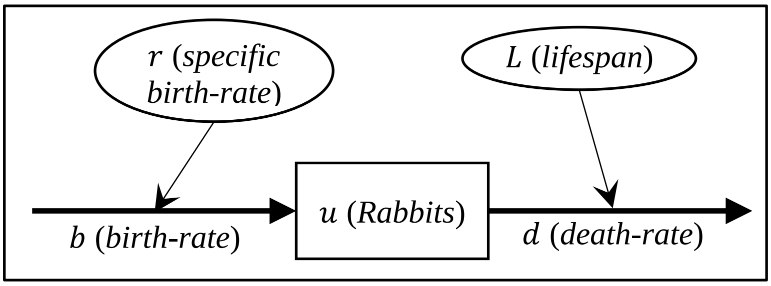
200. Dynamical narratives: How do systems develop over time?

Specifying the Rabbits dynamical system

Stock-and-flow diagram:



Designing the Rabbits dynamical system

Stocks (or ***state variables***):

$u(t)$, where $u_0 \equiv u(0) = 1000$ rabbit (*Here, units are important!*)

Dynamical ***flows***:

$$\dot{u} \equiv \frac{du}{dt} = (b - d)\ \text{rabbit/month}$$

Structural ***relations***:

$$b = r\,u$$
$$d = u/L$$

Numerical ***parameters***:

$$r \equiv \frac{\dot{u}}{u} = \underline{\qquad\qquad}\ (\text{month})^{-1}$$
$$L = 48\ \text{month}$$

Differential equations (DE):

$$\left.\begin{aligned} \dot{u} &= (r - L^{-1})\,u \\ u_0 &= 1000 \\ r &= \underline{\qquad\qquad} \\ L &= 48 \end{aligned}\right\}$$ (*From now on, we will ignore units for convenience!*)

Implementing the Rabbits dynamical system

Behaviour over time (***BOT***) graph:

$$u(t) = \int_{\tau=0}^{\tau=t} \dot{u}\,dt = \int_{\tau=0}^{\tau=t} (r - L^{-1})\,u\,dt$$

In general, biological systems are *never* exactly integrable, but we have simplified the Rabbits system so greatly (by ignoring migration and predation) that we can calculate this integral exactly:

$$\begin{aligned} & \frac{du}{dt} = (r - L^{-1})\,u \\ \Rightarrow\quad & \int \frac{du}{u} = \int (r - L^{-1})\,dt \\ \Rightarrow\quad & \ln u = (r - L^{-1})\,t + c \\ \Rightarrow\quad & u = \exp(c)\cdot \exp\big((r - L^{-1})\,t\big) \\ \Rightarrow\quad & u(t) = u_0 \exp\big((r - L^{-1})\,t\big) = 1000\, e^{\underline{\quad}\, t} \end{aligned}$$

We are lucky: This is almost the *only* time we will be able to integrate our DEs exactly! From now on, we will need to calculate BOTs *numerically* using the julia DynamicalSystems package.