Homework 6—Due Tuesday, March 13 at 11:55 PM

Instructions: Complete the following tasks. Copy and paste your code and analysis into a DOC/PDF/ODT document (like a lab report). Also upload a script and/or functions files with your code. You will submit this work online through the CROPS assignment page. For the plots, you must use the `xlabel`, `ylabel`, and `title` parameters on every plot to receive full credit.

1. **Normalization** Write a MATLAB function that takes a vector as an input, and outputs the unit vector (i.e. the vector of length one) in the one-norm, the two-norm, and the infinity-norm—that is, your function will have 3 outputs. Your program also needs to be robust enough to simply output a zero vector (of the same length as the input) if the input vector has virtually zero length. Note: you may use the `norm` command in MATLAB.

2. **Eig** Type "`help eig`" into MATLAB, and answer the following questions.

   (a) How do you find eigenvalues with this MATLAB function?

   (b) How do you find eigenvectors with this MATLAB function?

   (c) What is a unit vector?

   (d) How do you have MATLAB output eigenvectors that are not normalized—that is, that are not unit vectors?

3. Find the eigenvalues and eigenvectors (without normalization) of the following matrices

$$A = \begin{bmatrix} -3 & 2 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & -2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 0 & 6 & 0 \\ 0 & -5 & 0 & -4 \\ -1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \end{bmatrix}$$

4. The **spectral radius** of a matrix $A \in \mathbb{R}^{n \times n}$ is defined as the largest eigenvalue in magnitude

$$\rho(A) \equiv \max_{1 \le i \le n} |\lambda|$$

   For example, considering the matrix

$$A = \begin{bmatrix} 3 & -7 & 8 \\ 0 & -4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

   the eigenvalues are $\lambda = -4, 1, 3$ (why?), so the spectral radius is $\rho(A) = 4$. That is, we found the eigenvalues, took the absolute value of them, and then looked for the maximum.

   Write a MATLAB function that computes the spectral radius of a matrix.

5. **Color Wheel** Use one of your favorite, digital images (the funnier the better, but please abide by the school's code of conduct). The `imread` command will load an image into a m-by-n by 3 tensor. Just like with a m-by-n matrix, a command like $A(:,:,1)$ will retrieve the first matrix in that third-dimension. Use the `subplot` command to display the original image and its red-green-blue components in a 2-by-2 grid.

6. **Image Inversion** Let A be the matrix representing a grayscale image. For each pixel, the number 0 represents black, and the number 255 represents white—while the numbers in between are shades of gray. Use the code `abs(255 - A)` to invert the shades of gray. Use the `subplot` command to display the grayscale image and its inversion side-by-side.

7. **Introduction to image compression.** Use the enclosed `Math50imagery.m` script to practice some image compression. Pick a number r of principal components that you think would yield a visually-pleasing compressed image, and report on your findings.

8. **Polynomial Interpolation** Use the `polyfit` command to find the quadratic function $f(x) = ax^2 + b(x) + c$ whose graph passes through the points

$$(-1,-4), \quad (1,-2), \quad (2,5)$$

9. **Linear Regression** Use the `polyfit` and `polyval` commands to find the line of best fit $\hat{y} = mx+b$ for the data. The enrollment (in thousands) and number of burgalaries per year at Ohio State are shown in the following table. Considering that OSU had 51,800 students this past year, is the predicted value close to the true value of 329 burgalaries?

Enrollment	32	31	53	28	27	36	42	30	34	46
Burglaries	103	103	86	57	32	131	157	20	27	161

10. **Limit Function** Plot the following functions

$$y = x, \quad y = x^2, \quad y = x^3, \quad y = x^4, \quad y = x^5$$

over the interval $[0,1]$ and arrange the plots into a 2-by-3 grid using the `subplot` command. Be sure to label all 6 plots. What do you think the limit function $f_\infty(x) = \lim_{n\to\infty} x^n$ looks like?