School of Computer Science Engineering and Technology

Course- BTech	Type- Core
Course Code- CSET302	Course Name- Automata Theory and Computability
Year- Third	Semester- Odd Batch- BTech 5th Semester

1. Show that $L = \{ww^R : w \in \Sigma^*\}$ is not regular.

Whatever m the opponent picks on Step 1, we can always choose a w as shown in Figure 4.5. Because of this choice, and the requirement that $|xy| \leq m$, the opponent is restricted in Step 3 to choosing a y that consists entirely of a's. In Step 4, we use $i = 0$. The string obtained in this fashion has fewer a's on the left than on the right and so cannot be of the form ww^R. Therefore, L is not regular.

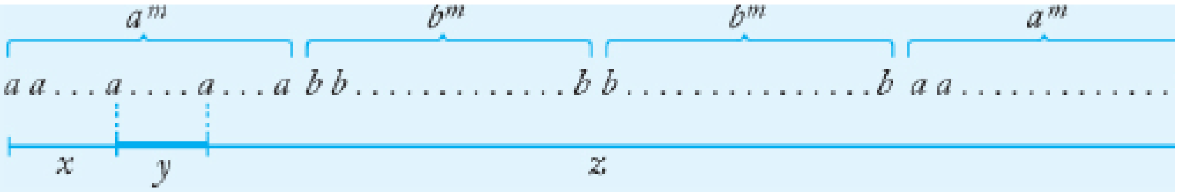


FIGURE 4.5

Note that if we had chosen a w too short, then the opponent could have chosen a y with an even number of b's. In that case, we could not have reached a violation of the pumping lemma on the last step. We would also fail if we were to choose a string consisting of all a's, say,

$$w = a^{2m},$$

which is in L. To defeat us, the opponent need only pick

$$y = aa.$$

Now w_i is in L for all i, and we lose.

To apply the pumping lemma we cannot assume that the opponent will make a wrong move. If, in the case where we pick $w = a^{2m}$, the opponent were to pick

$$y = a,$$

then w_0 is a string of odd length and therefore not in L. But any argument that assumes that the opponent is so accommodating is automatically incorrect.

2. Let $\Sigma = \{a, b\}$. Show the language $L = \{w \in \Sigma^* : n_a(w) < n_b(w)\}$ is not regular.

Suppose we are given m. Since we have complete freedom in choosing w, we pick $w = a^m b^{m+1}$. Now, because $|xy|$ cannot be greater than m, the opponent cannot do anything but pick a y with all a's, that is

$$y = a^k, \ 1 \le k \le m.$$

We now pump up, using $i = 2$. The resulting string

$$w_2 = a^{m+k} b^{m+1}$$

is not in L. Therefore, the pumping lemma is violated, and L is not regular.

3. Show that the language $L = \{a^n b^l : n \neq l\}$ is not regular.

Here we need a bit of ingenuity to apply the pumping lemma directly. Choosing a string with $n = l + 1$ or $n = l + 2$ will not do, since our opponent can always choose a decomposition that will make it impossible to pump the string out of the language (that is, pump it so that it has an equal number of a's and b's). We must be more inventive. Let us take $n = m!$ and $l = (m + 1)!$. If the opponent now chooses a y (by necessity consisting of all a's) of length $k < m$, we pump i times to generate a string with $m!+(i - 1)\,k$ a's. We can get a contradiction of the pumping lemma if we can pick i such that

$$m!+(i-1)\,k = (m+1)!$$

This is always possible since

$$i = l + \frac{m\,m!}{k}$$

and $k \le m$. The right side is therefore an integer, and we have succeeded in violating the conditions of the pumping lemma.

However, there is a much more elegant way of solving this problem. Suppose L were regular. Then, by Theorem 4.1, L and the language

$$L_1 = \bar{L} \cap L\left(a^* b^*\right)$$

would also be regular. But $L_1 = \{a^n b^n : n \ge 0\}$, which we have already classified as nonregular. Consequently, L cannot be regular.

4. Pumping lemma for regular languages is generally used to prove:

A) Two regular expressions are equivalent

B) A grammar is ambiguous

C) A grammar is regular

D) A grammar is **not** regular

Answer: D) A grammar is not regular

Explanation: The lemma is a tool to prove **non-regularity** by contradiction (assuming regular and showing violation).

