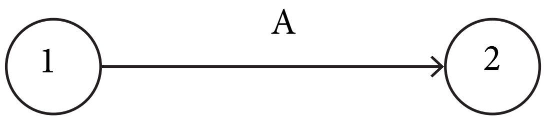
Problem 2

Develop a network diagram for the project specified below:

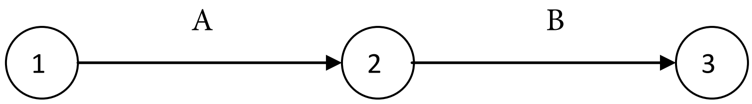
Activity	Immediate Predecessor Activity
A	-
B	A
C, D	B
E	C
F	D
G	E, F

Solution

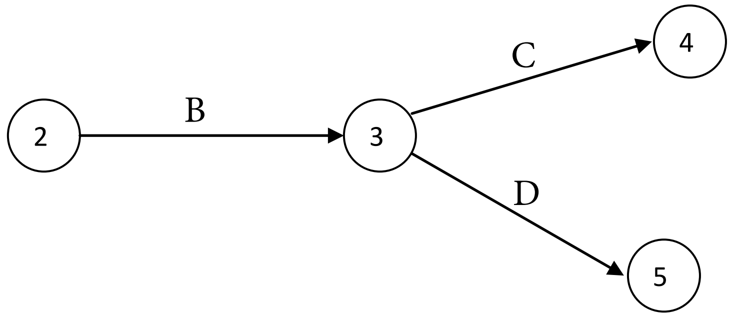
Activity A has no predecessor activity. i.e., It is the first activity. Let us suppose that activity A takes the project from event 1 to event 2. Then we have the following representation for A:



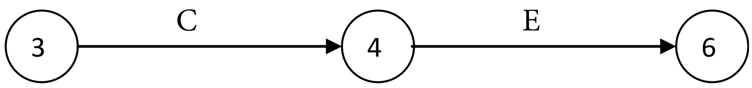
For activity B, the predecessor activity is A. Let us suppose that B joins nodes 2 and 3. Thus we get



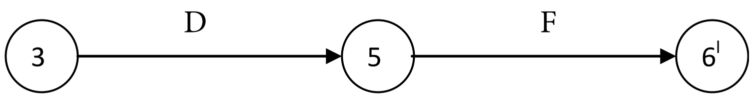
Activities C and D have B as the predecessor activity. Therefore we obtain the following:



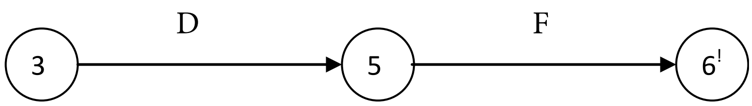
Activity E has D as the predecessor activity. So we get



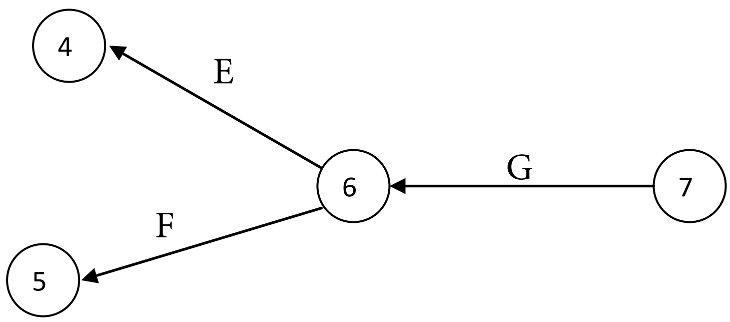
Activity F has D as the predecessor activity. So we get



Activity G has E and F as predecessor activities. This is possible only if nodes 6 and 6l are one and the same. So, rename node 6l as node 6. Then we get

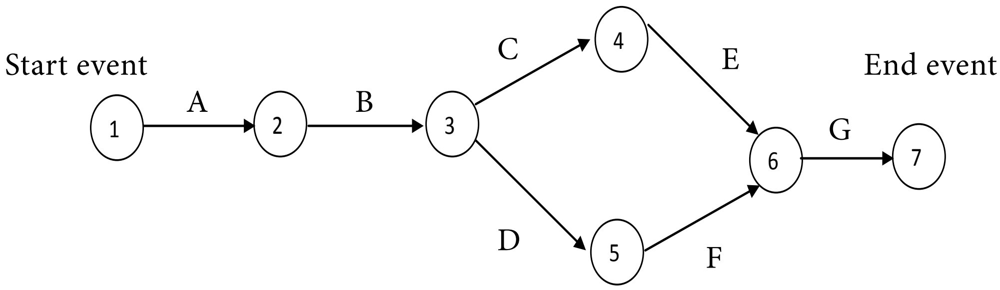


and



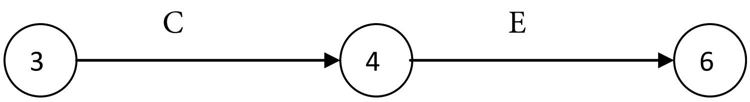
G is the last activity.

