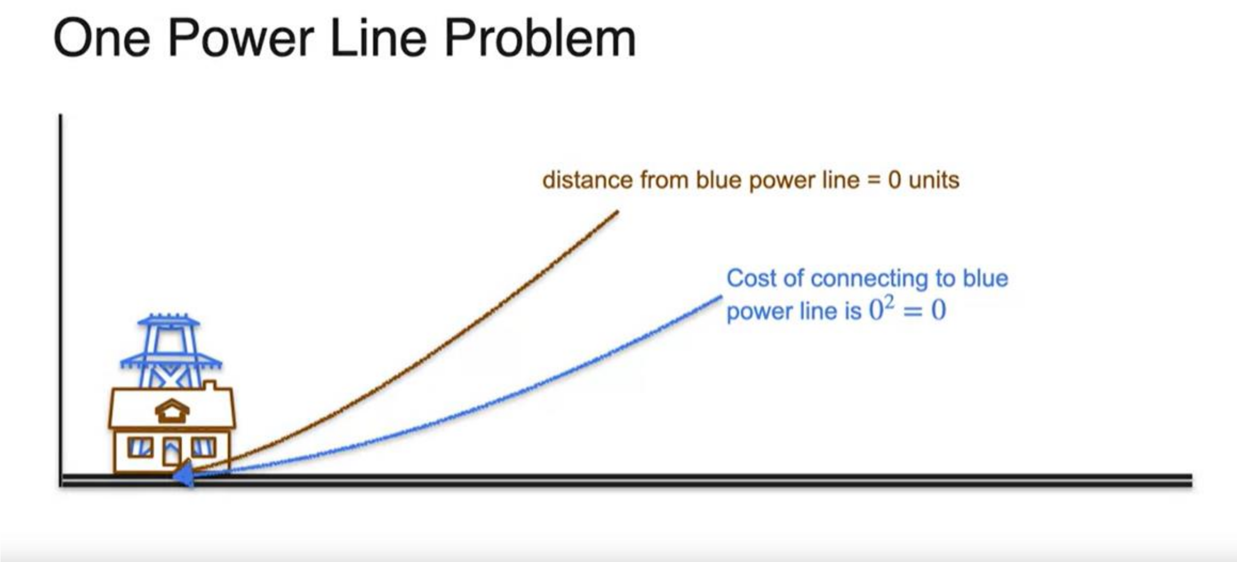
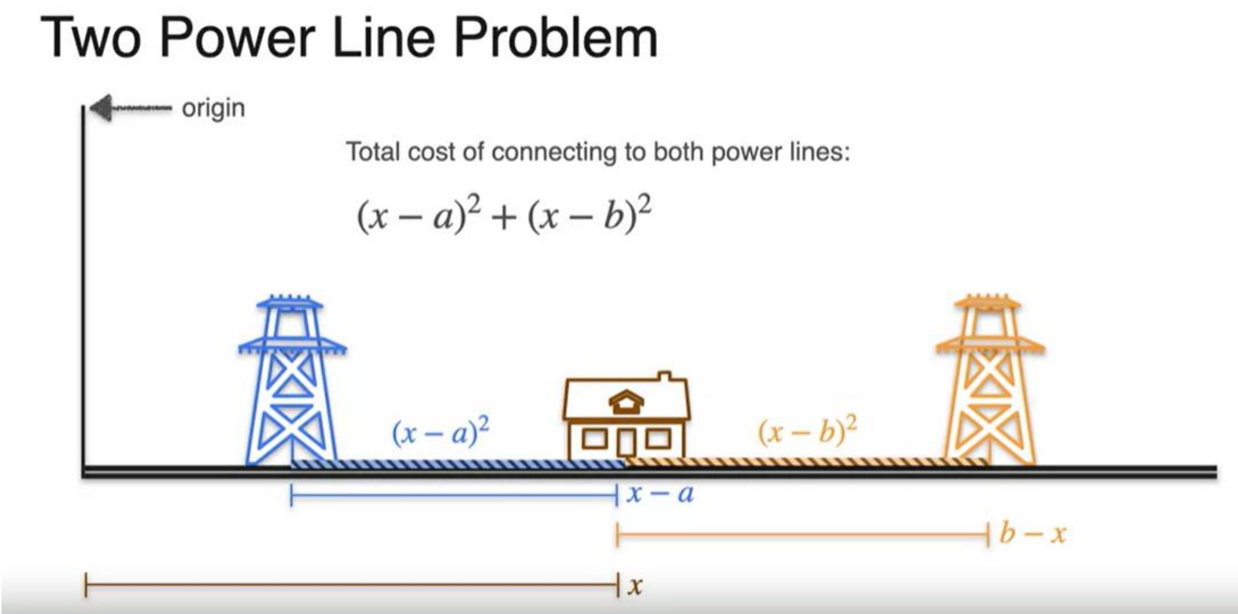
Optimization

