

# Statistical Natural Language Processing

## ML intro & regression

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## Why machine learning?

- Majority of the modern computational linguistic tasks and applications are based on machine learning
  - Tokenization
  - Part of speech tagging
  - Parsing
  - ...
  - Speech recognition
  - Named Entity recognition
  - Document classification
  - Question answering
  - Machine translation
  - ...

## Machine learning is ...

*The field of machine learning is concerned with the question of how to construct computer programs that automatically **improve with experience**.* —Mitchell (1997)

*Machine Learning is the study of data-driven methods capable of mimicking, understanding and aiding **human and biological information processing tasks**.* —Barber (2012)

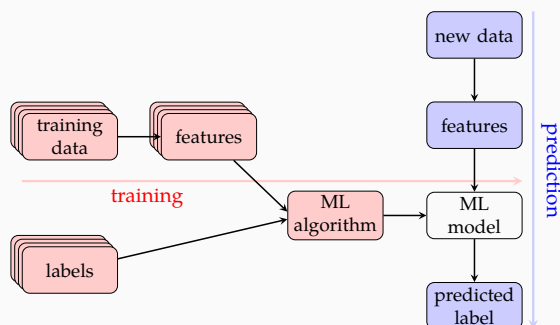
*Statistical learning refers to a vast set of tools for **understanding data**.* —James et al. (2013)

## Supervised or unsupervised

- Machine learning methods are often divided into two broad categories: *supervised* and *unsupervised*
- Supervised methods rely on *labeled* (or *annotated*) data
- Unsupervised methods try to find regularities in the data without any (direct) supervision
- Some methods do not fit any (or fit both):
  - *Semi-supervised* methods use a mixture of both
  - *Reinforcement learning* refers to the methods where supervision is indirect and/or delayed

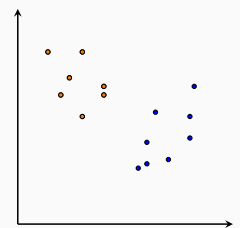
In this course, we will mostly discuss/use supervised methods.

## Supervised learning



## Unsupervised learning

- In unsupervised learning we do not have any labels
- The aim is discovering some 'latent' structure in the data
- Common examples include
  - Clustering
  - Density estimation
  - Dimensionality reduction
- In NLP, methods that do not require (manual) annotation are sometimes called unsupervised

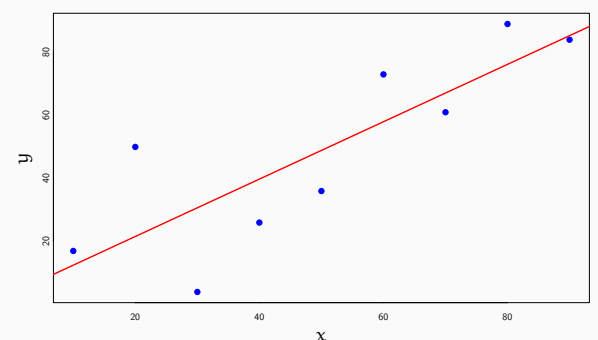


## Supervised learning

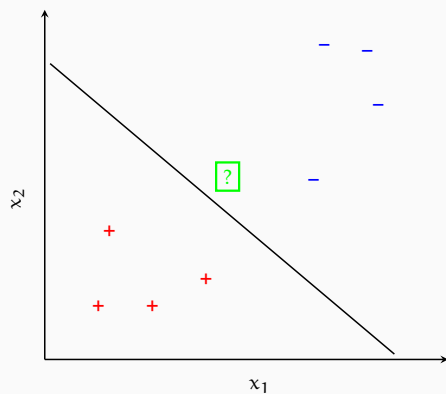
### two common settings

An ML algorithm is called  
*regression* if the outcome to be predicted is a numeric (continuous) variable  
*classification* if the outcome to be predicted is a categorical variable

## Regression



## Classification



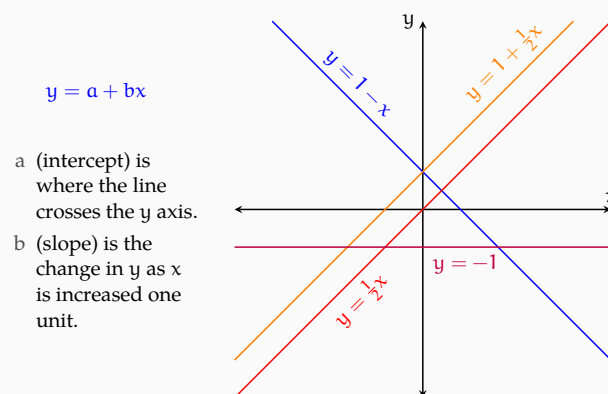
## ML topics we will cover in this course