Putting all the pieces together, we obtain the following diagram the project network:

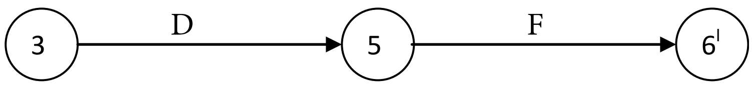


The diagram is validated by referring to the given data.

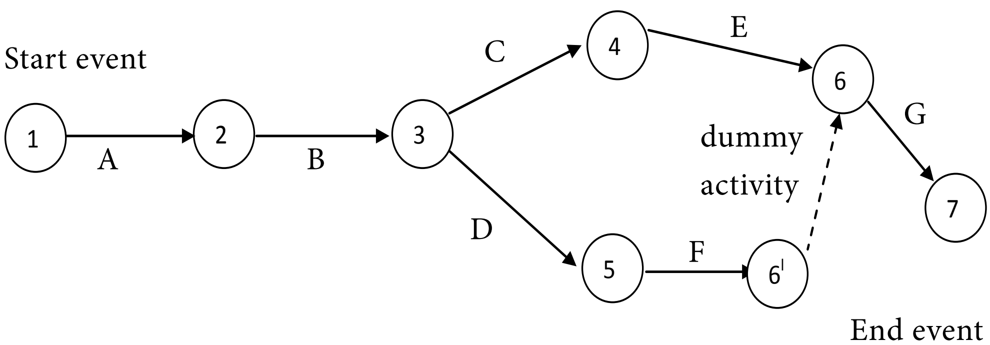
Note: An important point may be observed for the above diagram. Consider the following parts in the diagram



and



We took nodes 6 and 6l as one and the same. Instead, we can retain them as different nodes. Then, in order to provide connectivity to the network, we join nodes 6l and 6 by a dummy activity. Then we arrive at the following diagram for the project network:



Questions

1. Explain the terms: event, predecessor event, successor event, activity, dummy activity, network.
2. Construct the network diagram for the following project:

Activity	Immediate Predecessor Activity
A	-
B	-
C	A
D	B
E	A
F	C, D
G	E
H	E
I	F, G
J	H, I

****

Lesson 4 - Critical Path Method (CPM)

Lesson Outline

- The concepts of critical path and critical activities
- Location of the critical path
- Evaluation of the project completion time

Learning Objectives

After reading this lesson you should be able to

- understand the definitions of critical path and critical activities
- identify critical path and critical activities
- determine the project completion time

Introduction

The critical path method (CPM) aims at the determination of the time to complete a project and the important activities on which a manager shall focus attention.

Assumption For Cpm

In CPM, it is assumed that precise time estimate is available for each activity.

Project Completion Time

From the start event to the end event, the time required to complete all the activities of the project in the specified sequence is known as the project completion time.

Path In A Project

A continuous sequence, consisting of nodes and activities alternatively, beginning with the start event and stopping at the end event of a network is called a path in the network.

Critical Path And Crtical Activities

Consider all the paths in a project, beginning with the start event and stopping at the end event. For each path, calculate the time of execution, by adding the time for the individual activities in that path.

The path with the largest time is called the critical path and the activities along this path are called the critical activities or bottleneck activities. The activities are called critical because they cannot be delayed. However, a non-critical activity may be delayed to a certain extent. Any delay in a critical activity will delay the completion of the whole project. However, a certain permissible delay in a non –critical activity will not delay the completion of the whole project. It shall be noted that delay in a non-critical activity beyond a limit would certainly delay the completion the whole project. Sometimes, there may be several critical paths for a project. A project manager shall pay special attention to critical activities.

Problem 1

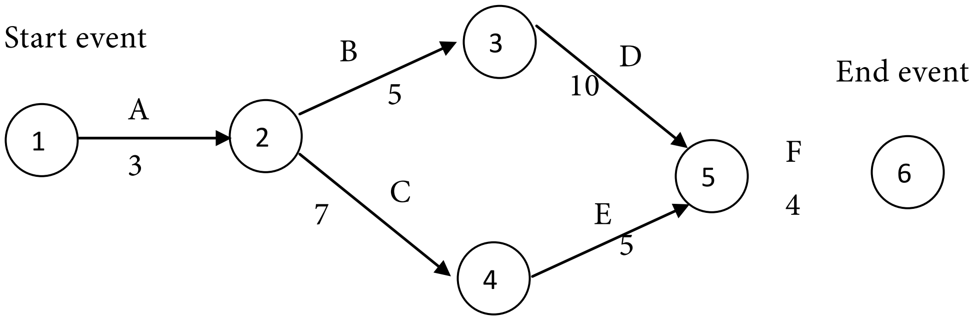
The following details are available regarding a project:

Activity	Predecessor Activity	Duration (Weeks)
A	-	3
B	A	5
C	A	7
D	B	10
E	C	5
F	D,E	4

Determine the critical path, the critical activities and the project completion time.

