Prob Exercises

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Advanced Computational Linguistics

September 22, 2024

1

i) If the probability of A and B happening is the same as the probability of A times the probability of B, then A and B are independent.

ii) Similarly if the probability of A happening given B happening is the equal to the probability of A, then A and B are independent

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = P(A)$$

$$P(A \wedge B) = P(A) \times P(B)$$

2

	mw	$\neg mw$
hh	14	1
$\neg hh$	70	15

a) $P(mw|hh) = \frac{14}{15}$
Counts irrelevant were 70 and 15, where Halaand did not score a hat-trick.

b) $P(hh|mw) = \frac{14}{84} = \frac{1}{6}$
Counts irrelevant were 1 and 15, where Man City did not win.

3

a) $p(yng) = 0.35$
$p(rizz|yng) = 0.01$
$p(rizz|\neg yng) = 0.001$
$p(rizz|yng) \times p(yng)$
$0.01 \times 0.35 = 0.0035$
$p(rizz|\neg yng) \times p(\neg yng)$
$0.001 \times (1 - 0.35) = 0.00065$
$0.0035 > 0.00065$
yng is likelier.

b) $p(yng) = 0.05$
$p(rizz|yng) = 0.01$
$p(rizz|\neg yng) = 0.001$
$p(rizz|yng) \times p(yng)$
$0.01 \times 0.05 = 0.0005$
$p(rizz|\neg yng) \times p(\neg yng)$
$0.001 \times (1 - 0.05) = 0.00095$
$0.0005 < 0.00095$
$\neg yng$ is likelier.

c) $p(yng) = 0.05$
$p(rizz|yng) = 0.01$
$p(rizz|\neg yng) = 0.0005$
$p(rizz|yng) \times p(yng)$
$0.01 \times 0.05 = 0.0005$
$p(rizz|\neg yng) \times p(\neg yng)$
$0.0005 \times (1 - 0.05) = 0.000475$
$0.0005 > 0.000475$
yng is likelier.

4

	$noisy:+$	$noisy:-$
$cool:+$	62	108
$cool:-$	38	292

$$p(cool:+) = \frac{170}{500} = \frac{17}{50} = 0.34$$

$$p(cool:+|noisy:+) = \frac{62}{100} = \frac{31}{50} = 0.64$$

The formula for Independence: $P(A|B) = P(A)$. $0.34 \neq 0.64$
$\therefore$ $cool:+$ is not independent of $noisy:+$

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$open:+$	$noisy:+$	$noisy:-$
$cool:+$	54	36
$cool:-$	6	4

$open:-$	$noisy:+$	$noisy:-$
$cool:+$	8	72
$cool:-$	32	288

$$p(cool:+|open:+) = \frac{90}{100} = 0.9$$

$$p(cool:+|open:+, noisy:+) = \frac{54}{60} = 0.9$$

$$p(cool:+|open:+) = p(cool:+|open:+, noisy:+)$$

Conditional Independence: $P(X|Y,Z) = P(X|Y)$
$cool:+$ is conditionally independent of $noisy:+$ given $open:+$

6

H = *heads*, T = *tails*
θ_h is the probability of a coin flipping heads.
For $\theta_h = 0.1$
H H H T $= 0.1 \times 0.1 \times 0.1 \times (1 - 0.1) = 0.0009$
For $\theta_h = 0.5$
$0.5 \times 0.5 \times 0.5 \times (1 - 0.5) = 0.0625$
For $\theta_h = 0.75$
$0.75 \times 0.75 \times 0.75 \times (1 - 0.75) = 0.1055$
For $\theta_h = 0.9$
$0.9 \times 0.9 \times 0.9 \times (1 - 0.9) = 0.0729$