Animation & Simulation

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Before the course

- What is computer animation?
- Where/How is it used?
- How to do computer animation?
- Expected you have learnt:
  - Math (linear algebra, calculus, discrete mathematics, numerical)
  - Physics
  - Computer graphics (rendering, geometry)
  - C++, OpenGL, GUI library (e.g. Qt)
  - Complexity analysis, numerical, optimisation, statistics, machine learning

During the course

- 10 weeks, 20 lectures (approx. 45-60 min) – *20 hours*
- 12 hour labs (week 4, 7-11) – *12 + X hours*
    - Assignment feedback labs online/in-person
    - Normal Labs online/in-person
    - Asynchronous lab activities (do the lab before coming to the lab).
    - More online labs will be scheduled for more support on a random basis.
- *Week 5 - reading week*
- 3 assignments & self study (2/3 weeks each, 100% total!) – *112 hours*
- *No exams*
- Totally *150 hours*
- *Online resources-Minerva, Yammer, Email, Teams.*

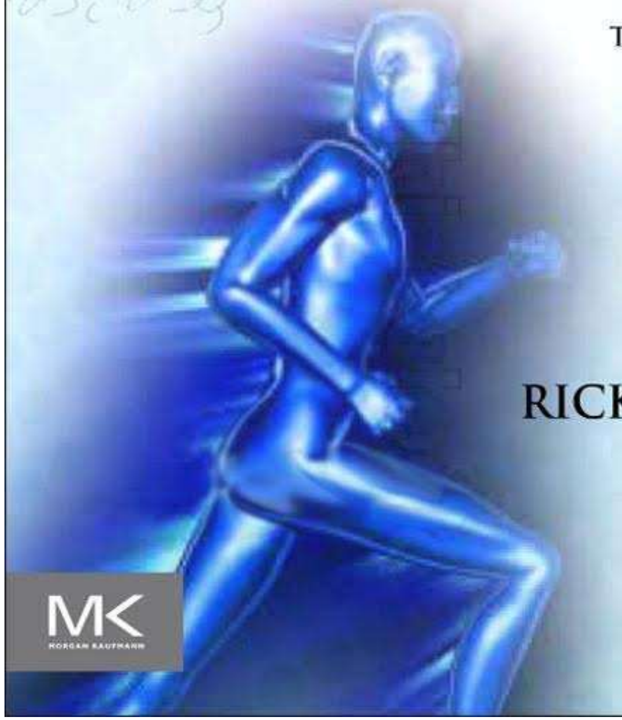
After the course

- Basic knowledge of computer animation
  - Interpolation, Free-form deformation, physical simulation, etc.
- Hands-on experience on four kinds of animations
  - FFD tool, Inverse Kinematics for character animation, Cloth Simulation and Fluid Simulation.
- In-depth knowledge to some animation techniques
  - Character Animation, Facial Animation, Collision Detection, etc.

COMPUTER ANIMATION

ALGORITHMS & TECHNIQUES

THIRD EDITION



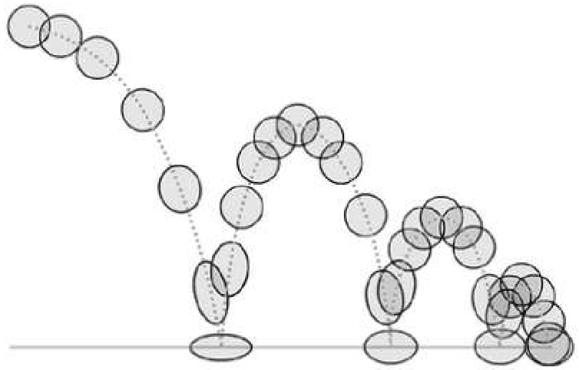
RICK PARENT

MORGAN KAUFMANN

Interpolating Values

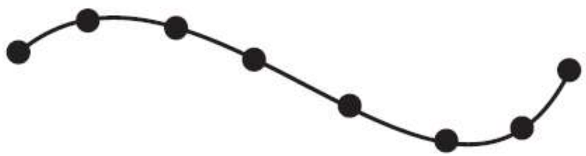
- Background (skip chapter 2)
  - Representations of rotation
  - Homogeneous transformation
- Foundation of animation
  - Show a series of static images quickly so that objects look moving
- Key elements (Key frame)

- Function descriptor
  - Automatically generate trajectories?
  - How to describe a trajectory?
  - Starting from animating a point.