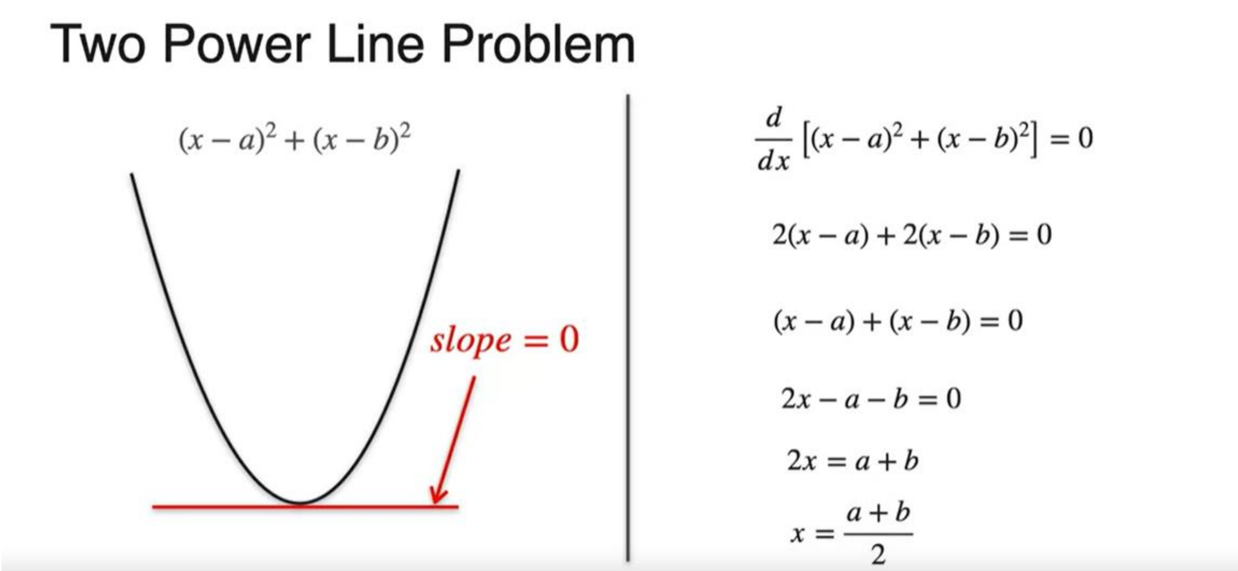
Optimization is the process of finding maximum and minimum values given constraints using calculus. For example, you'll be given a situation where you're asked to find: The Maximum Profit. The Minimum Travel Time. Or Possibly the Least Costly Enclosure.

