Using LSTM to predict stock price and apply Black-Litterman Model to allocate asset

1. Introduction

In stock market, it is important to spread the risk. Therefore, I apply Black-Litterman Model in my work. Black-Litterman Model(BL Model) is one of the best way of Asset Allocation. By computing the rate of return of each stock as input, then we can get the configuration of asset allocation. According to the estimate of rate of return, this is the best combination. However, what is the method to get the estimate of rate of return? There are two methods. First, I used LSTM model to predict. Using the history stock price from 2018/3 to 2021/3 to train model, then predicting the whole month on April, 2021. Second, I used the target price from investment company/bank.

2. Problem Statement

In stock market, there are a lot of investment company such as some foreign bank, JPMorgan, UBS, Deutsche Bank, Nomura securities, etc., or native bank, Mega International Investment Trust Co.,Ltd., Yuanta Securities, etc. These investment companies will evaluate the company, may analyze the company's financial status and future prospects, and then write a report to give the company's stock a target price. Investors can evaluate whether to invest in this company based on these research reports. However, investment companies do not necessarily evaluate every company, so when investors want to know more about these companies, there will be less reference data. Therefore, deep learning can be used to help judge at this time. You can use LSTM to predict the stock price trend, and the LSTM trained with historical stock prices can effectively judge the future trend, and you can make up for the shortcomings of the target price of the investment company to invest.

Therefore, We have prepared 4 stocks, all of which have target prices obtained by the investment company's analysis of them. Then compare it with the future stock price predicted by LSTM. Then put the expected rate of return obtained by these two different methods into the BL model that deals with asset allocation, and compare two different portfolios to verify whether this investment method is feasible.

3. Related Research

Before BL model created, The mainstream asset allocation model is the Markowitz Mean-Variance model. This model define how to diversify investment so as to minimize risks and maximize returns. This is the first time that mathematical statistics methods have been introduced into portfolio theory. It is based on the expected return and standard deviation of various portfolios. The following function can help investor get the optimal asset allocation:

$$\max_{\omega} \omega'\mu - \frac{1}{2}\delta\omega'\Sigma\omega,$$

where μ is expected rate of return on asset, Σ is covariance matrix of asset, δ is investor risk aversion coefficient, and ω is weight of assets in the portfolio, and the best solution is: $\omega = (\delta\Sigma)^{-1}\mu$.

Although Markowitz Model can get the best solution, it is still not suitable for real-world applications. Because this model is sensitive to expected rate of return on asset. For example, if the return rate of one asset A is increased by 1%, the Markowitz model originally calculated that it was a small-volume sale, but it would become a large-volume purchase. So this model is unstable, and the BL model later improved this problem.

4. Model Architecture

In this project, I will use the BL model to find the best asset portfolio. Its input is divided into two types. One is to use LSTM to predict the stock price and calculate the rate of return; the other is the return calculated from the target price researched by the investment bank. Then put the two into the model and compare the returns in April with the asset portfolio obtained.

Input   Output
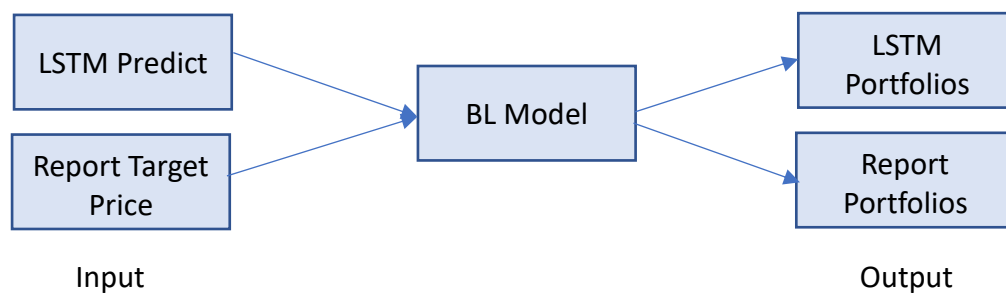
5. Black-Litterman Model

Black-Litterman model is a mathematical model for portfolio allocation developed in 1990 at Goldman Sachs by Fischer Black and Robert Litterman, and published in 1992. It seeks to overcome problems that institutional investors have encountered in applying modern portfolio theory in practice. The model starts with an asset allocation based on the equilibrium assumption[1] and then calculate that allocation[2] by taking into account the opinion of the investor regarding future asset performance[3].

