Fortgeschrittenenpraktikum Physik
Zeeman-Effekt

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THEORETICAL PREREQUISITES

1.1 SELECTION RULES FOR DIPOLE TRANSITIONS

Question 1

A transition is (in dipole approximation) possible, only if the following selection rules are obeyed.[3, Tab. 7.2]

$$\Delta J = 0, \pm 1 \quad J = 0 \to 0 \text{ is forbidden} \tag{1.1}$$
$$\Delta M_J = 0, \pm 1 \quad M = 0 \to 0 \text{ is forbidden if } \Delta J = 0 \tag{1.2}$$
$$\Delta l = \pm 1 \tag{1.3}$$

1.2 CLEBSCH-GORDAN COEFFICIENTS

Question 2

We are considering transitions between the states $\mathrm{P}_{\frac{3}{2}} = |L = 1, J = \frac{3}{2}, M_J\rangle$, $\mathrm{P}_{\frac{1}{2}} = |L = 1, J = \frac{1}{2}, M_J\rangle$ and $\mathrm{S}_{\frac{1}{2}} = |L = 0, J = \frac{1}{2}, M_J\rangle$. Since a photon has the spin quantum number $s_{ph} = 1$, we can write the states before the transition as a coupling of a state with angular momentum $J_{ph} = 1$ and another state with angular momentum $J_2 = \frac{1}{2}$. Using the respective Clebsch-Gordan coefficients and the notation $|L, J, M_J\rangle = \sum |J_{ph}, M_{J_{ph}}\rangle\, |J_2, M_{J_2}\rangle$, we get [4, Tab. 4.8]

$$|1, \tfrac{3}{2}, -\tfrac{3}{2}\rangle = \quad |1, -1\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle \tag{1.4}$$
$$|1, \tfrac{3}{2}, -\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |1, \ 0\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\, |1, -1\rangle\, |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \tag{1.5}$$
$$|1, \tfrac{3}{2}, +\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\, |1, +1\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\, |1, \ 0\rangle\, |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \tag{1.6}$$
$$|1, \tfrac{3}{2}, +\tfrac{3}{2}\rangle = \quad |1, +1\rangle\, |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \tag{1.7}$$

$$|1, \tfrac{1}{2}, -\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\, |1, \ 0\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\, |1, -1\rangle\, |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \tag{1.8}$$
$$|1, \tfrac{1}{2}, +\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\, |1, +1\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\, |1, \ 0\rangle\, |\tfrac{1}{2}, +\tfrac{1}{2}\rangle\,. \tag{1.9}$$

1.3 ANGULAR DISTRIBUTION OF DIPOLE RADIATION

Question 3

The spherical basis vectors are defined as follows:

$$\boldsymbol{e}_0 = \boldsymbol{e}_z \tag{1.10}$$
$$\boldsymbol{e}_+ = -\frac{1}{\sqrt{2}}\boldsymbol{e}_x - \frac{i}{\sqrt{2}}\boldsymbol{e}_y \tag{1.11}$$
$$\boldsymbol{e}_- = +\frac{1}{\sqrt{2}}\boldsymbol{e}_x - \frac{i}{\sqrt{2}}\boldsymbol{e}_y \tag{1.12}$$

where $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ are the usual cartesian basis vectors. Therefore we have

$$\boldsymbol{e}_0 \exp(-i\omega_{ba}t) = \boldsymbol{e}_z \exp(-i\omega_{ba}t) \tag{1.13}$$

$$\begin{aligned}\boldsymbol{e}_+ \exp(-i\omega_{ba}t) &= -\frac{1}{\sqrt{2}}\left(\boldsymbol{e}_x \exp(-i\omega_{ba}t) + \boldsymbol{e}_y i \exp(-i\omega_{ba}t)\right) \\ &= -\frac{1}{\sqrt{2}}\left(\boldsymbol{e}_x \exp(-i\omega_{ba}t) + \boldsymbol{e}_y \exp(-i\omega_{ba}t + \tfrac{\pi}{2})\right) \qquad (1.14)\end{aligned}$$

$$\begin{aligned}\boldsymbol{e}_- \exp(-i\omega_{ba}t) &= +\frac{1}{\sqrt{2}}\left(\boldsymbol{e}_x \exp(-i\omega_{ba}t) - \boldsymbol{e}_y i \exp(-i\omega_{ba}t)\right) \\ &= +\frac{1}{\sqrt{2}}\left(\boldsymbol{e}_x \exp(-i\omega_{ba}t) - \boldsymbol{e}_y \exp(-i\omega_{ba}t + \tfrac{\pi}{2})\right) \qquad (1.15)\end{aligned}$$

From this we can see, that eq. (1.13) describes an oscillation along the $\boldsymbol{e}_z$-axis, whereas eqns. (1.14) and (1.15) describe a circular oscillation in the $\boldsymbol{e}_x$-$\boldsymbol{e}_y$-plane. The rotation described by eq. (1.14) ((1.15)) is in the positive (negative) direction, i.e counter clockwise (clockwise) when viewed from the positive $\boldsymbol{e}_z$-axis.

