Trường Đại học Tôn Đức Thắng

Machine Learning
Support Vector Machine

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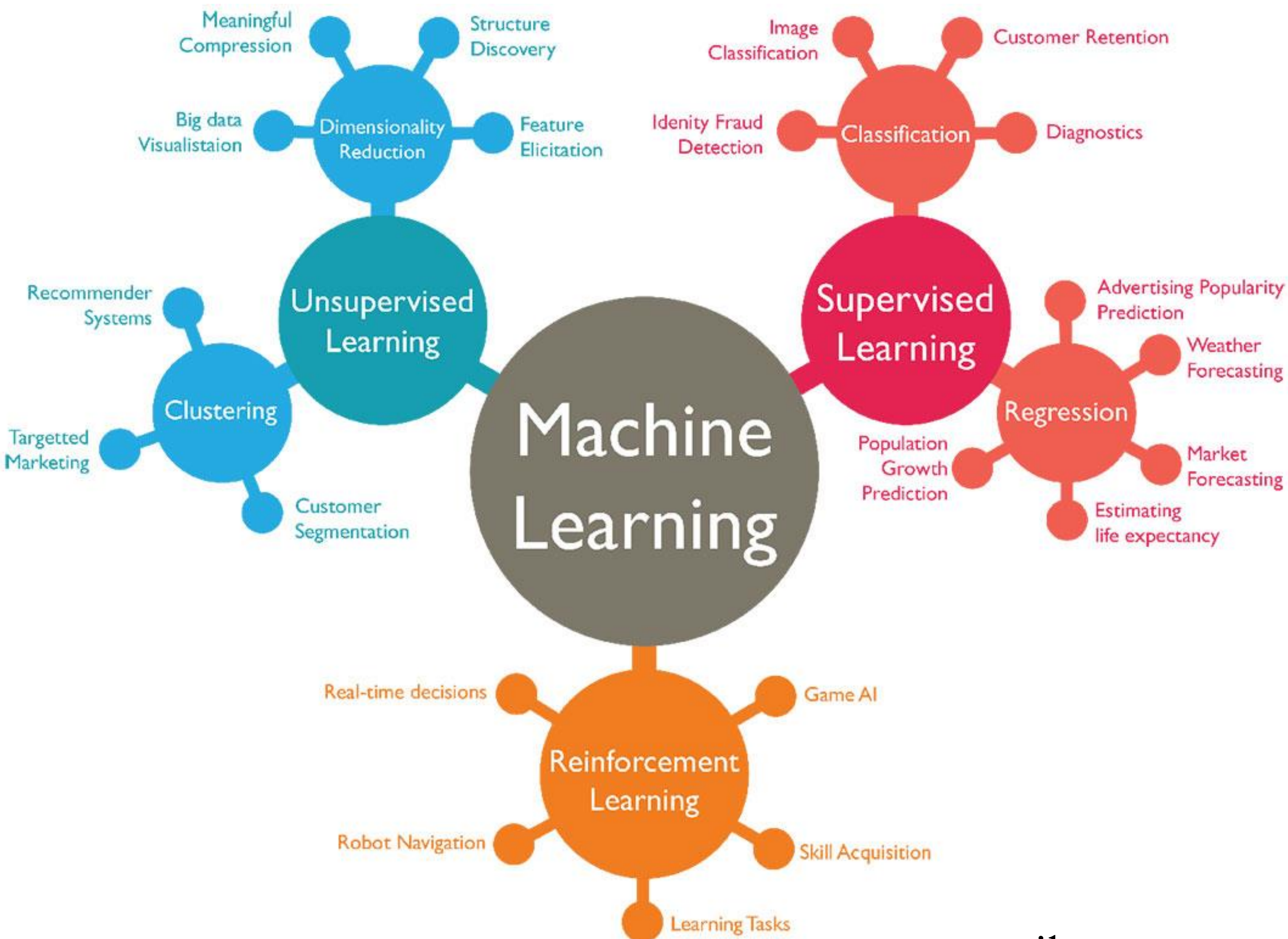
3- Machine Learning