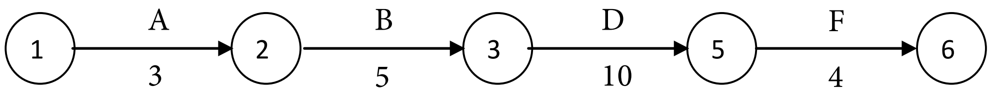
Solution

First let us construct the network diagram for the given project. We mark the time estimates along the arrows representing the activities. We obtain the following diagram:



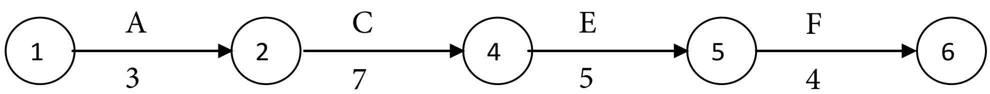
Consider the paths, beginning with the start node and stopping with the end node. There are two such paths for the given project. They are as follows:

Path I



with a time of 3 + 5 + 10 + 4 = 22 weeks.

Path II



with a time of 3 + 7 + 5 + 4 = 19 weeks.

Compare the times for the two paths. Maximum of {22,19} = 22. We see that path I has the maximum time of 22 weeks. Therefore, path I is the critical path. The critical activities are A, B, D and F. The project completion time is 22 weeks.

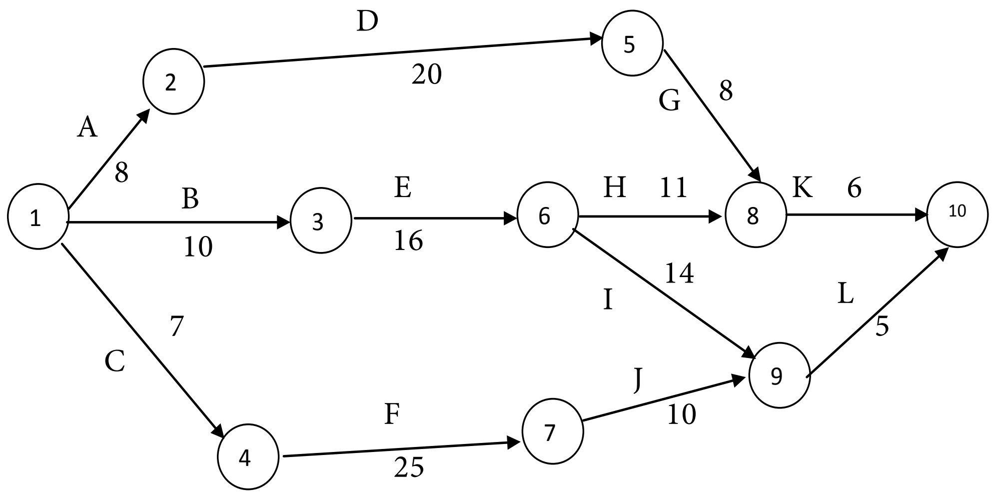
We notice that C and E are non- critical activities.

Time for path I - Time for path II = 22- 19 = 3 weeks.

Therefore, together the non- critical activities can be delayed upto a maximum of 3 weeks, without delaying the completion of the whole project.

Problem 2

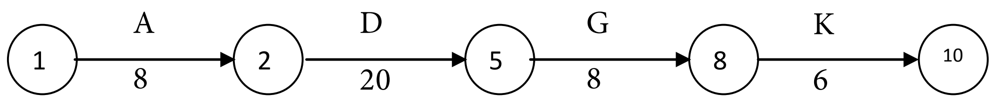
Find out the completion time and the critical activities for the following project:



Solution

In all, we identify 4 paths, beginning with the start node of 1 and terminating at the end node of 10. They are as follows:

Path I



Time for the path = 8 + 20 + 8 + 6 = 42 units of time.

Path II

$$1 \xrightarrow[10]{B} 3 \xrightarrow[16]{E} 6 \xrightarrow[11]{H} 8 \xrightarrow[6]{K} 10$$

Time for the path = 10 + 16 + 11 + 6 = 43 units of time.