1. Assets will perform in the future as they have in the past 2.Using the formula below to calculate 3. Expected return on assets

The formula is:

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} [(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \quad \text{-(1)}$$

τ: scale factor, mean uncertainty, const

Σ: Covariance matrix of Excess rate of return[1], size N*N

P: Investor's view matrix, size K*N

Π: Implied equilibrium rate of return vector, size is N*1

Q: Point of view earnings vector[2], size K*1

Ω: Covariance matrix of opinion error, used to express the difference between the investor's opinion and the actual situation, size K*K

K: The number of view N: The number of asset

In my work, τ is 0.05, Σ is the covariance matrix calculated using the historical daily rate of return of the four stocks, the formula is as follows:

$$\Sigma = \delta * F + (1 - \delta) * S$$

,where S is identity matrix of the historical daily rate of return matrix, and F is a diagonal matrix which size is 4*4. Each row of matrix P represents a viewpoint, and

each column represents a stock. In my work, I make the matrix equal in the two method. P is 4*4 identity matrix, this mean that computing the rate of return of the stocks without discussing the relationship between stocks. Π is none. Matrix Q is the rate of return for each stock based on two methods. In Report method, Q is [0.1969112, 0.10732538, 0.27472527, 0.10915493]$_T$. In LSTM method, Q is [-0.0124535, 0.03225983, 0.25351432, 0.06221124]. The formula of matrix Ω is: $\tau P'\Sigma P$.

1. Compared with Taiwan weighted index rate of return 2.The rate of return of each asset as a vector

6. Stock Used

I use the stocks of 4 different company. 2002, China Steel Corporation, this company is the largest steel production company in Taiwan. 2330, Taiwan Semiconductor Manufacturing Co., it is an internationally famous company for foundry, what's more its market value is the largest in Taiwan, it is the company with the highest weight in Taiwan's weighted index. 2603, Evergreen Marine Corporation, this is the largest shipping company in Taiwan and the seventh largest in the world. Benefited from the impact of the epidemic, the world's dependence on shipping has increased. The stock price has also risen from single digits in the past to a staggering increase of two hundred. 2881, Fubon Financial Holding Co., Ltd. It is the largest market value and the second largest financial holding Co. Recently expanded the company's strength by merging with other banks.

Stock	2002	2330	2603	2881
Name	中鋼	台積電	長榮	富邦金
Capital(Billion)	157.731	259.304	52.908	134.05

7. Using LSTM to Predict stock price

7.1 Data Collection

I used yahoo finance API on python to collect 3 years stock data to train model, and use this model to predict April, 2021.

Finding the maximal stock price in the predicted data, then calculate the rate of return to apply it on BL model as a parameter matrix(Investor viewpoint matrix). Therefore, we can get the asset allocation from BL model.

7.2 Data Preprocess

I made data split into training data and testing data. First, the training data was normalized between 1 and 0; the purpose is that avoiding the data exceeding situation. Then, I made training data 20-day cycle, because 20-day is a month working days, it belongs mid-to-long term view. Training Label is daily open price.

7.3 What is LSTM?

LSTM consists of four units, input gate, output gate, memory cell, forget gate. Input gate can decide whether to refer to during data input. Output gate can control whether to output the value, if not, output 0. Memory cell can memorize the value

for the next cell. Forget cell can decide whether to forget the value.
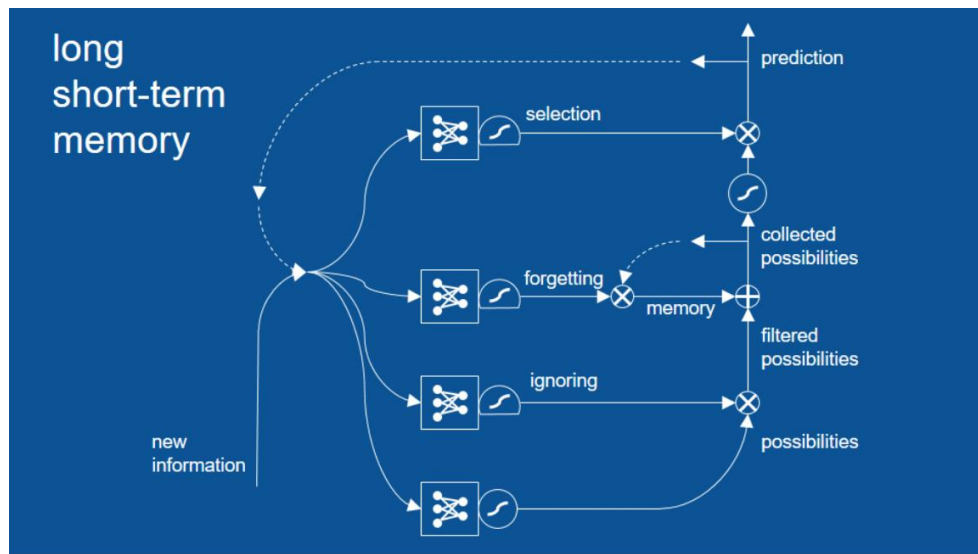
Brandon Rohrer(2017)

7.4 Building Model

I used keras "Sequential" to build a model. First, I added a "LSTM" layer, units = 100, then I added a Dense layer to make output dimension=1. In the end, compiling the model, using 'Adam' optimizer, loss function is 'mean_square_error'. We use it to fit data, epochs = 30, batch size = 16.

