

## **Systems of sentences**

1. Complete, which is also called Non-singular

eg:- The dog is black

The cat is orange

Both the sentence give two different information.

2. Redundant, Which is Singular

eg:- The dog is black

The dog is black

3. Contradictory, which is also a singular.

eg:- The dog is black

The dog is white

## **Likewise, Systems of information is:-**

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.





Solution 1:

The bird is red.

And System of equations is:-

## Systems of equations

### System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 






Unique solution:

$$\begin{aligned} \text{apple } a &= 8 \\ \text{banana } b &= 2 \end{aligned}$$

Complete

Non-singular

### System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  






Infinite solutions

$$\begin{aligned} \text{apple } a &= 8, 7, 6, \dots \\ \text{banana } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

### System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

No solution

Contradictory

Singular

## What is a linear equation?

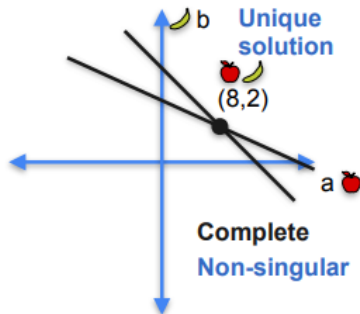
Linear:-  $a+b=10$

Non – linear :-  $a^2 + b^2 = 10$

## Systems of equations as lines

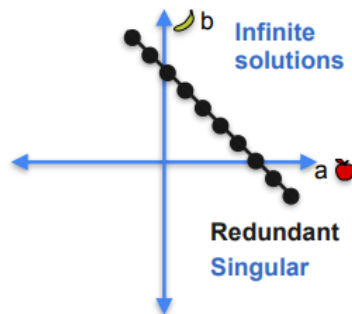
System 1

- $a + b = 10$
- $a + 2b = 12$



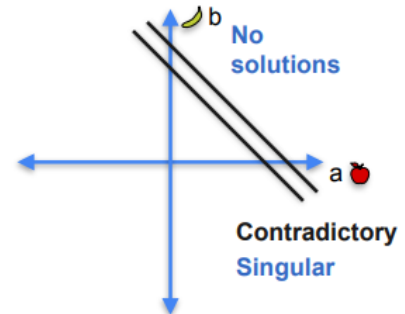
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

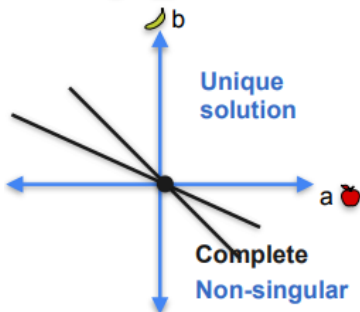
- $a + b = 10$
- $2a + 2b = 24$



## Systems of equations as lines

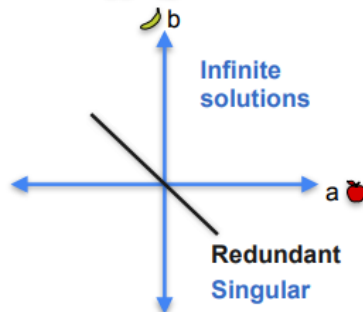
System 1

- $a + b = 0$
- $a + 2b = 0$



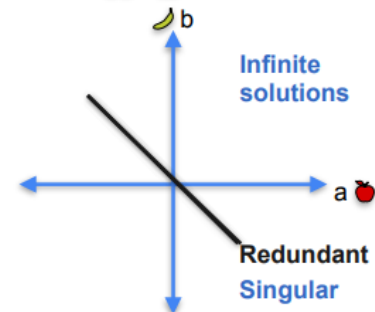
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $a + b = 0$
- $2a + 2b = 0$




# Systems of equations as matrices

System 1

$$\begin{aligned} \bullet a + b &= 0 \\ \bullet a + 2b &= 0 \end{aligned}$$

Non-singular  
system

1	1
1	2

Non-singular  
matrix

(Unique solution)

System 2

$$\begin{aligned} \bullet a + b &= 0 \\ \bullet 2a + 2b &= 0 \end{aligned}$$

Singular  
system

1	1
2	2

Singular  
matrix


(Infinitely many solutions)

## Linear dependence between rows

Non-singular

$$\begin{aligned} \bullet a + b &= 0 \\ \bullet a + 2b &= 0 \end{aligned}$$

No equation is a  
multiple of the  
other one

1	1
1	2

No row is a  
multiple of the  
other one

Rows are  
*linearly independent*

Singular system

$$\begin{aligned} \bullet a + b &= 0 \\ \bullet 2a + 2b &= 0 \end{aligned}$$

Second equation is  
a multiple of the  
first one



 

1	1
2	2

Second row is a  
multiple of the first  
row

Rows are  
*linearly dependent*

# Determinant

a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{array}{c} a \quad b \\ d \quad c \end{array} -$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \cdot k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

Non-singular matrix




1	1
1	2

Determinant

$$\begin{array}{c} 1 \quad 1 \\ 2 \quad 1 \end{array} -$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix




1	1
2	2

Determinant

$$\begin{array}{c} 1 \quad 1 \\ 2 \quad 2 \end{array} -$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

## Solutions: More systems of equations

### System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinitely many sols.**

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0, 5, 5), (1, 4, 5), (2, 3, 5), \dots\end{aligned}$$

### System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\ \text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

### System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

**Infinitely many solutions**

$$\begin{aligned}\text{Any 3 numbers that add} \\ \text{to 10 work.} \\(0, 0, 10), (2, 7, 1), \dots\end{aligned}$$

## Constants don't matter for singularity

### System 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$

**Unique solution**

**Complete**

**Non-singular**

### System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

**Infinite solutions**

**Redundant**

**Singular**

### System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

**No solutions**

**Contradictory**

**Singular**

### System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 15 \\3a + 3b + 3c &= 20\end{aligned}$$

**Infinite solutions**

**Redundant**

**Singular**

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

No relations

$$2\text{Row1} = \text{Row3}$$

Dependent (singular)

Dependent (singular)

Independent  
(Non-singular)

Dependent (singular)

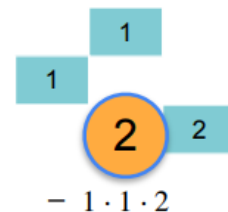
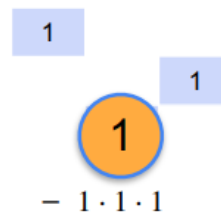
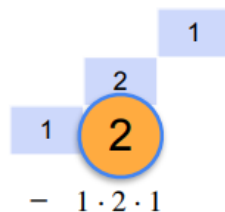
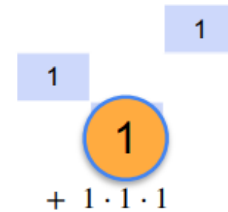
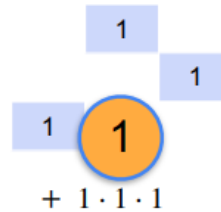
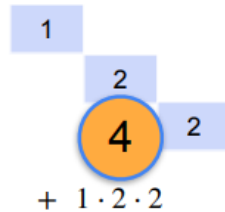
## The determinant

1	1	1
1	2	1
1	1	2

$$\text{Det} = 4 + 1 + 1$$

$$- 2 - 1 - 2$$

$$= 1$$



## Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz).  
Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

**Singular**

1	1	1
1	1	2
0	0	-1

Determinant = 0

**Singular**

1	1	1
0	2	2
0	0	3

Determinant = 6

**Non-singular**

1	2	5
0	3	-2
2	4	10

Determinant = 0

**Singular**