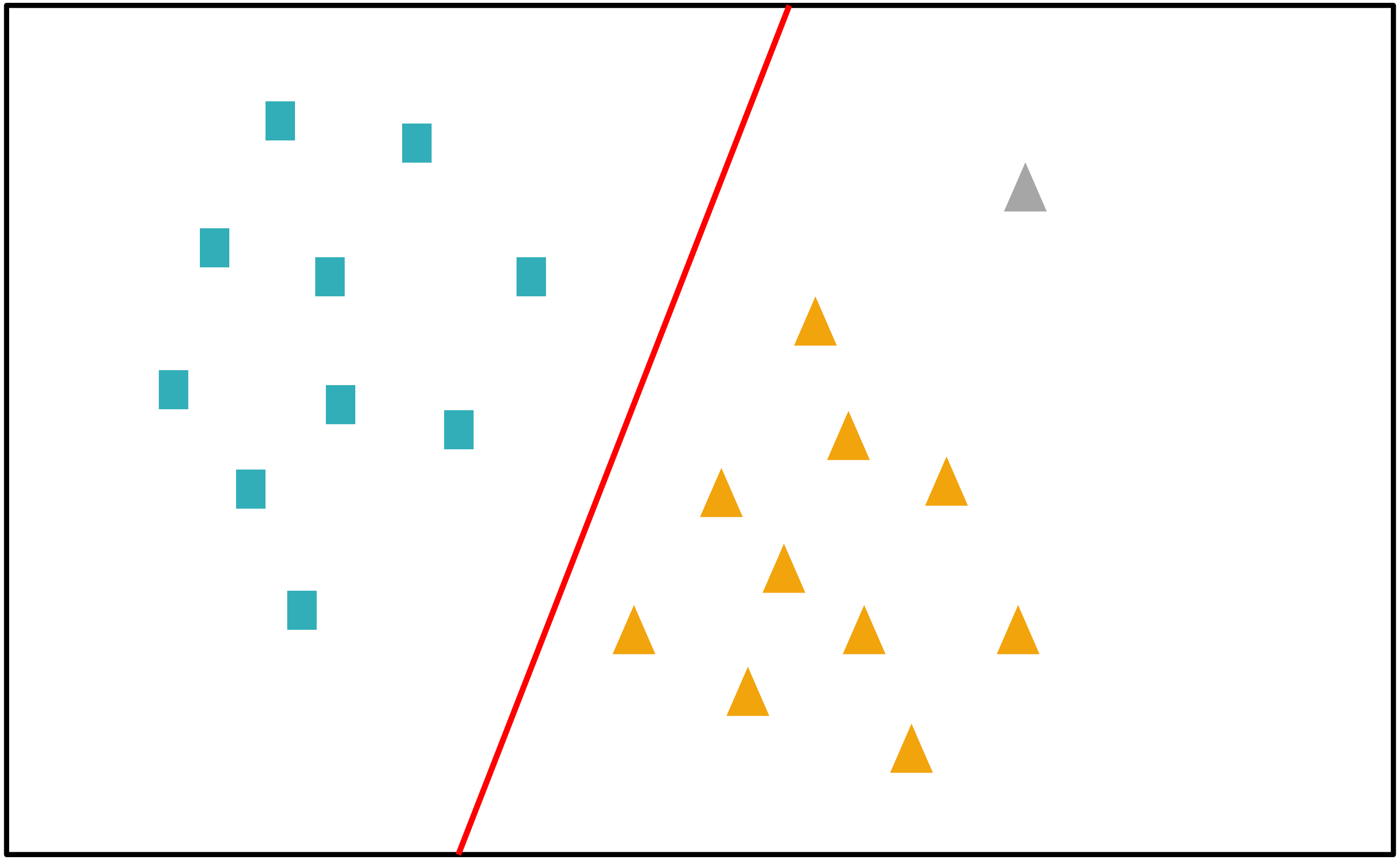


Support Vector Machine

