Digital Communications - EEEN40060

Assignment 1 - Simulation of an 8-ary Digital Communication System

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0.1 Bit to Symbol Mapping

The most likely error to occur is where a symbol is pushed over the decision threshold into the neighbouring symbol decision region. As a result it is sensible to choose a bit to symbol mapping which minimises the effects of this error.

Gray coding can be used to ensure that each adjacent symbol differs by only one bit in their bit to symbol mapping. The result of this is the most likely error only results in a one bit error in the data.

This Gray coding technique results in the bit to symbol mapping as outlined below.

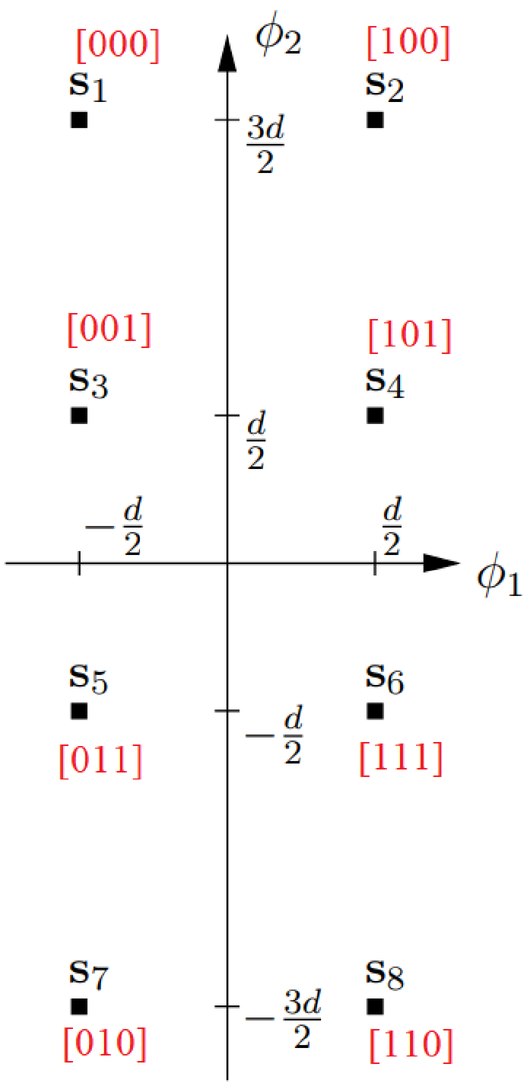

Figure 1: Bit to Symbol Mapping

0.2 Theoretical SER expression Derivation

We can think of the 8-ary QAM as a 2-PAM system, operating along the ϕ_1 axis of Figure 1 above, in combination with a 4-PAM system operating along the ϕ_2 axis .

The probability of error for an M-ary PAM is

$$P_e = 2\left(1 - \frac{1}{M}\right)Q\left(\sqrt{\frac{6E}{(M^2-1)N_0}}\right) \tag{1}$$

Where E is the average symbol energy, N_0 noise PSD, and Q is the Q-function.

Because we have 2 PAM systems, special attention is required when it comes to the energy of the symbols. We can think of the average symbol energy of the QAM system (E_s) as the average energy of the 2 PAM systems.

$$E_s = \frac{1}{6}(E_{\phi 1} + E_{\phi 2}) \tag{2}$$

Where $E_{\phi 1} = (d/2)^2$ and $E_{\phi 2} = (2(d/2)^2 + 2(3d/2)^2)/4$, the energy contributions of each of the PAM systems to the average symbol energy of the QAM system can thus be summarised as follows.

$$E_{\phi 1} = \frac{1}{6}(E_s), E_{\phi 2} = \frac{5}{6}(E_s) \tag{3}$$

Taking the expressions above and analysing for the different PAM systems.

