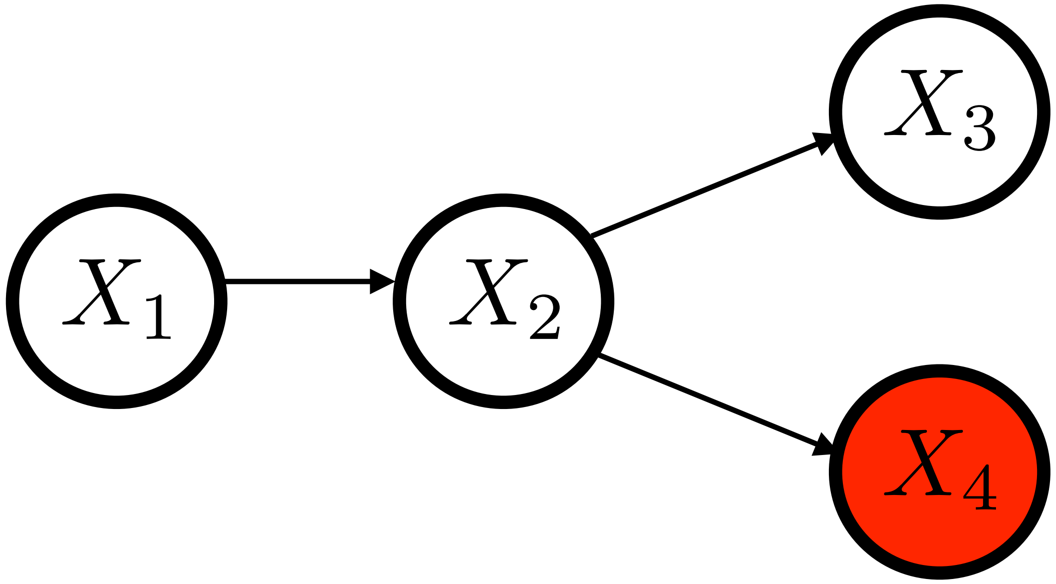
Machine Learning for Data Science (CS4786)
Lecture 20

Graphical Models

Course Webpage :
http://www.cs.cornell.edu/Courses/cs4786/2016sp/

Variable Elimination: Examples



Variable Elimination: Bayesian Network

Initialize List with conditional and marginal probability distributions

Pick an order of elimination I for remaining variables

For each $X_i \in I$

Find distributions in List containing X_i, remove them from List and add them to list^i

