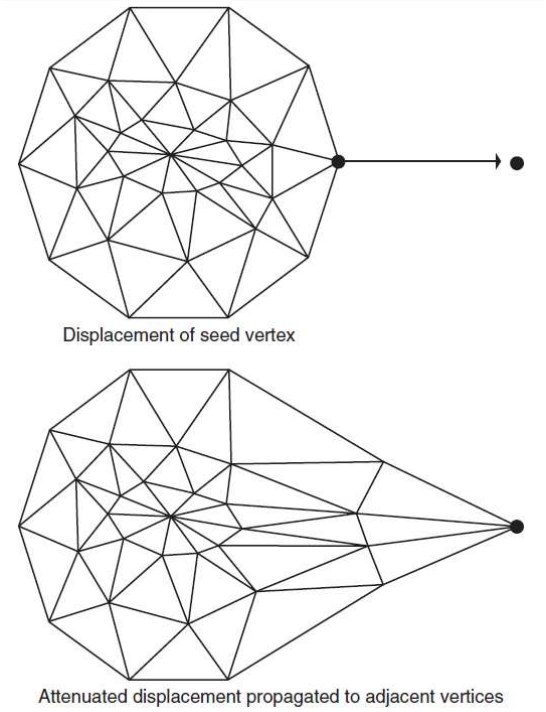
Animation & Simulation

He Wang (王鹤)

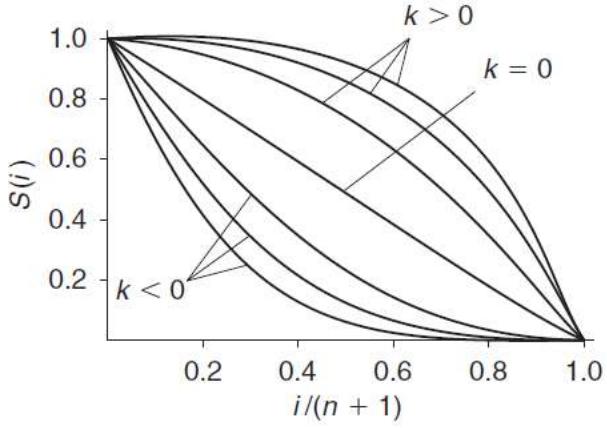
Interpolation-Based Animation

- Picking and pulling



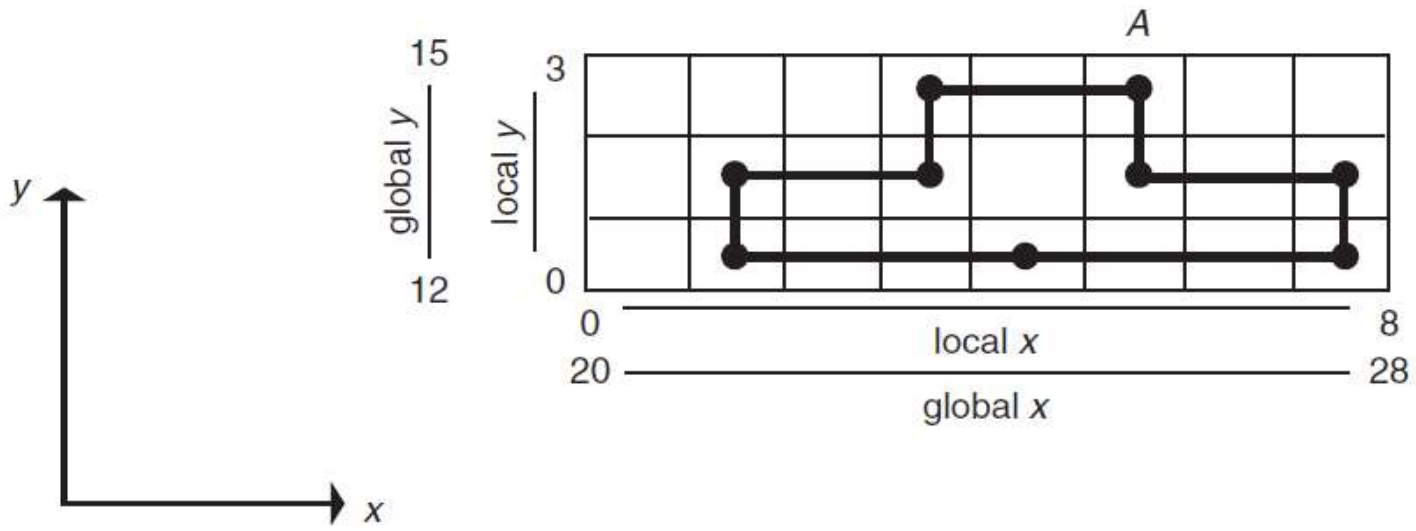
$$S(i) = 1 - \left(\frac{i}{n+1}\right)^{k+1} \qquad k \geq 0$$

$$= \left(1 - \left(\frac{i}{n+1}\right)\right)^{-k+1} \qquad k < 0$$



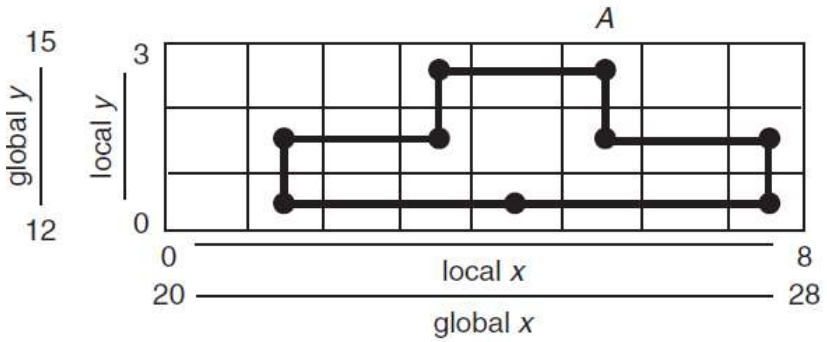
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - 2D grid



Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - 2D grid

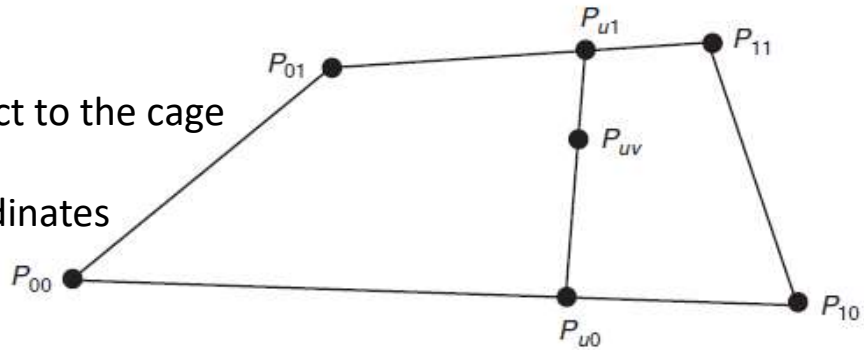


$$P = (0.6)(0.7)P_{00} + (0.6)(1.0 - 0.7)P_{01} + (1.0 - 0.6)(0.7)P_{10} + (1.0 - 0.6)(1.0 - 0.7)P_{11}$$

$$\begin{aligned} P_{u0} &= (1-u)P_{00} + uP_{10} \\ P_{u1} &= (1-u)P_{01} + uP_{11} \\ P_{uv} &= (1-v)P_{u0} + vP_{u1} \\ &= (1-u)(1-v)P_{00} + (1-u)vP_{01} + u(1-v)P_{10} + uvP_{11} \end{aligned}$$

General Principle:

1. Find local coordinates for mesh vertices with respect to the cage
2. Deform the cage
3. Compute the new coordinates using the local coordinates



Interpolation-Based Animation

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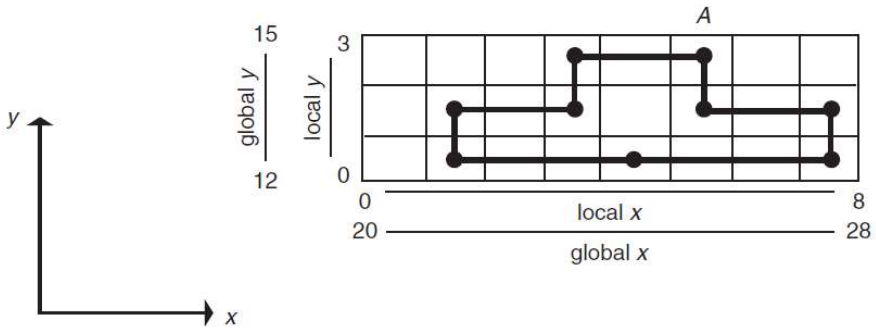
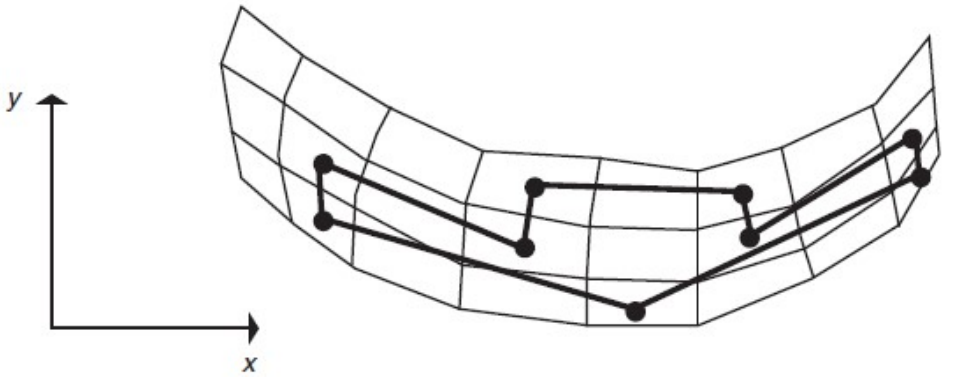
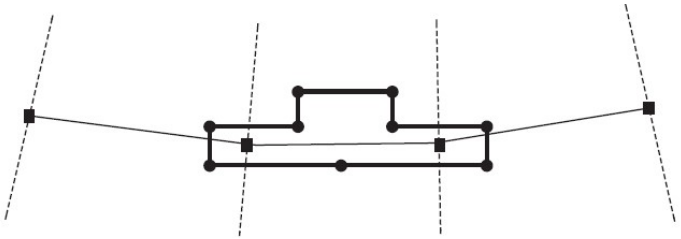
  - 2D grid

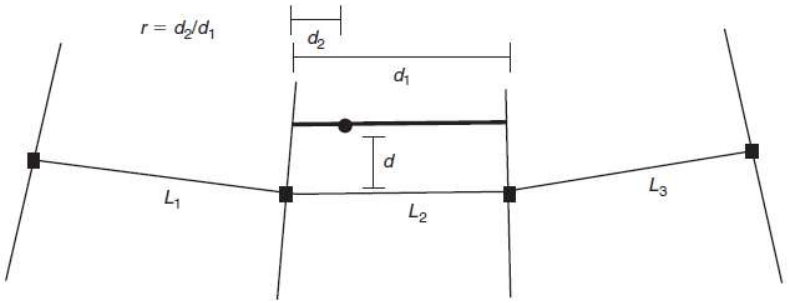


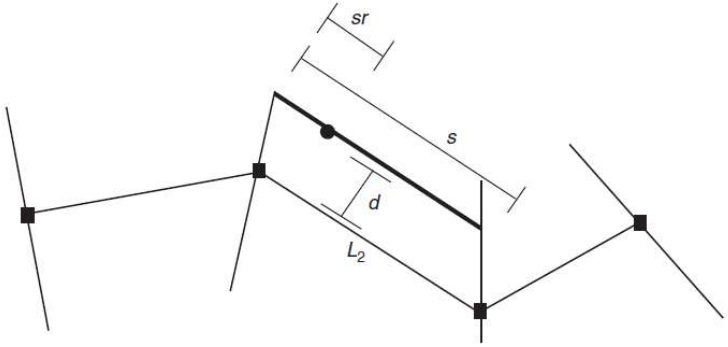


Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Polyline deformation

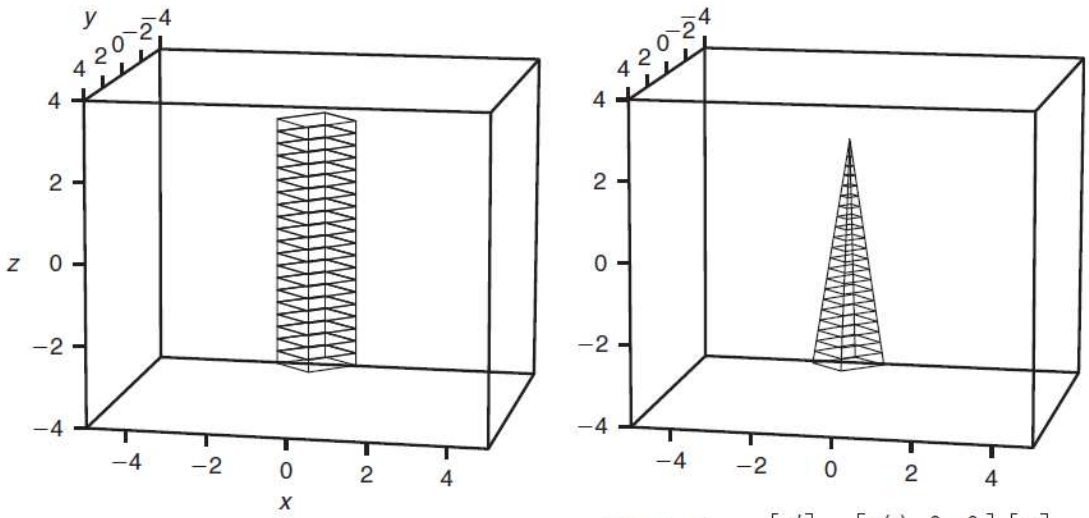






Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Global deformation



$$s(z) = \frac{(\max z - z)}{(\max z - \min z)}$$

$$x' = s(z)x$$
$$y' = s(z)y$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} s(z) & 0 & 0 \\ 0 & s(z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

