Exercise-8.1

Question 1: Expand the expression $(1-2x)^5$

Answer: - By using Binomial Theorem, the expression $(1-2x)^5$ *can be expanded as*

$$(1-2x)^5 = {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(1)^0(2x)^5$$

$$= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - 32x^5$$

$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

Question 2: Expand the expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Answer:- *By using Bionomial Theorem, the expression* $\left(\frac{2}{x} - \frac{x}{2}\right)^5$ *can be expanded as*

$$\left(\frac{2}{x} - \frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$

$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \left(\frac{x^5}{32}\right)$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^3}{8} - \frac{x^5}{32}$$

Question 3: Expand the expression $(2x-3)^6$

Answer: *By using Bionomial Theorem, the expression* $(2x-3)^6$ *can be expanded as*

$$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 + {}^6C_4(2x)^2(3)^4$$

$$- {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6$$

$$= 64x^6 - 6 \times 32x^5 \times 3 + 15 \times 16x^4 \times 9 - 20 \times 8x^3 \times 27$$

$$+15 \times 4x^2 \times 81 - 6 \times 2x \times 243 + 729$$

$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

Question 4: Expand the expression $\left(\frac{x}{3} + \frac{1}{x}\right)^5$

Answer: By using Binomial Theorem, the expression $\left(\frac{x}{3} + \frac{1}{x}\right)^5$ *can be expanded as*

$$\left(\frac{x}{3} + \frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$$

$$+ {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5$$

$$= \frac{x^5}{243} + 5 \times \frac{x^4}{81} \times \frac{1}{x} + 10 \times \frac{x^3}{27} \times \frac{1}{x^2} + 10 \times \frac{x^2}{9} \times \frac{1}{x^3}$$

$$+5 \times \frac{x}{3} \times \frac{1}{x^4} + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

Question 5: Expand $\left(x + \frac{1}{x}\right)^6$

Answer: $By\ using\ Bionomial\ Theorem, the\ expression\ \left(x + \frac{1}{x}\right)^6\ can\ be\ expanded\ as$

$$\left(x + \frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 + {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 + {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6 \times x^5 \times \frac{1}{x} + 15 \times x^4 \times \frac{1}{x^2} + 20 \times x^3 \times \frac{1}{x^3} + 15 \times x^2 \times \frac{1}{x^4} + 6 \times x \times \frac{1}{x^5} + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

Question 6: Using Binomial Theorem, evaluate $(96)^3$

Answer: $(96)^3 = (100-4)^3 = {}^3C_0(100)^3 - {}^3C_1(100)^2 4 + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3$

$$= 1000000 - 3 \times 10000 \times 4 + 3 \times 100 \times 16 - 64$$

$$= 1000000 - 120000 + 4800 - 64$$

$$= 884736$$

Question 7: Using Binomial theorem, evaluate $(102)^5$

Answer:
$(102)^5 = (100+2)^5 = {}^5C_0(100)^5 + {}^5C_1(100)^4 2 + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4$

$$+{}^5C_5(2)^5$$

$$= 10000000000 + 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4 + 10 \times 10000 \times 8 + 5 \times 100 \times 16$$

$$+32$$

$$= 11040808032$$

Question 8: Using Binomial Theorem, evaluate $(101)^4$

Answer: $(101)^4 = {}^4C_0(100)^4 + {}^4C_1(100)^3 + {}^4C_2(100)^2 + {}^4C_3(100) + {}^4C_4$

$$= 100000000 + 4 \times 1000000 + 6 \times 10000 + 4 \times 100 + 1$$

$$= 104060401$$

Question 9: Using Binomial Theorem, evaluate $(99)^5$

Answer: $(99)^5 = (100-1)^5 = {}^5C_0(100)^5 - {}^5C_1(100)^4 + {}^5C_2(100)^3 - {}^5C_3(100)^2 + {}^5C_4(100) - {}^5C_5$

$$= 10000000000 - 5 \times 100000000 + 10 \times 1000000 - 10 \times 10000 + 5 \times 100 - 1$$

$$= 9509900499$$

Question 10: Using Binomial Theorem, indicate which number $(1.1)^{10000}$ or 1000