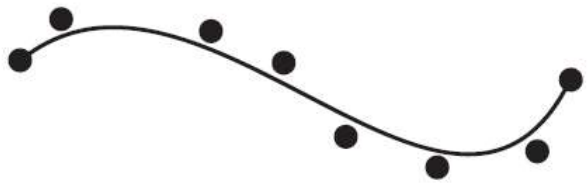


Interpolating Values

- Interpolation
  - Interpolation vs approximation
    - Motion as a function, given key positions
    - The function needs to go through (interpolation) or not (approximation)
      - Position constraints
      - Joint angles for walking (if a skeletal pose is seen as a high-dimensional point)
      - Hybrid (position constraints plus velocity constraints)



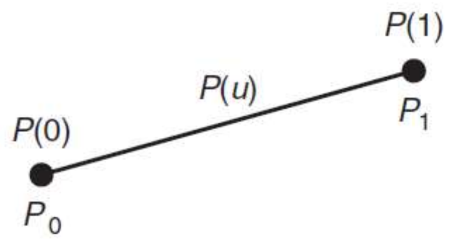
An interpolating spline in which the spline passes through the interior control points



An approximating spline in which only the endpoints are interpolated; the interior control points are used only to design the curve

Interpolating Values

- Interpolation
  - Interpolation
    - Linear Interpolation



Geometric form

$$P(u) = F_0(u)P_0 + F_1(u)P_1$$

$$P(u) = \begin{bmatrix} F_0(u) \\ F_1(u) \end{bmatrix} [P_0 P_1] = FB^T$$

$$P(u) = (1-u)P_0 + uP_1$$

Algebraic form

$$P(u) = (P_1 - P_0)u + P_0$$

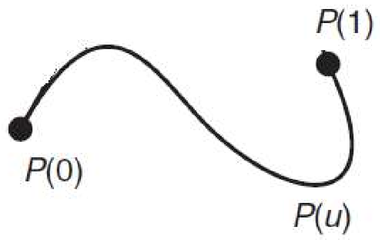
$$P(u) = a_1 u + a_0$$

$$P(u) = [u \quad 1] \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = U^T A$$

$$P(u) = [u \quad 1] \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \end{bmatrix} = U^T MB = FB = U^T A$$

Interpolating Values

- Interpolation
  - Parameterised by arcs



e.g. quadratic form $P(u) = P_0 + ((1-u)u + u)(P_1 - P_0)$

Or cubic $P(u) = U^T MB = |u^3 \; u^2 \; u \;\; 1|MB$

Or higher order

$$P(u) = [u \quad 1]\begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = U^T A$$

$$P(u) = [u \quad 1]\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} P_0 \\ P_1 \end{bmatrix} = U^T MB = FB = U^T A$$

Interpolating Values

- Interpolation
  - Parameterised by arcs
  - Derivatives of interpolate functions (higher order behaviours)

