K-Nearest-Neighbors Algorithm

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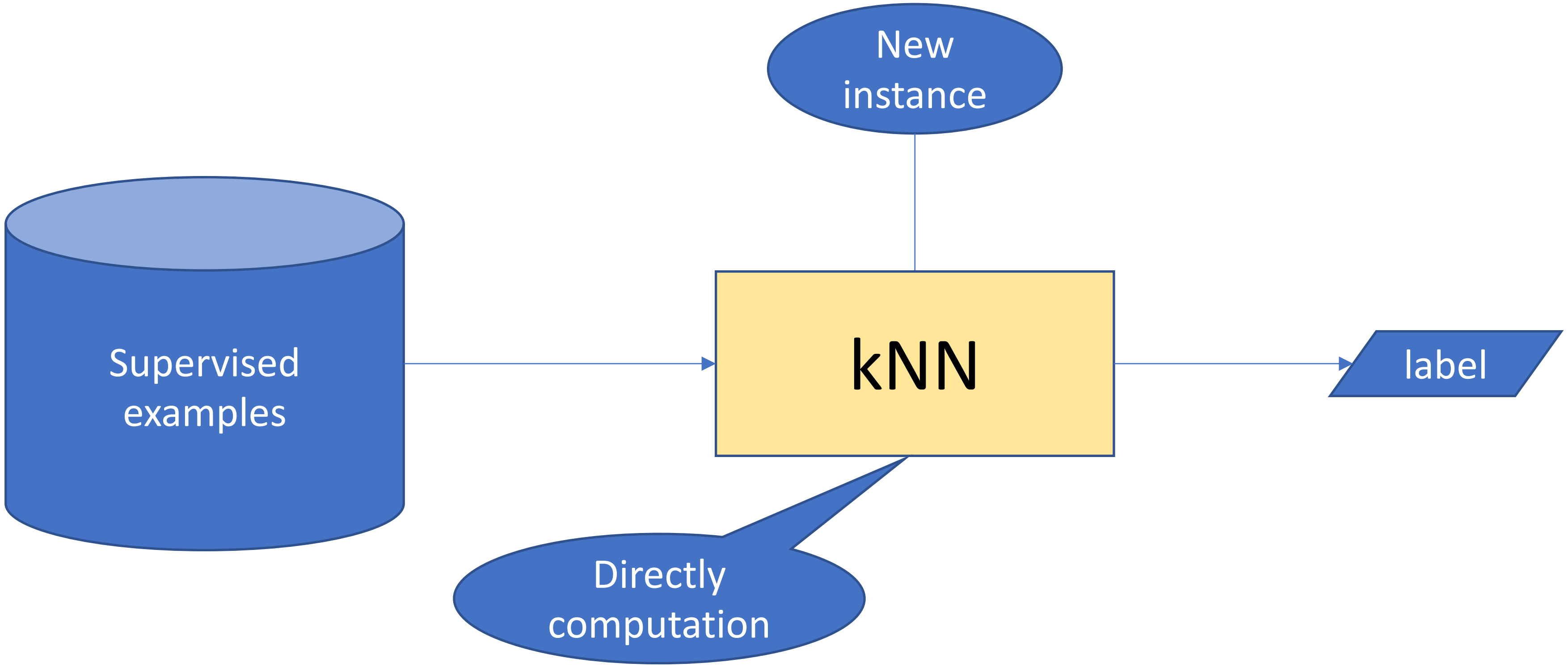
KNN is a Method in Instance-Based Learning

- Instance-based learning is often termed *lazy* learning, as there is typically no “transformation” of training instances into more general “statements”