Define new distribution

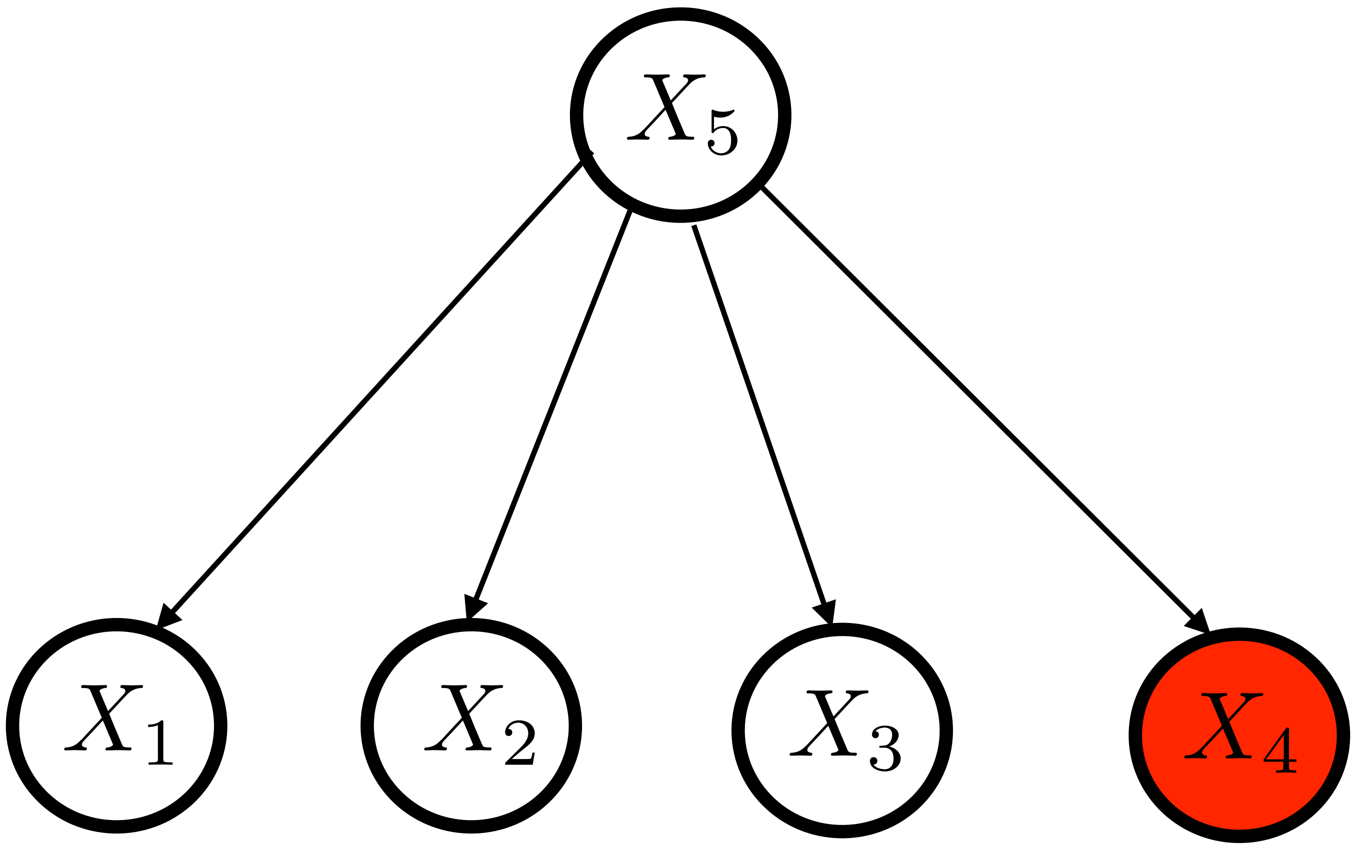
$$m_{X_i} = \sum_{X_i} \prod_j \text{list}_j^i[X_i]$$