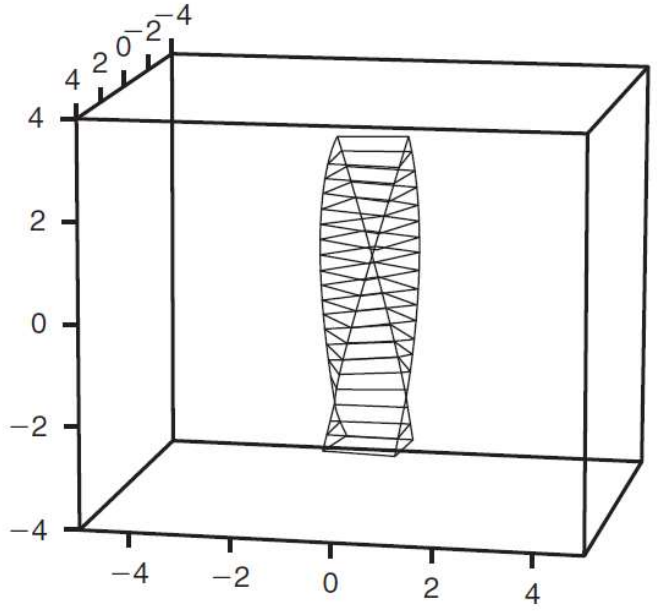
$$P' = M(p)p$$

Original object

Tapered object

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Global deformation



k = twist factor

$x' = x\cos(kz) - y\sin(kz)$

$y' = x\sin(kz) + y\cos(kz)$

$z' = z$

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
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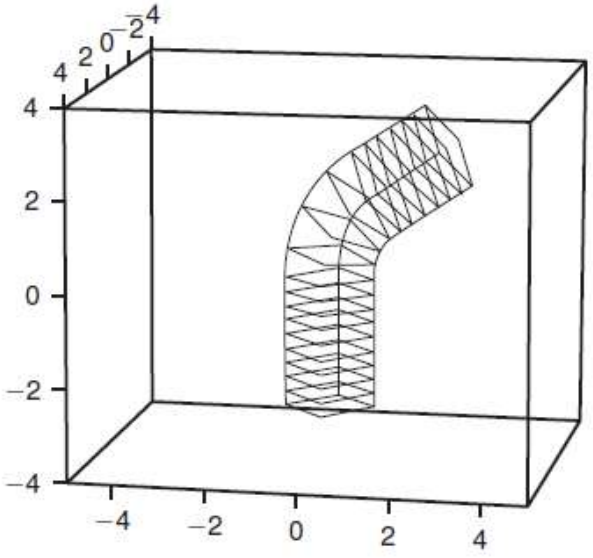


$(z_{min} : z_{max}) - bend\ region$
$(y_0, z_{min}) - center\ of\ bend$

$$x' = x$$

$$y' = \begin{cases} y & z < z_{max} \\ y_0 - (RC_\theta) & z_{min} \le z \le z_{max} \\ y_0 - (RC_\theta) + (z - z_{max})S_\theta & z > z_{max} \end{cases}$$

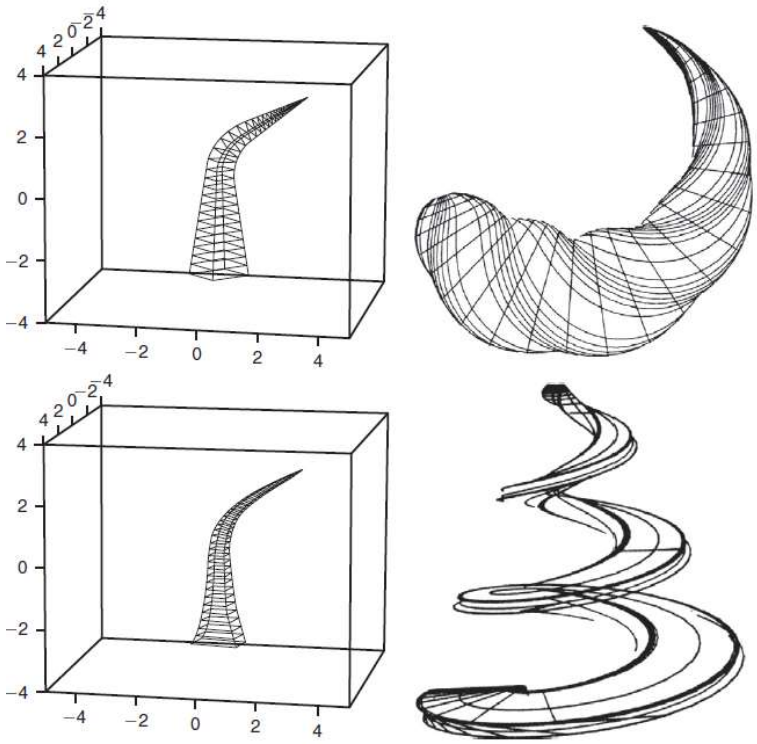
$$z' = \begin{cases} z & z < z_{max} \\ z_{min} - (RS_\theta) & z_{min} \le z \le z_{max} \\ z_{min} - (RS_\theta) + (z - z_{max})C_\theta & z > z_{max} \end{cases}$$

$$\theta = \begin{cases} 0 & z < z_{min} \\ z_{max} - z_{min} & z > z_{max} \\ z - z_{min} & otherwise \end{cases}$$

$$C_\theta = \cos\theta$$
$$S_\theta = \sin\theta$$
$$R = y_0 - y$$

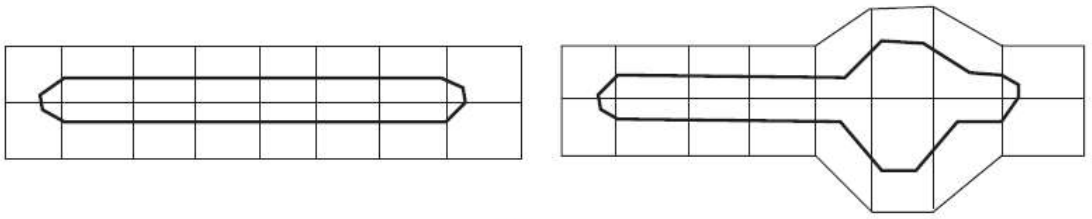
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Global deformation