1. Optimization of squared loss - The one powerline problem

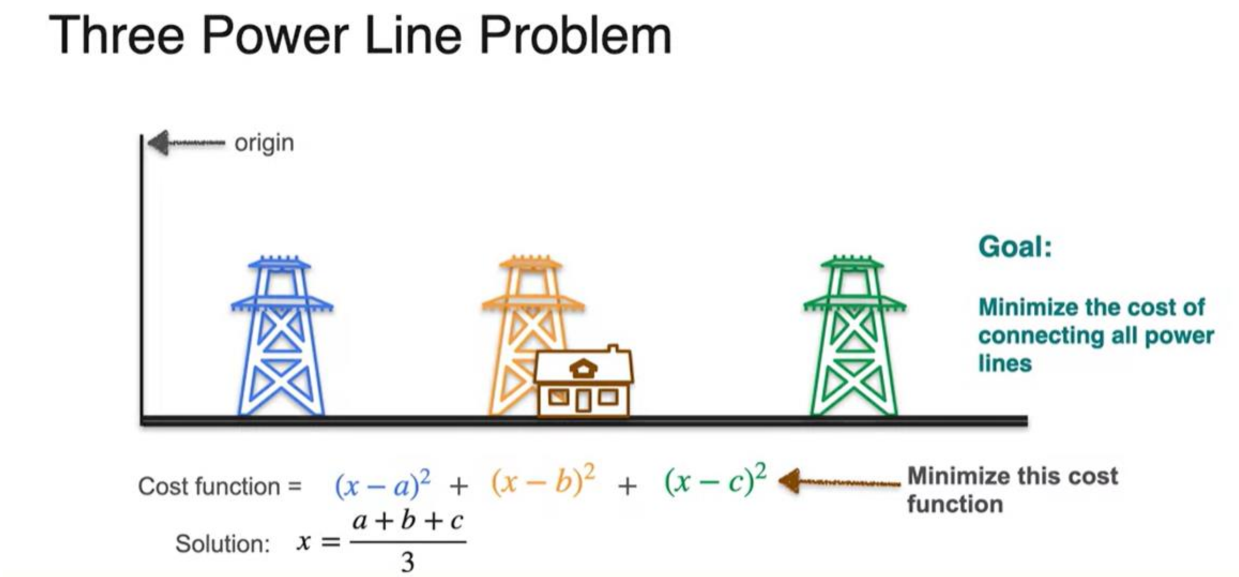


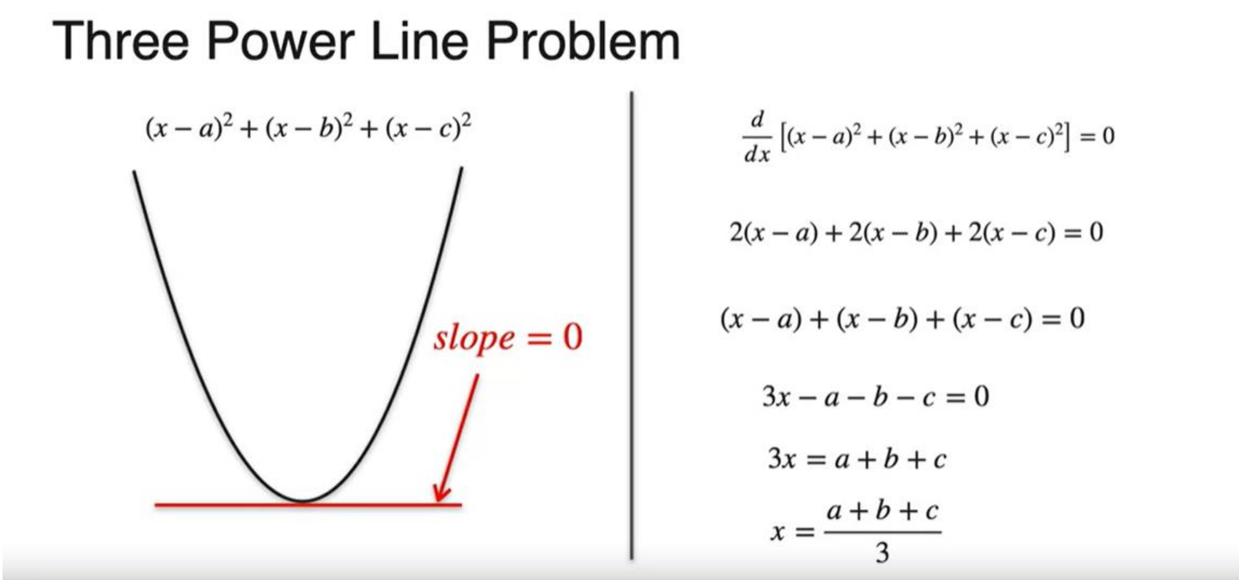
2. Optimization of squared loss - The two-powerline problem





