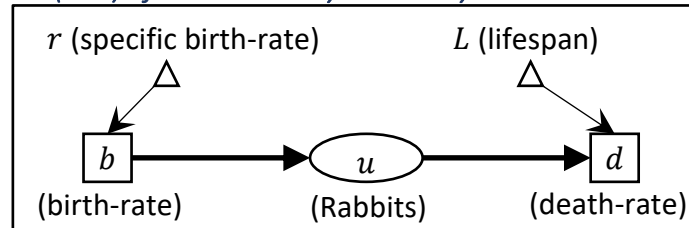


200. Dynamical narratives: How do systems develop over time?

Fundamental Principle of *system dynamics (SD)*: *Narrative emerges from structure!*

Structure-process diagram (SPD) of the Rabbits dynamical system:



Designing the Rabbits dynamical system

Stocks (or state variables):

$$u(t), \text{ where } u_0 \equiv u(0) = 1000 \text{ rabbit (Here, units are important!)}$$

Dynamical processes:

$$\dot{u} \equiv \frac{du}{dt} = (b - d) \text{ rabbit/month}$$

Structural relations:

$$\begin{aligned} b &= r u \\ d &= u/L \end{aligned}$$

Numerical parameters:

$$\begin{aligned} r &\equiv \frac{\dot{u}}{u} = \text{_____ (month)}^{-1} \\ L &= 48 \text{ month} \end{aligned}$$

Differential equations (DE):

$$\left. \begin{aligned} \dot{u} &= (r - L^{-1}) u \\ u_0 &= 1000 \\ r &= \text{_____} \\ L &= 48 \end{aligned} \right\} \text{ (From now on, we will ignore units for convenience!)}$$

Implementing the Rabbits narrative

Behaviour over time (BOT):

$$u(t) = \int_{\tau=0}^{\tau=t} \dot{u} dt = \int_{\tau=0}^{\tau=t} (r - L^{-1}) u dt$$

In general, biological systems are *never* exactly integrable, but we have simplified the Rabbits system so greatly (by ignoring migration and predation) that we can calculate this integral exactly:

$$\begin{aligned} \frac{du}{dt} &= (r - L^{-1}) u \\ \Rightarrow \int \frac{du}{u} &= \int (r - L^{-1}) dt \\ \Rightarrow \ln u &= (r - L^{-1}) t + c \\ \Rightarrow u &= \exp(c) \cdot \exp((r - L^{-1}) t) \\ \Rightarrow u(t) &= u_0 \exp((r - L^{-1}) t) = 1000 e^{\text{---} t} \end{aligned}$$

We are very lucky: This is probably the *only* time we will be able to integrate our DEs exactly! From now on, we will need to calculate BOTs *numerically* using the julia DynamicalSystems package.