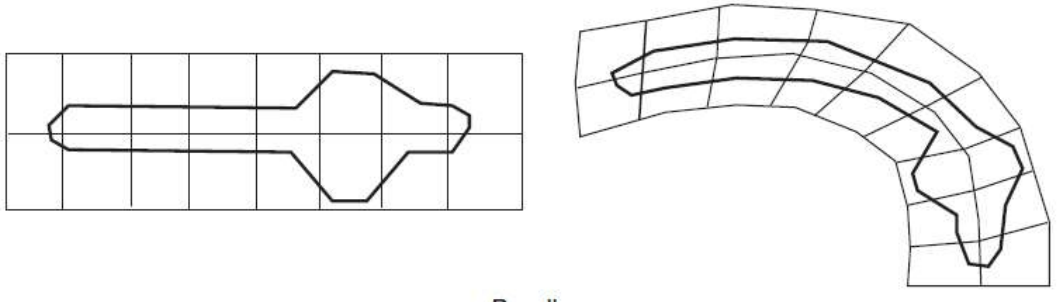


Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Composite FFDs – sequential and hierarchical
    - sequential

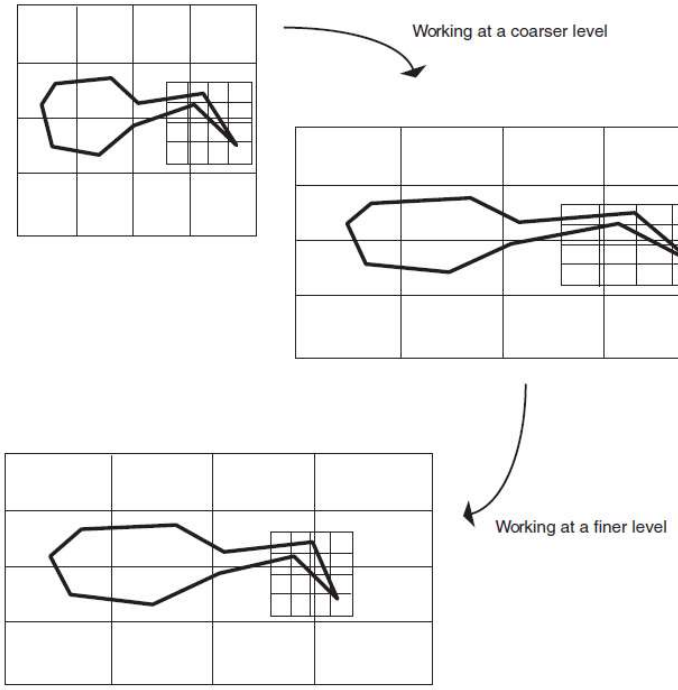


Bulging



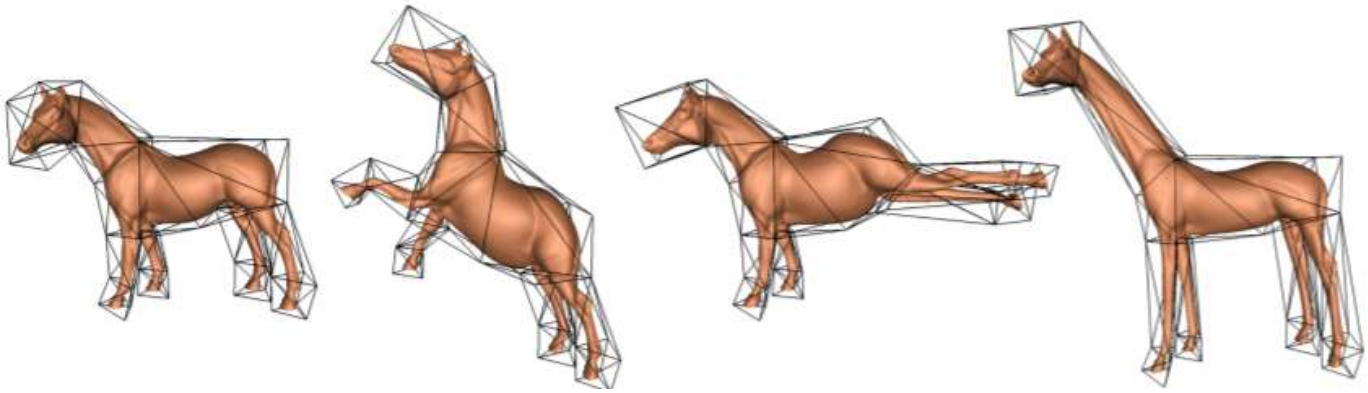
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Composite FFDs – sequential and hierarchical
    - hierarchical



Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates (Siggraph 2005)

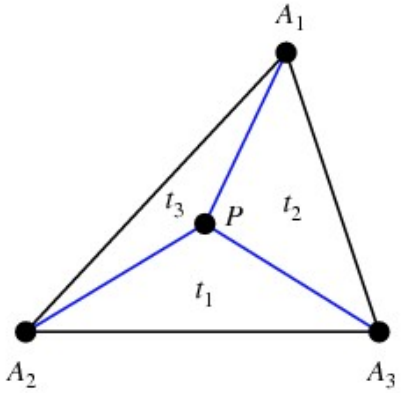


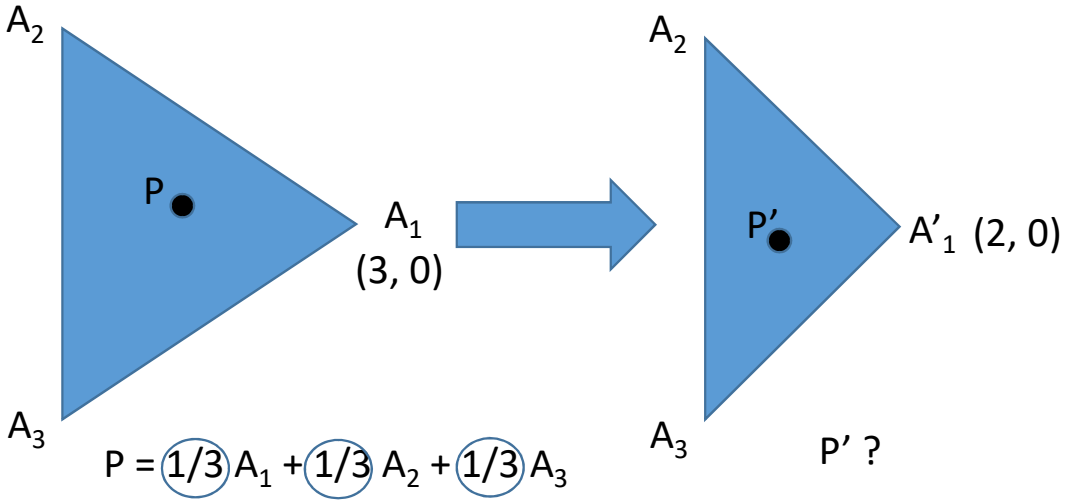
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates (Siggraph 2005)
      - Barycentric coordinates