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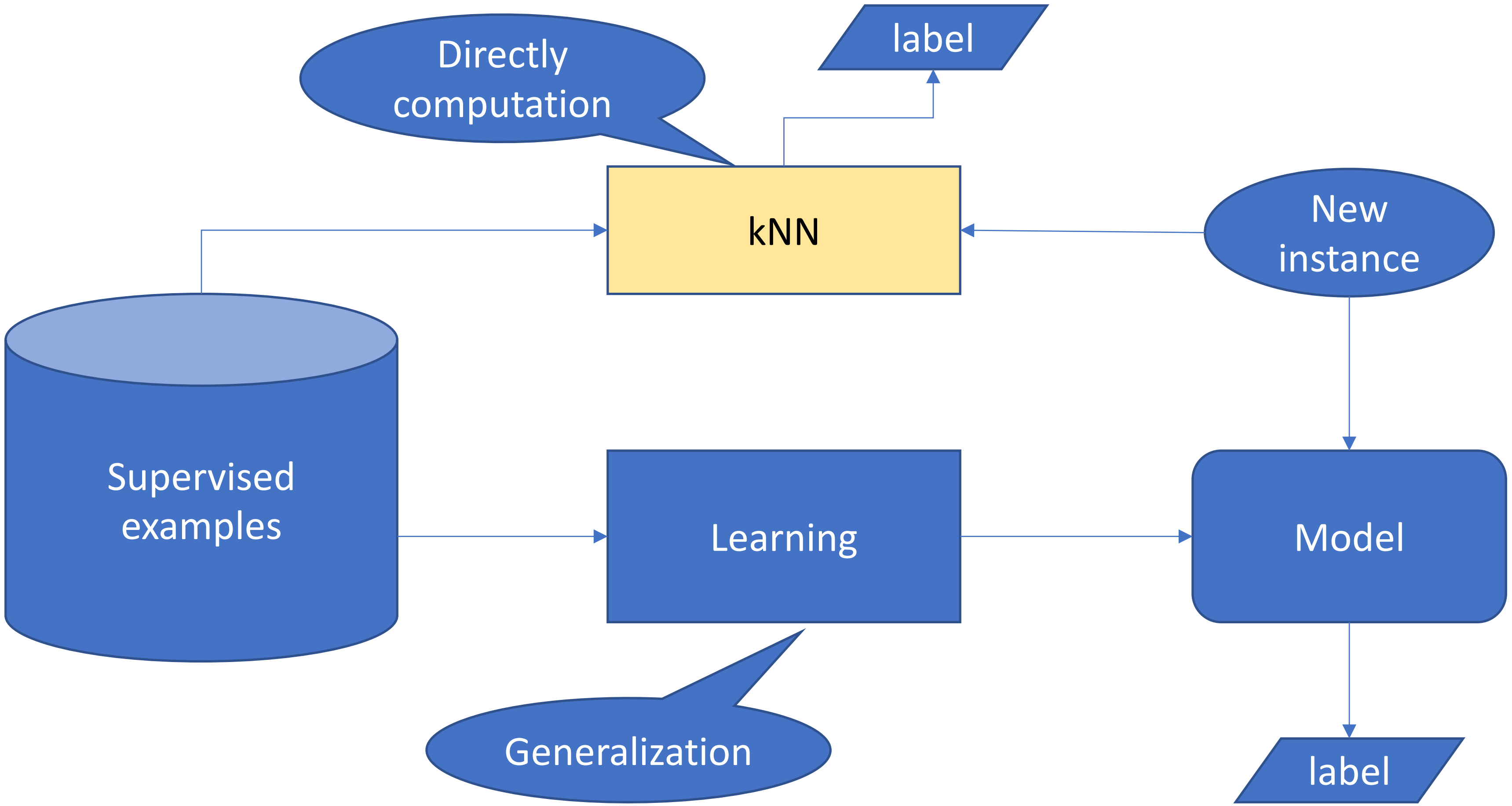


