Q2:

E-step:

$$Q_t^{(i)}(c_t) = P\left(c_t = k \middle| x_t, \theta^{(i-1)}\right)$$

$$\propto P\left(x_t \middle| c_t = k, \theta^{(i-1)}\right) \cdot P\left(c_t = k \middle| \theta^{(i-1)}\right)$$

$$\propto \Phi(x_t, \lambda) \cdot \pi_k^{(i-1)}$$

$$Q_t^{(i)}(c_t) = \frac{\lambda_k^{x_t} e^{-\lambda_k}}{\sum_{k=1}^{K} e^{-\lambda_k}}$$

M-step for π_k:

$$\pi_k = \underset{\pi_k}{argmax} \left(\sum_{t=1}^{n} \sum_{k=1}^{K} Q_t^{(i)}(k) \, log(\pi_k) \right)$$

$$\pi_k = \frac{\sum_{t=1}^{n} Q_t^{(i)}(k)}{n}$$

M-step for λ_k:

$$\lambda_k = \underset{\lambda_k}{argmax} \left(\sum_{t=1}^{n} \sum_{k=1}^{K} Q_t^{(i)}(k) \, log\left(P(x_t | c_t = k, \theta)\right) \right)$$

$$= \underset{\lambda_k}{argmax} \left(\sum_{t=1}^{n} \sum_{k=1}^{K} Q_t^{(i)}(k) \, log\left(\frac{\lambda_k^{x_t} e^{-\lambda_k}}{x_t!} \right) \right)$$

$$= \underset{\lambda_k}{argmax} (\sum_{t=1}^{n} \sum_{k=1}^{K} Q_t^{(i)}(k) (x_t log(\lambda_k) - \lambda_k - log(x_t!)))$$

Take the derivative of the RHS and equate it to 0

Then we get:

$$\sum_{t=1}^{n} Q_t^{(i)}(k) \left(x_t \frac{1}{\lambda_k} - 1 \right) = 0$$

So:

$$\lambda_k = \frac{\sum_{t=1}^{n} Q_t^{(i)}(k) \, x_t}{\sum_{t=1}^{n} Q_t^{(i)}(k)}$$