# Redundant Bureaucracy or Efficient Supervision: Collusion-Proof Contracts in Princial-Agent Problems with Multiple Supervisors

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May 2022

**Abstract**

We consider the principal-agent problem with multiple supervisors. We investigate collusion-proof equilibria in two structures, the vertical structure where each supervisor verifies the report of the supervisor one level below him, and the parallel structure where multiple supervisors simultaneously monitor the agent. We first show that compared to having only one supervisor, hiring another supervisor is always optimal in the parallel structure. On the other hand, the optimality of hiring a supervising supervisor is independent of his own monitoring ability but depends on the monitoring ability of the existing supervisor and the cost of side transfer. We further show that the optimal number of supervisors in the parallel structure is infinity, while it is never optimal to expand the vertical structure beyond the supervising supervisor. We also compare the maximum and limit efficiencies in both structures.

# 1 Introduction

The principal-agent problem is a classic model in contract theory that illustrates the effects of asymmetric information. Tirole (1986) studies the strategic tension when adding a supervisor to monitor the agent's effort, which is unobservable by the principal. In that paper, Tirole assumes that "it is not efficient to divide the supervisory job among several supervisors" and that "there exists ex-ante a competitive supply of supervisors." This paper attempts to analyze the efficiency of the organization when there exists a competitive supply of supervisors and the principal could hire multiple of them, each doing the complete supervisory work rather than part of it. We mainly consider two structures. The first one is vertical, in which the principal hires a supervisor together with a supervising supervisor who verifies the truthfulness of the supervisor's report. The principal could further expand the vertical structure by, for example, adding a supervising supervising supervisor to monitor the truthfulness of the supervising supervisor's report. The second is parallel, where the principal hires multiple supervisors simultaneously, all monitoring the action of the agent and reporting it to the principal.

We study the following questions in this paper: Under what conditions would the principal prefer to add one more supervisor/supervising supervisor to the existing system with one supervisor? When would the vertical structure be more cost-efficient than the parallel structure when there are two supervisors? If the principal could hire many (or even infinitely many) supervisors/supervising supervisors, what would be the optimal number of supervisors in each structure, and when both structures have optimal numbers of supervisors, under what conditions would the vertical structure be more efficient? Under what conditions would the vertical structure be more efficient when the number of supervisors approach infinity in both structures?

The questions above are inspired by many real-world phenomena. The typical hierarchy in investment banks consists of analysts, associates, vice presidents, senior vice presidents, and managing directors. Similarly, the hierarchy in consulting firms involves consultants, senior consultants, senior managers, and directors. In both industries, much of the work of people above associates/consultants are supervisory. Therefore, a natural question to investigate is whether such hierarchy is redundant. In this paper, we derive the striking result that in the collusion-proof equilibrium, the optimality condition for hiring a supervising supervisor is independent of the monitoring ability of the supervising supervisor himself, but only dependent on the monitoring ability of the supervisor as well as the cost of side transfers (i.e., bribes). We further show that it is never efficient to have any hierarchical level above the supervising supervisor. The academia, on the other hand, often assigns two advisors to a graduate student as in the parallel structure. In this paper, we show that having the second supervisor is always cost-efficient for the principal in the collusion-proof equilibrium.

## 2 Literature Review

In this section, we discuss literature related to the principal-supervisor-agent model. Tirole (1986) introduces a supervisor into the classic principal-agent problem. In his model, the output produced by the agent is the sum of his effort and the productivity parameter. The output is observed by the principal, but he does not know the productivity parameter. With some probability, the true productivity parameter is revealed to the supervisor, and the agent would know such revelation when it happens. Tirole assumes that information is "hard," which means that the supervisor could not make up a false report of high productivity parameter when he does not observe anything, and therefore the only possible kind of collusion between the supervisor and the agent is that the supervisor hides his information of low productivity parameter when he observes that. Baliga (1999) extends Tirole's model to study the optimal contract when information is "soft," which means the supervisor's report is unverifiable. He shows that it is profitable for the principal to employ a supervisor even if information is "soft."

Relatively few studies analyze the case where two or more supervisors are employed. Koffman and Lawarree (1996) develop a model in which two supervisors are simultaneously sent to write a report of the agent's effort. They assume that the true report could be identified when two reports differ. In this game, supervisors' reports are always conclusive (i.e., the report either says low productivity or high productivity). Also, supervisors do not benefit from limited liability in their model: the supervisor who is detected of writing a false report would suffer from a severe punishment like imprisonment.

Bohn (1987) analyzes the case where the principal could hire multiple supervisors with various hierarchical levels to monitor many risk-averse agents. Each supervisor could divide his monitoring effort among supervisors in lower levels and agents. Due to this rather general setting, most of the results are existential rather than constructive. For example, he proved the existence of a unique optimal allocation of monitoring efforts. He also assumed that side transfer is impossible, while collusion is the main theme in our paper.

An important issue with the existing literature is that these papers have different setups when analyzing different aspects of firm hierarchy, making the direct comparison between the vertical and parallel structures impossible. For example, Tirole (1986) models the output as an observable sum of productivity parameter and the agent's effort. On the other hand, other literature, including Tirole (1992) and Bolton (2004), models the principal-supervisor-agent problem with an observable output that equals 1 if the agent exerts effort and 0 otherwise, together with an unobservable private cost faced by the agent when he exerts effort. Some papers assume limited liability of the supervisor and the agent (i.e., their wages could not be negative), while others like Koffman and Lawarree (1996) include outside punishment like jail time to deter collusion.