$\underline{2-PAM}$

Using equation 1, for the case $M = 2$.

$$P_{e2} = 2\left(1 - \frac{1}{2}\right)Q\left(\sqrt{\frac{6E_{\phi 1}}{3N_0}}\right) \tag{4}$$

Using the substitution $E_{\phi 1} = \frac{1}{6}(E_s)$

$$P_{e2} = Q\left(\sqrt{\frac{6E_s}{(3N_0)6}}\right) = Q\left(\sqrt{\frac{E_s}{3N_0}}\right) \tag{5}$$

$\underline{4-PAM}$

Using equation 1, for the case $M = 4$.

$$P_{e4} = 2\left(1 - \frac{1}{4}\right)Q\left(\sqrt{\frac{6E_{\phi 2}}{15N_0}}\right) \tag{6}$$

Using the substitution $E_{\phi 2} = \frac{5}{6}(E_s)$

$$P_{e4} = \left(\frac{3}{2}\right)Q\left(\sqrt{\frac{(6E_s)5}{(15N_0)6}}\right) = \left(\frac{3}{2}\right)Q\left(\sqrt{\frac{E_s}{3N_0}}\right) \tag{7}$$

Combining $2-PAM$ and $4-PAM$

In order for the correct symbol decision to be made, we must be correct in the horizontal and vertical directions.

$$P_{correct} = (1 - P_{e2})(1 - P_{e4}) \tag{8}$$

From this we obtain our error probability for a symbol.

$$P_{error} = 1 - P_{correct} \tag{9}$$

Written explicitly in its full form using equations 5 and 7.

$$P_{error} = 1 - \left(1 - Q\left(\sqrt{\frac{E_s}{3N_0}}\right)\right)\left(1 - \left(\frac{3}{2}\right)Q\left(\sqrt{\frac{E_s}{3N_0}}\right)\right) \tag{10}$$

Using the above expression we can compute the SER for a given Es/N_0.

0.3 SER simulation

In order to simulate the 8-ary QAM system and calculate the SER, a MATLAB simulation randomly generated symbols and added random noise. At the receiver the distances between the received symbols and the symbol set are computed, the received symbol is assumed to be that which closest matches one of the members of the symbol set. The error is then computed by comparing the transmitted symbols to those estimated by the receiver.

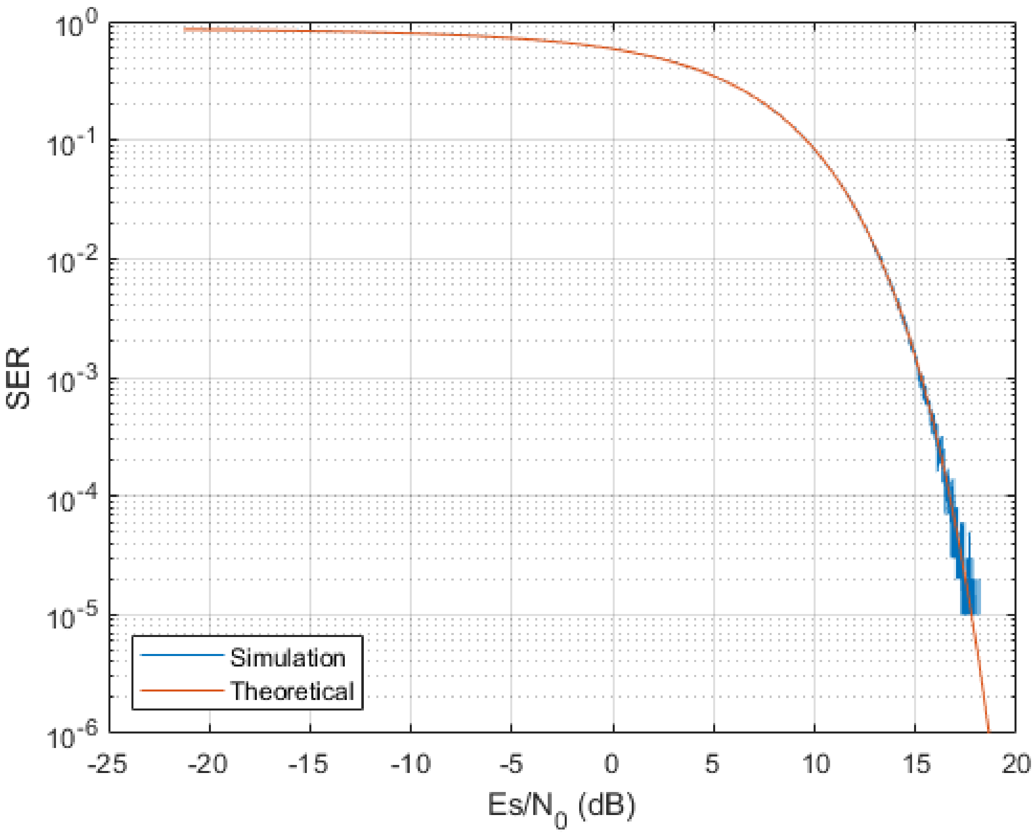

Figure 2: Symbol Error Rate vs. E_s/N_0

From figure 2 we note that as E_s/N_0 increases our SER decreases. It is also clear that there is a close relationship between the simulated SER and the derived theoretical SER expression. There is a slight discrepancy at high E_s/N_0 due to very few incorrect symbols, simulating with more symbols would remedy this at the expense of a quick simulation.