e.g. quadratic form $P(u) = P_0 + ((1-u)u + u)(P_1 - P_0)$

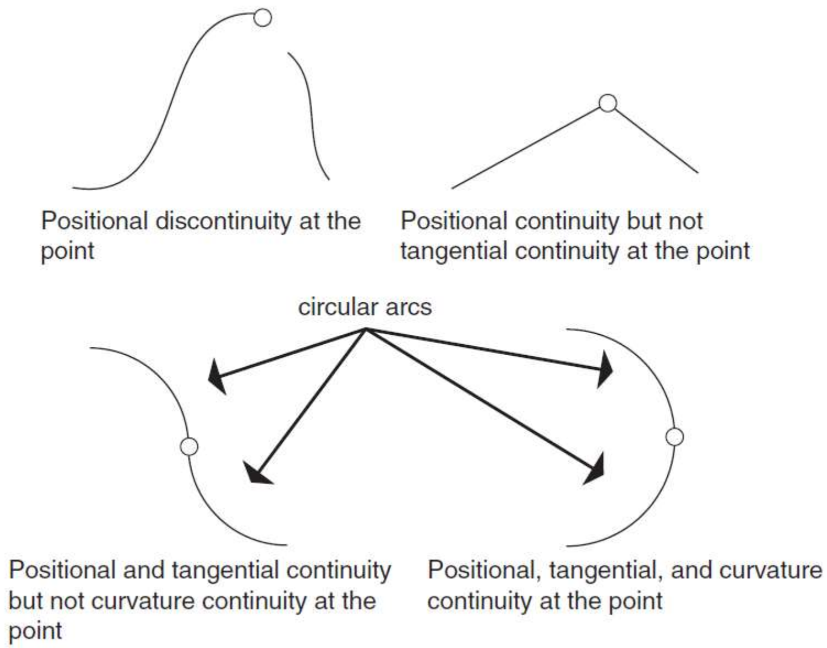
Or cubic $P(u) = U^T MB = |u^3 \; u^2 \; u \;\; 1\,|MB$

Or higher order

$$P(u) = U^T MB = |u^3 \; u^2 \; u \;\; 1\,|MB$$
$$P'(u) = U'^T MB = |3u^3 \; 2u \; 1 \;\; 0\,|MB$$
$$P''(u) = U''^T MB = |6u \; 2 \;\; 0 \;\; 0\,|MB$$

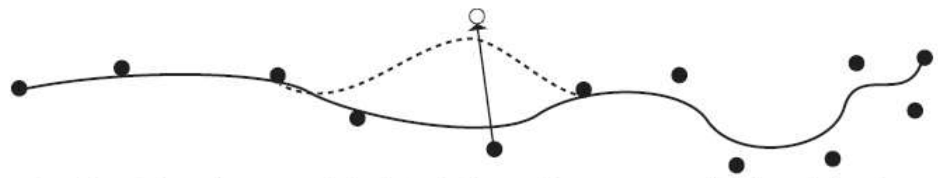
Interpolating Values

- Interpolation
  - Complexity
    - Order of polynomials (related to linear regression)
  - Continuity
    - Information higher than zero-order
  - Local vs global

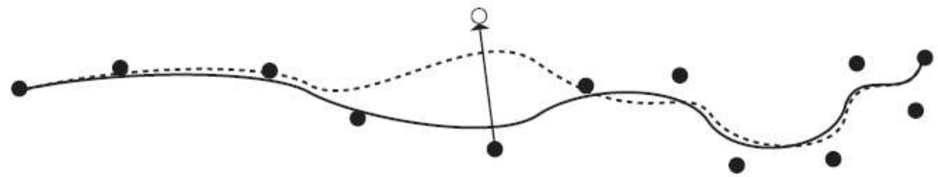


Interpolating Values

- Interpolation
  - Complexity
  - Continuity
  - Local vs global
    - Global
      - Lagrange Interpolation
    - Local
      - Parabolic blending
      - Catmull-Rom
      - Cubic Bezier
      - Cubic B-spline



Local control: moving one control point only changes the curve over a finite bounded region



Global control: moving one control point changes the entire curve; distant sections may change only slightly

Interpolating Values

- Basic concepts
  - Explicit
  - Implicit
  - Parametric

Interpolating Values

- Basic concepts
  - Explicit
    - $y = f(x)$, i.e. $y = x^2$
    - Pros: a y for any x
    - Cons: depend on choice of coordinate axes; Ambiguous $y = sqrt(x)$
  - Implicit
  - Parametric