        For P, $t_1 + t_2 + t_3 = 1$, and they are proportional to $\triangle A_2A_3P$, $\triangle A_1A_3P$ and $\triangle A_1A_2P$

        All $t > 0$





$P = (1/3)\,A_1 + (1/3)\,A_2 + (1/3)\,A_3$

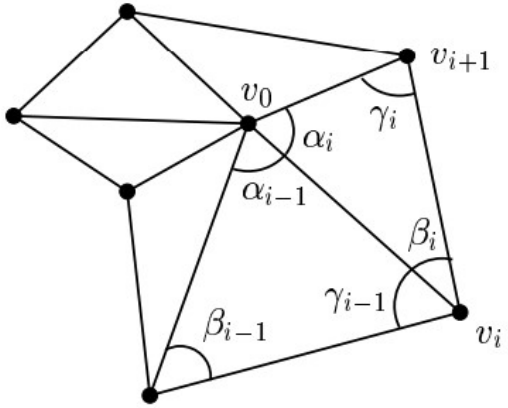
Barycentric coordinates

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates (T Ju et al, Siggraph 2005)
      - Barycentric coordinates Generalised- Mean value coordinates (Floater 2003)

$$\sum_{i=1}^{k} \lambda_i v_i = v_0,$$

$$\sum_{i=1}^{k} \lambda_i = 1.$$



$$\lambda_i = \frac{w_i}{\sum_{j=1}^{k} w_j}, \qquad w_i = \frac{A(v_{i-1}, v_i, v_{i+1})}{A(v_{i-1}, v_i, v_0) A(v_i, v_{i+1}, v_0)} = \frac{\cot \gamma_{i-1} + \cot \beta_i}{||v_i - v_0||^2},$$

Interpolation-Based Animation

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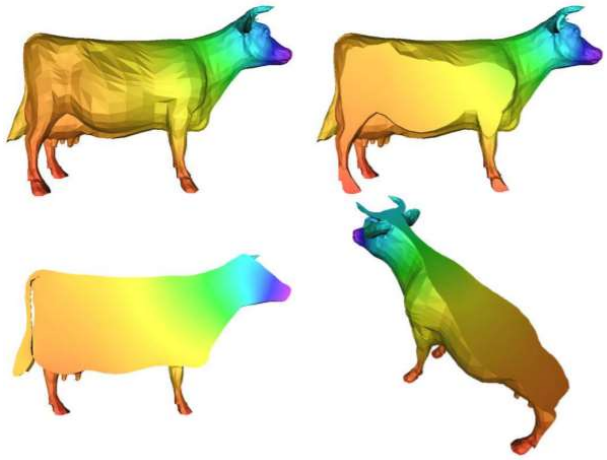
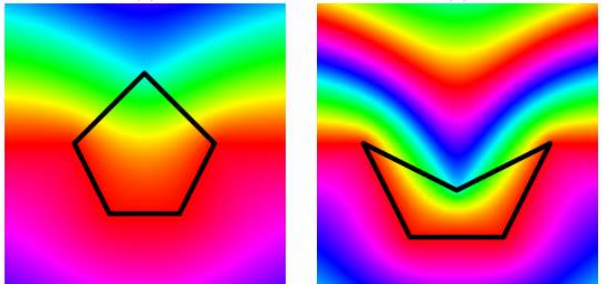
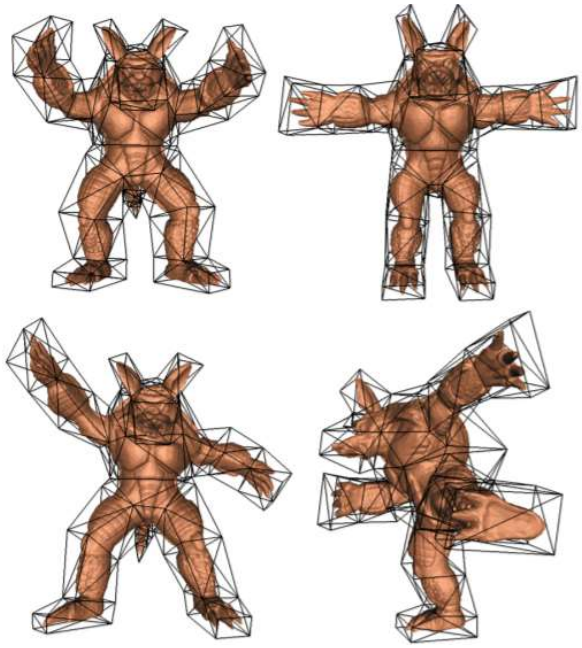




// Robust evaluation on a triangular mesh
for each vertex p_j with values f_j
  $d_j \leftarrow \|p_j - x\|$
  if $d_j < \varepsilon$ return f_j
  $u_j \leftarrow (p_j - x)/d_j$
$totalF \leftarrow 0$
$totalW \leftarrow 0$
for each triangle with vertices p_1, p_2, p_3 and values f_1, f_2, f_3
  $l_i \leftarrow \|u_{i+1} - u_{i-1}\|$ *// for* $i = 1,2,3$
  $\theta_i \leftarrow 2\arcsin[l_i/2]$
  $h \leftarrow (\sum \theta_i)/2$
  if $\pi - h < \varepsilon$
    // x lies on t, use 2D barycentric coordinates
    $w_i \leftarrow \sin[\theta_i] d_{i-1} d_{i+1}$
    return $(\sum w_i f_i)/(\sum w_i)$
  $c_i \leftarrow (2\sin[h]\sin[h-\theta_i])/(\sin[\theta_{i+1}]\sin[\theta_{i-1}]) - 1$
  $s_i \leftarrow \text{sign}[det[u_1, u_2, u_3]]\sqrt{1 - c_i^2}$
  if $\exists i, |s_i| \leq \varepsilon$
    // x lies outside t on the same plane, ignore t
    continue
  $w_i \leftarrow (\theta_i - c_{i+1}\theta_{i-1} - c_{i-1}\theta_{i+1})/(d_i \sin[\theta_{i+1}] s_{i-1})$
  $totalF += \sum w_i f_i$
  $totalW += \sum w_i$
$f_x \leftarrow totalF/totalW$