To solve this issue, in this paper we will adopt the model used by Tirole (1992), Baliga (1999), and Bolton (2004) and then add supervisors/supervising supervisors to the model. In each enriched model, we focus on finding a collusion-proof mechanism, where it is an equilibrium for no side transfers to occur. We also assume limited liability of both the agent and the supervisors/supervising supervisors.

# 3 The Models

## 3.1 The Classic Principal-Supervisor-Agent Model

We first discuss the canonical model analyzed in Tirole (1992), Baliga (1999), and Bolton (2004). A principal buys 0 or 1 unit of a good from an agent, and the principal's utility of having 1 unit of that good is $V > 1$, and his utility of having 0 unit is 0. Producing this good would induce a private cost of $c \in \{c_L, c_H\}$ from the agent, where $c_H > c_L$. The principal's prior belief of the cost is $\mathbb{P}(c = c_H) = \mu$ and $\mathbb{P}(c = c_L) = 1 - \mu$. Throughout the paper, we assume that $V > c_H$, that is, the principal would prefer to have this good even if it induces a high cost. The principal offers the agent a wage $w^a$ if he receives the good, and 0 otherwise. The risk-neutral agent has reservation utility 0, and his utility is $U^a = w^a - c$ when he produces the good. Without having a supervisor, the principal faces a classic screening problem, and he would pay the agent

$$w^a = \begin{cases} c_H & \text{if } V - c_H > (1-\mu)(V - c_L) \\ c_H \text{ or } c_L & \text{if } V - c_H = (1-\mu)(V - c_L) \\ c_L & \text{if } V - c_H < (1-\mu)(V - c_L). \end{cases}$$

We also make the assumption throughout the paper that $V - c_H > (1-\mu)(V - c_L)$. Therefore, without having a supervisor, the principal has utility $U^p = V - c_H$.

Now we introduce a supervisor into the model. The supervisor is also risk-neutral with reservation utility 0. He learns a signal $\sigma \in \{\emptyset, \sigma_L\}$. If $c = c_L$, then he learns $\sigma = \sigma_L$ with probability $p$, and $\sigma = \emptyset$ with probability $1 - p$. If $c = c_H$, then he learns $\sigma = \emptyset$ with probability 1. Therefore, $p$ measures his monitoring ability. Although this information structure seems unusual at the first glance, it is in fact the standard one used in auditing and regulation literature. The principal offers the supervisor a wage $w^s \in \{w^s_\emptyset, w^s_L\}$ conditional on his report $r^s \in \{r^s_\emptyset, r^s_L\}$, where $r^s_\emptyset$ is a report of inconclusive private cost, and $r^s_L$ is a report of low private cost. $w^s_\emptyset$ is his wage when he files an inconclusive report, and $w^s_L$ is his wage when he files a report of agent's low cost. His utility is $U^s = w^s$. The principal gives the agent a wage $w^a \in \{w^a_L, w^a_\emptyset\}$, where $w^a_L$ is his wage when the supervisor reports a low cost, and $w^a_\emptyset$ is his wage when the supervisor gives the principal an inconclusive report. Following the setup of Tirole (1986, 1992) and Baliga (1999), we assume that the agent's information is finer than that of the supervisor, which means that the agent also learns $\sigma$. We focus on the case where information is "hard," so the report of low cost could be later verified by the principal. Therefore, the supervisor could not forge a report of low cost when he observes no signal. The timeline of the game is as follows:

**Time 1**: The principal offers the supervisor and the agent a contract. The game ends immediately if either party does not accept the contract.

**Time 2**: The agent learns his cost $c$.

**Time 3**: The supervisor and the agent learn the signal $\sigma^s$.

**Time 4**: The agent chooses to produce the good or not. The supervisor and the agent discuss and sign side contracts and make side-transfers.

**Time 5**: The supervisor sends the report $r^s$ to the principal.

**Time 6**: The output is observed by the principal, and the contract is implemented.

In this model, collusion would take place in Time 4 in the form of a side monetary transfer. If the agent tries to transfer 1 dollar to the supervisor, the supervisor would receive $k < 1$ dollar. The idea is that this hidden monetary transfer is costly for the agent (Bolton 2004). We now give a canonical result in Tirole (1992), Baliga (1999), and Bolton (2004).

**Proposition 1**: Compared to having no supervisor, the principal finds it less costly to implement his optimum when having a supervisor.

*Proof.* Firstly, as we assume limited liability of the agent and the supervisor, we have

$$w^a_L \geq 0, w^a_\varnothing \geq 0, w^s_L \geq 0, w^s_\varnothing \geq 0.$$

If the principal receives a report of low cost, then he would give the agent $w^a_L \geq c_L$, because otherwise he would receive 0 as output. Clearly this condition must be binding in equilibrium, so ${w^a}^*_L = c_L$. In the collusion-proof equilibrium, if the principal sees an inconclusive report, he believes that the agent faces low cost with probability

$$\mathbb{P}(c = c_L | r^s = r^s_\varnothing) = \frac{(1-\mu)(1-p)}{(1-\mu)(1-p) + \mu}$$

If he gives $w^a_\varnothing \geq c_H$, then he would get $V - w^s_\varnothing - w^a_\varnothing$, and if he gives $c_L \leq w^a_\varnothing \leq c_H$, then he would get $\frac{(1-\mu)(1-p)}{(1-\mu)(1-p)+\mu}(V - w^s_\varnothing - w^a_\varnothing)$. Notice that a positive $w^s_\varnothing$ only lowers the utility of the principal without giving him any informational gain. Therefore, ${w^s}^*_\varnothing = 0$. Now, since

$$\frac{(1-\mu)(1-p)}{(1-\mu)(1-p)+\mu}(V - c_L) < (1-\mu)(V - c_L) < V - c_H,$$

the optimal contract would give $w^a_\varnothing = c_H$.