Interpolating Values

- Basic concepts
    - Explicit
    - Implicit
        - $f(x, y) = 0$, $x^2 + y^2 = 1$
        - Pros: good for testing if a point is on the curve
        - Cons: not generative, if a series of points are desired, not easy
    - Parametric

Interpolating Values

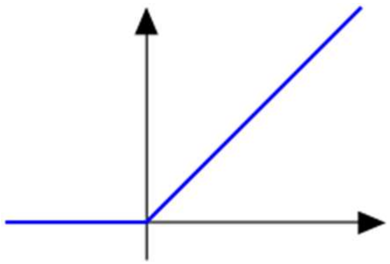
- Basic concepts
  - Explicit
  - Implicit
  - Parametric
    - $x = f(t)$, $y = g(t)$, t does not have to be time, i.e. $x = t$, $y = t^2$
    - Pros: an order list of points given t; also useful for multi-value x functions

Interpolating Values

- Basic concepts
  - Polynomials
    - Named by the highest order, $ax + b$ is *linear*, $ax^2 + bx + c$ is *quadratic*
    - Highest order also refereed as the *degree*
  - Other functions: trigonometrics, logs, called *transcendental* here

Interpolating Values

- Basic concepts
  - Polynomials
  - Other functions: trigonometrics, logs, called *transcendental* here
  - Continuity, number of continuous derivatives
    - Zeroth-order, C^0, for a value arbitrarily close to x, if the function is close to f(x)



$$f(x) = \begin{cases} x & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases}$$

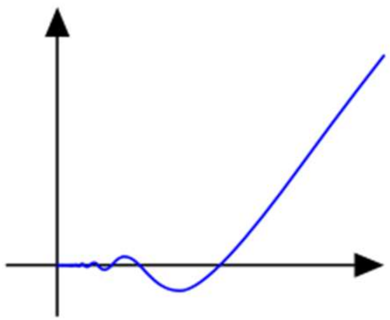
is continuous, but not differentiable at $x = 0$, so it is of class C^0 but not of class C^1

Continuous but not differentiable at x=0

Interpolating Values

- Basic concepts
  - Polynomials
  - Other functions: trigonometrics, logs, called *transcendental* here
  - Continuity, number of continuous derivatives
    - Zeroth-order, C^0, for a value arbitrarily close to x, if the function is close to f(x)

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases} \qquad g'(x) = \begin{cases} -\cos(\frac{1}{x}) + 2x\sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$



Differentiable but not continuous at x=0

Interpolating Values

- Basic concepts
  - Polynomials
  - Other functions: trigonometrics, logs, called *transcendental* here
  - Continuity, nth-order continuous derivatives
    - Zeroth-order, C^0, for a value arbitrarily close to x, if the function is close to f(x)
    - C^1, continuous first-order derivative
    - C^2, continuous first and second-order derivative
    - What about sin(x)?

Interpolating Values

- Basic concepts
  - Polynomials
  - Other functions: trigonometrics, logs, called *transcendental* here
  - Continuity, nth-order continuous derivatives
  - A function formed by curve segments has *piece-wise* properties

Interpolating Values

- Basic concepts
  - Polynomials
  - Other functions: trigonometrics, logs, called *transcendental* here
  - Continuity, nth-order continuous derivatives
  - A function formed by curve segments has *piece-wise* properties
  - Distinction between parametric continuity and geometric continuity
    - Geometric continuity is less restrictive
    - Parametric: the end tangent of the first segment equal to the one in the beginning of the second segment
    - Geometric: the direction should be the same, not the magnitude

Interpolating Values

- Interpolation
  - Parameterised by arcs
  - Derivatives of interpolate functions (higher order behaviours)

e.g. quadratic form $P(u) = P_0 + ((1-u)u + u)(P_1 - P_0)$

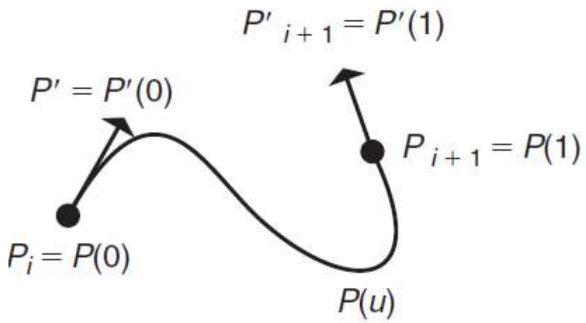
Or cubic $P(u) = U^T MB = |u^3\ u^2\ u\ \ 1\,|MB$

