HW3 CS 5786

Yan Deng, Haonan Liu, Nianze Liu, Yunzhou Zhang

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Q1

a) write down steps for variable elimination in terms of $P(variable|Parents)$

For marginal probability of X_1:

Initialize list with conditional probability distribution:

$$List = [P(X_1), P(X_2|X_1), P(X_3|X_2, X_1)]$$

Pick an order of elimination I for remaining variables: I=[3, 2],

$$\begin{aligned}
P(X_1) =& \sum_{X_2} \sum_{X_3} P(X_1)P(X_2|X1)P(X_3|X_2, X1) \\
=& P(X_1) \sum_{X_2} P(X_2|X1) \sum_{X_3} P(X_3|X_2, X1) \\
m_{X_3}(X_1, X_2) =& \sum_{X_3} P(X_3|X_2, X1) = 1 \\
m_{X_2}(X_1) =& \sum_{X_2} P(X_2|X1) = 1
\end{aligned}$$

Return the List: List=$[P(X_1)]$

For marginal probability of X_2:

Initialize list with conditional probability distribution:

$$List = [P(X_1), P(X_2|X_1), P(X_3|X_2, X_1)]$$

Pick an order of elimination I for remaining variables: I=[3, 1],

$$\begin{aligned}
P(X_2) =& \sum_{X_1} \sum_{X_3} P(X_1)P(X_2|X1)P(X_3|X_2, X1) \\
=& \sum_{X_1} P(X_1)P(X_2|X1) \sum_{X_3} P(X_3|X_2, X1) \\
m_{X_3}(X_1, X_2) =& \sum_{X_3} P(X_3|X_2, X1) = 1 \\
m_{X_1}(X_2) =& \sum_{X_1} P(X_1)P(X_2|X1)
\end{aligned}$$

Return the List: List=[$m_{X_1}(X_2)$]

For marginal probability of X_3:

Initialize list with conditional probability distribution:

$$List = [P(X_1), P(X_2|X_1), P(X_3|X_2, X_1)]$$

Pick an order of elimination I for remaining variables: I=[2, 1],

$$P(X_3) = \sum_{X_1} \sum_{X_2} P(X_1)P(X_2|X1)P(X_3|X_2, X1)$$
$$= \sum_{X_1} P(X_1) \sum_{X_2} P(X_2|X1)P(X_3|X_2, X1)$$
$$m_{X_2}(X_1, X_3) = \sum_{X_2} P(X_2|X1)P(X_3|X_2, X1)$$
$$m_{X_1}(X_3) = \sum_{X_1} P(X_1)m_{X_2}(X_1, X_3)$$

Return the List: List=[$m_{X_1}(X_3), m_{X_2}(X_1, X_3)$]

b) Plug in values from the table and compute marginal probabilities $P(X_1|X_2 = 1)$ and $P(X_3|X_2 = 1)$ for observation X2 = 1

$$P(X_1|X2 = 1) = \frac{P(X_1, X2 = 1)}{P(X2 = 1)} = \frac{P(X2 = 1|X_1,)P(X_1)}{P(X2 = 1)} \propto P(X2 = 1|X_1)P(X_1)$$
$$P(X_1 = 1|X2 = 1) \propto P(X2 = 1|X_1 = 1)P(X_1 = 1) = 0.3 * 0.1 = 0.03$$
$$P(X_1 = 0|X2 = 1) \propto P(X2 = 1|X_1 = 0)P(X_1 = 0) = 0.4 * 0.9 = 0.36$$

Normalize:

$$P(X_1 = 1|X2 = 1) = 0.077$$
$$P(X_1 = 0|X2 = 1) = 0.923$$

$$P(X_3|X2 = 1) = \frac{P(X_3, X2 = 1)}{P(X2 = 1)} \propto P(X_3, X2 = 1)$$
$$P(X_3, X2 = 1) = \sum_{X_1} P(X_1)P(X_2 = 1|X1)P(X_3|X_2 = 1, X1)$$
$$P(X_3 = 1|X2 = 1) \propto \sum_{X_1=0,1} P(X_1)P(X_2 = 1|X_1)P(X_3 = 1|X_2 = 1, X_1)$$
$$= 0.9 * 0.4 * 0.5 + 0.1 * 0.3 * 0.2$$
$$P(X_3 = 0|X2 = 1) \propto \sum_{X_1=0,1} P(X_1)P(X_2 = 1|X_1)P(X_3 = 0|X_2 = 1, X_1)$$
$$= 0.9 * 0.4 * 0.5 + 0.1 * 0.3 * 0.8$$

Normalize:

$$P(X_3 = 1|X2 = 1) = 0.478$$
$$P(X_3 = 0|X2 = 1) = 0.522$$

Q3

a) Use the Likelihood weighted (importance weighted) sampling procedure for this Bayesian Network. Submit 100 samples in file sample.csv where each line is a vector of length 3 separated by commas and you will provide 100 lines, one for each sample.

   See the attached appendix for the code.

b) Based on the sample, compute empirical marginal distributions for $P(X1|X2 = 1)$, $P(X3|X2 = 1)$ and report them.

   For one simulation, we got the following results, which is close to what we got from Q1.

$$P(X_1 = 1|X2 = 1) = 0.061$$
$$P(X_1 = 0|X2 = 1) = 0.939$$
$$P(X_3 = 1|X2 = 1) = 0.474$$
$$P(X_1 = 0|X2 = 1) = 0.526$$

Appendix

Matlab Code:

```
N=100;
w=zeros(N,1);
x=zeros(N,3);
x(:,2)=1;
for t=1:N
    w(t)=1;

    for i=1:3

        if i==1

            if rand()<=0.1
            x(t,i)=1;
            else
            x(t,i)=0;
            end

        elseif i==2

            if x(t,1)==1;
               w(t)= w(t)*0.3;
            else
               w(t)= w(t)*0.4;
            end

        elseif i==3

            if x(t,1)==1
                if rand()<=0.2
                    x(t,i)=1;
                else
                    x(t,i)=0;
                end

            elseif x(t,1)==0
                if rand()<=0.5
                    x(t,i)=1;
                else
                    x(t,i)=0;
                end
            end
```

```
        end

    end
end

s1=0;
s2=0;
s3=0;
s4=0;

for t=1:N
    if x(t,1)==1
        s1=s1+w(t);
    elseif x(t,1)==0
        s2=s2+w(t);
    end
    if x(t,3)==1
        s3=s3+w(t);
    elseif x(t,3)==0
        s4=s4+w(t);
    end
end

p1=s1/sum(w);
p2=s2/sum(w);
p3=s3/sum(w);
p4=s4/sum(w);
csvwrite('sample.csv',x);
```