Animation & Simulation

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Behavioural Animation

- Crowd Simulation
  - Velocity optimisation – Reciprocal Velocity Obstacles (RVOs), [Van de Berg et al. 2008]
    - An agent that is moving creates an area-block (depending on its velocity) for other agents

velocity obstacle $VO_B^A(\mathbf{v}_B)$ of obstacle B to agent A is then the set consisting of all those velocities $\mathbf{v}_A$ for A that will result in a collision at some moment in time with obstacle B moving at velocity $\mathbf{v}_B$.

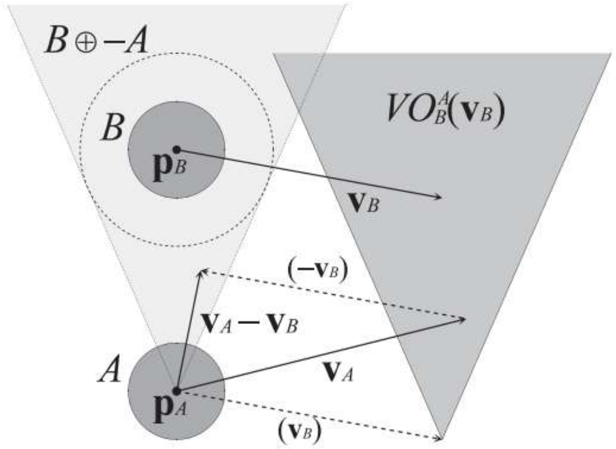

Fig. 2. The Velocity Obstacle $VO_B^A(\mathbf{v}_B)$ of a disc-shaped obstacle B to a disc-shaped agent A.

$$\lambda(\mathbf{p}, \mathbf{v}) = \{\mathbf{p} + t\mathbf{v} \mid t \geq 0\}$$

If the ray starting at $\mathbf{p}_A$ and heading in the direction of the relative velocity of A and B (which is $\mathbf{v}_A - \mathbf{v}_B$) intersects the Minkowski sum of B and $-A$ centered at $\mathbf{p}_B$, velocity $\mathbf{v}_A$ is in the velocity obstacle of B. Hence, the velocity obstacle of B to A is defined as follows:

Definition 1 (Velocity Obstacle).

$$VO_B^A(\mathbf{v}_B) = \{\mathbf{v}_A \mid \lambda(\mathbf{p}_A, \mathbf{v}_A - \mathbf{v}_B) \cap B \oplus -A \neq \emptyset\}.$$

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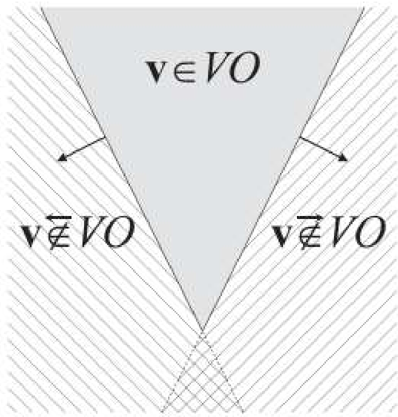

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$$RVO^A_B(\mathbf{v}_B, \mathbf{v}_A) = \{\mathbf{v}'_A \mid 2\mathbf{v}'_A - \mathbf{v}_A \in VO^A_B(\mathbf{v}_B)\}.$$

The reciprocal velocity obstacle $RVO^A_B(\mathbf{v}_B, \mathbf{v}_A)$ of agent B to agent A contains all velocities for agent A that are the average of the current velocity $\mathbf{v}_A$ and a velocity inside the velocity obstacle $VO^A_B(\mathbf{v}_B)$ of agent B. It can geometrically be interpreted as the velocity obstacle $VO^A_B(\mathbf{v}_B)$ that is translated such that its apex lies at $\frac{\mathbf{v}_A+\mathbf{v}_B}{2}$ (see Fig. 4).

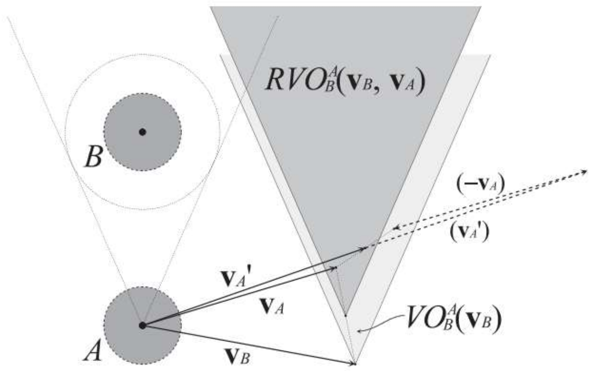

Fig. 4. The Reciprocal Velocity Obstacle $RVO^A_B(\mathbf{v}_B, \mathbf{v}_A)$ of agent B to agent A.