Which of the following statements is FALSE about the pumping lemma for regular languages?

A) Every regular language satisfies the pumping lemma

B) If a language satisfies the pumping lemma, then it is regular

C) If a language does not satisfy the pumping lemma, it is not regular

D) The lemma gives a necessary condition, not sufficient

Answer: B) If a language satisfies the pumping lemma, then it is regular

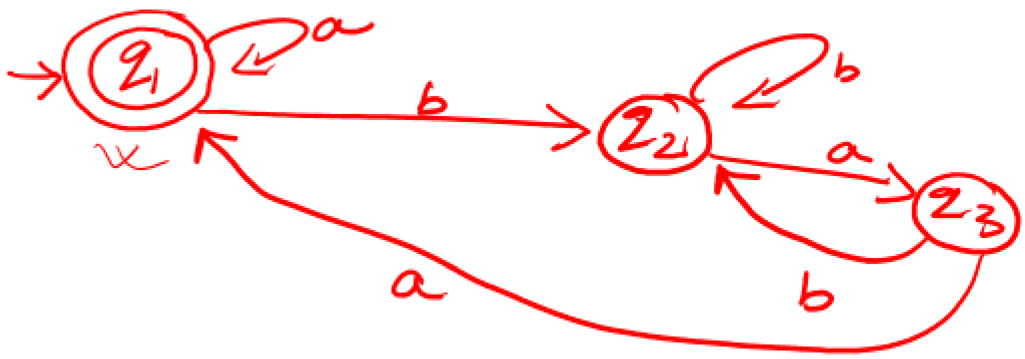
Explanation: Pumping lemma is **necessary but not sufficient**. Some non-regular languages also satisfy it.

5. Which of the following is **not** a condition of the pumping lemma for regular languages?

   A) There exists $p \geq 1$ such that for every $w \in L$ with $|w| \geq p$, w can be written as xyz
   B) $|y| \geq 1$
   C) $|xy| \leq p$
   D) For some $i \geq 0$, $xy^i z \in L$

   Answer: D) For some $i \geq 0$, $xy^i z \in L$
   Explanation: The condition is **for all $i \geq 0$**, not just for some i.

6. Find out the Regular expression for the given finite automata.



$$q_1 = \epsilon + q_1 \cdot a + q_3 \cdot a \quad \text{(i)}$$

$$q_2 = q_1 \cdot b + q_2 \cdot b + q_3 \cdot b \quad \text{(ii)}$$

$$q_3 = q_2 a \quad \text{(iii)}$$

$$q_2 \cdot a \cdot b$$

$$R = Q + RP$$

$$R = QP^*$$

$$q_2 = q_2 b + q_2 ab + q_1 b$$

$$\underset{R}{q_2} = \underset{R}{q_2}\underset{P}{(b+ab)} + \underset{Q}{q_1 b}$$

$$q_2 = q_1 b (b+ab)^*$$

$$q_1 = \epsilon + q_1 a + q_3 a$$

$$q_1 = \epsilon + q_1 a + q_2 a a$$

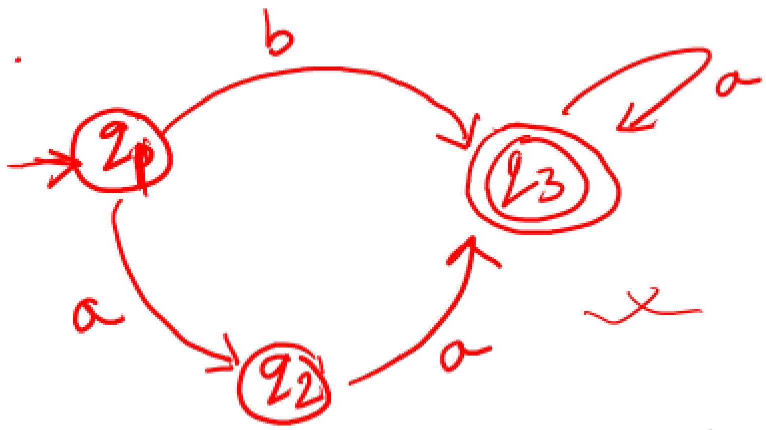
$$q_1 = \epsilon + q_1 a + q_1 b (b+ab)^* aa$$

$$\underset{R}{q_1} = \underset{Q}{\epsilon} + \underset{R}{q_1} \underset{P}{(a + b(b+ab)^* aa)}$$