When the supervisor receives $\sigma = \sigma_L$, the agent is willing to give up at most $c_H - c_L$ to bribe the supervisor, who will in turn receive $k(c_H - c_L)$. Therefore, assuming that the supervisor and the agent would choose the principal's desired action when they are indifferent with another action, in the collusion-proof equilibrium, the incentive compatibility requirement for the supervisor to reveal his information is

$$w^s_L - w^s_\varnothing \geq k(c_H - c_L) \text{ (CP-IC-S)}$$

This condition also must be binding in equilibrium, so ${w^s}^*_L = k(c_H - c_L)$.

Thus, in this game, with probability $\mu$, the agents faces a high cost, and $w^a = c_H$ and $w^s = 0$. With probability $p(1-\mu)$, the agent faces a low private cost that is reported by the supervisor. In this case, $w^a = c_L$ and $w^s = k(c_H - c_L)$. With probability $(1-\mu)(1-p)$, the agent faces a low private cost that is not detected by the supervisor. In this case, $w^a = c_H$ and $w^s = 0$. Therefore, the expected utility of the principal is

$$\mathbb{E}(U^p\{P, S, A\}) = V - \{\mu(c_H + 0) + (1-\mu)p[c_L + k(c_H - c_L)] + (1-\mu)(1-p)(c_H + 0)\}.$$

Notice that the principal gets $V - c_H$ without a supervisor, and therefore the supervisor brings the principal an additional expected utility of

$$\begin{aligned}\mathbb{E}(U^p) - (V - c_H) &= c_H - \{\mu c_H + (1-\mu)p[c_L + k(c_H - c_L)] + (1-\mu)(1-p)c_H\} \\ &= p(1-\mu)(1-k)(c_H - c_L) > 0.\end{aligned}$$

■

## 3.2 The Principal-Two Supervisors-Agent Model

Now we add one more supervisor into the model. Suppose the principal employs two supervisors to monitor the agent, where each supervisor has the same characteristics as the supervisor in the previous models. The signals they receive are independent, and the signals are common knowledge among the two supervisors and the agent. So $\mathbb{P}(\sigma_i^s = \sigma_L | \sigma_{-i}^s = \sigma_L) = \mathbb{P}(\sigma_i^s = \sigma_L | \sigma_{-i}^s = \emptyset) = \mathbb{P}(\sigma_i^s = \sigma_L) = p$ for $i = 1, 2$. We also assume that when both supervisors receive low signal and decide to collude with the agent, they would share the bribe equally. In this game, the principal would offer each supervisor a wage conditional on both of their reports. For example, $w^{s_2}(r_L, r_L)$ is the wage of supervisor 2 when both report $r_L$, and $w^{s_1}(r_\emptyset, r_L)$ is the wage of the supervisor 1 who reports $r_\emptyset$ when the other reports $r_L$. The agent receives $w_L^a$ if at least one report is $r = r_L$, and $w_\emptyset^a$ otherwise. The timeline of the new game is as follows:

**Time 1**: The principal offers the supervising supervisor, the supervisor, and the agent a contract. The game ends immediately if either party does not accept the contract.

**Time 2**: The agent learns his cost $c$.

**Time 3**: The agent as well as supervisors $i$ and $-i$ learn the signal $\sigma_i^s$.

**Time 4**: The agent chooses to produce the good or not. Each supervisor discusses with the agent and signs side contracts and makes side transfers.

**Time 5**: Supervisor $i$ sends the report $r_i^s$ to the principal.

**Time 6**: The output is observed by the principal, and the contract is implemented.

**Theorem 1**: Compared to having one supervisor, the principal finds it less costly to implement his optimum when having two supervisors.

*Proof.* In this game, to ensure collusion-proofness, we need to give each supervisor incentives to tell the truth when they see a low cost. Therefore, we need

$$w_i^s(r_L, r_L) - w_i^s(r_\emptyset, r_L) \geq \frac{1}{2}k(c_H - c_L) \text{ (CP-IC-S-1)}$$

and

$$w_i^s(r_L, r_\emptyset) - w_i^s(r_\emptyset, r_\emptyset) \geq k(c_H - c_L) \text{ (CP-IC-S-2)}$$

Observe that in equilibrium, these conditions, as well as each party's participation constraints as derived in the previous proposition, are binding. So $w_i^{s*}(r_\emptyset, r_L) = w_i^{s*}(r_\emptyset, r_\emptyset) = 0$, and ${w^a}^*_L = c_L$. Conditional on receiving two inconclusive reports, the probability that the agent faces a low cost is

$$\mathbb{P}(c = c_L | (r_\emptyset, r_\emptyset)) = \frac{(1-\mu)(1-p)^2}{(1-\mu)(1-p)^2 + \mu}$$

From here, we can see that giving the agent $c_H$ is better than $c_L$ when receiving two inconclusive reports:

$$\frac{(1-\mu)(1-p)^2}{(1-\mu)(1-p)^2 + \mu}(V - c_L) < (1-\mu)(V - c_L) < V - c_H.$$

