Methods of optimization

Roman Nichiporuk

Lab number 2

1 Task

(4(17 + 13) mod 54) + 1 = 13
It is necessary to check whether a function f is convex (concave) on a given set X, or indicate the regions in which f is convex or concave:
$f(x) = 4x_1^3 - x_2^4 - \frac{1}{2}x_3^4 + 3x_1 + 8x_2 + 11$, $X = \{x \in R^3 : x \leq 0\}$.

$$\frac{df}{dx} = \begin{bmatrix} 12x_1^2 + 3 \\ -4x_2^3 + 8 \\ -2x_3^3 \end{bmatrix}$$

$$\frac{df^2}{d^2x} = \begin{bmatrix} 24x_1 & 0 & 0 \\ 0 & -12x_2^2 & 0 \\ 0 & 0 & -6x_3^2 \end{bmatrix}$$

$\Delta_{1.1} = 24x_1 \leq 0$, $\Delta_{1.2} = -12x_2 \leq 0$, $\Delta_{1.3} = -6x_3 \leq 0$.
$\Delta_{2.1} = -288x_1x_2^2 \geq 0$, $\Delta_{2.2} = -144x_1x_3^2 \geq 0$, $\Delta_{2.3} = 72x_2^2x_3^2 \geq 0$.
$\Delta_3 = 1728x_1x_2^2x_3^2 \leq 0$.
Function isn't convex, let's check it for concave:
$-f(x) = -4x_1^3 + x_2^4 + \frac{1}{2}x_3^4 - 3x_1 - 8x_2 - 11$, $X = \{x \in R^3 : x \leq 0\}$.

$$\frac{d(-f)}{dx} = \begin{bmatrix} -12x_1^2 - 3 \\ 4x_2^3 - 8 \\ 2x_3^3 \end{bmatrix}$$

$$\frac{d(-f)^2}{d^2x} = \begin{bmatrix} -24x_1 & 0 & 0 \\ 0 & 12x_2^2 & 0 \\ 0 & 0 & 6x_3^2 \end{bmatrix}$$

$\Delta_{1.1} = -24x_1 \geq 0$, $\Delta_{1.2} = 12x_2 \geq 0$, $\Delta_{1.3} = 6x_3 \geq 0$.
$\Delta_{2.1} = 288x_1x_2^2 \geq 0$, $\Delta_{2.2} = -144x_1x_3^2 \geq 0$, $\Delta_{2.3} = 72x_2^2x_3^2 \geq 0$.
$\Delta_3 = 1728x_1x_2^2x_3^2 \leq 0$.

Function is concave.
ANSWER: Function is concave.