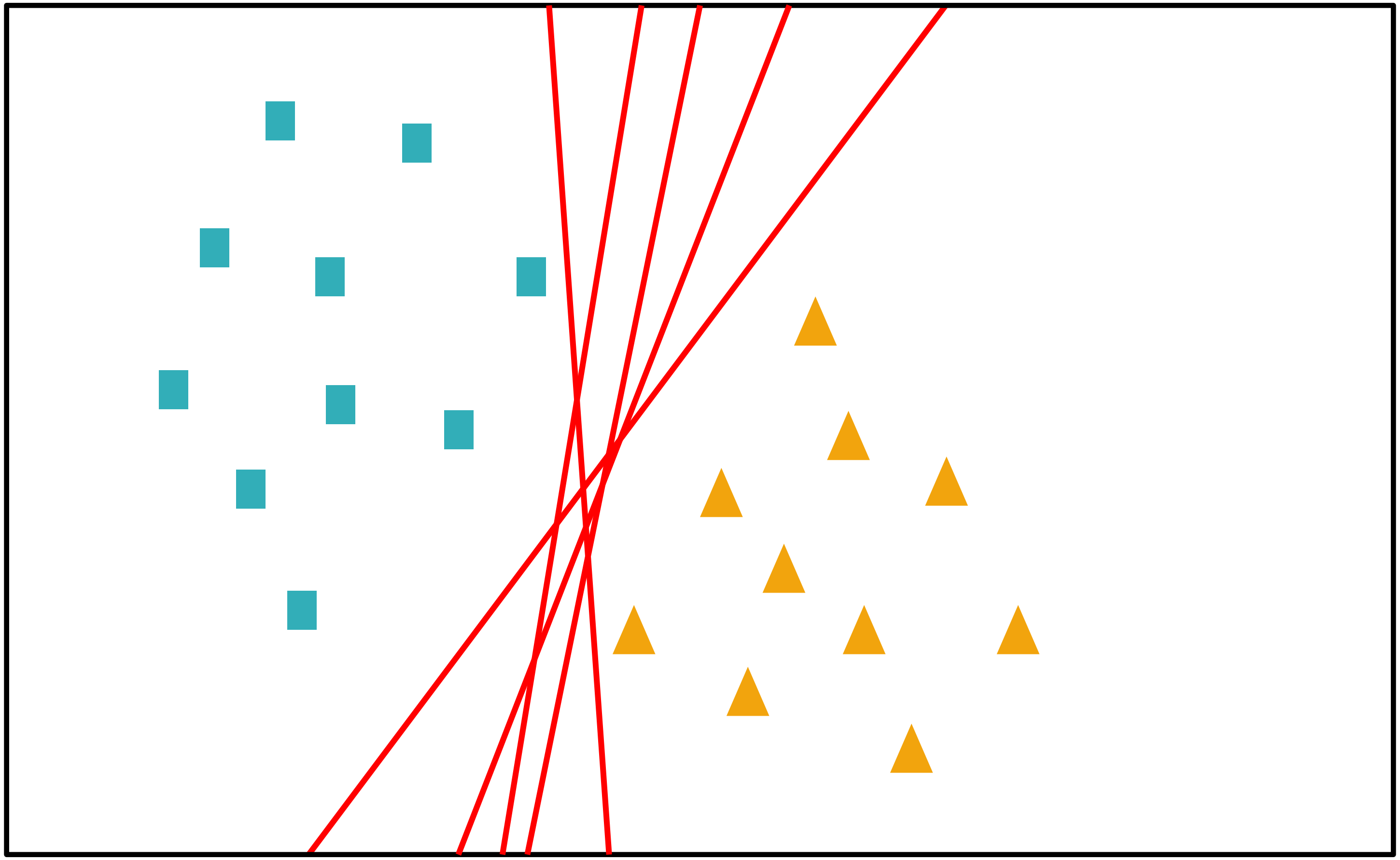
Why using SVM?