Question 4

We now consider a electromagnetic wave propagating at an angle θ to the $\boldsymbol{e}_z$-axis. This means that it's wavevector is of the form:

$$\boldsymbol{k} = k(\sin\theta \boldsymbol{e}_x + \cos\theta \boldsymbol{e}_z)^1$$

This wave is polarized in the plane perpendicular to $\boldsymbol{k}$ which is spanned by $\boldsymbol{e}_1 = \cos\theta \boldsymbol{e}_x - \sin\theta \boldsymbol{e}_z$ and $e_2 = \boldsymbol{e}_y$. We can obtain the components along $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ by projecting the oscillator components from eqns. (1.13)-(1.15) onto these vectors [5, Eq. 16.65]. Thus we obtain:

$$\boldsymbol{e}_1 \cdot \boldsymbol{e}_0 = -\sin\theta \tag{1.16}$$

$$\boldsymbol{e}_2 \cdot \boldsymbol{e}_0 = 0 \tag{1.17}$$

$$\boldsymbol{e}_1 \cdot \boldsymbol{e}_+ = -\frac{1}{\sqrt{2}}\cos\theta \tag{1.18}$$

$$\boldsymbol{e}_2 \cdot \boldsymbol{e}_+ = -\frac{i}{\sqrt{2}} \tag{1.19}$$

$$\boldsymbol{e}_1 \cdot \boldsymbol{e}_- = +\frac{1}{\sqrt{2}}\cos\theta \tag{1.20}$$

$$\boldsymbol{e}_2 \cdot \boldsymbol{e}_- = -\frac{i}{\sqrt{2}} \tag{1.21}$$

Observing the radiation perpendicular to the quantization axis corresponds to the case $\theta = \frac{\pi}{2}$. In this case we get for the $\boldsymbol{e}_0$ oscillation:

$$E\left(\theta = \frac{\pi}{2}\right) \propto \boldsymbol{e}_1 = \boldsymbol{e}_z \tag{1.22}$$

and for the $\boldsymbol{e}_\pm$ oscillations:

$$E\left(\theta = \frac{\pi}{2}\right) \propto \boldsymbol{e}_2 = \boldsymbol{e}_y \tag{1.23}$$

[1] Since the setup is rotationally symmetric we can assume without loss of generality that $\varphi = 0$.

So in both cases the observed light in the plane perpendicular to $\boldsymbol{e}_z$ is linearly polarized. In the case of oscillation along $\boldsymbol{e}_z$ the light is polarized parallel to the quantization axis, whereas it is polarizad perpendicularly for the $\boldsymbol{e}_\pm$ oscillations.

1.4 ZEEMAN EFFECT

Question 5

One can distinguish between the normal and the anomalous Zeeman effect. The normal Zeeman effect occurs, when the total spin of the atom vanishes, so that the atom's magnetic moment only stems from the orbital angular momentum. If the atom has non-zero total spin, the spin also contributes to the atom's magnetic moment and thus to the energy shift in an external field. The name "anomalous Zeeman effect" stems from the fact, that it was discovered before electron spin and therefore could not be explained classically.

Question 6

Sodium has the electron configuration [Ne]$3s^1$, therefore it has non-zero total spin $S = \frac{1}{2}$ which leads us to expect that sodium will show the anomalous Zeeman effect when placed in a magnetic field.

Question 7

We have the following Hamiltonian:

$$H = H_0 + H_Z \tag{1.24}$$

Where H_0 is the Hamiltonian of the atom without the external field which is diagonalized by $|n, J, M_J, L, S\rangle$. H_Z describes the interaction with the external magnetic field $\boldsymbol{B} = B\boldsymbol{e}_z$ and is given by:

$$\begin{aligned} H_Z &= \frac{e}{2m_e}(g_L \boldsymbol{L} + g_S \boldsymbol{S}) \cdot \boldsymbol{B} \\ &= \frac{e}{2m_e}(g_L \boldsymbol{J} + (g_S - g_L)\boldsymbol{S}) \cdot \boldsymbol{B} \\ &= \frac{e}{2m_e}(g_L \boldsymbol{J} + (g_S - g_L)\boldsymbol{S}) \cdot \boldsymbol{B} \\ &= \frac{eB}{2m_e}(g_L J_z + (g_S - g_L)S_z). \end{aligned} \tag{1.25}$$

