Methods of optimization

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Lab number 1

1 Task

N 5.18 ((17 + 6) mod 20) + 1 = 4
Prove that the set X = $\{2x_1^2 - 5x_1x_2 - 3x_2^2 \le 0, x_2 \ge 0\}$ is convex cone.

Solution:
We can notice, that: $2x_1^2 - 5x_1x_2 - 3x_2^2$ equals $(x_1 - 3x_2)(2x_1 + x_2)$
From here we have:

$$\begin{cases} 2x_1^2 - 5x_1x_2 - 3x_2^2 \le 0, \\ x_2 \ge 0; \end{cases} \tag{1}$$

equals:

$$\begin{cases} x_1 - 3x_2 \le 0, \\ 2x_1 + x_2 \ge 0; \end{cases} \tag{2}$$

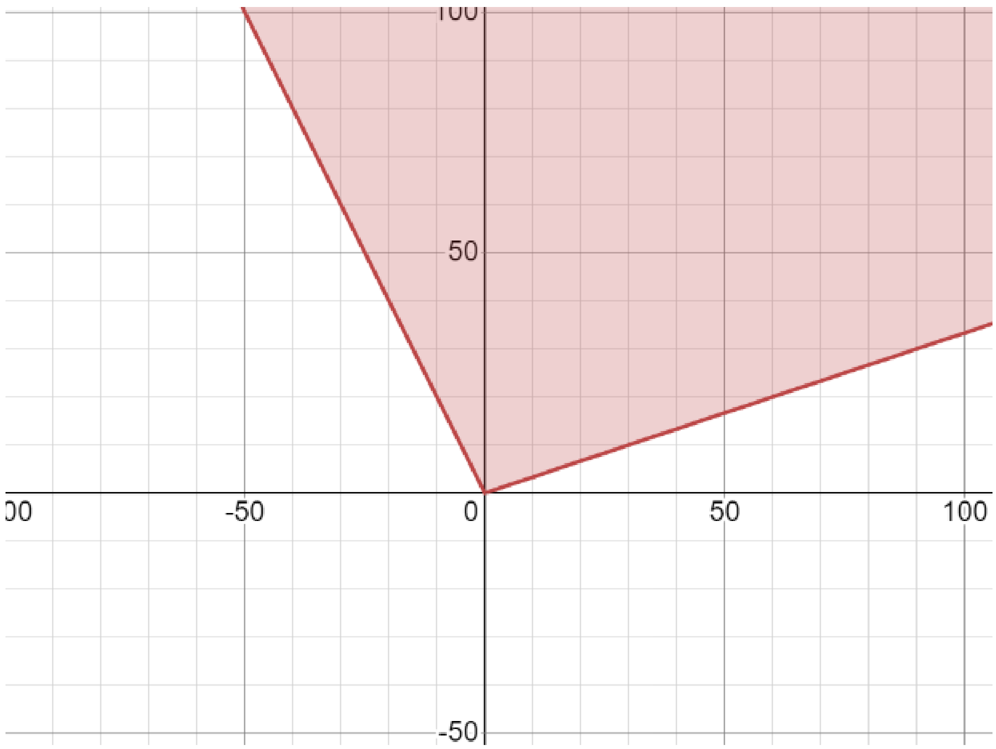
Now let's substitute $x = (x_1, x_2)$ with $(y_1 + z_1, y_2 + z_2)$ where $(y_1, y_2), (z_1, z_2) \in X$ and after that we have:

$$\begin{cases} y_1 + z_1 - 3(y_2 + z_2) \le 0, \\ 2(y_1 + z_1) + y_2 + z_2 \ge 0; \end{cases} \tag{3}$$

equals:

$$\begin{cases} (y_1 - 3y_2) + (z_1 - 3z_2) \le 0, \\ (2y_1 + y_2) + (2z_1 + z_2) \ge 0; \end{cases} \tag{4}$$

From here we can see that: $(y_1 - 3y_2), (z_1 - 3z_2) \le 0, (2y_1 + y_2), (2z_1 + z_2) \ge 0$ this means that $(y_1 + z_1, y_2 + z_2) \in X$. and that means that X is convex cone.