- High-Dimensionality
- Memory Efficiency
- Versatility

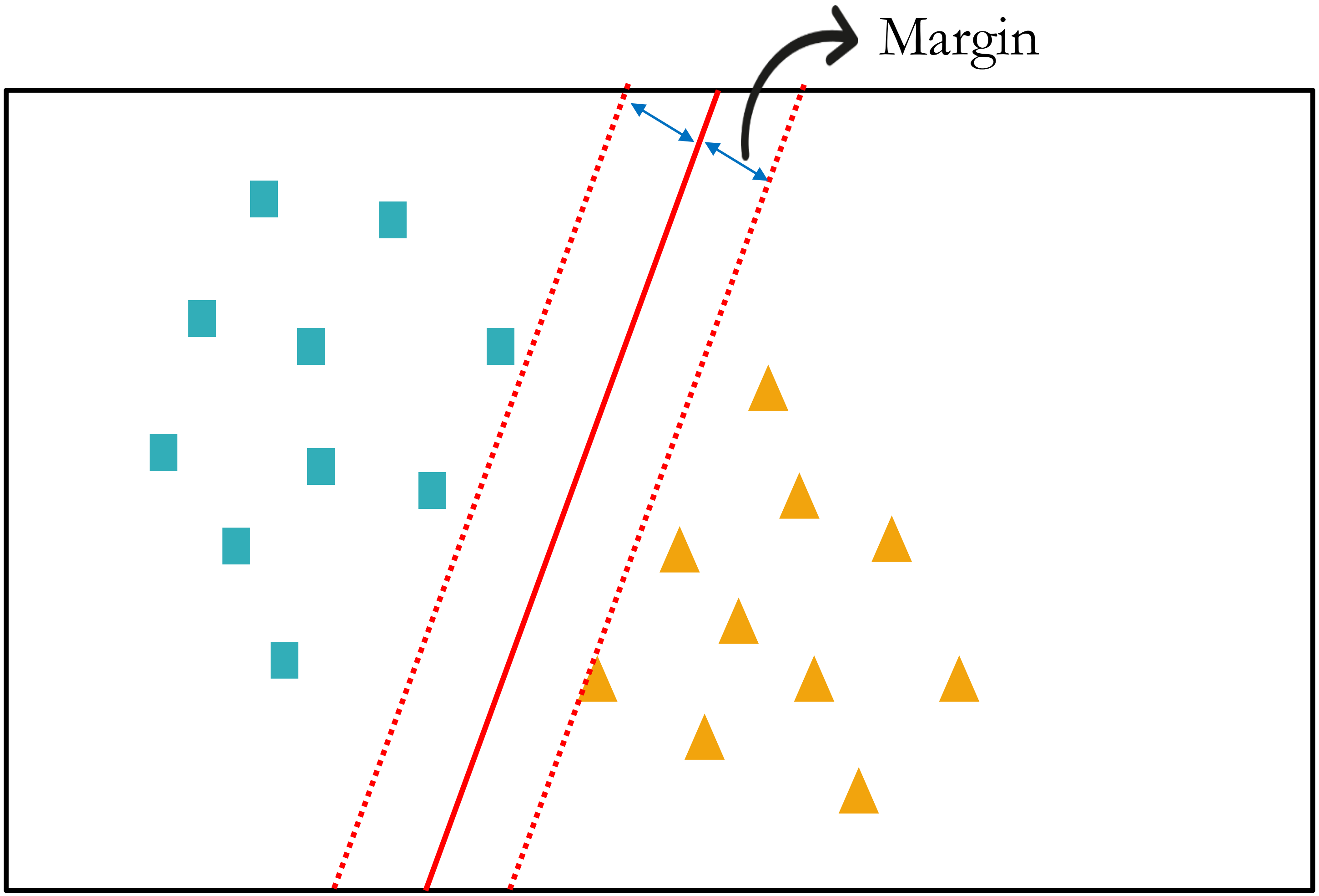
How SVM works



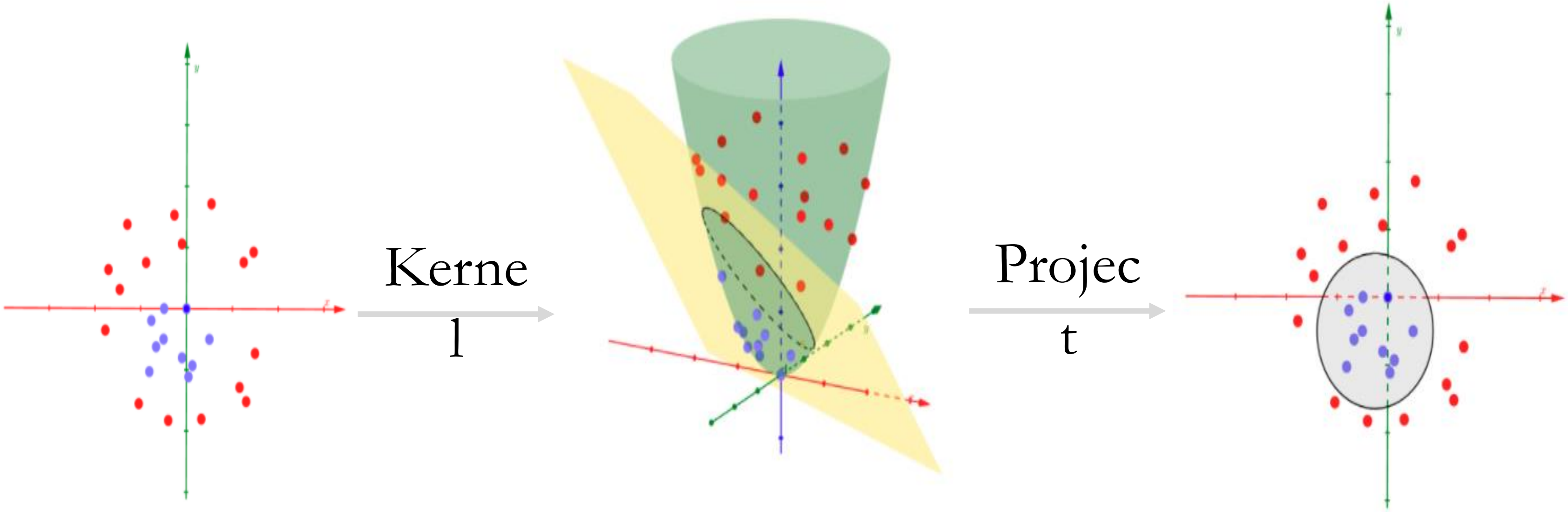
How SVM works



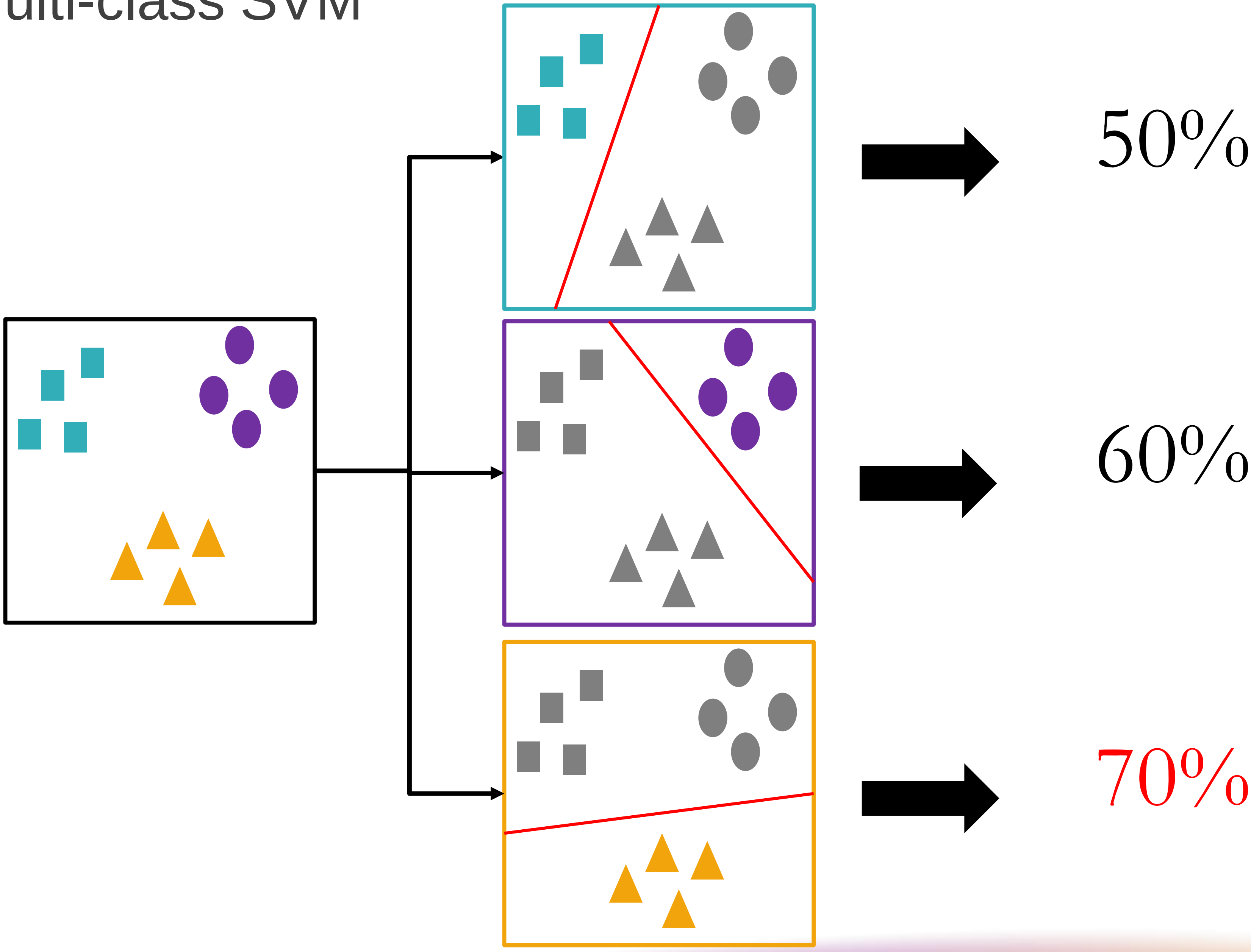
How SVM works



How SVM works



Multi-class SVM



Finding the hyperlane

Hyperplane: $w^T x + b = 0$

$$margin = \min_n \frac{y_n(w^T x_n + b)}{\|w\|_2}$$

Finding w and b:

$$(w, b) = \arg\max_{w,b} \left\{ \min_n \frac{y_n(w^T x_n + b)}{\|w\|_2} \right\} \rightarrow \boxed{(w, b) = \arg\min_{w,b} \frac{1}{2} \|w\|_2^2}$$

