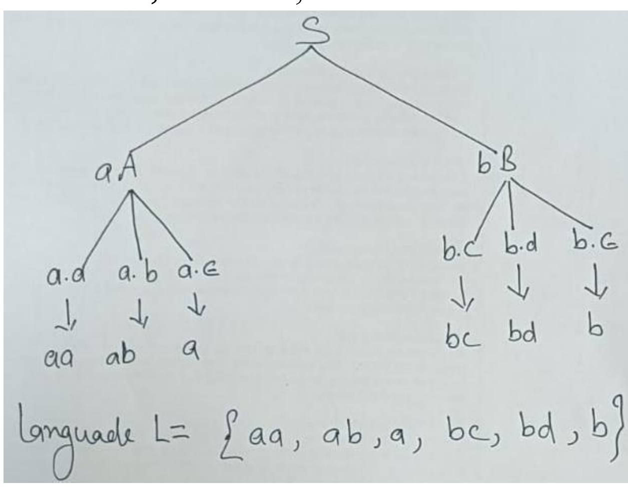
School of Computer Science Engineering and Technology

Course- BTech	Type- Core
Course Code- CSET302	Course Name- Automata Theory and Computability
Year- Third	Semester- Odd Batch- BTech 5th Semester

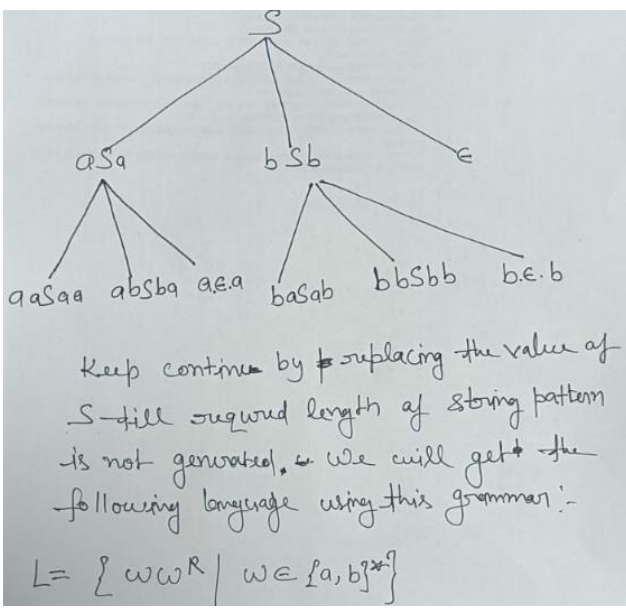
Tutorial 9

Q1. Form a language from the given grammar:

(a) S→aA/bB, A→a/b/ε, B→c/d/ε



(b) S→aSa/bSb/ε



(c) S→AB/CD, A→aAb/ε, B→CB/ε, C→bC/ε, D→bDc/ε

(C) CFG

$S \rightarrow AB \mid CD$

$A \rightarrow aAb \mid \epsilon \quad \xRightarrow{L(A)} \quad a^n b^n \mid n \geq 0$ ⇒ Equal No of a's & b's

$B \rightarrow CB \mid \epsilon \quad \xRightarrow{L(B)} \quad b^{n_1} \mid n_1 \geq 0$

$C \rightarrow bC \mid \epsilon \quad \xRightarrow{L(C)} \quad b^m \mid m \geq 0$

$D \rightarrow bDc \mid \epsilon \quad \xRightarrow{L(D)} \quad b^n c^n \mid n \geq 0$

⇓ Equal No of b's and c's.

For $S \rightarrow AB$

$L(AB) = \{a^n b^n\} \cdot b^{n_1} \Rightarrow a^n b^{n+n_1} \mid n, n_1 \geq 0$

or No of b's ≥ No of a's

$\therefore L_1 = a^m b^p \mid p \geq m$

For $S \rightarrow CD$

$L(CD) = \{b^m\}\{b^n c^n\} = b^{m+n} c^n \mid m, n \geq 0$

or No of b's ≥ No of c's.

$\therefore L_2 = b^p c^n \mid p \geq n$

Required Language $L = L_1 \cup L_2$

$$L = \{a^m b^p \mid p \geq m\} \cup \{b^p c^n \mid p \geq n\}$$

The Grammar generates strings where either
No of b's ≥ No of a's
or
No of b's ≥ No of c's.

