

# Aufgaben Statistical Models

Prof. Dr.-Ing. Steffen Schober

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## 1 Biased Coin Flipping

1. Consider flipping a biased coin with parameter  $\theta$ . Prove that

$$\hat{\theta}_{MAP} = \frac{k + a - 1}{N + a + b - 2}.$$

for a  $\text{Beta}(\theta|a, b)$  prior,  $N$  trials, with  $k$  HEADs. Hint: Start from the proof of the maximum likelihood solution discussed in the lecture.

2. Now let  $\theta = 0.25$ . Draw  $N = 5$  samples and compute and plot the posterior distribution, if you start with a prior  $\text{Beta}(\theta|a, b)$  with  $a = 2$  and  $b = 2$ . Compare this with the situation where you use  $a = 4$  and  $b = 4$ .

## 2 ML for the Gaussian

Given data  $\mathcal{D} = (\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)})$  assumed to be independently drawn from a multivariate Gaussian distribution with dimension  $p$ .

1. Give the formula of the log likelihood function.
2. Let  $p = 1$ , derive the maximum likelihood solution.