So we conclude ${w^a}^*_\emptyset = c_H$. Therefore, with probability $(1-\mu)p^2$, the principal could get two reports of low cost from the supervisors, and with probability $(1-\mu)2p(1-p)$, he could get one such report. In each case, he would pay $k(c_H - c_L)$ in total as the monitoring fee, and also pay the agent $c_L$. With probability $\mu + (1-\mu)(1-p)^2$, he receives inconclusive reports from both supervisors and then pays each supervisor 0 and the agent $c_H$. Therefore,

$$\mathbb{E}(U^p\{P, S_1, S_2, A\}) = V - \{\mu c_H + (1-\mu)[(p + 2p(1-p))(c_L + k(c_H - c_L)) + (1-p)^2(c_H + 0 + 0)]\}.$$

Comparing with the case where there is only one supervisor, we have

$$\begin{aligned}\mathbb{E}(U^p\{P, S_1, S_2, A\}) - \mathbb{E}(U^p\{P, S, A\}) &= \Big\{V - \{\mu c_H + (1-\mu)[(p^2 + 2p(1-p))(c_L + k(c_H - c_L)) + (1-p)^2 c_H]\}\Big\} \\ &\quad - \Big\{V - \{\mu c_H + (1-\mu)p[c_L + k(c_H - c_L)] + (1-\mu)(1-p)c_H\}\Big\} \\ &= (1-\mu)p(1-p)(1-k)(c_H - c_L) > 0.\end{aligned}$$

Therefore, the principal would always find it optimal to get two supervisors when possible. ■

### 3.3 The Principal-Supervising Supervisor-Supervisor-Agent Model

Now we introduce a supervising supervisor, denoted $S^2$. The risk-neutral supervising supervisor has reservation utility 0. His utility is $U^{s^2} = w^{s^2}$, where $w^{s^2}$ is his wage. If the supervisor sends a report of low cost, then by the "hard" information assumption, the supervising supervisor does not take any actions and receives his reservation utility. If the supervisor sends an inconclusive report, then with probability $q$, the supervising supervisor could detect the truthfulness of this inconclusive report. So he would observe a signal $\sigma^{s^2} \in \{\emptyset, T, F\}$, where $T$ stands for a true report of inconclusiveness, $F$ stands for a false report of inconclusiveness (i.e., he obtains verifiable evidence of the collusion between the supervisor and the agent), and

$\emptyset$ means the supervising supervisor does not receive a signal about the truthfulness of the report. The supervising supervisor sends a report $r^{s^2} \in \{r^{s^2}_{\emptyset}, r^{s^2}_{T}, r^{s^2}_{F}\}$ to the principal. The principal gives the supervising supervisor a wage conditional on his report: $w^{s^2} \in \{w^{s^2}_{T}, w^{s^2}_{F}, w^{s^2}_{\emptyset}\}$. The wage of the agent depends on the supervisor's report and the supervising supervisor's report. The wage of the supervisor also depends on both reports. The timeline of the new game is as follows:

**Time 1**: The principal offers the supervising supervisor, the supervisor, and the agent a contract. The game ends immediately if either party does not accept the contract.

**Time 2**: The agent learns his cost $c$.

**Time 3**: The supervisor and the agent learn the signal $\sigma^s$.

**Time 4**: The agent chooses to produce the good or not. The supervisor and the agent discuss and sign side contracts and make side transfers.

**Time 5**: The supervisor sends the report $r^s$ to the principal and the supervising supervisor.

**Time 6**: The supervising supervisor and the supervisor learn the signal $\sigma^{s^2}$.

**Time 7**: The supervising supervisor and the supervisor discuss and sign side contracts and make side transfers.

**Time 8**: The supervising supervisor sends his report $r^{s^2}$ to the principal.

**Time 9**: The output is observed by the principal, and the contract is implemented.

**Theorem 2**: Compared to only having one supervisor, the optimality of having a supervising supervisor is independent of the monitoring ability of the supervising supervisor himself. In fact, having him is optimal for the principal if and only if

$$p > \frac{k}{1+k}.$$

*Proof.* When the realization of the cost is low and the supervisor observes this low cost, the agent is willing to bribe him with $k(c_H - c_L)$. However, in Time 5, the supervisor does not know the realization of the signal $\sigma^{s^2}$. His belief is that with probability $q$, the supervising supervisor will learn the signal $F$ when he has accepted the bribe. When detected, the supervisor would earn 0. Therefore, if he is in collusion with the agent and has gained $k(c_H - c_L)$, the supervising supervisor would receive a total bribe of at most $k^2(c_H - c_L)$ when he knows that the report is false. The expected payoff of the supervisor when he colludes with the agent upon seeing a low signal is

$$(1-q)k(c_H-c_L)+0\cdot q=(1-q)k(c_H-c_L).$$

Then the collusion-proof incentive compatibility condition for the supervisor is

$$w_L^s-w_\varnothing^s\geq(1-q)k(c_H-c_L)\text{ (CP-IC-S)}$$

Similarly, the collusion-proof incentive compatibility requirements for the supervising supervisor are