- (Linear) Regression (today)
- Classification (perceptron, logistic regression)
- Evaluation ML methods / algorithms
- Unsupervised learning
- Sequence learning
- Neural networks / deep learning

## Regression

- Regression is a (supervised) method for predicting the value of a continuous response variable based on a number of predictors
- We estimate the conditional expectation of the outcome variable given the predictor(s)
- It is the foundation of many models in statistics and machine learning
- If the outcome is a label, the problem is called classification

## The linear equation: a reminder



## The simple linear model

$$y_i = a + bx_i + \epsilon_i$$

$y$  is the *outcome* (or response, or dependent) variable. The index  $i$  represents each unit observation/measurement (sometimes called a 'case')

$x$  is the *predictor* (or explanatory, or independent) variable

$a$  is the *intercept* (called *bias* in the NN literature)

$b$  is the *slope* of the regression line.

$a$  and  $b$  are called *coefficients* or *parameters*

$a + bx$  is the *deterministic* part of the model. It is the model's prediction of  $y$  ( $\hat{y}$ ), given  $x$

$\epsilon$  is the *residual*, error, or the variation that is not accounted for by the model. Assumed to be normally distributed with 0 mean

## Notation differences for the regression equation

$$y_i = \mathbf{w}\mathbf{x}_i + \epsilon_i$$

- Sometimes, Greek letters  $\alpha$  and  $\beta$  are used for intercept and the slope, respectively
- Another common notation to use only  $b$ ,  $\beta$ ,  $\theta$  or  $w$ , but use subscripts, 0 indicating the intercept and 1 indicating the slope
- In machine learning it is common to use  $w$  for all coefficients (sometimes you may see  $b$  used instead of  $w_0$ )
- Sometimes coefficients wear hats, to emphasize that they are estimates
- Often, we use the vector notation for both input(s) and coefficients:  $\mathbf{w} = (w_0, w_1)$  and  $\mathbf{x}_i = (1, x_i)$

## Estimating model parameters: reminder

In least-squares regression, we find

$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}} \sum_i (y_i - \hat{y}_i)^2$$

In general, we define an objective (or loss) function  $J(\mathbf{w})$  (e.g., negative log likelihood), and minimize it with respect to the parameters

$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}} J(\mathbf{w})$$

Then,

- take the derivative of  $J(\mathbf{w})$
- set it to 0
- solve the resulting equation(s)

## Least-squares regression

$$y_i = \underbrace{w_0 + w_1 x_i}_{\hat{y}_i} + \epsilon_i$$

- Find  $w_0$  and  $w_1$ , that minimize the prediction error:

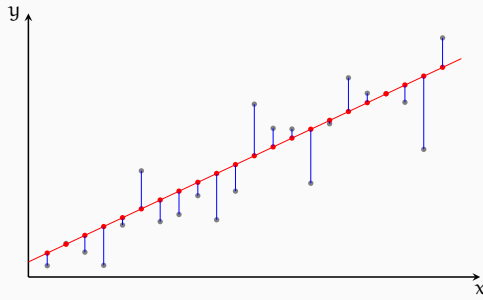
$$J(\mathbf{w}) = \sum_i \epsilon_i^2 = \sum_i (y_i - \hat{y}_i)^2 = \sum_i (y_i - (w_0 + w_1 x_i))^2$$

- We can minimize  $J(\mathbf{w})$  analytically

$$w_1 = \frac{\text{sd}_y}{\text{sd}_x} \quad w_0 = \bar{y} - w_1 \bar{x}$$

\* See appendix for the derivation.

