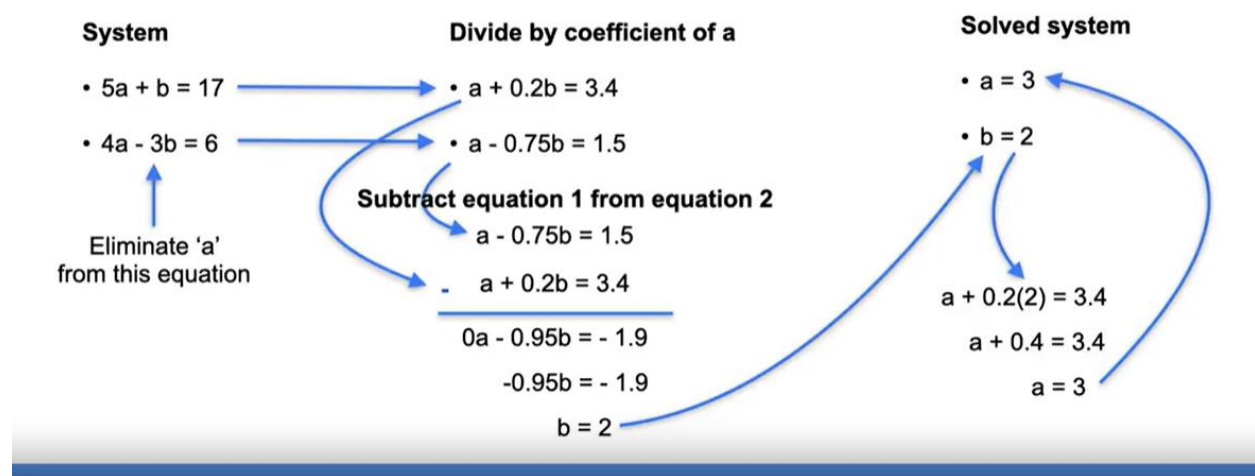


Solving System of linear equations: Elimination

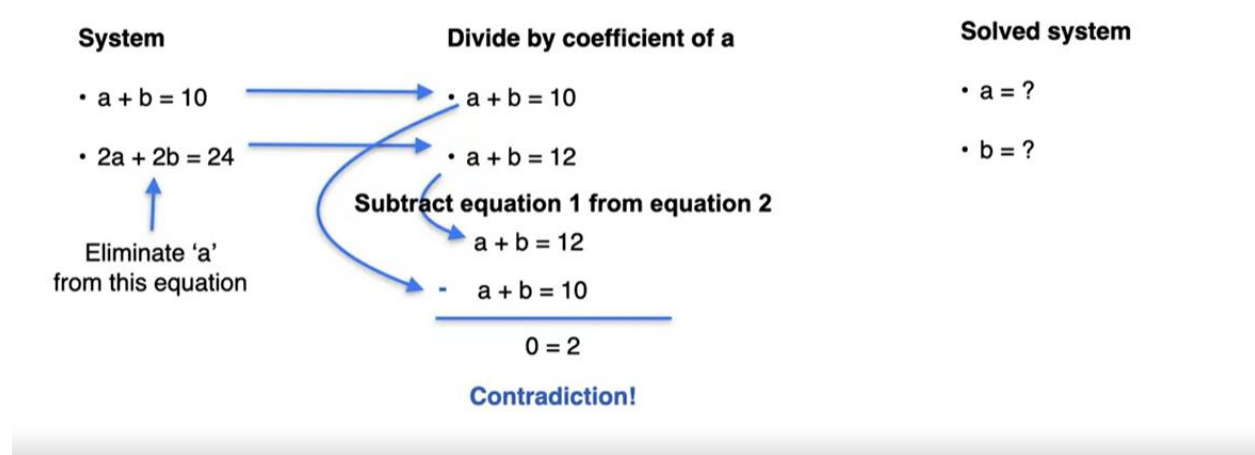
1. Solving non-singular systems of linear equations. (Non-singular system has the unique solution)

Systems of equations



2. Solving singular system of linear equations

What if the system is singular (contradictory)?



3. Solving Systems of equations with more variable

Elimination method

System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$

Elimination method

System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $c = 3$

$\xrightarrow{\quad\quad\quad} a + 2 + 6 = 12$
 $a = 4$

$\xrightarrow{\quad\quad\quad} b + 7/2 = 11/2$
 $b = 2$

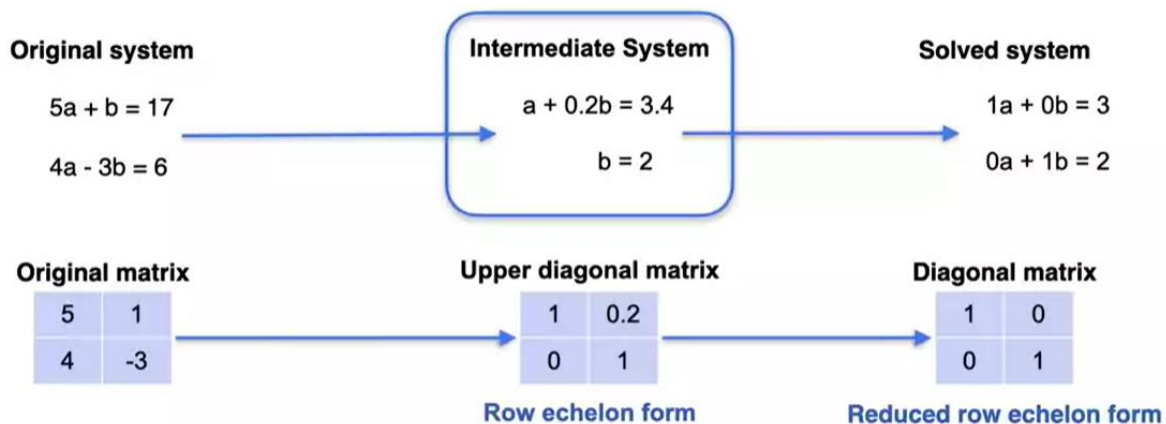
Replace $c = 3$ in the second equation, get $b = 2$