0.4 BER Simulation

In the same MATLAB simulation that produced the plot above, we also compute the Bit Error Rate (BER). This is done by transforming symbols to bits using the bit mapping outlined in figure 1. The number of incorrect bits is then computed, from this we calculate the BER.

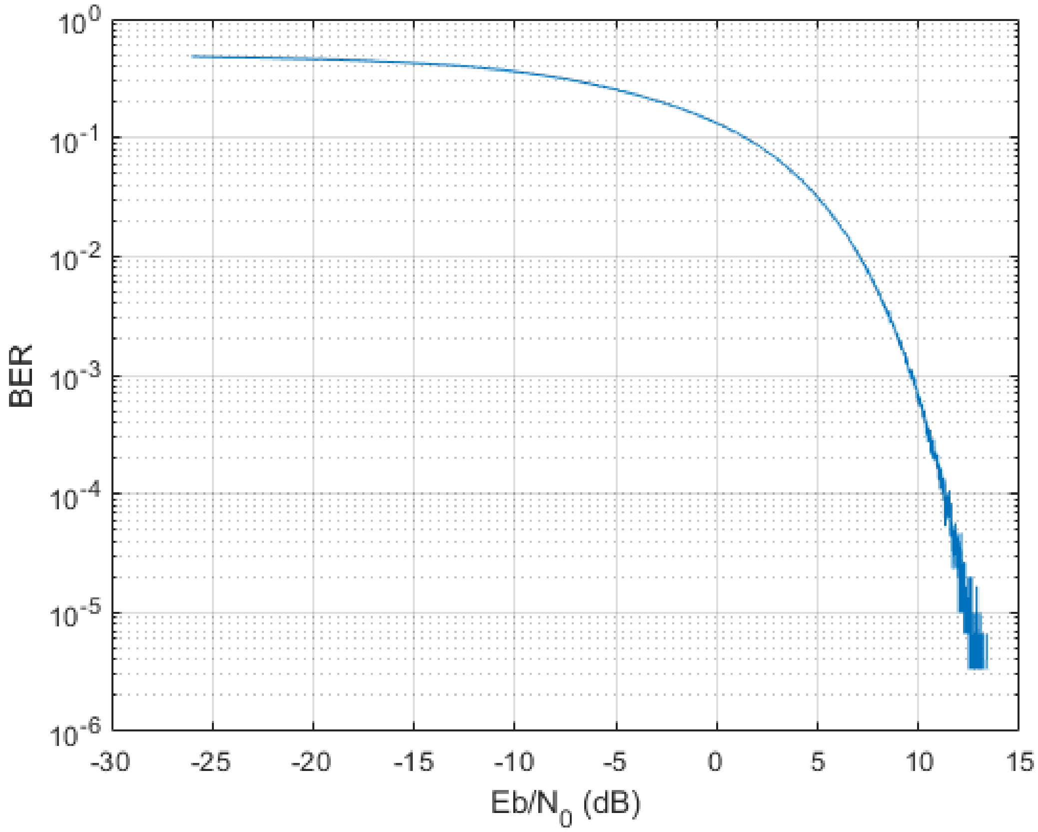

Figure 3: Bit Error Rate vs. E_b/N_0

From the figure above, it is clear that the value of E_b/N_0 for which the BER lies below 10^{-4} is approximately 11.3dB

As expected, with increasing E_b/N_0, BER decreases. This is inline with what we expect from the theory and also agrees with the simulation and theory presented in above sections.

Summary

A theoretical and simulated SER analysis for 8-ary QAM shows that it can be analysed as a 2-PAM and 4-PAM systems combined. The results from the simulation show tight agreement between the theory and simulation. BER is shown to follow a similar trend as SER, decreasing as SNR increases.