## Visualization of least-squares regression



## What is special about least-squares?

- Minimizing MSE (or  $SS_R$ ) is equivalent to MLE estimate under the assumption  $\epsilon \sim \mathcal{N}(0, \sigma^2)$
- Working with 'minus log likelihood' is more convenient

$$J(\mathbf{w}) = -\log \mathcal{L}(\mathbf{w}) = -\log \prod_i \frac{e^{-\frac{(y_i - \hat{y}_i)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}} (-\log \mathcal{L}(\mathbf{w})) = \arg \min_{\mathbf{w}} \sum_i (y_i - \hat{y}_i)^2$$

- There are other error functions, e.g., absolute value of the errors, that can be used (and used in practice)
- One can also estimate regression parameters using Bayesian estimation

## Short digression: minimizing functions

In least squares regression, we want to find  $w_0$  and  $w_1$  values that minimize

$$J(\mathbf{w}) = \sum_i (y_i - (w_0 + w_1 x_i))^2$$

- Note that  $J(\mathbf{w})$  is a *quadratic* function of  $\mathbf{w} = (w_0, w_1)$
- As a result,  $J(\mathbf{w})$  is *convex* and have a single extreme value
  - there is a unique solution for our minimization problem
- In case of least squares regression, there is an analytic solution
- Even if we do not have an analytic solution, if our error function is convex, a search procedure like *gradient descent* can still find the *global minimum*

## Measuring success in Regression

- Root-mean-square error* (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_i (y_i - \hat{y}_i)^2}$$

measures average error in the units compatible with the outcome variable.

- Another well-known measure is the *coefficient of determination*

$$R^2 = \frac{\sum_i^n (\hat{y}_i - \bar{y})^2}{\sum_i^n (y_i - \bar{y})^2} = 1 - \left( \frac{\text{RMSE}}{\sigma_y} \right)^2$$

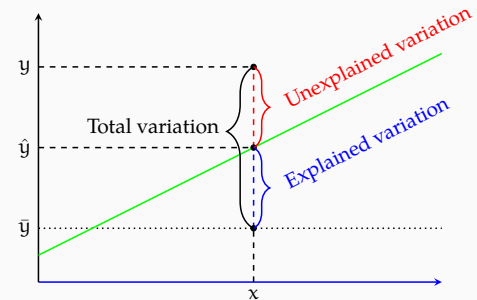
## Assessing the model fit: $r^2$

We can express the variation explained by a regression model as:

$$\frac{\text{Explained variation}}{\text{Total variation}} = \frac{\sum_i^n (\hat{y}_i - \bar{y})^2}{\sum_i^n (y_i - \bar{y})^2}$$

- This value is the square of the correlation coefficient
- The range of  $r^2$  is  $[0, 1]$
- $100 \times r^2$  is interpreted as 'the percentage of variance explained by the model'
- $r^2$  shows how well the model fits to the data: closer the data points to the regression line, higher the value of  $r^2$

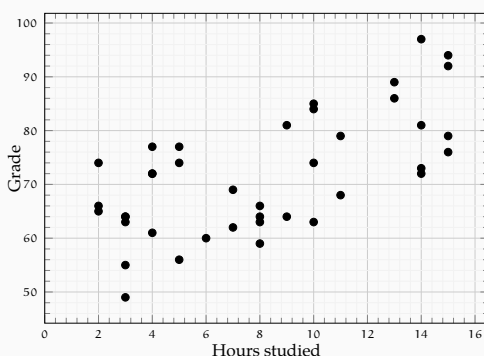
## Explained variation



$$\begin{array}{lcl} \text{Total variation} & = & \text{Unexplained variation} + \text{Explained variation} \\ y - \bar{y} & = & y - \hat{y} + \hat{y} - \bar{y} \end{array}$$

## A hands-on exercise

Draw a regression line over the plot

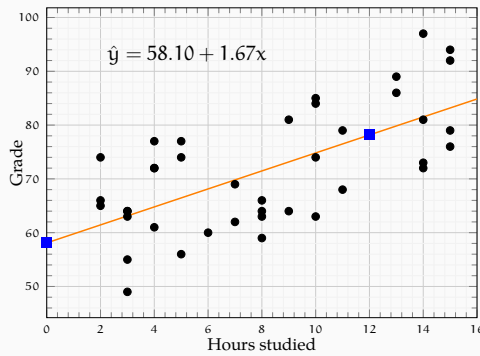


## A hands-on exercise (cont.)

- What is the regression equation?
- What is the expected grade for a student who did not study at all?
- What is the expected grade for a student who studied 12 hours?
- What is the expected grade for a student who studied 40 hours?

## A hands-on exercise

### The regression line



## Regression with multiple predictors

$$y_i = \underbrace{w_0 + w_1 x_{i,1} + w_2 x_{i,2} + \dots + w_k x_{i,k}}_{\hat{y}} + \epsilon_i = w x_i + \epsilon_i$$

$w_0$  is the intercept (as before).