Replace $c = 3$ and $b = 2$ in the first equation, get $a = 4$

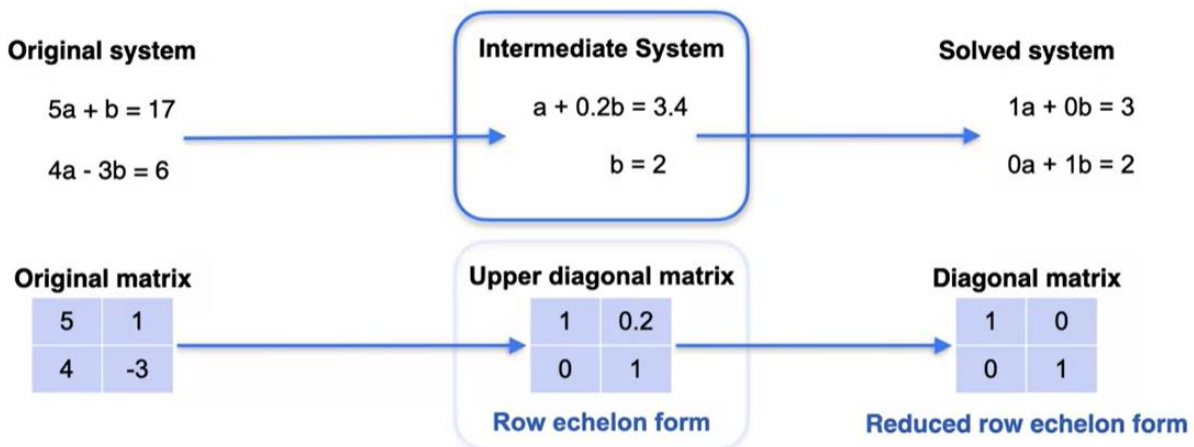
The solution is
 $a = 4$
 $b = 2$
 $c = 3$

4. Matrix row-reduction

Systems of equations to matrices



Systems of equations to matrices



For echelon matrix, On the main diagonal, we can have a bunch of ones followed by perhaps a bunch of zeros. You could potentially have all ones, but you could also have all zeros. Below the diagonal, everything is a zero, to the right of the ones any number is allowed and finally to the right of the zeros, everything must be zero.



5. Row operations that preserve singularity of matrix.
 - a. Switching rows
 - b. Multiplying a row by a (non-zero) scalar
 - c. Adding a row to another row

Solving System of linear equations: Row echelon form and rank

1. The rank of a matrix

Systems of information

System 1



 The dog is **black**
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

System 2

 The dog is **black**
 The dog is **black**

Two sentences

One piece of information

Rank = 1

System 3



 The dog
 The dog

Two sentences

Zero pieces of information

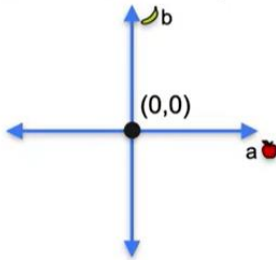
Rank = 0



Rank and solutions to the system

	
1	1
1	2

Rank = 2

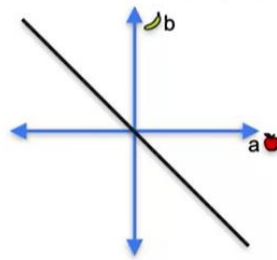
Dimension of solution space = 0





	
1	1
2	2

Rank = 1

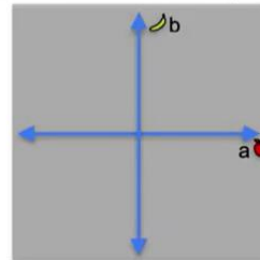
Dimension of solution space = 1





	
0	0
0	0

Rank = 0

Dimension of solution space = 2





Rank of a matrix

	
1	1
1	2



Rank = 2

Dimension of solution space = 0

	
1	1
2	2

Rank = 1

Dimension of solution space = 1

	
0	0
0	0

Rank = 0

Dimension of solution space = 2

$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 \quad \checkmark \\ a + 2b + c &= 0 \quad \checkmark \\ a + b + 2c &= 0 \quad \checkmark \end{aligned}$$

3 Equations
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 \quad \checkmark \\ a + b + 2c &= 0 \quad \times \\ a + b + 3c &= 0 \quad \checkmark \end{aligned}$$

3 Equations
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \quad \checkmark \\ 2a + 2b + 2c &= 0 \quad \times \\ 3a + 3b + 3c &= 0 \quad \times \end{aligned}$$

3 Equations
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \quad \times \\ 0a + 0b + 0c &= 0 \quad \times \\ 0a + 0b + 0c &= 0 \quad \times \end{aligned}$$

3 Equations
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank 0

0	0	0
0	0	0
0	0	0

Is there an easier way to calculate the rank?

>> Yes! It has to do with the row echelon form of the matrix

2. Row echelon form

Row echelon form, singularity, and rank

Non-singular matrix	$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$	Rank 2 2 ones in the diagonal
Singular matrix	$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$	Rank 1 1 one in the diagonal
Singular matrix	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	Rank 0 0 ones in the diagonal

Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0