Or higher order

$$P(u) = U^T MB = |u^3\ u^2\ u\ \ 1\,|MB$$
$$P'(u) = U'^T MB = |3u^3\ 2u\ 1\ \ 0\,|MB$$
$$P''(u) = U''^T MB = |6u\ 2\ \ 0\ \ 0\,|MB$$

Interpolating Values

- Interpolation
  - Hermite interpolation

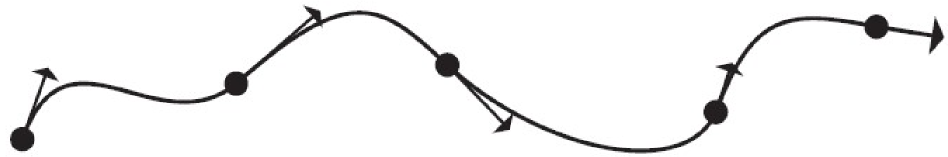


$$P(u) = U^T M B = |u^3\ u^2\ u\ \ 1\,| MB$$

$$U^T = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} P_i \\ P_{i+1} \\ P'_i \\ P'_{i+1} \end{bmatrix}$$

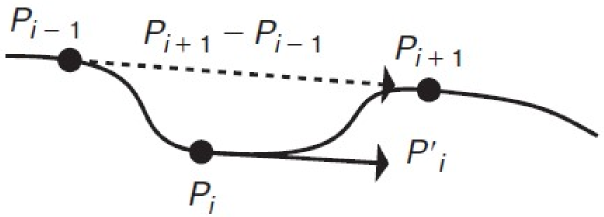
Need to provide first order on every P



A composite Hermite, piecewise cubic

Interpolating Values

- Interpolation
  - Catmull-Rom Spline



$$P(u) = U^T MB = |u^3\ u^2\ u\ \ 1\ |MB$$

$$P'_i = (1/2)(P_{i+1} - P_{i-1})$$

$$U^T = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$$

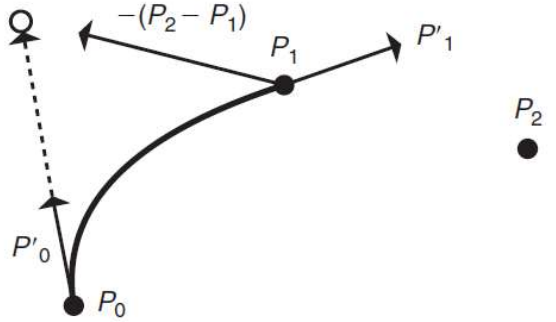
$$M = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix}$$

First order for the middle point is computed

Interpolating Values

- Interpolation
  - Catmull-Rom Spline



Initial Tangent

$$P'(0.0) = \frac{1}{2}(P_1 - (P_2 - P_1) - P_0) = \frac{1}{2}(2P_1 - P_2 - P_0)$$

an internal tangent vector is not dependent on the position of the internal point relative to its two neighbours

Interpolating Values

- Interpolation
  - Four-point form (fit a cubic curve to four points)

$$P(u) = [u^3 \quad u^2 \quad u \quad 1] \begin{bmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

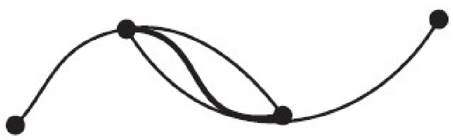
$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} u_0^3 & u_0^2 & u_0 & 1 \\ u_1^3 & u_1^2 & u_1 & 1 \\ u_2^3 & u_2^2 & u_2 & 1 \\ u_3^3 & u_3^2 & u_3 & 1 \end{bmatrix} \begin{bmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