$w_{1..k}$  are the coefficients of the respective predictors.

$\epsilon$  is the error term (residual).

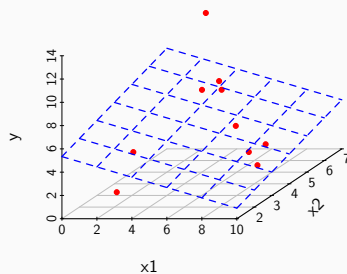
- using vector notation the equation becomes:

$$y_i = w x_i + \epsilon_i$$

where  $w = (w_0, w_1, \dots, w_k)$  and  $x_i = (1, x_{i,1}, \dots, x_{i,k})$

It is a generalization of simple regression with some additional power and complexity.

## Visualizing regression with two predictors



## Input/output of liner regression: some notation

A regression with  $k$  input variables and  $n$  instances can be described as:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_y = \underbrace{\begin{bmatrix} 1 & x_{1,1} & x_{1,2} & \dots & x_{1,k} \\ 1 & x_{2,1} & x_{2,2} & \dots & x_{2,k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n,1} & x_{n,2} & \dots & x_{n,k} \end{bmatrix}}_X \times \underbrace{\begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_k \end{bmatrix}}_w + \underbrace{\begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \vdots \\ \epsilon_n \end{bmatrix}}_\epsilon$$

$$y = Xw + \epsilon$$

## Estimation in multiple regression

$$y = Xw + \epsilon$$

We want to minimize the error (as a function of  $w$ ):

$$\begin{aligned} \epsilon^2 &= J(w) = (y - Xw)^2 \\ &= \|y - Xw\|^2 \end{aligned}$$

Our least-squares estimate is:

$$\begin{aligned} \hat{w} &= \arg \min_w J(w) \\ &= (X^T X)^{-1} X^T y \end{aligned}$$

Note: the least-squares estimate is also the maximum likelihood estimate under the assumption of normal distribution of errors.

## Categorical predictors

- Categorical predictors are represented as multiple binary coded input variables
- For a binary predictor, we use a single binary input. For example, (1 for one of the values, and 0 for the other)

$$x = \begin{cases} 0 & \text{for male} \\ 1 & \text{for female} \end{cases}$$

- For a categorical predictor with  $k$  values, we use  $k - 1$  predictors (various coding schemes are possible). For example, for 3-values

$$x = \begin{cases} (0, 0, 1) & \text{neutral} \\ (0, 1, 0) & \text{negative} \\ (1, 0, 0) & \text{positive} \end{cases} \quad \text{one-hot coding}$$

$$x = \begin{cases} (0, 0) & \text{neutral} \\ (0, 1) & \text{negative} \\ (1, 0) & \text{positive} \end{cases} \quad \text{'treatment' encoding}$$

## Dealing with non-linearity

- Least squares works, because the loss function is linear with respect to parameter  $w$
- Introducing non-linear combinations of inputs does not affect the estimation procedure. The following are still linear models

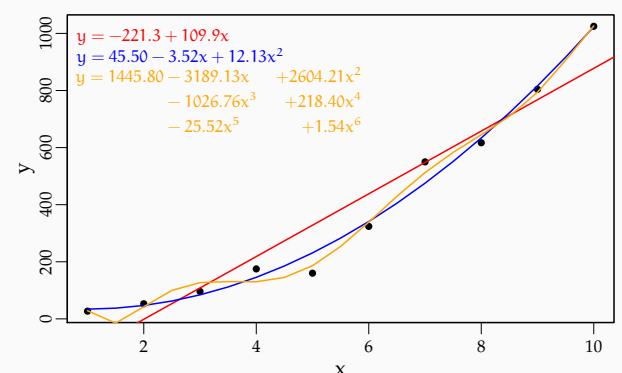
$$y_i = w_0 + w_1 x_i^2 + \epsilon_i$$

$$y_i = w_0 + w_1 \log(x_i) + \epsilon_i$$

$$y_i = w_0 + w_1 x_{i,1} + w_2 x_{i,2} + w_3 x_{i,1} x_{i,2} + \epsilon_i$$

- These *transformations* allow linear models to deal with some non-linearities
- In general, we can replace input  $x$  by a function of the input(s)  $\Phi(x)$ .  $\Phi()$  is called a *basis function*

