all pixels  $f[x,y]$ , left to right, top to bottom
  - If  $f[x,y]=0$ , do nothing.
  - If  $f[x,y]=1$  and  $counter[label[x,y]] < S$ , set  $f[x,y]=0$
- Removes all regions smaller than  $S$  pixels

# Hole filling as dual to small region removal

Mask with holes



After NOT operation, (background) region labeling, small region removal, and second NOT operation



# Region moments

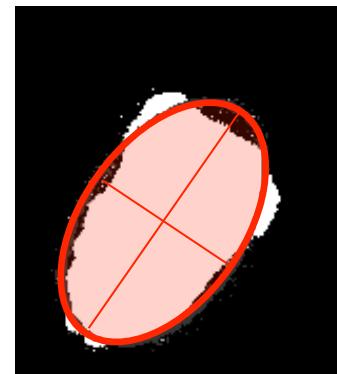
- Raw moments  $M_{pq} = \sum_{x,y \in \text{Region}} x^p y^q$

- Central moments

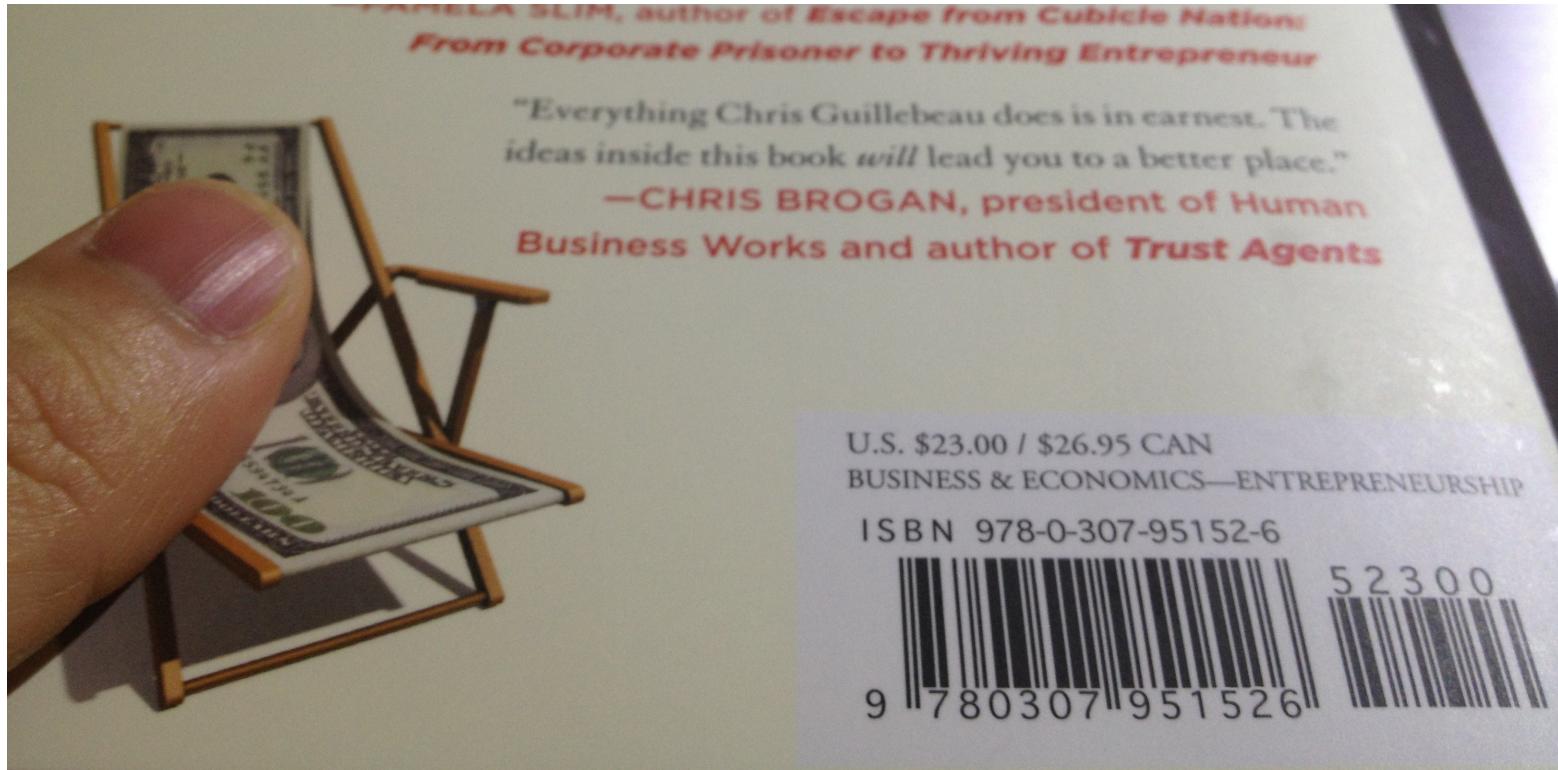
$$\mu_{pq} = \sum_{x,y \in \text{Region}} (x - \bar{x})^p (y - \bar{y})^q \quad \text{with } \bar{x} = \frac{M_{10}}{M_{00}} \text{ and } \bar{y} = \frac{M_{01}}{M_{00}}$$