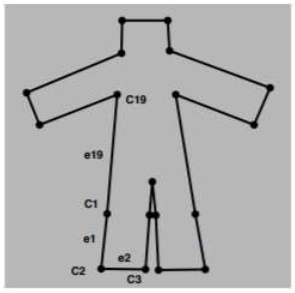
Interpolation-Based Animation

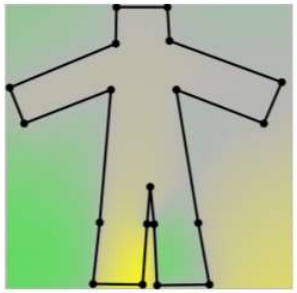
- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates (Siggraph 2005)



Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates
    - Harmonic Coordinates (Pixar, Siggraph 2007)






Mean Value Coordinates Harmonic Coordinates

Interpolation-Based Animation

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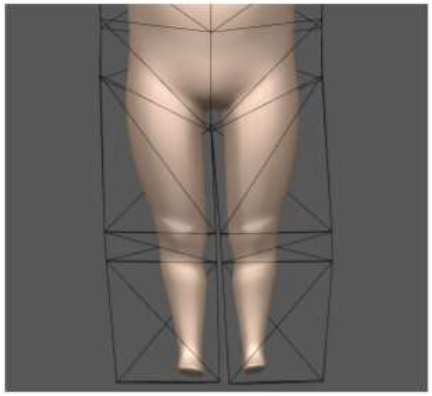
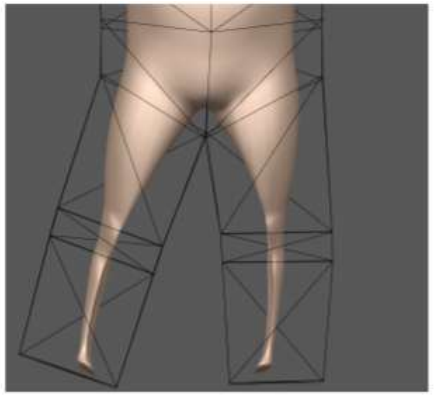
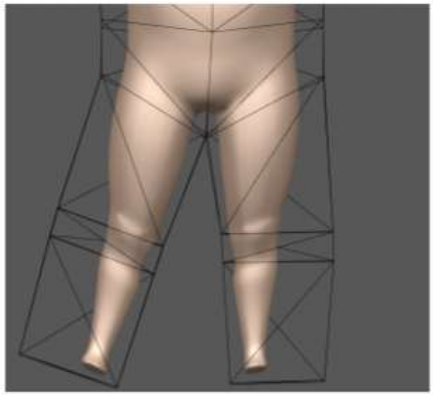




Mean Value Coordinates



Harmonic Coordinates

Interpolation-Based Animation

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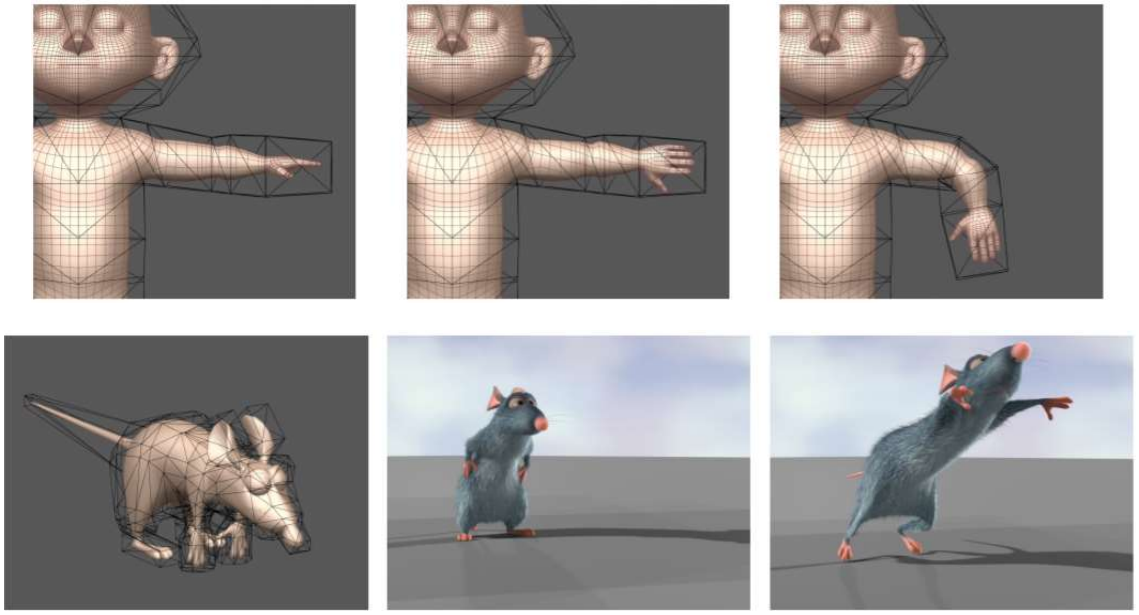
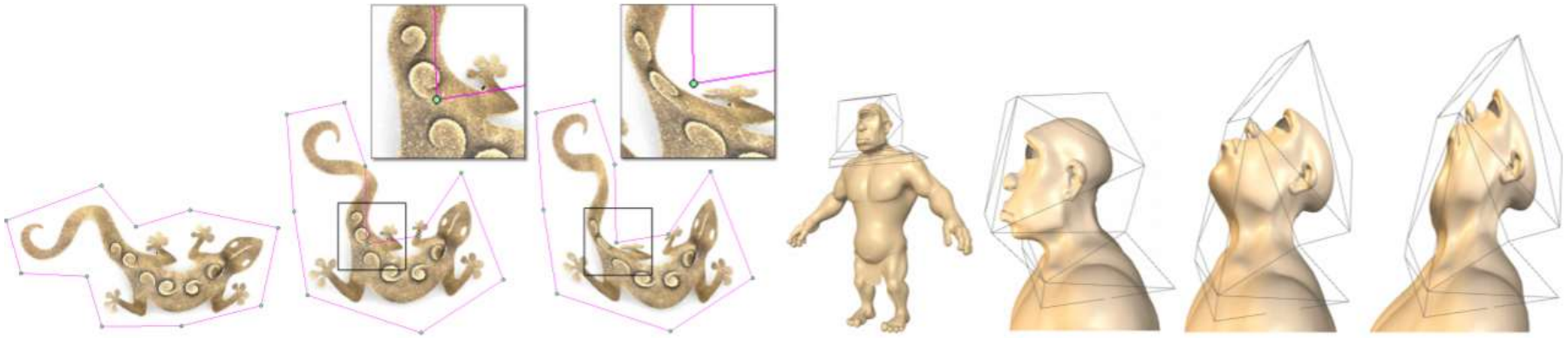


Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates
    - Harmonic Coordinates (Pixar, Siggraph 2007)
    - Green Coordinates (Siggraph 2008)



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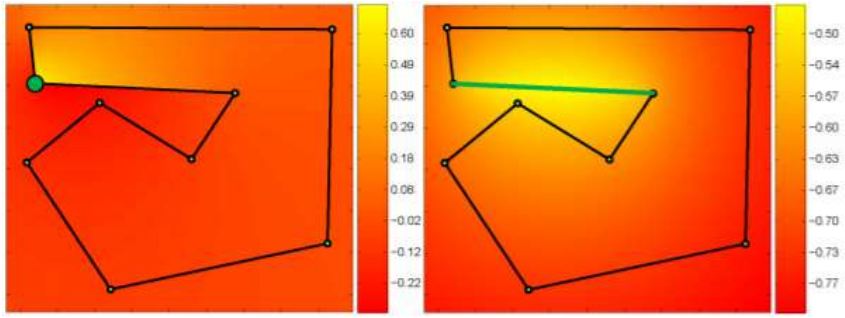

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
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    - Harmonic Coordinates (Pixar, Siggraph 2007)
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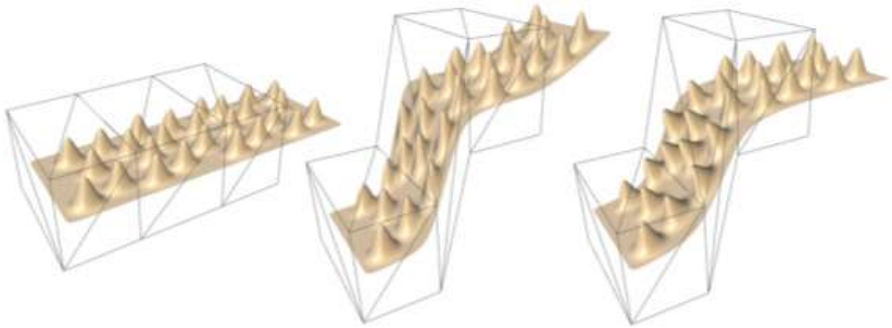
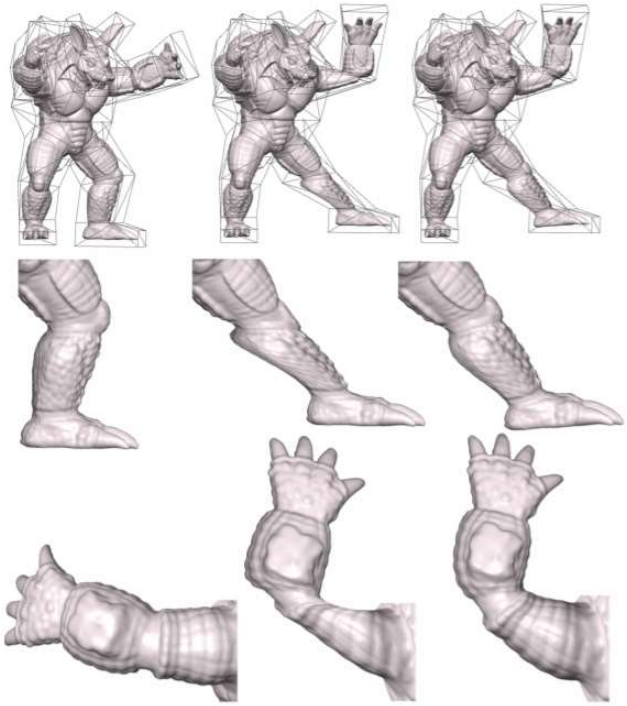


Original (left), Mean Value Coordinates(middle), Green Coordinates (right)



Original (left), Mean Value Coordinates(middle), Green Coordinates (right)

Interpolation-Based Animation

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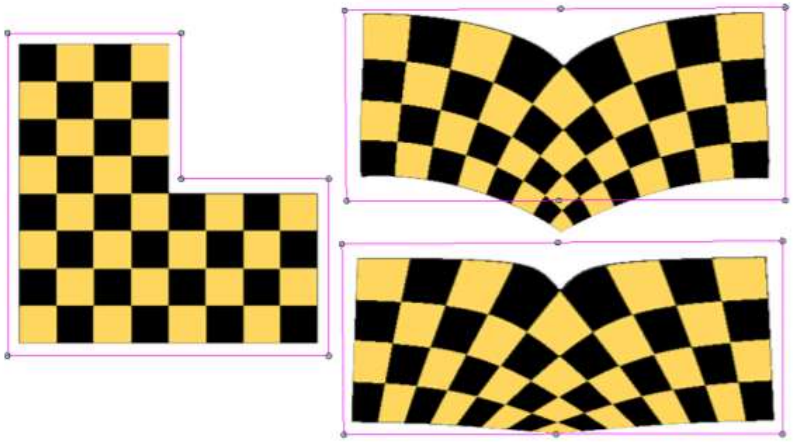
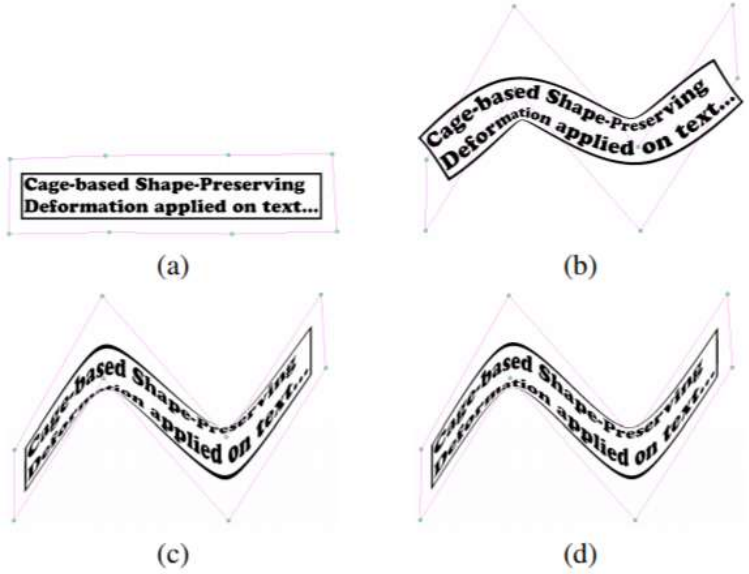