Path III

$$1 \xrightarrow[10]{B} 3 \xrightarrow[16]{E} 6 \xrightarrow[14]{I} 9 \xrightarrow[5]{L} 10$$

Time for the path = 10 + 16 + 14 + 5 = 45 units of time.

Path IV

$$1 \xrightarrow[7]{C} 4 \xrightarrow[25]{F} 7 \xrightarrow[10]{J} 9 \xrightarrow[5]{L} 10$$

Time for the path = 7 + 25 + 10 + 5 = 47 units of time.

Compare the times for the four paths. Maximum of {42, 43, 45, 47} = 47. We see that the following path has the maximum time and so it is the critical path:

$$1 \xrightarrow[7]{C} 4 \xrightarrow[25]{F} 7 \xrightarrow[10]{J} 9 \xrightarrow[5]{L} 10$$

The critical activities are C, F, J and L. The non-critical activities are A, B, D, E, G, H, I and K. The project completion time is 47 units of time.

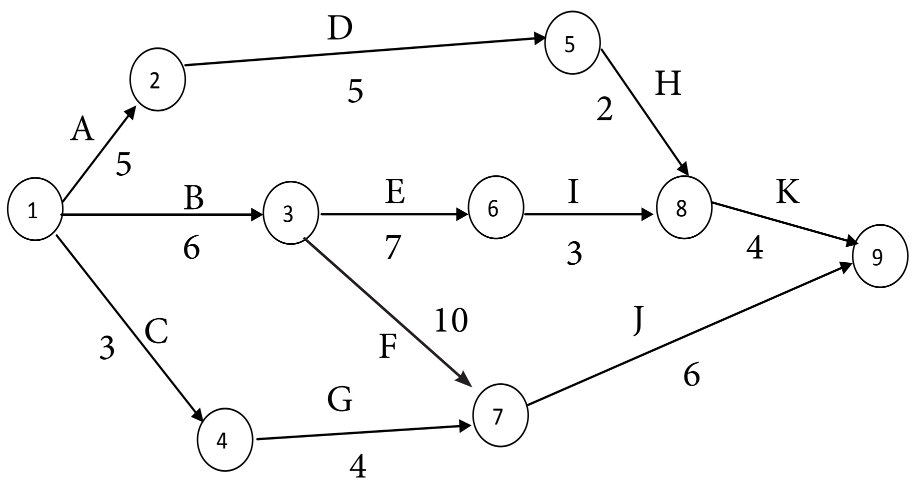
Problem 3

Draw the network diagram and determine the critical path for the following project:

Activity	Time estimate (Weeks)
1- 2	5
1- 3	6
1- 4	3
2 -5	5
3 -6	7
3 -7	10
4 -7	4
5 -8	2
6 -8	5
7 -9	6
8 -9	4

Solution

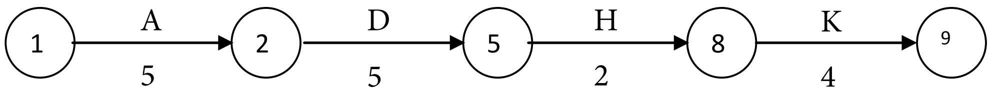
We have the following network diagram for the project:



Solution

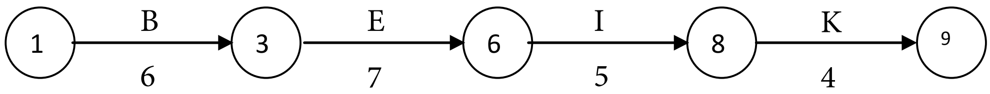
We assert that there are 4 paths, beginning with the start node of 1 and terminating at the end node of 9. They are as follows:

Path I



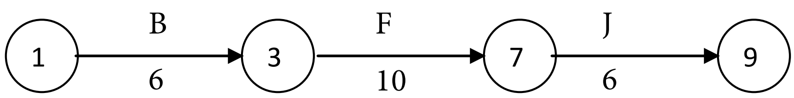
Time for the path = 5 + 5 + 2 + 4 = 16 weeks.

Path II



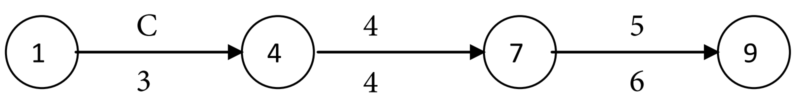
Time for the path = 6 + 7 + 5 + 4 = 22 weeks.

Path III



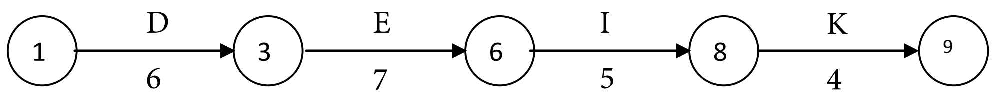
Time for the path = 6 + 10 + 6 = 16 weeks.