Because S_z does not commute with $\boldsymbol{J}^2$, we need to compute the Energy shift due to the magnetic field via first order perturbation theory. This means that we need to compute the following matrix elements:

$$\langle H_Z \rangle_{J,M_J,L,S} = \frac{eB}{2m_e}(g_L M_J + (g_S - g_L) \langle S_z \rangle_{J,M_J,L,S}) \tag{1.26}$$

To calculate $\langle S_z \rangle_{J,M_J,L,S}$, we note that $[S_i, L_j] = 0$, $[S_i, S_j] = -i\hbar\varepsilon_{ijk}S_k$[2], from which follows

$$S_i L_j S_j - L_j S_j S_i = S_i L_j S_j - S_i L_j S_j - i\hbar\varepsilon_{ijk} L_j S_k = -i\hbar\varepsilon_{ijk} L_j S_k \tag{1.27}$$

[2] In the following we will use Einstein summation notation

This is equivalent to

$$\boldsymbol{S}(\boldsymbol{L}\cdot\boldsymbol{S})-(\boldsymbol{L}\cdot\boldsymbol{S})\boldsymbol{S}=-i\hbar\boldsymbol{S}\times\boldsymbol{L}. \tag{1.28}$$

From the vector product of this identity with $\boldsymbol{J}$ follows:

$$\begin{aligned}\boldsymbol{S}\times\boldsymbol{J}(\boldsymbol{L}\cdot\boldsymbol{S})-(\boldsymbol{L}\cdot\boldsymbol{S})\boldsymbol{S}\times\boldsymbol{J}&=-i\hbar(\boldsymbol{S}\times\boldsymbol{L})\times\boldsymbol{J}\\&=-i\hbar(\boldsymbol{L}(\boldsymbol{S}\cdot\boldsymbol{J})-\boldsymbol{S}(\boldsymbol{L}\cdot\boldsymbol{J}))\\&=\quad i\hbar(-\boldsymbol{J}(\boldsymbol{S}\cdot\boldsymbol{J})+\boldsymbol{S}\boldsymbol{J}^2).\end{aligned} \tag{1.29}$$

Because the states $|n,J,M_J,L,S\rangle$ diagonalize $\boldsymbol{L}\cdot\boldsymbol{S}$, the left side will vanish when taking the expected value of of eq. (1.29) in those states. From this we obtain:

$$\langle\boldsymbol{S}\boldsymbol{J}^2\rangle=\langle\boldsymbol{J}(\boldsymbol{S}\cdot\boldsymbol{J})\rangle\,. \tag{1.30}$$

If we now use $\boldsymbol{S}\cdot\boldsymbol{J}=\frac{1}{2}(\boldsymbol{J}^2+\boldsymbol{S}^2-\boldsymbol{L}^2)$, which follows from the expansion of $\boldsymbol{L}^2=(\boldsymbol{J}-\boldsymbol{S})^2$, eq. (1.30) simplifies to

$$\langle S_z\rangle=\langle J_z\rangle\,\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}. \tag{1.31}$$

Inserting eq. (1.31) into eq. (1.26) yields:

$$\begin{aligned}\langle H_Z\rangle&=\frac{eB}{2m_e}\,\langle J_z\rangle\left(g_L+(g_S-g_L)\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}\right)\\&=\frac{eB}{2m_e}\,\langle J_z\rangle\left(\frac{g_L+g_S}{2}-\frac{g_S-g_L}{2}\,\frac{L(L+1)-S(S+1)}{J(J+1)}\right)\\&=g_J\mu_B B\,M_J\end{aligned} \tag{1.32}$$

[5, Chap. 14.2]

Question 8

The energy shift of a transition follows from eq. (1.32) and is given by

$$\Delta E=\mu_B B(g_{J_1}M_{J_1}-g_{J_2}M_{J_2}) \tag{1.33}$$

where g_{J_i} and M_{J_i} are the Landé-factors and quantum numbers of the states before and after the transition. In the case of the sodium D_1-line the initial state is $P_{1/2}=|J=\frac{1}{2},M_J,L=1,S=\frac{1}{2}\rangle$ and the final state is $S_{1/2}=|J=\frac{1}{2},M_J,L=0,S=\frac{1}{2}\rangle$. The Landé-factors are:

$$g_{S_{1/2}}=2,\qquad g_{P_{1/2}}=\frac{2}{3},\qquad g_{P_{3/2}}=\frac{4}{3}$$

The radiation frequency is then given by

$$\Delta\nu=\frac{\Delta E}{h} \tag{1.34}$$

We can obtain the shift in wavelength from

$$\Delta\lambda=\lambda'-\lambda_0=\frac{ch}{E_0+\Delta E}-\lambda_0 \tag{1.35}$$