Harmonic Coordinates

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Other examples of FFD
    - Mean Value Coordinates
    - Harmonic Coordinates (Pixar, Siggraph 2007)
    - Green Coordinates (Siggraph 2008)



Green Coordinates

Mean Value Coordinates

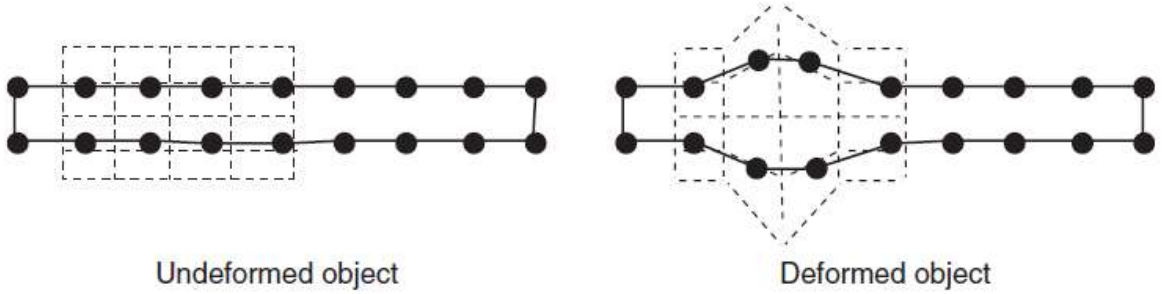
Harmonic Coordinates

Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Animated FFDs
    - A deformed space-a traversal of an object through the FFD space deforms it continuously
    - Control points animated

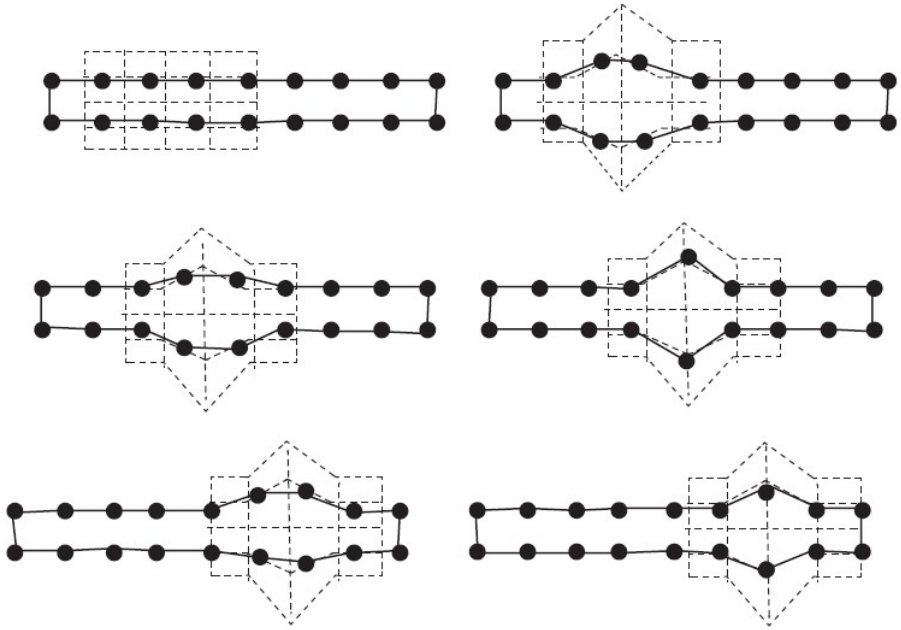
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Animated FFDs
    - A deformed space-a traversal of an object through the FFD space deforms it continuously
      - Deformation tool



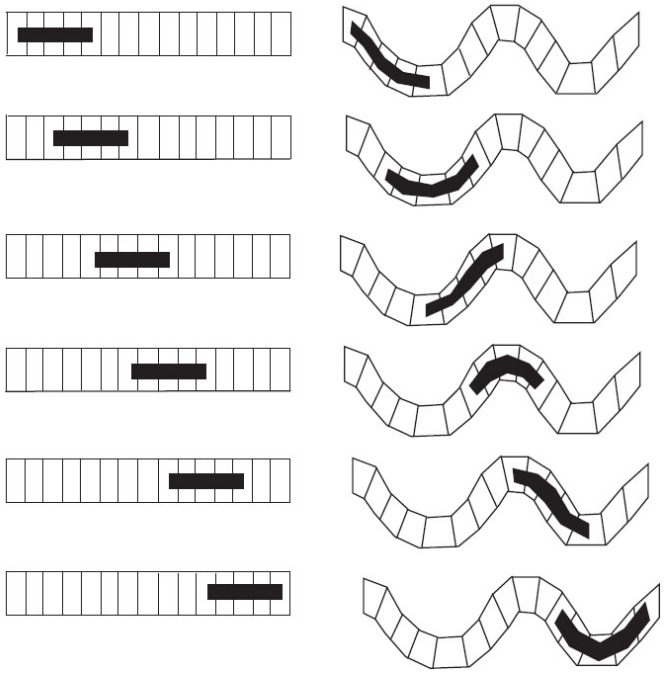
Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Animated FFDs
    - A deformed space-a traversal of an object through the FFD space deforms it continuously
      - Deformation tool
      - Move the tool



Interpolation-Based Animation

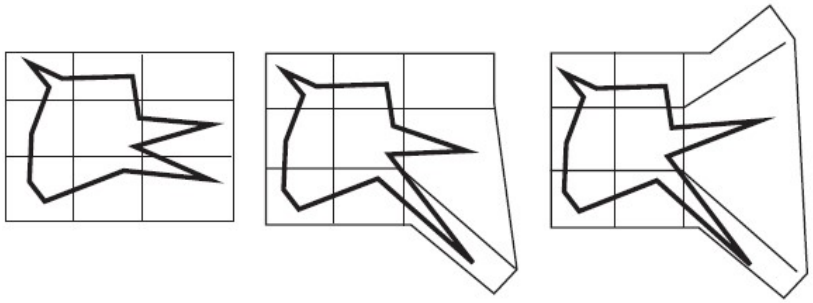
- Deforming an embedding space (Free-form deformation, FFD)
  - Animated FFDs
    - A deformed space-a traversal of an object through the FFD space deforms it continuously
      - Deformation tool
      - Move the tool
      - Move the object



Interpolation-Based Animation

- Deforming an embedding space (Free-form deformation, FFD)
  - Animated FFDs
    - Control points animated

Motion



Skinning