$$w_T^{s^2}-w_\varnothing^{s^2}\geq k^2(c_H-c_L)\text{ (CP-IC-}S^2\text{-1)}$$

and

$$w_F^{s^2}-w_\varnothing^{s^2}\geq k^2(c_H-c_L)\text{ (CP-IC-}S^2\text{-2).}$$

It is clear that these conditions are binding in equilibrium, and then the optimal contract would give the agent $c_L$ if the supervisor reports $r_L$, $c_H$ if the supervisor reports $r_\varnothing$ and the supervising supervisor reports $r_\varnothing^{s^2}$ or $r_T^{s^2}$. If the supervisor reports $r_\varnothing$ and the supervising supervisor reports $r_F^{s^2}$, then the agent would receive $c_L$, but this scenario would not take place in equilibrium. Therefore, in this setting, the expected utility of the principal is

$$\begin{aligned}\mathbb{E}(U^p\{P,S^2,S,A\})&=V-\{\mu(c_H+0+0)+(1-\mu)p[c_L+(1-q)k(c_H-c_L)+0]\\&+(1-\mu)(1-p)q[c_H+0+k^2(c_H-c_L)]+(1-\mu)(1-p)(1-q)(c_H+0+0)\}\\&=V-\{\mu c_H+(1-\mu)pc_L+(1-\mu)[p(1-q)k+(1-p)qk^2](c_H-c_L)+(1-\mu)(1-p)c_H\}.\end{aligned}$$

We want to compare the expression above with

$$\mathbb{E}(U^p\{P,S,A\})=V-\{\mu c_H+(1-\mu)p[c_L+k(c_H-c_L)]+(1-\mu)(1-p)c_H\}.$$

Then

$$\mathbb{E}(U^p\{P,S^2,S,A\})>\mathbb{E}(U^p\{P,S,A\})$$

if and only if

$$p(1-q) + (1-p)qk < p,$$

which simplifies to

$$p > \frac{k}{1+k}.$$

This means the principal's expected utility in the collusion-proof equilibrium with a supervisor and a supervising supervisor is only optimal if the monitoring ability of the supervisor is high enough or if the side transfer is costly enough. This result is striking because it says that the optimality of hiring a supervising supervisor is independent of his own monitoring ability. Notice that $0 < \frac{k}{1+k} < \frac{1}{2}$ for $0 < k < 1$, which means a sufficient condition for having a supervising supervisor being optimal for the principal is that $p \geq \frac{1}{2}$. The intuition is that in equilibrium, the supervising supervisor only possibly receives a nonzero wage when the supervisor observes nothing, which means if $p$ becomes higher, then the chance of the supervising supervisor receiving a wage would become lower. The constraint on the monitoring ability of the supervisor is also increasing in $k$. The intuition is that the agent's bribe $c_H - c_L$ would be transferred twice in order to arrive at the hand of the supervising supervisor, which means his equilibrium wage $w_T^{s^2}$ would decrease drastically when side transfers become more costly. ■

## 3.4 Comparing Equilibrium Utilities

We now compare the additional expected utility brought to the principal by a supervising supervisor versus another supervisor. As derived above,

$$\mathbb{E}(U^p\{P, S^2, S, A\}) - \mathbb{E}(U^p\{P, S, A\}) = (1-\mu)[pk - (p(1-q)k + (1-p)qk^2)](c_H - c_L),$$

and

$$\mathbb{E}(U^p\{P, S_1, S_2, A\}) - \mathbb{E}(U^p\{P, S, A\}) = (1-\mu)p(1-p)(1-k)(c_H - c_L).$$

Then we have

$$\mathbb{E}(U^p\{P, S^2, S, A\}) > \mathbb{E}(U^p\{P, S_1, S_2, A\})$$

if and only if

$$pk - (p(1-q)k + (1-p)qk^2) > p(1-p)(1-k),$$

which simplifies to

$$q > \frac{p(1-p)(1-k)}{pk - k^2 + pk^2}.$$

An interesting special case is when $k \to 1$. In this case, the condition simply becomes

$$p > \frac{1}{2}.$$

This means that if side transfer is costless, then adding a supervising supervisor would be more efficient than another supervisor if the monitoring ability of the supervisor is relatively high, and this optimality is independent of the monitoring ability of the supervising supervisor himself. The intuition is that when side transfer is costless, the supervisor would demand a very high wage without the supervising supervisor. In fact, the equilibrium expected utility of the principal would be the same with or without having any supervisors, because the supervisor's collusion-proof wage would be equal to all the surplus, $c_H - c_L$, when detecting the low cost. On the other hand, the supervising supervisor poses a threat to the supervisor that his bribe might be taken away if he detects the report being false. Adding one more supervisor in the parallel structure does not help, because the new supervisor would also demand a high wage. If $p$ is relatively high, then since the supervising supervisor only receives a nonzero wage when the supervisor receives a zero wage, the expected wage of the supervising supervisor would be low, which makes hiring him optimal.

Theorems 1 and 2 could be viewed as a complement to the canonical paper written by Tirole (1986). Tirole assumes that "it is not efficient to divide the supervisory job among several supervisors" and that "there exists ex-ante a competitive supply of supervisors." The theorems show that if there exists ex-ante a competitive supply of supervisors, then hiring more than one supervisor and asking each to do the complete supervisory job (rather than part of it) could bring the principal additional utility.

### 3.5 Limit Efficiencies with More Supervisors or Supervising Supervisors

We extend the models above to compare the limiting efficiencies of the parallel and vertical structures when the number of supervisors or supervising supervisors are higher than 2. The following theorem summarizes the optimal number of supervisors in each structure, and provides a a comparison of their maximum potential efficiencies (i.e., when both structures are having the optimal number of supervisors) as well as efficiencies when the number of supervisors approach infinity.