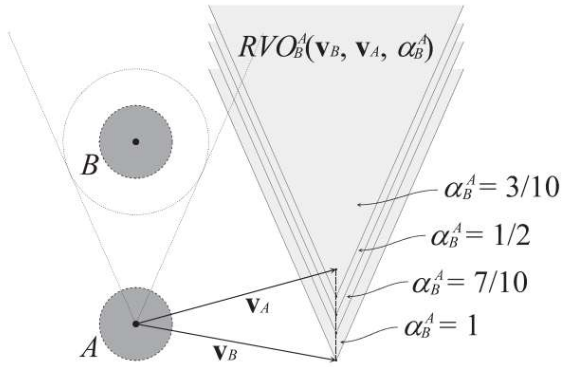

Fig. 5. The Generalized Reciprocal Velocity Obstacle $RVO^A_B(\mathbf{v}_B, \mathbf{v}_A, \alpha^A_B)$ of agent B to agent A for various values of α^A_B.

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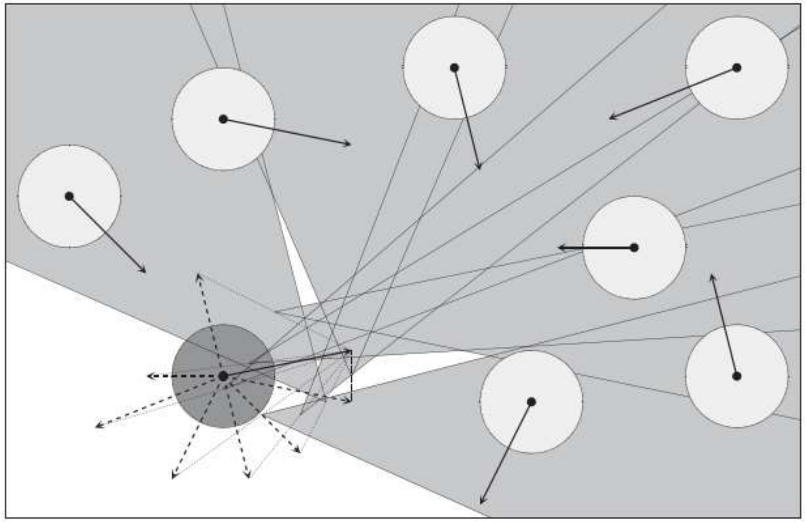

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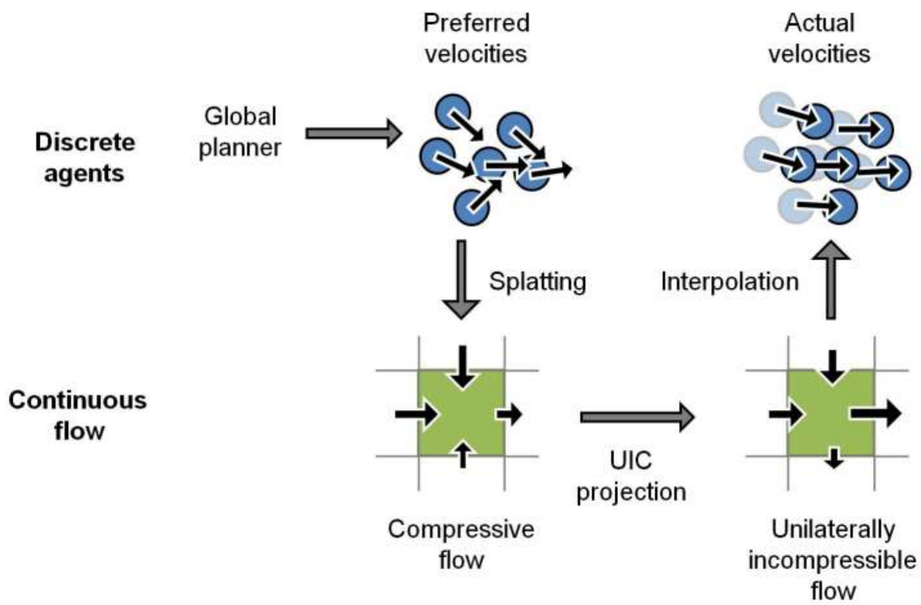
https://www.youtube.com/watch?v=1Fn3Mz6f5xA

