

Проверить, является ли функция f выпуклой (вогнутой) на заданном множестве X , или указать области, в которых f является выпуклой или вогнутой:

1. $f(x) = x_1^6 + x_2^2 + x_3^2 + x_4^2 + 10x_1 + 5x_2 + 3x_4 + 20$; $X = \mathbf{R}^4$;
2. $f(x) = e^{2x_1+x_2}$; $X = \mathbf{R}^2$;
3. $f(x) = -x_1^3 - x_2^3 - x_3^3 + 10x_1 - x_2 + 15x_3 + 6$; $X = \{x \in \mathbf{R}^3 : x \geq 0\}$;
4. $f(x) = x_1^2 + x_2^2 + \frac{1}{2}x_3^2 + x_4^2 + x_1x_2 - x_3 + 10$; $X = \mathbf{R}^4$;
5. $f(x) = -x_1^2 - x_2^2 - 2x_3^2 + x_1x_2 + x_1x_3 + x_2x_3 + 5x_2 + 25$; $X = \mathbf{R}^3$;
6. $f(x) = -x_2^5 + \frac{1}{2}x_3^2 + 7x_1 - x_3 + 6$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
7. $f(x) = 3x_1^2 + x_2^2 + 2x_3^2 + x_1x_2 + 3x_1x_3 + x_2x_3 + 3x_2 - 6$; $X = \mathbf{R}^3$;
8. $f(x) = 5x_1^2 + \frac{1}{2}x_2^2 + 4x_3^2 + x_1x_2 + 2x_1x_3 + 2x_2x_3 + x_3 + 1$; $X = \mathbf{R}^3$;
9. $f(x) = -2x_1^2 - \frac{1}{2}x_2^2 - 5x_3^2 + \frac{1}{2}x_1x_2 + 2x_1x_3 + x_2x_3 + 3x_1 - 2x_2 + 6$; $X = \mathbf{R}^3$;
10. $f(x) = x_1^3 + 2x_3^3 + 10x_1 + x_2 - 5x_3 + 6$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
11. $f(x) = 5x_1^4 + x_2^6 + x_3^2 - 13x_1 + 7x_3 - 8$; $X = \mathbf{R}^3$;
12. $f(x) = -3x_1^2 + x_2^2 + 2x_3^2 + 3x_1x_2 + x_1x_3 + 2x_2x_3 + 17$; $X = \mathbf{R}^3$;
13. $f(x) = 4x_1^3 - x_2^4 - \frac{1}{2}x_3^4 + 3x_1 + 8x_2 + 11$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
14. $f(x) = 10 - (x_2 - x_1)^2$; $X = \{x \in \mathbf{R}^2 : -1 \leq x_i \leq 1, i = 1, 2\}$;
15. $f(x) = 8x_1^3 - 12x_3^2 - 3x_1x_2 + 6x_2 + 17$; $X = \{x \in \mathbf{R}^3 : x \geq 0\}$;
16. $f(x) = -2x_1^2 - 2x_2^2 - 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3 + 10$; $X = \mathbf{R}^3$;
17. $f(x) = -\frac{1}{2}x_1^7 + \frac{1}{2}x_3^4 + 2x_2x_3 + 11x_1 + 6$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
18. $f(x) = \frac{5}{2}x_1^2 + x_2^2 + 4x_3^2 + \frac{3}{2}x_1x_2 + 2x_1x_3 + \frac{1}{2}x_2x_3 + 8x_3 + 13$; $X = \mathbf{R}^3$;
19. $f(x) = -3x_1^2 + \frac{1}{2}x_2^3 + 2x_1x_2 + 5x_1x_3 + 7x_1 + 16$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
20. $f(x) = -2x_1^2 - x_2^2 + \frac{3}{2}x_3^2 + x_1x_2 + \frac{1}{2}x_1x_3 + 2x_2x_3 + 10$; $X = \mathbf{R}^3$;
21. $f(x) = 2x_1^2 + \frac{3}{2}x_3^2 + \frac{5}{2}x_1x_3 + 12x_2 + 18$; $X = \mathbf{R}^3$;
22. $f(x) = 6x_1^2 + x_2^3 + 6x_3^2 + 12x_1 - 8x_3 + 7$; $X = \{x \in \mathbf{R}^3 : x \geq 0\}$;
23. $f(x) = -x_1^2 - \frac{3}{2}x_2^2 - 2x_3^2 + x_1x_2 + x_1x_3 - 2x_2x_3 + 8x_2$; $X = \mathbf{R}^3$;
24. $f(x) = \frac{7}{2}x_1^2 - \frac{4}{3}x_2^3 - \frac{1}{2}x_3^3 + 13x_1 - 9$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
25. $f(x) = \frac{7}{2}x_1^2 - \frac{4}{3}x_2^3 - \frac{1}{2}x_3^3 + 13x_1 - 9$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
26. $f(x) = -\frac{5}{6}x_1^3 - \frac{1}{4}x_2^5 - \frac{3}{2}x_3^3 + 22x_2 + 17$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;