Where $E_0=3.369\cdot10^{-19}\mathrm{J}$ and $\lambda_0=589.5924\,\mathrm{nm}$ are the energy and wavelength of the transition in the absence of an external magnetic field. The Thus we obtain in the case of $B=0.5\,\mathrm{T}$:

$M_J \to M_J{}'$	$-\frac{1}{2},-\frac{1}{2}$	$-\frac{1}{2},+\frac{1}{2}$	$+\frac{1}{2},-\frac{1}{2}$	$+\frac{1}{2},+\frac{1}{2}$
$\Delta\nu$ [GHz]	$+4.67$	-9.33	$+9.33$	-4.67
$\Delta\lambda$ [pm]	-5.40	$+10.8$	-10.8	$+5.42$

Tab. 1: Zeeman shift of the $P_{1/2} \to S_{1/2}$ transitions in the case of $B = 0.5$ T.

$M_J \to M_J{}'$	$-\frac{3}{2},-\frac{1}{2}$	$-\frac{1}{2},-\frac{1}{2}$	$+\frac{1}{2},-\frac{1}{2}$	$-\frac{1}{2},+\frac{1}{2}$	$+\frac{1}{2},+\frac{1}{2}$	$+\frac{3}{2},+\frac{1}{2}$
$\Delta\nu$ [GHz]	-7.00	$+2.33$	$+11.6$	-11.6	-2.33	$+7.00$
$\Delta\lambda$ [pm]	$+8.13$	-2.69	-0.135	$+0.135$	$+2.718$	-8.10

Tab. 2: Zeeman shift of the $P_{3/2} \to S_{1/2}$ transitions in the case of $B = 0.5$ T.

Question 9

The D_1 and D_2 lines have the wavelengths

$$\lambda_{D_1} = 589.5970\,\text{nm}, \qquad \lambda_{D_2} = 588.9997\,\text{nm} \tag{1.36}$$

in Air [6, Tabs. 3&4]. Since both of these transitions have $S_{\frac{1}{2}}$ as the final state, we can calculate the energy difference between $P_{1/2}$ and $P_{3/2}$ using these wavelengths. We obtain

$$\Delta E_{P_{3/2},P_{1/2}} = hc\left(\frac{1}{\lambda_{D_2}} - \frac{1}{\lambda_{D_1}}\right) = 3.42 \cdot 10^{-22}\,\text{J} = 2.14\,\text{meV} \tag{1.37}$$

To achieve a Zeeman separation of equal magnitude as $\Delta E_{P_{3/2},P_{1/2}}$, B would have to fulfill

$$\Delta E_{P_{3/2},P_{1/2}} = \mu_B g_J B \Leftrightarrow B = \frac{\Delta E_{P_{3/2},P_{1/2}}}{\mu_B g_J} = 55.3\,\text{T}. \tag{1.38}$$

Which corresponds to a very strong magnetic field.

1.5 THE LYOT FILTER

Question 10

Because the crystal has two different indices of refraction, the two orthogonal components of a lightwave gain a relative difference in phase $\Delta\varphi$ after passing through the crystal. The speed of light in a crystal with refractive index n is given by:

$$c' = \frac{c}{n} \tag{1.39}$$

where c is the speed of light in vacuum. Therefore the time a lightwave needs to fully pass through a crystal of length l is given by:

$$t = \frac{l}{c'} = \frac{l}{c} \times n \tag{1.40}$$

which corresponds to a change in phase of magnitude:

$$\varphi = 2\pi\frac{t}{T} = 2\pi\nu t = 2\pi\frac{l}{\lambda}n. \tag{1.41}$$

The phase difference between both components of the wave is thus given by:

$$\Delta\varphi = 2\pi\frac{l}{\lambda}\Delta n. \tag{1.42}$$

The light is linearly polarized after passing through the crystal if $\Delta\varphi = m\pi$, $m \in \mathbb{Z}$. We can use the fact that these linearly polarized waves with even m are orthogonal to those with odd m, to distinguish between the D_1 and D_2 lines. [2, Chap. 4.2.8]

Question 11

The Jones vectors for horizontal and vertical polarization are::

$$\boldsymbol{H} = \begin{pmatrix}1\\0\end{pmatrix}, \qquad \boldsymbol{V} = \begin{pmatrix}0\\1\end{pmatrix} \tag{1.43}$$

The Jones matrix for a birefractive crystal with axes parallel to $\boldsymbol{H}$ and $\boldsymbol{V}$ is given by:

$$\mathbf{M} = \begin{pmatrix}e^{i\varphi_1} & 0\\ 0 & e^{i\varphi_2}\end{pmatrix} \tag{1.44}$$