- Region orientation and eccentricity:  
calculate eigenvectors of covariance  
matrix

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$



# Example: Detecting bar codes

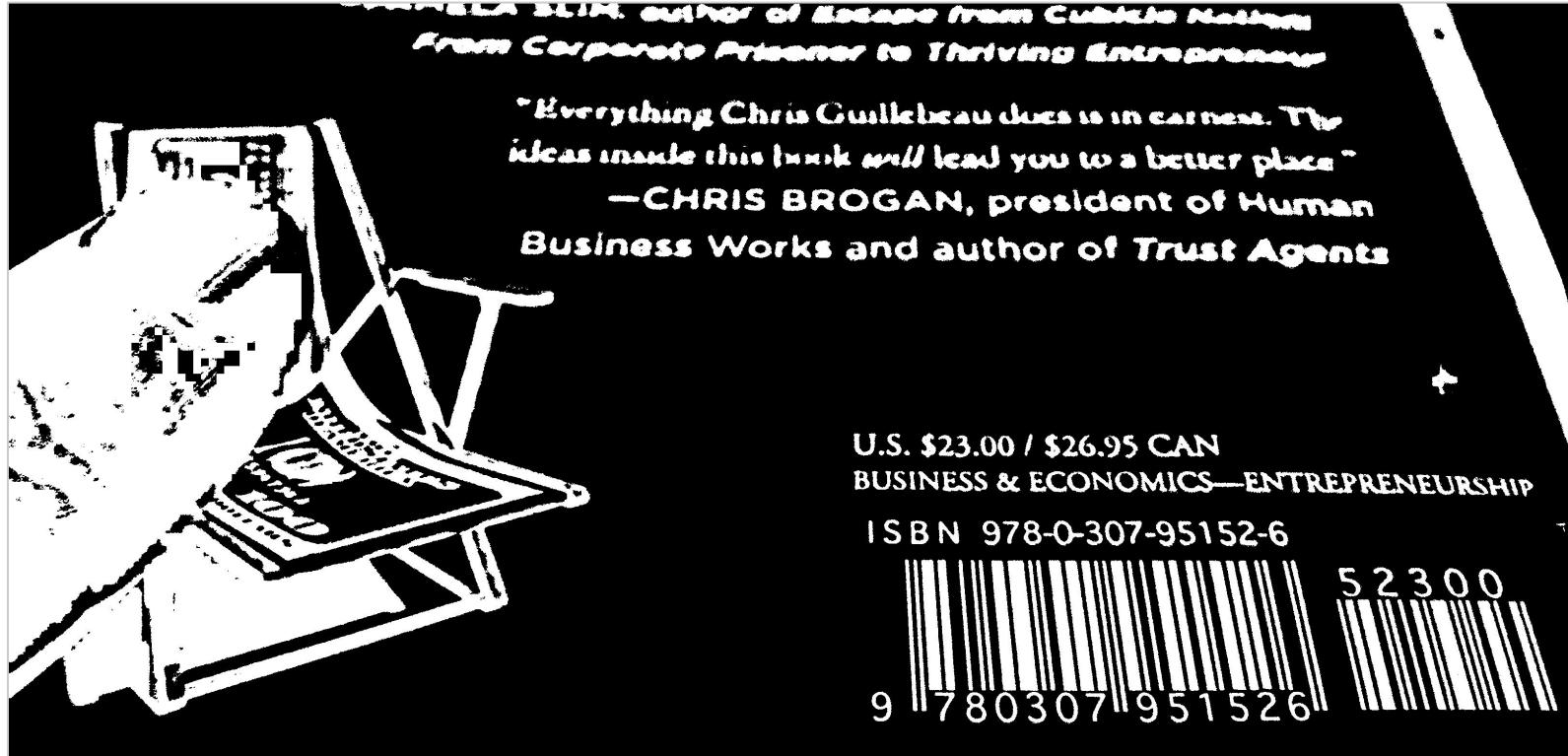


Original Image



# Example: Detecting bar codes

Locally adaptive  
thresholding



# Example: Detecting bar codes

Locally adaptive  
thresholding