Answer: By splitting 1.1 and then applying Binomial theorem, the first few terms of $(1.1)^{10000} can\ be\ obtained\ as$

$$(1.1)^{10000} = (1+0.1)^{10000} = {}^{10000}C_0 + {}^{10000}C_1(0.1) + other\ positive\ terms.$$

$$= 1 + 10000 \times 0.1 + other\ positive\ terms.$$

$$= 1001 + other\ positive\ terms$$

$$and\ therefore\ (1.1)^{10000} > 1000$$

Question 11: Find $(a+b)^4 - (a-b)^4$ $Hence\ , evaluate$ $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$

Answer: $(a+b)^4 - (a-b)^4 = ({}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4)$

$$-({}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4)$$

$$= (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) - (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)$$

$$= 8a^3b + 8ab^3$$

$$= 8ab(a^2 + b^2)$$

$$Now\ by\ putting\ a = \sqrt{3}\ and\ b = \sqrt{2}, we\ obatain$$

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\sqrt{3}\sqrt{2}(3+2)$$

$$= 40\sqrt{6}$$

Question 12: $Find\ (x+1)^6 + (x-1)^6\ Hence\ or\ otherwise\ evaluate\ \left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$

Answer: $By\ Binomial\ Theorem,$

$$(x+1)^6 + (x-1)^6 = ({}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6) + ({}^6C_0x^6 - {}^6C_1x^5$$

$$+ {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$$

$$= 2x^6 + 30x^4 + 30x^2 + 2 = 2(x^6 + 15x^4 + 15x^2 + 1)$$

Now putting x=$\sqrt{2}$, *we obtain*

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6=2\left(\left(\sqrt{2}\right)^6+15\left(\sqrt{2}\right)^4+15\left(\sqrt{2}\right)^2+1\right)$$

$$=2(8+15\times 4+15\times 2+1)=2\times 99=198$$

Question 13: $Show\ that\ 9^{n+1}-8n-9\ is\ divisible\ by\ 64, whenever\ n\ is\ a\ positive\ integer.$

Answer: In order to show that $9^{n+1}-8n-9\ is\ divisible\ by\ 64, it\ has\ to\ be\ proved\ that$

$$9^{n+1}-8n-9=64k, where\ k\ is\ some\ natural\ number.$$

$$By\ Binomial\ Theorem,$$

$$(1+a)^m={}^mC_0+{}^mC_1a+{}^mC_2a^2+{}^mC_3a^3+\cdots+{}^mC_ma^m$$

$$For\ a=8\ and\ m=n+1$$

$$9^{n+1}={}^{n+1}C_0+{}^{n+1}C_1(8)+{}^{n+1}C_2(8)^2+{}^{n+1}C_3(8)^3+\cdots+{}^{n+1}C_{n+1}(8)^{n+1}$$

$$9^{n+1}=1+8(n+1)+(8)^2(({}^{n+1}C_2(8)+\cdots+(8)^{n-2}))$$

$$9^{n+1}-8n-9=64(({}^{n+1}C_2(8)+\cdots+(8)^{n-2}))$$

$$Thus, 9^{n+1}-8n-9\ is\ divisible\ by\ 64.$$

Question 14: $Prove\ that\ \sum_{r=0}^{n}3^{r\,n}C_r=4^n$

Answer: By Binomial Theorem,

$$\sum_{r=0}^{n}(({}^nC_ra^{n-r}b^r))=(a+b)^n$$

$Putting\ b=3\ and\ a=1\ we\ get\ ,$

$$\sum_{r=0}^{n}(({}^nC_r(1)^{n-r}3^r))=4^n$$

$$\sum_{r=0}^{n}(({}^nC_r3^r))=4^r\ Hence\ Proved.$$

Exercise-8.2

Question 1: Find the Coefficient of $x^5\ in\ (x+3)^8$.

Answer: It is known that

$$(r+1)^{th}\ term\ ,(T_{r+1}),\qquad in\ the\ binomial\ expansion\ of\ (a+b)^n\ is\ given\ by$$

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

$$(r+1)^{th} term\ of\ the\ expansion\ of\ (x+3)^8 is$$

$$T_{r+1} = {}^8 C_r x^{8-r} 3^r$$

$$Comparing\ the\ indices\ of\ x\ in\ x^5 and\ in\ T_{r+1}, we\ obtain,$$

$$8-r=5 \therefore r=3$$

$$Thus\ the\ coefficient\ of\ x^5\ is\ \ ({}^8C_3 3^3) = \left(\frac{8!}{5!\,3!}\right) \times 27 = 56 \times 27 = 1512$$

Question 2: $Find\ the\ coefficient\ of\ a^5 b^7 in\ (a-2b)^{12}$

Answer :