$$R \quad Q \quad P^*$$

$$q_1 = \epsilon \cdot (a + b(b+ab)^* aa)^*$$

7. Find out the Regular expression for the given finite automata.



$$q_1 = \epsilon \qquad \text{(i)}$$

$$q_2 = q_1 a \qquad \text{(ii)}$$

$$q_3 = q_1 \cdot b + q_2 \cdot a + q_3 \cdot a \qquad \text{(iii)}$$

$$\epsilon \cdot b + q_1 \cdot a \cdot a$$

$$q_3 = b$$

*

$$q_3 = q_1 \cdot aa + q_3 a + \epsilon b$$

$$q_3 = \epsilon aa + q_3 a + b$$

$$\underset{R}{q_3} = \epsilon(\underbrace{aa+b}_{Q}) + \underset{R}{q_3} \cdot \underset{P}{a}$$

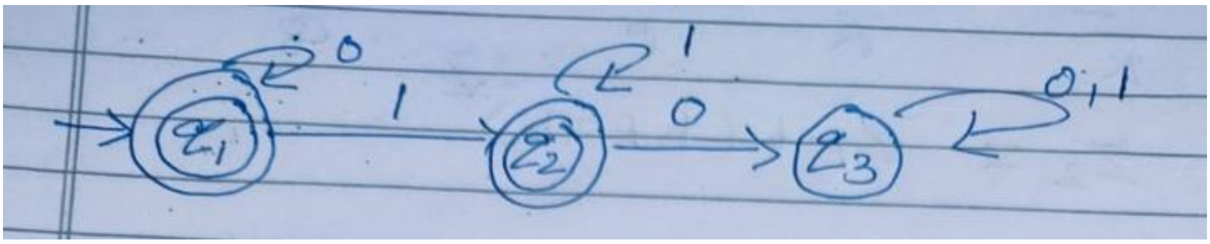
$$q_3 = (aa+b)a^*$$

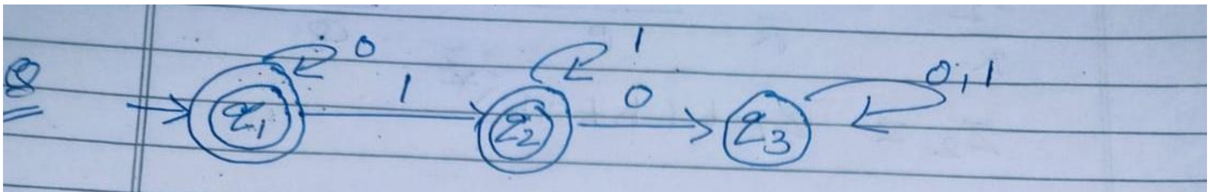
$$q_3 = (aa+b)a^*$$

$$R = Q + RP$$

$$R = QP^*$$

8. Find out the Regular expression for the given finite automata.





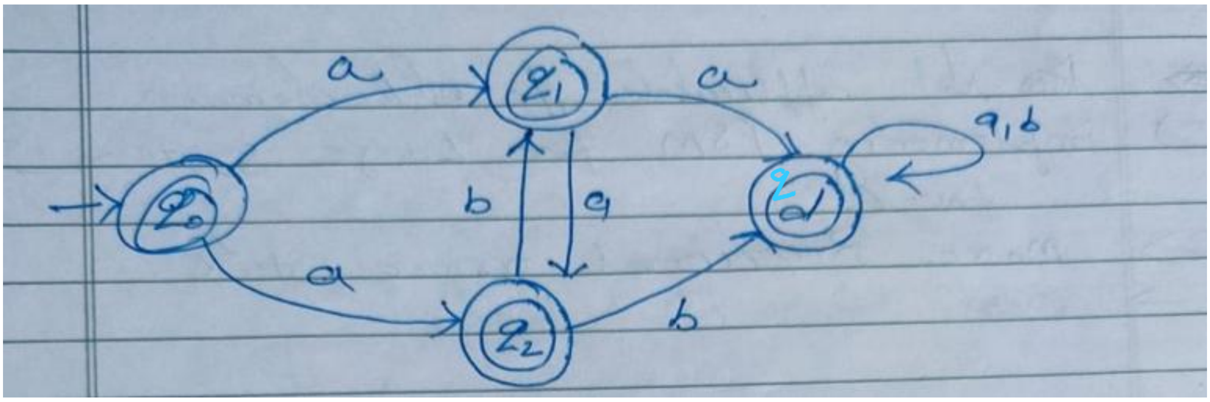
Sol^n: $q_1 = \epsilon + q_1 \cdot 0 \Rightarrow r_1 = 0^*$