Behavioural Animation

- Crowd Simulation
  - Fluid-based [Narain et al. Siggraph 2009]



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Kernel function w

$$\rho(\mathbf{x}) = \sum_i w_{\mathbf{x}}(\mathbf{x}_i) m_i,$$ mass

$$\tilde{\mathbf{v}}(\mathbf{x}) = \frac{\sum_i w_{\mathbf{x}}(\mathbf{x}_i)\tilde{\mathbf{v}}_i}{\sum_i w_{\mathbf{x}}(\mathbf{x}_i)}$$

Current Velocity

$$\mathbf{v}_i = \tilde{\mathbf{v}}_i + \frac{\rho(\mathbf{x}_i)}{\rho_{max}} \left(\mathbf{v}(\mathbf{x}_i) - \tilde{\mathbf{v}}_i\right)$$

Preferred Velocity

Unilateral Incompressibility

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

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1. At the beginning of each timestep, we know the position $\mathbf{x}_i$ of each agent.
2. Global planning is performed to determine the preferred velocity $\tilde{\mathbf{v}}_i$ of each agent, taking into account environmental obstacles but not neighboring agents. (For efficiency, the global planner may cache results from earlier timesteps, so that the full planning cost is not paid at each timestep.)
3. The agent positions $\mathbf{x}_i$ and preferred velocities $\tilde{\mathbf{v}}_i$ are transferred to the simulation grid by (1) and (2).
4. If there are moving obstacles in the environment, the free area f of each grid cell is recomputed.
5. The UIC solve is performed, giving the corrected velocity field $\mathbf{v} = \mathsf{psolve}(\mathsf{renorm}(\mathsf{psolve}(\tilde{\mathbf{v}})))$.
6. Each agent determines its actual velocity $\mathbf{v}_i$ taking the corrected flow into account via (3), and updates its position for the next timestep as $\mathbf{x}_i := \mathbf{x}_i + \mathbf{v}_i \Delta t$.
7. Finally, pairwise collision resolution is performed on the new $\mathbf{x}_i$ to handle inter-agent collisions.

$$\rho(\mathbf{x}) = \sum_i w_{\mathbf{x}}(\mathbf{x}_i) m_i, \tag{1}$$

$$\tilde{\mathbf{v}}(\mathbf{x}) = \frac{\sum_i w_{\mathbf{x}}(\mathbf{x}_i)\tilde{\mathbf{v}}_i}{\sum_i w_{\mathbf{x}}(\mathbf{x}_i)} \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\mathbf{v}_i = \tilde{\mathbf{v}}_i + \frac{\rho(\mathbf{x}_i)}{\rho_{max}} \left(\mathbf{v}(\mathbf{x}_i) - \tilde{\mathbf{v}}_i\right). \tag{3}$$

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https://youtu.be/pqBSNAOsMDc

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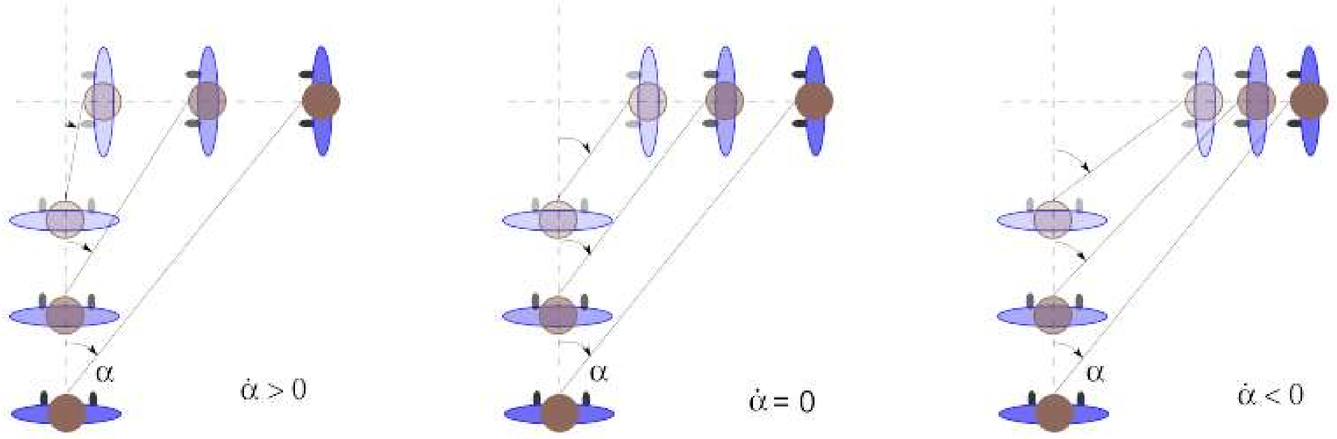
  - Vision-based [Ondrej et al. Siggraph 2010]



