Homework 4—Due Friday, February 21 at 11:55 PM

Instructions: Complete the following tasks. Copy and paste your code and analysis into a DOC/PDF/ODT document (like a lab report). Also upload a script and/or functions files with your code. You will submit this work online through the CROPS assignment page. For the plots, you must use the `xlabel`, `ylabel`, and `title` parameters on every plot to receive full credit.

1. Write a MATLAB function that takes a vector of positive numbers as its input, and outputs their geometric mean. Note: since this is a training exercise, try to complete this problem without using the `geomean` function in MATLAB.

2. Write a trio of MATLAB functions that each take a vector of numbers. The outputs are

   (a) one-norm

   (b) two-norm

   (c) infinity-norm

   Note: since this is a training exercise, try to complete this problem without using the `norm` function in MATLAB.

3. Write a MATLAB function that takes the side lengths a, b, and c of a triangle, and outputs the triangle's area using Heron's area formula.

4. **Gasket** Use a MATLAB script file for the following.

   (a) Initialize a variable n with the value of 10 (this will be the number of rows)

   (b) Initialize an n-by-n matrix full of zeroes

   (c) Set the first column as a n-by-1 vector of ones

   (d) Construct Pascal's Triangle: for j from 2 to n ... for k from 2 to j

   $$A_{jk} = A_{j-1,k-1} + A_{j-1,k}$$

   (e) We will only need the ones' digit for this adventure. Furthermore, we only need to know if each number of Pascal's Triangle is even or odd, so use modulo arithmetic to retain only the even/odd aspect.

   (f) What does the `spy(A)` command do in MATLAB? Run that command on your matrix.

   Run your script for various numbers of rows, such as $n = 50, 100, 150, 200$. Comment on the results.

5. **Finite Differences—One-Step Method** Write a function that approximates a derivative of a function by using the one-step, left-side, finite difference scheme

   $$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

   The input of this function is a vector of values, and the output is a vector of values with the same length. Test your program on the math function

   $$f(x) = \left(\frac{x-1}{x+1}\right)^3$$

   and plot the results on an interval of your choice.

6. **Finite Differences—Two-Step Method** Write a function that approximates a derivative of a function by using the two-step, centered, finite difference scheme

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

The input of this function is a vector of values, and the output is a vector of values with the same length. Test your program on the math function

$$g(\theta) = |\sin\theta|$$

and plot the results on an interval of your choice that covers $\theta = 0$. Note: you're allowed to use one-step methods on the endpoints.

7. **Numeric Integration** Write a function that approximates the definite integral $\int_a^b f(x)\,dx$ using the Trapezoidal Rule. Test your program on the integral

$$\int_0^{\pi/4} \frac{\tan t}{1+\cos t}\,dt$$

8. **Adaptive Methods** Run the Adaptive Quadrature Demo script on the function

$$f(x) = e^{-|x|}\sin\frac{1}{x}$$

over an interval of your choice that covers the singularity at $x = 0$. Note: you need to call your own Trapezoidal Rule function.