$$6/\lim_{x\to+\infty} x\left(e^{\frac{1}{x}}-1\right)=\lim_{x\to+\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}}=\lim_{x\to+\infty}\frac{\left(e^{\frac{1}{x}}-1\right)'}{\left(\frac{1}{x}\right)'}=\lim_{x\to+\infty}\frac{\left[e^{\frac{1}{x}}.\left(\frac{1}{x}\right)'-1'\right]}{-\frac{1}{x^2}}$$

$$=\lim_{x\to+\infty}\frac{e^{\frac{1}{x}}.\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}=\lim_{x\to+\infty} e^{\frac{1}{x}}=e^0=1.$$

$$7/\lim_{x\to0^+} e^{-\frac{1}{x}}.\ln x=\lim_{x\to0^+}\frac{\ln x}{e^{\frac{1}{x}}}=\lim_{x\to0^+}\frac{(\ln x)'}{\left(e^{\frac{1}{x}}\right)'}=\lim_{x\to0^+}\frac{\frac{1}{x}}{e^{\frac{1}{x}}.\left(\frac{1}{x}\right)'}=\lim_{x\to0^+}\frac{\frac{1}{x}}{e^{\frac{1}{x}}.\left(-\frac{1}{x^2}\right)}=-\lim_{x\to0^+}\frac{1}{e^{\frac{1}{x}}.\left(\frac{1}{x}\right)}$$

$$=-\lim_{x\to0^+}\frac{e^{-\frac{1}{x}}}{\frac{1}{x}}=-\lim_{x\to0^+}\frac{\left(e^{-\frac{1}{x}}\right)'}{\left(\frac{1}{x}\right)'}=-\lim_{x\to0^+}\frac{e^{-\frac{1}{x}}.\left(-\frac{1}{x}\right)'}{\left(-\frac{1}{x^2}\right)}=-\lim_{x\to0^+}\frac{e^{-\frac{1}{x}}.\frac{1}{x^2}}{\left(-\frac{1}{x^2}\right)}=\lim_{x\to0^+} e^{-\frac{1}{x}}$$

$$=\lim_{x\to0^+}\frac{1}{e^{\frac{1}{x}}}=0.$$

Bài 11.

Câu b. Khai triển chuỗi Maclaurin của hàm $f(x)=\frac{\sin x}{x}$.

$$\sin x=f(0)+\frac{f'(0)}{1!}.x+\frac{f''(0)}{2!}.x^2+\frac{f'''(0)}{3!}.x^3+\frac{f^{(4)}(0)}{4!}.x^4+...+\frac{f^{(n)}(0)}{n!}.x^n+R_n(x)$$

$$f(x)=\sin x\Rightarrow f'(x)=\cos x\Rightarrow f''(x)=-\sin x\Rightarrow f'''(x)=-\cos x$$

$$\Rightarrow...\Rightarrow f^{(n)}(x)=(\sin x)^{(n)}=(-1)^n\sin\left(x+\frac{n\pi}{2}\right)$$

$$\sin x=x-\frac{1}{3!}.x^3+\frac{1}{5!}.x^5-\frac{1}{7!}.x^7+\frac{1}{9!}.x^9-\frac{1}{11!}.x^{11}+...+(-1)^m\frac{1}{(2m+1)!}.x^{2m+1}+\theta\left(x^{2m+2}\right)$$

$$\Rightarrow\frac{\sin x}{x}=1-\frac{1}{3!}.x^2+\frac{1}{5!}.x^4-\frac{1}{7!}.x^6+\frac{1}{9!}.x^8-\frac{1}{11!}.x^{10}+...+(-1)^m\frac{1}{(2m+1)!}.x^{2m}+\theta\left(x^{2m+1}\right)$$