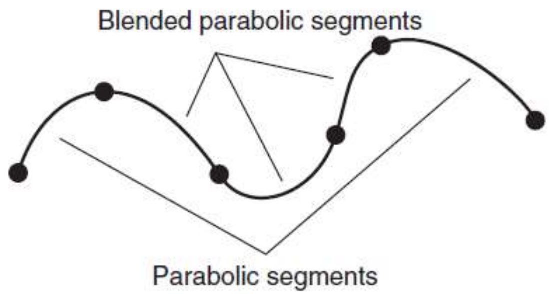
$$M = \frac{1}{2} \begin{bmatrix} -9 & 27 & -27 & - \\ 18 & -45 & 36 & -9 \\ -11 & 18 & -9 & 2 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

No way to guarantee the continuity

Interpolating Values

- Interpolation
  - Blended parabolas
    - A parabola for each triplets of points
    - A cubic segment by linearly interpolating two overlapping parabolas
      - Reparameterise the overlapped region to 0 and 1, then linearly blend the two



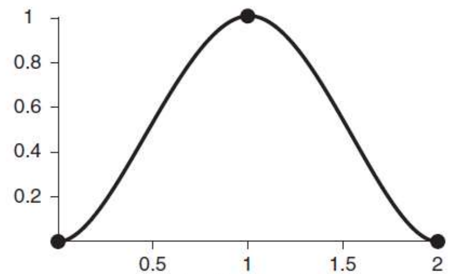


Interpolating Values

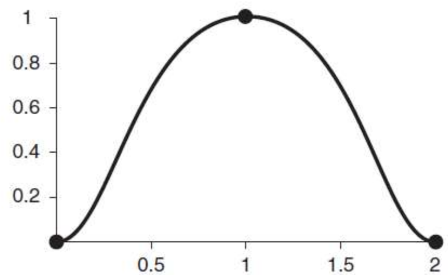
- Interpolation
  - Tension, continuity and bias control

Tension

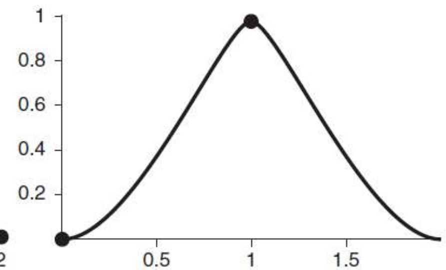
$$T_i^L = T_i^R = (1-t)\frac{1}{2}((P_{i+1} - P_i) + (P_i - P_{i-1}))$$