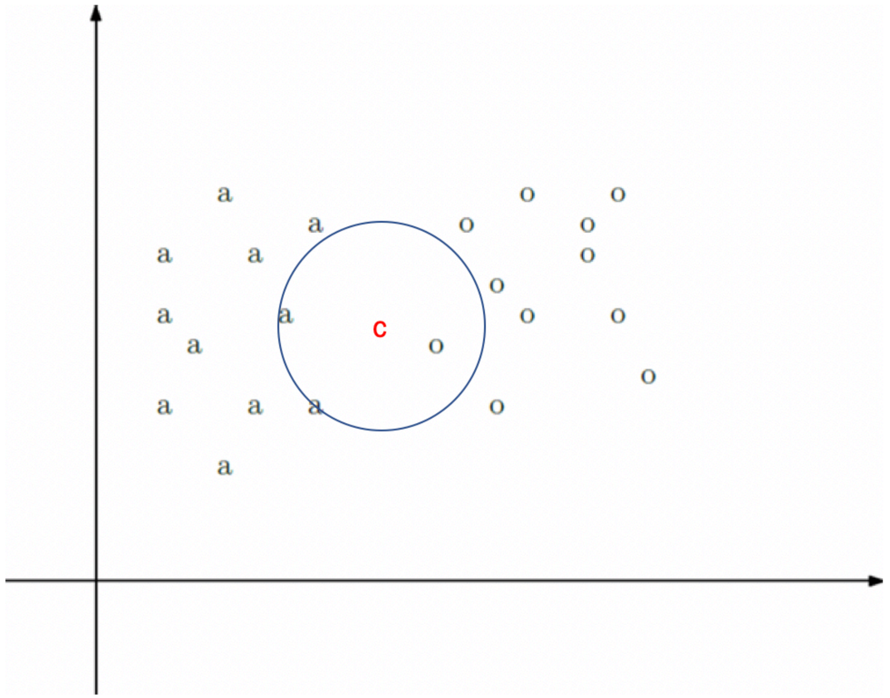
K-Nearest-Neighbors Algorithm

- A case is classified by a majority voting of its neighbors, with the case being assigned to the class most common among its K nearest neighbors measured by a Distance Function.

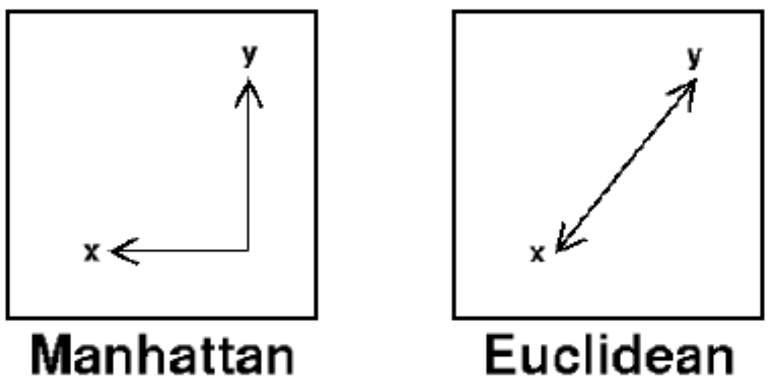
- If K=1, then the case is simply assigned to the class of its nearest neighbor

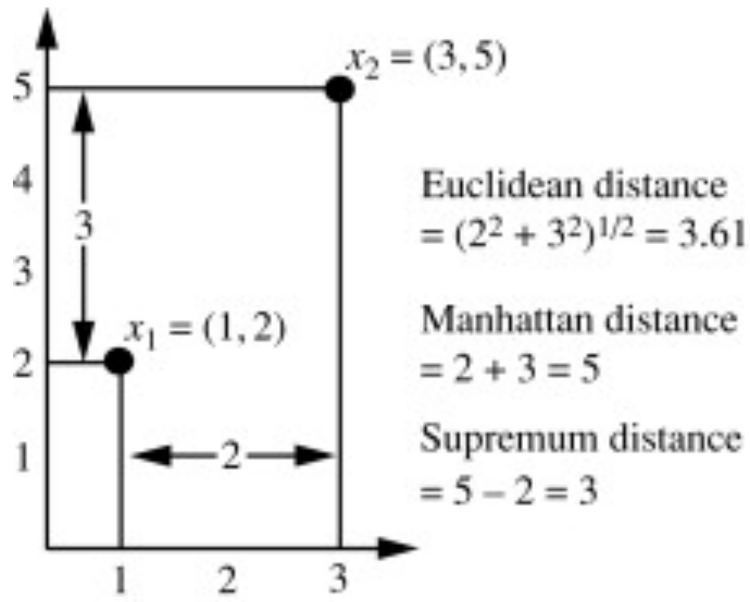
What is the most possible label for c?