**Theorem 3**: I) **The Optimal Number of Supervisors in the Parallel Structure**: In the parallel structure, adding new supervisors into the system would always bring more efficiency into the system, but the additional utility of the principal brought by the $n^{th}$ supervisor is decreasing in $n$.

II) **The Optimal Number of Supervisors in the Vertical Structure**: In the vertical structure, it is never optimal to hire a supervising supervising supervisor or anyone above this level. Thus the optimal structure is a supervisor with a supervising supervisor if $p > \frac{k}{1+k}$ and otherwise just a supervisor.

III) **Comparing Maximum Potential Efficiencies**: Suppose adding a supervising supervisor is optimal (i.e., $p > \frac{k}{1+k}$). When the principal has the freedom of choosing any number of supervisors/supervising supervisors, the maximum potential efficiency of the vertical structure is higher than that of the parallel structure if and only if

$$k - p(1-q)k - (1-p)qk^2 - (1-p) > 0.$$

In particular, when $k \to 1$, this condition becomes

$$p > \frac{1}{2}.$$

IV) **Comparing Limit Efficiencies**: When the numbers of supervisors/supervising supervisors approach infinity in both structures, the vertical system is more efficient than the parallel system if and only if

$$k - p(1-q)k - 1 + p > \frac{(1-p)q(1-q)k}{1-k+qk}.$$

When $k \to 1$, this condition becomes

$$p > 1 - q.$$

*Proof.* In the parallel structure, suppose the principal simultaneously sends $n$ supervisors to monitor the agent, and each supervisor could receive the signal $\sigma_L$ with probability $p$ when $c = c_L$. Then to ensure collusion-proofness, the supervisor would offer a wage $\frac{k(c_H - c_L)}{t}$ when there are $t$ supervisors who report the low signal. With probability $\mu$, the agent will have high private cost, and then wage to each supervisor would be 0. With probability $(1-p)^n$, the agent has low private cost but none of the supervisor receives the low signal, and then the agent would receive $c_H$ and supervisors receive 0. With probability $\binom{n}{t}p^t(1-p)^{n-t}$, there are $t$ supervisors who receive the low signal, and then they divide $k(c_H - c_L)$ equally. Therefore, we have

$$\mathbb{E}(U^p\{P, S_1, ..., S_n, A\}) = V - \Big\{\mu c_H + (1-\mu)\{(\sum_{t=1}^{n}[\binom{n}{x}p^t(1-p)^{n-t})(c_L + \frac{tk(c_H - c_L)}{t})] + (1-p)^n(c_H + 0 + 0)\}\Big\}$$
$$= V - \{\mu c_H + (1-\mu)[(1-(1-p)^n)(c_L + k(c_H - c_L)) + (1-p)^n c_H]\}$$

From the expression above, we can see that

$$\mathbb{E}(U^p\{P, S_1, ..., S_n, A\}) - \mathbb{E}(U^p\{P, S_1, ..., S_{n-1}, A\})$$
$$= p(1-p)^{n-1}(1-k)(c_H - c_L).$$

This means that no matter how many supervisors the principal is having, adding more would always increase his expected utility, but the marginal utility brought by the new supervisor is decreasing in the number of existing supervisors. When we take the limit, we have

$$\lim_{n\to\infty} \mathbb{E}(U^p\{P, S_1, ..., S_n, A\}) = V - \{\mu c_H + (1-\mu)[c_L + k(c_H - c_L)]\}.$$

Intuitively, when there are infinitely many supervisors, the probability that the agent's low private cost being reported approaches 1, and then he would always receive a wage equal to $c_L$ while the supervisors reporting the low cost share $k(c_H - c_L)$.

Let us also consider the efficiency in the vertical structure. Suppose there are $n$ supervisors assigned vertically (a supervisor, a supervising supervisor, a supervising supervising supervisor, etc.). As usual, the supervisor has a chance $p$ of receiving $\sigma_L$ when $c = c_L$. We denote the supervisor as $S$, the supervising supervisor as $S^2$, the supervising supervising supervisor as $S^3$, and the rest similarly. When receiving an inconclusive report from $S^{t-1}$, $S^t$ has a $q$ chance of detecting the truthfulness of such inconclusive report. Following the same logic as the case with one supervisor and one supervising supervisor, for $t \leq n-1$, conditional on the event that $S$ receives a low signal and decides to collude, and $S^2, ..., S^{t-1}$ all realizing that the report from one level below is false but deciding to accept the bribe, if $S^t$ also receives a low signal, then he could get a maximum bribe equal to $k^t(c_H - c_L)$. He would be able to keep this bribe with probability $(1-q)$. Therefore, the optimal wages for $S^t$ reporting a true/false report (denoted $w_T^{s^t}$ and $w_T^{s^t}$ respectively) are $(1-q)k^t(c_H - c_L)$ and for reporting an inconclusive report is 0, because to ensure collusion-proofness, we need

$$w_T^{s^t} - w_\varnothing^{s^t} \geq (1-q)k^t(c_H - c_L) \text{ (CP-IC-}S^t\text{-1)}$$

and

$$w_F^{s^t} - w_\varnothing^{s^t} \geq (1-q)k^t(c_H - c_L) \text{ (CP-IC-}S^t\text{-2)}$$

for all $t \leq n-1$. Similarly, for $S^n$ (the supervisor one level below the principal), we need

$$w_T^{s^n} - w_\varnothing^{s^n} \geq k^n(c_H - c_L) \text{ (CP-IC-}S^n\text{-1)}$$

and

$$w_F^{s^n} - w_\varnothing^{s^n} \geq k^n(c_H - c_L) \text{ (CP-IC-}S^n\text{-2)}$$