Path IV



Time for the path = 3 + 4 + 6 = 13 weeks.

Compare the times for the four paths. Maximum of {16, 22, 16, 13} = 22. We see that the following path has the maximum time and so it is the critical path:



The critical activities are B, E, I and K. The non-critical activities are A, C, D, F, G, H and J. The project completion time is 22 weeks.

Questions

1. Explain the terms: critical path, critical activities.
2. The following are the time estimates and the precedence relationships of the activities in a project network:

Activity	IMMEDIATE Predecessor Activity	time estimate (weeks)
A	-	4
B	-	7
C	-	3
D	A	6
E	B	4
F	B	7
G	C	6
H	E	10
I	D	3
J	F, G	4
K	H, I	2

Draw the project network diagram. Determine the critical path and the project completion time.

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Lesson 5 - Pert

Lesson Outline

- The Concept Of Pert
- Estimates Of The Time Of An Activity
- Determination Of Critical Path
- Probability Estimates
- Normal Probability Distribution Table

Learning Objectives

After reading this lesson you should be able to

- Understand the importance of PERT
- Locate the critical path
- Determine the project completion time
- Find out the probability of completion of a project before a stipulated time
- Use the normal probability distribution table

Introduction

Programme Evaluation and Review Technique (PERT) is a tool that would help a project manager in project planning and control. It would enable him in continuously monitoring a project and taking corrective measures wherever necessary. This technique involves statistical methods.

Assumptions for Pert

Note that in CPM, the assumption is that precise time estimate is available for each activity in a project. However, one finds most of the times that this is not practically possible.

In PERT, we assume that it is not possible to have precise time estimate for each activity and instead, probabilistic estimates of time alone are possible. A multiple time estimate approach is followed here. In probabilistic time estimate, the following 3 types of estimate are possible:

Pessimistic time estimate (t_p)

Optimistic time estimate (t_o)

Most likely time estimate (t_m)

The optimistic estimate of time is based on the assumption that an activity will not involve any difficulty during execution and it can be completed within a short period. On the other hand, a pessimistic estimate is made on the assumption that there would be unexpected problems during the execution of an activity and hence it would consume more time. The most likely time estimate is made in between the optimistic and the pessimistic estimates of time. Thus the three estimates of time have the relationship

$$t_o \leq t_m \leq t_p$$

Practically speaking, neither the pessimistic nor the optimistic estimate may hold in reality and it is the most likely time estimate that is expected to prevail in almost all cases. Therefore, it is preferable to give more weight to the most likely time estimate.

We give a weight of 4 to most likely time estimate and a weight of 1 each to the pessimistic and optimistic time estimates. We arrive at a time estimate (t_e) as the weighted average of these estimates as follows:

$$t_e = \frac{t_o + 4\ t_m + t_p}{6}$$

Since we have taken 6 units (1 for t_p , 4 for t_m and 1 for t_o), we divide the sum by 6. With this time estimate, we can determine the project completion time as applicable for CPM.

Since PERT involves the average of three estimates of time for each activity, this method is very practical and the results from PERT will be have a reasonable amount of reliability.

Measure Of Certainty

The 3 estimates of time are such that

$$t_o \le t_m \le t_p$$

Therefore the range for the time estimate is $t_p - t_o$.

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

i.e., The standard deviation = $\sigma = \frac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

The certainty of the time estimate of an activity can be analysed with the help of the variance. The greater the variance, the more uncertainty in the time estimate of an activity.

Problem 1

Two experts A and B examined an activity and arrived at the following time estimates.

Expert	Time Estimate		
	t_o	t_m	t_p
A	4	6	8
B	4	7	10

Determine which expert is more certain about his estimates of time:

Solution

Variance (σ^2) in time estimates = $\left(\frac{t_p - t_o}{6}\right)^2$

In the case of expert A, the variance = $\left(\frac{8-4}{6}\right)^2 = \frac{4}{9}$

As regards expert B, the variance = $\left(\frac{10-4}{6}\right)^2 = 1$

So, the variance is less in the case of A. Hence, it is concluded that the expert A is more certain about his estimates of time.