3. Optimization of squared loss - The three-powerline problem





4. Optimization of log-loss

Coins Toss

Coin 1 $0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3 \times 0.3 \times 0.3 = 0.7^7 0.3^3$

$= 0.00222$

Coin 2 $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 0.5^7 0.5^3$

$= 0.00097$

Coin 3 $0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.7 \times 0.7 \times 0.7 = 0.3^7 0.7^3$

$= 0.00008$

Coin Toss

p $(1-p)$

Chances of winning: $p^7(1-p)^3 = g(p)$

Goal: maximize $g(p)$

Coin Toss

Product rule

$$\frac{dg}{dp} = \frac{d}{dp}(p^7(1-p)^3) = \frac{d(p^7)}{dp}(1-p)^3 + p^7 \boxed{\frac{d\left((1-p)^3\right)}{dp}}$$

Chain rule

$$= 7p^6(1-p)^3 + p^7 \boxed{3(1-p)^2(-1)}$$

$$= p^6(1-p)^2\left[7(1-p) - 3p\right]$$

$$= \boxed{p^6}\boxed{(1-p)^2}\boxed{(7-10p)} = 0$$

$p = 0$

$p = 1$

$p = 0.7$

Coin Toss

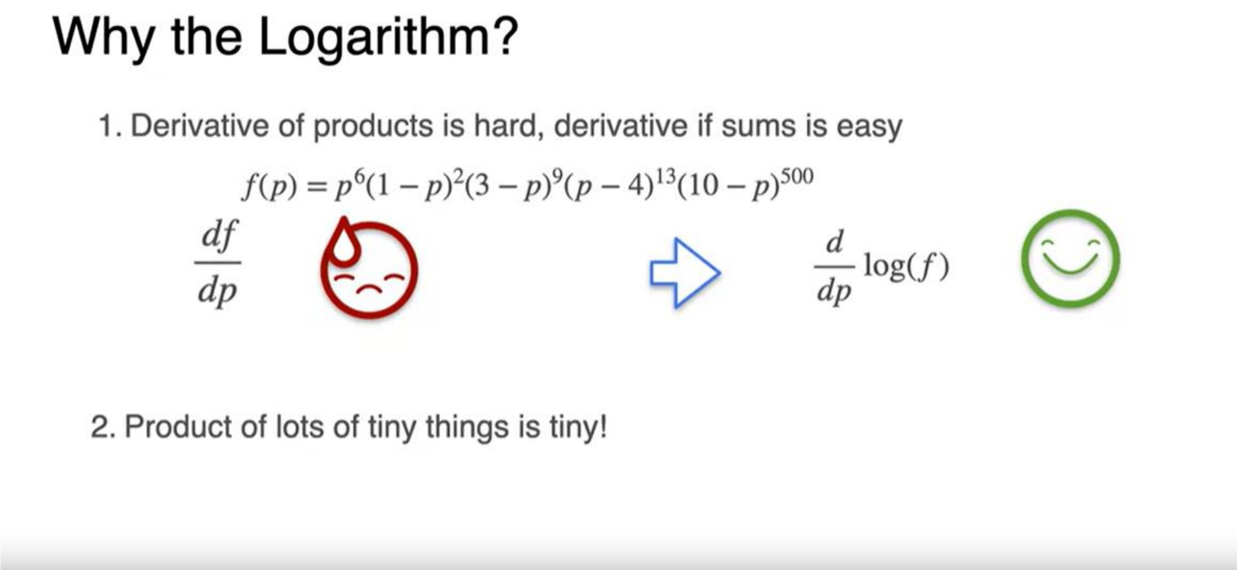
$$\log(g(p)) = \log(p^7(1-p)^3) = \log\left(p^7\right) + \log\left((1-p)^3\right)$$

$$= 7\log(p) + 3\log(1-p) = G(p)$$

$$\frac{dG(p)}{dp} = \frac{d}{dp}(7\log(p) + 3\log(1-p)) = 7\frac{1}{p} + 3\frac{1}{1-p}(-1)$$