Default tension, $t = 0$



Low tension, $t = -3/4$



High tension, $t = 3/4$

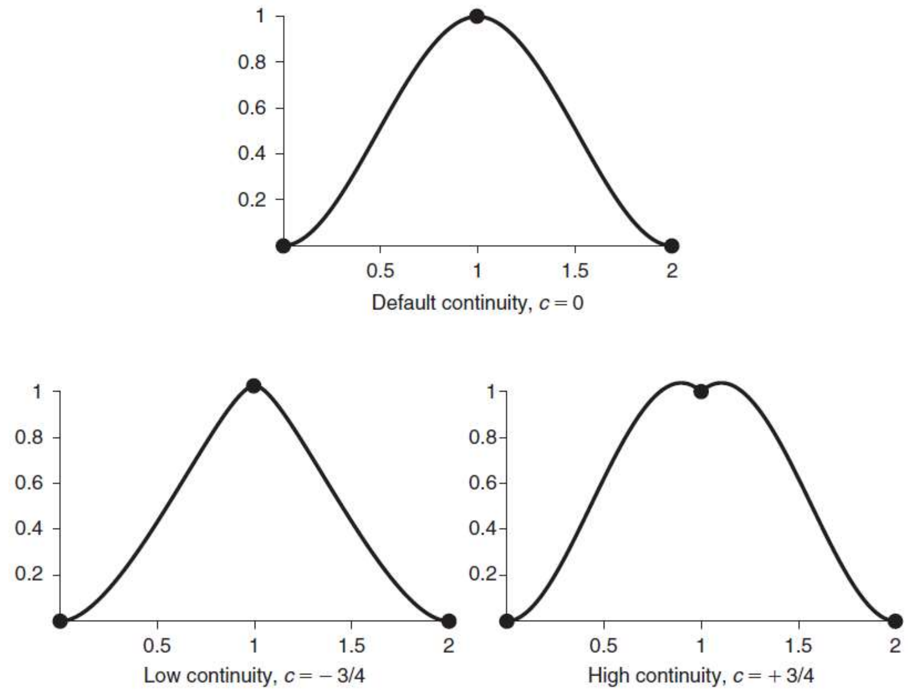
Interpolating Values

- Interpolation
  - Tension, continuity and bias control

Continuity

$$T_i^L = \frac{1-c}{2}(P_i - P_{i-1}) + \frac{1+c}{2}(P_{i+1} - P_i)$$

$$T_i^R = \frac{1+c}{2}(P_i - P_{i-1}) + \frac{1-c}{2}(P_{i+1} - P_i)$$

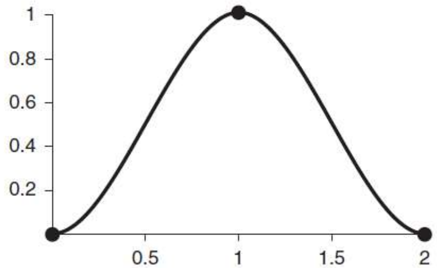


Interpolating Values

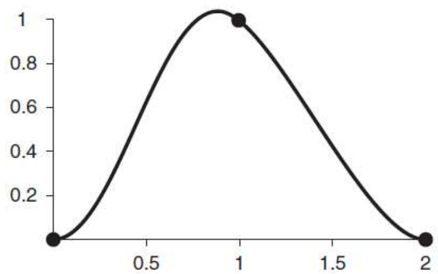
- Interpolation
  - Tension, continuity and bias control

Bias

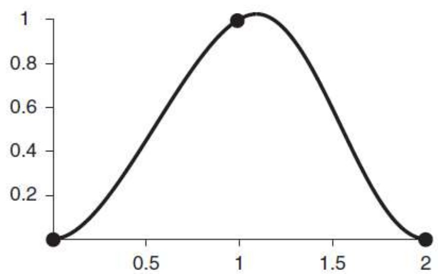
$$T_i^R = T_i^L = \frac{1+b}{2}(P_i - P_{i-1}) + \frac{1-b}{2}(P_{i+1} - P_i)$$



Default bias, $b = 0$



Low bias, $b = 23/4$



High bias, $b = 13/4$

Interpolating Values

- Interpolation
  - Tension, continuity and bias control

Tension

$$T_i^L = T_i^R = (1-t)\frac{1}{2}((P_{i+1}-P_i)+(P_i-P_{i-1}))$$

Continuity

$$T_i^L = \frac{1-c}{2}(P_i-P_{i-1}) + \frac{1+c}{2}(P_{i+1}-P_i)$$

$$T_i^R = \frac{1+c}{2}(P_i-P_{i-1}) + \frac{1-c}{2}(P_{i+1}-P_i)$$

Bias

$$T_i^R = T_i^L = \frac{1+b}{2}(P_i-P_{i-1}) + \frac{1-b}{2}(P_{i+1}-P_i)$$

$$T_i^R = \frac{((1-t)(1+c)(1+b))}{2}(P_i-P_{i-1}) + \frac{((1-t)(1-c)(1-b))}{2}(P_{i+1}-P_i)$$

