Math 425 Spring 2021 Project 1 due: 5PM on Fri March 12

Project Directions

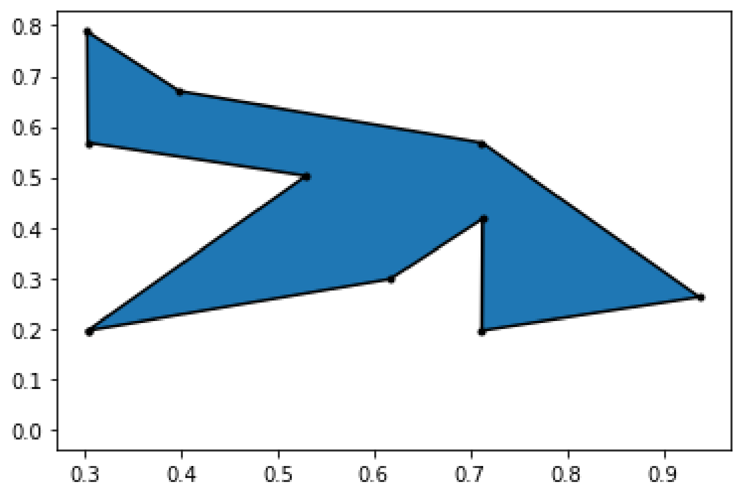
- Include a report on every group member's contribution.
- Submit the group's well commented code used for the project with instructions on how to compile and run.
- Make an **8** to **15** minute video presentation of your results.

The project consists of three problems

Problem 1

Escher in the Matrix - Tiling the plane

Consider the tile in the figure below. We'll call this the model bird.



The ten endpoints of the line segments are (0.3036, 0.1960), (0.6168, 0.2977), (0.7128, 0.4169), (0.7120, 0.1960), (0.9377, 0.2620), (0.7120, 0.5680), (0.3989, 0.6697), (0.3028, 0.7889), (0.3036, 0.5680), and (0.5293, 0.5020). Our goal is to create an Escher like artwork by tiling the model bird.

(a) Take four tiles of the model bird and fit them together according to the instructions below:

   i. Create the first tile by rotating the model bird through π radians about the point $(0.7120, 0.4320)$. Provide the matrix for the transformation in homogeneous coordinates.

   ii. Form the second tile by reflecting the model bird through the horizontal line $y = 0.6180$ and then translating this image by 0.4084 units along the x-axis. Provide the matrix for the transformation in homogeneous coordinates.

iii. To create the third tile, reflect the model bird through the vertical line $x = 0.5078$ and translate the image by 0.1000 along the y-axis. Provide the matrix for the transformation in homogeneous coordinates.

iv. Create the fourth tile by translating the model bird along the y-axis by 0.4720. Provide the matrix for the transformation in homogeneous coordinates.

v. Graph all four tiles together to produce the base pattern.

(b) Repeat the pattern of four model birds in part (a) but translate the entire pattern by $0.7441n$, $n = 1, 2, 3$ along the y-axis to produce a column of the tilings.

(c) Repeat parts (b) and translate the column of tilings by $-0.8168n$, $n = 1, 2, 3, 4, 5$ to produce the final pattern.

Problem 2

To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every second, from $t = 0$ to $t = 12$. The positions (in feet) were: 0, 8.8, 29.9, 62.0, 104.7, 159.1, 222.0, 380.4, 471.1, 571.7, , 686.8, 809.2 .

(a) Find the least-squares cubic curve $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ for these data.

(b) Use the result of (a) to estimate the velocity of the plane when $t = 4.5$ seconds.

(c) Suppose the possible measurement errors become greater as the speed of the airplane increases, and let W be the diagonal weighting matrix whose diagonal entries are 1, 1, 1, 0.9, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, ,0.2, 0.1. Find the cubic curve that fits the data with minimum weighted least-squares error, and use it to estimate the velocity of the plane when $t = 4.5$ seconds.

Problem 3

Download the Human Weight/Height Dataset
http://wiki.stat.ucla.edu/socr/index.php/SOCR_Data_Dinov_020108_HeightsWeights

(a) Divide the data into a training set (15000 data points), validation set (8000 data points) and a test set (2000 points). Make a scatter plot of the training set.

(b) Implement batch gradient descent to determine the linear regression parameters θ for the dataset. Set the initial parameters to 0 and use a learning rate $\alpha = 0.01$. Use the validation set to tune your parameters.

(c) Apply the results to the test set and report the 2-norm error.

(d) Use the QR algorithm to find the least-squares linear model for the data.