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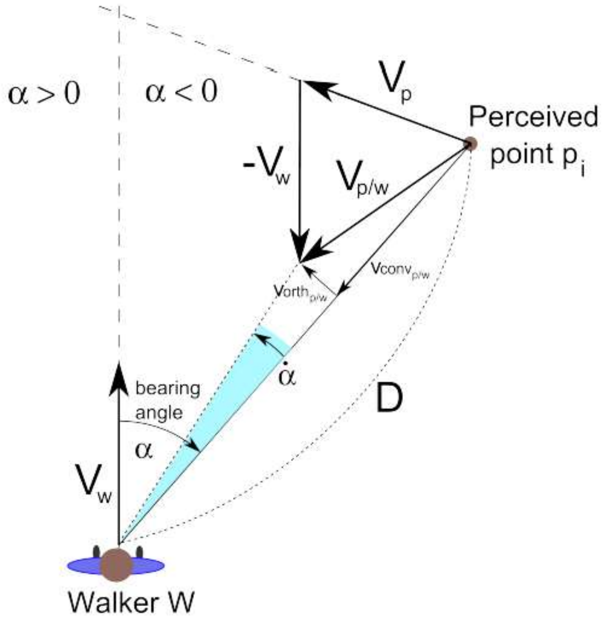
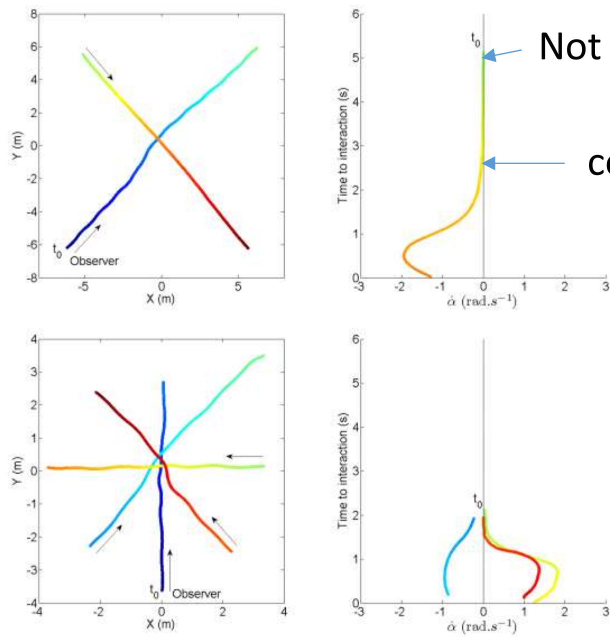


Cutting's theory, estimated time-to-collision (tti)



Not going to collide soon

collide soon!

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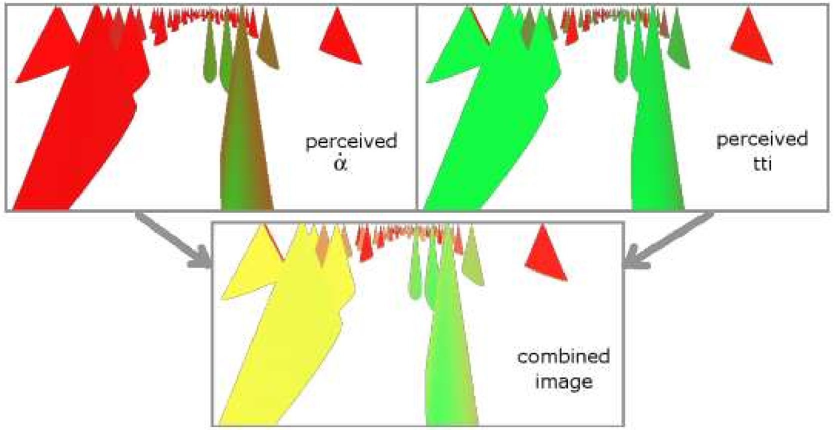

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https://www.youtube.com/watch?v=586qhaDwr24