27. $f(x) = -\frac{5}{6}x_1^3 - 2x_2^2 + \frac{3}{2}x_3^2 + 2x_1x_2 + 3x_1x_3 + x_2x_3$; $X = \{x \in \mathbf{R}^3 : x \leq 0\}$;
28. $f(x) = 2x_1^2 + 4x_2^2 + x_3^2 - x_1x_2 + 9x_1x_3 + x_2x_3 - 9$; $X = \mathbf{R}^3$;
29. $f(x) = \frac{3}{2}x_1^2 + \frac{5}{2}x_2^2 - \frac{9}{2}x_3^2 - 3x_1x_3 + 7x_2x_3$; $X = \mathbf{R}^3$;
30. $f(x) = -\frac{7}{6}x_1^3 + \frac{5}{2}x_2^2 + \frac{5}{12}x_3^4 + \frac{1}{2}x_4 - 3x_2$; $X = \{x \in \mathbf{R}^4 : x \geq 0\}$;
31. $f(x) = 4x_1^2 + x_2^2 - 2x_1x_2 + 6x_1 - x_2 - 2$; $X = \mathbf{R}^2$;
32. $f(x) = \sqrt{1 + x_1^2 + x_2^2}$; $X = \mathbf{R}^2$;
33. $f(x) = x_1^2 + x_2^2 - \cos \frac{x_1 - x_2}{2}$; $X = \mathbf{R}^2$;
34. $f(x) = \frac{x_1^2}{x_2}$; $X = \{x \in \mathbf{R}^2 : -1 \leq x_2 \leq 1, x_2 \leq x_1^2 + 1\}$;
35. $f(x) = \sin(x_1 + x_2)$; $X = \{x \in \mathbf{R}^2 : -2\pi \leq x_i \leq 2\pi, i = 1, 2\}$;
36. $f(x) = x_1^2 + 2x_2^2 - \sin(x_1 - x_2)$; $X = \{x \in \mathbf{R}^2 : x \geq 0\}$;
37. $f(x) = x_1^2 + 2x_2^2 + \frac{1}{x_1 - x_2}$; $X = \{x \in \mathbf{R}^2 : -\infty < x_1 \leq 0, 1 \leq x_2 < +\infty\}$;
38. $f(x) = -2x_1^4 - 2x_2^2 - 5x_3^2 + 3x_2x_3 + 10x_2 + 22$; $X = \{x \in \mathbf{R}^3 : x \geq 0\}$;
39. $f(x) = -3x_1^2 - 4x_2^4 - 5x_3^2 + 2x_1x_3 + 2x_2 - 7$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0, x_3 \geq 0\}$;
40. $f(x) = -2x_1^3 - 3x_2^2 - x_3^3 + 4x_1x_2 + 7x_3 + 18$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
41. $f(x) = -2x_1^2 - \frac{1}{2}x_2^3 - 3x_3^2 + 2x_1x_2 - 3$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0, x_2 \geq 0\}$;
42. $f(x) = \frac{1}{3}x_1^3 + \frac{3}{2}x_2^2 + \frac{5}{2}x_3^2 - 10x_3$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
43. $f(x) = -\frac{1}{2}x_1^3 - x_2^2 - \frac{3}{2}x_3^2 + 2x_2x_3 - 15$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
44. $f(x) = \frac{1}{3}x_1^4 + 3x_2^2 + 2x_3^3 + 2x_2x_3 + 7x_1$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0\}$;
45. $f(x) = -x_1^3 - 2x_2^2 - 3x_3^2 + 2x_2x_3 + 5$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
46. $f(x) = -\frac{5}{6}x_1^3 - 4x_2^2 + \frac{1}{3}x_3^3 + 10$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0\}$;
47. $f(x) = -\frac{1}{2}x_1^3 - x_2^2 - \frac{5}{2}x_3^5 + 2x_2x_3 - 4$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
48. $f(x) = -\frac{7}{2}x_1^2 - \frac{5}{6}x_2^3 + \frac{1}{5}x_3^5 - 5$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0\}$;
49. $f(x) = -x_1^7 - \frac{1}{2}x_2^4 - 2x_3^2 - 5$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0, x_3 \geq 0\}$;
50. $f(x) = x_1^6 + 4x_2^2 + 2x_3^5 + 3x_4^2$; $X = \{x \in \mathbf{R}^3 : x_2 \geq 0, x_3 \geq 0\}$;
51. $f(x) = \frac{5}{3}x_1^3 + \frac{5}{2}x_2^2 + 2x_3^5 + 2x_2x_3$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0\}$;
52. $f(x) = \frac{1}{3}x_1^3 + 2x_2^2 + \frac{3}{20}x_3^5 - x_4^2 + x_1x_4 - 4$; $X = \{x \in \mathbf{R}^3 : x_1 \geq 0, x_2 \geq 0\}$;

$$53. \ f(x) = \frac{9}{4}x_1^4 + \frac{1}{3}x_2^5 + x_3^2 - x_1x_2; \ X = \{x \in \mathbf{R}^3 : \ x_1 \geq 0, \ x_3 \geq 0\};$$

$$54. \ f(x) = 2x_1^5 + 4x_2^2 + \frac{3}{2}x_3^3 - x_1x_2; \ X = \{x \in \mathbf{R}^3 : \ x_1 \geq 0, \ x_2 \geq 0\}.$$