Q2. Remove **Unit Productions** from the following **Context-Free Grammar (CFG)** and write the simplified grammar in equivalent form:

$S \to AB$, $B \to C \mid \cancel{a}b$, $D \to E$
$S \to a$, $C \to D$, $E \to a$

Solution:

Sol^n

unit Production = $B \to C$
$C \to D$
$D \to E$

$S \to AB$
$A \to a$
$B \to C \mid b$ a
$C \to \cancel{D}$ a
$D \to \cancel{E}$ a
$E \to a$

$\Rightarrow$

$S \to AB$
$A \to a$
$B \to a \mid b$
$\boxed{\begin{array}{l} C \to a \\ D \to a \\ E \to a \end{array}}$ $\to$ un reachable productions

$\therefore$ Simplified Grammar is

$S \to AB$
$A \to a$
$B \to a \mid b$

Q3. Remove **Unit Productions** from the following **Context-Free Grammar (CFG)** and write the simplified grammar in equivalent form:

$S \rightarrow aA \mid B$
$A \rightarrow ba \mid bb$
$B \rightarrow A \mid bba$

Solution:

Sol^n unit production — $S \rightarrow B$
$B \rightarrow A$

$S \rightarrow aA \mid \cancel{B} \mid bba \mid ba \mid bb$
$A \rightarrow ba \mid bb$
$B \rightarrow bba \mid \cancel{A} \mid ba \mid bb$

Q4. Construct the Context Free Grammar for the following language:

(a) $L = \{a^m b^n \mid m >= n\}$

ⓐ $S \rightarrow aSB / aAB / \epsilon$

$A \rightarrow aA / \epsilon$

$B \rightarrow b / \epsilon$

OR

(a) $S \rightarrow aS \mid aSb \mid \epsilon$

(b) $L = \{a^m b^n \mid n = 2m\}$

ⓑ $S \rightarrow aSbb / \epsilon$

(c) $L = \{a^m b^n \mid m \neq n\}$

(c) $S \rightarrow AS_1 \mid S_1 B$

$S_1 \rightarrow aS_1 b \mid \lambda$

$A \rightarrow aA \mid a$

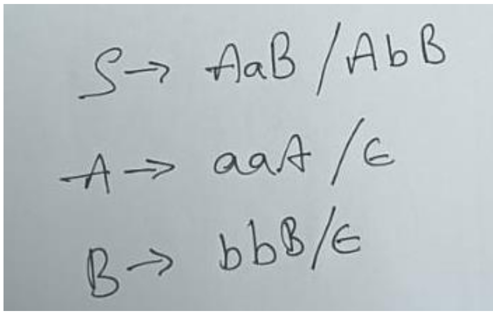
$B \rightarrow bB \mid b$

(d) $L = \{a^m b^n \mid m + n = \text{even}\}$

$S \rightarrow AB / AaBb$

$A \rightarrow aaA / \epsilon$, $B \rightarrow bbB / \epsilon$

(e) $L = \{a^m b^n \mid m + n = \text{odd}\}$



Q5. Find string *abbbb using leftmost and rightmost derivation for the given CFG:*

$$
\begin{aligned}
S &\rightarrow aAB, \\
A &\rightarrow bBb, \\
B &\rightarrow A|\lambda.
\end{aligned}
$$

Solution:

Then

$$S \Rightarrow aAB \Rightarrow abBbB \Rightarrow abAbB \Rightarrow abbBbbB \Rightarrow abbbbB \Rightarrow abbbb$$

is a leftmost derivation of the string *abbbb*. A rightmost derivation of the same string is

$$S \Rightarrow aAB \Rightarrow aA \Rightarrow abBb \Rightarrow abAb \Rightarrow abbBbb \Rightarrow abbbb.$$

Q6. Construct the **parse trees** and find the **yields (derived strings)** for **abBbB** and **abbbb** using the following production:

$$
\begin{aligned}
S &\rightarrow aAB, \\
A &\rightarrow bBb, \\
B &\rightarrow A|\lambda.
\end{aligned}
$$

Solution:

$$
\begin{aligned}
S &\rightarrow aAB, \\
A &\rightarrow bBb, \\
B &\rightarrow A|\lambda.
\end{aligned}
$$