In our case, $\Delta\varphi$ is dependent on wavelength and can have the values $\Delta\varphi_1 = 2m\pi$, $\Delta\varphi_2 = (2k+1)\pi$ with $m, k \in \mathbb{Z}$. Thus we have the following two cases for $\mathbf{M}$:

$$\mathbf{L}_1 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \mathbf{L}_2 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{1.45}$$

Where $\mathbf{L}_i$ is the transmission matrix for a wave of wavelength λ_i. Because the Lyyot filter in the experiment is at an angle of 45°to $\boldsymbol{H}$ and $\boldsymbol{V}$. We need to use the transformation

$$\mathbf{L}(\theta) = \mathbf{R}(\theta)\mathbf{L}\mathbf{R}^{-1}(\theta). \tag{1.46}$$

Where $\mathbf{R}(\theta)$ is given by:

$$\mathbf{R}(\theta) = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} \tag{1.47}$$

Thus we obtain:

$$\mathbf{L}_1(\theta) = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \mathbf{L}_2(\theta) = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \tag{1.48}$$

The Jones matrices for the the final polarizer are given by:

$$\mathbf{P}_H = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \qquad \mathbf{P}_V = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \tag{1.49}$$

Where H and V correspond to wether the polarizer is set to horizontal or vertical polarization. The Matrices for the system as a whole are then given by

$$\mathbf{M}_{ij} = \mathbf{P}_j\mathbf{L}_i \tag{1.50}$$

Thus we have the following four possibilities for $\mathbf{M}_{ij}$

$$\mathbf{M}_{1H} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \qquad \mathbf{M}_{1V} = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \tag{1.51}$$

$$\mathbf{M}_{2H} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \qquad \mathbf{M}_{2V} = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix} \tag{1.52}$$

From which we can see, that for a given setting of the input and output polarizers only one of the two wavelengths is getting transmitted.

1.6 FABRY–PÉROT INTERFEROMETER

Question 12

Eqns. (19) and (20) from the experiment instructions [1] are:

$$\cos\theta_k = 1 - (k+\varepsilon)\frac{\lambda}{2d} \tag{1.53}$$

$$\tan\theta_k = \frac{R_k}{f} \tag{1.54}$$

In the case of small angles θ_k, we can use the approximations

$$\cos\theta_k = 1 - \frac{\theta_k^2}{2} + \mathcal{O}(\theta_k^4) \tag{1.55}$$

$$\tan\theta_k = \theta_k + \mathcal{O}(\theta_k^2) \tag{1.56}$$

From which we obtain

$$\theta_k^2 \approx (k+\varepsilon)\frac{\lambda}{d} \tag{1.57}$$

$$R_k^2 \approx f^2\theta_k^2 \tag{1.58}$$

Thus:

$$R_k^2 \approx \lambda\frac{f^2}{d}(k+\varepsilon) \tag{1.59}$$

Question 13

From eqn. (1.59) we obtain

$$\lambda_i = R_i^2\frac{1}{f^2}\frac{d}{k_i+\varepsilon}, \qquad i \in \{1,2\} \tag{1.60}$$

If we assume that $k_1 = k_2 = k$ this implies:

$$\Delta\lambda = \frac{1}{f^2}\frac{d}{k+\varepsilon}(R_1^2 - R_2^2) \tag{1.61}$$

Because

$$\Delta l = 2d - (k+\varepsilon)\overline{\lambda} \tag{1.62}$$

and Δl is in the order of magnitude of a few wavelengths, i.e. approximately zero when compared to d, we get

$$d = \lambda\frac{k+\varepsilon}{2} \tag{1.63}$$

and thus

$$\Delta\lambda = \frac{\overline{\lambda}}{2f^2}(R_1^2 - R_2^2). \tag{1.64}$$

Question 14

In eqn. (1.37) we calculated the difference in energies between the D_1 and D_2 lines to be $\Delta E = 3.42 \cdot 10^{-22}$ J. From which we can compute their difference in frequencies

$$\Delta\nu = \frac{\Delta E}{h} = 516\,\text{GHz}. \tag{1.65}$$

Question 15

Eqns. (23) and (24) from the instructions are:

$$\delta\nu = \frac{c}{2d} \tag{1.66}$$

$$\Delta\nu = n\delta\nu + \Delta x \tag{1.67}$$

Thus if $\Delta x = 0$, we obtain

$$\begin{aligned} \Delta\nu &= n\delta\nu \\ &= n\frac{c}{2d} && (1.68) \\ \Leftrightarrow \quad d &= n\frac{c}{2\Delta\nu} && (1.69) \\ &= n \cdot 290.767\,\mu\text{m} && (1.70) \end{aligned}$$