Filtering by  
eccentricity

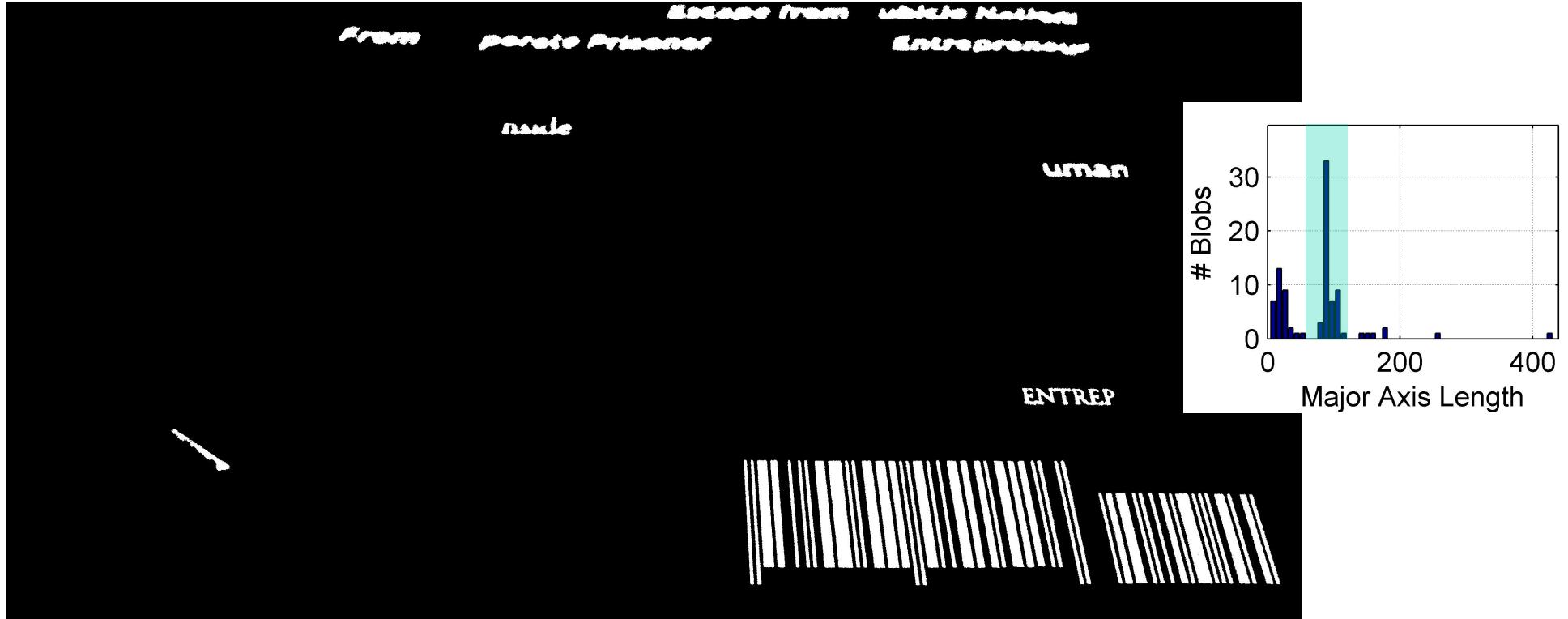


# Example: Detecting bar codes

Locally adaptive  
thresholding

Filtering by  
eccentricity

Filtering by major  
axis length



# Example: Detecting bar codes

Locally adaptive  
thresholding

Filtering by  
eccentricity

Filtering by major  
axis length

Filtering by  
orientation

