Internship in Ordinal Arithmetic on Isabelle

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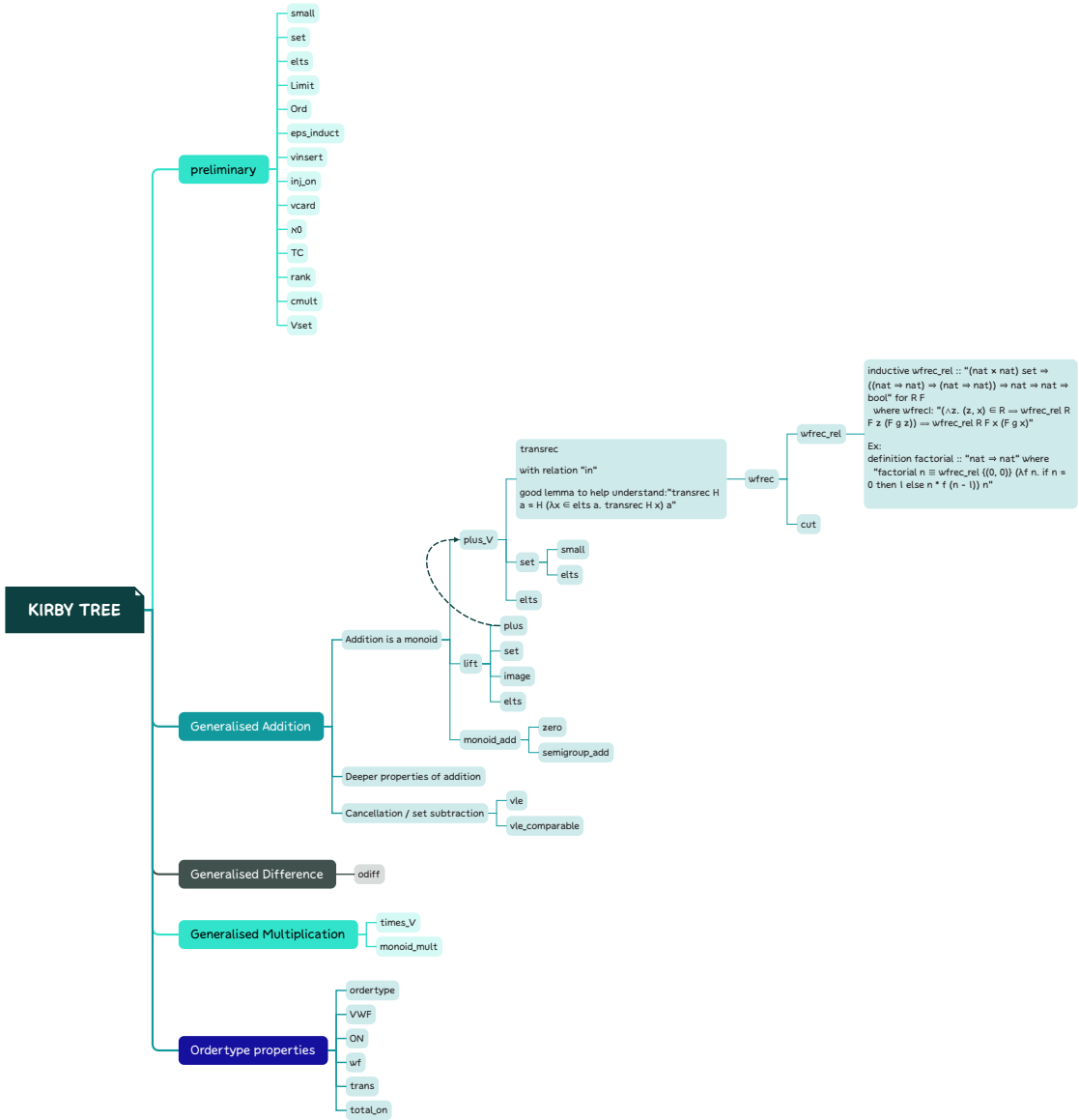
framework

1. Make a dependency graph of all the required definitions
2. Have a semi-decent understanding how the proofs work
3. Clone Isabelle/Set
https://github.com/kappelmann/Isabelle-Set/ and build it.
4. Explore the ‘HOTG‘ directory. It contains basic set-theoretic constructions and axioms that you will have to use.
5. Explore the ‘Isabelle_Set‘ directory. It contains set-theoretic constructions with soft-types. Soft-types are just predicates that describe a type.

It is not yet clear whether we will have to use them. Explore, particularly, the files ‘Ordinals.thy‘ and the ‘Nat‘ folder.

definition tree



proof methodology

Lemma (eps_induct)

Assumes: $\forall x.(\forall y.y \in \mathit{elts}(x) \Rightarrow P(y)) \Rightarrow P(x)$
Shows: $P(a)$

Lemma (mem_induction)

$(\forall X.(\forall x.\mathit{mem}(x, X) \Rightarrow P(x)) \Rightarrow P(X)) \Rightarrow (\forall X.P(X))$

Both follows *wf_induct_rule* form.

Problems

- Why Tarski-Grothendieck set theory, how much should I know.
- How can I use "Jump to Definition" in Isabelle?
- Which tutorials explain the function of some statements? i.e., bundle, instantiation.

Further plan

Reading HOTG paper, or starting to build the addition part