Determination of Project Completion Time in PERT

Problem 2

Find out the time required to complete the following project and the critical activities:

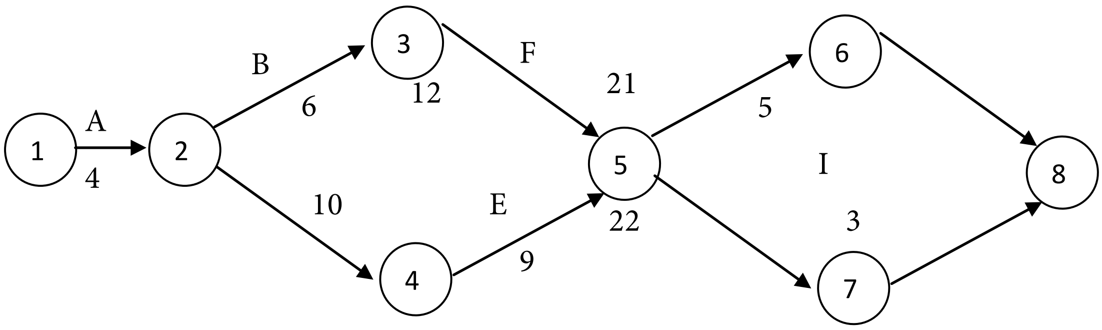
Activity	Predecessor Activity	Optimistic time estimate (to days)	Most likely time estimate (tm days)	Pessimistic time estimate (tp days)
A	-	2	4	6
B	A	3	6	9
C	A	8	10	12
D	B	9	12	15
E	C	8	9	10
F	D, E	16	21	26
G	D, E	19	22	25
H	F	2	5	8
I	G	1	3	5

Solution

From the three time estimates t_p, t_m and t_o, calculate t_e for each activity. We obtain the following table:

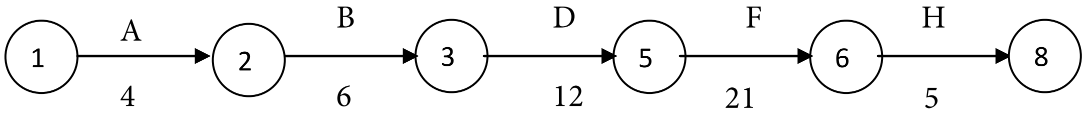
Activity	Optimistic time estimate (to)	4 x Most likely time estimate	Pessimistic time estimate (tp)	to+ 4tm + tp	Time estimate $t_e = \frac{t_o + 4\ t_m + t_p}{6}$
A	2	16	6	24	4
B	3	24	9	36	6
C	8	40	12	60	10
D	9	48	15	72	12
E	8	36	10	54	9
F	16	84	26	126	21
G	19	88	25	132	22
H	2	20	8	30	5
I	1	12	5	18	3

Using the single time estimates of the activities, we get the following network diagram for the project.



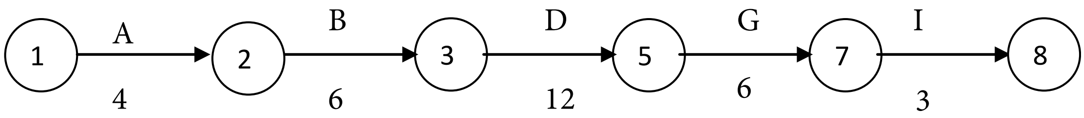
Consider the paths, beginning with the start node and stopping with the end node. There are four such paths for the given project. They are as follows:

Path I



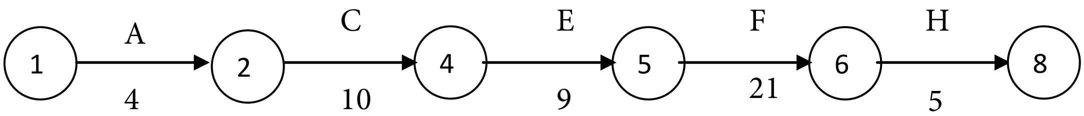
Time for the path: 4+6+12+21+5 = 48 days.

Path II



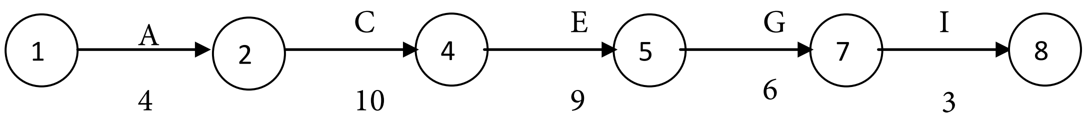
Time for the path: 4+6+12+ 6+3 = 31 days.

Path III



Time for the path: 4+10+9+ 21+5 = 49 days.

Path IV



Time for the path: 4+10+9+ 6+3 = 32 days.

Compare the times for the four paths.

Maximum of {48, 31, 49, 32} = 49.

We see that Path III has the maximum time.

Therefore the critical path is Path III. i.e., $1 \to 2 \to 4 \to 5 \to 6 \to 8$.

The critical activities are A, C, E, F and H.

The non-critical activities are B, D, G and I.

Project time (Also called project length) = 49 days.