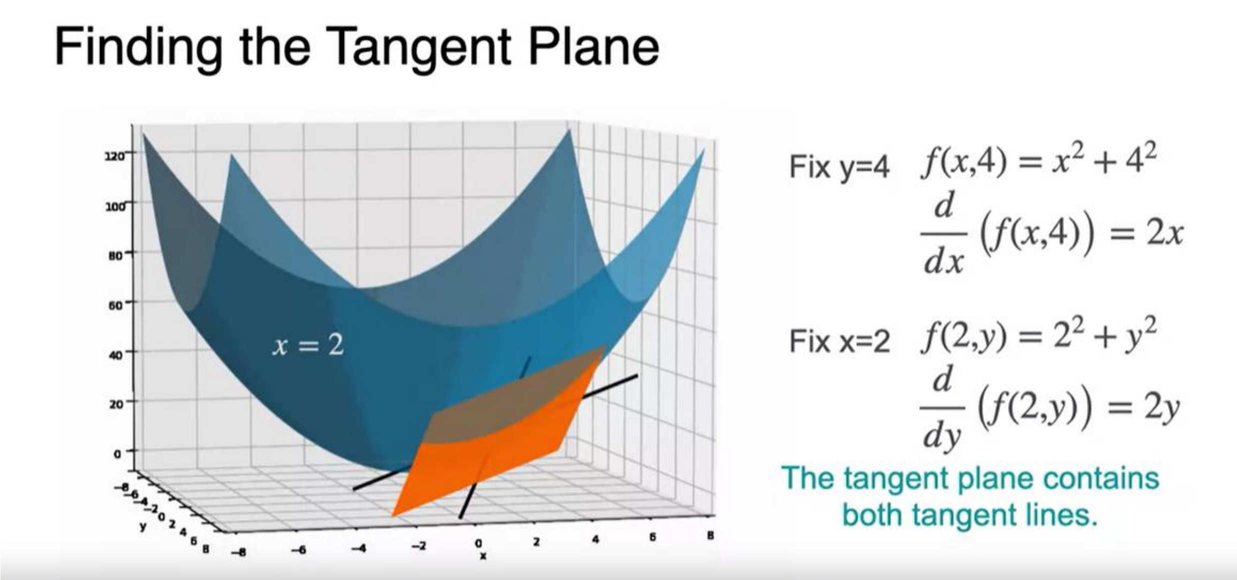
$$= \frac{7(1-p) - 3p}{p(1-p)} = 0$$

$$7(1-p) - 3p = 0 \quad p = 0.7$$

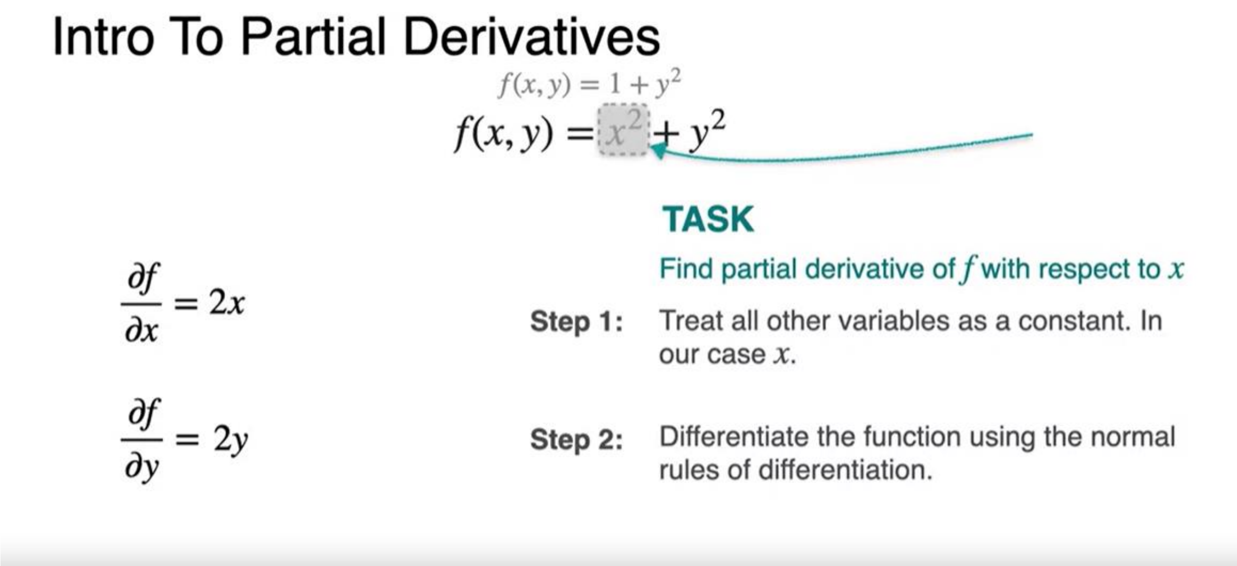