Quadratic Programming Solver

Problem: $\boldsymbol{\lambda} = \boldsymbol{arg}\ \mathbf{min}_{\lambda} \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\lambda}^{\boldsymbol{T}} \boldsymbol{K} \boldsymbol{\lambda} + \boldsymbol{p}^{\boldsymbol{T}} \boldsymbol{\lambda}$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

- $\boldsymbol{\lambda}$ is Lagrange multiplier
  - $\boldsymbol{K}$ is square matrix
- $\boldsymbol{G}, \boldsymbol{A} \in R^{m \times n}$; $\boldsymbol{h}, \boldsymbol{b} \in R^{m}$; $\boldsymbol{p} \in R^{n}$

Solve problem using CVXOPT

$$\boldsymbol{w} = \sum_{n=1}^{N} \boldsymbol{\lambda}_n\, y_n x_n \longrightarrow \mathbf{b}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\lambda = \arg\min_{\lambda} \frac{1}{2}\lambda^T K\lambda + p^T\lambda$$

Subject to: $G\lambda \leq \mathbf{h}$

$A\lambda = \mathbf{b}$

Training data

X =[[2.3, 1.7] y = [1
[1.5, 1.4] 1
[2.4, 2.1] 1
[3.4, 0.7] -1
[4.2 2.3]] -1]

$V = [y_1 X_1 + y_2 X_2 + \ldots + y_5 X_5]$ → V = [[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[-3.4, -0.7]
[-4.2, -2.3]]