- Solution: Looking for the nearest K neighbors of c.
- Take the majority label as c's label
- Let's suppose k = 3:



Distance Function Measurements



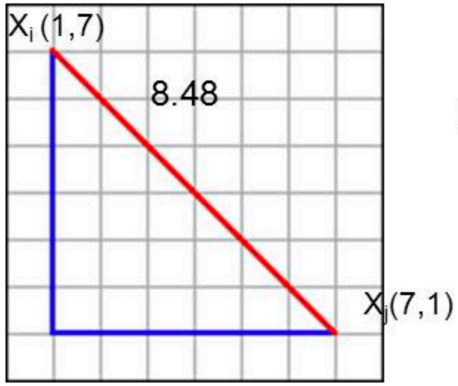


Minkowski Distance

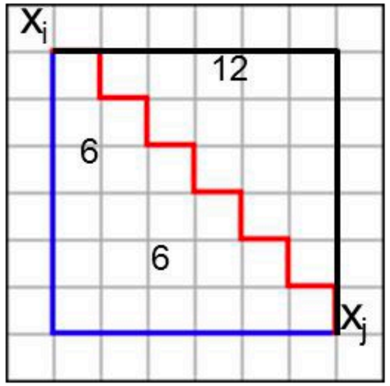
- Minkowski distance: a generalization

$$d(i,j) = \sqrt[q]{|x_{i_1} - x_{j_1}|^q + |x_{i_2} - x_{j_2}|^q + \dots + |x_{i_p} - x_{j_p}|^q} \quad (q > 0)$$

- If q = 2, d is Euclidean distance
- If q = 1, d is Manhattan distance



q=2



q=1

Example

ID	Height	Age	Weight	
1	5	45	77	H
2	5.11	26	47	L
3	5.6	30	55	M
4	5.9	34	59	M
5	4.8	40	72	H
6	5.8	36	60	M
7	5.3	19	40	L
8	5.8	28	60	M
9	5.5	23	45	L
10	5.6	32	58	M
11	5.5	38	?	

kNN for Classification

- A simple implementation of **KNN classification** is a majority voting mechanism.

- Given: training examples D={x_i, y_i}, and a new example x

where
  - x_i: attribute-value representation of the example i^{th}
  - y_i: corresponding label or class of example i^{th}
- Algorithm:
  - Compute distance D(x,x_j) for every x_j of the training data D
  - Select k closest instances x_{i1}, …, x_{ik} with their labels are y_{i1},..y_{ik}
  - y = majority (y_{i1},..y_{ik}) is the predicted label of x

Example

Weight Category

ID	Height	Age	Weight
1	5	45	H
2	5.11	26	L
3	5.6	30	M
4	5.9	34	M
5	4.8	40	H
6	5.8	36	M
7	5.3	19	L
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kNN for Regression

- A simple implementation of **KNN regression** is to calculate the average of the numerical target of the K nearest neighbors.

- Given:
  - training examples $\{x_i, y_i\}$
    - x_i ... attribute-value representation of examples
    - y_i ... real-valued target (profit, rating on YouTube, etc)
  - testing point x that we want to predict the target
- Algorithm:
    - compute distance $D(x, x_i)$ to every training example x_i
    - select k closest instances $x_{i1} \ldots x_{ik}$ and their labels $y_{i1} \ldots y_{ik}$
    - output the mean of $y_{i1} \ldots y_{ik}$:

$$\hat{y} = f(x) = \frac{1}{k} \sum_{j=1}^{k} y_{i_j}$$

Example

ID	Height	Age	Weight	
1	5	45	77	H
2	5.11	26	47	L
3	5.6	30	55	M
4	5.9	34	59	M
5	4.8	40	72	H
6	5.8	36	60	M
7	5.3	19	40	L
8	5.8	28	60	M
9	5.5	23	45	L
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Distance-weighted *k*-NN