Question 17

If we move the mirror by an amount Δd, the free spectral range changes to

$$\delta\nu' = \frac{c}{2(d + \Delta d)}. \tag{1.71}$$

This corresponds to a relative change of

$$\frac{\delta\nu'}{\delta\nu} = \frac{d}{d + \Delta d} \tag{1.72}$$

2

EXECUTION AND EVALUATION OF THE EXPERIMENT

2.1 MAGNETIC FIELD

2.1.1 *Uniformity of the magnetic field*

Because our theoretical treatment of the Zeeman effect assumed that the atoms are placed inside a homogenous magnetic field, we need to confirm if this is indeed the case. To achieve this we used a Hall effect sensor to measure the magnitude of the magnetic field at several positions between the solenoids of the electromagnet. From a measurement along the horizontal axis perpendicular to the solenoid we obtained:

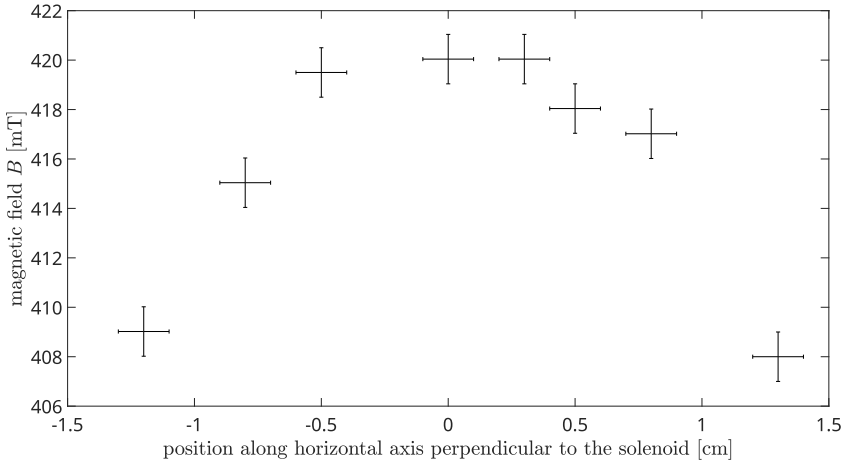

Fig. 1: Measurement of the magnetic field's homogenity for a current of 5 A.

As we can see from figure 4 the magnetic field only varies by about 3% over the complete length of the magnet and is almost constant for points closer than 0.5 cm to the centre. Thus the field can be approximately treated as homogenous.

Because of the constraints of the experimental setup we were not able to make the same quantitative measurement of the magnetic field's dependency on the vertical position as for the horizontal direction, but we were able to confirm that qualitatively it is of the same form.

2.1.2 *Calibration of the magnet*

We needed to determine the magnetic field's dependence on the current in the solenoid before conducting the actual Zeeman effect experiment because then the space between the solenoids was taken up by the sodium lamp and the field could thus not be measured directly.

To do this we inserted the Hall probe into the centre of the electromagnet and measured the magnetic field for several values of current. From figure 2 we can see that for sufficiently small currents B increases linearly with I but

for larger currents the relationship starts to noticeably deviate from a linear one and we can also see, that B saturates at about 1.3 T.

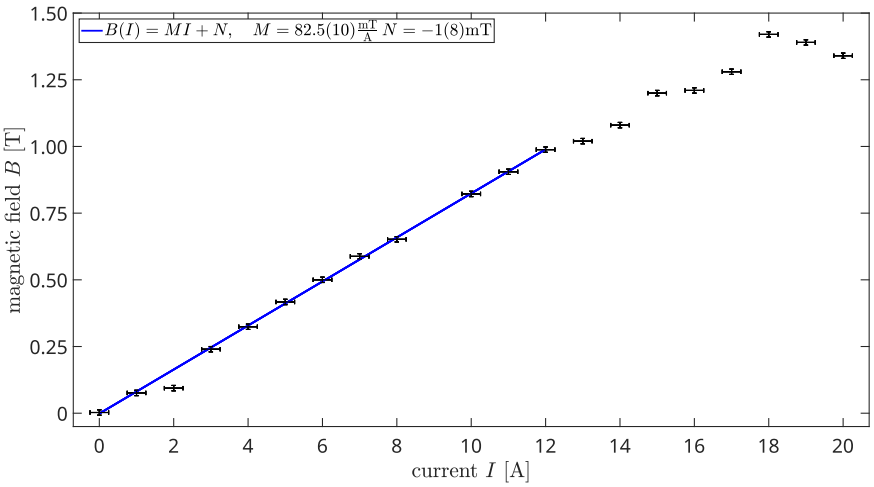

Fig. 2: Measurement of the magnetic field's dependency on current.