K = $V^T V$ → K=

$$\begin{bmatrix} 8.18 & 5.83 & 9.09 & -9.01 & -13.57 \\ 5.83 & 4.21 & 6.5 & -6.08 & -9.52 \\ 9.09 & 6.54 & 10.17 & -9.63 & -14.91 \\ -9.01 & -6.08 & -9.63 & 12.05 & 15.9 \\ -13.57 & -9.52 & -14.91 & 15.9 & 22.93 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\boldsymbol{\lambda} = \boldsymbol{arg}\ \min_{\boldsymbol{\lambda}} \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\lambda}^{T} \boldsymbol{K} \boldsymbol{\lambda} + \boldsymbol{p}^{T} \boldsymbol{\lambda}$$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

Training data

X =[[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[3.4, 0.7]
[4.2 2.3]]

y = [1
1
1
-1
-1]

p is all-one vector:

$$p = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\lambda = arg\min_{\lambda} \frac{1}{2}\lambda^T K \lambda + p^T \lambda$$

Subject to: $G\lambda \le \mathbf{h}$

$A\lambda = \mathbf{b}$

Training data

X =[[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[3.4, 0.7]
[4.2 2.3]]

y = [1
1
1
-1
-1]

G is a diag matrix:

For all λ >= 0

$$G = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\lambda = arg\ \min_{\lambda} \frac{1}{2} \lambda^T K \lambda + p^T \lambda$$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

Training data

X =[[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[3.4, 0.7]
[4.2 2.3]]

y = [1
1
1
-1
-1]

$$h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\boldsymbol{\lambda} = \boldsymbol{arg}\ \min_{\lambda} \frac{1}{2} \boldsymbol{\lambda}^T \boldsymbol{K} \boldsymbol{\lambda} + \boldsymbol{p}^T \boldsymbol{\lambda}$$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

Training data

X =[[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[3.4, 0.7]
[4.2 2.3]]

y = [1
1
1
-1
-1]

$$\text{A} = y^T = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\lambda = \arg\min_{\lambda} \frac{1}{2} \lambda^T K \lambda + p^T \lambda$$

Subject to: $G\lambda \leq \mathbf{h}$

$A\lambda = \mathbf{b}$

Training data

X =[[2.3, 1.7]
[1.5, 1.4]
[2.4, 2.1]
[3.4, 0.7]
[4.2 2.3]]

y = [1
1
1
-1
-1]

b =[0]

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\boldsymbol{\lambda} = \boldsymbol{arg}\,\min_{\lambda} \frac{1}{2}\boldsymbol{\lambda^T K \lambda} + \boldsymbol{p^T \lambda}$$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

Training data

X =[[2.3, 1.7] y = [1
[1.5, 1.4] 1
[2.4, 2.1] 1
[3.4, 0.7] -1
[4.2 2.3]] -1]

Apply K, p, G, h, A, b to below equation:

$$\boldsymbol{\lambda} = \begin{bmatrix} 9.61127142e-02 \\ 2.92683333e-09 \\ 8.97157531e-01 \\ 2.68704898e-08 \\ 9.93270222e-01 \end{bmatrix}$$

Build K

Create p

Build G

Build h

Build A

Build b

Find λ

Find w, b

$$\lambda = \arg\min_{\lambda} \frac{1}{2}\lambda^T K \lambda + p^T \lambda$$

Subject to: $\boldsymbol{G\lambda} \leq \mathbf{h}$

$\boldsymbol{A\lambda} = \mathbf{b}$

Training data

X =[[2.3, 1.7]   y = [1
[1.5, 1.4]   1
[2.4, 2.1]   1
[3.4, 0.7]   -1
[4.2 2.3]]   -1]

Apply $\boldsymbol{\lambda}$, X, y to $\boldsymbol{w} = \sum_{n=1}^{N} \lambda_n y_n x_n$

w = [-1.34287034, 0.42806505]

b = 3.45

Support Vector Machine (Classification)

- Pima dataset (ratio 6:4)
- SVM (sklearn)
- Evaluation: Accuracy

Time: 15 minutes

SVM (Regresion)

Time: 15 minutes