7.5 Why is LSTM

I compare this model with SVR(Support Vector Regression). SVR uses SVM to do regression analysis. SVM is good to solve classification problem, but in regression, this algorithm is very sensitive to parameters and kernel function. Because the parameter 'c' in kernel function 'rbf' which can decide the difficulty of decision function. If c is small, the decision function would be simpler; however, the accuracy will be unstable. Moreover, the parameter 'gamma' defines the influence of a single training reaches, low values means 'far' high values means 'close'. Therefore, SVR is very sensitive to gamma, if gamma is too large, the model would overfitting. But if gamma is too small, the model cannot capture the complexity of the data

So the performance of SVR is worse than that of LSTM. We look at it from the perspective of root mean square error.
We used the data from March, 2018 to March, 2021 to train model. Root mean square error (RMS)formula is: sqrt(mean((real_data - predicted_data)^2))

RMS of LSTM: 1.72, RMS of SVR: 10.14. Therefore, we can find that LSTM model is more better than SVR in this view.
Figure 1 and Figure 2 are stock 2603, Figure 1 is using LSTM to predict, and Figure 2 is using SVR to predict, we can compare the forecast for April.
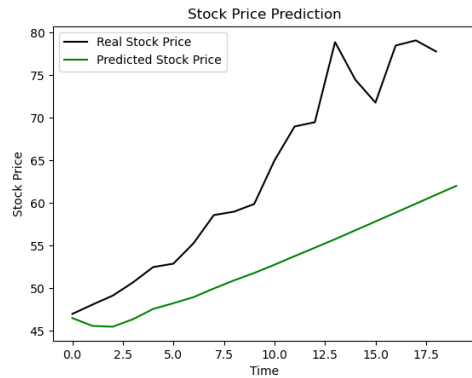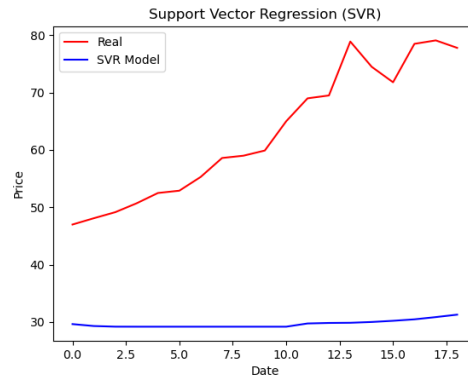
Figure 1

Figure 2

8. Compare with the report of the Investment Company/Bank

I got the target price of the stock from 4 credible investment companies. I used it to calculate the rate of return and apply it on BL model as a parameter matrix(Investor viewpoint matrix). Then, we could get a asset allocation that is different from the using LSTM. Then, we discuss the rate of return in April, 2021.

Stock	2002	2330	2603	2881
Investment Company/Bank	兆豐	瑞士信貸	兆豐	宏遠
Target Price	31.0	650.0	58.0	63.0

9. BL Asset Allocation

I used python API pypfopt.black_litterman to build BL Model. The calculation method I have explained in the second point. We only need to compute Q matrix. The Q matrix uses the obtained rate of return as input. The following are the configurations obtained after applying the BL model respectively:

LSTM

Stock ID	2002	2330	2603	2881
Config	-0.08	-0.49	1.5	0.07

Report

Stock ID	2002	2330	2603	2881
Config	0.74	0.015	0.195	0.05

10. The Rate of Return

We calculate the stock price on the last day of April to calculate the rate of return, We buy all stocks when stock market opened on April 1st, and then sell stocks on April 29th. The formula of Rate of Return is:

$$Rate\ of\ Return = \frac{Sell\ Price - Purchase\ Price}{Purchase\ Price} \times 100\%$$

The stock price on the last day of April (Open price)

Stock ID	2002	2330	2603	2881
4/01	26	598	47	57.6
4/29	39.5	609	77.8	64.9
Rate of Return(%)	51.9	1.5	65.5	12.7

Assume we have NTD20,000,000 initial funds. And we can long(Buy before Sell) or short(Sell before Buy) stock. Let's calculate the rate of return of each method.

9.1 LSTM

Stock ID	2002	2330	2603	2881
	Short	Short	Long	Long
Money(NTD)	800,000	4,900,000	15,000,000	700,000
Earn	-415,200	-73,500	9,825,000	88,900

Total profit: (-415,200) + (-73,500) + 9,825,000 + 88,900 = 9,425,200
The Rate of Return: Total profit / Initial Funds = 9,425,200 / 20,000,000 = 47.13%

10.2   Report of Investment Company

Stock ID	2002	2330	2603	2881
	Long	Long	Long	Long
Money(NTD)	14,800,000	300,000	3,900,000	1,000,000
Earn	7,681,200	4,500	2,554,500	127,000

Total profit: 7,681,200 + 4,500 + 2,554,500 + 127,000 = 10,367,200
The Rate of Return: Total profit / Initial Funds = 51.84%

11. Conclusion

We can find that the rates of return of the two are very close.

The research report of the investment company is the conclusion obtained after a series of precise analysis of the company. The target price is set by a very professional person who specializes in accounting, international situations, economic conditions, business cycles, etc. To give an evaluation.

The LSTM model is based on historical stock price trend data as training data, and the trained artificial intelligence can predict future stock trends.

The LSTM model is suitable for providing a reference basis for investment in addition to the research report of the investment company, combining with the BL model to form a sound investment portfolio, and then earning profits in the stock market.

12. Reference

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