Using matlab we fitted the data[1] to a linear model function of the Form

$$B(I) = MI + N \tag{2.1}$$

and obtained the parameters

$$M = 82.5(10)\,\frac{\mathrm{mT}}{\mathrm{A}} \tag{2.2}$$

$$N = -1(8)\,\mathrm{mT}. \tag{2.3}$$

Because we used the Gaussmeter in the 1000 G range the uncertainty for the B measurements is given by $u(B) = 1$ mT. We approximated the uncertainty for the current measurements to be $u(I) = 0.25\,\mathrm{A}$.

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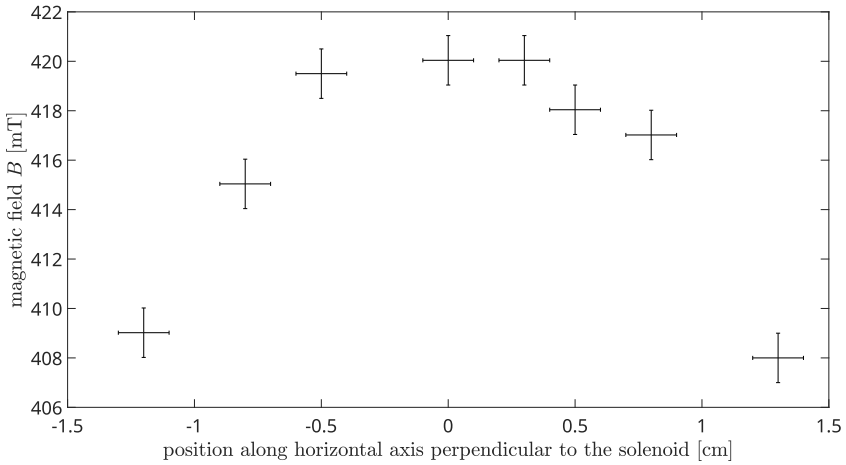

Fig. 3: Measurement of the magnetic field's homogenity for a current of 5 A.

[1] We only used the data for currents for which B depends linearly on I i.e. currents up to 12 A. We also excluded the outlier at 2 A.

2.2 CALIBRATION OF THE FABRY-PÉROT INTERFEROMETER

We measured the frequency shift due to the Zeeman effect using a Fabry-Pérot interferometer (FPI). To get any kind of quantitative information out of these measurements, it is therefore crucial to correctly calibrate the FPI.

2.2.1 *Determining the correction function for the FPI movement*

In the experiment, one of the FPI's mirrors is moved by a piezoelectric crystal. This is achived by applying a linear voltage ramp, which is represented by integer values, to the crystal. Therefore we must determine a function to translate between the voltage signal i.e. the integer values and the position of the mirror and thus the transmitted Frequency. Furthermore the response of the crystal to this voltage is slightly non-linear, which we correct by approximating the movement with a second degree polynomial.

To determine the coefficients of this polynomial we took a scan of the light without a magnetic field. Because we know that the signal repeats for frequencies that are integer multiples of the free spectral range (FSR), we can use the positions of the repeated signal in the scan to determine the calibration function.

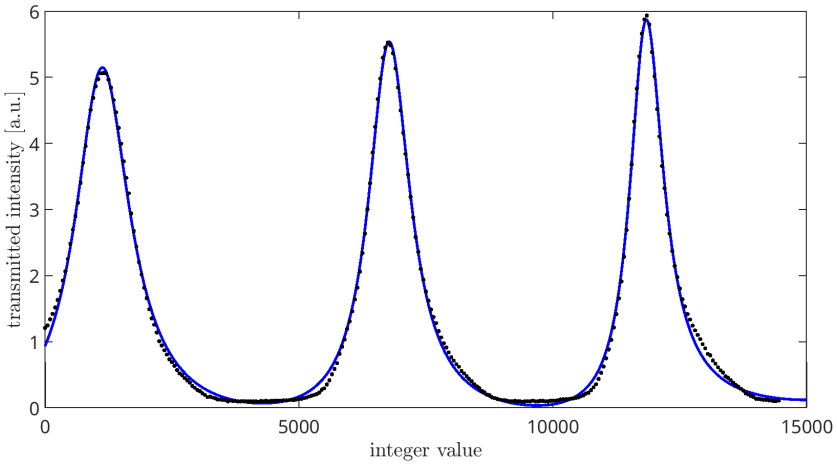

Fig. 4: FPI scan to determine the calibration function.

The data from this scan was fitted to a sum of three Lorentzians of the form

$$f_j(x) = \frac{a_j}{(x^2 - \mu_j^2)^2 + \gamma_j} \tag{2.4}$$

using the matlab. From this fit we obtained the following parameters μ_j for the positions of the three peaks

Number of Peak j	Position μ_j
0	1 130(10)
1	6 778(5)
2	11 842(5)

Tab. 3: Fit parameters to determine the FSR.