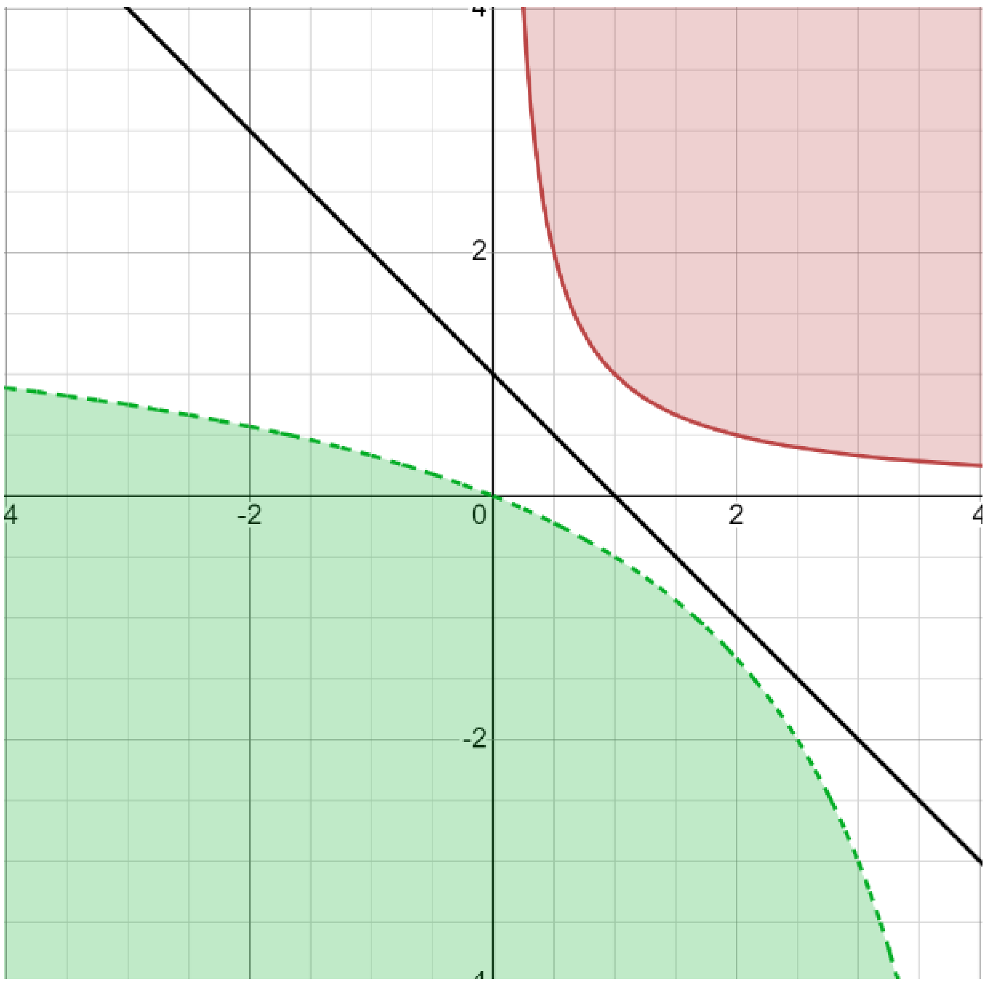
2 Task

N 5.23 ((17 + 7) mod 30) + 1 = 25
Write down the equation of the hyperplane separating the sets:
$X_1 = \{x : x_1x_2 \geq 1, x_1 \geq 0\}$,
$X_2 = \{x : x_2 \leq \frac{10}{x_1 - 5} + 2, x_1 < 5\}$.

Solution:
We need to find a hyperplane that separates the sets:

$$\begin{cases} X_1 = \{x : x_1x_2 \geq 1, x_1 \geq 0\}, \\ X_2 = \{x : x_2 \leq \frac{10}{x_1 - 5} + 2, x_1 < 5\}; \end{cases} \tag{5}$$

Let's assume that this is a plane with the formula: $x_2 = 1 - x_1$



Now let's check whether this hyperplane not intersects two sets:
1)X_1: $x_1 x_2 = 1$ from this we have $x_2 = \frac{1}{x_1}$ that means $x_1 = 0$ is not a solution.

$$\begin{cases} x_2 = \frac{1}{x_1}, \\ x_2 = 1 - x_1; \end{cases} \tag{6}$$

$1 - x_1 = \frac{1}{x_1}$
$\frac{x_1^2 - x_1 + 1}{x_1} = 0$
Discriminant is negative, there are no non-complex solutions.

2)X_2: $x_2 = \frac{10}{x_1 - 5} + 2$ that means $x_1 = 5$ is not a solution.

$$\begin{cases} x_2 = \frac{1}{x_1}, \\ x_2 = \frac{10}{x_1 - 5} + 2; \end{cases} \tag{7}$$

$x_2 = \frac{10}{x_1 - 5} + 2 = \frac{1}{x_1}$
$\frac{2x_1^2 - x_1 + 5}{x_1(x_1 - 5)} = 0$
Discriminant is negative, there are no non-complex solutions.
ANSWER: $x_2 = 1 - x_1$.