We have

$$\begin{gathered}\mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) = V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L)) \\ + \sum_{t=1}^{n-1}(1-\mu)(1-p)(1-q)^{t-1}q[c_H + (1-q)k^t(c_H - c_L)] + (1-\mu)(1-p)(1-q)^n c_H\}\end{gathered}$$

From this result, we could find the additional efficiency (or inefficiency) brought by the $n^{th}$ supervising supervisor:

$$\begin{gathered}\mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) - \mathbb{E}(U^p\{P, S^{n-1}, ..., S^2, S, A\}) \\ = \Big\{V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L)) \\ + \sum_{t=1}^{n-1}(1-\mu)(1-p)(1-q)^{t-1}q[c_H + (1-q)k^t(c_H - c_L)] + (1-\mu)(1-p)(1-q)^n c_H\}\Big\} \\ -\Big\{V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L)) \\ + \sum_{t=1}^{n-2}(1-\mu)(1-p)(1-q)^{t-1}q[c_H + (1-q)k^t(c_H - c_L)] + (1-\mu)(1-p)(1-q)^{n-1} c_H\}\Big\}\end{gathered}$$

$$= (1-\mu)(1-p)(1-q)^{n-1}c_H - (1-\mu)(1-p)(1-q)^n c_H - (1-\mu)(1-p)(1-q)^{n-2}q[c_H + (1-q)k^{n-1}(c_H - c_L)],$$

and then

$$\mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) > \mathbb{E}(U^p\{P, S^{n-1}, ..., S^2, S, A\})$$

if and only if

$$q(1-q)k^{n-1}c_L > (q^2 + q(1-q)k^{n-1})c_H,$$

which is impossible as

$$q(1-q)k^{n-1}c_L < q(1-q)k^{n-1}c_H.$$

Therefore, it is never optimal to have a supervising supervising supervisor or anyone above this level in the vertical structure.

Now we compare the maximum potential efficiencies in both structures. Clearly the parallel structure is more optimal if $p \leq \frac{k}{1+k}$. If $p > \frac{k}{1+k}$, then the maximum potential efficiency of the vertical structure is attained when the principal hires a supervisor and a supervising supervisor, while in the parallel structure he would hire infinitely many supervisors. Then

$$\begin{aligned}
&\mathbb{E}(U^p\{P, S^2, S, A\}) - \lim_{n\to\infty} \mathbb{E}(U^p\{P, S_1, ..., S_n, A\}) \\
&= \Big\{V - \{\mu c_H + (1-\mu)pc_L + (1-\mu)[p(1-q)k + (1-p)qk^2](c_H - c_L) + (1-\mu)(1-p)c_H\}\Big\} \\
&\quad - \Big\{V - \{\mu c_H + (1-\mu)[c_L + k(c_H - c_L)]\}\Big\} \\
&= (1-\mu)[k - p(1-q)k - (1-p)qk^2 - (1-p)](c_H - c_L).
\end{aligned}$$

Then the maximum potential efficiency of the vertical structure is higher if and only if

$$k - p(1-q)k - (1-p)qk^2 - (1-p) > 0.$$

If $k \to 1$, then the condition becomes

$$p > \frac{1}{2}.$$

We now compute the limit efficiency of the vertical structure as $n \to \infty$. We have

$$\lim_{n\to\infty} \mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) = V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L))$$
$$+ \lim_{n\to\infty} \sum_{t=1}^{n-1}(1-\mu)(1-p)(1-q)^{t-1} q c_H + \lim_{n\to\infty} \sum_{t=1}^{n-1}(1-\mu)(1-p)q(1-q)^t k^t (c_H - c_L)\}$$
$$= V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L)) + (1-\mu)(1-p)c_H + \frac{(1-\mu)(1-p)q(1-q)k(c_H - c_L)}{1-(1-q)k}\}$$

Then

$$\lim_{n\to\infty} \mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) - \mathbb{E}(U^p\{P, S_1, ..., S_n, A\})$$
$$= \Big\{V - \{\mu c_H + (1-\mu)p(c_L + (1-q)k(c_H - c_L)) + (1-\mu)(1-p)c_H + \frac{(1-\mu)(1-p)q(1-q)k(c_H - c_L)}{1-(1-q)k}\}\Big\}$$
$$-\Big\{V - \{\mu c_H + (1-\mu)[c_L + k(c_H - c_L)]\}\Big\}$$
$$= (1-\mu)\{[1 - k - p + p(1-q)k + \frac{(1-p)q(1-q)k}{1-k+qk}]c_L + [k - p(1-q)k - 1 + p - \frac{(1-p)q(1-q)k}{1-k+qk}]c_H\}$$
$$= (1-\mu)[k - p(1-q)k - 1 + p - \frac{(1-p)q(1-q)k}{1-k+qk}](c_H - c_L).$$

Therefore,

$$\lim_{n\to\infty} \mathbb{E}(U^p\{P, S^n, ..., S^2, S, A\}) > \lim_{n\to\infty} \mathbb{E}(U^p\{P, S_1, ..., S_n, A\})$$

if and only if

$$k - p(1-q)k - 1 + p > \frac{(1-p)q(1-q)k}{1-k+qk}.$$

If $k \to 1$, then the condition above simplifies to

$$p > 1 - q.$$

■

### 3.6 Discussions and Extensions

In theorem 3, we showed that the optimal number of supervising levels in the vertical hierarchy could not be higher than two. This result seems to explain real world phenomena well. Admittedly, there are firms whose hierarchy includes more than two supervisory levels, for example, there are associates, vice presidents, and senior vice presidents above analysts in investment banks. But the jobs of people on the higher levels of the hierarchy are often not merely in monitoring the work of analysts. A major component of their job is to manage client relationships, and a good relationship with the clients often brings the "principal" a considerable amount of additional utility.