We now need to find a function f that satisfies $f(\mu_j) = j, \forall j \in \{0,1,2\}$. If we assume f to be a second degree polynomial $f(x) = ax^2 + bx + c$, we obtain the following linear system of equations

$$\begin{pmatrix} \mu_0^2 & \mu_0 & 1 \\ \mu_1^2 & \mu_1 & 1 \\ \mu_2^2 & \mu_2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}. \tag{2.5}$$

Inserting the respective values of μ_j and numerically solving this system of equations yields

$$a = \quad 1.906\,136 \cdot 10^{-9} \tag{2.6}$$

$$b = \quad 1.619\,801 \cdot 10^{-4} \tag{2.7}$$

$$c = -1.854\,714 \cdot 10^{-1} \tag{2.8}$$

As we can see from the values of the coefficients and figure 5 the relationship between the integer values and the frequency shift deviates only slightly from a linear one.

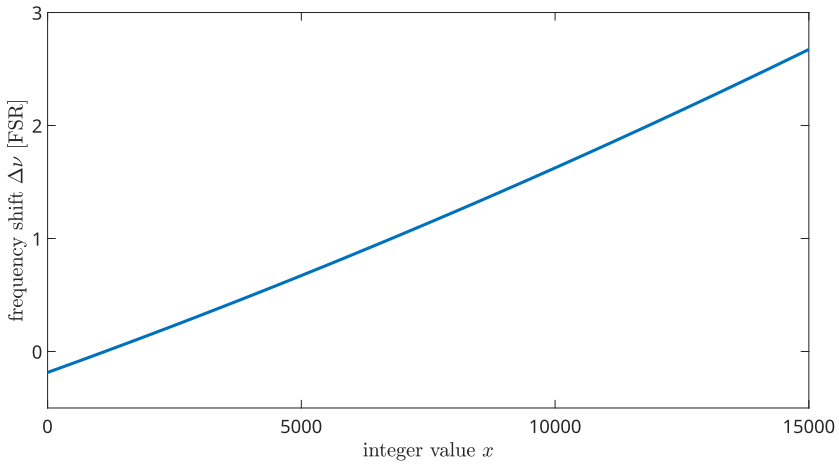

Fig. 5: Plot of the function $f(x) = ax^2 + bx + c$.

2.2.2 *Determining the free spectral range*

In the previous section we determined the relation between the integer values and the frequency shift measured in units of the free spectral range. It now remains for us to find the FPI's free spectral range to translate this information into "useful units".

According to (1.66) the free spectral range $\delta\nu$ is given by

$$\delta\nu = \frac{c}{2d} \tag{2.9}$$

and is thus a function of the distance d between the FPI's mirrors.

We determined this distance in two steps. First adjusted the FPI such that the interference rings of both D-lines overlap. Then took an approximate mechanical measurement of d using a micrometer screw and then used (1.70) to determine the exact distance.

The micrometer reading was $\xi = 6.82$ mm. Using the fact that the mirrors are 1.4 mm apart for a micrometer reading of 5.94 mm, we obtain

$$d_{\text{micrometer}} = 6.82\,\text{mm} - 5.94\,\text{mm} + 1.4,\text{mm} = 2.28\,\text{mm} \tag{2.10}$$

According to (1.70) this would correspond to

$$n_{\text{meas}} = \frac{d_{\text{micrometer}}}{290.767\,\mu\text{m}} = 7.84 \tag{2.11}$$

Since we expect n to be a natural number, we assume in the following that $n = 8$. The relative difference between n_{meas} and n is about 2% and can thus be attributed to the inaccuracy of the different measurements of lengths.

Thus we have from (1.70)

$$d_{\text{calc}} = n \cdot \frac{c}{2\Delta\nu} = 2.326\,\text{mm} \tag{2.12}$$

which leads to

$$\delta\nu = \frac{c}{2d} = 64.440\,\text{GHz}. \tag{2.13}$$

2.3 MEASURING THE ZEEMAN SHIFT OF THE SODIUM D LINES

We determined the Zeeman shift by using the Lyot filter to select either the D_1 or the D_2 line [2] and using the FPI to take scans of both lines for several different magnetic field strengths. Our work from the previous section enabled us to calculate the frequency shifts from this data.

2.3.1 *D_1 line*

[2] Since we expect D_1 (D_2) to split into 4 (6) individual lines, it is possible to distinguish them by the number of their respective Zeeman lines. We found that D_1 (D_2) corresponds to a setting of the Lyot filter's second polarizer of approximately 310°(220 °).

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