Problem 3

Find out the time, variance and standard deviation of the project with the following time estimates in weeks:

Activity	Optimistic time estimate (to)	Most likely time estimate (tm)	Pessimistic time estimate (tp)
1-2	3	6	9
1-6	2	5	8
2-3	6	12	18
2-4	4	5	6
3-5	8	11	14
4-5	3	7	11
6-7	3	9	15
5-8	2	4	6
7-8	8	16	18

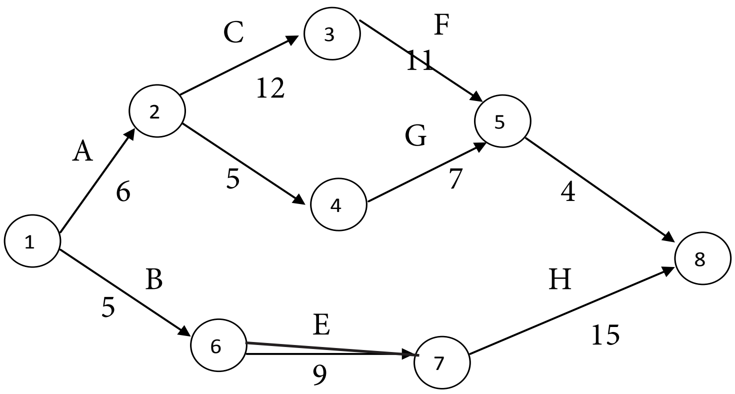
Solution

From the three time estimates t_p , t_m and t_o , calculate t_e for each activity. We obtain the following table:

Activity	Optimistic time stimate (to)	4 x Most likely time estimate	Pessimistic time estimate (tp)	to+ 4tm + tp	Time estimate $t_e = \frac{t_o + 4\ t_m + t_p}{6}$
1-2	3	24	9	36	6
1-6	2	20	8	30	5
2-3	6	48	18	72	12
2-4	4	20	6	30	5
3-5	8	44	14	66	11

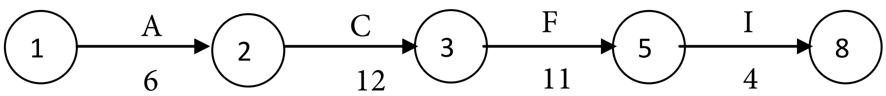
4-5	3	28	11	42	7
6-7	3	36	15	54	9
5-8	2	16	6	24	4
7-8	8	64	18	90	15

With the single time estimates of the activities, we get the following network diagram for the project.



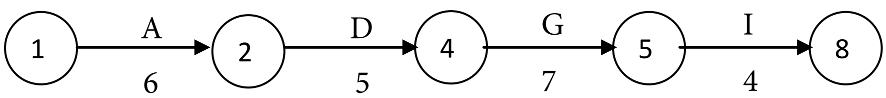
Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I



Time for the path: 6+12+11+4 = 33 weeks.

Path II



Time for the path: 6+5+7+ 4= 22 weeks.

Path III

$$1 \xrightarrow[5]{B} 6 \xrightarrow[9]{E} 7 \xrightarrow[15]{H} 8$$

Time for the path: 5+9+15 = 29 weeks.

Compare the times for the three paths.

Maximum of {33, 22, 29} = 33.

It is noticed that Path I has the maximum time.

Therefore the critical path is Path I. i.e., $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 8$

The critical activities are A, C, F and I.

The non-critical activities are B, D, G and H.

Project time = 33 weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (to)	Most likely timeestimate (tm)	Pessimistic time estimate (tp)	Range (tp - to)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: 1 2	3	6	9	6	1	1
C: 2 3	6	12	18	12	2	4
F: 3 5	8	11	14	6	1	1
I: 5 8	2	4	6	4	2/3	4/9

Variance of project time (Also called Variance of project length) =

Sum of the variances for the critical activities = 1+4+1+ 4/9 = 58/9 Weeks.

Standard deviation of project time = √Variance = √58/9 = 2.54 weeks.

Problem 4

A project consists of seven activities with the following time estimates. Find the probability that the project will be completed in 30 weeks or less.

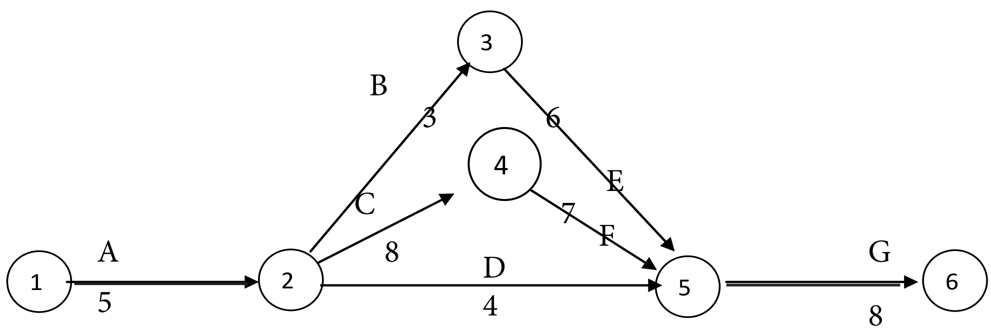
Activity	Predecessor Activity	Optimistic time estimate (to days)	Most likely time estimate (tm days)	Pessimistic time estimate (tp days)
A	-	2	5	8
B	A	2	3	4
C	A	6	8	10
D	A	2	4	6
E	B	2	6	10
F	C	6	7	8
G	D, E, F	6	8	10