Instert m_{X_i} into List

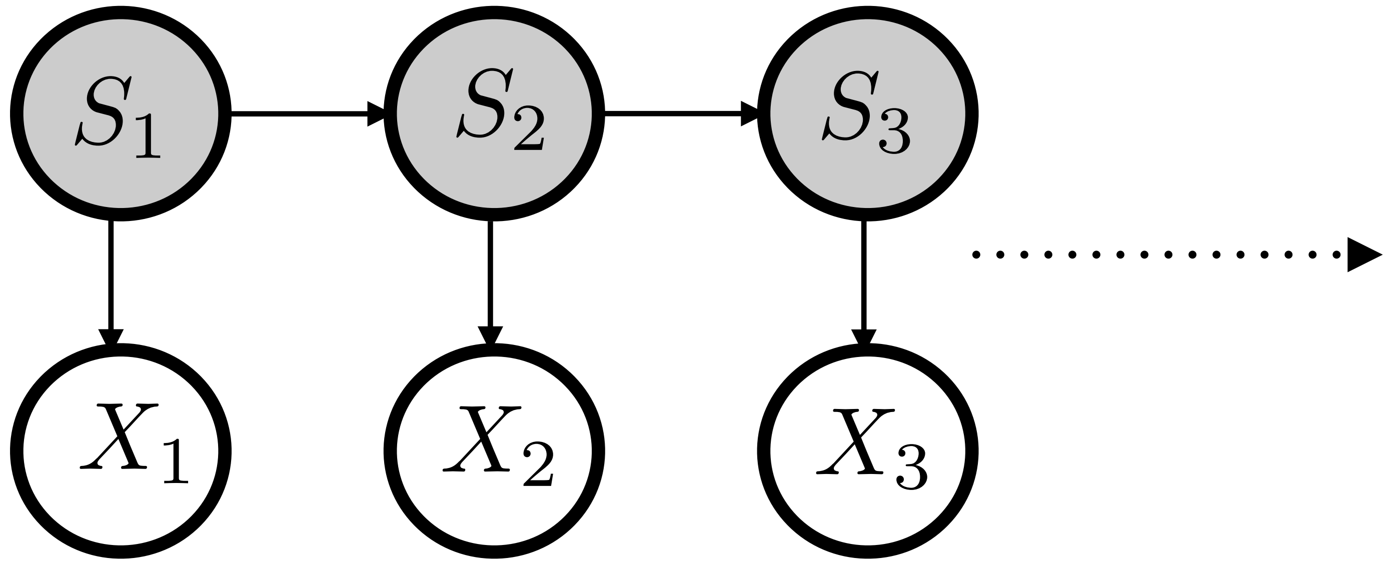
End

Return List

Variable Elimination: Order Matters



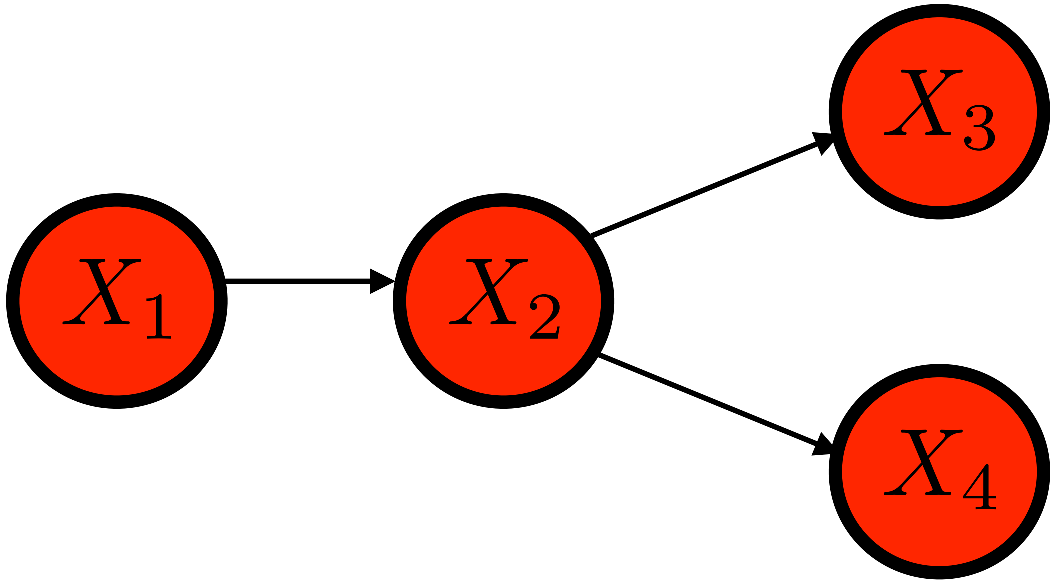
Variable Elimination: Examples



Message Passing

- Often we need more than one marginal computation
- Over variables we need marginals for, there are many common distributions/potentials in the list
- Can we exploit structure and compute these intermediate terms that can be reused?

Message Passing Example



Belief Propagation

- Think of variables as nodes in a network, each node is allowed to chat with its neighbors
- Adjacent nodes receive messages from neighbors telling the node how to update its belief
- Each node in turn sends messages to its neighbors:
  based on observation, previous received messages, marginal and conditional distributions telling the other how to update beliefs
- (Hopefully) All the nodes converge on their beliefs

Belief Propagation

1. For every observation $X_j = x_j$ define $E_{X_j}(x) = \mathbf{1}\left\{x = x_j\right\}$, for unobserved variables set $E_{X_j}(x) = 1$
2. At round 0, all messages between nodes are 1

Belief Propagation

For any node X_i

- Incoming message to node from children:

$$\lambda(x) = E_{X_i}(x) \prod_{j \in \text{children}(X_i)} \lambda_{X_j}(x)$$

- Incoming message from Parents:

$$\pi(x) = \sum_u P(X_i = x | \text{Parent}(X_i) = u) \prod_{k \in \text{Parent}(X_i)} \pi_{X_i}(u_k)$$

- Outgoing message to Parent X_j:

$$\lambda_{X_i}(u_i) \propto \sum_x \lambda(x) \sum_{u \setminus u_i} P(X_i = x | \text{Parent}(X_i) = u) \prod_{k \neq i} \pi_{X_i}(u_k)$$

- Outgoing message to child X_j:

$$\pi_{X_j}(x) \propto \pi(x) E_{X_i}(x) \prod_{k \neq j} \lambda_{Y_j}(x)$$

Belief Propagation on Trees

Hidden Markov Model Example