Gradients

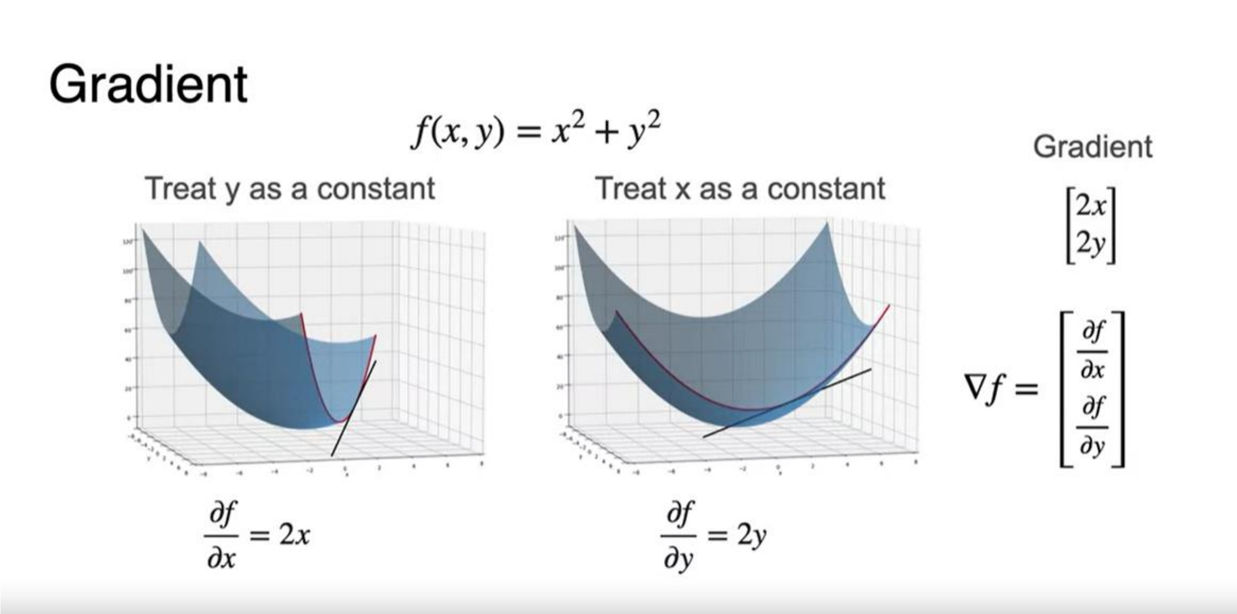
1. Introduction to tangent planes



2. Partial derivatives



3. Gradients



4. Gradients and maxima/minima