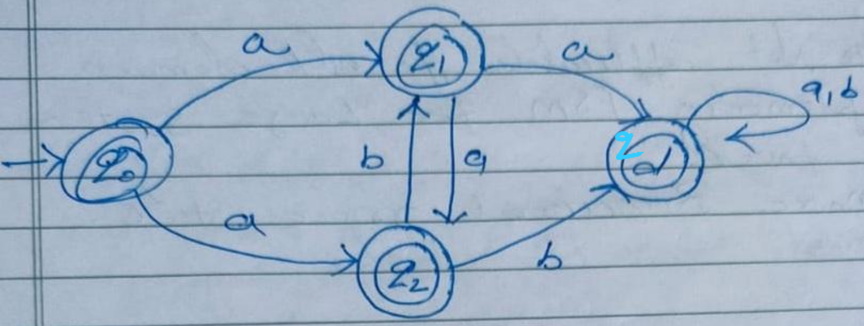
$q_2 = 0^*1 + q_2 \cdot 1 \Rightarrow r_2 = 0^*1 \cdot 1^*$

$\therefore$ Final Sol^n = $r = r_1 + r_2 = 0^* + 0^*11^*$

$= 0^* + 0^*1^+$

9. Find out the Regular expression for the given finite automata.





$q_0 = \epsilon$ —(I)

$q_1 = q_0 \cdot a + q_2 \cdot b$

$q_2 = q_0 \cdot a + q_1 \cdot a$

$q_d = q_d \cdot a + q_d \cdot b + q_2 \cdot b + q_1 \cdot a$

$q_1 = a + (a + q_1 \cdot a) \cdot b$

$q_1 = a + ab + q_1 ab = (a+ab) + q_1 ab$ (II)

$q_1 = (a+ab)(ab)^*$

$q_2 = a + (a + q_2 b) \cdot a = a + aa + q_2 ba$ (III)

$q_2 = (a+aa)(ba)^*$

$q_d = (a+aa)(ba)^* \cdot b + (a+ab)(ab)^* \cdot a + q_d (a+b)$

$\therefore q_d = [(a+aa)(ba)^* \cdot b + (a+ab) \cdot (ab)^* a] (a+b)^*$

$\therefore$ final R.E

$\epsilon + (a+ab)(ab)^* + (a+aa)(ba)^* + [(a+aa)(ba)^* \cdot b + (a+ab) \cdot (ab)^* a] \cdot (a+b)^*$

10. Prove that

$(0110+01)(10)^* = 01(10)^*$

11. Prove that
$(0^*1^*)^* = (0+1)^*$

12. Given two regular expressions R1 and R2, the problem of checking if L(R1) = L(R2) is:

A) Decidable

B) Undecidable

C) Semi-decidable but not decidable

D) None of the above

Answer: A) Decidable

Explanation: Convert to DFAs, minimize, and compare → decidable.

13. Which of the following is decidable for regular languages?

A) Membership problem

B) Emptiness problem

C) Finiteness problem

D) All of the above

Answer: D) All of the above

14. Which of the following languages over the alphabet {0,1} does the regular expression
`(0 + 1)* 0 (0 + 1)`
represent?

A) All strings containing exactly one `0`
B) All strings ending with `0`
C) All strings having at least two symbols and the second last symbol is `0`
D) All strings having even number of `0`s

Answer: C) All strings having at least two symbols and the second last symbol is `0`.
Explanation: `(0+1)*` allows any prefix, then `0` is fixed, followed by one symbol `(0+1)`. Hence second last must be `0`.

15. Which of the following operations do not preserve regularity of languages?

A) Homomorphism

B) Reversal

C) Infinite Union

D) Concatenation

Answer: C) Infinite Union

16. Let L_1 and L_2 be regular languages. Which of the following languages is **guaranteed to be regular**?

A) $L_1 \cap L_2^c$

B) $L_1 \cap L_2^{ccc}$

C) $(L_1 \cup L_2)^c$

D) All of the above

Answer: D) All of the above