Digital Communications - EEEN40060

Assignment 2 - Simulation of a Hamming-coded BPSK System

Niall Donohoe
Student ID: 17340571

Lecturer: Professor Mark Flanagan

UCD School of Electrical and Electronic Engineering
University College Dublin

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Encoding

In order to compare (7,4) Hamming coded BPSK with conventional BPSK a MATLAB simulation was used. Information bits were randomly generated, this was then multiplied by the generator matrix G.

$$G = (P|I_k) = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

By multiplying the generated information bits, by the matrix G, we obtain the codeword.

Modulation

In order to modulate the codeword into BPSK, we map 0s to -1 and 1s to 1. So where there is a 0 in the data we transmit a -1, when there is a 1 in the data we transmit a 1.

Noise

The channel adds AWGN to the modulated signal. The energy of this noise is varied to change the E_c/N_0 ratio.

Demodulation

At the receiver we must make a decision on the data received. As this is BPSK modulation our decision is simple, if the received symbol phase is greater than 0, assume a 1 was sent. If the received symbol phase is less than 0, assume a 0 was sent. We denote this received data as the vector r.

Decoding

At this stage we can use the Hamming decoder to correct one-bit errors that may have occurred. This is done by extracting the syndrome denoted by s.

$$H = (I_{n-k}|P^T) \tag{2}$$

$$s = rH^T \tag{3}$$

Using the decoding table provided in the assignment document we may attempt to correct the error which may have occurred.

$$\hat{r} = r \oplus e \tag{4}$$

$\hat{r}$ can be compared to the information bits generated at the transmitter to compute BER. The theoretical BER for an uncoded BPSK system is also computed using the equation

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{5}$$

The above procedure is repeated many times (at least 10^5) and for various E_b/N_0 values, in order to achieve the plot below.

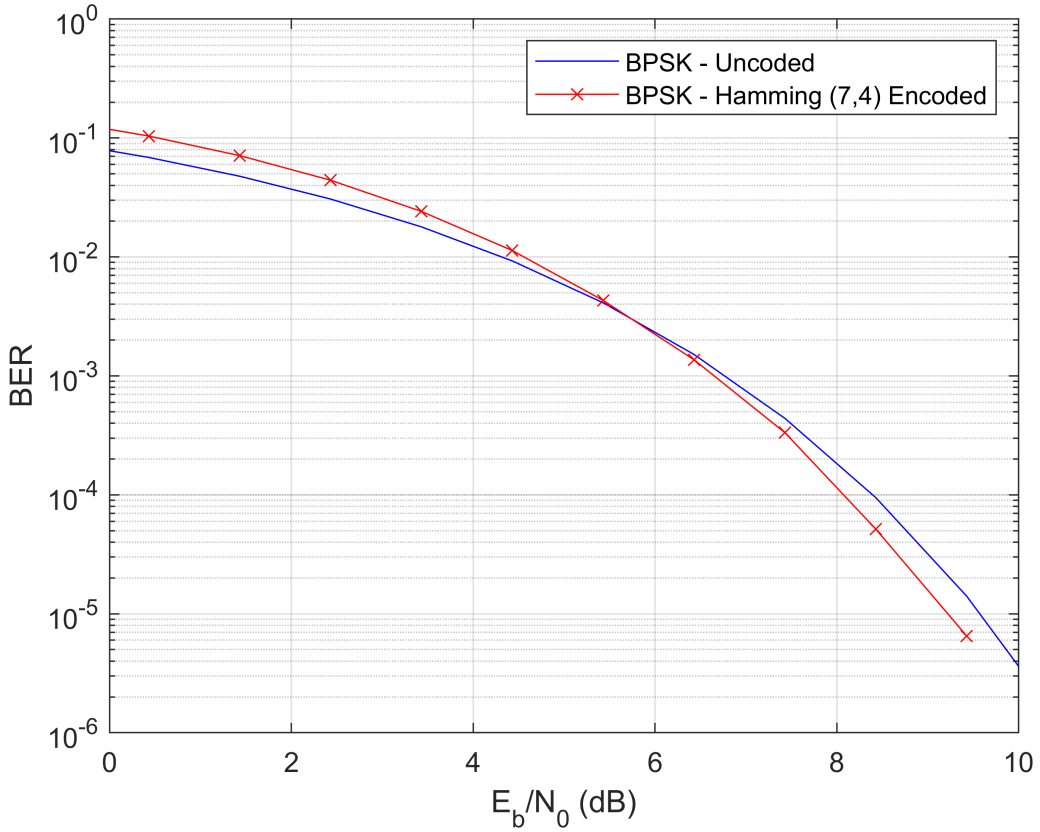

Figure 1: BER vs.E_b/N_0

From Figure 1 we note that the crossover point is at about 5.75dB, this corresponds to a BER of $2.5x10^{-3}$. Furthermore we also note that at a BER of 10^{-5}, the Hamming encoded system has an coding gain of about 0.4dB.

Conclusion

We have shown that at E_b/N_0 greater than about 5.75dB, there is a coding gain achievable by Hamming code in a BPSK system. This means is we require a system with a BER of 10^{-5}, using (7,4) Hamming code we can achieve this with 0.4dB less energy per bit than BPSK.