Solution

From the three time estimates , and , calculate for each activity. The results are furnished in the following table:

Activity	Optimistic time estimate (to)	4 x Most likely time estimate	Pessimistic time estimate (tp)	to+ 4tm + tp	Time estimate $t_e = \frac{t_o + 4\,t_m + t_p}{6}$
A	2	20	8	30	5
B	2	12	4	18	3
C	6	32	10	48	8
D	2	16	6	24	4
E	2	24	10	36	6
F	6	28	8	42	7
G	6	32	10	48	8

With the single time estimates of the activities, the following network diagram is constructed for the project.



Consider the paths, beginning with the start node and stopping with the end node. There are three such paths for the given project. They are as follows:

Path I

$$1 \xrightarrow[5]{A} 2 \xrightarrow[3]{B} 3 \xrightarrow[6]{E} 5 \xrightarrow[8]{G} 6$$

Time for the path: 5+3+6+8 = 22 weeks.

Path II

$$1 \xrightarrow[5]{A} 2 \xrightarrow[8]{C} 4 \xrightarrow[7]{F} 5 \xrightarrow[8]{G} 6$$

Time for the path: 5+8+7+ 8 = 28 weeks.

Path III

$$1 \xrightarrow[5]{A} 2 \xrightarrow[4]{D} 5 \xrightarrow[8]{G} 6$$

Time for the path: 5+4+8 = 17 weeks.

Compare the times for the three paths.

Maximum of {22, 28, 17} = 28.

It is noticed that Path II has the maximum time.

Therefore the critical path is Path II. i.e., $1 \to 2 \to 4 \to 5 \to 6$.

The critical activities are A, C, F and G.

The non-critical activities are B, D and E.

Project time = 28 weeks.

Calculation of Standard Deviation and Variance for the Critical Activities:

Critical Activity	Optimistic time estimate (to)	Most likely time estimate (tm)	Pessimistic time estimate (tp)	Range (tp -to)	Standard deviation = $\sigma = \frac{t_p - t_o}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A: 1 2	2	5	8	6	1	1
C: 2 4	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$
F: 4 5	6	7	8	2	$\frac{1}{3}$	$\frac{1}{9}$
G: 5 6	6	8	10	4	$\frac{2}{3}$	$\frac{4}{9}$

Standard deviation of the critical path = √2 = 1.414

The standard normal variate is given by the formula

$$Z = \frac{Given\ value\ of\ t\ -\ Expected\ value\ of\ t\ \text{in}\ the\ critical\ path}{SD\ for\ the\ critical\ path}$$

So we get $Z = \frac{30\ -28}{1.414} = 1.414$

We refer to the Normal Probability Distribution Table.

Corresponding to Z = 1.414, we obtain the value of 0.4207

We get 0.5 + 0.4207 = 0. 9207

Therefore the required probability is 0.92

i.e., There is 92% chance that the project will be completed before 30 weeks.

In other words, the chance that it will be delayed beyond 30 weeks is 8%

Questions

1. Explain how time of an activity is estimated in PERT.
2. Explain the measure of certainty in PERT.
3. The estimates of time in weeks of the activities of a project are as follows:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	2	4	6
B	A	8	11	20
C	A	10	15	20
D	B	12	18	24
E	C	8	13	24
F	C	4	7	16
G	D,F	14	18	28
H	E	10	12	14
I	G,H	7	10	19

Determine the critical activities and the project completion time.

4. Draw the network diagram for the following project. Determine the time, variance and standard deviation of the project.:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	12	14	22
B	-	16	17	24
C	A	14	15	16
D	A	13	18	23
E	B	16	18	20
F	D,E	13	14	21
G	C,F	6	8	10

5. Consider the following project with the estimates of time in weeks:

Activity	Predecessor Activity	Optimistic estimate of time	Most likely estimate of time	Pessimistic estimate of time
A	-	2	4	6
B	-	3	5	7
C	A	5	6	13
D	A	4	8	12
E	B,C	5	6	13
F	D,E	6	8	14

Find the probability that the project will be completed in 27 weeks.