## Example: polynomial basis functions



## Regularized parameter estimation

- To avoid overfitting and high variance, one of the common methods is *regularization*
- With regularization, in addition of minimizing the cost function, we simultaneously constrain the possible parameter values
- For example, the regression estimation becomes:

$$\hat{\mathbf{w}} = \arg \min_{\mathbf{w}} \sum_i (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^k w_j^2$$

- The new part is called the regularization term, where  $\lambda$  is a *hyperparameter* that determines the effect of the regularization.
- In effect, we are preferring small values for the coefficients
- Note that we do not include  $w_0$  in the regularization term

## L2 regularization

The form of regularization, where we minimize the regularized cost function,

$$J(\mathbf{w}) + \lambda \|\mathbf{w}\|$$

is called L2 regularization.

- Note that we are minimizing the L2-norm of the weight vector
- In statistic literature this L2-regularized regression is called *ridge regression*
- The method is general: it can be applied to other ML methods as well
- The choice of  $\lambda$  is important
- Note that the scale of the input becomes important

## L1 regularization

In L1 regularization we minimize

$$J(\mathbf{w}) + \lambda \sum_{j=1}^k |w_j|$$

- The additional term is the L1-norm of the weight vector (excluding  $w_0$ )
- In statistic literature the L1-regularized regression is called *lasso*
- The main difference from L2 regularization is that L1 regularization forces some values to be 0 – the resulting model is said to be ‘sparse’

## Regularization as constrained optimization

L1 and L2 regularization can be viewed as minimization with constraints

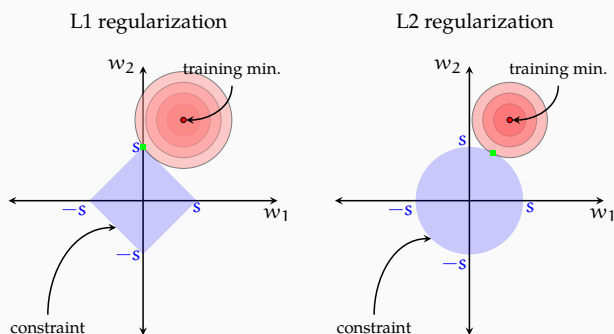
L2 regularization

Minimize  $J(\mathbf{w})$  with constraint  $\|\mathbf{w}\| < s$

L1 regularization

Minimize  $J(\mathbf{w})$  with constraint  $\|\mathbf{w}\|_1 < s$

## Visualization of regularization constraints



## Regularization: some remarks

- Regularization prevents overfitting and reduces variance
- The *hyperparameter*  $\lambda$  needs to be determined
  - best value is found typically using a *grid search*, or a *random search*
  - it is tuned on an additional partition of the data, *development set*
  - development set cannot overlap with training or test set**
- The regularization terms can be interpreted as *priors* in a Bayesian setting
- Particularly, L2 regularization is equivalent to a normal prior with zero mean

## Summary

What to remember:

- Supervised vs. unsupervised learning
- Regression vs. classification
- Linear regression equation
- Least-square estimate
- MSE,  $r^2$
- non-linearity & basis functions
- L1 & L2 regularization (lasso and ridge)

Next:

Mon classification

Wed exercises

Fri classification / ML evaluation

## Additional reading, references, credits

- Hastie, Tibshirani, and Friedman (2009) discuss introductory bits in chapter 1, and regression on chapter 3 (sections 3.2 and 3.4 are most relevant to this lecture)
- Jurafsky and Martin (2009) has a short section (6.6.1) on regression
- You can also consult any machine learning book (including the ones listed below)



Barber, David (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press. isbn: 9780521518147.




Hastie, Trevor, Robert Tibshirani, and Jerome Friedman (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Second. Springer series in statistics. Springer-Verlag New York. isbn: 9780387848587. url: <http://web.stanford.edu/~hastie/ElemStatLearn/>.




James, G., D. Witten, T. Hastie, and R. Tibshirani (2013). *An Introduction to Statistical Learning: with Applications in R*. Springer Texts in Statistics. Springer New York. isbn: 9781461471387. url: <http://www-bcf.usc.edu/~gareth/ISL/>.

# Additional reading, references, credits (cont.)



Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second. Pearson Prentice Hall. isbn: 978-0-13-504196-3.



Mitchell, Thomas (1997). *Machine Learning*. 1st. McGraw Hill Higher Education. isbn: 0071154671,0070428077,9780071154673,9780070428072.