- Replace $\hat{f}(q) = \arg\max_{v \in V} \sum_{i=1}^{k} \delta(v, f(x_i))$ by:

$$\hat{f}(q) = \arg\max_{v \in V} \sum_{i=1}^{k} \frac{1}{d\left(x_i, x_q\right)^2} \delta(v, f(x_i))$$

Issues with Distance Metrics

- Most distance measures were designed for linear/real-valued attributes
- Two important questions in the context of machine learning:
  - How to handle nominal attributes
  - What to do when attribute types are mixed

Nominal data type

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Hamming Distance

- For category variables, Hamming distance can be used.

Hamming Distance

$$D_H = \sum_{i=1}^{k} \left| x_i - y_i \right|$$

$$x = y \Rightarrow D = 0$$

$$x \neq y \Rightarrow D = 1$$

X	Y	Distance
Male	Male	0
Male	Female	1

Normalization

Age	Loan	Default	Distance	
25	$40,000	N	102000	
35	$60,000	N	82000	
45	$80,000	N	62000	
20	$20,000	N	122000	
35	$120,000	N	22000	2
52	$18,000	N	124000	
23	$95,000	Y	47000	
40	$62,000	Y	80000	
60	$100,000	Y	42000	3
48	$220,000	Y	78000	
33	$150,000	Y	8000	1
48	**$142,000**	**?**		

Euclidean Distance

$$D = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

Normalization

Age	Loan	Default	Distance
0.125	0.11	N	0.7652
0.375	0.21	N	0.5200
0.625	0.31	N	0.3160
0	0.01	N	0.9245
0.375	0.50	N	0.3428
0.8	0.00	N	0.6220
0.075	0.38	Y	0.6669
0.5	0.22	Y	0.4437
1	0.41	Y	0.3650
0.7	1.00	Y	0.3861
0.325	0.65	Y	0.3771
0.7	**0.61**	**?**	

Standardized Variable

$$X_s = \frac{X - Min}{Max - Min}$$

Exercise

Customer	Age	Loan	Default	E
John	25	40000	N	
Smith	35	60000	N	
Alex	45	80000	N	
Jade	20	20000	N	
Kate	35	120000	N	
Mark	52	18000	N	
Anil	23	95000	Y	
Pat	40	62000	Y	
George	60	100000	Y	
Jim	48	220000	Y	
Jack	33	150000	Y	
Andrew	**48**	**142000**	**?**	

What to do when attribute types are mixed

- Convert categorical values into numerical values
  - Binary values: convert to 0 and 1, for example ’male’, ‘female’
  - Multiple degress, such as ‘low’, ‘average’, and ‘high’: convert to 1, 2, 3
  - if the values are “Red”, “Green”, “Blue” (or more generally, something that has no intrinsic order): convert to [1,0,0], [0,1,0], [0,0,1]
- Normalize or scale the data into the same interval.

Exercise

case ID | predictors | target

CUST_ID	CUST_GENDER	EDUCATION	OCCUPATION	AGE	AFFINITY_CARD
101501	F	Masters	Prof.	41	0
101502	M	Bach.	Sales	27	0
101503	F	HS-grad	Cleric.	20	0
101504	M	Bach.	Exec.	45	1
101505	M	Masters	Sales	34	1
101506	M	HS-grad	Other	38	0
101507	M	< Bach.	Sales	28	0
101508	M	HS-grad	Sales	19	0
101509	M	Bach.	Other	52	0
101510	M	Bach.	Sales	27	1

Discussions

- kNN can deal with complex and arbitrary decision boundaries.
- Despite its simplicity, researchers have shown that the classification accuracy of kNN can be quite strong and in many cases as accurate as those elaborated methods.
- kNN is slow at the classification time
- kNN does not produce an understandable model