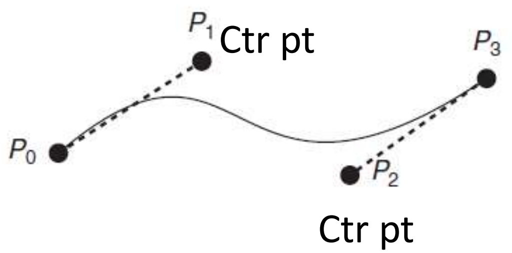
$$T_i^L = \frac{((1-t)(1-c)(1+b))}{2}(P_i-P_{i-1}) + \frac{((1-t)(1+c)(1-b))}{2}(P_{i+1}-P_i)$$

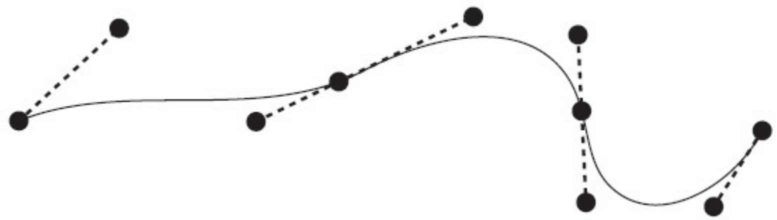
Interpolating Values

- Interpolation
  - Bezier interpolation/approximation
    - A cubie Bezier is similar to Hermite, but use auxiliary ctr points to control tangents

$P'(0) = 3(P_1 - P_0)$ and $P'(1) = 3(P_3 - P_2)$.

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

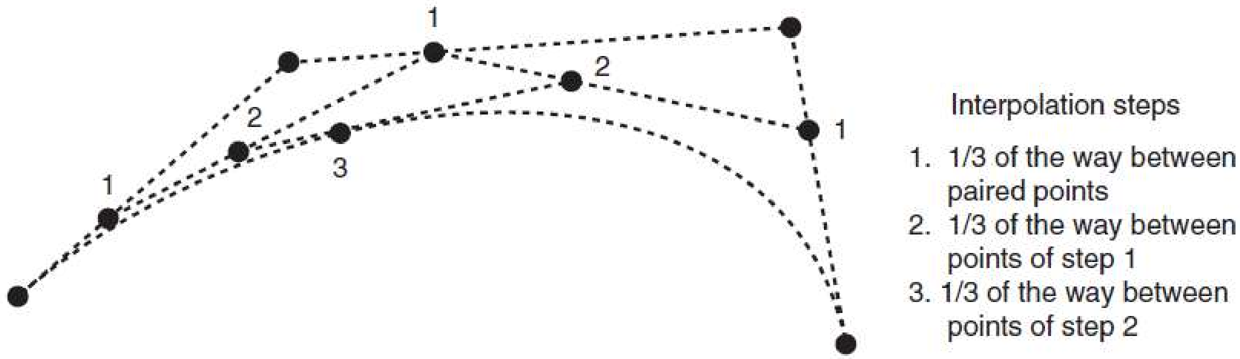




$$P(u) = U^T MB = |u^3 \ u^2 \ u \ \ 1|MB$$

Interpolating Values

- Interpolation
  - De Casteljau construction of Bezier Curve



$$P(u) = U^T MB = \left| u^3 \ u^2 \ u \ \ 1 \right| MB$$

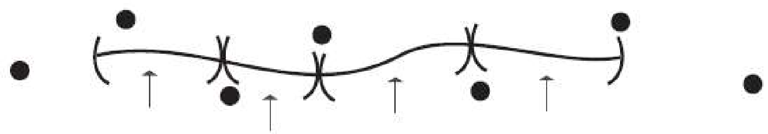
Interpolating Values

- Interpolation
  - B-splines
    - Decouple the ctr point number from the degree of the polynomial
    - Knot vector [0, 1, 2, …, n + k -1], n the ctr point number, k the degree
    - Values vary between the first and last knot value
    - The knot vector establishes a relationship between the parametric value and the control points.

Interpolating Values

- Interpolation
    - B-splines

$$P(u) = \frac{1}{6}\begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} P_i \\ P_{i+1} \\ P_{i+2} \\ P_{i+3} \end{bmatrix}$$



Segments of the curve defined by different sets of four points

None of the control point is interpolated