1/ Tính $\int_0^{+\infty} \frac{dx}{1+x^2}$.

Giải. Đặt $I = \int_0^{+\infty} \frac{dx}{1+x^2}$

Ta có $I = \int_0^{+\infty} \frac{dx}{1+x^2} = \lim_{a\to+\infty} \int_0^{a} \frac{dx}{1+x^2}$.

Ta có $\int_0^{a} \frac{dx}{1+x^2} = \arctan x\Big|_0^a = \arctan a - \arctan 0 = \arctan a$.

Suy ra $I = \lim_{a\to+\infty} \int_0^{a} \frac{dx}{1+x^2} = \lim_{a\to+\infty} \arctan a = \frac{\pi}{2}$.

2/ Tính $\int_2^{+\infty} \frac{dx}{x^2+x-2}$

Đặt $K = \int_2^{+\infty} \frac{dx}{x^2+x-2} = \lim_{t\to+\infty} \int_2^{t} \frac{dx}{x^2+x-2}$

Ta có $\int_2^{t} \frac{dx}{x^2+x-2} = \int_2^{t} \frac{dx}{(x-1)(x+2)}$.

Giả sử

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$
$$\Leftrightarrow 1 = A(x+2) + B(x-1)$$
$$\Leftrightarrow 1 = x(A+B) + (2A-B)(*)$$

Đồng nhất 2 vế của (*) ta được hệ phương trình:

$$\begin{cases} A+B=0 \\ 2A-B=1 \end{cases} \Leftrightarrow \begin{cases} A=\frac{1}{3} \\ B=-\frac{1}{3} \end{cases}$$

Suy ra $\frac{1}{(x-1)(x+2)} = \frac{1/3}{x-1} + \frac{-1/3}{x+2}$

Ta có

$$\int_2^t \frac{dx}{x^2+x-2} = \int_2^t \frac{dx}{(x-1)(x+2)} = \int_2^t \left(\frac{1/3}{x-1} + \frac{-1/3}{x+2} \right) dx = \left(\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| \right) \Bigg|_2^t$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| \Bigg|_2^t = \frac{1}{3}\ln\left|\frac{t-1}{t+2}\right| - \frac{1}{3}\ln\left|\frac{1}{4}\right| = \frac{1}{3}\ln\left|\frac{t-1}{t+2}\right| + \frac{1}{3}\ln 4$$

Suy ra $K = \lim\limits_{t\to+\infty} \left(\frac{1}{3}\ln\left|\frac{t-1}{t+2}\right| + \frac{1}{3}\ln 4 \right) = \frac{1}{3}\ln 1 + \frac{1}{3}\ln 4 = \frac{1}{3}\ln 4.$