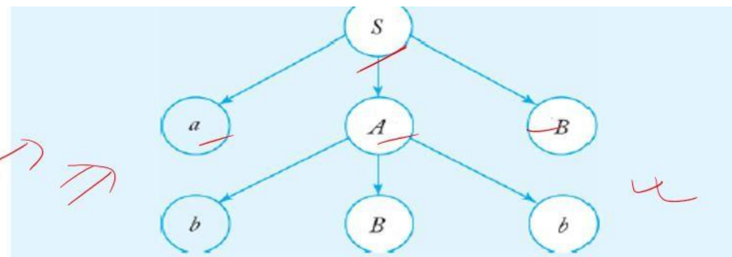
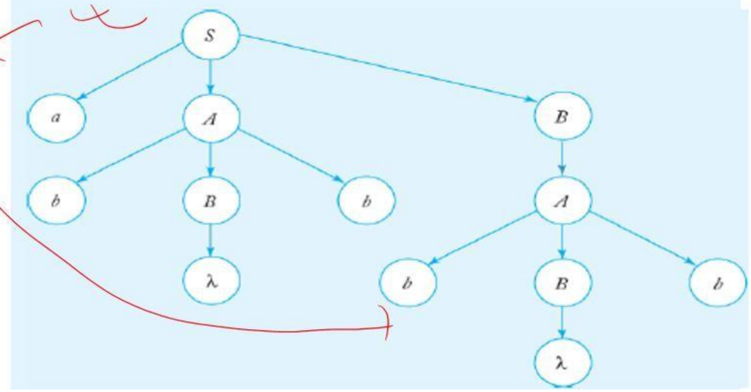


FIGURE 5.2



a b λ b b λ b

abbbb

Q7 Check whether the given grammar is ambiguous or not:

S → AB / C

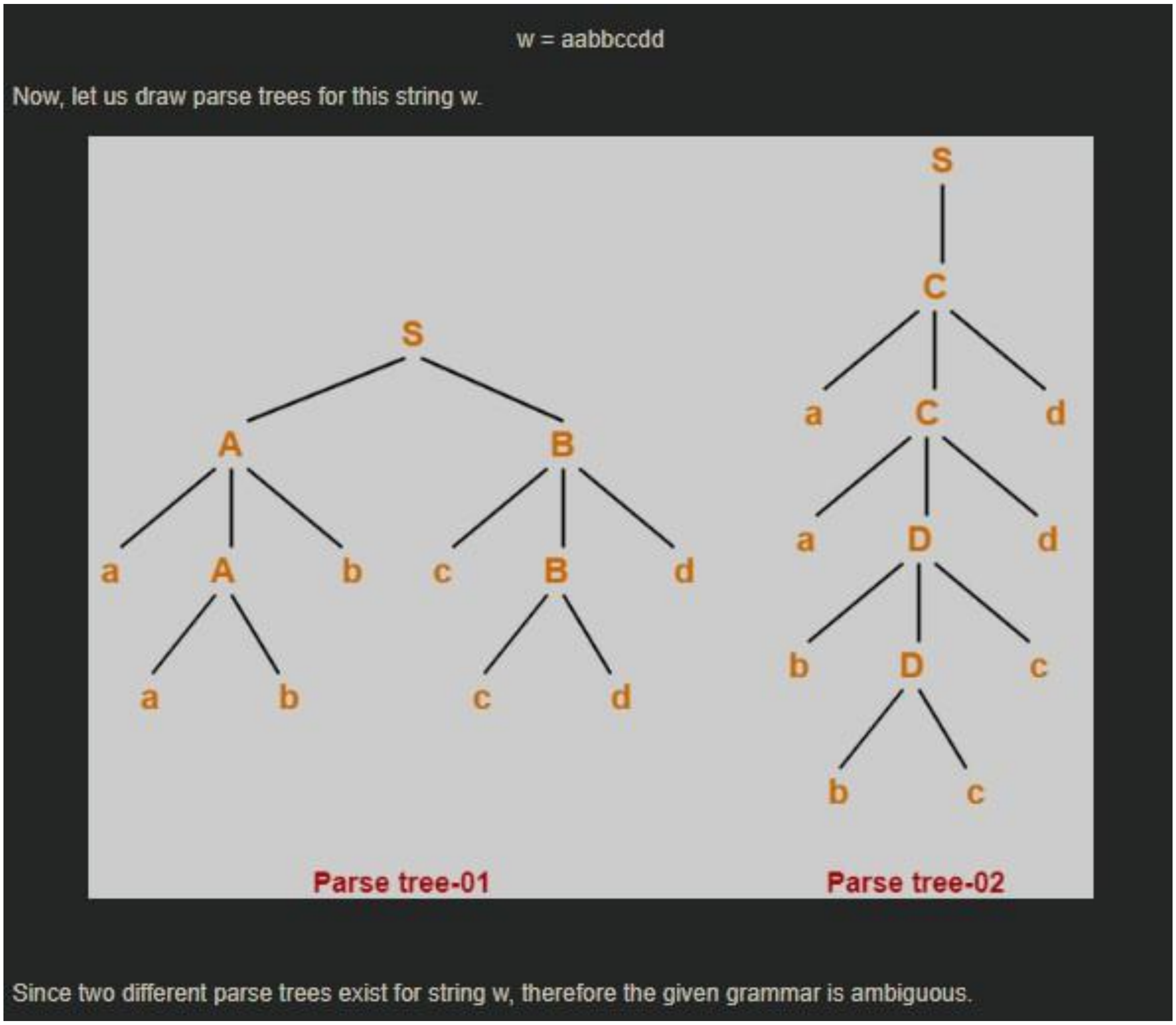
A → aAb / ab

B → cBd / cd

C → aCd / aDd

D → bDc / bc

Solution:


w = aabbccdd

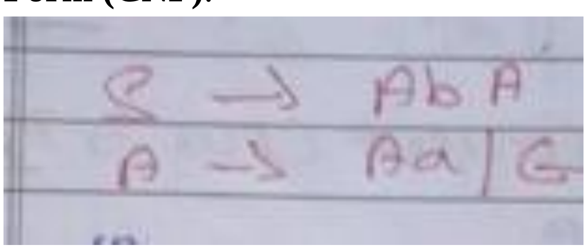
Now, let us draw parse trees for this string w.

Parse tree-01

Parse tree-02

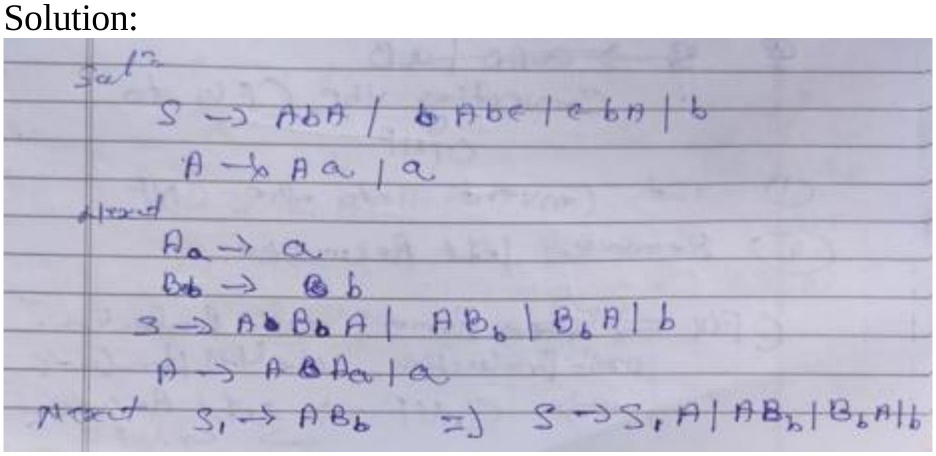
Since two different parse trees exist for string w, therefore the given grammar is ambiguous.

Q 8. Convert the following Context-Free Grammar (CFG) into **Chomsky Normal Form (CNF)**.


$S \rightarrow AbA$

$A \rightarrow Aa \mid a$

Solution:


Sol^n

$S \rightarrow AbA \mid Ab \mid bA \mid b$

$A \rightarrow Aa \mid a$

Next

$A_a \rightarrow a$

$B_b \rightarrow b$

$S \rightarrow A B_b A \mid A B_b \mid B_b A \mid b$

$A \rightarrow A A_a \mid a$

Next $S_1 \rightarrow A B_b \Rightarrow S \rightarrow S_1 A \mid A B_b \mid B_b A \mid b$

Q 9. Convert the following Context-Free Grammar (CFG) into **Chomsky Normal Form (CNF)**.

$S \to ABa$

$A \to aab$

$B \to Ac$

Convert into CNF

Solution:

$S \to ABA_a$

$A \to A_0 B A_0 B_0$

$B \to A C_0$

$A_0 \to a$

$B_0 \to b$

$C_0 \to c$

$S \to A A_1$

$A \to A_0 A_2$

$B \to A C_0$

$A_1 \to B A_0$

$A_2 \to A_0 B_0$

$A_0 \to a$

$B_0 \to b$

$C_0 \to c$

Q 10. Convert the following Context-Free Grammar (CFG) into **GNF**.

$$S \to abSb \mid aa$$

Solution:

the equivalent grammar $S \to aBSB \mid aA$,

$$A \to a,$$

$$B \to b,$$