On the other hand, the optimal number of supervisors in the parallel structure is infinity, but this is not consistent with what we observe in the real world. A natural explanation is that when there are more supervisors, the principal needs to provide extra space and resources for them to work, which would be particularly costly if their only role is supervision. We could extend the model to incorporate the cost of hiring $n$ supervisors, modeled as $C = m(n-1)^2$, where $m$ is a constant that does not depend on other parameters of the model. Then denoting the principal's expected utility in the collusion-proof equilibrium with $n$ supervisors as $\hat{U}^p(n)$, he faces the following problem when deciding on how much supervisors to hire under the parallel structure:

$$\max_{n\in\mathbb{N}} \mathbb{E}[\hat{U}^p(n)] = V - \{\mu c_H + (1-\mu)[(1-(1-p)^n)(c_L + k(c_H - c_L)) + (1-p)^n c_H]\} - m(n-1)^2.$$

When we extend the domain of $\hat{U}^p(n)$ to $n \in \mathbb{R}^+$ and take the partial derivative, we have

$$\begin{aligned}\frac{\partial \mathbb{E}[\hat{U}^p]}{\partial n} &= \frac{\partial}{\partial n}\Big\{V - \{\mu c_H + (1-\mu)[(1-(1-p)^n)(c_L + k(c_H - c_L)) + (1-p)^n c_H]\} - m(n-1)^2\Big\} \\ &= (1-\mu)(1-p)^n(1-k)(c_L - c_H)\ln(1-p) - 2m(n-1).\end{aligned}$$

Notice that

$$\frac{\partial \mathbb{E}[\hat{U}^p]}{\partial n}\Big|_{n=1} > 0,$$

while

$$\frac{\partial \mathbb{E}[\hat{U}^p]}{\partial n}\Big|_{n=\frac{(c_L-c_H)\ln(1-p)}{2m}+1} < 0.$$

Since the function $\hat{U}^p(n)$ with the extended domain is continuous, by Bolzano's intermediate value theorem, we know that there exists $1 < n^* < \infty$ such that $\frac{\partial \mathbb{E}[\hat{U}^p]}{\partial n}\Big|_{n=n^*} = 0$. The uniqueness of $n^*$ can be seen from the fact that

$$\frac{\partial^2 \mathbb{E}[\hat{U}^p]}{\partial n^2} = -(1-\mu)(1-p)^n(1-k)(c_H - c_L)(\ln(1-p))^2 - 2m < 0$$

for all $n > 1$. Because the the principal must chooses an integer $n$, we know that there are at most two candidates of the optimal number of supervisors, $\lfloor n^* \rfloor$ and $\lceil n^* \rceil$, and then the principal could compute and compare $\mathbb{E}[\hat{U}^p(\lfloor n^* \rfloor)]$ and $\mathbb{E}[\hat{U}^p(\lceil n^* \rceil)]$. This extended model could serve as a basis for empirical works that estimate $m$ and help firms hire optimally.

# 4 Conclusion

In this paper, we discussed two supervisory structures in firms, the vertical structure and the parallel structure. We showed that compared to having one supervisor, the principal would always find it optimal to have two supervisors in the parallel structure. In the vertical structure, the optimality of adding a supervising supervisor does not depend on the monitoring ability of the supervising supervisor, but rather depends on the monitoring ability of the supervisor and the cost of side transfer.

We also analyzed the optimal number of supervisors in both structures. We found a striking contrast: in the parallel structure, the optimal number is infinity; in the vertical structure, it is never efficient to have another level above the supervising supervisor. We gave a condition under which the vertical structure has a higher maximum potential efficiency (i.e., when the principal employs the optimal numbers of supervisors in each structure), and another condition under which the vertical structure has a higher limit efficiency (i.e., when the number of supervisors approach infinity in both cases).

There are many dimensions that future work could analyze. The first is to re-design collusion-proof mechanisms when information is “soft” rather than “hard.” That is, the supervisor’s report is not verifiable and he could report low cost when observing nothing. In this case, the mechanism in the classic-principal-supervisor-agent is no longer efficient, because the supervisor always has incentives to report low cost when he observes nothing. Baliga (1999) gives a new mechanism that is collusion-proof under “soft” information. It would be interesting to consider such mechanisms with multiple supervisors in both structures.

Another dimension is to consider the sequential sending of supervisors in contrast to the simultaneous case analyzed in this paper. Some interesting problems in this dimension include discussing the pooling and separating equilibria where the agent offers the same or different bribes to the first and second supervisors respectively.

Lastly, it is also worth considering the case where the principal could adopt a mixture of the two structures. For example, he might want two supervising supervisors with three supervisors. Analyzing the division of supervisory effort and equilibrium wages would be an interesting problem with many real world applications.

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